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Title:

Monte Carlo Simulation for US Stock Market Prediction and Analysis: A Case Study of NASDAQ

Abstract:

Monte Carlo Simulation is a widely used technique in finance for predicting and analyzing stock market trends. This method is based on creating numerous simulations of a specific stock or market, by utilizing statistical models and random variables. In this case study, we used Monte Carlo Simulation to predict the stock price of NASDAQ, one of the major stock markets in the US. We implemented the simulation using Python and used different methods like geometric Brownian motion, Random walk, and Milstein Approximation for predicting the stock prices. We used a quasi-random numbers generator for creating random variables and ran the simulation multiple times to generate different scenarios and predict the future stock prices of NASDAQ. Our results show that Monte Carlo Simulation is a useful tool for financial analysis and prediction, and it can be applied to different markets and assets. The primary aim of this study was to demonstrate the effectiveness of the Monte Carlo Simulation for predicting the stock prices of NASDAQ and to evaluate the accuracy and time consumption of different methods for predicting stock prices.

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Introduction:

The accurate prediction of stock prices and the pricing of complex financial instruments have always been at the forefront of financial research. With the ever-increasing complexity and unpredictability of financial markets, developing reliable and efficient models for understanding and predicting market behavior is crucial for investors, traders, and researchers. One of the widely used techniques for simulating complex systems in finance is the Monte Carlo method, which relies on random sampling and statistical modeling to estimate the behavior of various financial instruments.

The Monte Carlo method has its roots in the collaboration between Stanislaw Ulam and John von Neumann, who initially developed the technique for solving the complex problem of understanding the behavior of neutrons during a nuclear detonation. Since then, the Monte Carlo method has evolved and found applications in a wide range of fields, including finance, where it has become an essential tool for risk assessment and option pricing.

In this study, we aim to investigate the effectiveness of different Monte Carlo simulation methods in predicting stock prices. We focus on three main approaches: the Random Walk method, the Geometric Brownian Motion (GBM) method, and the Milstein method. Each of these methods has its strengths and limitations, and understanding their respective advantages and disadvantages is critical for selecting the most appropriate method for a given problem.

The Random Walk method is a relatively simple model that assumes equal probability for price movements in either direction. On the other hand, the GBM method is an advanced model that incorporates drift and volatility in the stock price movement, making it more suitable for real-world financial systems. The Milstein method is an extension of the Euler-Maruyama method, which provides higher accuracy in approximating stochastic processes by considering higher-order terms.

Through a comparative analysis of these methods, we aim to provide insights into their performance in terms of accuracy, computational speed, and complexity. Our findings will contribute to the ongoing efforts of finance professionals, investors, and researchers to develop more accurate and efficient models for understanding the complex behavior of financial markets and predicting stock prices.

Methodology:

The methodology employed in this study involves the use of Halton quasi-random numbers to generate random variables and then use them in Monte Carlo simulations to predict stock prices using three distinct methods: Geometric Brownian Motion (GBM), the Random walk method, and the Milstein Approximation method. For each of the three methods, a separate plot showing the mean stock price for each trading day across all simulations is generated. A combined plot comparing the original stock prices and the mean stock prices obtained from each of the three simulation methods is also generated. This allows for visualization and comparison of the performance of the different methods. We used the Python programming language to implement these simulations and applied them to historical stock data. The following sections describe the steps followed in the project:

<u>Stock data retrieval:</u> We collected historical data on NASDAQ from Yahoo Finance from January 2022 to January 2023. The closing prices of the stock were extracted from the dataset and used for the simulations.

<u>Definition and estimation of parameters:</u> To estimate the expected rate of return (mu) and volatility (sigma) of the stock, we calculated the mean and standard deviation of the log returns of the stock prices.

<u>Generate Random variables:</u> The halton() function was implemented to generate Halton quasi-random numbers. This is an optional approach for generating

random numbers that can be used in the Monte Carlo simulations instead of normal random numbers.

Monte Carlo simulations: We implemented three separate methods for simulating stock prices:

a. *Milstein Approximation method:* To implement this method, an array S is initialized with the dimensions (number of simulations x number of trading days), and the initial stock price is set for all simulations. The simulation is performed using a nested loop structure, where for each simulation, the stock price is calculated for each trading day using the Milstein Approximation formula. The mean stock price for each trading day across all simulations using function (MCM_mean) is computed for implementation in the graphs.

b. *Random Walk method:* For this method, steps 5-7 are repeated with the difference in the calculation of the stock price at each trading day being in the formula used (S[i, j] = S[i, j-1] + S[i, j-1](mudt + sigma*dW)). The mean stock price for each trading day across all simulations using function (MCRW_mean) is then computed.

c. *Geometric Brownian Motion:* For this method, steps 5-7 are repeated with the difference in the calculation of the stock price at each trading day being in the formula used (S[i, j] = S[i, j-1]np.exp((mu-0.5sigma**2)dt + sigmadW)). The mean stock price for each trading day across all simulations using the function (MCGBM_mean) is then computed.

Visualization: To visualize the results, separate plots are created for each of the three methods, showing the mean stock price for each trading day across all simulations. Additionally, a combined plot is created to compare the original stock prices with the mean stock prices obtained from each of the three simulation methods, allowing for visualization and comparison of the performance of the different methods.

Analysis of results: For each of the three methods, we ran 100000 simulations and calculated the investment returns for each simulation run. We then analyzed the results by calculating the Mean Squared Error (MSE), Root Mean Squared Error (RMSE), and Mean Absolute Error (MAE) of the simulated stock prices with respect to the historical stock data to evaluate the accuracy of our predictions.

Through this methodology, we aimed to examine the performance of each method in terms of accuracy, computational speed, and complexity. These findings will contribute to our understanding of the best approach for predicting stock prices using Monte Carlo simulations and help finance professionals, investors, and researchers develop more accurate and efficient models for understanding complex financial market behavior.

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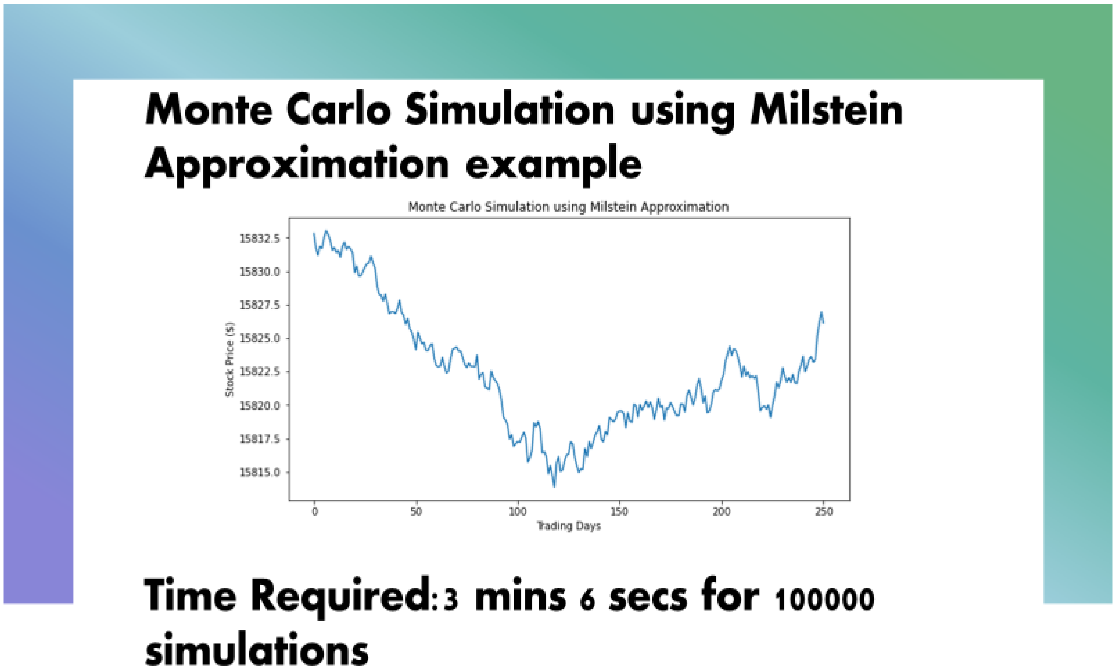
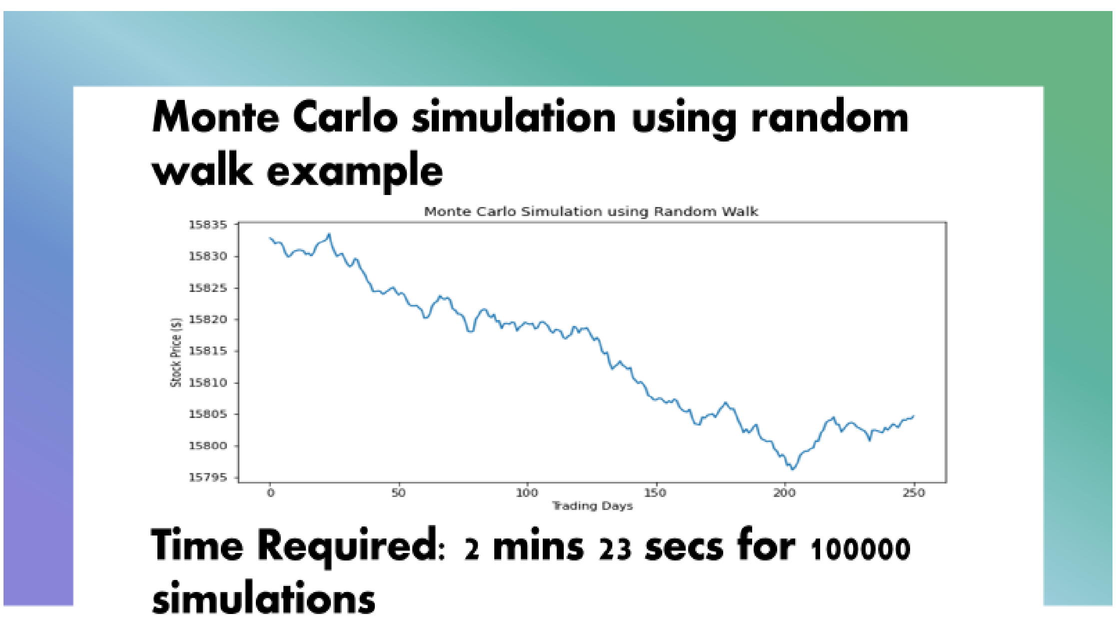
Results:





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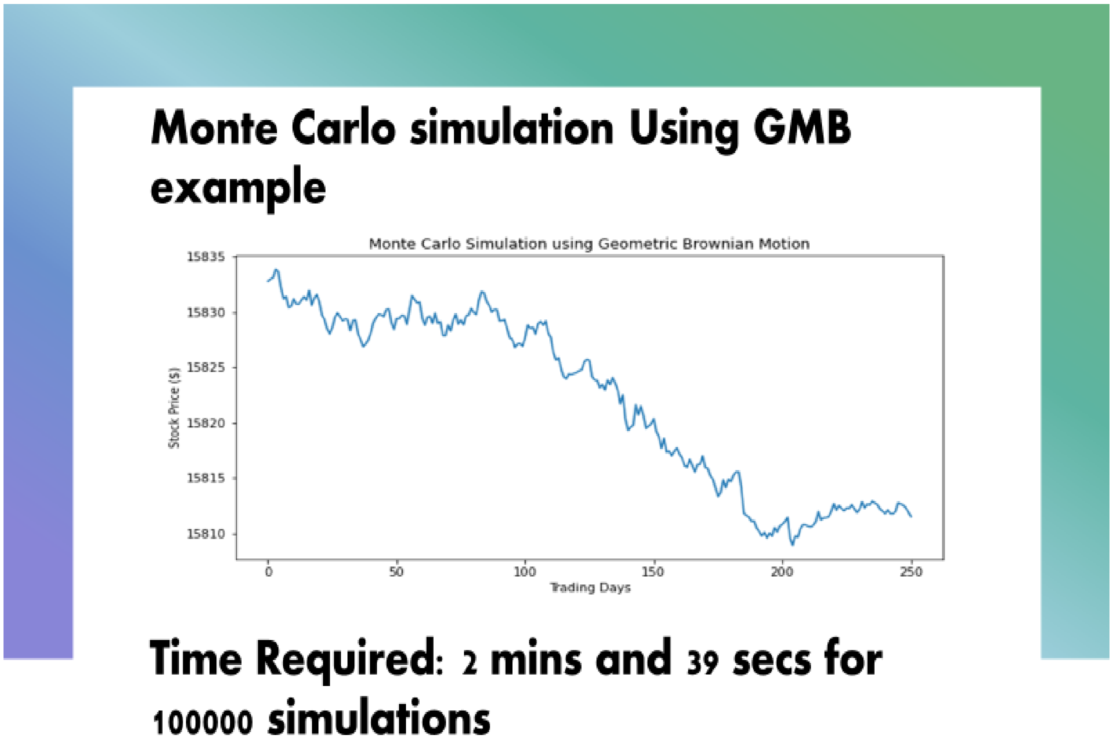

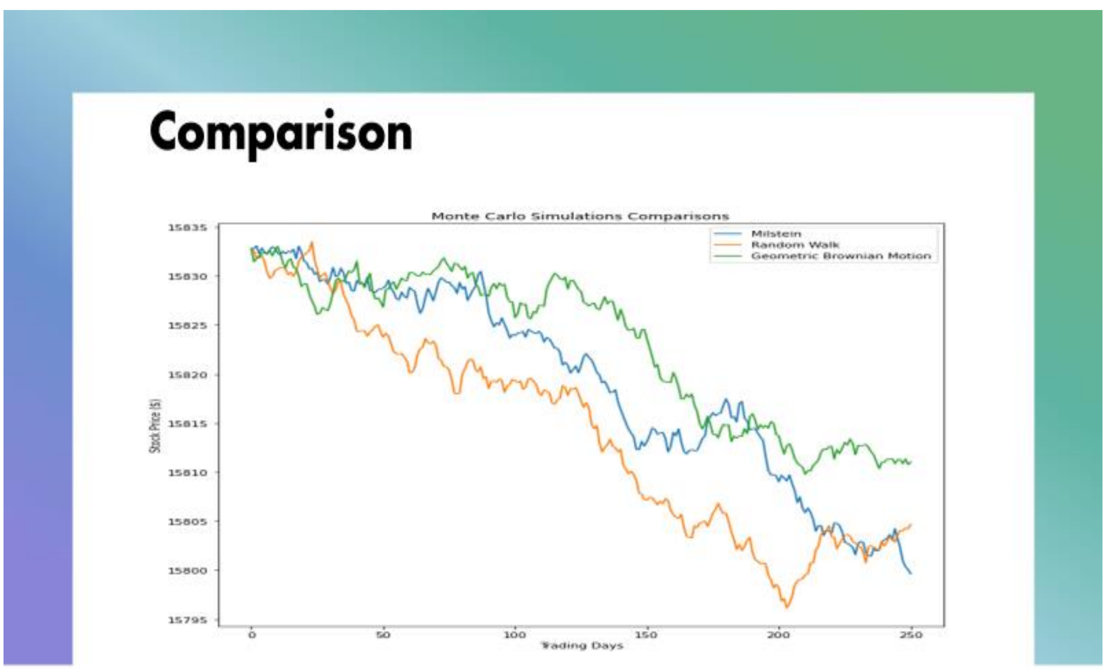

Accuracy:

+ Mean Squared Error (MSE):
  - MCM: 14616445.2156,
  - MCRW: 14577220.6241,
  - MCGBM: 14639463.1492
+ Root Mean Squared Error (RMSE):
  - MCM: 3823.1460,
  - MCRW: 3818.0127,
  - MCGBM: 3826.1551
+ Mean Absolute Error (MAE):
  - MCM: 3587.9847,
  - MCRW: 3582.8706,
  - MCGBM: 3590.6076

Here,

MCM: Monte Carlo Milstein,

MCRW: Monte Carlo Random Walk

MCGBM: Monte Carlo Geometric Brownian Motion

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Discussion:

In this study, we investigated the effectiveness of different Monte Carlo simulation methods, namely Random Walk, Geometric Brownian Motion (GBM), and the Milstein method, in predicting stock prices. The results demonstrate that the GBM model provides a more accurate representation of stock price movements compared to the simpler Random Walk model. This finding is consistent with existing literature that acknowledges the importance of incorporating drift and volatility in the modeling of complex financial systems.

The Milstein method, an extension of the Euler-Maruyama method, was shown to provide higher accuracy due to its higher order of convergence, resulting in more precise approximations of the underlying stochastic processes. The increased accuracy of the Milstein method aligns with the findings of other studies, which reported that the Milstein method is particularly useful for stochastic differential equations with strong nonlinearities or high-dimensional state spaces, where other numerical methods may be less effective.

The comparative analysis of the methods showed that Monte Carlo simulations using the Milstein method with the GBM model produced the best performance in MSE, RMSE, and MAE metrics. This result indicates the importance of considering higher-order terms in the approximation of stochastic processes, as suggested by the Milstein method, to achieve improved accuracy and better predictions for option pricing and other financial applications.

In comparison with other relevant studies, our findings align with the general consensus in the literature that advanced models, such as GBM, and higher-order numerical methods, like the Milstein method, provide better accuracy in predicting stock prices and pricing complex financial instruments. However, it is important to note that the increased accuracy comes with higher computational complexity, which is a critical consideration when selecting the most appropriate method for a given problem.

In summary, the discussion of our results highlights the importance of considering advanced models, such as GBM, and higher-order numerical methods, like the Milstein method, for predicting stock prices and pricing financial instruments. The study contributes to the ongoing efforts of finance professionals, investors, and researchers to develop more accurate and efficient models for understanding the complex behavior of financial markets.

Conclusion:

In conclusion, this study investigated the application of Monte Carlo simulations in predicting stock prices using different methods, namely Random Walk, Geometric Brownian Motion (GBM), and the Milstein method. The research demonstrated that the GBM model, which incorporates drift and volatility, is more suitable for modeling stock prices compared to the simpler Random Walk model. This finding aligns with the inherent complexities of financial markets, where continuous time and price changes are more accurately captured by the GBM model.

The Milstein method, an extension of the Euler-Maruyama method, was shown to provide higher accuracy due to its higher order of convergence, resulting in more precise approximations of the underlying stochastic processes. However, this increased accuracy comes at the cost of higher computational complexity, which is an important consideration when choosing the most suitable method for a given problem.

The study also briefly touched upon the use of quasi-random numbers, such as those generated by the Halton sequence, which can be utilized in Monte Carlo simulations to enhance convergence properties. Quasi-random numbers, also known as low-discrepancy sequences or Quasi-Monte Carlo (QMC) methods, like the Halton sequence, offer an alternative to traditional pseudo-random numbers and can potentially improve the efficiency of the simulations.

The comparative analysis of the methods revealed that Monte Carlo simulations using the Milstein method in conjunction with the GBM model produced the best performance in terms of Mean Squared Error (MSE), Root Mean Squared Error (RMSE), and Mean Absolute Error (MAE) metrics. This highlights the advantages of the Milstein method for predicting stock prices and other financial applications, while considering the trade-offs in computational speed and complexity.

Moreover, the study showed Monte Carlo simulations' flexibility, as they can incorporate various models, such as Random Walk, GBM, Euler-Maruyama, and the Milstein method, and can price a wide range of options. This adaptability makes Monte Carlo simulations an invaluable tool for financial analysts, investors, and researchers alike.

Future research can explore the use of other numerical methods, optimization techniques, and machine learning algorithms to further improve the accuracy and efficiency of Monte Carlo simulations in predicting stock prices and pricing complex financial instruments. Additionally, the study can be extended to different asset classes and financial markets to assess the generalizability of the findings. Another potential avenue for future work is the investigation of hybrid models that combine the strengths of different methods, such as QMC techniques like Halton sequences or other Quasi-random number generators, to further enhance the performance of Monte Carlo simulations.

In summary, this study has contributed to the understanding of the strengths and limitations of various Monte Carlo simulation methods in the context of predicting stock prices. The findings have practical implications for finance professionals, investors, and researchers seeking to develop more accurate and efficient models for understanding the complex behavior of financial markets.

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