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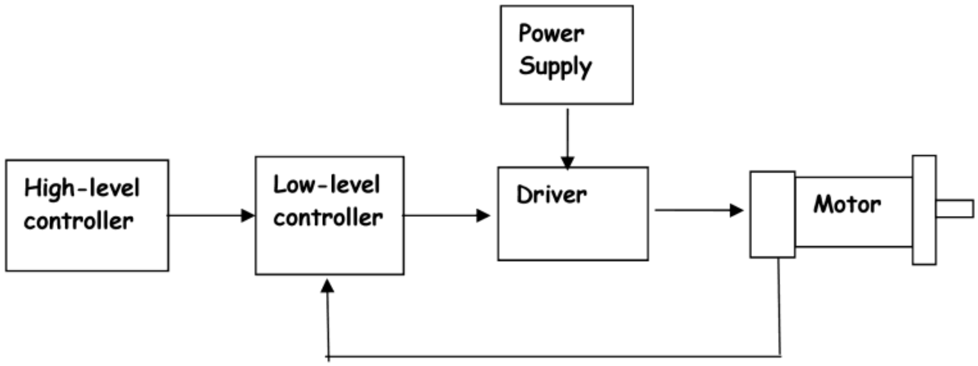
Wednesday, March 22, 2017 1:30 PM

Overview of motors and motion control

1. Elements of a motion-control system



2. Types of motors discussed here;

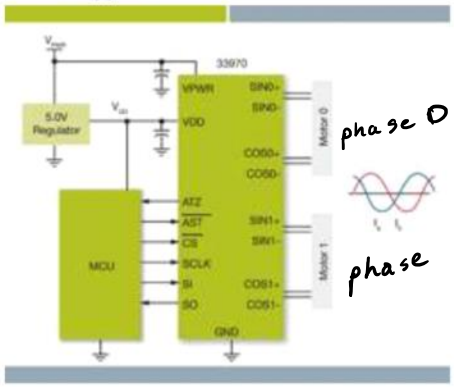
a little

Brushed, PM DC Motors	**Stepper Motors**	**Brushless DC Motors**	**Permanent magnet synchronous motors**	**AC induction motors**
Cheap, rugged, high-reliability	Cheap, rugged, high-reliability	No brushes, suitable for any environment		No brushes, suitable for any environment
Low Torque ripple	No brushes, suitable for any environment			
	Do not require feedback			
	At low speeds, provide up to 5x torque of brushed motor, 2x torque of brushless motor			
	Suffers from resonance and long settling times			

Page 2

Wednesday, March 22, 2017 1:30 PM

Motor Types

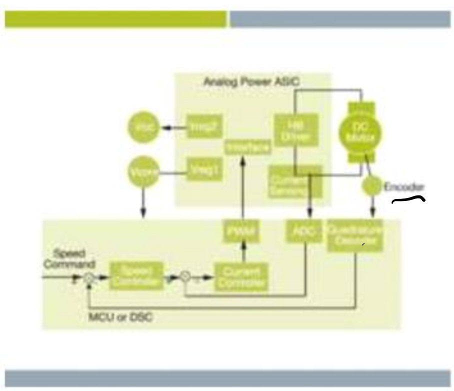


windings. → stator
permanent
→ rotor - magnet.
& open loop posit.

Stepper Motor

Stepper motors consist of a doubly salient structure (teeth on both the rotor and stator) and are used primarily in applications requiring precise position control which cannot justify the cost of expensive position feedback sensors. Stepper ~~motors are a fairly new motor type,~~ designed as a replacement for expensive servo motors. As current is switched from one set of stator coils to the next, the magnetic attraction between rotor and stator teeth results in the rotor moving by a small amount to the next stable position, or "step". Since it takes time for the current to be removed from one coil and established in the next (commutation), and since this process results in very little angular displacement of the rotor, stepper motors are typically limited to low speed position control applications.

mechanical.
• commutation



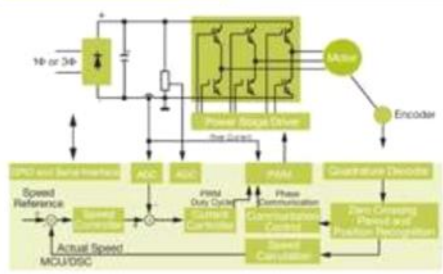
Brushed DC Motor

DC motors typically consist of a rotating armature coil inside of a stationary magnetic field which is generated by either a permanent magnet or a stationary electromagnet connected in series or parallel with the armature coil (the series connection often being referred to as a Universal Motor). The fact that these motors can be driven by DC voltages and currents makes them very attractive for low cost applications. To convert the armature current from DC into AC (which is required for rotation), a mechanical solution consisting of brushes and a commutator is employed. However, the arcing produced by the armature coils on the brush-commutator surface generates heat, wear, and EMI, and represents the most significant drawback of this motor type.

Page 3

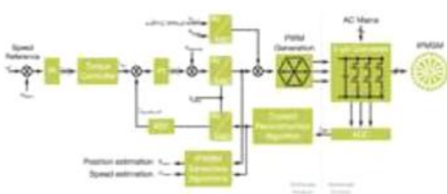
Wednesday, March 22, 2017 1:30 PM

• stepper but. better
• less pole 3 phases.



Brushless DC Motor (BLDC)

Although the name implies a DC motor, it is actually an AC motor. Concentrated coil windings on the stator work in conjunction with surface mounted magnets on the rotor to generate a nearly uniform flux density in the airgap. This permits the stator coils to be driven by a constant DC voltage (hence the name brushless DC), which is simply switched from one stator coil to the next. This process (referred to as COMMUTATION) must be electronically synchronized to the rotor angular position, and results in an AC voltage waveform which resembles a trapezoidal shape. Since there are no brushes or commutator, the BLDC motor does not exhibit the arcing problems associated with a brushed DC motor.

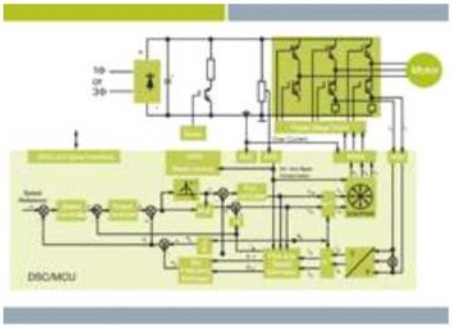


Permanent Magnet Synchronous Motor

Very similar to their BLDC cousins, PMSM motors are driven with sinusoidal voltages and currents which can achieve lower torque ripple than BLDC motors. A sinusoidal flux density exists in the airgap which has been traditionally generated by sinusoidally distributed multi-phase stator windings. However, newer designs achieve this flux density with concentrated stator windings and a modified rotor structure. Rotor magnets may be surface mounted for lowest torque ripple, or buried inside the rotor structure for increased saliency, which increases the reluctance torque of the machine. Field Oriented Control (FOC) is often employed to control these motors, which requires precise knowledge of the rotor angular position.

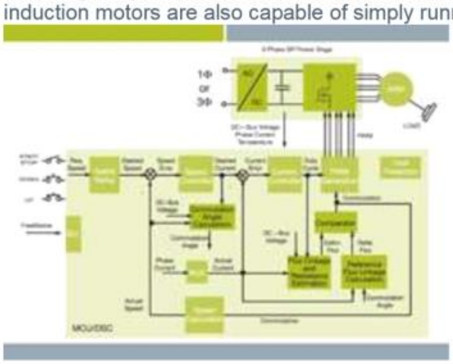
Page 4

Wednesday, March 22, 2017 1:30 PM



AC Induction Motor (ACIM)

Invented at the tail end of the nineteenth century, AC Induction Motors were the electric workhorse of the industrial revolution. The rotor consists either of multiphase windings, or the more popular copper or aluminum bars arranged in a structure that resembles a squirrel cage. Essentially a rotating transformer, currents are "induced" in the rotor conductors (secondary) from the stator coils (primary). The absence of permanent magnets makes AC induction motors extremely rugged and robust. Sinusoidal flux density is created in the airgap which is generated by sinusoidally distributed multi-phase stator windings. Field Oriented Control (FOC) is often employed to control these motors, which requires precise knowledge of the rotor angular position. However, due to the damping action provided by the moving rotor conductors, AC induction motors are also capable of simply running open loop from a multi-phase AC supply.



Switched Reluctance (SR) Motor

One of the oldest motor topologies, SR motors utilize concentrated stator windings and contain no permanent magnets. The rotor is a very simple construction of soft iron laminates with no coils. Since the rotor cannot generate its own magnetic field, there is no reactive torque (magnet to magnet) in an SR machine. Instead, both rotor and stator poles demonstrate salient protrusions (doubly salient design) where the flux length is made to vary as a function of angle. This results in the magnetic reluctance also changing as a function of angle, which gives rise to saliency torque. This is the only torque producing mechanism in an SR motor, which tends to result in high torque ripple. However, due to their simple design, SR motors are very economical to build, and are perhaps the most robust motor available. Unfortunately, the high torque ripple also gives rise to audible noise during operation, which has limited the application of SR motors in many applications.

Page 5

Wednesday, March 22, 2017 1:30 PM

Example: RC Servo Motor:

From http://www.princeton.edu/~mae412/TEXT/NTRAK2002/292-302.pdf

Servos have their own proprietary circuitry built inside the servo case. This circuitry consists of a pulse width comparator, which compares the incoming signal from the receiver with a one-shot timer whose period depends on the resistance of a potentiometer connected to the servo's drive shaft. This feedback is what provides the stability for the control circuitry. The difference between the control signal and the feedback signal is the error signal. This error signal is used to control a flip-flop that toggles the direction the current flows through the motor. The outputs of the flip-flop drive an HBridge circuit that handles the high current going through the motor.

RC servo-motor control, see: http://en.wikipedia.org/wiki/Servo_control
The RC motor rotation is determined by the duration of a pulse that is applied to the input wire (generally white or orange). This is a form of pulse width modulation. However, the servo position is not defined by the PWM duty cycle but only by the duration of the pulse. The servo expects to see a pulse every 20 ms. The length of the pulse will determine how far the motor turns with a typical range is from 1 ms to 2 ms.. For example, a 1.5 ms pulse will make the motor turn to the 90 degree position (neutral position).

Page 6

Wednesday, March 22, 2017 1:30 PM

Brushed DC Motor Basics:

Overview of Brushed DC motor systems (images from Siemens.com):

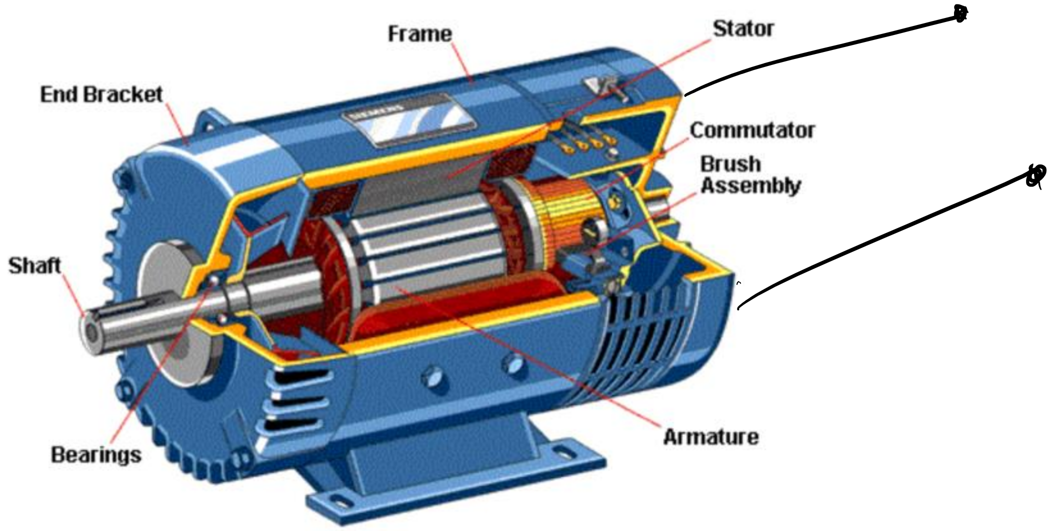
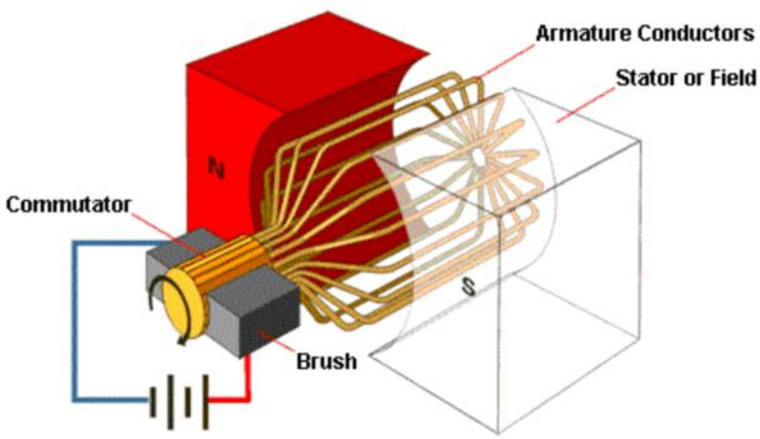


Figure 1: CAD Drawing of DC motor with cut-away

Main components:

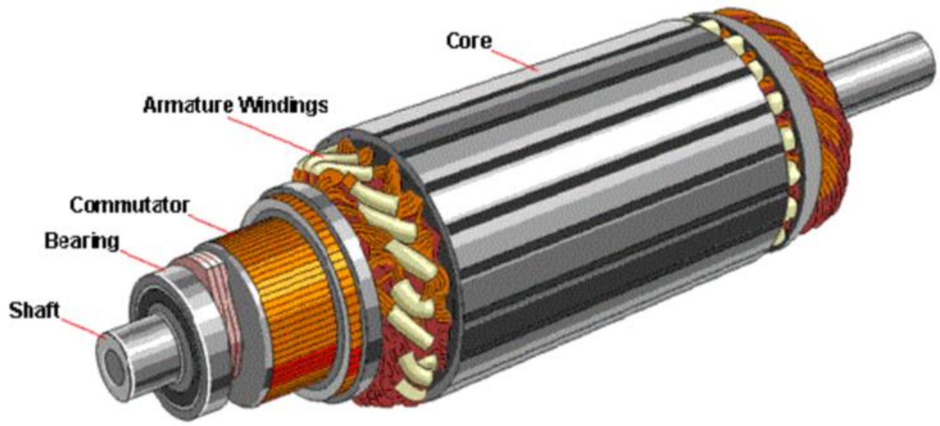
1) Armature (conductors) – Attached to the output shaft - rotor.
2) Stator (Field) – Attached to the frame
3) Commutator and Brushes – Connect armature to the input voltage source

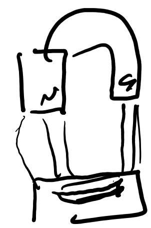


Page 7

Wednesday, March 22, 2017 1:30 PM

Detailed view of the Armature:

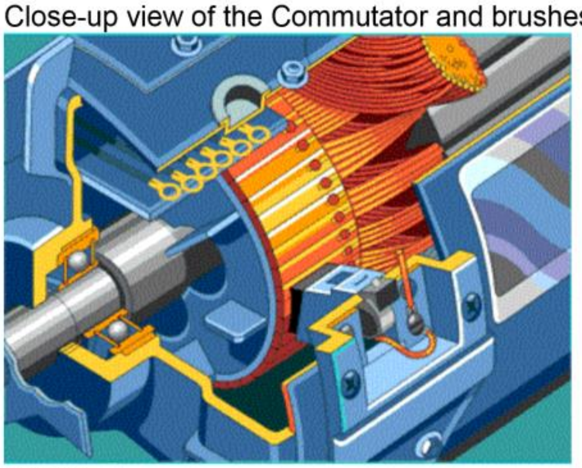




Notes:

1) Windings – coils of conductors,
2) Pass through the core – Made of Iron to conduct magnetic field,
3) attached to the shaft (mounted in bearings),
4) conductors directly connect to commutator, also connected to shaft.

Close-up view of the Commutator and brushes



<u>Notes:</u>

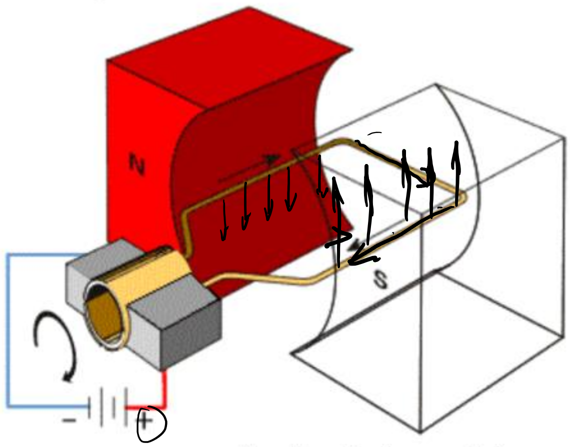
1) Commutator – bands of copper
2) Brushes – typically carbon, spring loaded
3) Sliding connection (often called a slip ring).

Page 8

Wednesday, March 22, 2017 1:30 PM

How the Motor Works:

Part I: Torque

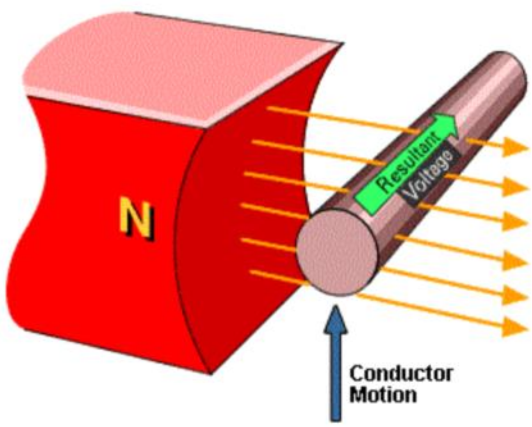


$\mathbf{F} = i\mathbf{L} \times \mathbf{B}$, $\mathbf{t} = \underline{\mathbf{r}} \times \mathbf{F}$, $\underline{\mathbf{t}} = \mathbf{r} \times i\mathbf{L} \times \mathbf{B} = k\underline{i}$ $= r * i * L * B$

$\underbrace{r * L * B}_{\underline{K_t}} \; i$

$K_t \; i$

Part II: Back emf



$v = \omega \times r$

From Lenz's law and Faraday's law of induction:

$V_{(\text{back emf})} = BLv$ ← velocity $= B\frac{L}{r}\omega = \underline{K_b\,\omega}$

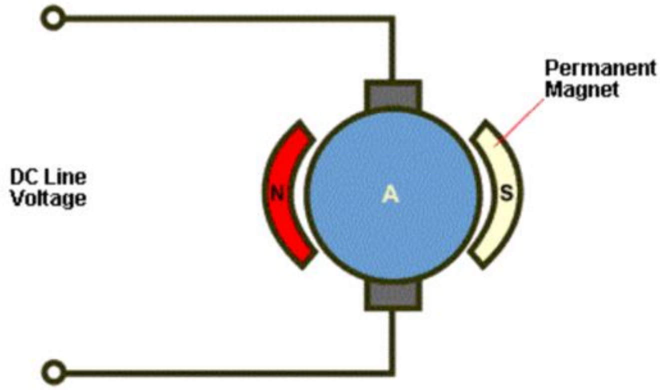
↑ voltage

Page 9

Wednesday, March 22, 2017 1:30 PM

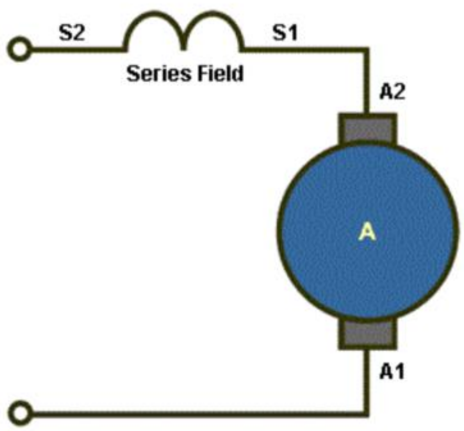
Types of brushed DC Motors:

1. PM DC motors



Use Permanent magnets to apply flux.
Maximum torque limited

2. Series Wound DC motors

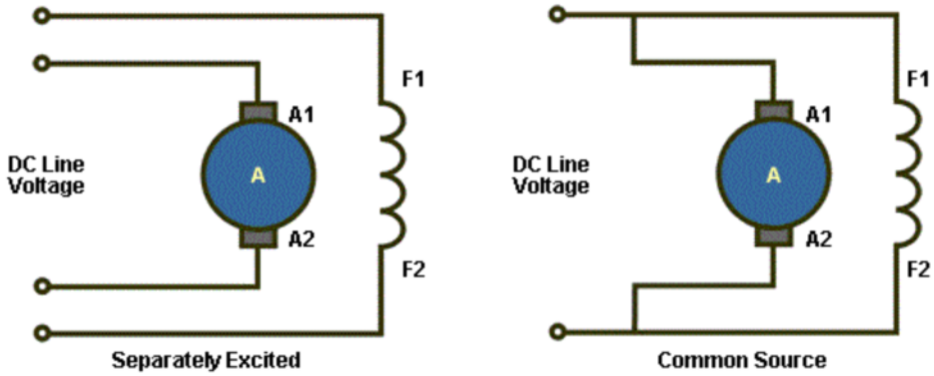


Stator is a field winding in series with the Armature.
Very large starting toque, difficult to control speed over varying loads, can operate too fast under no-load speeds.

Page 10

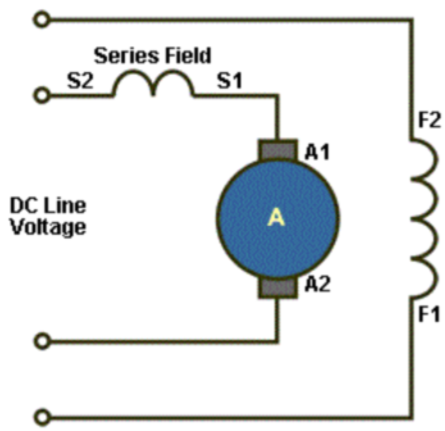
Wednesday, March 22, 2017 1:30 PM

3. Shunt Wound DC Motors:



The stator is made of field windings in parallel with the armature
Solves the problems associated with series wound motors
Separate voltages can be applied to the stator and armature allowing more control – good for regenerative drives.

4. Compound DC Motors:

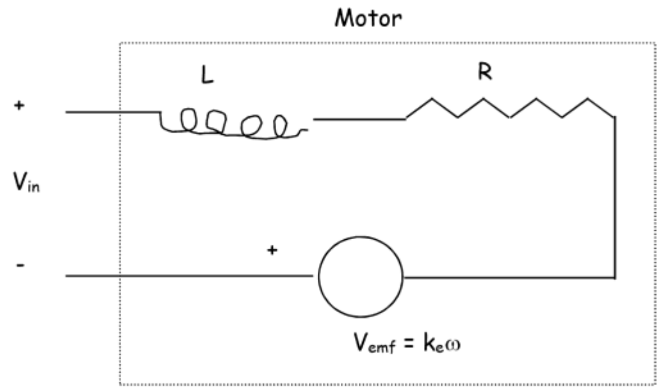


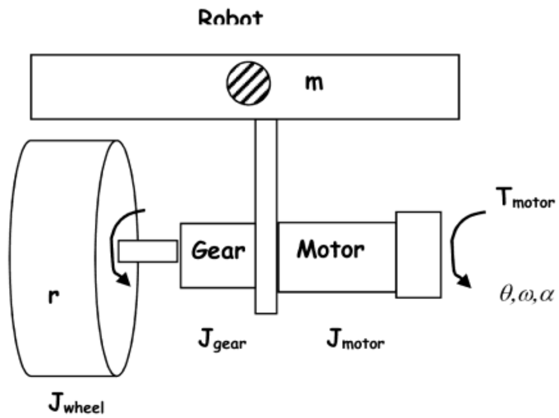
The stator is made of a combination of series and parallel windings.

Page 11

Wednesday, March 22, 2017 1:30 PM

Simplified Model of a DC PM Motor:





Model is: (details shown below)

$$Li + Ri + k_e\dot{\theta} = V_{in}$$
$$J\ddot{\theta} + C\dot{\theta} - k_t i = -T_{load}$$

Ignoring higher-order effects:

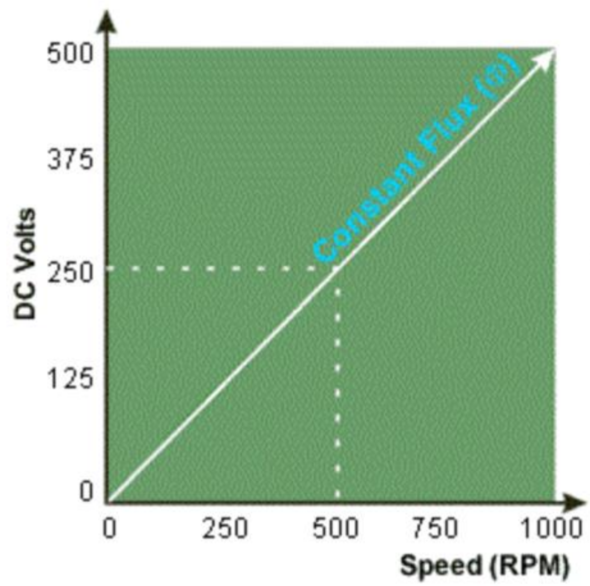
$$Ri + k_e\dot{\theta} = V_{in}$$
$$k_t i = T$$

Page 12

Wednesday, March 22, 2017 1:30 PM

Sample Motor (selected from www.clickautomation.com):
PM DC motor with gearhead:

<u>Speed:</u>



$$\text{Speed}(n) = \frac{V_a - (I_a R_a)}{Kt\Phi}$$

or

$$\text{Speed}(n) \approx \frac{\text{CEMF}}{\Phi}$$

$V_a = i_a{*}R_a + K_e{*}\text{omega}$

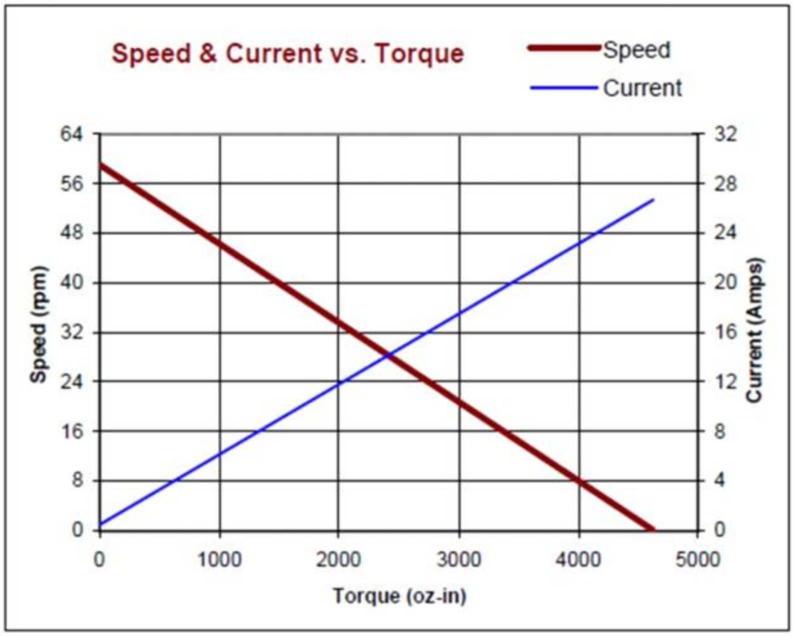
Assumptions:

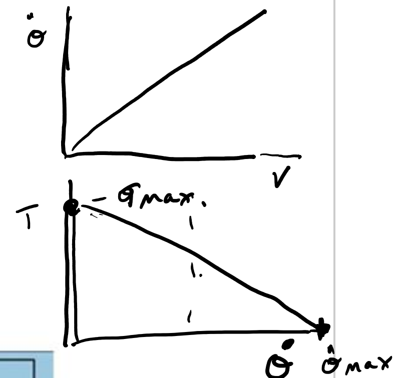
Constant magnetic flux, constant current

Page 13

Wednesday, March 22, 2017 1:30 PM

Torque:



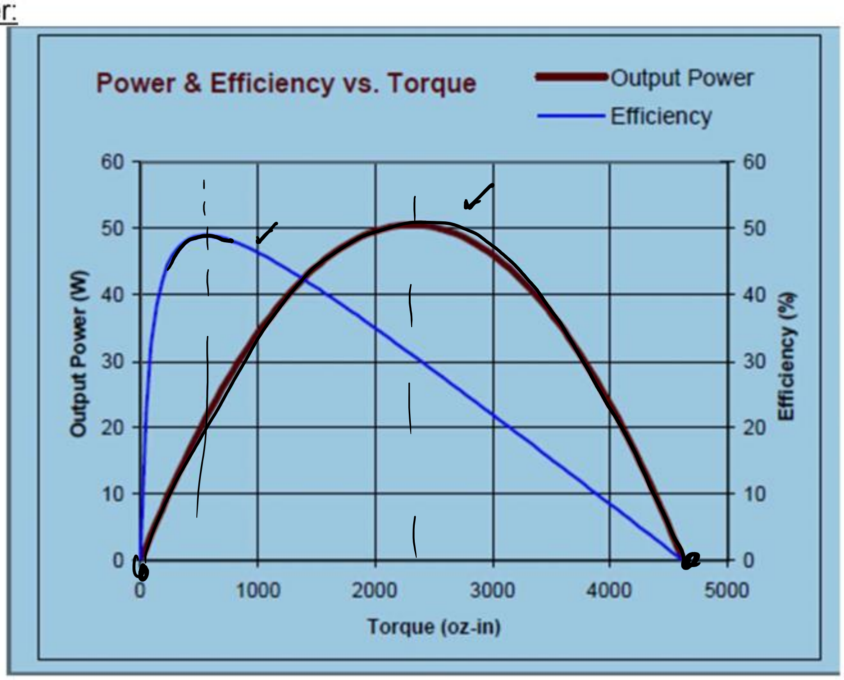


$T = K_t * i$

Assumptions:

Constant magnetic flux, constant voltage

Power:



13

Page 14

Wednesday, March 22, 2017 1:30 PM

Motor Control Applications

Technology	Refrigeration Compressors	Washers/Dryers	Pumps/Fans/ Blowers	Industrial Factory Floor	CNC Tool and Dye, Health Care Scanners	Garage Door Openers	Hand Tools	Kitchen Contertop Appliances	Computers, Office Equipment	Toys
Universal Motor		8-bit				8-bit	8-bit	8-bit		
Brush DC Motor						8-bit	8-bit	8-bit		8-bit
Low-Performance DC Servo					8-bit				8-bit	8-bit
Switched Reluctance		8-bit	8-bit							
Stepper Control					8-bit 16-bit				8-bit 16-bit	8-bit
High-Performance DC Servo				16-bit	16-bit				16-bit	16-bit
AC Induction Scalar-Slip Control	8-bit	8-bit 16-bit	8-bit 16-bit	8-bit 16-bit		8-bit				
BLDC Commutated Control	8-bit 16-bit	8-bit 16-bit	8-bit 16-bit			8-bit	8-bit		8-bit	8-bit
Permanent Magnet AC Field Oriented Control	16-bit	16-bit 32-bit	16-bit 32-bit	16-bit 32-bit	16-bit 32-bit					
AC Induction Field Oriented Control	16-bit	16-bit 32-bit	16-bit 32-bit	16-bit 32-bit	16-bit 32-bit	16-bit				

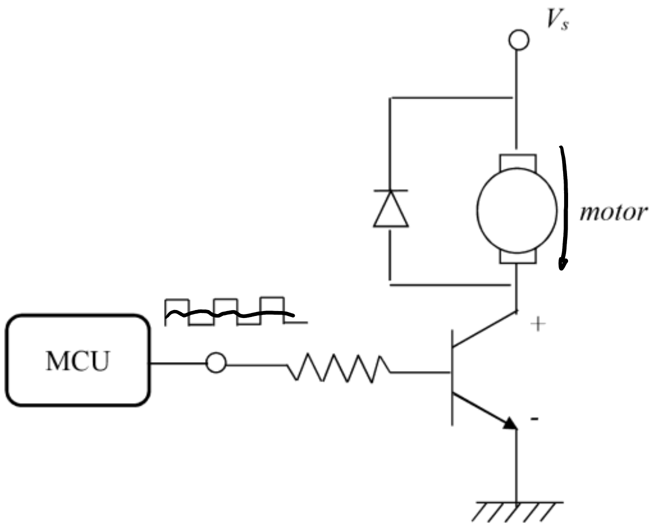
Technology/Application Match		Good		Moderate		Poor

Page 15

Wednesday, March 22, 2017 1:30 PM

Electronic Control of DC Motors:

Open Loop Control:



Slow Speed:

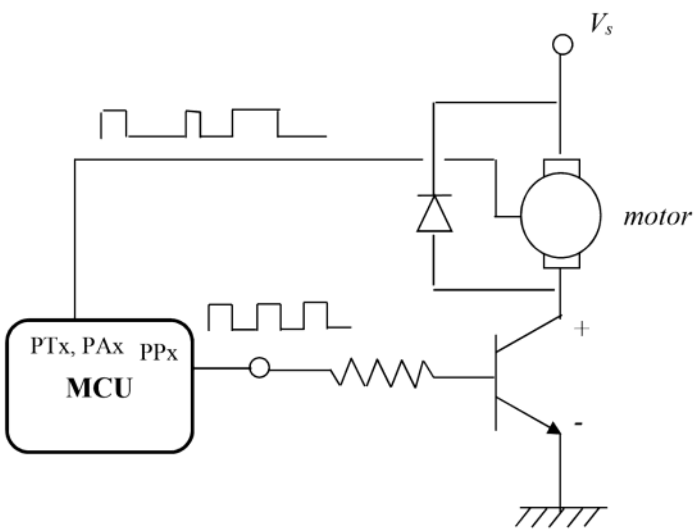
High Speed:

15

Page 16

Wednesday, March 22, 2017 1:30 PM

Closed-Loop Control:



16

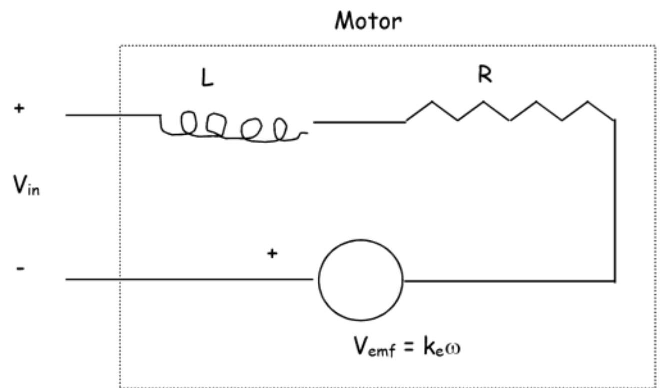
Page 17

Wednesday, March 22, 2017 1:30 PM

MOTOR Model

The behavior of this motor can be described through a system of equations that relates the input to the motor, voltage, to the motor output, motion (position, velocity, acceleration) under a given load. This model will require; 1) a description of the current flow in the motor, 2) equations of motion defining rotation of the motor, 3) electro/mechanical relation in the motor.

Motor Electronics:
A circuit of the motor drive system is constructed and evaluated using Kirchoff's voltage law.



Summing voltages throughout the circuit gives

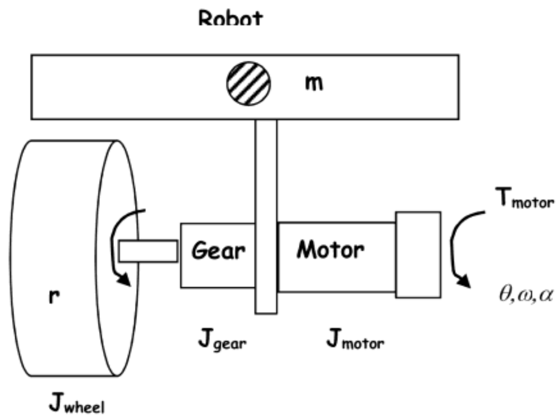
$$V_{in} = L\,{}^{di}\!/_{dt} + Ri + k_e\omega \qquad (1)$$

a first order ODE in current, *i,* with *L*, *R* the motor inductance and resistance, k_e the electric constant, V_{emf} the back emf and ω the rotational speed of the motor (rad/s).

Motor Dynamics:
The equations of motion for the robot will consider the simple case of single-degree-of-freedom motion of the robot, moving forward and reverse. A free-body diagram of a symmetric half of the robot is constructed and used to write the equations of motion:

Page 18

Wednesday, March 22, 2017 1:30 PM



$$J\ddot{\theta} + C\dot{\theta} = T - T_{load} \tag{2}$$

where J is the equivalent inertia as seen by the motor, C is the equivalent viscous damping seen by the motor, T_{motor} is the input torque from the motor and T_{load} all other loads on the system. The total inertia of the system as seen by the motor is given by the following equation,

$$J_{equiv} = J_{motor} + J_{gear} + \left(J_{wheel} + mr^2\right)\left(\frac{1}{GR}\right)^2 \tag{3}$$

← relating energies.

100:1 → 10e3 : 1

where J_{motor}, J_{gear} are the inertias of the motor and gear train relative to the motor, J_{wheel} the robot wheel inertia, m the robot mass, r the wheel radius and GR the transmission ratio expressed as the ratio of input rotation to unit output rotation. Equation 2 gives a second order ODE in motor rotation, θ.

Electro-Mechanical Relation

The electrical and mechanical components are coupled in two ways. First, an approximate relation is generally used that describes motor torque as a linear function of current in the motor,

$$T = k_t i \tag{4}$$

where k_t the motor torque constant. In addition, the back emf in the motor is linearly related to the motor rotational velocity,

$$V_{emf} = k_e \dot{\theta} \tag{5}$$

Page 19

Wednesday, March 22, 2017 1:30 PM

Combined system model

The electrical and dynamic relationships are now combined to result in a system of equations that govern the robot response. Equations 4 and 5 are substituted into Eqs. 2 and 1 respectively to result in the final system equations; a 1st and 2nd order ODE with two unknowns, i and θ:

$$J\ddot{\theta} + C\dot{\theta} - k_t i = -T_{load} \tag{6}$$

$$L\dot{i} + Ri + k_e\dot{\theta} = V_{in} \tag{7}$$

State Space Model:

This system can be cast into state-space form to result in a system of three 1st order ODE's as,

$$x_1 = \theta \qquad \dot{x}_1 = x_2$$

$$x_2 = \dot{\theta} \qquad \dot{x}_2 = -\frac{C}{J}x_2 + \frac{k_t}{J}x_3 - T_{load}$$

$$x_3 = i \qquad \dot{x}_3 = -\frac{k_b}{L}x_2 - \frac{R}{L}x_3 + \frac{V_{in}}{L} \tag{8}$$

or;

$$\begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{Bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -\frac{C}{J} & \frac{k_t}{J} \\ 0 & -\frac{k_b}{L} & -\frac{R}{L} \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} + \begin{Bmatrix} 0 \\ \frac{T_{load}}{J} \\ \frac{VR}{L} \end{Bmatrix}$$

$$\begin{Bmatrix} y_1 \\ y_2 \\ y_3 \end{Bmatrix} = \begin{bmatrix} r/GR & 0 & 0 \\ 0 & r/GR & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} \tag{9}$$

following the traditional state-space form for linear systems,

$$\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{Bu}$$

$$\mathbf{y} = \mathbf{Cx} \tag{10}$$

where **x** is the state vector, **A** the state coefficient matrix, **B** the input coefficient matrix, **u** the input vector, **y** the output vector and **C** the linear transformation from state to output variables. In this case, the output variables are robot linear position and velocity, and motor current.

Matlab provides a variety of tools to easily model this system. A simple first step could be to observe a step response of this system (step input of the motor voltage) using the command;

>step(A,B,C,D)

with A,B,C the matrices defined above.

Page 20

Wednesday, March 22, 2017 1:30 PM

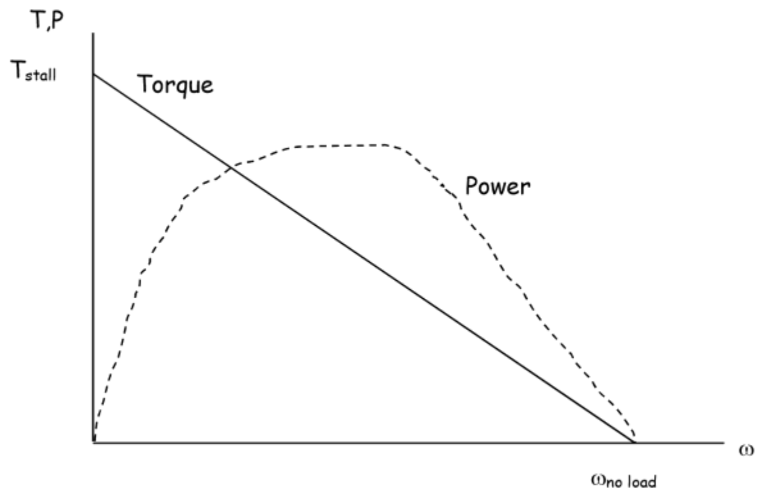
Steady-State Motor Behavior:

At steady state, the dynamic motor model above can be greatly simplified (*i_dot*, ~~*θ_dot*~~ $\ddot{\theta}$ = 0) to yield the equations;

$$Ri + k_e\omega = V_{in} \; + \; T = k_t i \rightarrow$$

$$T = \left(k_t/R\right)V_{in} - \left(k_e k_t/R\right)\omega \, ,$$

This equation represents a linear torque/speed relation for a PM dc motor. The response (assuming a constant input applied voltage) looks like;



The maximum generated torque occurs at rest (stall), and decreases to zero at the maximum motor speed under no load. At this speed, the back-emf voltage in the motor is equal to the input voltage (minus a small amount to overcome inefficiencies in the motor).

If we consider power in the motor, write a linear equation for T as a function of motor speed;

$$T(\omega) = T_s\left(1 - \omega/\omega_{\max}\right)$$

and then express power as;

$$P(\omega) = T(\omega) = \omega T_s\left(1 - \omega/\omega_{\max}\right).$$

The maximum power occurs at;

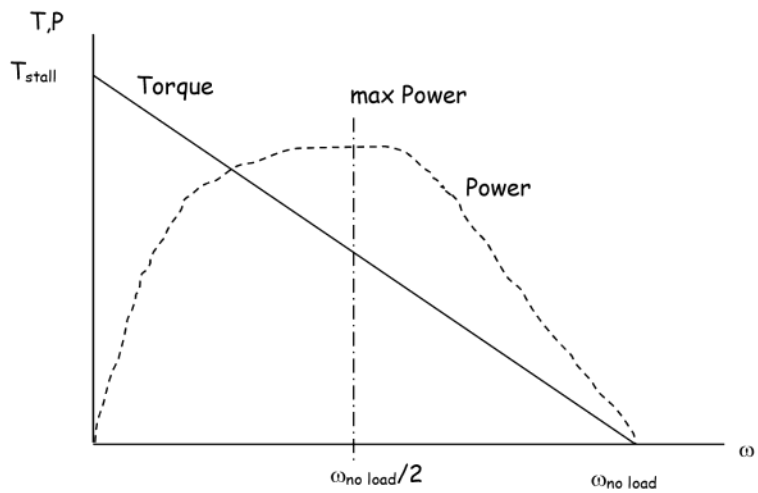
$$dP(\omega)/d\omega = T_s\left(1 - 2\omega/\omega_{\max}\right) = 0$$

or

$$\omega = \tfrac{1}{2}\omega_{\max}$$

Page 21

Wednesday, March 22, 2017 1:30 PM


T,P
T_{stall}
Torque
max Power
Power
ω
$\omega_{no\ load}/2$
$\omega_{no\ load}$

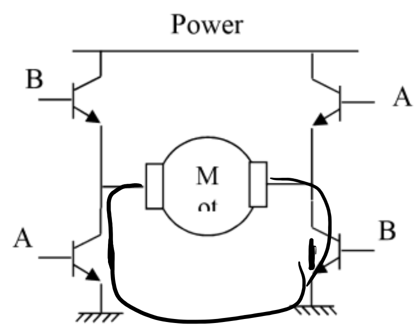
Page 22

Wednesday, March 22, 2017 1:30 PM

Motor Drives:

H-bridge:

Transistors/Mosfets are highly efficient when acting as a simple on/off switch. These can be combined through a circuit made of four such devices to direct power in two directions through the DC motor as shown below. This arrangement is called the H-bridge.



(You can see how this device gets its name). This ability has made the H-bridge an extremely popular and diverse driver. A standard Table can be created based on the inputs, A & B:

A	B	Output
1	0	Forward
0	1	Reverse
0	0	Open-circuit
1	1	Locked

Page 23

Wednesday, March 22, 2017 1:30 PM

The H-bridge is so useful that it comes in many packages. The table below lists some common H-bridges along a few of their characteristics

Product	Voltage	Current	Frequency	Cost	Comments
L293/D updated	36V	.6/1 A	High	$2-3	Bundled in a 16dip IC, 2-bridges per chip, can be used as half-bridges
SN754410	36V	.6/1 A 1/1.3	High	$2-3	Bundled in a 16dip IC, 2-bridges per chip, can be used as half-bridges
LM15200	55V	3A	High	$20	Thermal protection
MC33886 With TTU board*	55V	5A	High	$5	Rugged chip when combined with TTU board Current and thermal sensing
Tecel D200	48V	15A+	Low (1kHz)	$36	Follow directions given be manuf. Small-production PCB system
Devantech	55V	5+A	High	$70+	Allows multiple inputs
AMD 50AT (Servo systems)	50V	30A	High	$500	Commercial driver with closed-loop control

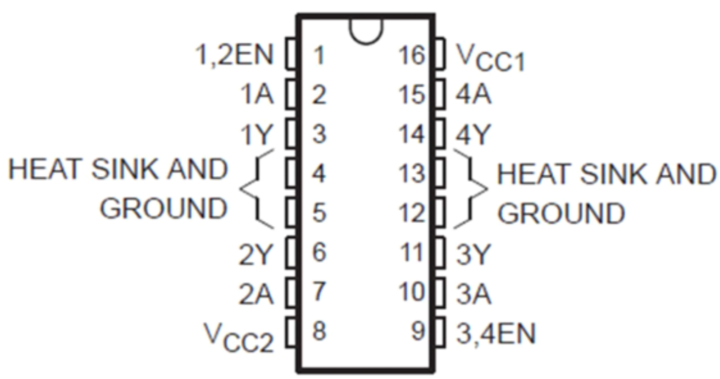
Prototypic

Commercial.

12V
12A
144.W.

The following schematic shows a typical layout for the SN754410 IC-based driver

NE PACKAGE
(TOP VIEW)



Page 24

Wednesday, March 22, 2017 1:30 PM

Exercises:

1. How can electric motors affect other nearby circuits
2. Given the following specifications for a PM DC motor,
    a. Torque Constant = 0.12 Nm/A
    b. Back emf constant = 12V/1000 RPM
    c. Ir = 1.5 ohm

    What is the motor's no-load speed, stall current, starting torque, and maximum power if input voltage is 12V.
3. Solve problem 2 assuming Vs = 24V and input PWM duty cycle = 70%
4. Draw a diagram/circuit to connect a small motor to an L293 IC
5. Draw a diagram/circuit to connect a small motor to an LMD 15200T

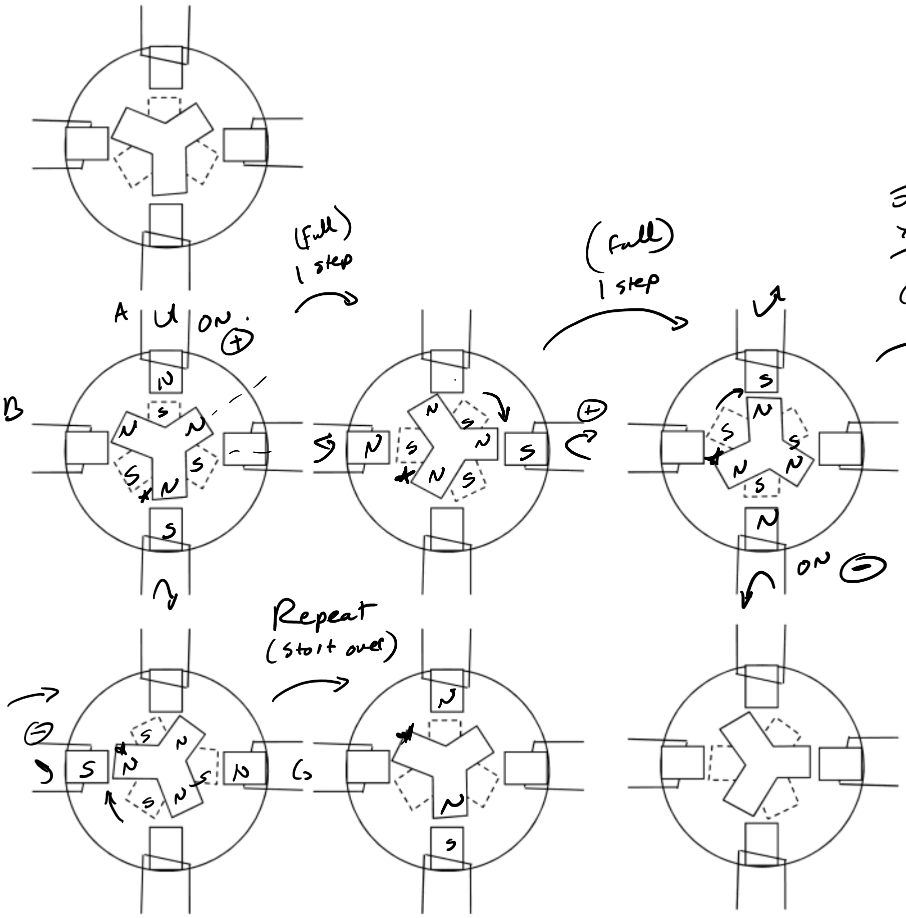
<u>Matlab code:</u>

```
M=10;          %Payload mass
g=9.81;        %gravity
Jconv=.0001;   %Conveyor inertia
rw=.08;    %Wheel radius
dg1=1;dg2=25;GR=dg2/dg1;
J_motor=1.3e-4; %Motor inertia
J=J_motor+Jconv*(1/GR)^2+M*rw^2*(1/GR)^2;    %Total inertia at motor
C=1.0791e-5;    %Viscous damping
theta=30*pi/180;
kt=.415;    % torque constant
kb=50.68*60/(2*pi*1000);    %Back emf constant
L=4.8*1e-3;     %Inductance
R=9.65;         %Resistance
V=12;           %Input voltage
Tf=.5;          %frictional torque as a percent of dead load
A=[0,1,0
   0,-C/J,kt/J
   0,-kb/L,-R/L];
B=[0;(-M*g*rw*sin(theta)/GR)*(1+Tf)/J;V/L];
C=[rw/GR,0,0;0,rw/GR,0;0,0,1];
D=[0;0;0];
step(A,B,C,D)
```

Page 25

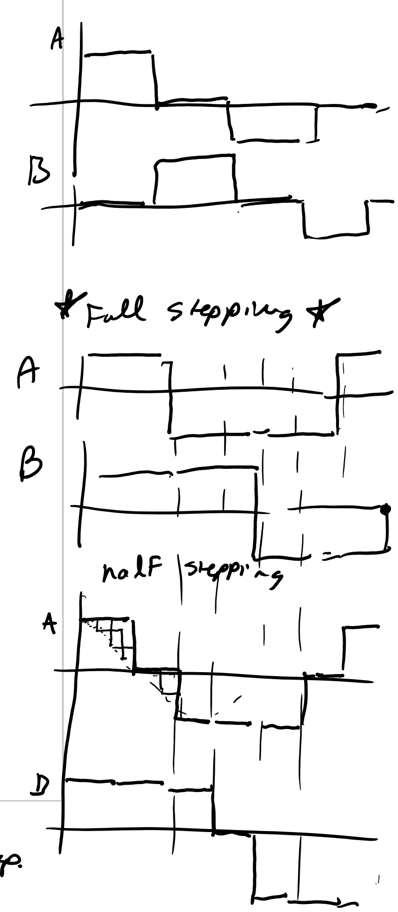
Wednesday, March 22, 2017 1:30 PM

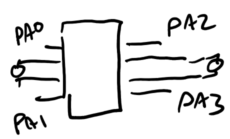
Stepper Motors



50
×4
200 steps / Rev.

$\frac{360}{200} = 1.8$ deg/step.





PORTA = 0b0000 ; // step 1
delay(10);
PORTA = 0b0000

25

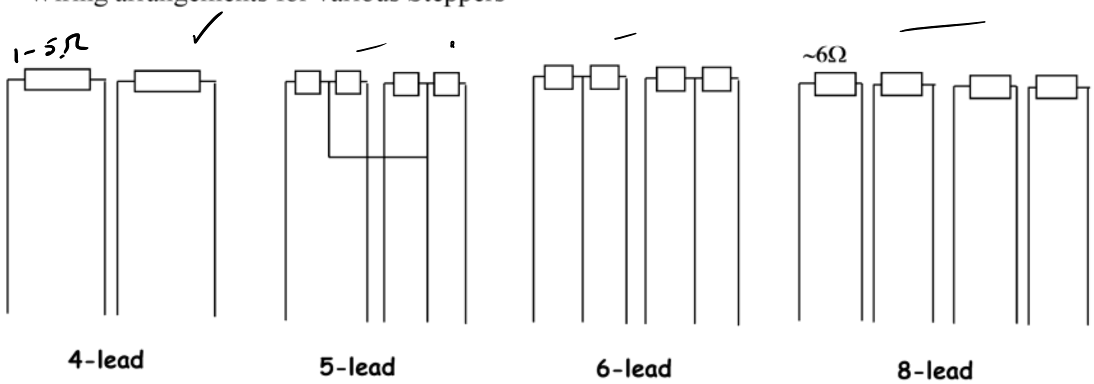
$8 \times 50 = 400 \approx .9$ deg/step.

Page 26

Wednesday, March 22, 2017 1:30 PM

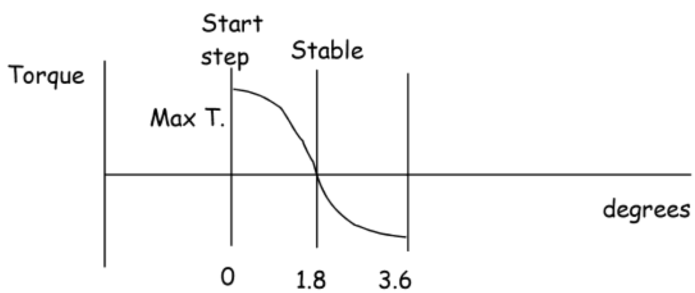
Stepper (cont.)

Wiring arrangements for various Steppers



Behavior of stepping-motor

1) A typical step motor has 200 full step positions. Thus, the typical stepper will move 1.8 degrees in one step.
2) When the stator portion of the stepper makes a step (but prior to the rotor actually moving), the stepper sees its maximum torque. As the rotor moves under the influence of this torque, the torque drops to a zero value when the rotor reaches the end of its step (1.8 degrees).
3) This results in a static torque curve that may look something like:

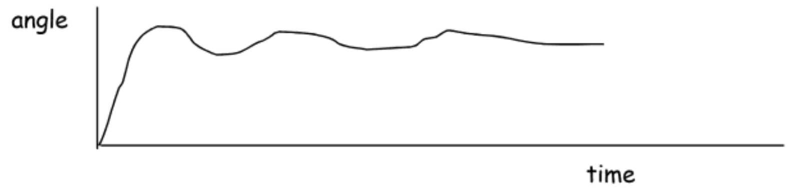


4) Considering the static torque curve above, the rotor on a step motor could lag commanded motion by as much as 1.8 degrees during an acceleration phase, or lead by as much as 1.8 degrees during deceleration.

Note that the static torque curve of a stepper behaves similar to a mass-spring system with a non-linear spring. From this model, the response to a single step move would look something like;

Page 27

Wednesday, March 22, 2017 1:30 PM



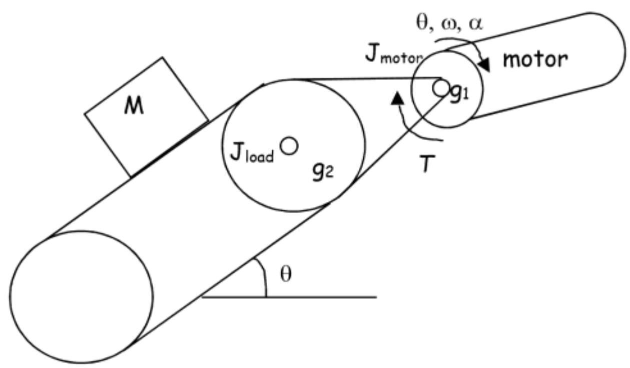
5) Due to this behavior of the step motor, driving the motor at a step speed near its natural frequency can greatly increase the oscillations during response. The natural frequency of the motor depends on the stiffness (electro-magnetic field) and rotational inertia. Higher inertia will decrease the natural frequency of the system and provide more separation between typical driving frequencies and ω_n. Typical ω_n for an unloaded stepper may be 100-200 Hz.
6) An additional consideration in driving steppers is the acceleration profile. Remember that the maximum torque of the system cannot be exceeded. Therefore, to operate at a desired rotation or slew rate, the motor must be started (and stopped) in a profiled manner, approaching the desired speed such that the motor is not driven past maximum acceleration.

Page 28

Wednesday, March 22, 2017 1:30 PM

Example Problem: Motor Selection

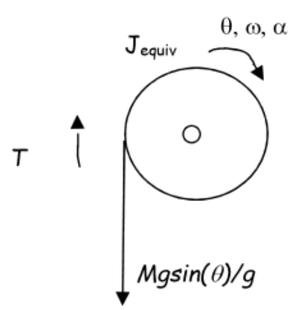
This example will demonstrate the process of analyzing a particular motor in the mechanical system demonstrated below. This system requires a DC brushed motor to move payloads along an inclined conveyor. The motor drives the conveyor through a gear reducer simply shown with pulleys g1 and g2.



Problem givens;
M=10kg; r_w=8cm; g2:g1=GR=50; J_{load} = .0001 kgm^2; θ = 30deg., consider friction in the system as a percentage of the static load.

Analysis Approach;
First we need to get our state equations. These will consist of equations of motion, our KVL equations and our electro-mechanical relations as derived above. The KVL relation will be identical to that above. Create a simple FBD of the motor system to get the equations of motion;

FBD:



Equations:

Page 29

Wednesday, March 22, 2017 1:30 PM

$$J_{equiv}\ddot{\theta} + C\dot{\theta} - k_t i = -Mg\sin(\theta)\left(\frac{1}{GR}\right) - T_{friction}\left(\frac{1}{GR}\right)$$

$$Li' + Ri + k_e\dot{\theta} = V_{in}$$

Note that the variable q is the motor rotation. *Jequiv.* is the equivalent inertia of the system as seen by the motor and determined using the relation described above. Remember that inertia's are magnified by the square of intermedi gear reductions. In this case, *Jequiv.* is given as:

$$J_{equiv} = J_{armature} + J_{conveyor}\left(\frac{1}{GR}\right)^2 + J_{payload}\left(\frac{1}{GR}\right)^2$$

$$J_{payload} = Mr_w^2$$

Equations in State-Space Form:

$$x_1 = \theta \quad x_2 = \dot{\theta} \quad x_3 = i$$

$$\begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{Bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -C/J & k_t/J \\ 0 & -k_b/L & -R/L \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} + \begin{Bmatrix} 0 \\ -Mg\sin(\theta)/J \cdot GR - T_{friction}/J \cdot GR \\ RV/L \end{Bmatrix}$$

$$\begin{Bmatrix} y_1 \\ y_2 \\ y_3 \end{Bmatrix} = \begin{bmatrix} r_w/GR & 0 & 0 \\ 0 & r_w/GR & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix}$$

Note that in describing that observers or outputs, *y* a conversion is made to output displacement and velocity in meters and m/s respectively, y_3 still represents current.

Solution approach:

Implement using the step function in Matlab. In this case, the system will be described in state space form with the matrices A, B, C (D=[]) as above.

Example:

Consider the SM233A motor from Parker automation (page 188 in '99 computmotor catalog). The specifications on this motor are as follows;
$J_{armature}$ = 1.3e-4 kg m^2; k_t = .415 N.A; k_b=50.68*60/(2*pi*1000) N m s; L= 4.8e-3 H, R = 9.65O, V=24

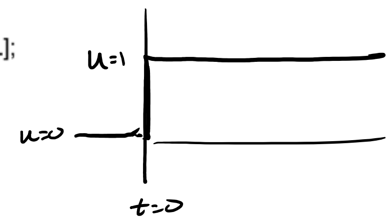
Matlab code:

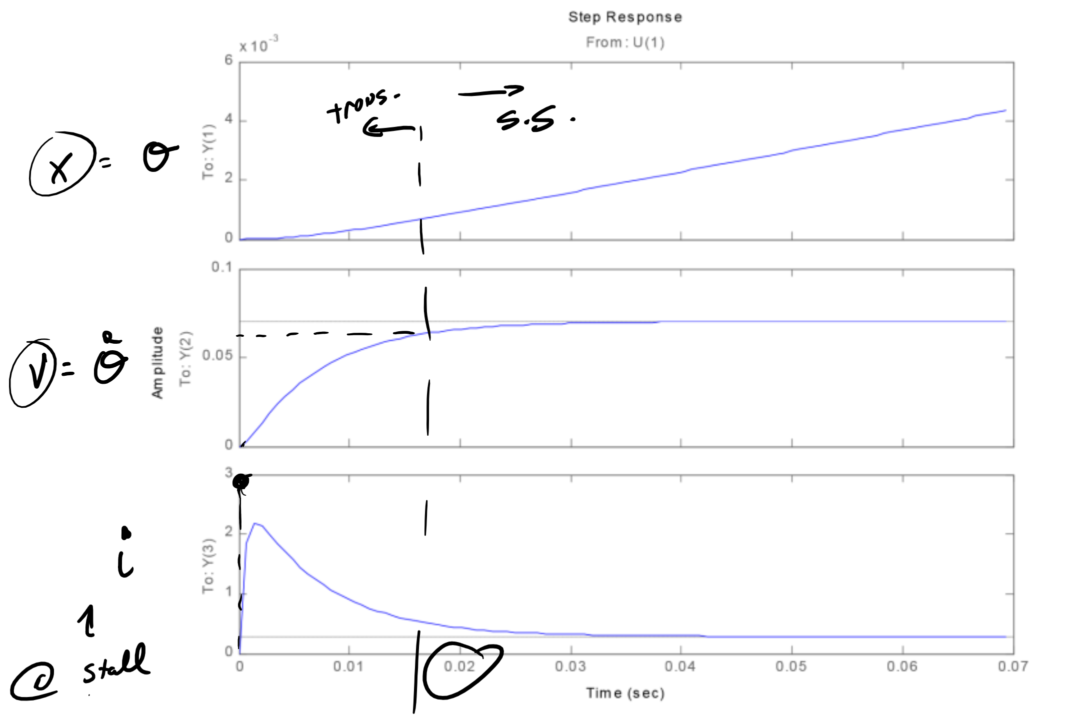
```
M=10;          %Payload mass
g=9.81;        %gravity
Jconv=.0001;   %Conveyor inertia
rw=.08;    %Wheel radius
```

Page 30

Wednesday, March 22, 2017 1:30 PM

```
dg1=1;dg2=25;GR=dg2/dg1;
J_motor=1.3e-4; %Motor inertia
J=J_motor+Jconv*(1/GR)^2+M*rw^2*(1/GR)^2;    %Total inertia at motor
C=1.0791e-5;    %Viscous damping
theta=30*pi/180;
kt=.415;    % torque constant
kb=50.68*60/(2*pi*1000);    %Back emf constant
L=4.8*1e-3;     %Inductance
R=9.65;         %Resistance
V=12;           %Input voltage
Tf=.5;          %frictional torque as a percent of dead load
A=[0,1,0
   0,-C/J,kt/J
   0,-kb/L,-R/L];
B=[0;(-M*g*rw*sin(theta)/GR)*(1+Tf)/J;V/L];
C=[rw/GR,0,0;0,rw/GR,0;0,0,1];
D=[0;0;0];
step(A,B,C,D)
```

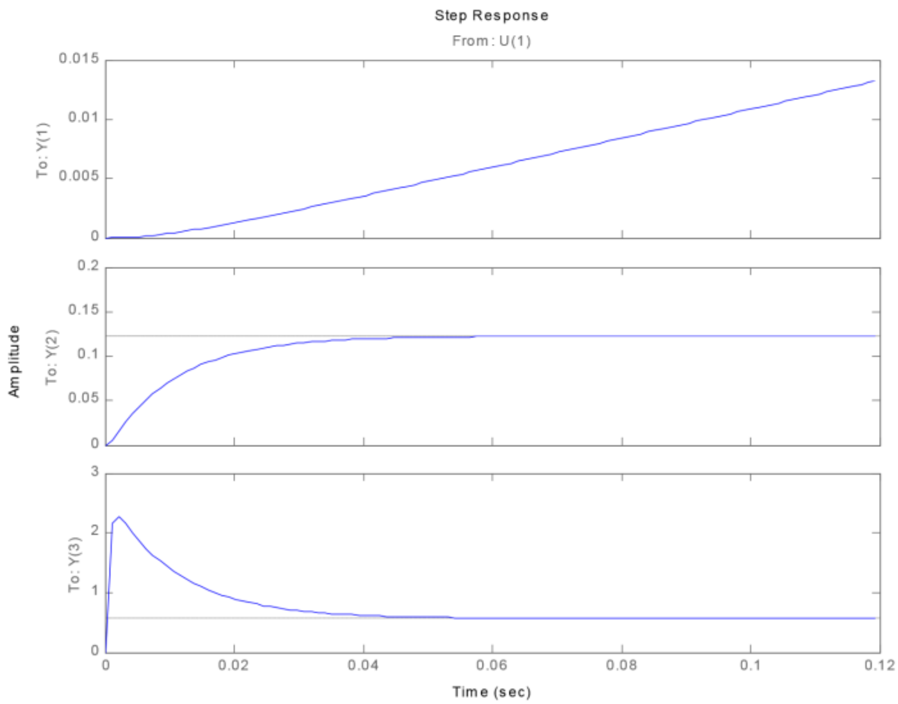




Note that this example shows the motor provides a reasonable response time, and low current at steady state. On the other hand, our final speed may be a little low (.065 m/s), so try decreasing the size of the gear ratio to 25:1 This yields the following results;

Page 31

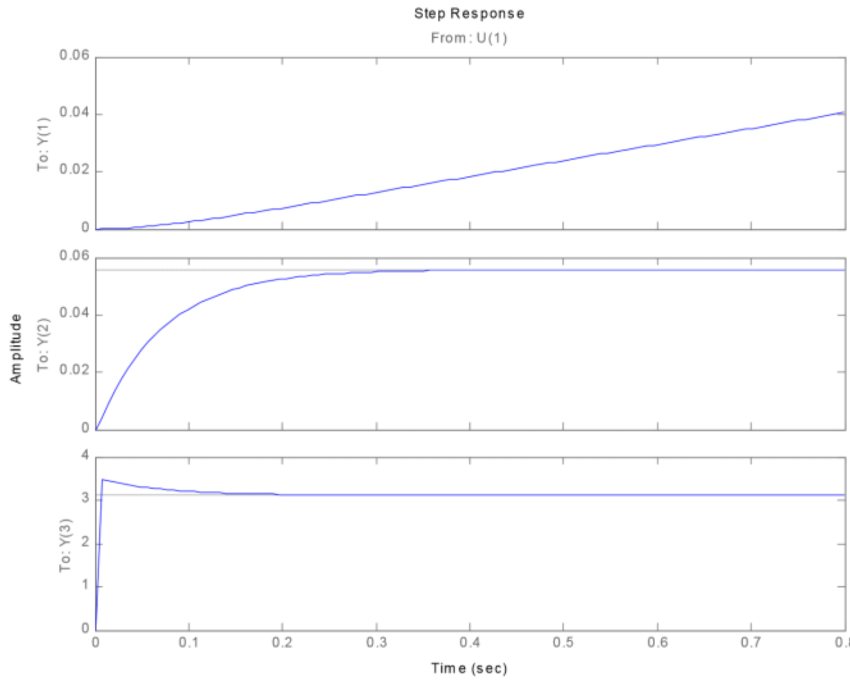
Wednesday, March 22, 2017 1:30 PM



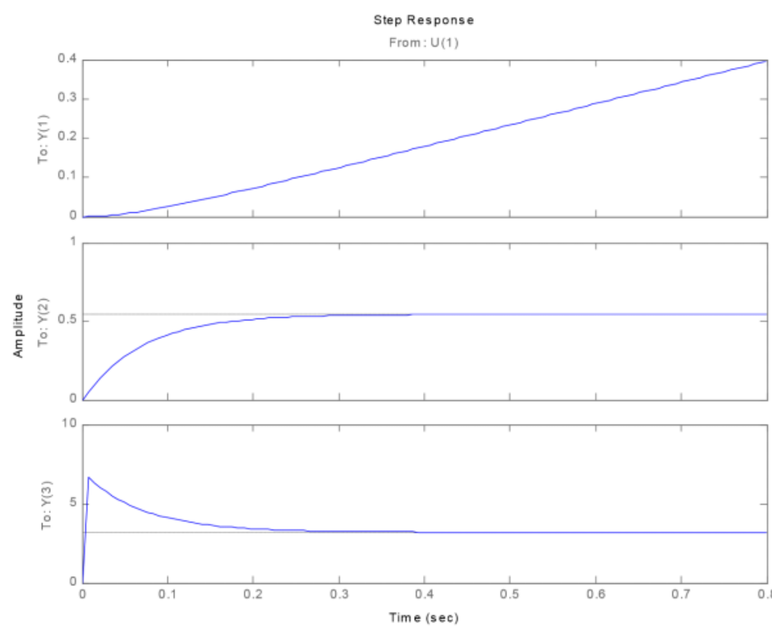
Now we have a maximum speed of .125 m/s, a maximum current draw of approx. 2.25 A and a fast response. Perhaps this motor is larger than needed (note also that this motor is intended to run at 120-170V). So, an SM160A motor is tried with the 24V input voltage; $J_{armature}$ = 5e-6 kg m^2; k_t = .038 N.A; k_b=4.02*60/(2*pi*1000) N m s; L= .53e-3 H, R=3.43 O

Page 32

Wednesday, March 22, 2017 1:30 PM



Notice that the response time is still acceptable, but the final speed is low and the current draw is high. If a higher motor volatage is tried (24V):



Page 33

Wednesday, March 22, 2017 1:30 PM

Final velocity is now acceptable, but the current may be to high, (near to slightly above the motor maximum ratings). Therefore, this motor is too small for this application.