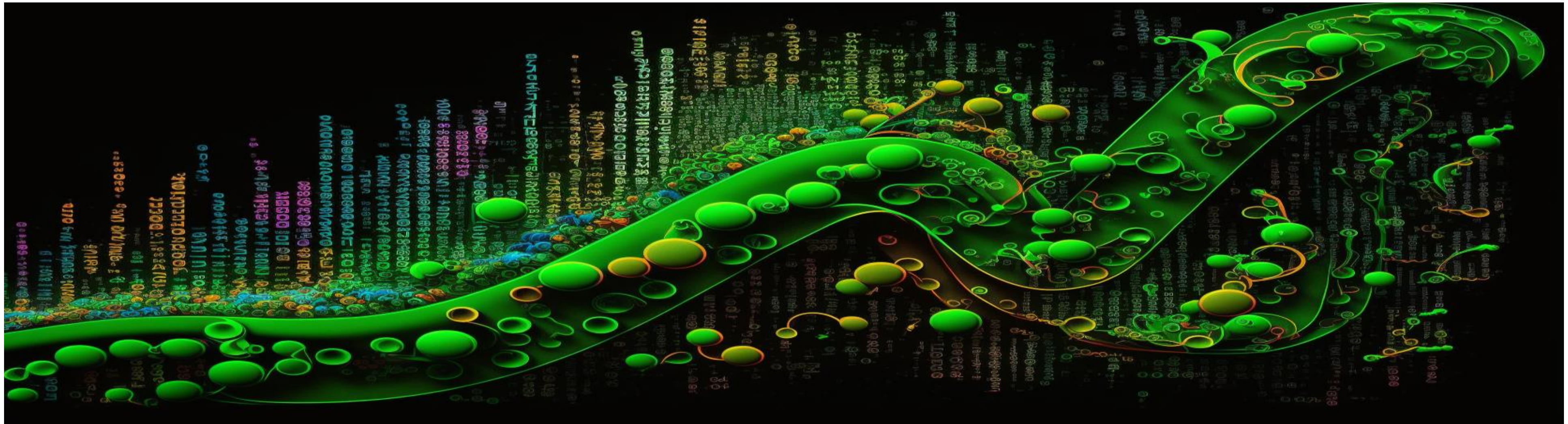
Monte Carlo Simulations for US Stock Market Analysis

Sayantan Manik, Grant Nelson, Joshua Ahrenholtz

Introduction

+ Monte Carlo Simulations (9.2)
+ Discrete and Continuous Brownian Motion (9.3)
+ Stochastic Differential Equations (SDEs) (9.4)
+ Application: to predict NASDAQ Stock prices



Monte Carlo Simulations

- Definition: computational techniques, model and simulate complex systems by using large numbers of random samples and analyzing their statistical properties.
- Applications: Used in finance, engineering, physics, biology, and computer science.
- Examples: risk analysis, optimization, and option pricing
- In General: Used to provide insights and inform decision making

Brief History of the Monte Carlo Simulation

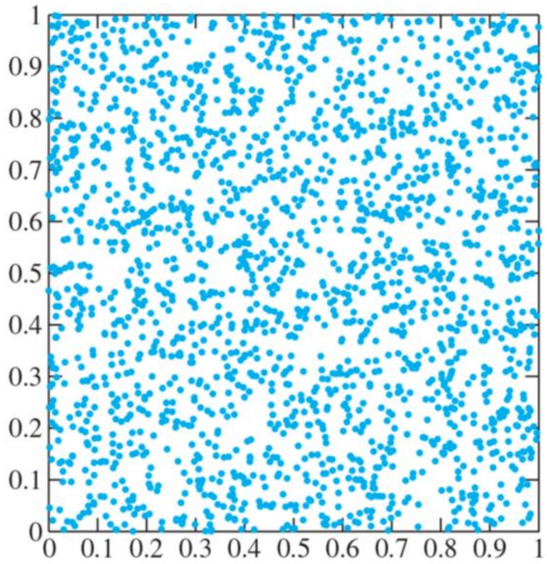
- The Monte Carlo simulation was developed by Stanislaw Ulam, a Polish-American mathematician, and John von Neumann, a Hungarian-American mathematician and polymath. The method was named after the Monte Carlo Casino in Monaco because of the random nature of the simulations, which is akin to gambling.
- Stanislaw Ulam (1909-1984) was born in Lwów, Poland (now Ukraine). He received his Ph.D. in mathematics from the University of Lwów in 1933. Ulam immigrated to the United States in 1935, and during World War II, he joined the Manhattan Project at Los Alamos, where he collaborated with John von Neumann.
- John von Neumann (1903-1957) was born in Budapest, Hungary. He completed his Ph.D. in mathematics at the University of Budapest in 1926. Von Neumann was a true polymath, making significant contributions to mathematics, physics, computer science, and economics. He also worked on the Manhattan Project during World War II, which is where he and Ulam began their collaboration.
- The Monte Carlo simulation method was developed in the 1940s, initially for solving the complex problem of understanding the behavior of neutrons during a nuclear detonation.

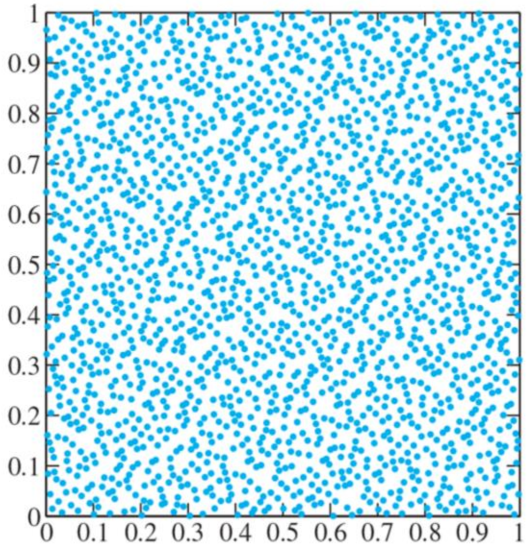
Motivation

- Financial markets are highly complex, difficult to predict
- Monte Carlo simulations are flexible
- Can be used to analyze the sensitivity of the price based on different parameters
- Allow for an analysis of different pricing models and assumptions, helping investors identify opportunities for profitable trading strategies

Pseudo-random vs Quasi-random

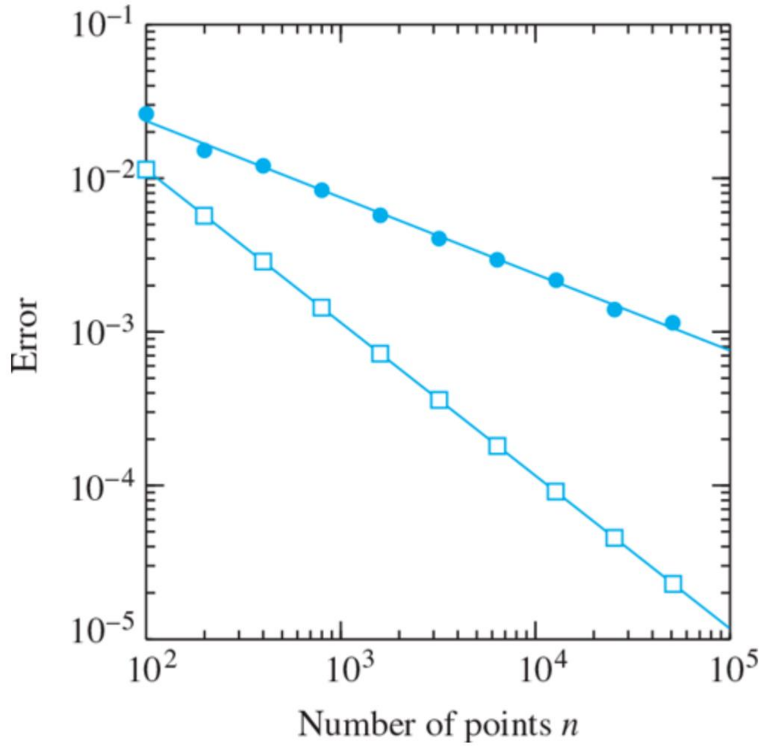
- Pseudo-random numbers are statistically random, but derived from a known starting point
- Quasi-random numbers sacrifices independence, meaning they not random as well as the fact that they don't pretend to be random, they are self-avoiding
- Avoids clustering which speeds up convergence





Monte Carlo Error Example

- Ex. 9.5
- Find a Monte Carlo estimate by using quasi-random numbers for the area under the curve of $y=x^2$ in [0, 1].
- Circles: Pseudo-random
- Squares: Quasi-random



Techniques used in Monte Carlo simulations for estimating

•The random walk method and the Geometric Brownian Motion (GBM) method are two techniques used in Monte Carlo simulations for estimating the behavior of complex systems, such as stock prices. Here's a brief comparison of the two methods:

•Random Walk Method:
•Assumes discrete time steps and discrete price changes
•Simpler model; each step has equal probability in either direction
•Less accurate in modeling real-world systems, like stock prices
•Useful for basic problems or as a starting point for more complex models

•Geometric Brownian Motion (GBM) Method:
•Assumes continuous time and continuous price changes
•More realistic model; based on the Brownian motion concept
•Incorporates drift (trend) and volatility (randomness) in the stock price movement
•More accurate for modeling real-world systems, like stock prices, and widely used in finance

Geometric Brownian Motion – Basics

Geometric Brownian Motion - Basics

- Stock prices follow a stochastic differential equation
- Model incorporates drift (μ) and volatility (σ)
- Drift represents the average return, and volatility represents the stock's price fluctuations
- The equation: $dS = \mu*S*dt + \sigma*S*dz$, where dS is the change in stock price, S is the stock price, dt is the time increment, and dz is a random term following standard normal distribution

Random Walk Method – Basics

- Assumes discrete time steps and discrete price changes
- Simpler model; each step has equal probability in either direction (up or down)
- Less accurate in modeling real-world systems, like stock prices
- The equation: $S_t = S_{(t-1)} \pm \Delta S$, where S_t is the stock price at time t, $S_{(t-1)}$ is the stock price at time t-1, and ΔS is the change in stock price

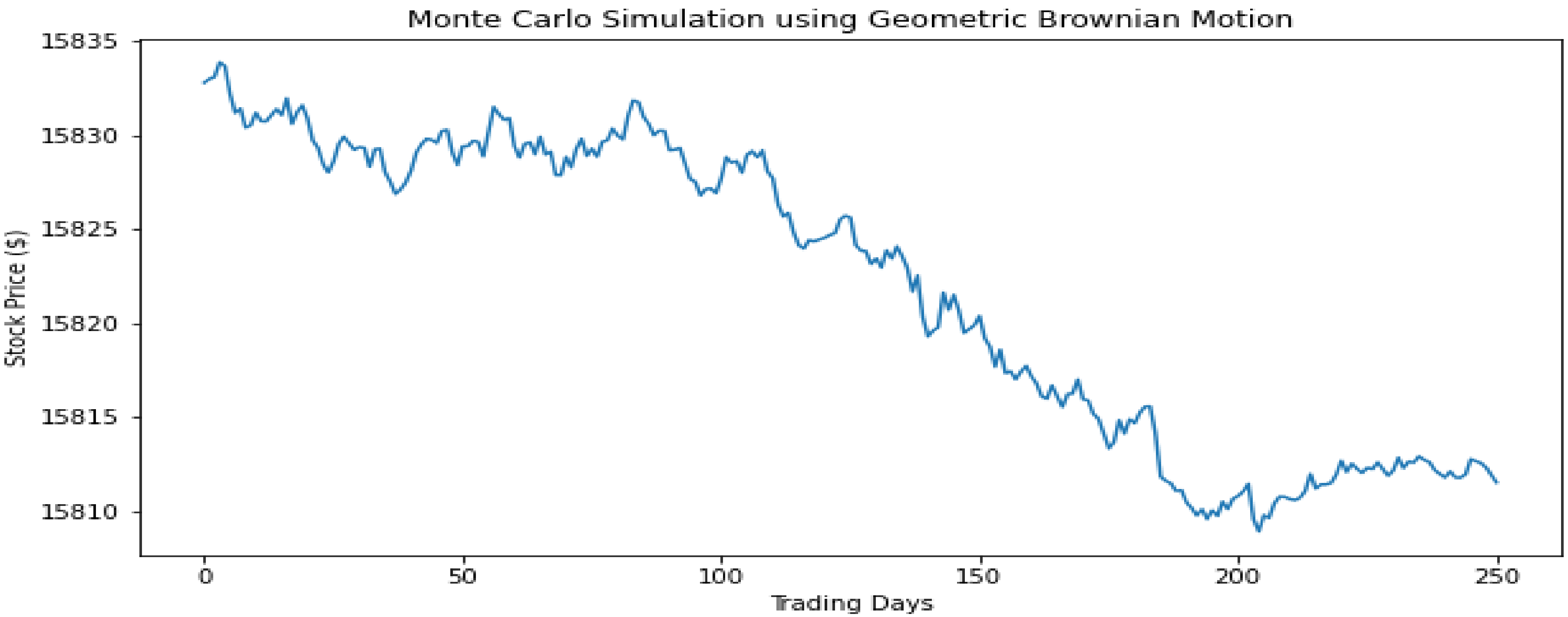
Milstein Method

- The Milstein method is a numerical algorithm used to approximate the solution of stochastic differential equations (SDEs) with additive noise. It is an extension of the Euler-Maruyama method, which is a simple and popular method to approximate SDEs.
- The Milstein method involves the addition of a correction term to the Euler-Maruyama method that accounts for the second-order effects of the stochastic process. This correction term involves taking the partial derivative of the drift term with respect to the state variable, and the partial derivative of the diffusion term with respect to both the state variable and the noise variable.
- The Milstein method is widely used in various fields, such as finance, physics, and engineering, where the dynamics of the system are influenced by random fluctuations. It is especially useful for SDEs that have strong nonlinearities or high-dimensional state spaces, where other numerical methods may be less effective.

Procedure:

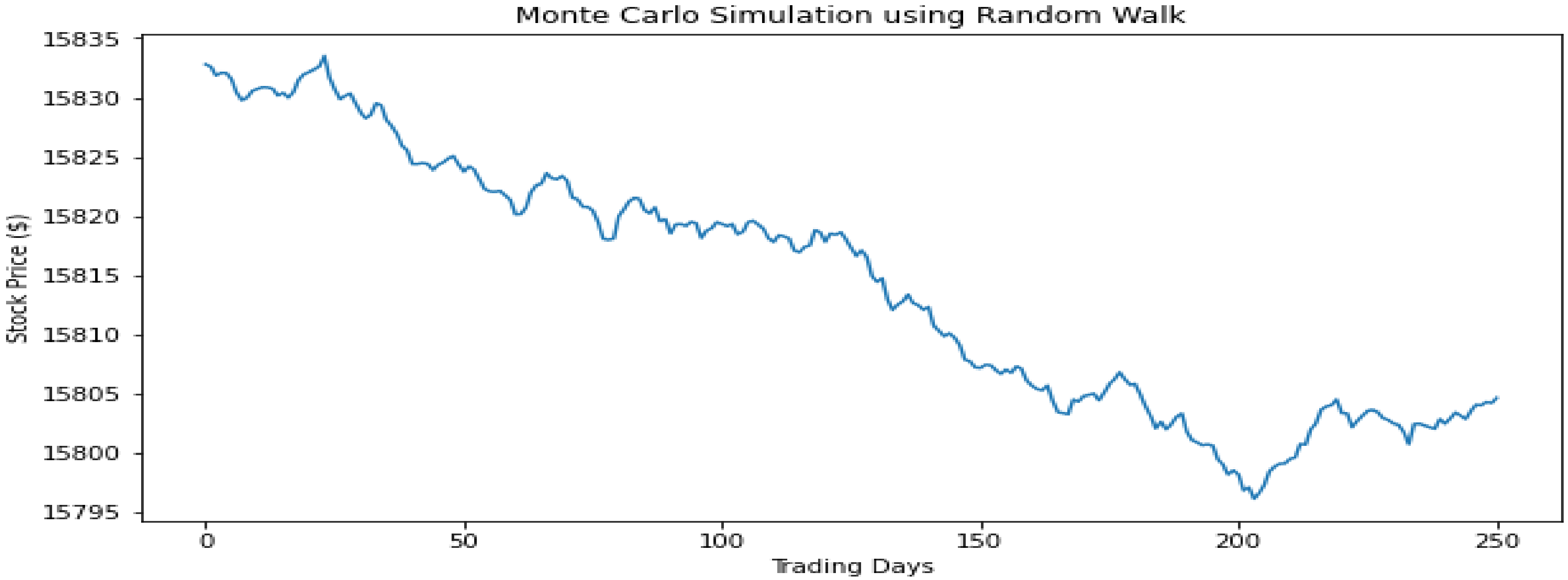
+ Download Nasdaq Stock Data from Yahoo Finance
+ Extract the Closing Prices
+ Define the parameters
+ Generate Halton quasi-random numbers
+ Perform Monte Carlo Simulation using Milstein Approximation
+ Perform Monte Carlo simulation using random walk
+ Perform Monte Carlo simulation using Geometric Brownian Motion
+ Plot graphs and compare
+ Compare accuracies

Monte Carlo simulation Using GMB example



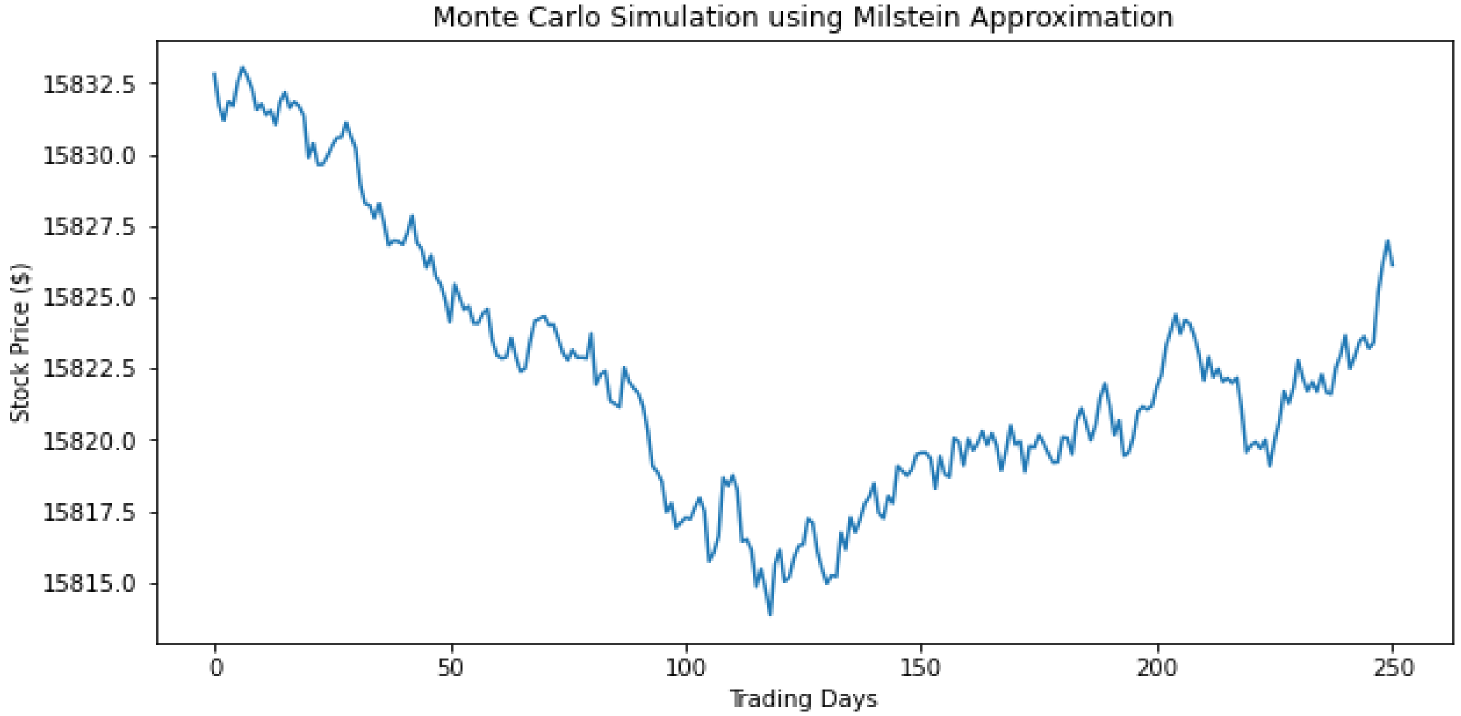
Time Required: 2 mins and 39 secs for 100000 simulations

Monte Carlo simulation using random walk example



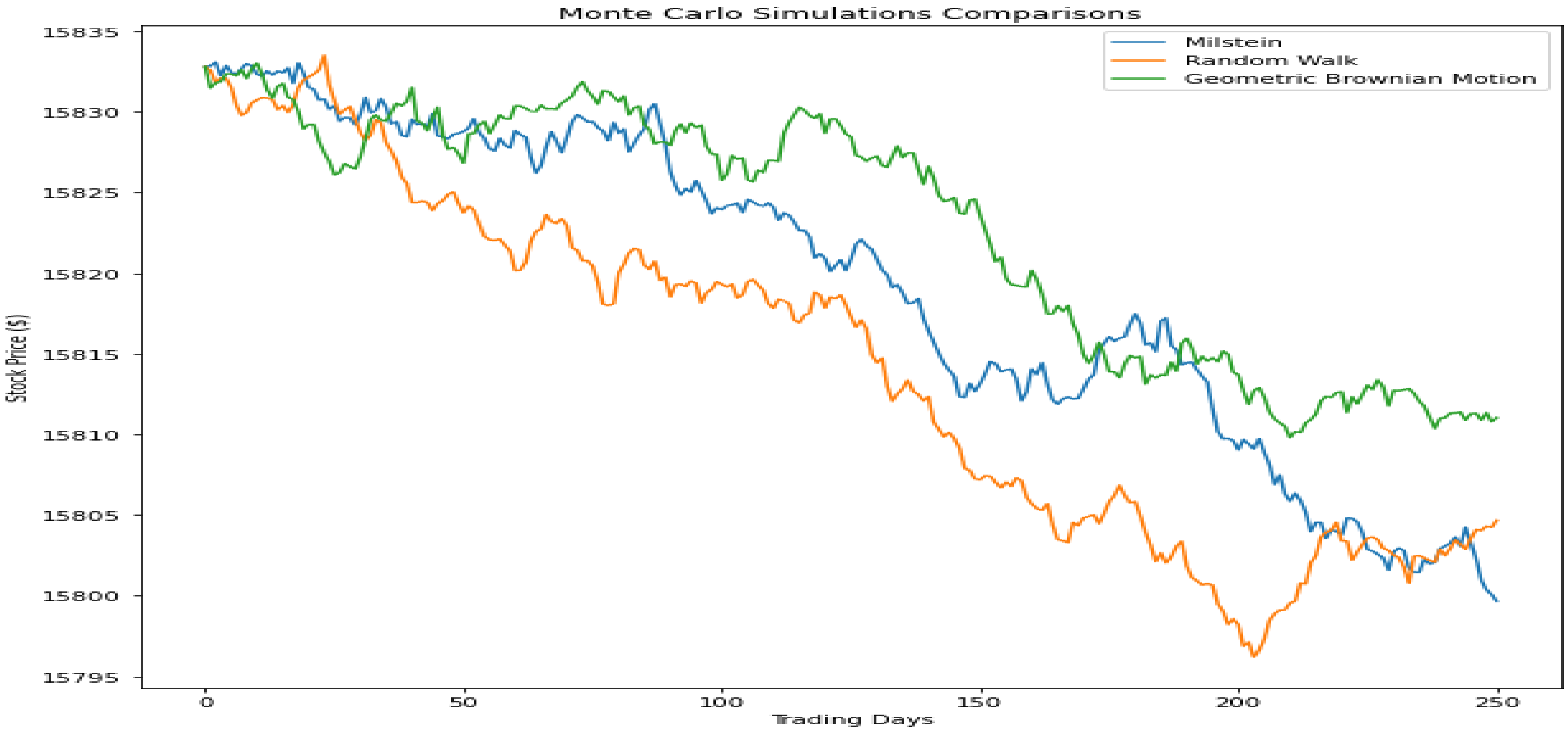
Time Required: 2 mins 23 secs for 100000 simulations

Monte Carlo Simulation using Milstein Approximation example



Time Required: 3 mins 6 secs for 100000 simulations

Comparison



Monte Carlo Simulations Comparisons
Milstein
Random Walk
Geometric Brownian Motion
Stock Price ($)
15835
15830
15825
15820
15815
15810
15805
15800
15795
0
50
100
150
200
250
Trading Days

Accuracy

- Mean Squared Error (MSE): This measures the average of the squared differences between the simulated prices and the actual prices. A lower MSE indicates better accuracy.
- Root Mean Squared Error (RMSE): This is the square root of the MSE, which measures the standard deviation of the differences between the simulated prices and the actual prices. A lower RMSE indicates better accuracy.
- Mean Absolute Error (MAE): This measures the average of the absolute differences between the simulated prices and the actual prices. A lower MAE indicates better accuracy.
- **MSE -**
- MCM: 14616445.2156, <u>MCRW: 14577220.6241</u>, MCGBM: 14639463.1492
- **RMSE -**
- MCM: 3823.1460, <u>MCRW: 3818.0127</u>, MCGBM: 3826.1551
- **MAE -**
- MCM: 3587.9847, <u>MCRW: 3582.8706</u>, MCGBM: 3590.6076

Conclusion

- In this presentation, we explored various methods for predicting stock prices using Monte Carlo simulations, including Random Walk, Geometric Brownian Motion (GBM), and the Milstein method. We discussed the key differences, strengths, and limitations of each approach to better understand their applications in the financial world. Random Walk is a simple model with equal probability of price movement in either direction, making it less accurate for real-world systems. GBM, on the other hand, is an advanced model that incorporates drift and volatility, making it better suited for stock price movements.
- When comparing numerical methods, the Milstein method provides higher accuracy when implemented with the GBM method due to its higher order of convergence, which leads to more precise approximations of the underlying stochastic processes. However, it comes with increased computational complexity.
- Lastly, the Monte Carlo simulation is a flexible and adaptable technique that can incorporate various models, such as Random Walk, GBM, Euler-Maruyama, and the Milstein method, and is capable of pricing a wide range of options. By employing the Milstein method in our Monte Carlo simulations, we can achieve improved accuracy and better predictions for option pricing and other financial applications while considering the inherent limitations of each modeling approach and their respective computational speed and complexity.

References

+ Sauer, Timothy. Numerical Analysis. Available from: VitalSource Bookshelf, (3rd Edition). Pearson Education (US), 2017.
+ "Nasdaq Composite Historical Data." *Yahoo! Finance*, Yahoo!, https://finance.yahoo.com

Thank You!