Mechanical Systems Review for Mechatronics Students:

Overview:

Materials ->Mechanics -> statics -> kinematics -> dynamics -> design

Mechanics of Materials:

1. Material Properties

Metal	Specific Gravity	Young's Modulus (E)	Shear Modulus (G)	Bulk Modulus	Poisson's Ratio	Thermal Conductivity 0 deg.C	Linear Expansion Coefficient	Melting Point	Proof/Yield Stress	Ultimate. Stress	Electrical Resistivity 20 deg.C
-		GPa	GPa	GPa		W / (m K)	$x10^{-6}$/ deg C	Deg K	x 10^7Pa	x 10^7Pa	$x10^{-8}$ ohm.m
Aluminium	2,7	68,95	26	75.	0,33	237	25	933	3-14	6-14	2,655
Copper	8,96	117,2	46	130	0,36	398.	16,6	1357	4,7-32	20-35	1,673
Gold	19,32	74,46	28.	167	0,42	315	14,2	1336	0-21	11-23	2,35
Iridium	22,42	517,1				147	6	2723			5,3
Iron	7,87	196,5	76		0,3	80,3	12	1809	16	35	9,7
Lead	11,35	13,79	6		0,43	35,2	29	600,7		1,5-1,8	20,6
Magnesium	1,74	44,13			0,35	156	25	923			4,45
Manganese	7,34	158,6					22	1517			185
Molybdenum	10,22	275,8			0,32	138	5	2893			5,2
Nickel	8,9	213,7	79.	176	0,31	90,5	13	1726	14-66	48-73	6,85
Silver	10,50	72,39	28.	100	0,37	427	19	1234	5,5-30	14-38	1,59
Sodium	0,97					134	70	370,98			4,2
Steel (Mild)	7,8	210	80		0,3	50	12	1630-1750	20-40	30-50	10
Tin	7,31	41,37	17.	52	0,33	67	20	505	0,9-1,4	1,5-20	11,0
Titanium	4,54	110,3	41.	110	0,3	22	8,5	1943	2-50	25-70	43
Tungsten	19,3	344,7	140		0,28	178	4,5	3673		100-400	5,65
Polylactide (PLA)	1.3 g/cm^3	3.5	2.4			.13		160	8		
Acrylonitrile Butadiene Styrene (ABS)	1.03	2.28							6.89		

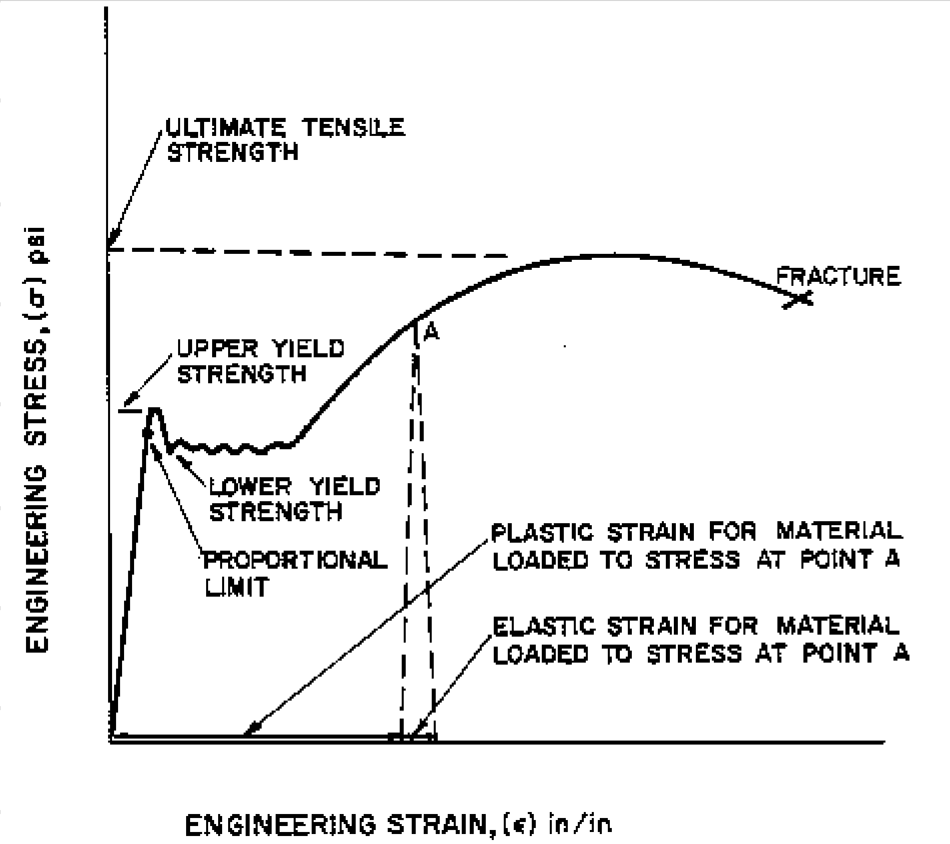
Table 1: Tensile strength and Young's modulus for selected materials

material	tensile strength MPa	modulus of elasticity GPa
304 stainless steel	500	200
copper	270	120
aluminium	90	70
epoxy resin	40	3
silicone rubber	10	0.003

Discussion:

- Key properties:
  - Modulus (ratio of force to displacement)
  - Specific gravity – density (mass)
  - Conductivity – thermal, electrical
  - Strength (yield, ultimate)
- Consider steel as our baseline metal for engineering design
  - Many variations in steel properties affecting, strength, workability
  - Modulus remains relatively constant
- Aluminum Vs Steel
  - 1/3 weight, 1/3 modulus (stiffness), 3+ price, lesser strength, but depends on alloy, treatment
- Other lightweight, high-strength metals commonly used: Magnesium, titanium
- Applications to AM (Additive Manufacturing): melting point, linear expansion coefficient

Macro Behavior of a typical metal



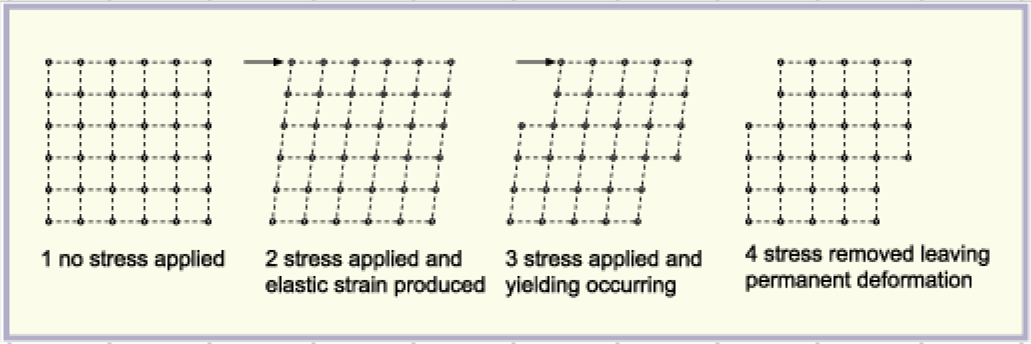
Discussion points:

- Linear region characterizes elastic strain (recoverable)

- Non-linear region characterizes plastic strain (non-recoverable)
- Loading up to (but not exceeding) yield strength is fully reversible
- Loading above the yield strength causes permanent deformation, see loading to point A in the figure

Micro-level behavior of the metals

Figure 7: The block slip model, showing behavior of metals under stress



- Atoms arranged in a lattice structure
- In elastic deformation, the bonds flex but are maintained
- In plastic deformation, the bonds between neighboring atoms are broken and reform with new neighbors

Materials for AM

Plastics – used in FDM methods

Resins – used in laser-sinter type processes

Metals – used first as powders in powder-bed methods, laser sintering,

Present a chart of plastic properties

Properties of plastics:

MATERIALS	TENSILE STRENGTH 73º F	FLEX MOD OF ELASTICITY 73º F	IZOD IMPACT (notched) 73º F	HEAT DEFLECT TEMP 66psi / 264psi		WATER ABSORPTION Immersion 24 Hours
				66psi	264psi	
ABS	4,100	304,000	7.7	200	177	0.30
ACRYLIC (Continuously Processed)	10,000	480,000	0.4	-	195	0.20
KYDEX® 100	6,100	335,000	18.0	-	173	0.05 - 0.08
NORYL® (Modified PPO)	9,600	370,000	3.5	279	254	0.07
PETG	7,700	310,000	1.7	164	157	0.20
POLYCARBONATE	9,500	345,000	12.0 - 16.0	280	270	0.15
POLYCARBONATE (20% Glass Filled)	16,000	800,000	2.0	300	295	0.16
POLYSTYRENE (High Impact)	3,500	310,000	2.0	-	185	-
POLYSULFONE	10,200	390,000	1.3	358	345	0.30
PVC (Rigid)	7,500	481,000	1.0	-	158	0.06
RADEL R®	10,100	350,000	13	-	405	0.37
ULTEM®	15,200	480,000	1.0	410	392	0.25
ULTEM® (30% Glass Filled)	24,500	1,300,000	1.6	414	410	0.16
ACETAL (Copolymer)	9,800	370,000	1.0	316	230	0.20
ACETAL (Homopolymer)	10,000	420,000	1.5	336	257	0.25
HDPE	4,000	200,000	-	172	-	0.10
LDPE	1,400	30,000	no break	122	-	0.10
NYLON (6 Cast)	10,000 - 13,500	420,000 - 500,000	0.7 - 0.9	400-430	200-400	0.60 - 1.20
NYLON (6/6 Extruded)	12,400	410,000	1.2	-	194	1.20
PBT	8,690	330,000	1.5	310	130	0.08
PEEK	14,000	590,000	1.6	-	306	0.50
PET (Semicrystalline)	11,500	400,000	0.7	240	175	0.10
PP (Homopolymer)	5,400	225,000	1.2	210	-	slight
PP (Copolymer)	3,800	215,000	12.5	190	-	slight
PPS	12,500	600,000	0.5	400	220	0.02
PTFE	1,500 - 3,000	72,000	3.5	250	-	<0.01
PVDF (Homopolymer)	7,800	310,000	3.0	300	235	0.02
UHMW-PE	3,100	110,000	18.0[1]	-	-	slight

<u>Discussion:</u>

- Important considerations are melting points, coefficient of thermal expansion, modulus and strength

2) Geometric Properties (of material cross-section):

a. Centroid

point about which mass is distributed – first integral of area

.
$$\frac{\int_{body} r dm}{\int_{body} dm}$$

Uses: Important concept in dynamic equilibrium

b. Moment of area

measure of geometric resistance to flexure – second integral of area

$$\int_{body} r^2 dA$$

Uses: Calculating bending stresses, strain

c. moment of inertia

rotational measure of inertia

$$\int_{body} r^2 dm$$

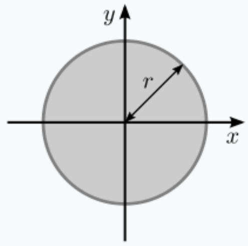
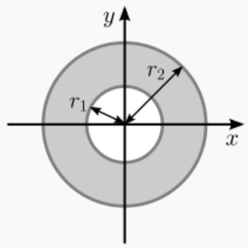
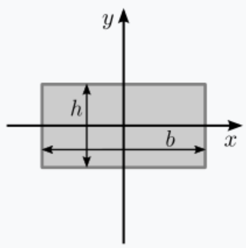
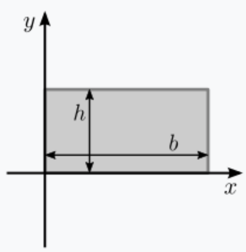
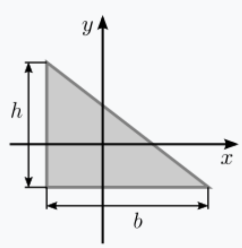
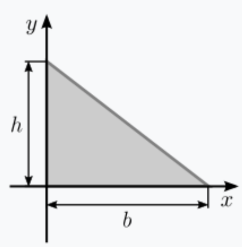
Uses: dynamic equations involving rotation

d. Parallel Axis Theorem

Used to find moment of inertia at various locations on the body

e. Polar moment of inertia

Moment of inertia measuring resistance to rotation in the plance/

Description	Figure	Area moment of inertia	Comment
A filled circular area of radius *r*		$I_x = \frac{\pi}{4} r^4$ $I_y = \frac{\pi}{4} r^4$ $I_z = \frac{\pi}{2} r^4$ [1]	I_z is the Polar moment of inertia.
An annulus of inner radius r_1 and outer radius r_2		$I_x = \frac{\pi}{4}\left({r_2}^4 - {r_1}^4\right)$ $I_y = \frac{\pi}{4}\left({r_2}^4 - {r_1}^4\right)$ $I_z = \frac{\pi}{2}\left({r_2}^4 - {r_1}^4\right)$	For thin tubes, $r \equiv r_1 \approx r_2$ and $r_2 \equiv r_1 + t$. So, for a thin tube, $I_x = I_y \approx \pi r^3 t$. I_z is the Polar moment of inertia.
A filled rectangular area with a base width of *b* and height *h*		$I_x = \frac{bh^3}{12}$ $I_y = \frac{b^3 h}{12}$ [4]	
A filled rectangular area as above but with respect to an axis collinear with the base		$I_x = \frac{bh^3}{3}$ $I_y = \frac{b^3 h}{3}$ [4]	This is a result from the parallel axis theorem
A filled triangular area with a base width of *b* and height *h* with respect to an axis through the centroid		$I_x = \frac{bh^3}{36}$ $I_y = \frac{b^3 h}{36}$ [5]	
A filled triangular area as above but with respect to an axis collinear with the base		$I_x = \frac{bh^3}{12}$ $I_y = \frac{b^3 h}{12}$ [5]	This is a consequence of the parallel axis theorem

3) Hookes Law; $\sigma = E\epsilon$

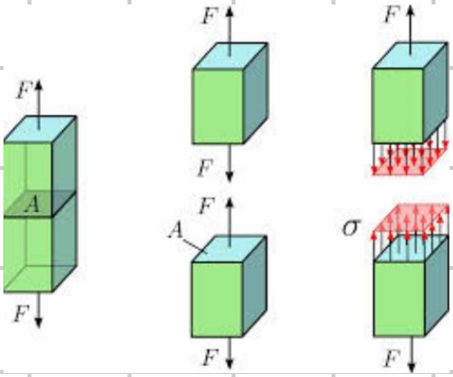
Fundamental concept stating proportional relationship between stress and strains Young Modulus (E) is that proportionality.

Uses: to go from stress to strain

4) Stress:

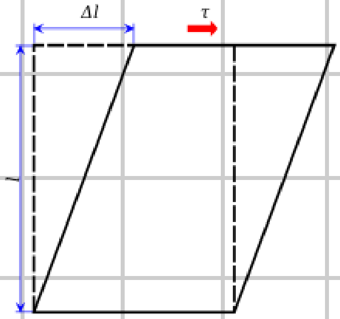
Measure of per-unit load within a body (unit is area). The stress varies at each location on the body

a. Axial Stress: $\sigma = \frac{F}{A}$ axial



b. Shear stress or Transverse shear stress: $\tau_{ave} = \left(\frac{F}{A}\right)$

A Stress parallel or in the plane of the cross section



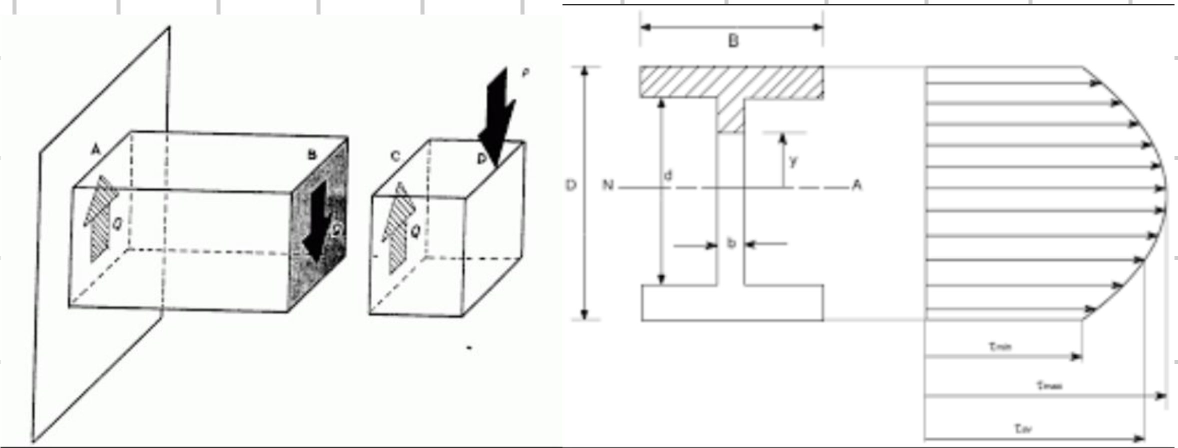
For uniform rectangular beam: $\tau_{max} = \frac{3}{2}\frac{F}{A}$ at the neutral axis

For circular beam: $\tau_{max} = \frac{4}{3}\frac{F}{A}$ at the neutral axis

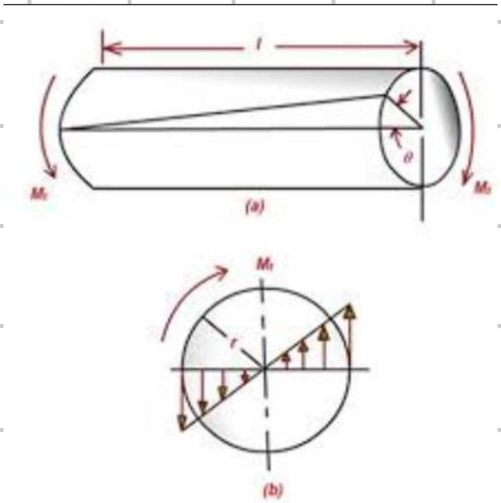
and

$$\tau = G\gamma$$

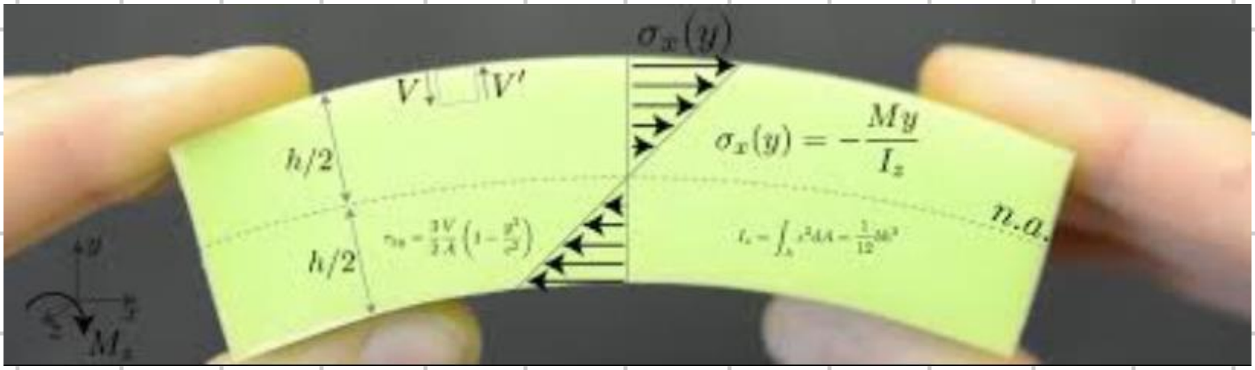
With G = shear modulus

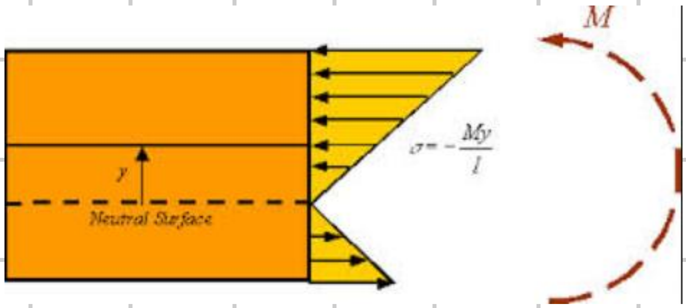


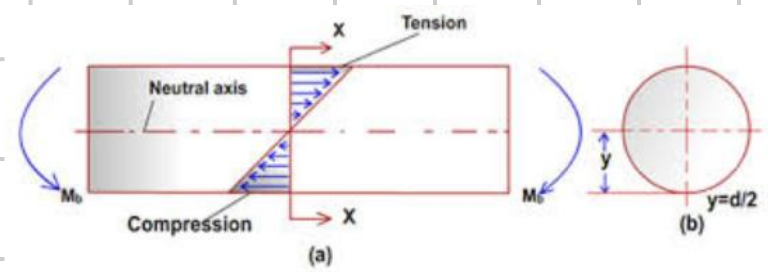
c. Torsional shear stress: $\tau = \frac{Tr}{J}$torsion



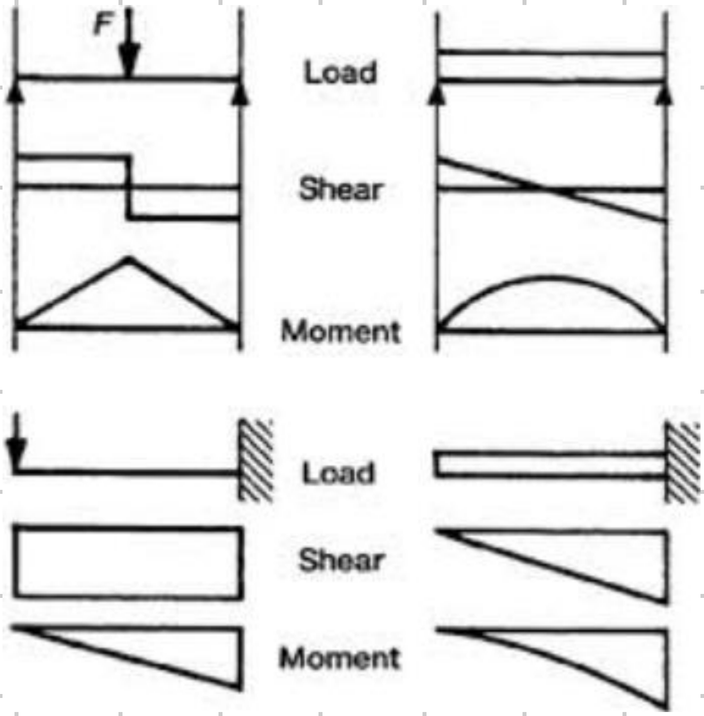
d. Bending Stress: $\sigma = \frac{My}{I}$ bending



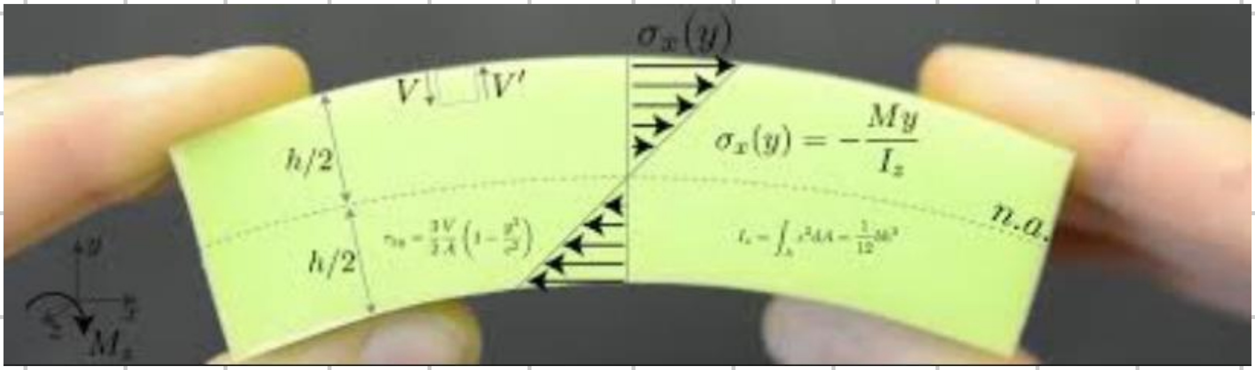

M
σ = − My / I
y
Neutral Surface


X
Tension
Neutral axis
Mb
Compression
X
Mb
y
y=d/2
(a)
(b)

5) Shear and Moment diagrams (Note convention – start at left and go right)



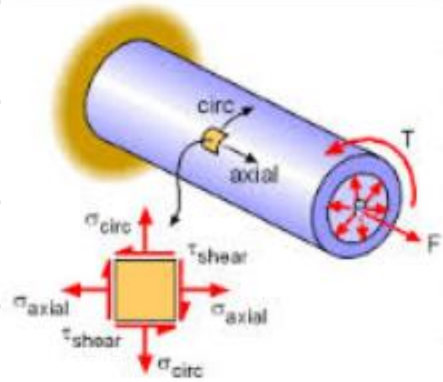
6) Beams:



a. $\sigma = \frac{My}{I}$ bending
   Compressive/zero/tension

b. $\tau = \frac{V}{A}$ shear
   Zero/max/zero

7) Combined Stresses



(sigma_circ = sigma_hoop =P*r/(l))

Stresses on any other plane:

$$\sigma_\theta = \frac{1}{2}(\sigma_x + \sigma_y) + \frac{1}{2}(\sigma_x - \sigma_y)\cos(2\theta) + \tau\sin(2\theta)$$

$$\tau_\theta = \frac{1}{2}(\sigma_x - \sigma_y)\sin(2\theta) + \tau\cos(2\theta)$$

Principal stresses

$$\sigma_1, \sigma_2 = \frac{1}{2}(\sigma_x + \sigma_y) \pm \tau_1$$

$$\tau_1 = \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + (2\tau)^2}$$

8) Allowable stress in design (Max Shear stress theory: $\tau < S_Y/(2n)$ or $\sigma_{max} < S_y/n$ with σ_{max} the max. $|\sigma_1|, |\sigma_2|, |\sigma_1 - \sigma_2|$ and *n* the factor of safety

Example 1:
Matthew is 3D printing a climbing robot, skid-steer with two tracks, total mass of 2 kg. He is using two 15W motors, one to drive each track with track sprocket pitch diameter of 6cm. He wants to 3D print a solid drive shaft from ABS plastic (S_{ult} = 40MPa), 1 cm diameter, 3 cm long from motor to center of track hub. Assuming a reduction in strength of 50% due to 3D printing, climbing speed of 10 cm/s, will his choice of plastic shaft work. Consider independently the shaft torque and bending. Extra note: plastic material doesn't present a yield strength but rather an ultimate. We saw the max-shear stress theory as a conservative form for ultimate materials. Use it here with caution noting we have an ult. Vs. yiel strength given.

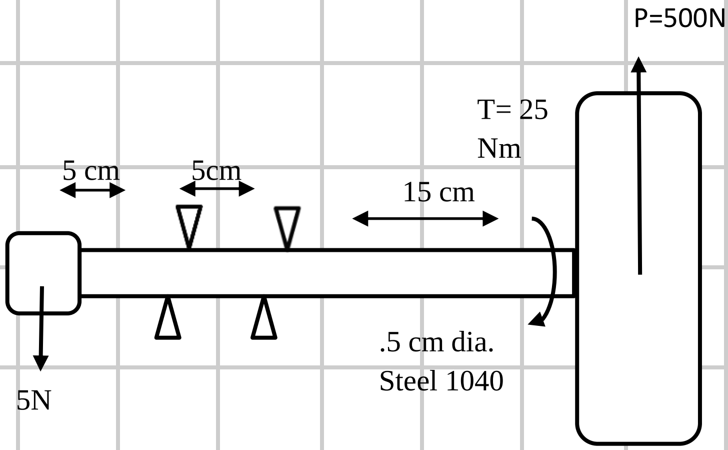
Example 2:
Determine the size shaft needed to run a 5000W generator. Assume the shaft is made out of 1040 steel, S_y = 415 MPa and safety factor of 1.5.

Example 3:
Back to Matthew's climbing robot. His 15W motors can generate a maximum torque of 50mN/m at stall and have a 250:1 gear reduction. Will his 1cm diameter shaft still work? (earlier problem: Matthew is 3D printing a climbing robot, skid-steer with two tracks, total mass of 2 kg. He is using two 15W motors, one to drive each track with track sprocket pitch diameter of 6cm. He wants to 3D print a solid drive shaft from ABS plastic (S_{ult} = 40MPa), 1 cm diameter. Assuming a reduction in strength of 50% due to 3D printing, climbing speed of 10 cm/s)

Example 4: Scott is 3D printing an anthropomorphic hand with plastic tendons made from extruded nylon 6/6 45MPa S_{ult}, 0.1cm diameter. They are connected to an RC servo motor horn with 1cm radius, RC servo MG995R 8 kg-cm torque. Will the nylon fail?

Example 5: (Combined Stress) Wheel axle on a mobile robot

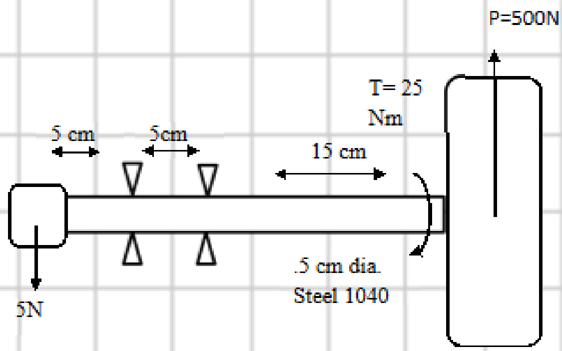
Will this axle work under static loading scenario



$\sigma = \frac{My}{I}$ $\tau = \frac{F}{A}$ $\tau = \frac{Tr}{J}$ $I_{cx} = \frac{\pi}{4}r^4, J = \frac{\pi}{2}r^4, A = \pi r^2$

Example: Wheel axle on a mobile robot

Will this axle work under static loading scenario



$$\sigma = \frac{My}{I}\ \tau = \frac{F}{A}\ \tau = \frac{Tr}{J}\ I_{cx} = \frac{\pi}{4}r^4, J = \frac{\pi}{2}r^4, A = \pi r^2$$

Find Reactions

R_1, R_2, T_m

$\Sigma F_x = 0$

$\Sigma F_y: -5 + R_1 + R_2 + 500 = 0$

$\Sigma M_{R_1}: 5(5) + 5 \cdot R_2 + 500 \cdot 20 = 0$

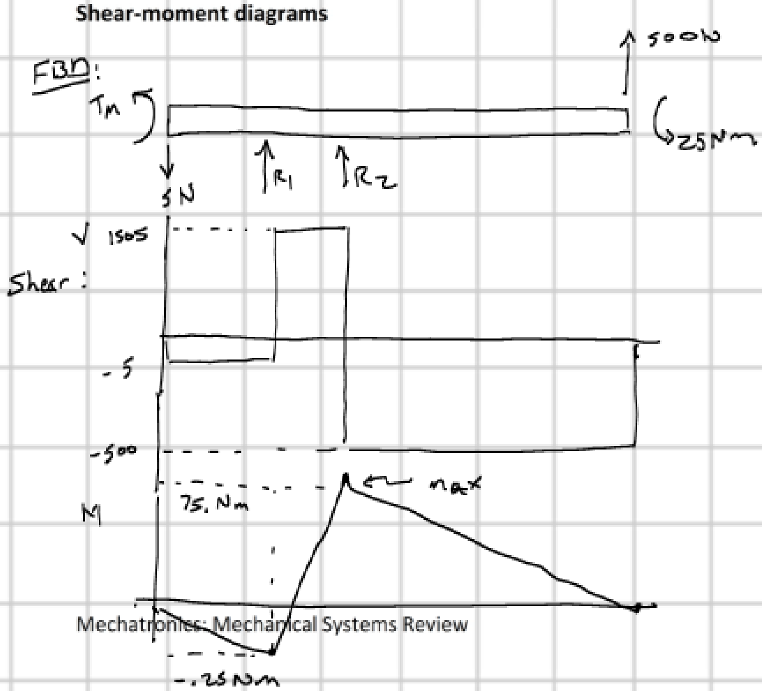
$R_2 = -2005\ N$

$R_1 = +1510\ N$

ΣT about shaft.

$T_m - 25 = 0 \rightarrow T_m = 25 N_m$

Shear-moment diagrams



Mechatronics: Mechanical Systems Review

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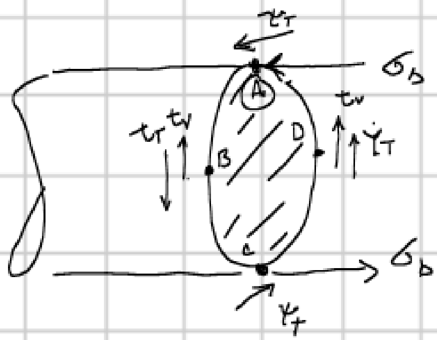
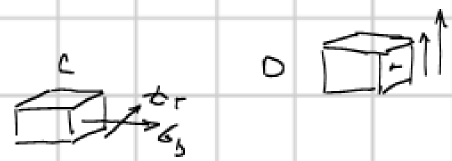
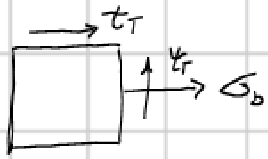


cut shaft @ Max M:

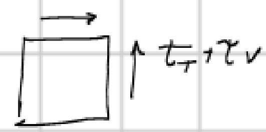


Look @ points C, D:

C



D



$$\sigma_b = \frac{My}{I} = \frac{75\,Nm * \frac{.005}{2}\,m}{\frac{\pi\left(\frac{.005}{2}\right)^4}{4}} = 6.11\ GPa$$

$$\tau_T = \frac{25\,Nm * \frac{.005}{2}\,m}{\frac{\pi\,(.005/2)^4}{2}} = 1.019\ GPa$$

$$\tau_V = \frac{4}{3}\frac{V}{A} = \frac{4}{3}\frac{1505\,N}{\pi\left(\frac{.005}{2}\right)^2} = 10.2\ MPa$$

Pricipal stresses: @C

$$\tau_{1,2} = \pm\frac{1}{2}\sqrt{6.11^2 + (2*1.019)^2} = \pm 3.17\ GPa$$

$$\sigma_{1,2} = \frac{1}{2}(6.11) \pm 3.17 = \begin{matrix} 6.225\ GPa \\ -.115\ GPa \end{matrix}$$

@D

$$\tau_{1,2} = \pm\frac{1}{2}\sqrt{0 + 2(1.02)^2} = \pm 1.02\ GPa$$

$$\sigma_{1,2} = \pm 1.02\ GPa$$

max shear stress theory:

$$\tau_1 = 3.17 \le \frac{S_y}{2} = \frac{.350}{2}\ GPa.$$

NO! Fails!!

Mechatronics: Mechanical Systems Review

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Statics:

1. Vectors

Things (in mechanics) that are vectors	Things that are scalars

2. Vector combinations:

Vector product (cross)	Scalar product (dot)

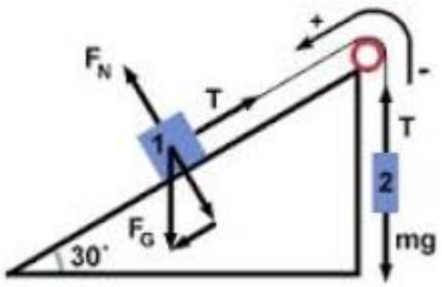
3. Basic principle in statics:

For a static object (object stationary and in equilibrium), the sum of all forces and moments must be zero.

- Statically determinant: number of inputs – number of constraints = 0

- Statically indeterminant: number of inputs – number of constraints < 0

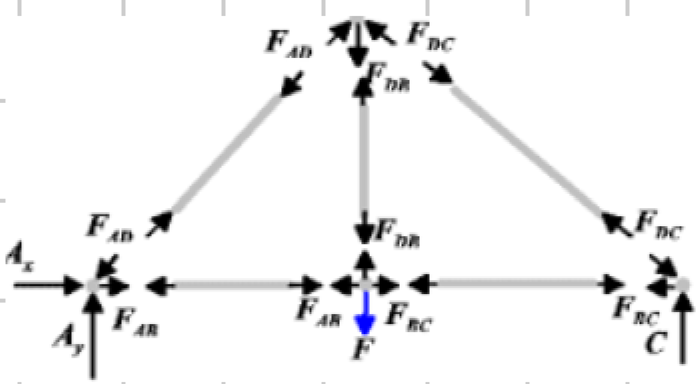
4. Free-body diagrams



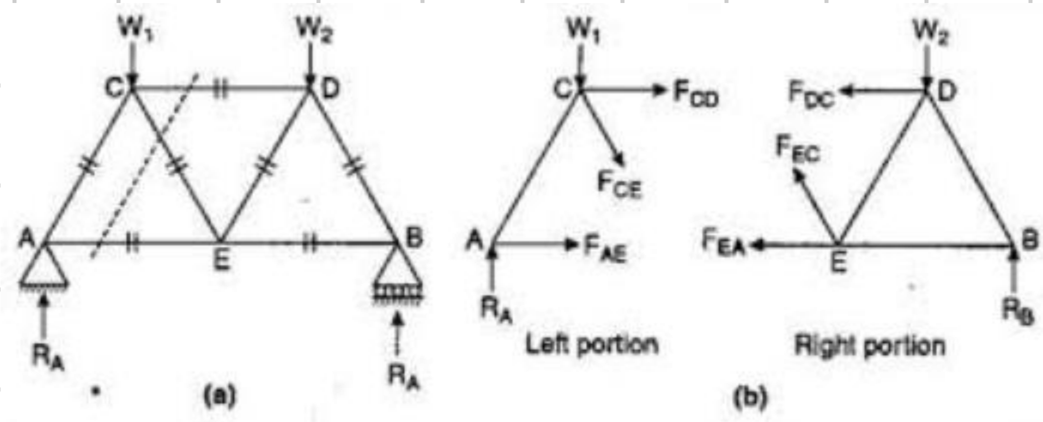
5. Finding External reactions

6. Finding internal reactions

a) method of joints (Like KCL): “For a truss to be in equilibrium, each of its joints must be in equilibrium” – Sum(Fx) = 0, Sum(Fy) = 0



b) method of sections: “Equilibrium required of every portion of truss”



Force, Torque, Work, Energy and Power

$$\vec{T} = \vec{r} \times \vec{F}$$

Units: F = N, T = N-m, and r is a vector from the point of interest out to application of torque

$$W = \int dW = \int \vec{F} \cdot d\vec{s} \ or \int \vec{T} \cdot d\vec{\theta}$$

Work (energy) is a scalar, units are Joules, J=N-m

$$P = \vec{F} \cdot \vec{v} \ or \ \vec{T} \cdot \vec{\omega}$$

Power is an instantaneous scalar, unites are Watt, W=N-m/s

Conversion from Force – Torque is through a moment arm (lever, link, gear, etc).

In many linkage mechanisms, Power is conserved: Po=Pi;

Or, the mechanism may have some associated efficiency, eta:

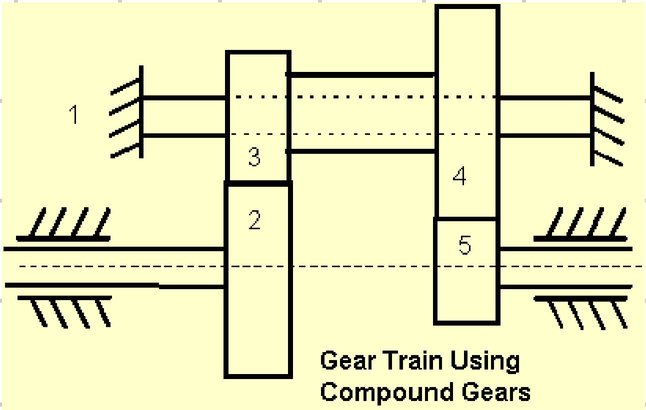
$$P_o = \eta P_i$$

This means that the force/torque relationship in a mechanism is inversely proportional to the velocity relationship:

$$F_o v_o = \eta F_i v_i \text{ or } \frac{F_o}{F_i} = \frac{\eta v_i}{v_o}$$

$$T_o \omega_o = \eta T_i \omega_i \text{ or } \frac{T_o}{T_i} = \frac{\eta \omega_i}{\omega_o} = \frac{\eta}{GR}$$

Where GR is the gear ratio if the mechanism is a gear train (ratio of output to input).



Example, in the gear train shown, Assume N2, N4=20, N3, N5=12, eta = 95%, Tin is 100N-cm, what is To?

Kinematics and Dynamics:

Particle Motion:

$$s = \int v(t)dt = \iint a(t)dt$$

a(t) = a, v(t) = v0 + at, s=so + vot + 1/2 at^2

For constant accelerations

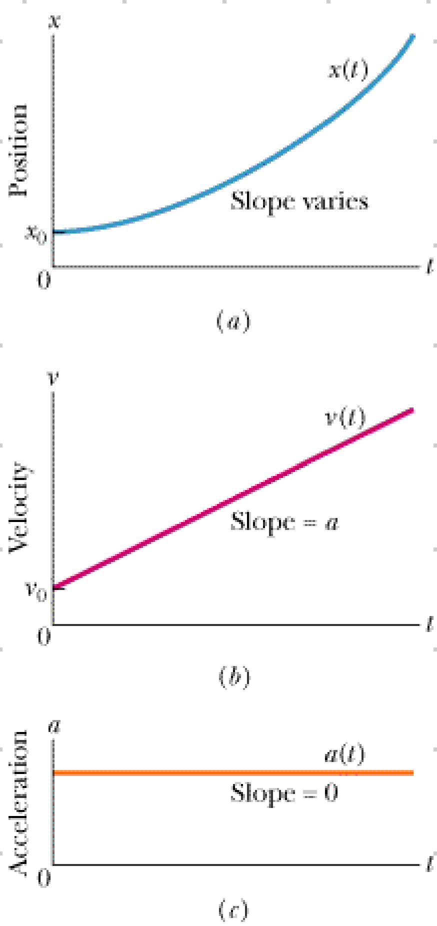


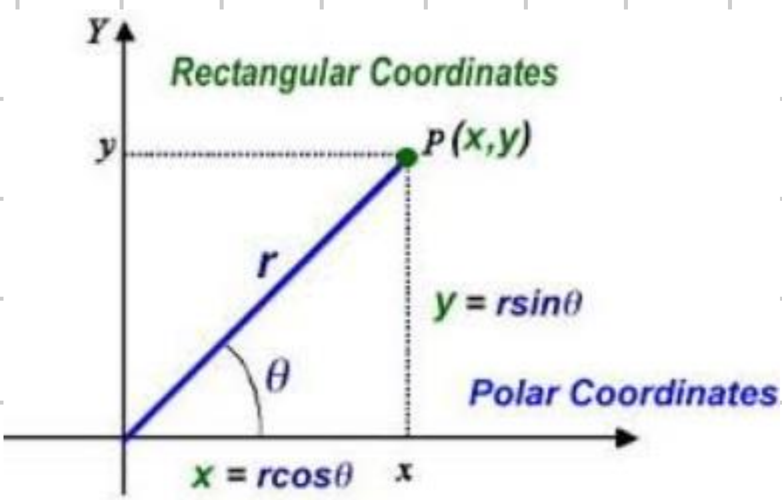
TABLE 1 Equations and Variables of Kinematics in One Dimension

		Variables				
Equation	***Information Given by Equation***	v_o	v_f	*t*	*a*	*d*
$v_f = v_o + at$	Velocity as a function of time	✓	✓	✓	✓	X
$d = ½(v_o + v_f)\,t$	Displacement varying with velocity and time	✓	✓	✓	X	✓
$d = v_o\,t + ½\,at^2$	Displacement as a function of time	✓	X	✓	✓	✓
$v_f^{\,2} = v_o^{\,2} + 2\,ad$	Velocity as a function of displacement	✓	✓	X	✓	✓

Describing Motion in multiple dimensions (Planar)

Cartesian Coordinates

Polar Coordinates:



Motion composed of two components:

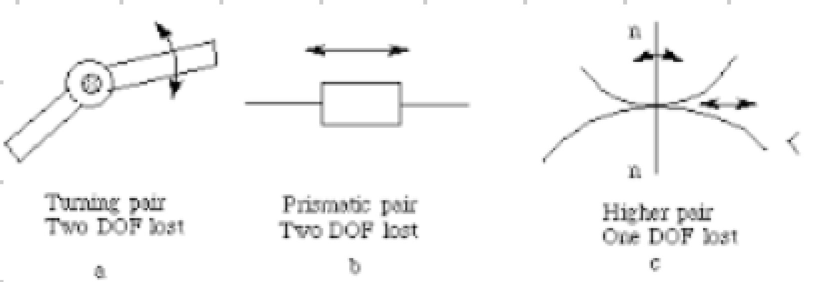
Translation

Rotation

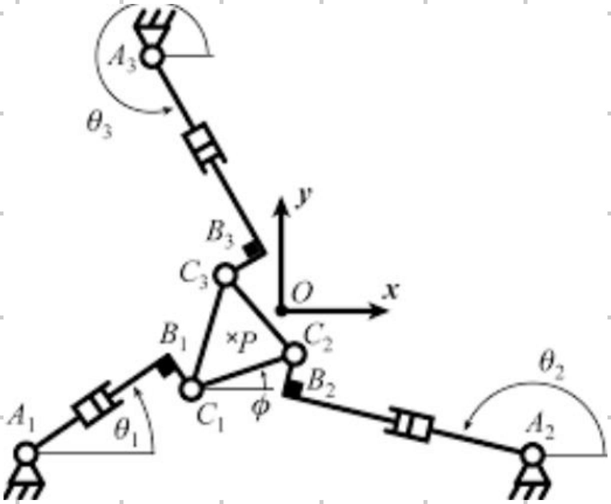
Kinematics of Multi Bodies

Links

Joints

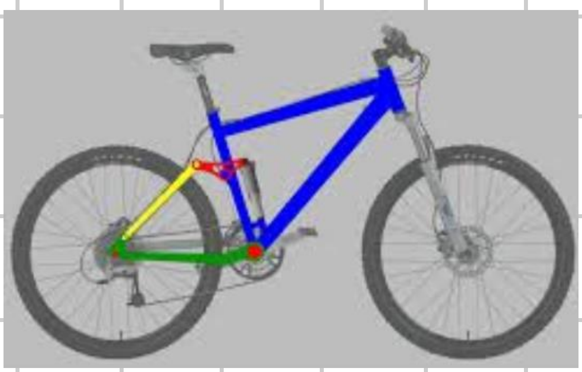


Degrees of Freedom / Mobility

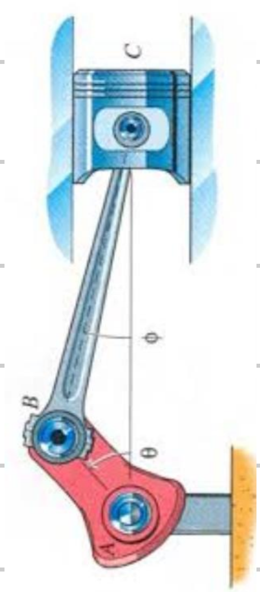


Linkages:

4-bar



Slider-crank



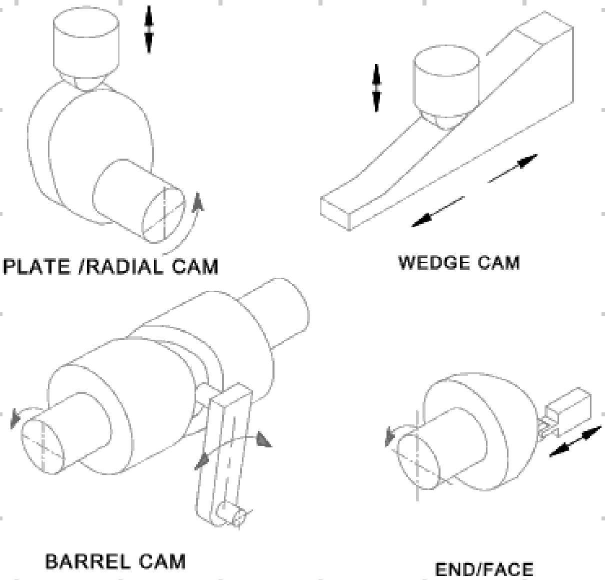
Notes:

1) Nonlinear
2) Toggle positions
3) Mulitple solutions
4) Forces governed related to kinematics through power assuming conservative machines

Cam systems:

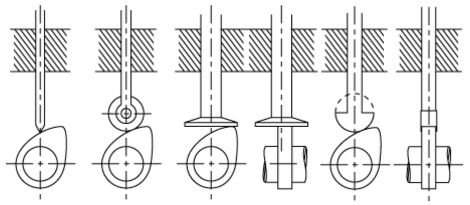
Automotive, manufacturing, rocking chairs, garage doors, door latches

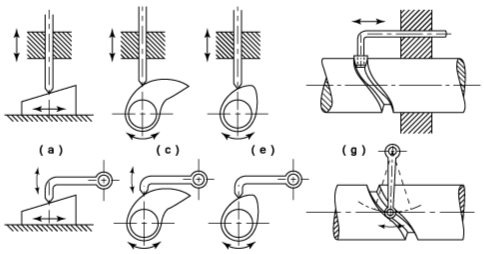
a. Terminology
   Types of cams:



Type of follower:

Follower Form:

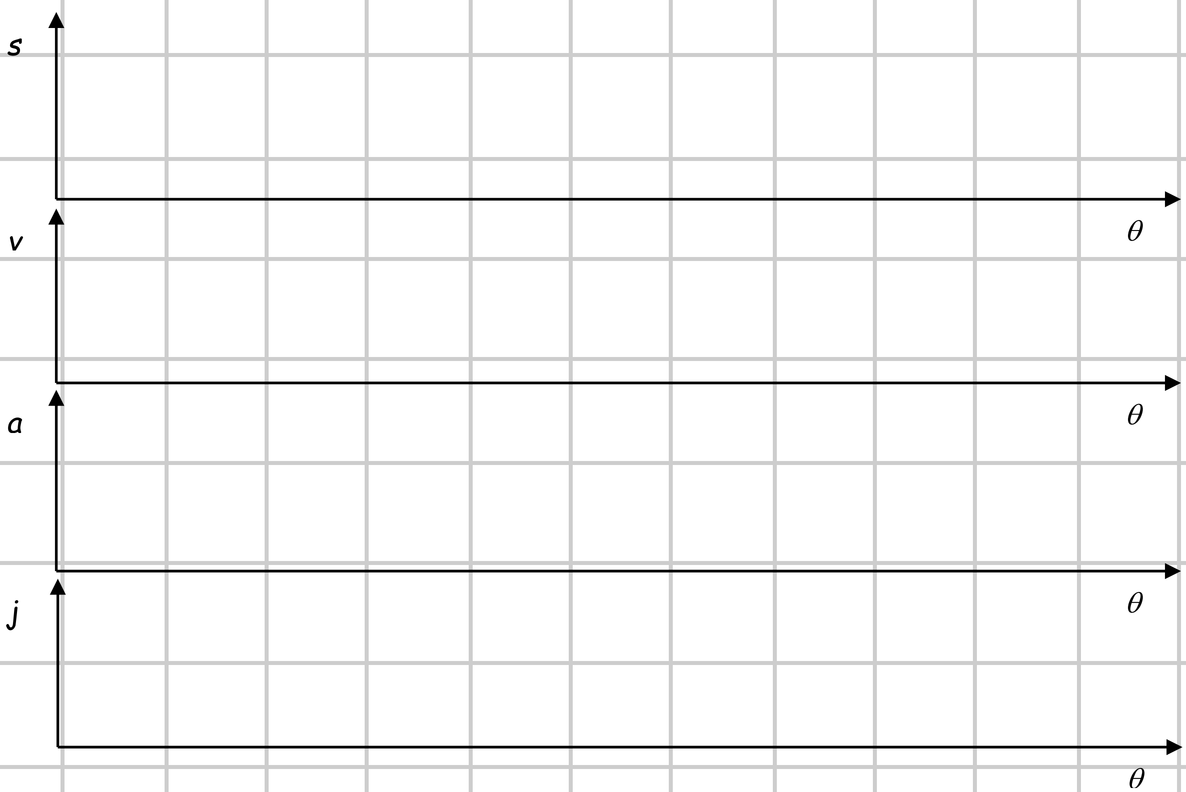

(a)
(b)
(c)
(d)


(a)
(c)
(e)
(g)
(b)
(d)
(f)
(h)

Cam Function Synthesis

a. "law of Cams"

b. Sample Cam functions



Constant-velocity transmissions:

Most common:

Gear trains, belts & pulleys, gears & sprockets

Governing equation: $\frac{\omega_1}{\omega_2} = -\frac{N_2}{N_1}$

Gears:

Gears 101: Gear tooth Terminology, Gears in mesh:

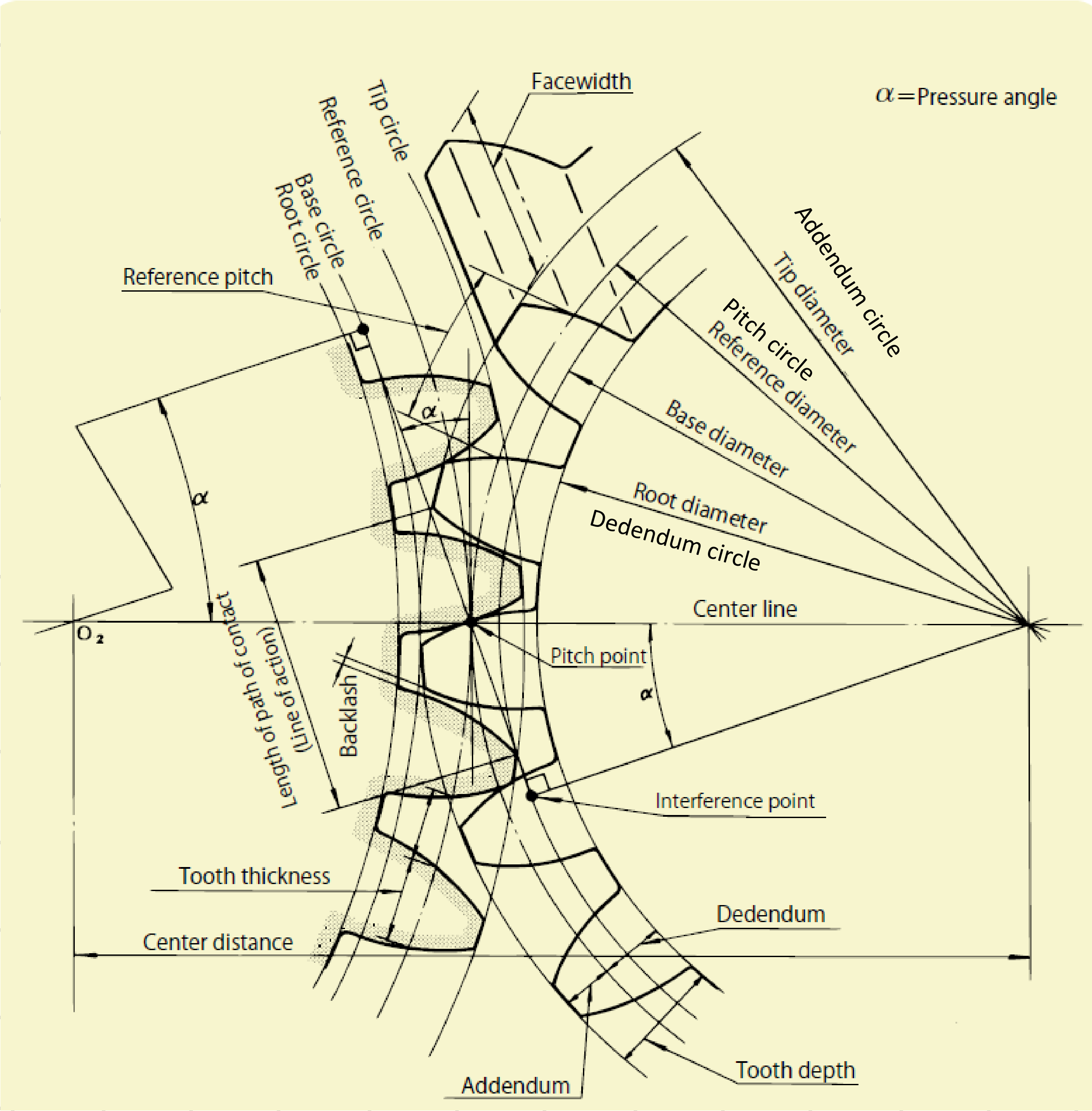
Based on this involute geometry of gear teeth, the geometry of a gear can be standardized and named, as in the following figures.

All gears have involute tooth profiles

For gears to mesh, they must have the same pitch and pressure angle.

Gear Pitch:

Pitch = Diametral Pitch = P = P_d = N/D



Facewidth
α=Pressure angle
Tip circle
Reference circle
Base circle
Root circle
Reference pitch
Addendum circle
Tip diameter
Pitch circle
Reference diameter
Base diameter
Root diameter
Dedendum circle
α
α
Center line
O₂
Length of path of contact
(Line of action)
Backlash
Pitch point
α
Interference point
Tooth thickness
Dedendum
Center distance
Tooth depth
Addendum

McMASTER-CARR

standard gears

Narrow By Clear All

Pressure Angle
20°

Number of Teeth
12, 14, 15, 16, 18, 20, 21, 24, 25, 28, 30, 32, 35, 36, 40, 48, 80

Pitch
8, 12, 16, 20, 24, 32, 48, 64

Material
Brass
Steel

Face Width
1/8", 3/16", 1/4", 1/2", 3/4", 1", 1 1/2"

Overall Width
0.315", 3/8", 0.438", 1/2", 0.56", 0.94", 1", 1 1/4", 1.37", 1 1/2", 1.62", 1.88", 2.25", 2.38"

Mount Type
Press Fit

46 Products

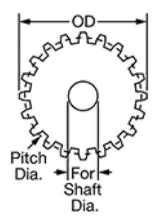
About Gears
More

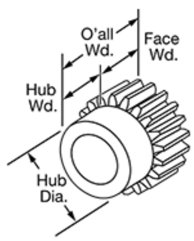
High-Load Metal Gears—20° Pressure Angle

Also known as spur gears, these gears have teeth with a 20° pressure angle, which have a wider profile
they have greater strength to handle high loads.

For gears and racks to mesh correctly, they must have the same pressure angle and pitch. Use the
components that have a 20° pressure angle.

CAD For technical drawings and 3-D models, click on a part number.

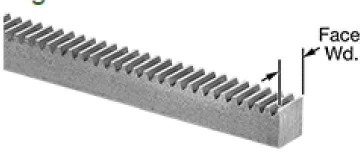




Press-Fit Mount

Pitch	Number of Teeth	Pitch Dia.	OD	Face Wd.	Overall Wd.	Bore Type	For Shaft Dia.	Hub Dia.	Hub Wd.		Each
Brass											
64	16	0.25"	0.28"	1/8"	0.315"	Plain	1/8"	0.19"	0.19"	7880K11	$11.37
64	24	0.375"	0.41"	1/8"	0.315"	Plain	1/8"	0.28"	0.19"	7880K13	13.45
64	28	0.438"	0.47"	1/8"	0.315"	Plain	1/8"	0.34"	0.19"	7880K14	14.65
48	12	0.25"	0.29"	1/8"	0.315"	Plain	1/8"	0.18"	0.19"	7880K17	11.37
48	15	0.312"	0.35"	1/8"	0.315"	Plain	1/8"	0.22"	0.19"	7880K18	12.33
48	18	0.375"	0.42"	1/8"	0.315"	Plain	1/8"	0.28"	0.19"	7880K19	13.45
48	24	0.5"	0.54"	1/8"	0.375"	Plain	3/16"	0.38"	0.25"	7880K21	15.78
48	36	0.75"	0.79"	1/8"	0.375"	Plain	3/16"	0.5"	0.25"	7880K23	18.71
48	48	1"	1.04"	1/8"	0.375"	Plain	1/4"	0.63"	0.25"	7880K24	29.43
32	12	0.375"	0.44"	3/16"	0.438"	Plain	1/8"	0.28"	0.25"	7880K25	13.45
32	14	0.438"	0.5"	3/16"	0.438"	Plain	1/8"	0.34"	0.25"	7880K26	14.22
32	16	0.5"	0.56"	3/16"	0.438"	Plain	3/16"	0.4"	0.25"	7880K27	15.08
32	20	0.625"	0.69"	3/16"	0.438"	Plain	3/16"	0.47"	0.25"	7880K29	16.96
32	24	0.75"	0.81"	3/16"	0.438"	Plain	3/16"	0.53"	0.25"	7880K31	19.73
32	28	0.875"	0.94"	3/16"	0.438"	Plain	3/16"	0.59"	0.25"	7880K32	21.22
32	36	1.125"	1.19"	3/16"	0.438"	Plain	1/4"	0.72"	0.25"	7880K34	27.12
32	40	1.25"	1.31"	3/16"	0.438"	Plain	1/4"	0.72"	0.25"	7880K35	29.51
32	48	1.5"	1.56"	3/16"	0.438"	Plain	1/4"	0.78"	0.25"	7880K36	34.73
24	12	0.5"	0.58"	1/4"	0.5"	Plain	3/16"	0.38"	0.25"	7880K37	22.45
24	15	0.625"	0.71"	1/4"	0.5"	Plain	3/16"	0.5"	0.25"	7880K38	22.45
24	18	0.75"	0.83"	1/4"	0.5"	Plain	3/16"	0.54"	0.25"	7880K39	25.08
24	21	0.875"	0.96"	1/4"	0.5"	Plain	3/16"	0.6"	0.25"	7880K41	23.88
24	36	1.5"	1.58"	1/4"	0.5"	Plain	1/4"	0.79"	0.25"	7880K44	37.96
24	48	2"	2.08"	1/4"	0.56"	Plain	5/16"	0.92"	0.31"	7880K45	53.14

High-Load Metal Gear Racks—20° Pressure Angle



These gear racks have teeth with a 20° pressure angle, which have a wider profile than 14 1/2° teeth, so they have greater strength to handle high loads.

For gears and racks to mesh correctly, they must have the same pressure angle and pitch. Use these racks with other components that have a 20° pressure angle.

Steel gear racks come longer than the length listed so you have room to finish the ends.

CAD For technical drawings and 3-D models, click on a part number.

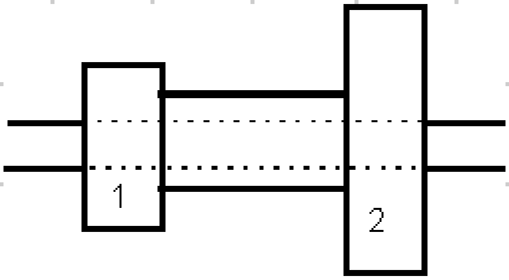
Pitch	Face Wd.	Ht.	2-ft. Length		4-ft. Length		6-ft. Length	
				Each		Each		Each
Brass								
64	1/8"	1/8"	7854K11	$31.98	____	—	____	—
48	1/8"	1/8"	7854K12	31.98	____	—	____	—
32	3/16"	3/16"	7854K13	33.73	____	—	____	—
24	1/4"	1/4"	7854K15	39.67	____	—	____	—
Steel								
20	1/2"	1/2"	5174T1	19.30	5174T11	$36.18	5174T21	$51.86
16	3/4"	3/4"	5174T2	23.46	5174T12	44.62	5174T22	64.46
12	1"	1"	5174T3	34.52	5174T13	63.14	5174T23	92.28
8	1 1/2"	1 1/2"	5174T4	67.08	5174T14	129.36	5174T24	189.86

Details of Involute Gears

Spur Gear Formulas

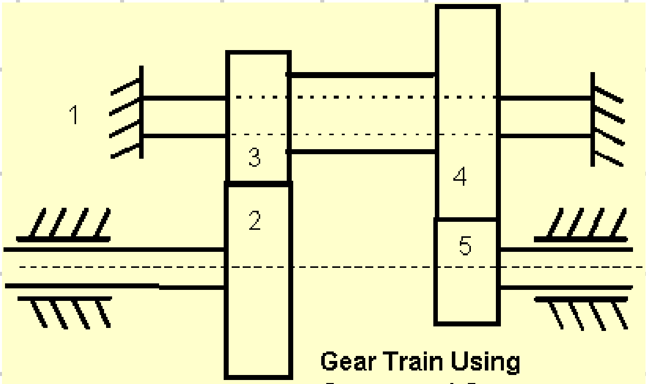
To Find	$14\frac{1}{2}$ degree Pressure Angle	20 and 25 degree Pressure Angles
Addendum, a	$a = \dfrac{1.0}{P}$	$a = \dfrac{1.0}{P}$
Dedendum, b	$b = \dfrac{1.157}{P}$	$b = \dfrac{1.250}{P}$
Pitch diameter, D	$D = \dfrac{N}{P}$	$D = \dfrac{N}{P}$
Outside diameter, D_o	$D_o = \dfrac{N + 2}{P}$	$D_o = \dfrac{N + 2}{P}$
Number of teeth, N	$N = D \times P$	$N = D \times P$
Tooth thickness, t	$t = \dfrac{1.5708}{P}$	$t = \dfrac{1.5708}{P}$
Whole depth, h_t	$h_t = \dfrac{2.157}{P}$	$h_t = \dfrac{2.250}{P}$
Clearance, c	$c = \dfrac{.157}{P}$	$c = \dfrac{.250}{P}$
Center distance, C	$C = \dfrac{N_1 + N_2}{2 \times P}$	$C = \dfrac{N_1 + N_2}{2 \times P}$
Working depth, h_k	$h_k = \dfrac{2}{P}$	$h_k = \dfrac{2}{P}$
Chordal tooth thickness, t_c	$t_c = D \sin\left(\dfrac{90 \text{ degrees}}{N}\right)$	$t_c = D \sin\left(\dfrac{90 \text{ degrees}}{N}\right)$
Chordal addendum, a_c	$a_c = a + \dfrac{t^2}{4D}$	$a_c = a + \dfrac{t^2}{4D}$
Diametral pitch, P	$P = \dfrac{N}{D}$	$P = \dfrac{N}{D}$
Center distance, C	$C = \dfrac{D_1 + D_2}{2}$	$C = \dfrac{D_1 + D_2}{2}$

Compound Gears:



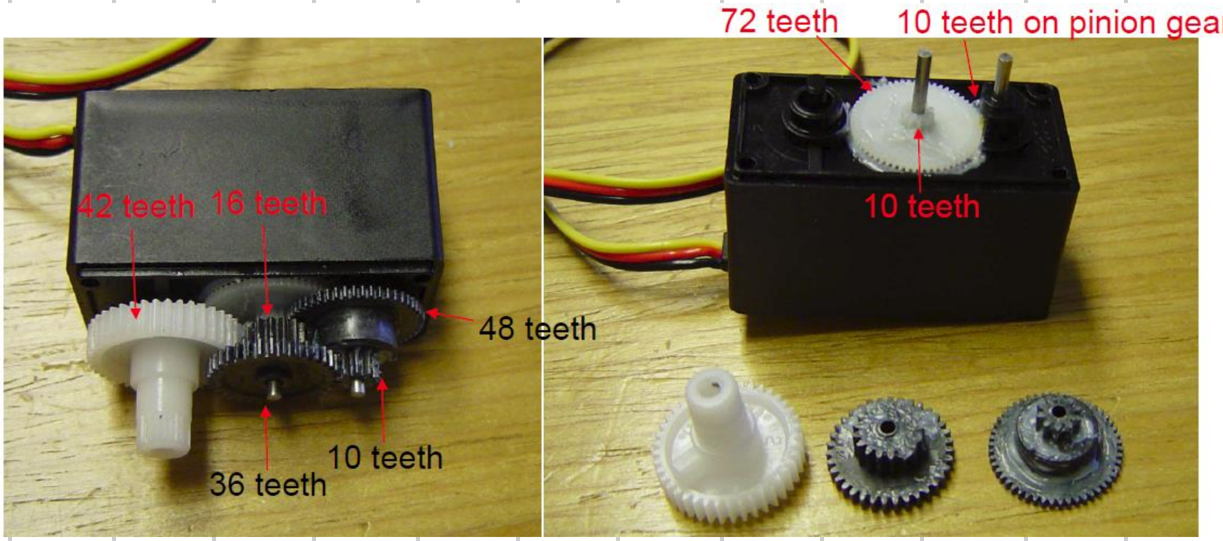
Compound Gears

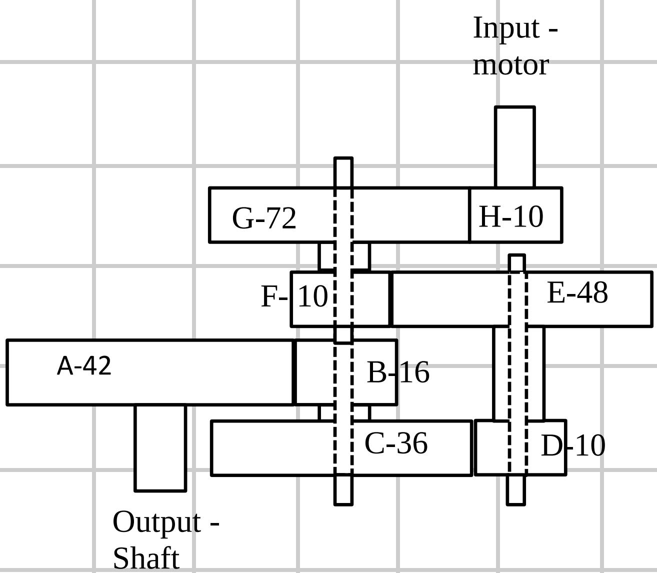
Example 1:



Gear Train Using Compound Gears

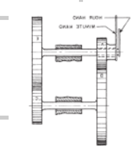
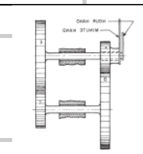
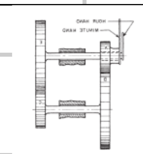
Example 2:


72 teeth
10 teeth on pinion gear
42 teeth
16 teeth
48 teeth
10 teeth
36 teeth
10 teeth


Input - motor
G-72
H-10
F- 10
E-48
A-42
B-16
C-36
D-10
Output - Shaft

Gear train design:

Functional requirements on a Gear train:

#	Requirement	Example	reference
1	Gear ratio	$m_v = \dfrac{N_3 N_5}{N_2 N_4}$	2-stage train 
2	Center distance	$C_1 = R_2 + R_3 = \dfrac{N_2 + N_3}{2p_d}$	
3	Center distance	$C_2 = R_4 + R_5 = \dfrac{N_4 + N_5}{2p_d}$	
4	Reverted (equal center distance)	$C_1 = C_2$ $\dfrac{N_2 + N_3}{2p_d} = \dfrac{N_4 + N_5}{2p_d}$ $N_2 + N_3 = N_4 + N_5$