Validating a model of the braking force in a linear eddy current brake

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A fundamental flaw in traditional braking systems is that they are prone to wear due to friction. Eddy current brakes are an alternative braking system in which the kinetic energy of the object is dissipated by an electric current induced in a conductor by a magnetic field, eliminating the need of friction thus reducing the wear on the brakes. This experiment aims to validate a model proposed by Heald [1] in regards to the relation between the braking force due to eddy currents and the magnetic field strength, as well as the relative velocity between the magnetic field and the conductor. This was done by repeatedly firing a cart supporting two magnets mounted on a C-channel along an air track whilst varying the magnetic field strength by increasing the magnet-to-magnet distance. An aluminium bar functioning as the conductor was suspended inside the C-channel whilst a high speed camera was used to film the runs. Tracking software was used to track the location of the cart along the track and to allow the data to be analysed. To validate the model, the measured cart trajectory and force-velocity relation were compared to those predicted by the model, by using them to determine the magnetic field strength between the magnets. The field strength determined using the trajectory differed by $(1.3 \pm 0.4)\%$ and the field strength determined using the force-velocity relation differed by $(1.7 \pm 0.8)\%$. Since these are both lower than 5% it is concluded that the model is validated for the used measurement setup.

I. INTRODUCTION

A fundamental flaw in traditional braking systems is that they are prone to wear due to friction. Eddy current brakes are an alternative braking system in which the kinetic energy of the object is dissipated by an electric current induced in a conductor by a magnetic field, eliminating the need of friction thus reducing the wear on the brakes. Because of this they are highly useful in high-speed applications such as high-speed trains and roller coasters. This experiment aims to validate a model proposed by Heald [1] describing the relation between the braking force and the relative velocity between the magnetic field and the conductor, as well as the relation between the braking force and the magnetic field strength in a linear eddy current brake. This will be done by shooting a cart supporting two magnets attached to a C-channel along an air track underneath a conductor beam. The experimentally determined cart trajectory and force-velocity relation will be compared to the one predicted by the model. The general criterion for the aforementioned research goal is that the model must not deviate more than 5% from the experimental results. If that is the case the model will be deemed insufficient to describe the measurement setup.

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II. THEORY

A model proposed by Heald [1] describes the relation between the braking force and the relative velocity between the magnetic field and the conductor in an eddy current brake, as well as the relation between the braking force and the magnetic field strength. The relation described by the model is as follows:

$$F = \alpha\sigma\delta B_0^2 lwv \tag{1}$$

with F the braking force, α a proportionality constant described by Equation (2), σ the conductor conductivity, δ the conductor thickness, B_0 the magnetic field strength applied to the conductor, l and w the length and width of the rectangular magnetic "footprint" (which is the area of the region where the highest density of magnetic flux crosses into the conductor) and v the relative velocity between the conductor and the magnetic field. The proportionality constant α is described as

$$\alpha = 1 - \frac{1}{2\pi}\left[4\arctan A + A\ln\left(1+\frac{1}{A^2}\right) - \frac{1}{A}\ln\left(1+A^2\right)\right] \tag{2}$$

with $A = l/w$.

The model makes the assumptions that the magnetic field is completely homogeneous, and that the conductor sheet is infinitely long.

III. METHOD

To validate the model in regards to the relation between the magnetic field strength and the braking force, as well as the relation between the force and the relative velocity between the magnetic field and the conductor, a cart supporting two magnets is shot along a horizontal air track to minimise friction whilst passing underneath an aluminium beam.

A. Experimental Setup

First a Lego C-channel with a magnet-to-magnet distance d (mtm distance) of $d = 16.7 \pm 0.05$ mm is mounted on an air track cart using double sided tape as illustrated by Figure 1. This is to allow an aluminium beam to pass between the magnets as is illustrated by Figure 2. Following this a black and yellow sticker is stuck on the side of the cart to allow the cart to be tracked by a computer.

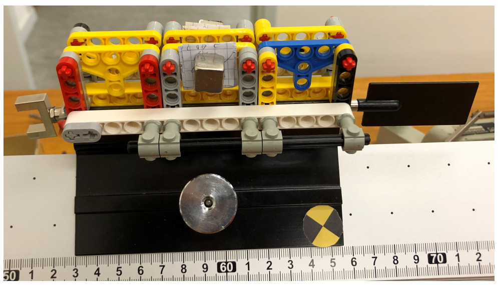

FIG. 1. Picture of the cart with C-channel, weights and sticker.

Next, an aluminium beam of thickness $\delta = 4.95 \pm 0.05$ mm is suspended above the air track using 2 stands and 4 clamps in such a way that the beam will be able to pass through the C-channel without touching it as illustrated by Figure 2.

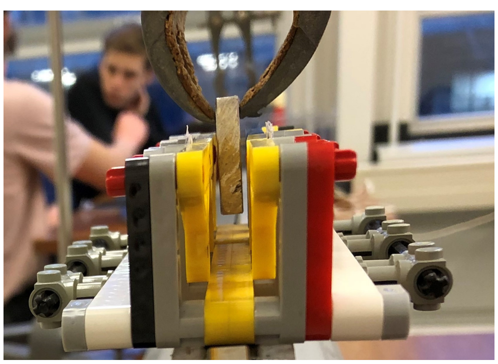

FIG. 2. Picture of the aluminium bar suspended inside the C-channel.

To continue a magnetic launcher is mounted on one side of the air track and a rubber band is attached to the front of the launcher (see Figure 3), this will be used to launch the cart with a constant initial speed. Finally a 480Hz 720p camera is placed to the side of the track facing the sticker so that the camera is parallel with the side of the track and able to record the entire trajectory of the cart. Figure 3 shows an illustration of the entire setup.

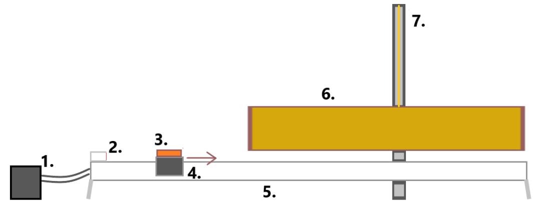

FIG. 3. Illustration of the measurement setup, side view. 1: Air pump, 2: Magnetic launcher with rubber band, 3: Magnets on C-channel, 4: Air track cart, 5: Air track, 6: Conductive plate, 7: Stand (with a clamp to hold the conductive plate in yellow).

B. Measurement Procedure

To determine the influence of the magnetic field and velocity on the braking force due to eddy currents 5 sets of 6 measurements with varying magnetic field strengths have been performed in addition to a control set consisting of 3 measurements without magnets. To carry out the measurements 2 neodymium magnets (N48, $(12 \times 12) \pm 0.05$ mm) are placed with opposite poles facing each other on the outside of the C-channel. In addition to this 0, 3, 5, 8 or 11 pieces of paper (0.09 ± 0.005 mm thick) depending on the set of measurements are placed in between the C-channel and the magnet on both sides to vary the magnetic field strength by increasing the mtm distance d. These pieces of paper are held in place due to the attracting forces acting on the magnets. Next, 4 weights of approximately 50 g each are added to the cart to increase the stopping distance. After the cart is ready, its mass is measured using a 5 digit scale with an uncertainty of 0.01 g.

After the mass of the cart has been measured, the cart is placed on the air track and attached to the magnetic launcher which is switched on. After a three second countdown the camera and magnetic launcher are simultaneously switched on and off respectively. The camera is switched off again after the cart has ceased moving. This process is repeated for all sets with the respective amount of measurements and paper. Please refer to Table I in Appendix A for a full list of the amounts of paper, mass, as well as the magnetic field strength for each set.

C. Data Analysis

Magnetic simulation

To analyse the data the magnetic field was simulated using the magnetic simulation software FEMM [4]. The parameters used to model the magnetic field are described here. Due to the planar nature of the setup the planar problem type was used with a depth of 12 mm. Next two neodymium magnets (N48, 12×12 mm) and an aluminium bar (1100 alloy) were added. The boundary condition parameters were set to 7 layers, a radius of 15 and a Dirichlet edge type, as suggested by the FEMM User's Manual [4] for this type of problem. Following this a mesh was generated and the simulation was run, after which a field density plot was added with the number of contours increased to 300 to increase the accuracy of the plot. Lastly the total magnetic field applied to the aluminium bar was calculated by integrating the magnetic flux density over a line, which was perpendicular to the magnetic field and between the outer edges of the magnets where the field is most homogeneous. For more information on the use of FEMM and explanations on simulation parameters please refer to the FEMM 4.2 User's Manual[4]. The uncertainty in the determined magnetic field due to the uncertainty in the magnet-to-magnet distance was determined to be 0.002 T.

Analysing the cart trajectory

To measure the cart trajectory, a Python script was used to track the position of the cart. The uncertainty in the position was found to be on the order of millimeters and is therefore assumed to be negligible.

To find the relation between braking force and magnetic field strength and the relation between braking force and relative velocity two different methods of analysing the cart trajectory were used.

First, method 1 was used to find the relation between the magnetic field strength and the braking force. Here the real trajectory of the cart was compared to the trajectory predicted by the model for varying magnetic field strengths. For every run, the trajectory was shifted and corrected as to make sure the cart first passes under the rod at a position x and a time t of $(x, t) = (0, 0)$. For every set an average run was calculated, by linearly interpolating the cart position at 2000 linearly spaced points from $t = 0$ to $t = \max(t_{\text{end}}$ of every run of that variation) and taking the average of the cart position at those points. To the average trajectory curve a curve was fitted generated from the model, by integrating Equation (1) twice with respect to time using Euler's method of numerical integration. All variables except for B_0 were kept constant. The value for the conductor conductivity σ was taken from literature to be $3.34 \cdot 10^{-7}\,\mathrm{S\,m^{-1}}$ [3]. The uncertainty in this value is assumed to be negligible. For each variation of the magnet-to-magnet distance, the magnetic field applied to the conductor B_0 was determined using FEMM [4]. Using this value for B_0, a predicted trajectory curve was generated from the model using the same method of numerical integration as mentioned above. Residual analysis was performed between the predicted trajectory and the measured trajectory and the final position of the cart was compared. The fitted curve yielded a value for the magnetic field strength as well, which was compared to the value determined using FEMM.

Next method 2 was used to find the relation between the relative velocity and the braking force. To do this the velocity of the cart was first obtained by numerically differentiating the average run with respect to time. Next the force on the cart was obtained by numerically differentiating the velocity with respect to time and multiplying by the cart mass. The force-velocity relation (F, v) was plotted and Equation (1) was fitted to it, keeping everything but B_0 constant. The fitted line yielded a value for the magnetic field strength, which was compared to the field strength determined using FEMM.

In both method 1 and 2, the uncertainty in B_0 determined by fitting Equation (1) to a curve was propagated using a functional approach, which is an approximation of a first order Taylor series expansion as described by Hughes and Hase [2].

IV. RESULTS

The experiment was conducted as was described in Section III. Table I in Appendix A shows the mass of the cart, the magnet-to-magnet distance, as well as the magnetic field strength calculated using the magnetic simulation software for respective variations in the configurations of the magnets. To determine the magnetic field strength between the magnets of the brake, magnetic simulation software was used. The magnetic flux density was integrated normal to a line through the region where the field is the most homogeneous. This line is illustrated by Figure 4.

In Figure 5 the real world trajectories of the cart and the trajectories predicted by the model were plotted. The fitted lines yielded values for the magnetic field strength B_0 which were compared to the field strength determined using FEMM. Please refer to Table II in Appendix A for the results for each individual run. The average deviation from the value determined using FEMM was $(1.3 \pm 0.4)\%$. Figure 6 shows the residual analysis of the real world data with the predicted data. The absolute deviation with uncertainty taken into account does not exceed 0.035 m.

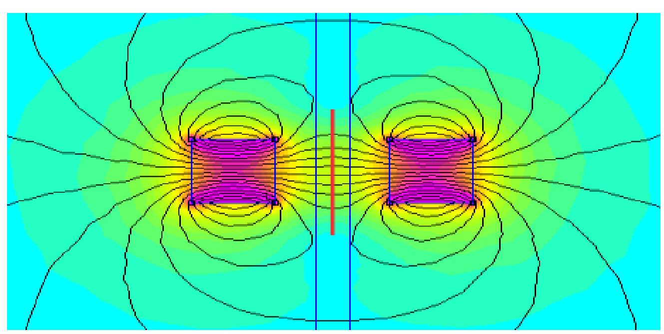

FIG. 4. Simulated magnetic field of 2 N48 neodymium magnets 16.7 mm apart with an aluminium (1100 alloy) plate in between. Darker colors correspond to a higher field density. The red line is the line over which was integrated to obtain the total field.

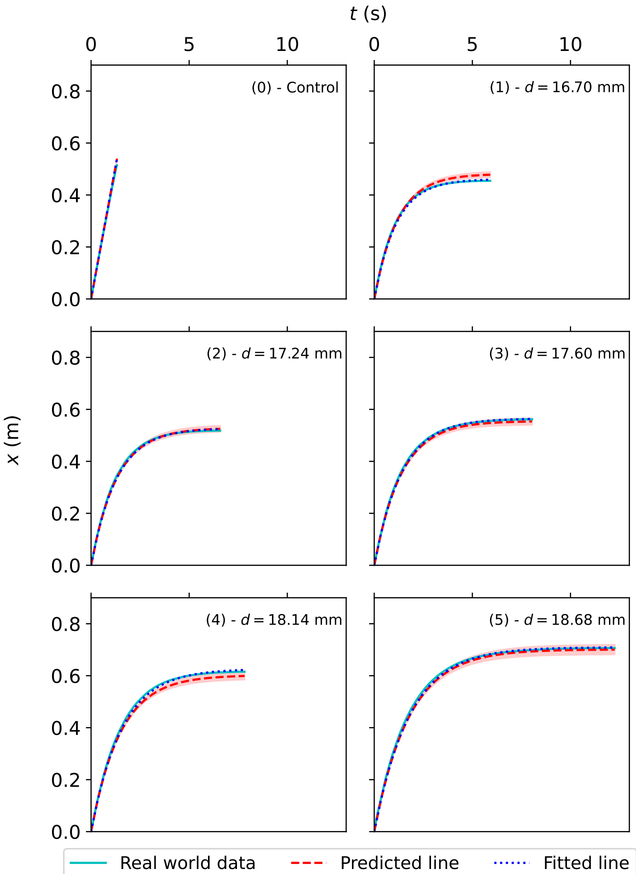

FIG. 5. Trajectory data for a control run (no magnets) and 5 variations of the magnet-to-magnet distance d. The solid cyan curve is the measured trajectory data. The dashed red curve is the trajectory predicted using Equation (1) with B_0 determined using FEMM. The red shaded area represents the uncertainty in the red curve. The dotted blue curve is fitted onto the real world data curve with Equation (1), keeping everything but B_0 constant.

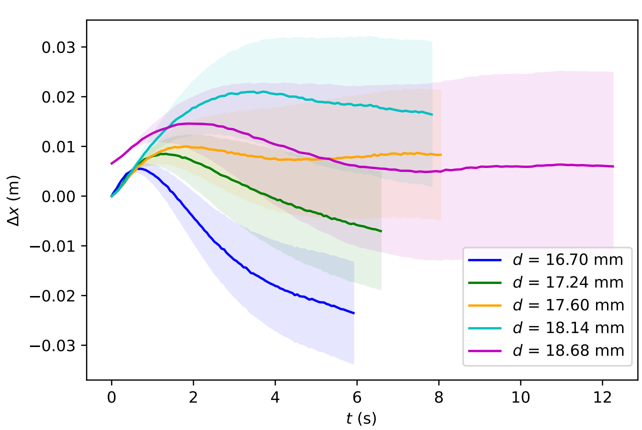

FIG. 6. Residual analysis of the real world data with the predicted data. d is the magnet-to-magnet distance. The shaded area around the curves represents the uncertainty in the residuals.

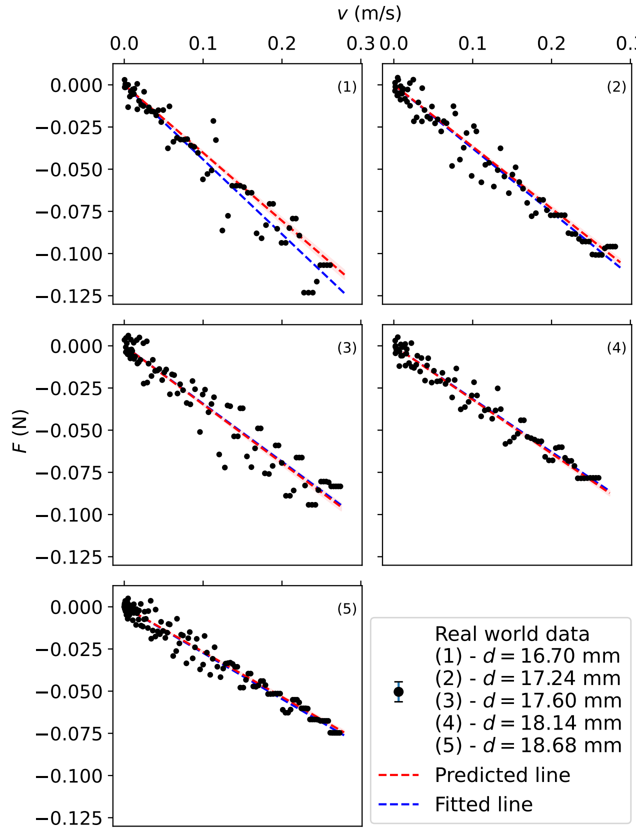

FIG. 7. Relation between velocity v of the cart and braking force F on the cart for variations in magnet-to-magnet distance d (and thus magnetic field strength). The solid dots are real world data with error bars, though invisible due to their size. The red dashed line is the (F, v)-relation predicted by Equation (1) using B_0 determined using FEMM. The red shaded area around the red line represents the uncertainty. The blue dashed line is fitted to the real world data with Equation (1), keeping everything but B_0 constant.

Additionally Figure 7 shows the relation between the cart velocity and the braking force on the cart for the 5 different magnetic field strengths with the predicted and fitted relations as well. The fitted lines yielded values for the magnetic field strength B_0 which were compared to the value determined using FEMM. Once again please refer to Table II in Appendix A for the results for each individual run. The average deviation from the value determined using FEMM was $(1.7 \pm 0.8)\%$.

V. DISCUSSION

The data introduced in Section IV will be discussed here. In Figure 5 it should be noted that the trajectory of the control set is almost a perfectly straight line. This illustrates that the drag on the cart in addition to any residual resistance from the air track is negligible. Furthermore, notice that the distance traveled increases as d increases, meaning that the braking force increases with magnetic field strength. In Figure 6 note that the deviation from the real world data does not exceed 0.035 m, showing that the model describes the trajectory of the cart within an accuracy of a few centimetres. Also note that the uncertainty increases with time, since the uncertainty in a quantity derived by numerical integration increases with every iteration.

Two methods were used to compare the measured data to the model data as described in Section III. The difference in B_0 found through FEMM and B_0 found by the curve fit determined using method 1 is $(1.3 \pm 0.4)\%$ on average over all 5 sets, with a maximum difference of $(2 \pm 1)\%$. The difference determined using method 2 is $(1.7{\pm}0.8)\%$ on average, with a maximum difference of $(5{\pm}2)\%$ Seeing as both these average values lie within the maximum of 5% requirement, it can be concluded that the model proposed by Heald [1] is sufficient to describe relation between the braking force, and the relative velocity between the magnetic field and the conductor in an eddy current brake, for the values used in the experimental setup.

In a future study it is advisable to be more careful with the magnets as small collisions can break off significant amounts which reduces the magnetic field strength, and makes it more difficult to accurately model the field strength. Additionally it would be better to use magnets with a much larger footprint as this generates a more homogeneous magnetic field which is closer to the idealised situation. Also it would be wise to use a camera with a higher resolution to increase accuracy on the trajectory measurement.

VI. CONCLUSION

In this experiment the validity of a magnetic braking force model proposed by Heald [1] has been tested. This was done by means of building a simple measurement setup using an air track, a cart, 2 magnets, a camera and an aluminium plate. With this setup the model could be tested against real world measurements.

To validate the model, the field strength calculated using simulation software was compared to the field strength determined by fitting the model onto two different curves: firstly by fitting onto the measured trajectory, which yielded a difference of $(1.3 \pm 0.4)\%$; secondly by fitting onto the measured (F, v)-relation, which yielded a difference $(1.7{\pm}0.8)\%$. Seeing as both these values lie within the maximum of 5% requirement to validate the model, it can be concluded that the proposed model is validated.

In a follow-up study, it would be wise to be more careful with the fragile neodymium magnets as they are prone to shattering. Furthermore using cameras with higher resolutions would be advisable.

Appendix A: Experiment data

TABLE I. Parameters of different sets of runs used in the experiment. (*mtm distance = magnet-to-magnet distance d)

	Mass m (g±0.01g)	Amount of paper per side	mtm distance* (mm±0.05mm)	B_0 by FEMM (T±0.002T)
Control	448.59	0	N/A	0
Set 1	473.51	0	16.70	0.184
Set 2	473.70	3	17.24	0.175
Set 3	473.84	5	17.60	0.171
Set 4	474.07	8	18.14	0.164
Set 5	474.33	11	18.68	0.150

TABLE II. Experiment results for different sets of runs. ΔB_0 is the difference between B_0 determined using FEMM and B_0 determined using a curve fit in the column to the left.

	B_0 by FEMM (T±0.02T)	B_0 curve fit (x,t) (T±0.0001T)	ΔB_0 ($\% \pm 1\%$)	B_0 curve fit (F,v) (T±0.002T)	ΔB_0 ($\% \pm 2\%$)
Set 1	0.184	0.1883	2	0.193	5
Set 2	0.175	0.1755	0	0.178	1
Set 3	0.171	0.1690	-1	0.170	-1
Set 4	0.164	0.1602	-2	0.162	-1
Set 5	0.150	0.1487	-1	0.151	1

[1] Mark A. Heald. Magnetic braking: Improved theory. *American Journal of Physics*, 56(6):521–522, 1988. doi:10.1119/1.15570. URL `https://doi.org/10.1119/1.15570`.

[2] Ifan G Hughes and Thomas P A Hase. Error propagation: A functional approach. *Journal of Chemical Education*, 89 (6):821–822, 2012. doi:10.1021/ed2004627. URL `https://doi.org/10.1021/ed2004627`.

[3] MatWeb. Aluminum 1100-o. URL `http://www.matweb.com/search/datasheet_print.aspx?matguid=db0307742df14c8f817bd8d62207368e`.

[4] D.C. Meeker. Finite Element Method Magnetics (FEMM) - User's Manual, 2020. URL `https://www.femm.info/wiki/Files/files.xml?action=download&file=manual.pdf`.