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Issues in the Development of Approaches to Container Loading

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The paper argues that existing approaches to container loading problems are each applicable only to a narrow part of the spectrum of situations encountered in practice and that there are many scenarios for which there are no adequate methodologies. A number of examples are given. Two approaches aimed at addressing some of the present limitations are discussed. One is designed to produce stable, evenly distributed packing patterns; the other caters for multi-drop loads. A comprehensive performance analysis using both published and randomly generated test problems is presented. The procedure used to create the latter is described in full to enable replication. It is shown that the two methods, when combined, provide a powerful and versatile tool.

Key words—containers, space utilization, multicriteria, benchmarks

1. INTRODUCTION

WHILST THE TERM 'container loading' has sometimes been used in the literature as if it were a self-explanatory and complete characterization of the problem under consideration, even a fairly superficial comparison of case study material reveals that there are many quite different practical scenarios which fall under this general heading.

A basic dichotomy exists between situations in which it is requisite that the whole of a given consignment of goods is loaded and cases where it is possible to leave some of the cargo behind. Typically, the former type of problem involves multiple containers, whereas the latter is usually concerned with a single one (cf. the classification in [7] of container loading problems into types 3/B/O/ and 3/V/I/). Another important characteristic is the mix of items to be loaded. At one extreme the cargo may be completely homogeneous—i.e. consist solely of identical items—at the other all the items might be different in type and size. An assortment consisting of a large number of different sizes relative to the total number of items is commonly referred to as 'strongly heterogeneous', whereas a cargo comprising only relatively few different types of articles is termed 'weakly heterogeneous' (e.g. [5]).

Of the various approaches to container loading which have been published each is applicable, or suitable, only within certain ranges of this spectrum of situations. With few exceptions (e.g. [15]) the literature has considered only the scenario where the cargo, or as much of it as possible, is to be loaded into a single container. A number of the approaches put forward also make specific assumptions about the composition of the cargo. The method of [9], for example, is aimed at problems involving a strongly heterogeneous cargo, whilst the methods suggested in [14] and [10] can be applied only if all items are of the same size. The majority of approaches assumed a weakly heterogeneous assortment of items. The methods of [12, 17–20] all fall into this category. Conceptually, an approach of this kind can also be used if the cargo is strongly heterogeneous or homogeneous. However, as demonstrated in [2], its performance may depend crucially on the cargo makeup. A method designed for packing a few hundred items of 10 or 15 different sizes, for instance, may produce poor results if there are only 2 or 3 types of items or, conversely, if there is a very wide range of different sizes.

This paper argues that the area of applicability of the existing approaches is limited in many other respects. The thesis put forward is that a number of factors which are frequently of importance in practical situations have not received sufficient attention in the OR literature. More precisely, it is suggested that a problem definition which often uses exclusively spatial data, coupled with a focus on loading space utilisation as the dominant decision criterion, renders the present methods inappropriate for many problem structures encountered in practice.

In addition to highlighting such limitations, the paper aims to make a contribution to overcoming some of them. Two specific methods are discussed which are capable of dealing with practical requirements not adequately catered for to date. It is argued that further development work of this kind, if it is to be directed towards appropriate goals, must be accompanied by systematic studies of the relative performance of different approaches. Potential benchmarks for such studies are provided by the results of extensive tests using the two approaches described.

2. PRACTICAL REQUIREMENTS

Much of the work published to date has been based on pure knapsack-type formulations of the problem structure. In the single-container case the only parameters of this kind of model are the dimensions of the cargo and container involved and, at least formally, some measure of the value of each item. The value of a piece, however, is usually defined as its volume. In this framework the task can be summed up simply as 'to find an allocation of given pieces within the container such that the sum of valuations is maximal' [25]. The few reported studies of multiple container problems have similarly focused on the efficiency of the loading arrangement. The approach of [15], for instance, is based on a knapsack model in which the objective function is the total cost of the containers required to pack the cargo.

The remainder of this section outlines some other considerations which may have a bearing on the problem. It is perhaps necessary to emphasize that no claim is made that the factors described are of importance in every case—or, indeed, that there are many practical situations where all of them are of importance. What is claimed, however, is that there are many cases where at least some of the factors listed below play an important role.

Orientation constraints

The familiar 'this way up' instruction on cardboard boxes is a simple example of this kind of restriction. It may, however, not only be the vertical orientation which is fixed—if, for instance, a two-way entry pallet is loaded by forklift truck even the orientation in the horizontal plane may have to be regarded as effectively pre-determined.

Load bearing strength of items

'Stack no more than x items high' is another instruction seen on many boxes in everyday situations. How far this translates into a straightforward figure for the maximum weight per unit of area which a box can support depends on its construction and also its contents. Often the load bearing strength of a cardboard box is provided primarily by its side walls, so that it might be acceptable to stack an identical box directly on top, whereas placing an item of half the size and weight in the centre of the top face causes damage. The load bearing ability of an item may, of course, also depend on its vertical orientation.

Handling constraints

The size or weight of an item and the loading equipment used may to some extent dictate the positioning within a container. It might be necessary, for instance, to put large items on the container floor or to restrict heavy ones to positions below a certain height. It may also be desirable from the viewpoint of easy/safe materials handling to place certain items near the door of the container.

Load stability

To ensure that the load cannot move significantly during transport is an obvious requirement if the cargo is easily damaged. Also, an unstable load can have important safety implications for loading and (especially) unloading operations. Straps, airbags and other devices can be used to restrict or prevent cargo movement, but the costs, especially in terms of time and effort spent, can be considerable.

Grouping of items

Checking of a load might be facilitated if items belonging to the same 'group'—defined, for example, by a common recipient or the item type—are positioned in close proximity. This may also have advantages in terms of the efficiency of loading operations.

Multi-drop situations

If a container is to carry consignments for a number of different destinations, it is desirable not only to place items within the same consignment close together, but also to order the consignments within the container so as to avoid, as far as possible, having to unload and re-load a large part of the cargo several times.

Separation of items within a container

In situations where the cargo compromises items which may adversely affect some of the other goods—e.g. if it includes both foodstuffs and perfumery articles or different chemicals which must not come into contact—it is necessary to ensure that the loading arrangement takes account of this.

Complete shipment of certain item groups

Sub-sets of the cargo may constitute functional entities—e.g. components for assembly into a piece of machinery—or may need to be treated as a single entity for administrative reasons. It is often necessary in these cases to ensure that if any part of such a sub-set is packed, then all the other items belonging to it are also included in the shipment.

Shipment priorities

The shipment of some items may be more important than that of others. More specifically, each item might—at least conceptually—have a certain priority rating, deriving from, for example, delivery deadlines or the shelf life of the product concerned. Depending on the practical context, this rating may represent an absolute priority—in the sense that no item in a lower priority class should be shipped if this causes items with higher ratings to be left behind—or it may have a relative character, reflecting merely the value placed on inclusion in the shipment without debarring trade-offs between priority classes.

Complexity of the loading arrangement

More complex packing patterns generally result in a greater materials handling effort. The additional effort required will be most significant if the complexity of a pattern necessitates a change to more labour intensive handling methods, such as from using clamp or forklift trucks to purely manual loading. Conversely, if the handling technology cannot be changed, the pattern must conform to the limitations of this technology.

Container weight limit

If the cargo to be loaded is fairly heavy, the weight limit of a container may represent a more stringent constraint than the loading space available.

Weight distribution within a container

From the viewpoint of transporting and handling the loaded container—such as lifting it onto a ship—it is desirable that its centre of gravity is close to the geometrical mid-point of the container floor. If the weight is distributed very unevenly, certain handling operations may be impossible to carry out. In cases where a container is transported by road at some stage of its journey, the implications of its internal weight distribution for the axle loading of the vehicle can be an important consideration. The same, of course, applies if the 'container' is actually a truck or trailer.

3. IMPLICATIONS FOR THEORETICAL WORK

The factors described can be classified in a number of different ways. One such division is into factors which represent constraints for the loading arrangement and those which have the character of an objective. The weight limit of the container has already been referred to as a constraint—no benefit usually derives from leaving some of the available weight capacity unused. Restrictions concerning permissable pack orientations and limits on the weight which may be placed on certain items clearly also represent straightforward constraints.

However, other factors are not simply limits. Placing items of the same type in 'close proximity' to one another, for instance, is not a clearly defined notion. It is likely that, up to a certain point, falling short of the theoretical ideal of placing identical items in separate blocks would be traded off against a sufficiently improved space utilization. Equally, a good utilization may be regarded as sufficient compensation for a somewhat uneven weight distribution. [13] refers to factors of this kind of 'soft constraints'. This, however, may be a somewhat misleading label, for it conveys only the fact that the boundary between what is and what is not acceptable is not rigid. The property not fully captured by this term is that such factors are of relevance not only on the boundary but also throughout the range of acceptable alternatives. A fairly complex loading arrangement, for example, may be acceptable in a given environment, but if there is a way of drastically simplifying it whilst maintaining all its other characteristics, such an alternative would usually be preferred.

It is, therefore, appropriate to consider such factors as additional objectives rather than constraints. However, whilst a multi-objective decision making perspective might represent an appropriate conceptual framework, it does not open up opportunities for applying the body of knowledge of this field for as long as suitable packing algorithms which are capable of fully combining the objectives/constraints concerned do not exist. In addition, many of the objectives are very difficult, if not impossible, to quantify adequately in a single measure.

It would be an exaggeration to state that the OR literature on container loading problems has completely ignored the existence of non-volumetric aspects. References to such factors are not confined to case-orientated articles, but can also be found in papers putting forward a general method. However, in cases where approaches are claimed to possess features which render them suitable for dealing with certain additional requirements, this property, with few exceptions (e.g. [22, 16]), is a propitious, but largely incidental, performance characteristic of the method, rather than one of the main criteria in its design. For this reason the methods are generally not able to cater adequately for situations in which the factor concerned is of significant importance (cf. also [11]). This shortcoming is exemplified by the procedure of [9], which tackles the objective of an even weight distribution in the container through providing for a degree of interchangeability of components of a loading pattern and a simple means for generating several different such patterns. The task of selecting an alternative and of rearranging the plan so as to achieve an acceptable weight distribution is left to the user. However, the procedure has no mechanisms for ensuring that the most suitable patterns—or indeed any appropriate ones—are produced.

For certain approaches it might be not too difficult to overcome some of the limitations. Relative priorities for shipment, for example, can be viewed in a knapsack model simply as objective function coefficients which define or adjust the value ratings of the items. For many approaches the incorporation of constraints on permissable box orientations might also be a fairly straightforward task. It is important to note, however, that constraints on the positioning of items cannot always be added to the rules of an algorithm simply as restrictions for filling free spaces in the container. The load bearing strength of an item is a good example. If this criterion was used merely to restrict the alternatives for packing a given space (depending on how this space is supported), then easily crushable items would not be prevented from being placed on or near the floor of the container, thus causing the space above them to become unusable. A more appropriate way of dealing with this factor, clearly, would be to devise mechanisms which cause items with a low load bearing strength to be placed near the top of the load. This, however, might be tantamount to developing a completely new algorithm.

Implied in this last argument, as well as several of the earlier comments, is the need for extensive comparative evaluation work alongside—or as part of—the development of new approaches in this area. In order to be able to judge the merits of different methods, and thus be in a position to choose the approach best suited to a particular situation, comprehensive performance analyses are required. The issue is of particular significance where non-spatial aspects play a role, because trade-offs between the way in which the additional problem-relevant factors are catered for and the container utilisation which can be obtained might influence the choice of method considerably.

Very little systematic evaluation work of this kind appears to have been carried out so far. A number of approaches have been put forward with no more than a few illustrative examples as evidence of their merits. Only very occasionally has the performance of different methods on identical test problems been examined. Container loading is not the only area where such comments apply. A recent paper [8] on approaches to one-dimensional stock-cutting problems make virtually the same points. In order to provide benchmarks for future studies they not only present the results obtained in a series of test runs, but also describe in detail the procedure used to generate the test problems. This enables others to replicate the problems and, where necessary, to produce further sets with similar characteristics.

The same philosophy is adopted here. The next section describes two container loading algorithms which are focused specifically on situations where, apart from space utilisation, other objectives are of importance. It also examines their performance on various test problems used by other authors. The subsequent section presents a more systematic performance analysis of the two methods on a large set of simulated test problems, which can be easily reproduced by others.

4. DEALING WITH NON-VOLUMETRIC CONSIDERATIONS: TWO CONCRETE EXAMPLES

4.1. Catering for stability as an objective

Many of the existing approaches to container loading are based on the philosophy of 'wall-building', i.e. the principle of constructing the loading pattern from a series of vertical layers across the width of the container. However, such arrangements may be quite unsuitable from the viewpoint of stability in transport, especially where the height of adjacent 'walls' varies significantly or where gaps exist between them.

The approach examined here is specifically designed to produce patterns which combine efficient space utilization with a high degree of stability. It was originally developed for pallet-loading problems where the stability objective—because of the absence of supporting sides—often takes on a crucial degree of importance. The method is fully described in [3], and [4] presents extensions to problems involving multiple pallets. This latter paper also comments that the approach appears to produce good results for container loading problems. An example of a problem originally described in [12] is given, for which the method produces a better result.

Briefly, the method uses the principle of building the loading pattern from the container floor upwards using layers of up to two different box types at a time. The choice of box type(s) and orientation(s) is determined by the utilization of the loading surface onto which a layer is placed. This procedure precludes situations where sections of the cargo overhang beyond the edge of the item(s) supporting it.

Figure 1 shows an example produced by the method. The container depicted is packed with 176 items of 8 different types. Each type of item was allowed to be placed in only one predefined vertical orientation. Despite this additional complication—which clearly reduces the options for creating dense packings—a compact arrangement is produced which allows very little room for any movement of items. The figure also illustrates how the pattern is constructed in layers from the bottom upwards. It is perhaps appropriate to add that the loading sequence used in practice is unlikely to correspond to the order in which the arrangement is generated.

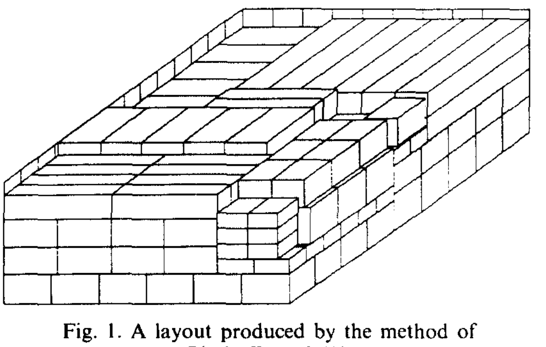

Fig. 1. A layout produced by the method of Bischoff *et al.* [2].

Table 1. Results obtained with test data of Loh and Nee [18]

	(a)		(b)		(c)		(d)	
Problem Nos.	Loh and Nee No. of boxes left out	Packing density	Ngoi *et al.* No. of boxes left out	Volume utilization	Bischoff *et al.* No. of boxes left out	Volume utilization	Method of Section 4.2 No. of boxes left out	Volume utilization
1	0	78.1	0	62.5	0	62.5	0	62.5
2	32	76.8	54	80.7	23	89.7	35	90.0
3	0	69.5	0	53.4	0	53.4	0	53.4
4	0	59.2	0	55.0	0	55.0	0	55.0
5	1	85.8	0	77.2	0	77.2	0	77.2
6	45	88.6	48	88.7	24	89.5	77	83.1
7	21	78.2	10	81.8	1	83.9	18	78.7
8	7	67.6	0	59.4	0	59.4	0	59.4
9	0	84.2	0	61.9	0	61.9	0	61.9
10	0	70.1	0	67.3	0	67.3	0	67.3
11	0	63.8	0	62.2	0	62.2	0	62.2
12	0	79.3	0	78.5	3	76.5	0	78.5
13	15	77.0	2	84.1	5	82.3	20	78.1
14	0	69.1	0	62.8	0	62.8	0	62.8
15	0	65.6	0	59.6	0	59.5	0	59.5
Average		74.2		69.0		69.5		68.6

However, the picking list for the actual loading operation can, of course, be generated independently from the algorithm itself, e.g. by scanning the pattern from the back of the container.

The example shown in Fig. 1 is not an arbitrary one, but represents the solution obtained for one (data set No. 6) of 15 problems used for testing purposes in [18], and later also in [20]. The results for the complete set of problems are summarized in column (c) of Table 1. Columns (a) and (b) list the results presented in [18] and [20], respectively. It should be noted that the figures shown in column (a) under the heading 'packing density' are not directly comparable to the volume utilization figures in the other columns, as they are quoted only on the basis of the smallest rectangular enclosure of the loaded boxes, rather than the actual container dimensions. Column (a), therefore, generally overstates the volume utilization achieved. As can be seen, the results produced by the approach of [3] compare well with those of the other two methods. For 7 of the problems it leads to a higher packed volume than at least one of the other approaches and in 3 cases it outperforms both. The results obtained are inferior to the others for only 2 problems—cases 12 and 13. [18] presents a diagram of the solution to problem 12 which clearly shows that the slightly higher volume utilization achieved entails a rather unstable arrangement.

Two further sets of potential test problems are provided in [15], one consisting of 6 single container examples, the other comprising 17 multiple container problems. For all of the former the proposed method matches the utilization figures achieved in [15]. The multiple container problems are actually concerned with an optimal mix of containers of different sizes, a problem which is outside the scope of the method examined here. However, [15] also provides the solutions obtained if the method there developed is applied to sets of containers of one size only. The solutions to 47 such problems are imbedded in the results. This type of situation can be dealt with by using the single container method sequentially until all the cargo is packed. [4] reports encouraging results for this approach applied in the context of pallet loading. The results obtained for the test problems of [15], summarized in terms of the number of containers required, are listed in Table 2.

As a comparison of columns (a) and (b) in Table 2 shows, the results produced by the method of [15] and the above approach, applied sequentially, are not always identical, but neither approach is generally superior. Taking the 47 problems together, the total number of containers required by the two methods is exactly the same.

These comparisons support the suggestion that the approach can be successfully applied to container loading problems. Even where stability is not an important issue, the results obtained are comparable in terms of volume utilization to those produced by the other two methods. The approach can also be adapted to cater for some of the other factors discussed in Section 2. [24], for example, uses the method as the basis of a procedure for dealing with load bearing strength considerations. However, it also has some drawbacks.

One obvious and important shortcoming is that the approach cannot easily be modified to deal with multi-drop situations. As the layers are built from the container floor upwards—and this is an integral part of the philosophy—there is no straightforward way of forcing items destined for later drop-off points to be placed towards the back of the container. In other words, even if items were pre-assigned to different destinations there would be a need for substantial re-loading after each drop-off.

4.2. Catering for multi-drop situations

In order to address the requirements of multi-drop situations an algorithm should ideally create distinct sections across the width of the container which correspond to the different destinations. One possible mechanism for attaining such a distribution of the cargo is to process the sub-sets concerned in sequence, starting with that for the final destination, and packing each separate consignment with a view to maximising the space remaining for the rest. This section describes a new method which can be used within an iterative procedure of this kind.

Each step in the proposed algorithm is concerned with packing a single box. Prior to every packing stage the potential loading surfaces in the container are identified. Only completely

Table 2. Results obtained with test data of Ivancic *et al.* [15]

Problem No.	Ivancic *et al.* Ref.	No. of box types	(a) Method of Ivancic *et al.*	(b) Method of Bischoff *et al.* applied sequentially	(c) Method of Section 4.2 applied sequentially
1	1a	2	26	27	27
2	1b	2	11	11	11
3	2a	4	20	21	26
4	2b	4	27	29	27
5	2c	4	65	61	59
6	3a	3	10	10	10
7	3b	3	16	16	16
8	3c	3	5	4	4
9	4a	2	19	19	19
10	4b	2	55	55	55
11	4c	2	18	19	25
12	5a	3	55	55	55
13	5b	3	27	25	27
14	5c	3	28	27	28
15	6a	3	11	11	15
16	6b	3	34	28	29
17	6c	3	8	8	10
18	7a	3	3	3	2
19	7b	3	3	3	3
20	7c	3	5	5	5
21	8a	5	24	24	26
22	8b	5	10	11	11
23	8c	5	21	22	22
24	9a	4	6	6	7
25	9b	4	6	5	5
26	9c	4	3	3	4
27	10a	3	5	5	5
28	10b	3	10	11	12
29	11a	4	18	17	23
30	11b	4	24	24	26
31	11c	4	13	13	14
32	12a	3	5	4	4
33	12b	3	5	5	5
34	12c	3	9	9	8
35	13a	2	3	3	3
36	13b	2	18	19	14
37	14a	3	26	27	23
38	14b	3	50	56	45
39	14c	3	16	16	18
40	15a	4	9	10	11
41	15b	4	16	16	17
42	16a	3	4	5	5
43	16b	3	3	3	3
44	16c	3	4	4	4
45	17a	4	3	3	3
46	17b	4	2	2	2
47	17c	4	4	3	4
Total			763	763	777

flat, contiguous, rectangular surfaces are considered. Such surfaces may overlap, but any which are wholly contained in others are not included. The placement rules of the algorithm do not allow a box to protrude beyond the loading surface onto which it is placed, i.e. all boxes are fully supported from below either by other boxes or the container floor. The space above a given surface, therefore, is always entirely empty and its height is simply the distance to the container roof.

Once the process of space identification is complete, the item to be placed and, simultaneously, the space into which it is to be packed are determined. All permissable orientations of all unpacked boxes are examined in relation to all the spaces. The selection from the feasible alternatives found is made on the basis of a four-tier ranking scheme. In order to explain this scheme it is useful to introduce some simple notation.

Let a coordinate system for positions within the container be defined in such a way that, when viewing the container from the front, its origin is the bottom left-hand corner of the back wall and the three coordinates refer to the position along the length, width and height, in that order. Reference points for empty spaces can then be defined in a similar way as the corner with the lowest coordinate values on the three axes. Let (a, b, c) be the reference point of a particular space. Also, let x, y and z be a permutation of the dimensions of a box, defined by its orientation relative to the container such that x points along the direction of the container length, y is the dimension placed along the container width and z refers to the height. Similarly, X, Y and Z are used to denote the dimensions of the empty space in these same directions. Let m, furthermore, stand for the number of boxes of the candidate type yet to be packed at this stage. The box will fit into the space in the given orientation if and only if $x \leqslant X$ and $y \leqslant Y$ and $z \leqslant Z$. If it fits then

$$k = \min\left(\left\lfloor \frac{Z}{z} \right\rfloor, m \right)$$

represents the maximum number of such boxes which can be placed into the space in the form of a homogeneous column. Whilst, as stated earlier, the method places only one box in each step, this measure k provides a limited look-ahead capability.

The ranking scheme employed can now be formulated as follows:

Main criterion:	Largest potential space utilization $u = k\ [xyz/XYZ]$.
Tie breaker 1:	Smallest lengthwise protrusion $p = a + x$.
Tie-breaker 2:	Largest box volume $v = xyz$
Tie-breaker 3:	Lowest value of b.

Both the main criterion and tie-breaker rule 1 involve parameters relating to the box, its orientation, and the space concerned. Tie-breaker 2, on the other hand, relates only to box characteristics and tie-breaker 3 only to those of the space. The combination with the highest rank is selected and the box involved positioned such that one corner coincides with the reference point of the space, i.e. it is placed as far back and to the left as possible within the relevant space.

It should be pointed out that, although the value u is based on a feasible option for the next packing stages, subsequent steps do not automatically lead to the creation of columns of identical boxes, because, having placed an item, higher ranking alternatives may present themselves. The first tie-breaker rule reflects a preference for items to be placed further back in the container. Such a strategy will preserve as much as possible the container length available for loading the remainder of the cargo and will tend to keep the profile of the front face of the load relatively even. Both aspects are clearly desirable from the viewpoint of dealing with multi-drop loads by processing the different parts in sequence. The other two rules are of less importance. Tie-breaker 2 is aimed at packing potentially awkward items early on, tie-breaker 3 primarily serves to produce a definitive selection when the other rules lead to identical scores.

Figure 2 depicts an example of a loading pattern produced with this method. It is assumed that the consignment shown represents the first part of a mixed destination cargo, i.e. the consignment destined for the final drop-off point. The data used was again that of set No. 6 of [18], already referred to in the previous sub-section, apart from the quantities for each box type being halved (and rounded down to the nearest integer where necessary). All 97 boxes involved have been packed and, as the

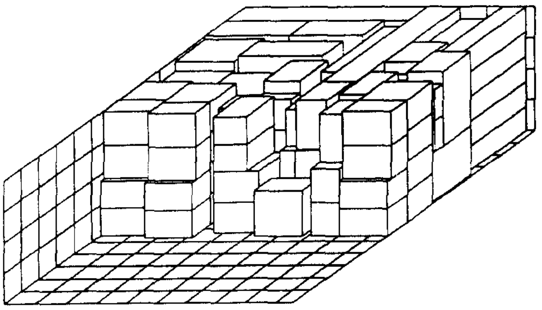

Fig. 2. Loading pattern for an individual consignment.

figure clearly illustrates, they have been arranged in such a way as to leave just under half the container completely free. Further consignments can thus be added to the container with a minimum of intermixing between items destined for different drop-off points.

The method proposed has many characteristics in common with the wall-building approaches referred to earlier on. Its main advantage over these methods is that by not trying to construct complete walls across the width, but only individual columns, it is able to produce efficient solutions even when the cargo is highly heterogeneous. This aspect is of particular importance in multi-drop situations where the separate sub-sets might each involve the same types of boxes, but in relatively small numbers.

It is obvious from Fig. 2 that the method has drawbacks from a stability viewpoint. Also, weight considerations are, of course, not taken into account. For example, where a cargo involves fragile items, the possibility of generating columns consisting of boxes with non-identical base dimensions may lead to problems in terms of product damage. Small modifications to the algorithmic rules, however, can easily eliminate this scenario.

The method was applied to all the test problems of Section 4.1. The implementation used employed the spatial representation technique of [20] to facilitate the search for candidate spaces. For the 6 single container problems described in [15] solutions of equal efficiency were obtained. The results for the other problems are listed in the final columns of Tables 1 and 2.

As can be seen, the results often match those of the other approaches and there are a number of instances where the method described generates solutions with a better space utilization than any of the other methods examined. The average performance, however, is slightly worse. On the data of [18], for instance, the average container utilization is 68.6% as opposed to 69.0% and 69.5% for the approaches of [20] and [3], respectively. The total number of containers needed for the 47 multi-container problems of Table 2 also is approx. 1.8% higher than for the other two methods applied.

5. SYSTEMATIC PERFORMANCE EVALUATION

The comparison of results gained from such a limited number of published data sets clearly represents an inadequate basis for the drawing of general inferences, especially since some of the test problems concerned are relatively undemanding. In order to obtain statistically reliable results, it is necessary to conduct tests on large data sets with similar characteristics. Various authors have carried out such tests for specific methods and problem domains. The drawback of these studies is that the experiments were designed independently.

The testing procedure proposed here uses a framework similar to that described in [3]. The main differences are concerned with providing for replicability of the data. One such aspect is that a standard random number generator is used which is capable of reproducing the same number streams on a wide range of platforms. Furthermore, non-random seed numbers are used and the values employed are quoted in this paper. Finally, the procedure for generating the data is fully described on a step-by-step basis.

The simulation procedures uses the following input parameters:

Cargo target volume T_c
Number of different box types n
Lower and upper limits on box dimensions a_j, b_j, $j = 1,3$
Box stability limit L
Seed number s.

Figure 3 contains a flowchart detailing the different steps involved in generating a single test example. As can be seen, the total cargo volume cannot exceed the target volume T_c and it will be as close as possible to this value in the sense that it cannot fall short of the target by more than the volume of the largest box. The procedure tends to lead to roughly similar numbers m_i for the different box types making up the cargo. Minor modifications—by introducing weighting factors into the formula defining the box type indicator k—would allow boxes to be generated in other (user-specified) proportions.

The random number generator used is a special variant, suggested in [21], of the multiplicative congruential method and, as pointed out in [8], can be implemented in most programming languages on almost any computer. The referenced papers discuss in detail the merits of

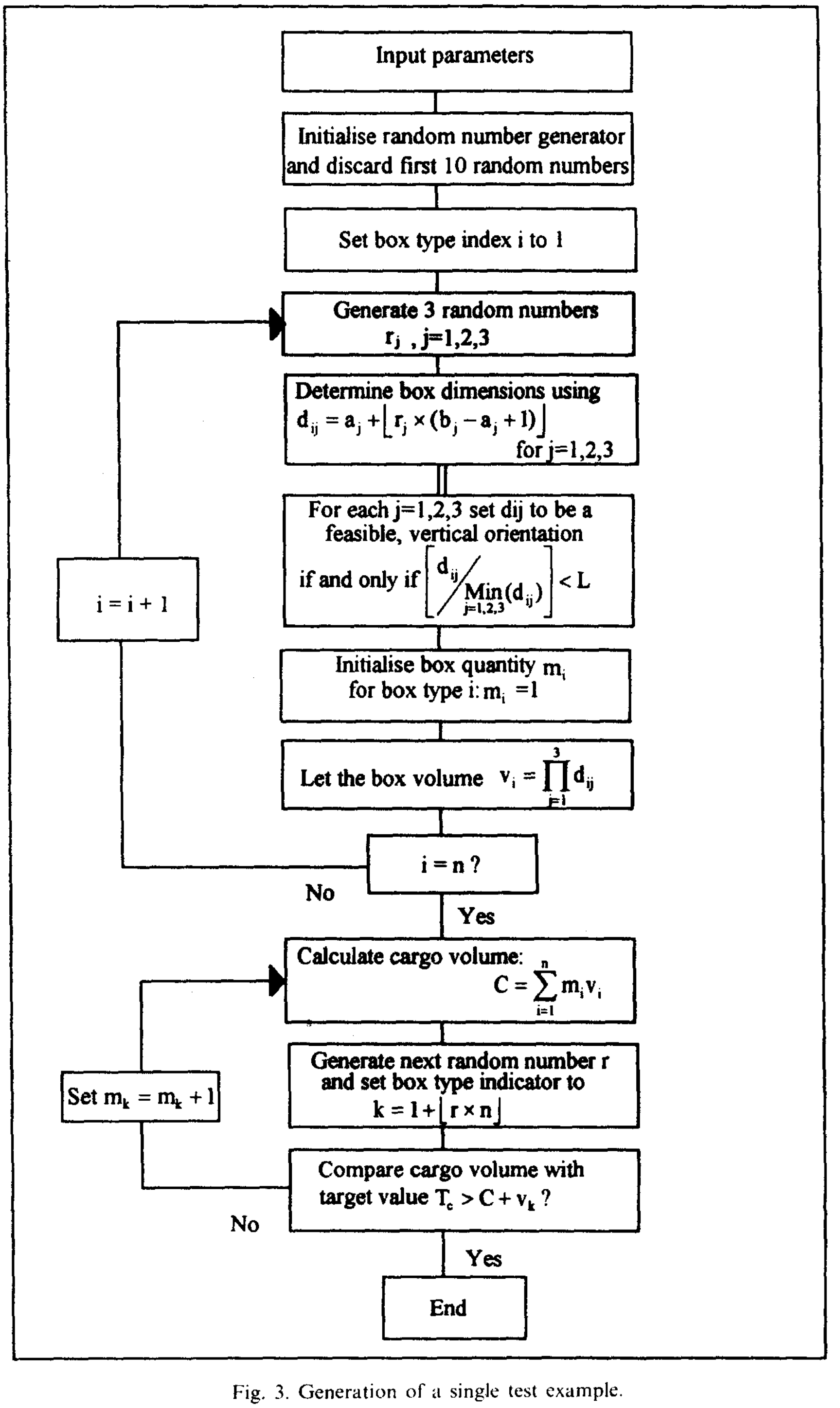

Fig. 3. Generation of a single test example.

Table 3. Summary of test results with 700 problems

No. of box types	Average vol. util.	SD	Highest	Problem No.	Lowest	Problem No.
(*a*) *Method of Section* 4.1						
3	81.76	5.98	94.36	39	64.96	79
5	81.70	6.54	93.76	33	66.70	91
8	82.98	5.44	92.63	49	66.91	72
10	82.60	4.11	88.89	60	66.46	55
12	82.76	4.03	90.39	41	70.38	74
15	81.50	3.88	89.15	58	64.86	7
20	80.51	3.75	88.28	51	70.50	34
(*b*) *Method of Section* 4.2						
3	83.79	4.67	93.58	13	72.05	20
5	84.44	3.97	91.85	39	67.98	11
8	83.94	3.12	89.65	37	75.33	46
10	83.71	3.07	90.06	22	73.11	18
12	83.80	2.88	89.87	2	74.87	89
15	82.44	3.05	88.39	72	72.29	3
20	82.01	2.58	86.88	14	75.57	93
(*c*) *Combined*						
3	85.40	4.30	94.36	39	73.72	44
5	86.25	3.49	93.76	33	73.79	11
8	85.86	3.27	92.63	49	75.33	46
10	85.08	2.47	90.06	22	78.38	93
12	85.21	2.49	90.39	41	78.71	70
15	83.84	2.62	89.15	58	75.22	93
20	82.95	2.43	88.28	51	75.73	74

this random number generator for producing replicable test data and the parameter settings they used in their experiments. The latter are identical to those proposed in [23] (cf. routine 'ran0') and the same values were also employed in the tests described here. The reason for discarding the first 10 random numbers in the sequence is to ensure that similar seed values do not lead to similar random numbers being used in the simulation process. In order to generate a succession of problems with identical characteristics, the seed values can, therefore, be simply incremented by some constant.

Throughout the tests conducted the value of T_c was set to the loading volume of $L \times W \times H$ provided by the container, with its dimensions defined as $L = 587$, $W = 233$ and $H = 220$ (the dimensions in cm of a 20 ft ISO container). The limits on box dimensions were also kept constant at $a_1 = 30$, $b_1 = 120$, $a_2 = 25$, $b_2 = 100$, $a_3 = 20$, $b_3 = 80$ and L, the stability ratio, was set to a value of 2. Seven different values for n, the number of box types, were used: $n = 3, 5, 8, 10, 12, 15$, and 20. For each of these values of n a total of 100 test problems were created, with the seed for problem No. p defined as $s = 2502505 + 100\,(p - 1)$. Individual problems can thus be reproduced without the need to generate the complete set. In order to allow other researchers to verify that their simulation procedure corresponds exactly to the one used here, 5 sample problems are listed in the appendix. (All 700 data sets generated are deposited in the OR-Library [see file thpackinfo] and can be obtained by anonymous ftp at mscmga.ms.ic.ac.uk or via WWW at http://mscmga.ms.ic.ac.uk/.)

Both the methods described in Section 4 were applied to all 700 problems. Table 3 summarizes the results obtained. It can be seen from sections (a) and (b) that the two approaches produce average volume utilizations in excess of 80% over the complete range of problems. It is also obvious that what might be inferred from the results of the previous section is not borne out by the more comprehensive evaluation: the method of Section 4.2 actually produces higher average utilization figures. An application of the matched pair t-test shows this difference in performance to be statistically significant at the 10% level for all 7 data sets at the 5% level for all sets apart from that for $n = 8$.

Section (c) of Table 3 refers to the better of the two results for each problem. The fact that the average utilization values here are considerably higher than those in sections (a) and (b) indicates that the two methods are to some extent complementary. This is confirmed by Fig. 4, which shows the proportion of cases where each method exceeded the space utilization achieved by the other. The diagram again highlights the slightly better performance of the method of Section 4.2.

The comparisons of Table 3 and Fig. 4 focus exclusively on volume utilization. The experimental set-up also allows the suggestions made earlier on concerning the performance of the two methods from a stability viewpoint to be verified in quantitative terms. Other studies making reference to stability considerations in packing problems (e.g. [1, 6]) have pointed out the difficulties in formulating a single measure of stability and have instead adopted a range of separate indicators. Two of these are the degree to which items are supported from below and the number of items which provide that support. As neither of the two methods considered allows overhang, only the second measure is of relevance here.

Table 4 shows the corresponding results. The figures headed 'Measure 1' are the average number of boxes by which items other than those on the floor are supported. As can easily be seen, the results are very much higher for the method of Section 4.1, indicating that there is a much greater degree of interlock within the arrangement. This is also reflected in the figures labelled 'Measure 1a' which are based on the same concept, but exclude insignificant areas of contact, defined here as those amounting to less than 5% of the top face of the supporting box. These two measures do not indicate the potential for lateral movement of a box. Also recorded for each box, therefore, were the number of sides which did not butt against some other object, i.e. another box or the container walls. 'Measure 2' of Table 4 is the average percentage of boxes not surrounded on at least three sides. Again, it is immediately obvious that the method of Section 4.1 consistently produces results which rate better on this criterion.

Table 4. Performance from a stability viewpoint

No. of box types	Measure 1	Measure 1a	Measure 2
(*a*) *Method of Section* 4.1			
3	2.02	1.85	8.50
5	2.22	1.98	11.21
8	2.20	1.94	15.93
10	2.10	1.87	17.51
12	2.09	1.88	21.60
15	2.04	1.81	22.13
20	1.92	1.72	27.07
(*a*) *Method of Section* 4.2			
3	1.13	1.09	10.36
5	1.10	1.07	14.60
8	1.08	1.06	19.67
10	1.07	1.05	23.53
12	1.06	1.04	26.03
15	1.06	1.04	31.04
20	1.04	1.02	35.99

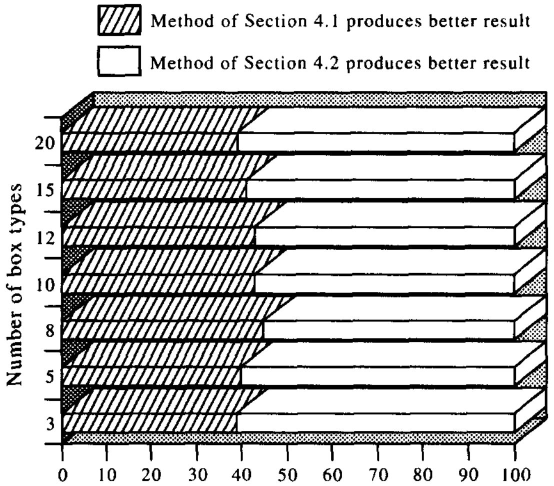

Fig. 4. Number of occasions where one method outperforms another.

6. SUMMARY AND CONCLUSIONS

The paper has highlighted some important shortcomings in the existing theoretical literature on container loading. It has been argued that fundamentally new approaches are required in order to be able to tackle adequately many of the situations arising in practice. It has also been suggested that the development of such approaches must incorporate extensive testing to pinpoint the strengths and weaknesses of methods and that, in order to provide a suitable basis for comparisons, the testing procedures used should be standardised as far as possible.

Two approaches have been examined in detail here. Both are heuristic in nature. One of the methods is specifically designed to produce stable, evenly distributed packing arrangements. The other has built into it features which make it particularly suitable for multi-drop situations. Tests using published data show that both methods are comparable, in terms of the space utilization achieved, to other methods which do not address additional practical requirements of this kind.

Comprehensive tests with randomly generated problems provide a more detailed picture of the performance characteristics of the two methods. In particular, the results show that the two approaches are complementary, not only with respect to their capabilities in dealing with different practical requirements, but also in terms of their performance in situations where only volumetric aspects play a role. Used as components in a computer support system for container load planning, the methods, therefore, provide for a degree of flexibility in allowing the system to be adapted to differing needs, as well as giving rise to more consistent overall performance.

The procedure used to generate test data for the experiments lends itself to easy replication. The results presented may thus be looked upon as potential benchmarks for further work in this area. However, for investigations involving objectives such as weight distribution or constraints on, for example, the load bearing strength of items additional parameters clearly need to be included in the data generation procedure.

APPENDIX

Test Examples

$n = 8$ $p = 49$ $s = 2507350$

Box type	Length	Vert.	Width	Vert.	Height	Vert.	Quantity
1	91	N	54	Y	45	Y	13
2	105	Y	77	Y	72	Y	15
3	79	Y	78	Y	48	Y	10
4	109	Y	76	Y	59	Y	12
5	48	Y	37	Y	30	Y	13
6	44	Y	37	Y	27	Y	9
7	79	Y	76	Y	54	Y	17
8	116	N	78	N	20	Y	16

$n = 10$ $p = 60$ $s = 2508405$

Box type	Length	Vert.	Width	Vert.	Height	Vert.	Quantity
1	78	Y	72	Y	58	Y	14
2	107	Y	57	Y	57	Y	11
3	72	Y	64	Y	37	Y	19
4	93	N	66	N	26	Y	13
5	116	N	67	Y	35	Y	13
6	101	N	56	Y	39	Y	12
7	108	N	100	N	39	Y	11
8	64	Y	55	Y	33	Y	10
9	73	Y	65	Y	53	Y	10
10	106	N	41	Y	35	Y	12

$n = 12$ $p = 41$ $s = 2506505$

Box type	Length	Vert.	Width	Vert.	Height	Vert.	Quantity
1	109	Y	80	Y	68	Y	6
2	9	N	45	Y	31	Y	15
3	66	Y	56	Y	34	Y	12
4	76	N	66	Y	34	Y	7
5	111	N	80	Y	53	Y	10
6	86	N	59	N	29	Y	16
7	80	Y	72	Y	63	Y	12
8	81	N	47	N	22	Y	12
9	114	Y	80	Y	64	Y	8
10	90	N	37	Y	32	Y	12
11	75	Y	48	Y	40	Y	12
12	67	Y	52	Y	45	Y	11

$n = 8$ $p = 37$ $s = 2506105$

Box type	Length	Vert.	Width	Vert.	Height	Vert.	Quantity
1	98	N	73	Y	44	Y	13
2	60	Y	60	Y	38	Y	12
3	105	Y	73	Y	60	Y	9
4	90	Y	77	Y	52	Y	21
5	66	N	58	N	24	Y	12
6	106	Y	76	Y	55	Y	20
7	55	Y	44	Y	36	Y	11
8	82	N	58	N	23	Y	14

$n = 10$ $p = 22$ $s = 2504605$

Box type	Length	Vert.	Width	Vert.	Height	Vert.	Quantity
1	89	Y	78	Y	50	Y	9
2	73	Y	65	Y	46	Y	10
3	94	N	66	Y	39	Y	5
4	66	Y	59	Y	42	Y	11
5	106	Y	93	Y	70	Y	18
6	80	Y	71	Y	62	Y	16
7	60	N	36	Y	26	Y	7
8	72	N	31	Y	31	Y	14
9	63	N	63	N	31	Y	16
10	74	N	39	Y	23	Y	5

$n = 12$ $p = 2$ $s = 2502605$

Box type	Length	Vert.	Width	Vert.	Height	Vert.	Quantity
1	49	N	25	Y	21	Y	13
2	60	Y	51	Y	41	Y	10
3	103	Y	76	Y	64	Y	5
4	95	Y	70	Y	62	Y	13
5	111	N	49	Y	26	Y	13
6	85	Y	84	Y	72	Y	15
7	48	Y	36	Y	31	Y	13
8	86	N	76	N	38	Y	13
9	71	Y	48	Y	47	Y	12
10	90	N	43	Y	33	Y	12
11	98	N	52	Y	44	Y	13
12	73	N	37	Y	23	Y	6

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