The Knowledge Discovery Toolbox (Version 0.1) User Guide

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Contents

1   Overview

The Knowledge Discovery Toolbox (KDT) provides subject-matter experts with a simple interface to analyze very large graphs quickly and effectively without requiring knowledge of the underlying graph representation or algorithms. Domain experts are defined as experts in a field of study that is not graphs and graph algorithms, though they may have some familiarity with graph algorithms by using them. Because KDT is open-source, it can be customized or extended by interested (and intrepid) users.

Version 0.1

This early release provides a tiny selection of functions on directed graphs ranging from simple exploratory functions to complex algorithms. The current version works on graphs contained in the memory of multiple computers in a cluster (while hiding data representation and partitioning from the user). Notes specific to this version will be denoted by "*Version 0.1*" and appear in blue text.

1.1   Context

While graphs represent many real-world relationships in a mathematically robust way, their analysis with current methods does not scale. The modern "data tsunami" has created graphs in critical scientific and societal domains that are large enough to be prohibitively time-consuming to analyze with well-known methods. This has led graph-analysis experts to create more efficient graph analysis algorithms, but has also led to a gap between those experts and the non-graph subject-matter experts who need to use them. KDT counters the trend by exposing an API through the Python language that is efficiently and scalably implemented on computer clusters, while remaining suitable for domain experts by hiding the underlying implementation.

KDT is intended to accelerate a virtuous cycle among (a) subject-matter experts who need to analyze graphs that don't fit in the memory of a single computer node; (b) researchers working on improved graph algorithms; and (c) developers of tool infrastructure. We envision that subject-matter experts will do more analysis of large graphs with the current algorithms in KDT and provide feedback on which algorithms are (not) most useful for the large graphs in their domains. This will spur algorithm researchers and tool developers to develop new variants to analyze the subject-matter experts' graphs better.

We believe that many of the subject-matter experts don't know exactly what analysis they need to perform on their data, so they need to explore different algorithms and analyses. The KDT's goal is for the subject-matter expert (or her graduate student) to be able to compose KDT building blocks at the Python level and explore interactively.

KDT's complex algorithms are difficult for even graph experts to verify, and so whenever possible KDT supports internal or user-driven verification of results. That variously consists of internal checks in KDT routines, companion routines that can validate a data structure (*i.e.*, `isBfsTree()` can validate the results of `bfsTree()`), and synthetic inputs whose metrics can be analytically derived (*e.g.*, each vertex of the graph created by `twoDTorus(nnodes)` has an identical betweenness centrality value of

0.5 * 2**(3*scale*0.5) – 2**scale + 1 for an even `nnodes` , where scale is equal to log2(nnodes**2)).

1.2 Intended use cases
Version 0.1

To tie KDT's development to the needs of potential users, this early release targets a small set of use cases. They all assume that the data about graph edges exists in "triplet" format, where the triplet is the source vertex, the destination vertex, and the attribute(s), such as weight and label, of the edge.

1.2.1 Creating a random power-law graph and calculating a breadth-first-search tree
Following the Graph500 benchmark, this use creates a random power-law graph in memory, calculates a breadth-first-search (BFS) tree from a starting vertex, and verifies the BFS tree.

1.2.2 Ranking the importance of Web pages
Analysis of the relative importance of Web pages led to the seminal PageRank algorithm, whose variations are the cornerstone of contemporary search engines.

1.2.3 Calculating the centrality of vertices of a graph
Centrality can be a step toward clustering that removes the most central vertices, but it is also an important metric in its own right for understanding which vertices most keep the graph connected. Exact betweenness centrality is simultaneously viewed by some researchers as not robust enough, because it provides information on only the single shortest path between two vertices, and by others as too computationally expensive, because its cost grows on the order of the number of vertices squared times the length of the longest path. For maximum flexibility, KDT provides a range of algorithms for calculating centrality from across the accuracy vs. execution-cost continuum.

1.3 Deferred use cases
Version 0.1

KDT is envisioned to grow over time to support capabilities not included in this early release. The list of the deferred features includes: undirected graphs, bipartite graphs, multigraphs, and hypergraphs; clustering; general-purpose attributes, such as labels on vertices and edges; visualization of resulting graphs; support on other than x86-64 Linux clusters; and a disk-based implementation, for problems that do not fit in the memory of a computational cluster. The KDT developers believe that general-purpose attributes on edges and vertices are particularly critical to the applicability of the KDT, but we don't yet understand exactly what those attributes and operations on them should look like, so they are not in v0.1.

1.4 Performance
KDT's goal is to enable quick and effective analysis of very large graphs, and thus performance is a vital aspect of KDT. Its performance has not yet been fully characterized. Performance of specific methods, where available, is provided in their descriptions.

Much of the early performance work with KDT has focused on the Graph500 benchmark, whose units are traversed-edges per second (TEPS). **On a 256-core cluster, the KDT implementation of Graph500 performs at over 1 GTEPS.** See section 4 for details.

1.5   Legend and Naming Convention

In the function interface descriptions, required arguments are shown in black text, optional arguments in square brackets, and (optional) expert arguments are shown in grey.

```
cl = cluster('Markov',G[, nclus=k][, power=r])
```

For example, in the `cluster` function, `'Markov'` and `G` are required arguments, `nclus` is an optional argument, and `power` is an optional expert argument.

Names follow the Python convention of generally using lower case and capitalizing the first letter of a class name (`DiGraph`, e.g.), sometimes referred to as Pascal case. Multi-word member names follow the so-called camel case, where the first letter is lower case but the first letter of subsequent words is capitalized, such as `sendFeedback`.

1.6   Giving feedback to the KDT developers

KDT includes `sendFeedback` , a built-in feedback method that enables users to type in code that they wish KDT could execute and then send it to the developers. It uses IPython's `%logstart` facility to capture the code snippet. In Python code interpreters other than IPython the feedback mechanism will not work but will not otherwise obstruct program execution.

For sites that cannot directly send email onto the Internet, the default email address (in `feedback.py`, variable name `_kdt_Alias`) can be changed to an internal collection point.

The feedback mechanism can be used as follows:

In the course of solving your problem, when you need a function not implemented by KDT, type the code that you want KDT to support (which will evoke an error) and then invoke its `sendFeedback` method. It will capture the most recent lines you've typed, create a file from them, and give you an opportunity to edit the file and add supporting comments. If you have feedback that is not directly related to a desired method, that can also be edited into the file. It will then prompt you for confirmation to send the file to the the KDT developers.

With a legend of user input / system responses / user annotation, an IPython session might look like:

```
In [43]: G = kdt.Graph500Edges(scale=32)
In [44]: bc = G.centrality('approxBC',sample=0.01)
In [45]: # delete the top 10 most central vertices; topK and
In [46]: # deleteverts() method doesn't exist
In [47]: discG = G.deleteverts(bc.topK(10))
In [48]: kdt.sendFeedback()
The code example you want to send to the KDT developers is
```

```
in /home/sam/graphdev/KDT_email.
If you wish, edit it with your favorite editor. Type 'Send'
when you are ready to send it or 'Cancel' to cancel sending it.
>>>
```

                    [… If desired, edit the file with an editor …]

```
Send
In [49]:
```

2   Graph500 example

The Graph500 benchmark has replaced SSCA #2 [SSCA] as the primary benchmark for a segment of the graph-analysis research community. This new benchmark is "needed in order to guide the design of hardware architectures and software systems intended to support such applications and to help procurements. Graph algorithms are a core part of many analytics workloads." The specification currently describes two kernels that, respectively, create the graph from the input edge tuples and perform a breadth-first search of the graph from a start vertex. (Future kernels are expected to calculate single-source shortest path and the maximal independent set.) This section illustrates the implementation of kernel 2 of the benchmark with KDT.

This section assumes that Python, IPython, and KDT have already been installed in their default locations and that the environment variable KDTINSTALL has been set to the location of the unzipped/untarred package (see section 5.4 for details). You can type the following to start a serial Python session to run the Graph500 script and understand how it works. For instructions on how to run KDT and Graph500 in parallel with Python, see section 5.4.

```
[sam@neumann ~]$ which ipython
/usr/bin/python
[sam@neumann ~]$ ipython –debug $KDTINSTALL/examples/Graph500.py
Activating auto-logging. Current session state plus future input
saved.
Filename      : .KDT_log    #This output is from the startup of the IPython logstart
Mode          : over        # mechanism used for the sendFeedback function
Output logging : False      # from Section 1.6
Raw input log  : False      #      ""
Timestamping   : False      #      ""
State          : active     #      ""

Generating a Graph500 RMAT graph with 2^15 vertices...
Duplicates removed (or summed): 78842 and self-loops removed: 0
Generation took 0.328309s.
iteration 1: start=28166, BFS took 0.027983s, verification took
2.482376s and succeeded, TEPS=31,832,419
iteration 2: start=27666, BFS took 0.028234s, verification took
2.441357s and succeeded, TEPS=31,549,367
iteration 3: start=27587, BFS took 0.028846s, verification took
2.421630s and succeeded, TEPS=30,879,990
```

```
iteration 4: start=2117, BFS took 0.029042s, verification took
2.409176s and succeeded, TEPS=30,671,607
       […]
iteration 61: start=18643, BFS took 0.028643s, verification took
2.409366s and succeeded, TEPS=31,098,729
iteration 62: start=26284, BFS took 0.029493s, verification took
2.411485s and succeeded, TEPS=30,202,494
iteration 63: start=7687, BFS took 0.030046s, verification took
2.421176s and succeeded, TEPS=29,646,722
iteration 64: start=30285, BFS took 0.029399s, verification took
2.412161s and succeeded, TEPS=30,299,244
Graph500 benchmark run for scale = 15
Kernel 1 time =   0.3283 seconds

Kernel 2 BFS execution times
            min_time: 2.79829502105712891e-02
  firstquartile_time: 2.86501049995422363e-02
         median_time: 2.90445089340209961e-02
  thirdquartile_time: 2.93260216712951660e-02
            max_time: 3.00459861755371094e-02
           mean_time: 2.89925746619701385e-02
         stddev_time: 4.45555267017642357e-04

Kernel 2 number of edges traversed
            min_nedge: 8.90765000000000000e+05
  firstquartile_nedge: 8.90765000000000000e+05
         median_nedge: 8.90765000000000000e+05
  thirdquartile_nedge: 8.90765000000000000e+05
            max_nedge: 8.90765000000000000e+05
           mean_nedge: 8.90765000000000000e+05
         stddev_nedge: 0.00000000000000000e+00

Kernel 2 TEPS
            min_TEPS: 2.96467220212343894e+07
  firstquartile_TEPS: 3.03745608150749356e+07
         median_TEPS: 3.06689642658791840e+07
  thirdquartile_TEPS: 3.10911601670094803e+07
            max_TEPS: 3.18324191444078088e+07
  harmonic_mean_TEPS: 3.07239012190395705e+07
harmonic_stddev_TEPS: 1.69827840156330167e+04
Python 2.4.3 (#1, Nov 11 2010, 13:30:19)
Type "copyright", "credits" or "license" for more information.

IPython 0.8.4 -- An enhanced Interactive Python.
?         -> Introduction and overview of IPython's features.
%quickref -> Quick reference.
help      -> Python's own help system.
object?   -> Details about 'object'. ?object also works, ?? prints
more.
In [5]: parents
Out[2]: Limiting print-out to first 100 elements
```

```
Elements stored on proc 0: {25802,1,1,19693,32686,624,21394,27838,
28156,-1,14783,8652,-1,-1,26729,-1,30564,29617,-1,-1,19294,-1,
6541,26552,23513,31138,2797,2567,30313,32502,-1,-1,8402,-1,32686,-1,
21311,30589,-1,16940,23819,-1,28403,32686,21906,25699,32645,30794,
16244,28393,30280,8146,16578,23171,23683,31583,26762,29294,-1,32010,
32686,27230,30280,-1,-1,28311,27501,-1,30589,30280,32736,-1,31834,
12570,31009,14818,28403,30280,5489,31834,27838,-1,-1,31280,-1,-1,
31009,9680,23819,21028,28403,16244,-1,-1,30279,30280,32100,32153,
21906,32700,}
```

Looking at the code in $KDTINSTALL/examples/Graph500.py, we can see how the Graph500 specification is implemented with KDT methods.

```
import time
import numpy as np
import scipy as sc
import kdt
#
#          […]
#
#     previous kernels have created the following live variables
#     scale:  the log base 2 of the number of edges in the graph
#     edges:  the edges originally used to build the graph
#     G:       the directed graph
```

The following code selects vertices, each to be used as the root of a BFS tree, as specified by the benchmark. KDT-specific functions are shown in **blue**; overloaded Python operators are not highlighted.

```
# find the vertices of degree > 2
deg3verts = (G.degree(dir=kdt.DiGraph.InOut) > 2).findInds()
nstarts = 64                        # #times to create a BFS
# randomly pick some of those vertices as start points
deg3verts.randPerm()
starts = deg3verts[kdt.ParVec.range(nstarts)]
```

The code segment below repeatedly calls the KDT bfsTree method, which creates a BFS tree for the graph and given starting vertex, and then validates the BFS tree via the user-written k2Validate function. Each element of the parents vector points to its (unique) parent in the tree.

```
for start in starts:
    start = int(start)
    before = time.time()

    # the actual BFS
    parents = G.bfsTree(start, sym=True)

    itertime = time.time() - before
    nedges = len((parents[origI] != -1).find())
```

```
K2elapsed.append(itertime)
K2edges.append(nedges)
K2TEPS.append(nedges/itertime)

i += 1
verifyInitTime = time.time()
verifyResult = "succeeded"
if not k2Validate(G, start, parents):
        verifyResult = "FAILED"
verifyTime = time.time() - verifyInitTime

if kdt.master():
        print "iteration %d: start=%d, BFS took %fs, verification
took %fs and %s, TEPS=%s"%(i, start, (itertime), verifyTime,
verifyResult, splitthousands(nedges/itertime))
```

The `k2Validate` user method calls the KDT `isBfsTree` method, which further illustrates the use of KDT with edge and vertex vectors. For instance, one section of `isBfsTree` code validates that each edge's endpoints are one level apart in the BFS tree. It achieves this by building a new `DiGraph` instance containing just the edges in the tree, validating in the process that no tree edge has the root as its destination. (Note that since `isBfsTree` is within the `DiGraph` class itself, which inherits the `Graph` class that contains the `ParVec` class, the code does not refer to the `DiGraph` module explicitly. Note further that this code can be viewed in its installed position (usually in `/usr/lib64/python2.4/site-packages/kdt/DiGraph.py`) and also in `$KDTINSTALL/kdt/DiGraph.py`).

```
    tmp2 = parents != ParVec.range(nvertG)
    treeEdges = (parents != -1) & tmp2
    treeI = parents[treeEdges.findInds()]
    treeJ = ParVec.range(nvertG)[treeEdges.findInds()]
    # root cannot be destination of any tree edge
    if (treeJ == root).any():
            ret = -1
            return ret
    # note treeJ/treeI reversed, so builtGT has reversed edges, as
    #    needed by SpMV
    builtGT = DiGraph(treeJ, treeI, 1, nvertG)
```

These very brief examples illustrate key points of KDT. First, the operations are graph operations, performed on graphs and (distributed edge- and vertex-) vectors. Second, to the extent practical, graph objects are accessible via standard Python methods such as subscripting, comparisons, and utility functions such as `len`.

3   Algorithms, Methods and Classes

Version 0.1

The v0.1 release of KDT supports only directed graphs with single edges between any two vertices, via the `DiGraph` class of the distributed memory version of KDT.

Collections of vertices and edges are represented as `ParVec` class instances. See section 3.4 for details about the class and method structure.

To save repetition, for the remainder of this section we assume that KDT has been imported as

```
import kdt
```

and that the `DiGraph` instance being operated on has the variable name `G`.

3.1   Execution Model

KDT supports parallel execution via the single-program-multiple-data (SPMD) model, meaning that every core runs its own copy of the Python/KDT program and communicates (within KDT) with the other cores as appropriate. The constraint this places on the KDT user is that the arguments passed to KDT functions must be the same on all cores. Most times this is trivial, as when an argument is the name of a variable. One place where this may require thought is in the use of random numbers, where having each core calculate its own random numbers locally may result in different sets of random numbers on each core. The `ParVec.randPerm` function is useful for ensuring random numbers are the same on all cores; the `examples/Graph500.py` file and `DiGraph:_approxBC` method have examples of its use.

3.2   Algorithms

The KDT `DiGraph` class includes the algorithms in this section.

3.2.1   Centrality

Centrality is the degree to which, by some measure, a vertex is *central* to a graph. There is a wide variety of measures used and means of calculating those measures and hence numerous centrality algorithms.

Syntax

```
c = G.centrality('<algorithm>'[, <algorithm-specific keyword arguments>])
```

Description

The `centrality` function takes as input a `DiGraph` object and an algorithm identifier (see below) and returns a `ParVec` (of length equal to the number of vertices in the graph) that contains, for each respective vertex of the graph, the vertex's centrality value. Optional algorithm-specific keyword arguments may also be specified as described by the algorithm-specific sections below.

3.2.1.1 Exact betweenness centrality algorithm

Betweenness centrality is the degree to which a vertex is *between* all other vertices in the graph, calculated as the fraction of shortest paths between two vertices that pass through the given vertex [Freeman 1977].

Syntax

```
cl = G.centrality('exactBC'[, normalize=True])
```

Description

The `exactBC` algorithm calculates exact betweenness centrality on the graph. The optional algorithm-specific `normalize` argument, which defaults to True, causes the function to normalize the betweenness values by dividing each value by (#vertices-1)*(#vertices-2).

Performance

Exact betweenness centrality can be prohibitively expensive for large graphs, while approximate betweenness centrality is less computationally expensive.

Version 0.1
The performance of exact and approximate BC have not been fully characterized, but the following fragmentary results may be useful. The input graph was generated by `twoDTorus`, whose diameter grows as the diameter of the torus. All times are in seconds.

N = # graph vertices [`twoDTorus(n)`]	N = 4096 (n = 64)			N = 16384 (n = 128)			N = 65536 (n = 256)
#cores	1	4	16	1	4	16	16
exactBC	229	75	33	5649	1970	1163	35010
approxBC (sample=0.05)	155	68		260		330	2717

3.2.1.2 Approximate betweenness centrality algorithm
This algorithm [Bader] approximates betweenness centrality for each vertex by using a sample of vertices between which to calculate the fraction of shortest paths, rather than using all vertices as in exact betweenness centrality.

Syntax

```
cl = G.centrality('approxBC'[, normalize=True, sample=])
```

Description

The `approxBC` algorithm performs approximate betweenness centrality on the graph. The optional algorithm-specific `normalize` argument, which defaults to True, causes the function to normalize the betweenness values by scaling each value by the inverse of the `sample` factor (# total vertices / (#vertices actually calculated) and the inverse of (#vertices-1)*(#vertices-2). The optional `sample` expert argument is a floating-point value between 0 and 1 that denotes the fraction of vertices whose connecting paths are calculated in the approximation; a value of 1.0 equates to exact betweenness centrality; the default value is 0.05.

Performance

See v0.1 timing results above in section 3.2.1.1. The numerical values of 'approxBC', sample=0.05 compared to 'exactBC' are as follows for a graph generated by twoDTorus(256), which has equal centrality for all vertices. Exact result: 0.001938. Approximate result: mean 0.001944, standard deviation 0.000784.

3.2.1 PageRank

The PageRank algorithm [Page] computes a Web page's importance based on the number and importance of links to it.

Syntax

```
c = G.pageRank([epsilon = 0.1][, dampingFactor = 0.85])
```

Description

The pageRank function takes as input a DiGraph object and optional arguments and returns a ParVec (of length equal to the number of vertices in the graph) that contains, for each respective vertex of the graph, the vertex's rank. The optional argument epsilon controls the stopping condition; iteration stops when the 1-norm of the difference in two successive result vectors is less than epsilon; the default is 0.1. The optional argument dampingFactor alters the results and speed of convergence, denoting the probability that the random surfer hops to an adjacent vertex (rather than hopping to a random vertex in the graph); the default value is 0.85.

The PageRank algorithm computes the percentage of time that a "random surfer" spends at each vertex in the graph. If the random surfer is at vertex V, she will take one of two actions:

- She will hope to another vertex to which vertex V has an outward edge (self loops are ignored).

- She will randomly hop to any vertex in the graph. This action is taken with probability (1 – dampingFactor)

3.2.2 Search

Several graph algorithms search for connectedness within the graph, such as trees, connected components, independent sets, paths, etc.

3.2.2.1 Breadth-first Search Tree

The bfsTree method creates a breadth-first search tree of a DiGraph instance from a starting vertex.

Syntax

```
parents = G.bfsTree(start[, sym=False])
```

Description

The `bfsTree` method takes as input a `DiGraph` instance and the index of the vertex from which to start the search. It returns a `ParVec` (of length equal to the number of vertices in the graph) that denotes, for each respective vertex of the graph, the vertex's parent in the BFS tree. The `start` vertex's parent is itself. A vertex that is unreachable from the `start` vertex has a parent of -1. The optional expert argument `sym` denotes whether the graph is a symmetric graph, which if True enables the operation to run faster.

Performance

The performance of the Graph500 benchmark is dominated by the performance of the `bfsTree` method. The fragmentary Graph500 performance available to date is described in section 1.4.

3.3   General-purpose methods

The KDT `DiGraph`, `ParVec`, and `SpParVec` classes include the general-purpose methods in this section.

3.3.1   Built-in methods for ParVec vectors

Version 0.1

The `ParVec` class represents vertex and directed-edge vectors. Its implementation for distributed memory includes a significant but not complete set of methods, which are mapped onto standard Python operators or methods consistent with clear understandability.

Note: In version 0.1, `ParVec` instances only support a 64-bit floating-point or integer elemental datatype, and no explicit Boolean datatype. Any `ParVec` whose elements are each either 0 or 1, such as "kdt.ParVec.range(n) < k", will be viewed as a Boolean vector for indexing purposes. The isBool() method tests this characteristic.

Syntax

```
# v, v2, and v3 are ParVec instances
# k is an integer scalar, m is an integer or floating-point scalar
v2 = v > k              # v2 is a Boolean vector, where each element
v2 = v >= k             #  is the comparison of the corresponding
v2 = v < k              #  element of v with the scalar k
v2 = v <= k
v2 = v == k
v2 = v != k
v2 = v > v3             # v2 is a Boolean vector, where each element
v2 = v >= v3            #  is the comparison of the corresponding
v2 = v < v3             #  elements of v and v3
v2 = v <= v3
v2 = v == v3
v2 = v != v3
v2 = ~v                 # negation (only for Boolean vectors)
v3 = v & v2
```

```
v3 = v | v2
v3 = v ^ v2

v3 = v + k
v3 = v + v2
v += k
v += v2
v3 = -v
v3 = v - k
v3 = v - v2
v -= k
v -= v2
v3 = v * k
v3 = v * v2
v3 = v / k
v3 = v / v2
v3 = v % k
v3 = v % v2

v[k] = m
m = v[k]
v[vec] = m              # vec an integer vector
v2 = v[vec]

v.abs()
v2 = abs(v)

print v
v.printAll()
```

Description

These methods have their standard definitions in Python with the following exceptions:

- Only a key set of indexing ("[]") modes are supported.   The right-hand side modes all return a ParVec instance with length equal to the number of requested elements, unless otherwise noted.

    - Indexing on the right-hand side of an equation by a scalar, returning a scalar.

    - Indexing on the right-hand side of an equation by a Boolean ParVec instance.

    - Indexing on the right-hand side of an equation by a non-Boolean ParVec instance.

    - Indexing on the right-hand side of an equation by a Boolean SpParVec instance.

- Indexing on the right-hand side of an equation by a non-Boolean `SpParVec` instance, returning a `SpParVec` instance.

- Indexing on the left-hand side by a scalar index with a scalar value, changing a single value.

- Indexing on the left-hand side by a Boolean `ParVec` index with a scalar value, changing the indicated values of the base `ParVec` instance to the value of the scalar.

- Indexing on the left-hand side by a Boolean `ParVec` index with a `ParVec` value, changing the values of the base `ParVec` instance corresponding to theTrue values of the `ParVec` index to the corresponding values of the `ParVec` value.

- Indexing on the left-hand side by a non-Boolean `SpParVec` index vector with a scalar value.

- Indexing on the left-hand side by a Boolean `SpParVec` index with a `SpParVec` value, changing the indicated values of the base `ParVec` instance to the corresponding values from the value vector.

- The modulus ("`%`") function implements the behavior of Python's modulus function for negative numbers, where the sign of the result is the same as the sign of the divisor (*e.g.*, "-1 % 8" yields "7"). This is different from the behavior of Python's `math.fmod` or C's `fmod`.

- The standard `print` statement will only print the first 100 (`_REPR_MAX`) elements of a `ParVec` instance. Printing all the elements can be achieved via `printAll`.

3.3.2   Non-built-in methods for ParVec vectors

The `ParVec` class implements several other methods, some of which are common Python names and some which are not.

Syntax

```
# v and v2 are ParVec instances, m a scalar
v2 = v.copy()              # deep copy
vSparse = v.toSpParVec()   # convert to a (sparse) SpParVec

v2 = v.ceil()
v2 = v.floor()
v2 = v.round()
v2 = v.sign()
v2 = v.allCloseToInt()

boolResult = v.any()  # boolResult is a logical scalar
boolResult = v.all()
boolResult = v.isBool()
```

```
v2 = v.logical_not()  # also accessible via not keyword

k = v.len()
k = v.nn()              # number of nulls
k = v.nnn()             # number of nonnulls

v = kdt.ParVec.ones(size)
v = kdt.ParVec.zeros(size)
v = kdt.ParVec.broadcast(size, value)
v = kdt.ParVec.range([start,] stop)
v2 = v.findInds()

m = v.max()
m = v.min()
m = v.sum()
m = v.mean()
m = v.std()

v.sort()
[sorted, perm] = v.sorted()
v2 = v.topK(k)

m = v.norm(ord=1)      # 1-norm

v.randPerm()
```

Description

These methods have their standard Python or SciPy definitions, with the following exceptions.

Assignment ("=") of a `ParVec` to another variable name does not create a copy of the object, following Python usage for complex objects. The `copy` method can be used if a copy is needed.

The `allCloseToInt` method returns a Boolean scalar denoting whether all the elements of the vector are within machine precision of an integer value.

The `broadcast` method creates a vector of constant value, equivalent to (but faster than) `ones(size)*value`.

The `nn` and `nnn` methods return the number of nulls and nonnulls, respectively, with `nnn` behaving the same as SciPy's `getnnz`. The `findInds` method returns the indices of nonnull elements of the input `ParVec` instance, the same as NumPy/SciPy's `nonzero` method. The distinct names were chosen to provide future flexibility for a sparse `ParVec` class with a configurable null (zero) element.

The `sort` method sorts the elements of the input vector, in-place, in ascending order. The `sorted` method returns a tuple whose first element is a copy of the input vector, sorted in ascending order, and whose second element is the indices of the sorted elements in the original vector. The `topK` method returns the k largest values of the input vector, sorted in ascending order, in a `ParVec` instance of length k.

The `norm` method (modeled on the NumPy method) calculates only the 1-norm.

The `randPerm` method randomly permutes the elements of the `ParVec` instance.

3.3.3 Built-in methods for SpParVec vectors

The `SpParVec` class implements data structures and operations for sparse distributed vectors; "sparse" in the sense that only non-null values are actually stored and represented in the vector.

Note: Operations on `SpParVec` instance(s) typically operate only on the non-null values, which is different from MATLAB behavior. Thus the `all` method, for instance, returns True if all the nonnull elements are True, ignoring the values of any null elements. Similarly, subtraction of a scalar from a `SpParVec` instance will subtract the scalar value only from the nonnull elements of the vector. Addition or logical Or of two `SpParVec` instances will yield nonnulls in all positions which had a nonnull in either of the inputs. These semantics are intended to reduce the likelihood of sparse vectors unintentionally becoming dense and thus much larger.

Version 0.1

The `SpParVec` class represents sparse vertex and directed-edge vectors. Its implementation for distributed memory includes a significant but not complete set of methods, which are mapped onto standard Python operators or methods consistent with clear understandability.

Note: In version 0.1, SpParVec instances only support a 64-bit floating-point or integer elemental datatype, and no explicit Boolean datatype. Any SpParVec whose nonnull elements are each either 0 or 1, such as "sparsevec < k", will be viewed as a Boolean vector for indexing purposes. The isBool() method tests this characteristic.

Syntax

```
# v, v2, and v3 are SpParVec instances
# k is an integer scalar, m is an integer or floating-point scalar
v2 = v > k            # v2 is a Boolean vector, where each element
v2 = v >= k           #  is the comparison of the corresponding
v2 = v < k            #  element of v with the scalar k
v2 = v <= k
v2 = v == k
v2 = v != k
v2 = ~v               # negation (only for Boolean vectors)
v3 = v & v2
v3 = v | v2
```

```
v3 = v ^ v2

v3 = v + v2
v += v2
v3 = -v
v3 = v - v2
v -= v2
v3 = v * v9          # only for v9 a ParVec instance
v3 = v / v9          # only for v9 a ParVec instance
v3 = v % v9          # only for v9 a ParVec instance

m = v[k]
v2 = v[vec]          # vec an integer SpParVec
v[k] = m
v[vec] = m           # see below
v[v2] = v3

del v[k]
del v[vec]           # vec an integer ParVec

v.abs()
v2 = abs(v)

print v
v.printAll()
```

Description

These methods have their standard definitions in Python with the following exceptions:

- Only a key set of indexing ("[]") modes are supported.

    o Indexing on the right-hand side of an equation by a scalar, returning a scalar.

    o Indexing on the right-hand side of an equation by a non-Boolean `ParVec` instance, with the elements of the result corresponding to the values of the base `SpParVec` instance indicated by the elements of the index. The length of the return is equal to the length of the index.

    o Indexing on the left-hand side by a scalar index (key) with a scalar value.

    o Indexing on the left-hand side by a Boolean `ParVec` index with a scalar value, which changes the elements of the base `SpParVec` corresponding to the True values in the `ParVec` index to the scalar value.

- o Indexing on the left-hand side by a Boolean `ParVec` index with a `ParVec` value, which changes the elements of the base `SpParVec` corresponding to the True values in the ParVec index with the values from the `ParVec` value.

- o Indexing on the left-hand side by a non-Boolean `SpParVec` index vector with a scalar value. Only the value elements corresponding to the nonnull elements in the index vector are changed.

- o Indexing on the left-hand side by a non-Boolean `SpParVec` index vector with a `SpParVec` value. Only the value elements corresponding to the nonnull elements in the index vector are changed, to the corresponding elements of the value vector.

- The standard `print` statement will only print the first 100 (_REPR_MAX) nonnull elements of a `SpParVec` instance. Printing all the nonnull elements can be achieved via `printAll`.

3.3.4 Non-built-in methods for SpParVec vectors

The `ParVec` class implements several other methods, some of which are common Python names and some which are not.

Syntax

```
# v and v2 are SpParVec instances, m a scalar
v2 = v.copy()          # deep copy
vDense = v.toParVec() # converts to a (dense) ParVec

boolResult = v.all()  # boolResult is a logical scalar
boolResult = v.any()
boolResult = v.isBool()
v.toBool()             # converts in-place to True/False (1.0/0.0)

v2 = v.logical_not()  # also accessible via not keyword

k = v.len()            #
k = v.nn()             # number of nulls
k = v.nnn()            # number of nonnulls
k = v.nnz()            # number of nonzeros

v = kdt.SpParVec(size)         # all nulls
v = kdt.SpParVec.ones(size)# all ones
v = kdt.SpParVec.range([start, ]stop))
v.spOnes()                     # each non-null is set to 1
v.set(m)                       # each non-null is set to m
v.spRange()                    # each non-null is set to its index

m = v.sum()
```

```
v.sort()
[sorted, perm] = v.sorted()
v2 = v.topK(k)          # the top k largest values
```

Description

These methods have their standard Python or SciPy definitions, with the following exceptions.

Assignment ("=") of a SpParVec object to another variable name does not create a copy of the object, following Python usage for complex objects. The copy method can be used if a copy is needed.

The all method returns True if all nonnull elements of the SpParVec instance are True (not equal to 0.0). The any method returns True if at least one nonnull element of the SpParVec instance is True (not equal to 0.0). The isBool method returns True if all nonnull elements of the SpParVec instance are either True (equal to 1.0) or False (equal to 0.0). The toBool method converts the input SpParVec instance in-place, with each nonnull element set to False (0.0) if it's input value was 0.0 or True (1.0) otherwise.

The len method returns the length (*i.e.*, the maximum number of non-nulls there could ever be) of the SpParVec object. The nn, nnn, and nnz methods return the number of nulls, nonnulls, and nonzeros, respectively. The sum of nn() and nnn() for any SpParVec instance will always be equal to the length of the instance. The distinct names were chosen to provide future flexibility for the SpParVec class' null (zero) element to be configurable.

The SpParVec constructor itself creates an instance of the length specified with all elements null. The ones method creates a SpParVec instance of the length specified with all elements set to 1. The range method creates a SpParVec instance of the length specified with each element set to its index in the vector, with the optional start argument as in standard Python. The spOnes method modifies the bound SpParVec instance by setting all its nonnull elements to 1.0. The set method modifies the bound SpParVec instance by setting all its nonnull elements to the passed value. The spRange method modifies the bound SpParVec instance by setting each of its nonnull elements to its index in the vector.

The sum method sums the nonnull elements of the bound SpParVec instance, returning the scalar result.

The sort method sorts the nonnull elements of the input vector, in-place, in ascending order. The sorted method returns a tuple whose first element is a copy of the input vector, with the nonnull elements sorted in ascending order, and whose second element is the indices of the sorted elements in the original vector. The topK method returns the k largest nonnull values of the input vector, sorted in ascending order, in a ParVec instance of length k.

3.3.5   Built-in methods for DiGraph objects

The `DiGraph` class represents directed graphs. Its implementation for distributed memory includes a small set of methods, some of which are mapped onto standard Python operators or methods consistent with clear understandability.

Version 0.1

`DiGraph` instances support a 64-bit floating-point or integer elemental datatype and a Boolean datatype, with an explicit conversion required to the Boolean form.

Syntax

```
# G1, G2 and G3 are DiGraphs
G3 = G1 + G2            # elemental addition
G3 += G2                # elemental addition in place
G3 = -G1                # elemental negation
G3 = G1 * G2            # elemental multiplication
G3 *= G2                # elemental multiplication in place
G3 = G1 / G2            # elemental division
G3 /= G2                # elemental division in place

print G
```

Description

These methods have their obvious definitions, with the caveat that the multiplication and division operators yield a null (zero) value in any element for which at least one of the graphs has a null entry. Because `DiGraph` are often large objects, in-place operators may often make sense to conserve the amount of memory consumed by temporary or transient objects.

The `toBool` method converts each nonnull element of a `DiGraph` instance to a Boolean True, thereby consuming less space and enabling faster operations.

The `print` method for `DiGraph` objects can be problematic, as they can often be extremely large (billions of elements), for which text display is rarely useful. For v0.1, for small graphs (defined as 100 edges or fewer), the print method will call the `toParVec` method and print those results. For larger graphs, you can accomplish this yourself by invoking the `toParVec` method manually and printing those results via `ParVec.printAll`.

3.3.6   Non-built-in methods for DiGraph objects

In addition to built-in methods, a number of other utility methods are implemented for the `DiGraph` class.

3.3.6.1   Simple methods

Syntax

```
# G and G2 are DiGraphs
G2 = G.copy()           # deep copy
k = G.nvert()           # number of vertices
```

```
k = G.nedge()          # number of edges
G.ones()               # set all non-null edge-weights to 1.0
G.toBool()             # set all non-null edge-weights to True
G.set(k)               # set all non-null edge-weights to k
G.mulNot(G2)           # see below
G.removeSelfLoops()    # remove all edges from a vertex to itself
```

Description

These methods have their obvious definitions with the following elaborations.

Assignment ("=") of a `DiGraph` object to another variable name does not create a copy of the object, following Python usage for complex objects. The `copy` method can be used if a copy is needed.

The `toBool` method converts each nonnull element of a `DiGraph` instance to a Boolean True, thereby consuming less space and enabling faster operations.

The `mulNot` method takes the logical inverse of each element of the second `DiGraph` before doing the multiplication. Because it does this elementally, there is no extra memory consumed by the inverse values.

3.3.6.2   toParVec

The `toParVec` method of the `DiGraph` class decomposes a `DiGraph` instance to its edges .

Syntax

```
[source, dest, weight] = G.toParVec()
```

Description

The `toParVec` method of the `DiGraph` class decomposes a `DiGraph` instance to its edges, returning `ParVec` instances containing the source vertices, destination vertices, and edge-weights, respectively. Other than the bound `DiGraph` instance, there are no input parameters.

The `toParVec` method is almost the converse of the `DiGraph` method, with the difference that, since the `DiGraph` method sums duplicate edges, the output of `toParVec` may have fewer edges, though the same set of vertices.

3.3.6.3   reverseEdges

The `reverseEdges` method of the `DiGraph` class reverses the direction of each edge of a `DiGraph` instance .

Syntax

```
G.reverseEdges()
```

Description

The `reverseEdges` method of the `DiGraph` class reverses the direction of each edge of a `DiGraph` instance. Other than the bound `DiGraph` instance, there are no input parameters. The method works on the DiGraph instance in place, so destroys its input contents.

3.3.6.4   subgraph

The `subgraph` method of the `DiGraph` class creates a new graph from a selected set of vertices of an existing `DiGraph` object and those vertices' incident edges.

Syntax

```
G2 = G1.subgraph(vertrange[, vertrange2])
```

Description

The `subgraph` method of the `DiGraph` class creates a new `DiGraph` instance by selecting a set of vertices of an existing `DiGraph` instance and all the edges incident to those vertices. The required input argument `vertrange` specifies a range (*i.e.*, a consecutive set of vertex indices) of vertices (and out-edges incident to them) to be used for the new `DiGraph` instance. The optional input argument `vertrange2` specifies another range of vertices (and in-edges incident to them) to be used as the in-vertices for the new `DiGraph` instance. If `vertrange2` is not specified, `vertrange` designates both out- and in-vertices.

Example

The code below creates a new `DiGraph` from the first half of the vertices in `G` and edges whose source and destination are both one of those vertices.

```
G2 = G.subgraph(kdt.DiGraph.range(G.nvert()/2))
```

The code below creates a new `DiGraph` with out-vertices of the first half of the vertices in `G`, in-vertices equal to the second half of the vertices in G, and edges whose source is in the first set and destination is in the second.

```
nvG = G.nvert()
G3 = G.subgraph(kdt.DiGraph.range(nvG/2), kdt.DiGraph.range(nvG/2,
nvG))
```

3.3.6.5   degree / sum / max / min

The `degree`, `sum`, `max`, and `min` methods of the `DiGraph` class calculate, respectively, the degree (count), sum, maximum, and minimum edge-weight of the edges of each vertex of a `DiGraph` instance.

Syntax

```
inoutdegs = G.degree([dir=kdt.InOut])
inoutsums = G.sum([dir=kdt.InOut])
inoutmaxs = G.max([dir=kdt.InOut])
inoutmins = G.min([dir=kdt.InOut])
```

Description

The sum, max, and min methods of the DiGraph class calculate, respectively, the sum, maximum, and minimum edge-weights of the edges of each vertex of a DiGraph instance, returning a ParVec object. The optional dir argument specifies whether the operation is performed on InOut (default), In, or Out edges.

Example

The code below calculates the sum of the edge-weights of the out-edges of the vertices of a DiGraph.

```
outmaxs = G.max(kdt.DiGraph.Out)
```

3.3.6.6   scale

The scale method of the DiGraph class multiplies the edge weights of each vertex of a DiGraph instance by the corresponding element of a vector of scale factors.

Syntax

```
G.scale(scaleV[, dir=kdt.Out])
```

Description

The scale method of the DiGraph class multiplies the edge weights of each vertex of a DiGraph instance by the corresponding element of a SpParVec vector of scale factors. The optional dir argument specifies whether the operation is performed on Out (default) or In edges.

Example

The code below normalizes the out-edge weights of each vertex of a DiGraph instance such that the sum of the edge-weights of each vertex is 1.0.

```
scalefac = kdt.ParVec.ones(G.nvert()) / G.sum()
G.scale(scalefac)
```

3.3.6.7   normalizeEdgeWeights

The normalizeEdgeWeights method of the DiGraph class normalizes the out-edge weights of each vertex V of a DiGraph instance such that each out-edge's weight is 1 / out-degree(V).

Syntax

```
G.normalizeEdgeWeights()
```

Description

The `normalizeEdgeWeights` method of the `DiGraph` class normalizes the out-edge weights of each vertex V of a `DiGraph` instance such that each out-edge's weight is 1 / out-degree(V). Besides the `DiGraph` instance there are no other arguments.

3.3.7 Advanced methods for DiGraph objects

The `DiGraph` class implements several advanced methods.

3.3.7.1 `neighbor`

The `neighbor` method of the `DiGraph` class calculates the neighbors of a set of starting vertices in a `DiGraph` instance.

Syntax

```
neighbors = G.neighbor(start[, nhop=1, sym=False])
```

Description

The `neighbor` method of the `DiGraph` class calculates, for a `ParVec` of input starting vertices, which vertices are neighbors; *i.e.*, connected by out-edges. The optional `nhop` argument determines how many hops from the starting vertices are used to calculate the neighbors; the default is 1. The optional expert argument `sym` denotes whether the graph is a symmetric graph, which if True enables the operation to run faster. The return value is a `ParVec` with the indices of neighboring vertices.

Example

If `start` is a Boolean `ParVec` of starting vertices, the call below will return all vertices connected via out-bound edges within one hop.

```
neighbors = G.neighbor(start)
```

3.3.7.2   `pathsHop`

The `pathsHop` method of the `DiGraph` class calculates the vertices that can be reached from a set of starting vertices in a `DiGraph` instance, also returning which of the start vertices has an edge to each new vertex. `pathsHop` is equivalent to one step in a breadth-first search.

Syntax

```
[fromV, toV] = G.pathsHop(start[, sym=False])
```

Description

For a `ParVec` of input starting vertices, the `pathsHop` method calculates which vertices can be reached across a single edge in the `DiGraph` instance and, for each reachable vertex, which starting vertex has an edge to it. The source vertices and the new vertices are returned in `ParVec` and Boolean

ParVec instances, respectively. The set of reachable vertices may include starting vertices. In the case of more than one start vertex having an edge to a reachable vertex, the highest-numbered vertex is selected. `pathsHop` can be used repeatedly to implement a breadth-first search. The optional expert argument `sym` denotes whether the graph is a symmetric graph, which if True enables the operation to run faster.

Example

If `start` is a Boolean `ParVec` of starting vertices, the code below finds all reachable vertices with the call, and then selects just the newly found vertices from the set of reachable vertices.

```
[fromV, toV] = G.pathsHop(start)
newV = toV & ~start
```

3.3.8   DiGraph

The `DiGraph` method creates a directed graph from the edges passed to it.

Syntax

```
G = kdt.DiGraph(source, dest, weight, nVertOut[, nVertIn])
G = kdt.DiGraph()
```

Description

The `DiGraph` method creates a `DiGraph` instance. The required input parameters `source` and `dest` are `ParVec` objects created by the program or by generators such as `genGraph500Edges`. The required input parameter `weight` can be a `ParVec` or a scalar. The required input parameter `nVertOut` is an integer defining the number of vertices that have out edges in the graph. The optional input parameter `nVertIn` is an integer defining the number of vertices that have in edges in the graph; if all vertices potentially have both in and out edges, `nVertIn` may be omitted. The output argument is a `DiGraph` object. The values of any duplicate edges (same source and destination) are summed in the creation of the `DiGraph` object. The `DiGraph` method is almost the converse of the `toParVec` method, with the difference that, since the `DiGraph` method sums duplicate edges, the output of `toParVec` may have fewer edges, though the same set of vertices.

The alternate form of calling the `DiGraph` method with no argument creates a `DiGraph` instance with an empty underlying graph object. This is useful for certain constructors, like `genGraph500Edges`, which populate the underlying graph object themselves.

`DiGraph` implements the functionality of Kernel 1 of the Graph500 benchmark.

Example

The code below creates a star graph with N vertices, with directed edges of weight 1 going only from vertex 0 to all vertices (including vertex 0).

```
source = kdt.ParVec.zeros(N)
dest = kdt.ParVec.range(N)
G = kdt.DiGraph(source, dest, 1, N)
```

3.3.9   Miscellaneous methods

The following methods exist in the KDT package but not in any class.

- `version`: returns the KDT version number as a string.
- `revision`: returns the SVN revision number (of the KDT release contents) as a string.
- `sendFeedback`: as described in section 1.6, sends feedback to the KDT development team.
- `master`: as described in section 3.6.4, can be used to restrict output to the master process.

3.4   Package and class structure

KDT is structured as shown in Table 1, with some methods omitted for brevity. The complete list of functions can be seen by executing (within IPython)

```
import kdt
dir(kdt)
dir(kdt.DiGraph)
dir(kdt.ParVec)
dir(kdt.SpParVec)
```

Entity	Name	Elements	Comments
Module	KDT	version, revision, sendFeedback, master	Methods unrelated to any specific data structure
Class	ParVec		Distributed parallel vector
		ParVec, [], =, +, -, += , -=, <>, >, findInds, abs, any, all, nnn, …	Methods
Class	SpParVec		Distributed parallel sparse vector
		SpParVec, [], =, +, -, += , -=, <>, >, findInds, abs, any, all, nnn, …	Methods
Class	DiGraph		Directed graph
		DiGraph, genGraph500Edges, load, fullyConnected	Constructors
		centrality, cluster	Algorithms
		bfsTree, isBfsTree, neighbors, pathsHop, toParVec, reverseEdges , subgraph	Graph primitives
		load	I/O
		[], nvert, nedge, degree, +, *, ones, set	General-purpose routines
		InOut, In, Out	Constants

Table 1. KDT library hierarchy

The KDT developers anticipate multiple implementations of the KDT interface, targeting distributed-parallel versions both memory-based and disk-based. Only the distributed in-memory version is implemented in v0.1.

3.5 Graph Generators

The `DiGraph` class includes the graph generators in this section. With v0.1, edges may be created as an `ParVec` object directly with a subsequent call to the `DiGraph` method, by the `load` or `UFget` method, by a constructor such as `fullyConnected`, or by using an application-specific method like `genGraph500Edges`.

3.5.1 `genGraph500Edges`

The `genGraph500Edges` function creates a graph following the specifications for the V1.1 Graph500 benchmark's input graph [Graph500]. The edges are inserted into the `DiGraph` object passed and represent an RMAT graph with specific values provided by the benchmark.

Syntax

```
time = G.genGraph500Edges(scale)
```

Description

The `genGraph500Graph` method creates an input graph as defined by the Graph500 benchmark. The required input parameter `scale` (logarithm base 2 of the number of desired vertices) defines the number of vertices. The edges are directed, though each edge has a twin going in the other direction because the specification requires the graph to be symmetric. Some vertices may have no edges incident to them. The time returned from `genGraph500Edges` includes the execution time of converting the edge vector to a graph but does not include the time to create the edge vector, which is exactly the time measured by Kernel 1 of the Graph500 benchmark.

Example

The following code will creates a `DiGraph` instance and inserts edges into it that match the Graph500 specification, of size `scale`.

```
G = kdt.DiGraph()
time = G.genGraph500Edges(scale)
```

3.5.2 `twoDTorus`

The `twoDTorus` method creates a `DiGraph` object reflecting the connectivity pattern of a 2D torus.

Syntax

```
G = kdt.twoDTorus(nnode)
```

Description

The `twoDTorus` method creates a `DiGraph` with the connectivity pattern of a 2D torus; *i.e.*, connections to its north, west, south, and east neighbors, where the neighbor may be wrapped around to the other side of the torus. The required input parameter `nnodes` defines the number of nodes along one dimension of the torus; the torus and the graph representing it will have `nnodes**2` vertices. The newly created `DiGraph` object is the return value from the method. A 2D torus has an

easily analyzed betweenness centrality value that can be useful for simple tests; specifically, each vertex of the `DiGraph` created by `twoDTorus(nnodes)` has an identical betweenness centrality value of 0.5 * 2**(3*`scale`*0.5) – 2**`scale` + 1 for an even `nnodes`.

Example

The following code creates a `DiGraph` instance `G` with `nnodes**2` vertices and inserts edges from every vertex to every other vertex, including itself.

```
G = kdt.DiGraph.twoDTorus(nnodes)
```

3.5.3 `fullyConnected`

The `fullyConnected` method creates a `DiGraph` object in which all vertices are directly connected to all other vertices. The edges are inserted into a newly created `DiGraph` object, which is the return value from the method.

Syntax

```
G = kdt.fullyConnected(nvert)
```

Description

The `fullyConnected` method creates a `DiGraph`. with all vertices having a directed edge to all vertices, including itself. The required input parameter `nvert` defines the number of vertices. The newly created `DiGraph` object is the return value from the method.

Example

The following code creates a `DiGraph` instance `G` with `nvert` vertices and inserts edges from every vertex to every other vertex, including itself.

```
G = kdt.DiGraph.fullyConnected(nvert)
```

3.6 I/O

The `DiGraph` class includes the I/O-realted methods in this section.

3.6.1 `load`: Reading a graph from a file

A Matrix Market file can be loaded directly into a `DiGraph` object with the standard `load` I/O operation.

Syntax

```
G = kdt.DiGraph.load(fname)
```

Description

The `load` method loads a file in the Coordinate Format of the Matrix Market Exchange Formats [MatrixMarket] directly into a `DiGraph` object.

Version 0.1

The source-vertex / destination-vertex information in the first two columns of the Matrix Market Exchange format is 1-based in the file, and is converted to 0-based in the load. With v0.1, the error-checking of a source-vertex or destination-vertex being out of bounds is not robust and can result in KDT aborting with Segmentation Faults or malloc errors.

Example

The following code will load the contents of the file `mymatrix.mtx` into a `DiGraph` instance named G.

```
G = kdt.DiGraph.load('mymatrix.mtx')
```

3.6.2 `save`: Writing a graph into a file

A `DiGraph` object can be saved directly into a Matrix Market file with the standard `save` I/O operation.

Syntax

```
G.save(fname)
```

Description

The `save` method save a a `DiGraph` object directly into a file in the Coordinate Format of the Matrix Market Exchange Formats [MatrixMarket].

Example

The following code will save the contents of a `DiGraph` instance named G into the file `mymatrix.mtx`.

```
G.save('mymatrix.mtx')
```

3.6.3 `UFget`: Fetching a file from the University of Florida Sparse Matrix Library and loading it as a DiGraph

The `UFget` method downloads a named Matrix Market file from the University of Florida Sparse Matrix Library ([link](#)) and loads it into a `DiGraph` instance.

Syntax

```
G.UFget(fname)
```

Description

The `UFget` method downloads a file from the UF Sparse Matrix Library, untars the contents, and loads the contained file (in the Coordinate Format of the Matrix Market Exchange Formats) directly into a `DiGraph` instance.

Note: See Errata #3 in section 5.7.

Example

The following code will load the contents of the file `Andrianov/ex3sta1.tar.gz` from the UF Sparse Matrix Library into a `DiGraph` instance named G.

```
G = kdt.DiGraph.UFget('Andrianov/ex3sta1')
```

3.6.4 `master`: Avoiding redundant print output

When running in parallel, each process will execute the Python code, including any `print` statements. For user output, this can lead to numerous copies of the output, often intermingled so as to make the output unintelligible. The `master` method is useful for avoiding this situation.

Note: Printing of `DiGraph`, `ParVec`, or `SpParVec` instances will, in general, require participation of all the active processes, and thus cannot be done solely on the master, and thus should not be done inside a `master` check. Doing so typically results in the program hanging.

Syntax

```
bool = kdt.master()
```

Description

The `master` method returns a Boolean result that is True in the master process of the underlying infrastructure and False in all other processes.

Example

The following code, taken from the `Graph500` script in KDT, restricts printing output just to the master process.

```
if kdt.master():
        print 'Graph500 benchmark run for scale = %2i' % scale
        print 'Kernel 1 time = %8.4f seconds' % K1elapsed
        print "\nKernel 2 BFS execution times"
        printstats(K2elapsed, "time", False)
```

3.7 Advanced Usage

KDT v0.1 supports only graphs with a single edge between any two vertices, which doesn't address nearly all graph types. While fully general graphs will have to wait for future releases, v0.1 does support (via built-in Python mechanisms) multiple graphs of different types of data. For instance, one potential KDT use case is to analyze data from protein-protein, DNA-protein, and other interactions. Data about a particular organism (*e.g., Shewanella*) could be collected in a single `DiGraph` instance which has other `DiGraph` instances as members, as portrayed by the following code.

```
shew = kdt.DiGraph()
shew.prot2prot = kdt.DiGraph.load('shewanella/protein_protein.mtx')
shew.DNA2prot = kdt.DiGraph.load('shewanella/DNA_protein.mtx')
d2p_bc = shew.DNA2prot.centrality('approxBC')
```

```
p2p_bc = shew.prot2prot.centrality('approxBC')
d2p_deg = shew.DNA2prot.degree()
p2p_deg = shew.prot2prot.degree()
```

In this example, each graph is still operated on independently, but where graph operations make scientific or mathematical sense, `DiGraph` graphs can be joined together.

4   Performance

KDT is intended to enable subject-matter experts who are not graph experts to solve large problems quickly, including both the development and execution phases of a program, and thus performance is a vital aspect of KDT. Its performance has not yet been fully characterized. Performance of specific methods, where available, is provided in their descriptions.

Today we have no means to execute KDT interactively in parallel, where "interactive" means having the full range of IPython's abilities (*e.g.,* breakpoints, stepping, variable inspection) at the user's disposal. Thus we consider interactive and parallel performance separately.

4.1   Interactive

Since interactive execution means serial execution and memory capacities of a single node for v0.1, users will likely limit the problem sizes they develop with interactively to get quick response times.

The Graph500 example in KDT (see `examples/Graph500.py`) is configurable for problem size and number of iterations. For scale=15 (equating to the number of graph vertices being 2**15 or 32K), the Graph500 kernel2 itself runs for a single starting vertex in about 0.3 seconds on a modern x86 core. The entire processing of the graph, including validation of the resulting BFS tree, takes about 3 seconds per iteration (starting vertex).

Betweenness centrality (*e.g.*, the class `CentralityTests` in file `test/TestDiGraph.py`) in KDT is also configurable for problem size and sample rate (for approximate BC). Using the `twoDTorus` graph generator with the number of torus nodes equal to 32 (equating to the number of graph vertices being 1024), exact BC runs in about 8 seconds on a modern x86 core, and for this scale approximate BC is only slightly faster. Note that a 2D torus is a poor performance case for BC, as one factor in BC's computational complexity is the diameter of the graph, which for a 2D torus is the square root of the number of graph vertices, whereas typical RMAT graphs have a diameter more like the log of the number of graph vertices.

4.2   Parallel

The parallel testing of KDT has occurred on three systems:

- Neumann, a 32-core cluster at UCSB

- Triton, a medium-sized 256-node cluster at the San Diego Supercomputer Center (configuration)

- Hopper II, a large 6392-node, 24 cores-per-node Cray XE6 system at the National Energy Research Supercomputer Center ([configuration](#))

The performance characterization of KDT at large scale is still in process. The following performance has already been demonstrated.

Much of the early performance work with KDT has focused on the Graph500 benchmark. In Table 1, initial Graph500 performance numbers from the Triton cluster at the San Diego Supercomputer Center are provided. The units are millions of traversed-edges per second (TEPS); the first number is for each data element being a 64-bit double-precision value; the second is for each data element being a Boolean value; the performance for Boolean values is often a factor of 5-10 faster than 64-bit double-precision values. On 256 cores, the KDT implementation of Graph500 performs at over 1 GTEPS.

Scale	20	23
Number of cores		
16	- / 200.5	25.9 / 171.4
64	- / 518.5	
256	- / 485.1	- / 1,133.2

Table 1. Graph500 performance

Betweenness centrality is a much more compute-intensive applet than the Graph500, so the problem sizes run so far are much smaller. On Neumann a 2D torus with 128 torus nodes (equating to 16K graph vertices) runs exact BC in 1163 seconds and approximate BC (sample rate of 0.05) in 884 seconds on 16 cores.

5 Practicalities

5.1 Downloading

To get the latest released package, go to `http://kdt.sourceforge.net/download.html` and download it.

5.2 System Requirements

We recommend Python 2.4 or newer. KDT has been tested primarily with OpenMPI 1.4 and IPython 0.8.4.

5.3 Installation and Build

The information in this section is repeated verbatim from the `README.txt` file in the base directory of the KDT package. In case of any discrepancies between this text and `README.txt`, `README.txt` should prevail.

KDT is distributed as a Python `distutils` package. It requires the MPI compiler to be specified in the `$CC` and `$CXX` environment variables. For example, in `bash`:

```
export CC=mpicxx
export CXX=mpicxx
```

The build and installation is performed by the standard `distutils setup` command:

```
$ python setup.py build
$ sudo python setup.py install
```

If the build step fails fails see the "Choosing a compiler" section below.

KDT makes use of some features from C++ TR1. If your compiler does not support TR1 then the free Boost C++ library (http://www.boost.org/) supplies the required headers as well. Make sure `boost/` is in your include path. If it is not, you can append the include path with the `-I` switch to the `setup.py` script. For example, if you installed Boost in `/home/username/include/boost`:

```
$ python setup.py build -I/home/username/include
```

The MPI library must be compiled with `-fPIC`. Python modules must be compiled with `-fPIC`, and that includes all libraries that get linked into the module, which includes the MPI library in the case of KDT. If your MPI was not compiled with `-fPIC` then the link step will fail. If this happens, contact your system administrator.

5.3.1   Choosing a Compiler

KDT consists of pure Python classes and a C++ extension. This C++ extension, called pyCombBLAS, is MPI code and must be compiled as such. However, it is also a Python module, so must be compiled with a compiler that is compatible with your Python installation. By default, `distutils` (i.e. `setup.py`) will use the same compiler with which Python was compiled. That was probably not an MPI compiler, so the proper compiler must be specified to `distutils` via the `CC` and `CXX` environment variables.

Note that your system may give you a choice between GNU and Portland Group (PGI) compilers. These are not fully binary compatible with each other, so you must use the same one that your Python was compiled with. Note that `mpicxx` is often just a wrapper around GNU or PGI compilers.

5.3.2   System Libraries

If you chose to use a non-default MPI compiler then be aware that the runtime may link to the default MPI libraries anyway. This is probably not what you want. The result may be something like this:

```
ImportError: ./kdt/_pyCombBLAS.so: undefined symbol: _ZN3MPI3Win4FreeEv
```

The solution is to set your LD_LIBRARY environment variable such that the MPI library you compiled with appears before the incorrect defaults.

5.3.3   Building without Distutils

If the `setup.py` script fails for you, you can build the C++ extension manually. The `kdt/pyCombBLAS` directory contains a Makefile (named `Makefile-dist`; rename as necessary) which you can tune for your system. The important things to change are:

- `COMPILER` - compiler to use.

- `INCADD` - include your Python build directory as well as your Boost installation if you need it.

- `OPT` - any flags you wish to change.

Once pyCombBLAS is built, you may manually install the `kdt/` directory in your systemwide Python `site-packages` directory, or simply set your `PYTHONPATH` environment variable to point to the parent directory of `kdt/` (*i.e.*, the base of the distribution).

5.4   Tests and Examples

The KDT package includes `test/` and `example/` directories at the base level of the distribution, and in particular the `example/Graph500.py` script runs the Graph500 benchmark, which may be a good way to exercise KDT after installation. Because these files embody the use of KDT functionality, not the core functionality itself, they are not installed in the standard `site-packages` directory with the core functionality. A user who downloads KDT for her own use may just leave them in the directory where the package was unzipped/untarred, or may put them in a system-wide directory for others' use as well. For the rest of this section the `test/` and `example/` directories are assumed to reside in a directory pointed to by the `KDTINSTALL` enviroment variable.

The `test/` directory contains three files: `TestParVec.py`, `TestSpParVec.py`, and `TestDiGraph.py`, which test the classes with the corresponding names. The current failures are noted in section 5.7 Errata.

The `examples/` directory contains the `Graph500.py` script, which implements the Graph500 benchmark specification, including all validation steps.

5.5   Execution with Python or IPython

This section provides details about how to execute KDT scripts via IPython and Python.

5.5.1   IPython

KDT has been used interactively only with IPython, namely version 0.8.4, on a single core.   The command-line and IPython commands to invoke it follow (note the same information appears in section 2).

```
[sam@neumann ~]$ PYTHONPATH="$PYTHONPATH:$KDTINSTALL/examples"
[sam@neumann ~]$ ipython
Python 2.4.3 (#1, Nov 11 2010, 13:30:19)
Type "copyright", "credits" or "license" for more information.
```

```
IPython 0.8.4 -- An enhanced Interactive Python.
?          -> Introduction and overview of IPython's features.
%quickref -> Quick reference.
help       -> Python's own help system.
object?   -> Details about 'object'. ?object also works, ?? prints
more.
```

In [1]: **import Graph500**
Activating auto-logging. Current session state plus future input
saved.
Filename   : .KDT_log   #This output is from the startup of the IPython logstart
Mode   : over   # mechanism used for the sendFeedback function
Output logging : False   # from Section 1.6
Raw input log : False   #   " "
Timestamping : False   #   " "
State   : active   #   " "

```
Generating a Graph500 RMAT graph with 2^15 vertices...
Duplicates removed (or summed): 78590 and self-loops removed: 0
Generation took 0.371470s.
iteration 1: start=9171, BFS took 0.030166s, verification took
2.803589s and succeeded, TEPS=29,545,933
iteration 2: start=12434, BFS took 0.030009s, verification took
2.762023s and succeeded, TEPS=29,700,392
iteration 3: start=20292, BFS took 0.029619s, verification took
2.818182s and succeeded, TEPS=30,091,517
iteration 4: start=29823, BFS took 0.029394s, verification took
2.728882s and succeeded, TEPS=30,321,679
      [...]
iteration 63: start=14394, BFS took 0.030498s, verification took
2.766191s and succeeded, TEPS=29,224,184
iteration 64: start=7008, BFS took 0.031285s, verification took
2.771903s and succeeded, TEPS=28,489,226
Graph500 benchmark run for scale = 15
Kernel 1 time =    0.3715 seconds

Kernel 2 BFS execution times
            min_time: 2.93109416961669922e-02
  firstquartile_time: 3.01442742347717285e-02
         median_time: 3.08154821395874023e-02
  thirdquartile_time: 3.10997962951660156e-02
            max_time: 3.19378376007080078e-02
           mean_time: 3.06311249732971191e-02
         stddev_time: 6.26722467712359749e-04

Kernel 2 number of edges traversed
           min_nedge: 8.91280000000000000e+05
  firstquartile_nedge: 8.91280000000000000e+05
        median_nedge: 8.91280000000000000e+05
  thirdquartile_nedge: 8.91280000000000000e+05
```

```
        max_nedge: 8.91280000000000000e+05
       mean_nedge: 8.91280000000000000e+05
     stddev_nedge: 0.00000000000000000e+00

Kernel 2 TEPS
           min_TEPS: 2.79067108782669082e+07
 firstquartile_TEPS: 2.86587086147099845e+07
        median_TEPS: 2.89231245181400143e+07
 thirdquartile_TEPS: 2.95671458547492772e+07
           max_TEPS: 3.04077572545734048e+07
 harmonic_mean_TEPS: 2.90972009933353513e+07
harmonic_stddev_TEPS: 1.99059058009882137e+04

In [2]:
```

5.5.2 Python, not via a job manager

KDT has been used in parallel only with the Python language processor, namely Python 2.4.3. The MPI used was OpenMPI 1.4. The Linux version was 2.6.18.

Note: The current Combinatorial BLAS, on which KDT is based, only supports core counts that are perfect squares, meaning that the argument to the `mpirun -n` command must be a perfect square.

For parallel execution with Python and MPI,you can use the following command-line.

```
 [sam@neumann test]$ mpirun -n 4 python $KDTINSTALL/examples/Graph500.py
Generating a Graph500 RMAT graph with 2^15 vertices...
Duplicates removed (or summed): 78883 and self-loops removed: 0
Generation took 0.080711s.
iteration 1: start=12627, BFS took 0.011984s, verification took
0.619967s and succeeded, TEPS=74,322,165
iteration 2: start=20502, BFS took 0.011391s, verification took
0.601116s and succeeded, TEPS=78,192,512
iteration 3: start=8839, BFS took 0.011701s, verification took
0.616723s and succeeded, TEPS=76,119,720
iteration 4: start=1999, BFS took 0.011484s, verification took
0.612136s and succeeded, TEPS=77,559,400
iteration 5: start=71, BFS took 0.011390s, verification took 0.619576s
and succeeded, TEPS=78,199,059
     […]
iteration 62: start=32066, BFS took 0.011214s, verification took
0.605297s and succeeded, TEPS=79,427,725
iteration 63: start=27903, BFS took 0.011867s, verification took
0.608496s and succeeded, TEPS=75,055,323
iteration 64: start=16201, BFS took 0.011442s, verification took
0.609218s and succeeded, TEPS=77,843,838
Graph500 benchmark run for scale = 15
Kernel 1 time =   0.0807 seconds
```

```
Kernel 2 BFS execution times
            min_time: 1.09930038452148438e-02
  firstquartile_time: 1.14405155181884766e-02
         median_time: 1.16879940032958984e-02
  thirdquartile_time: 1.18589401245117188e-02
            max_time: 1.21469497680664062e-02
           mean_time: 1.16281397640705109e-02
         stddev_time: 2.91774617905905386e-04


Kernel 2 number of edges traversed
            min_nedge: 8.90685000000000000e+05
  firstquartile_nedge: 8.90685000000000000e+05
         median_nedge: 8.90685000000000000e+05
  thirdquartile_nedge: 8.90685000000000000e+05
            max_nedge: 8.90685000000000000e+05
           mean_nedge: 8.90685000000000000e+05
         stddev_nedge: 0.00000000000000000e+00


Kernel 2 TEPS
            min_TEPS: 7.33258156991442293e+07
  firstquartile_TEPS: 7.51066276284680367e+07
         median_TEPS: 7.62052171539593935e+07
  thirdquartile_TEPS: 7.78535725269949883e+07
            max_TEPS: 8.10228953379023224e+07
  harmonic_mean_TEPS: 7.65973765427299738e+07
harmonic_stddev_TEPS: 6.64094336990017182e+04
[sam@neumann test]$
```

5.5.1 Python, via a job manager

Many large HPC systems require programs using a large number of cores to be submitted via a job management system. The script below works on SDSC's Triton cluster, using the TORQUE (formerly PBS) job management system.

```
$ qsub testscriptRun.sh
```

where `testscriptRun.sh` looks like

```
#!/bin/bash
#PBS -q batch
#PBS -N pyCombBLAS
#PBS -l nodes=72:ppn=8
#PBS -l walltime=1:20:00
#PBS -o out.testscript23-576-b
#PBS -e err.testscript23-576-b
#PBS -V
#PBS -M alugowski@gmail.com
#PBS -m abe
```

```
#PBS -A alugowski-ucsb

export LD_LIBRARY_PATH=/opt/openmpi/gnu/mx/lib:$LD_LIBRARY_PATH

cd ~/trunk/kdt/pyCombBLAS
/opt/openmpi/gnu/mx/bin/mpirun -v -machinefile $PBS_NODEFILE -np 576
python ./testscript.py -1 /home/alugowski-ucsb/matrices/rmat23.mtx
```

5.6 Debugging

The KDT developers have developed all their Python code interactively on a single core with IPython and then executed it in parallel with Python. When parallel execution encounters bugs that don't appear serially, the primary debugging tool has been printing, either via Python's `print` statement or via the `ParVec` and `SpParVec` `printAll` methods.

5.7 Errata

The following bugs or anomalies are known to exist in v0.1:

1. The definition of the distinction between nonnull and nonzero elements in `DiGraph` and `SpParVec` instances is not completely coherent. The most glaring instance of this is `SpParVec` instances that represent indices, in which zero is a valid nonnull element. Because there is not a sparse `Find` method exposed from the Combinatorial BLAS, KDT converts a `SpParVec` instance to a `ParVec` instance before calling the dense `Find` from the Combinatorial BLAS. This conversion obliterates the distinction between a nonnull element whose value is zero and a null element.

2. The `ParVec` and `SpParVec` `print` methods will sometimes cause the message
   `ADIOI_UFS_OPEN (line 83): **io Too many open filesMPI_FILE_CLOSE` `(line 51): **iobadfh`
   `Problem reading binary input file`
   to be printed on stderr and stdout. This appears to effect only the output itself and not the correct functioning of the rest of the program.

3. The `DiGraph` `UFget` method does not act properly on files that have the Matrix Market keyword `symmetric` in their first (comment) line, so the resulting graphs only have half as many edges as expected. The `Pajek/dictionary28` file illustrates this problem. Such a file can still be loaded properly by the following code:
   ```
   G = kdt.UFget('Pajek/dictionary28')
   G2 = G.copy()
   G2.reverseEdges()
   G = G+G2
   ```

4. In addition, a handful of unit tests (see files `TestParVec.py`, `TestSpParVec.py`, and `TestDiGraph.py` in directory `test`) fail:

a. Two tests expose that KDT does not always collapse multiple edges between the same source and destination to a single edge; `TestDiGraph:test_load` and `TestDiGraph:test_DiGraph_duplicates`.

b. `test_UFget_simple` exposes the issue noted in item #3 above

c. `TesteSpParVec:test_indexing_LHS_SpParVec_booleanParVec_scalar` exposes an issue that indexing of a `SpParVec` by a Boolean `ParVec` key and a scalar value does not always produce the right numerical result.

Appendix A. Implementing Graph Algorithms with the Combinatorial BLAS

The Combinatorial BLAS [Buluc] is described by its authors as:

> We describe the Parallel Combinatorial BLAS, which consists of a small but powerful set of linear algebra primitives specifically targeting graph and data mining applications. We provide an extendible library interface and some guiding principles for future development. The library is evaluated using two important graph algorithms, in terms of both performance and ease-of-use. The scalability and raw performance of the example applications, using the combinatorial BLAS, are unprecedented on distributed memory clusters.

It provides C++ interfaces that are appropriate for many HPC users, but no interface from the high-level productivity languages such as Python and the M language of MATLAB™. KDT provides such an interface via Python, but the current KDT exposes just a small fraction of the power of the Combinatorial BLAS. This section describes the implementation of graph algorithms with the Combinatorial BLAS' primitives, and describes a few of the Combinatorial BLAS primitives whose full power is not yet exposed. The Combinatorial BLAS' basic structure is a sparse matrix, which KDT uses with From vertices as rows and To vertices as columns. The Combinatorial BLAS' C++ interfaces are wrapped into Python as the `pyCombBLAS` module, with classes `pyDenseParVec`, `pySpParVec`, and `pySpParMat` that correspond directly with KDT's `ParVec`, `SpParVec`, and `DiGraph` classes.

This information is provided so that KDT users can understand the generality of what could be implemented eventually, and so that heroic KDT users can implement their own extensions to KDT by using the `pyCombBLAS` directly.

A.1 Implementing Breadth-first Search with the Combinatorial BLAS

One of the core algorithms implemented in KDT v0.1 is creating a breadth-first search tree from a graph and a root vertex. The code for this is found in `DiGraph.bfsTree`.

The abstract BFS algorithm starts with a "fringe" or "frontier" of vertices that has been first reached in the prior step of the algorithm, and in each step calculates all as-yet-unreached vertices that are reachable from the fringe and makes those vertices the new fringe. In sparse-matrix terms, the graph is the dual of the adjacency matrix, with destination vertices corresponding to rows and source vertices corresponding to columns as shown below. A standard sparse-matrix/vector multiplication multiplies corresponding elements of a row of the matrix and the vector and then adds those products. The operation needed for BFS is similar but different. We still want to identify positions where both the matrix and the vector have nonnulls, but instead of a multiplication the result just needs to be the position of the nonnull in the row/column (denoting which fringe vertex is the parent of the new vertex in the BFS tree), and instead of addition of multiple nonnulls in the row/column we just need to select one of them (*i.e.*, if a new vertex is reachable from multiple vertices in the fringe, in general it doesn't matter which one of those vertices is denoted as the parent in the BFS tree). The Combinatorial BLAS and KDT draw on the rich theory of linear algebra on semirings [Gilbert, p. 28-31] to implement this operation. Using the same computational structure and communication pattern but replacing the

multiplication and addition of standard matrix-vector multiplication with selection (of the column of the matrix element) and maximum, BFS achieves the needed computation.

Let's consider the example shown in, with a graph and its dual adjacency matrix, which is transposed to work properly with matrix-vector multiplication.
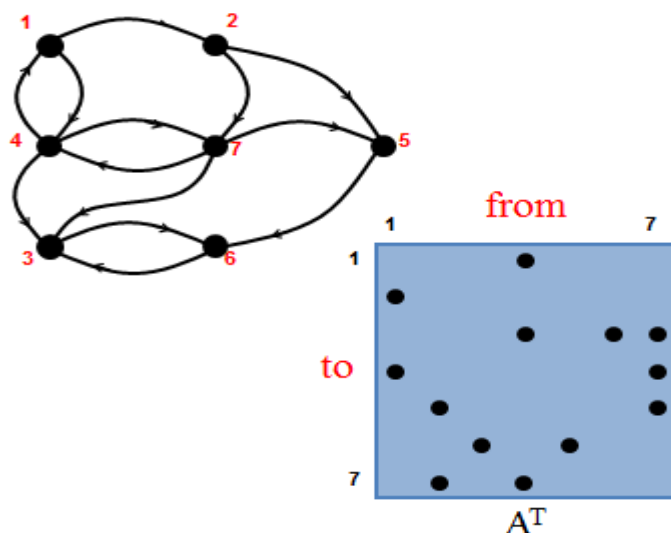

Figure 1. Illustration of bfsTree algorithm (initial state)

Let's make vertex 1 the root of the BFS tree and take the first step of the algorithm in . The fringe vector has a nonnull only in position 1. Since the "multiplication" of row 2 of the matrix with the fringe vector results in a nonzero in position 1, element 2 of the result matrix is set nonzero, with its value equal to the value of the nonzero in the vector "product". More precisely, the algorithm selects any position with nonzeros in both in the row and the fringe vector, with the result being the value of the fringe vector element (which is previously set to its position in the element). Applying this algorithm, vertices 2 and 4 are calculated as the next level of the BFS tree (with vertex 1 as each's parent), and the red edges are added to the BFS tree. (The algorithm truncates from the new fringe any vertices that have already been visited, as with the self-loop from vertex 1 to itself.)
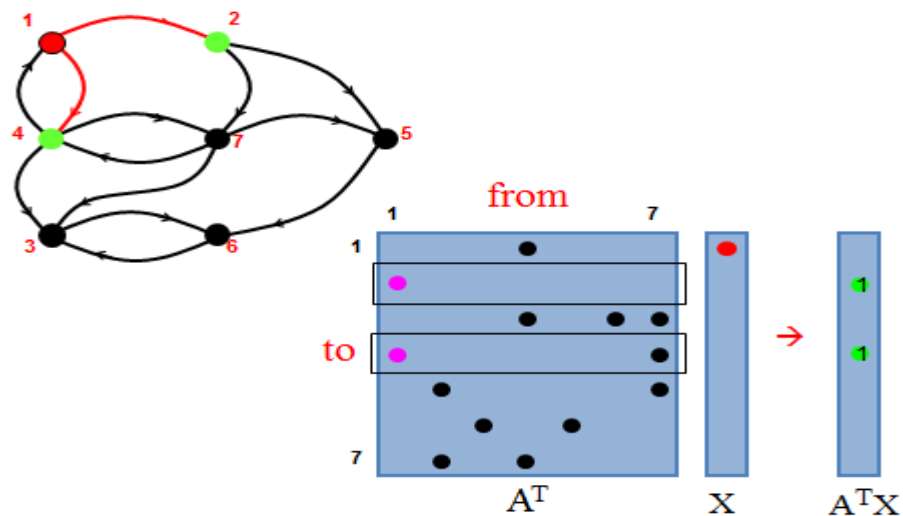
Figure 2. Illustration of bfsTree algorithm (after first iteration)

For the next step, the result column vector of the previous step is used as the fringe vector for the current step, with each nonnull element set to its position in the fringe. As illustrated in Figure 3Figure 3, the same algorithm is applied. The calculation of the edge to vertex 7 illustrates the maximum step (in place of the addition in standard matrix-vector multiplication).   Vertices 2 and 4 both have edges to vertex 7. The algorithm chooses the maximum-numbered vertex (vertex 4, denoted by the dark circle around the pink dot in the matrix) as the parent. Vertices 3, 5, and 7 are in the new fringe with parents of 4, 2, and 4, respectively. The resulting green edges are added to the BFS tree.
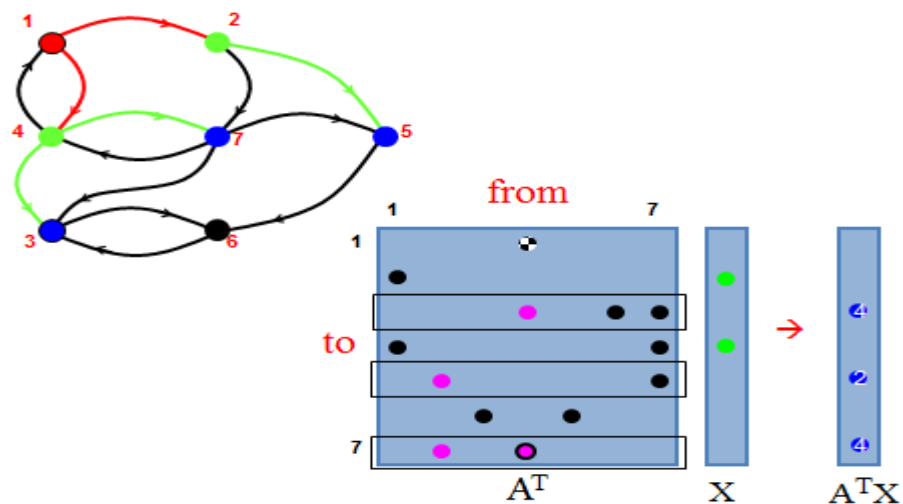

Figure 3. Illustration of bfsTree algorithm (after second iteration)

The next step of the algorithm for this graph is illustrated in Figure 4~~Figure 4~~. The only as-yet-unvisited vertex is vertex 6, which is reachable from either vertex 3 or 5. The tie-breaker (maximum vertex number) determines that vertex 6's parent is vertex 5, and the blue edge is added to the BFS tree.
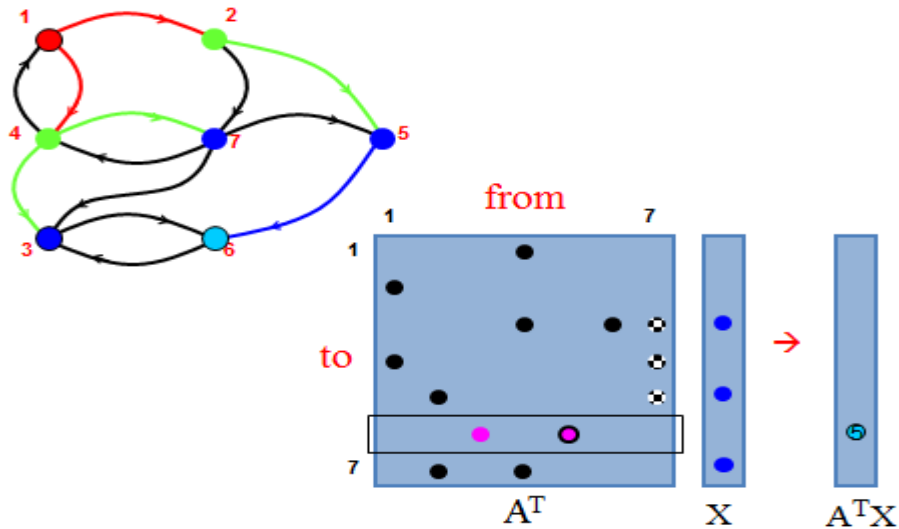

Figure 4. Illustration of bfsTree algorithm (after third iteration)

The last step of the algorithm uses the result vector of the third step, with only vertex 6 in the fringe, and detects no unreached vertices, at which point the algorithm terminates. The parent vector result is [0, 1, 1, 4, 1, 2, 3, 4]; note that the root vertex is its own parent and that in KDT vertices are numbered from 0, even though vertex 0 has no edges in this example.
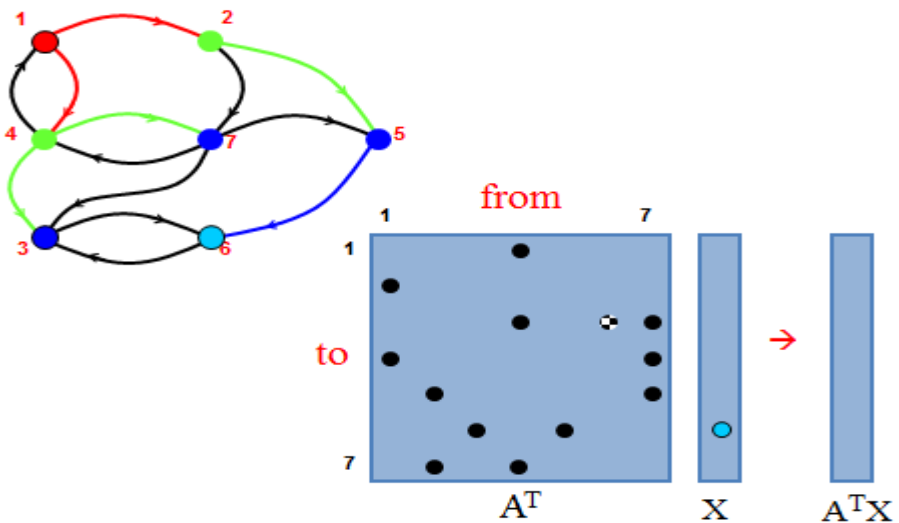

Figure 5. Illustration of bfsTree algorithm (after fourth iteration)

A.2 Powerful Operations in the Combinatorial BLAS

The Combinatorial BLAS has several operations which have powerful generality, of which sparse-matrix/vector multiplication on semirings, illustrated in the previous section, is one. The full complement of `pyCombBLAS` operations is specified in the file `pyCombBLAS.i` in the `kdt/pyCombBLAS` directory.

A.2.1 Sparse-matrix/vector multiplication on semirings

As illustrated in the previous section, replacing the element-wise multiplication and summation reduction of the standard sparse-matrix/vector multiplication operation with other semiring operations can perform powerful operations on the graph. The standard operation is available in `pyCombBLAS` as `SpMV_PlusTimes`. The operation used for the BFS tree calculation is available as `SpMV_SelMax`. Eventually this operation will be generalized in the style of the `Reduce` and `Apply` operations described below.

A.2.2 Applying an elemental operator

`pyCombBLAS` has a variety of elemental operations built-in (see the file `pyCombBLAS.i` for a complete list), both unary (*e.g.*, `abs`) and binary (*e.g.*, `greater`, or `fmod` with a constant). These operations can be applied to each element of a `pySpParMat` object by the `pySpParMat.Apply` method.

A.2.3 Applying an elemental operator with a column-specific value

`pyCombBLAS` can also apply these elemental operations with a second input that is column-specific. The `pySpParMat.ColWiseApply` method takes a second argument (a `pySpParVec` instance), which provides the second element for a binary operation. For instance, the `DiGraph.scale` method, which multiplies each out-/in-edge of a DiGraph by the corresponding element of a `SpParVec`, is implemented for the in-edge case as

```
self.spm.ColWiseApply(other.spv, pcb.multiplies())
```

A.2.4 Reducing the out-edges (rows) of a `DiGraph` (`pySpParMat`) with configurable operators

`pyCombBLAS` has a `pySpParMat.Reduce` method that is configurable with the same unary and binary operations. The `Reduce` method, in addition to its bound `pySpParMat` instance, accepts a dimension (columns or rows), a binary (reduction) function, and a unary function that's applied elementally before the binary function. For example, the row-wise sum of the absolute values of the nonnull elements could be implemented by

```
ret = self.spm.Reduce(pcb.pySpParMat.Row(),pcb.plus(), pcb.abs());
```

Appendix B.   Glossary

The following terms are used with their given meanings throughout this document.

Graph: A collection of **vertices** and **edges** connecting the vertices.

Edge vector: A vector of tuples, with each tuple containing the indices of the vertices upon which an edge is incident and the weight of the edge. Represented by classes specific to the type of graph, *e.g.* `DiEdgeV`.

Vertex vector: A vector of integers, each being the index of a vertex being referred to. Represented by the `DiEdgeV` class generic to all types of graphs.

DiGraph: A collection of vertices and directed edges connecting the vertices.

Appendix C.   References

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Appendix D.   Revision History

R0.01	December 16, 2010	Initial draft
R0.02	December 20, 2010	Revisions including comments from Aydin Buluc and Drew Waranis
R0.03	January 22, 2011	Bring current with code base (almost), documenting neighbors() and pathHop() as well as several other methods.
R0.04-05	February 12, 2011	Bring current with code base, documenting SpParVec class, many new ParVec and DiGraph methods, adding start-up and Implementing-graphs-on-CombBLAS information
R0.06	February 13, 2011	Finish Implementing-graphs-on-CombBLAS, add UFget, stipple cluster(), add DiGraph.save
R0.07	February 21, 2011	Add pageRank, toBool. Remove cluster. Fix several minor additions/deletions/errors.
R0.08	February 21, 2011	Add normalizeEdgeWeights, change names to spOnes/sprang, note that only serial work can be done inside a master() if-check.
R0.09	February 28, 2011	Clean up out-/in-edge directions and ParVec/SpParVec indexing modes.
R0.10	March 16, 2011	Add high-level note about SPMD execution; add note about UFget oddities; change SpParVec:__getitem__ to reflect CombBLAS changes; add sort/sorted/topK for ParVec/SpParVec; add mean/std for SpParVec. Include last directory clean-up before v0.1 release and errata.