

4 Orthogonalität und orthogonale Funktionen

4.1	Orthogonalität und Näherung
4.2	Harmonische Funktionen
4.2.1	Eindimensionale Funktionen
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Geschichtliches

Walsh, J.L.: A closed set of orthogonal functions. Amer. J. of Math. (1923)5-24

Harmuth, H.F.: On the transmission of information by orthogonal time functions. Trans. AIEE Com.and Electr. (1960)248-255

Harmuth, H.F.: Verallgemeinerung des Fourier-Integrales und des Begriffes Frequenz. Arch.Elektr. Übertr.(1964)439-451

Pichler, F.: Das System der sal- und cal-Funktionen als Erweiterung des Systems der Walshfunktionen und die Theorie der sal- und cal-Fouriertransformation. Dissertation, Universität Innsbruck (1967)

Erstes Symposium „Sequenztechnik“ 1968 im Fernmeldetechnischen Zentralamt der Deutschen Bundespost, Darmstadt



Joseph Leonard Walsh

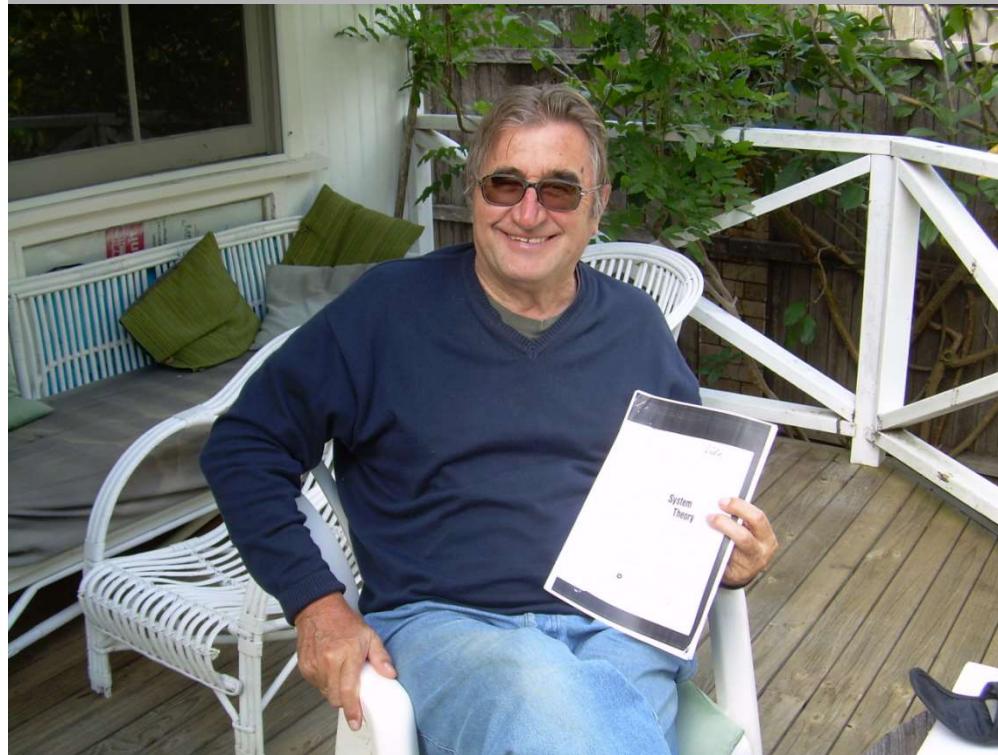
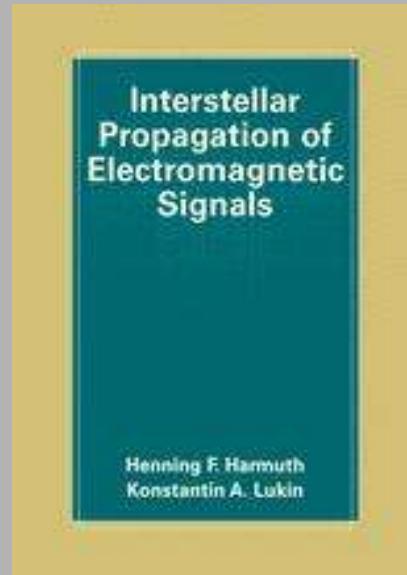
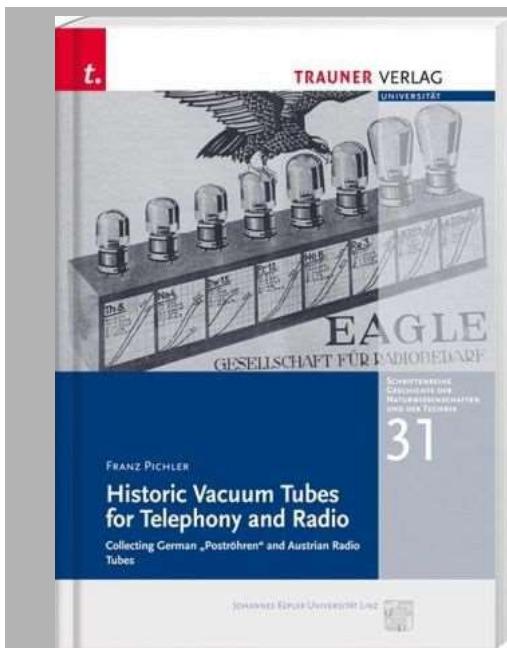
Born: 21 Sept 1895 in Washington DC

Died: 6 Dec 1973 in College Park, Md



http://www-history.mcs.st-andrews.ac.uk/Mathematicians/Walsh_Joseph.html

<http://www-history.mcs.st-andrews.ac.uk/BiogIndex.html>



THE TRANSPOSITION OF ELECTRICAL CONDUCTORS,
BY FRANK F. FOWLE.

The transmission of electromagnetic energy through the ether, by means of conducting wires whose function is that of directing and concentrating the energy flow, is accompanied by mutual interferences between transmission systems which employ in common any considerable portion of the ether. The extent to which the ether is energized, in a direction transverse to the direction of energy flow, is controlled by the design of the transmission circuit and the rate of energy flow. The energy storage-capacity of a transmission circuit is very small in comparison with the transmitted energy, but the great velocity of transmission renders possible the transmission of large energies. The conformations of the electric and the magnetic fields about a line circuit depend, in alternating-current systems, on the number of phases and on the number, size, and separation of the wires. The field at a given point due to a single-phase circuit is constant in direction but variable in intensity, from instant to instant throughout a cycle; the field due to a polyphase circuit varies both in direction and intensity.

Alternating-current transmission systems resolve themselves into two great classes, one for the transmission of power and the other for the transmission of intelligence. Power systems are characterized by: high pressure; large currents; low or moderate frequencies; only one fundamental frequency; relatively great reaction of terminal apparatus compared with line reactions; magnitude of inductive disturbances in the line usually small and important only as affecting regulation; line length less than a wave length, and the flow of energy usually

reduce the cross-talk. It is necessary to devise different types of transposed circuits, no two transposed alike, in order to treat the cases occurring in practice. The manner of doing this is shown in Fig. 11.

The "exposure," as it is termed, of circuit 1 to circuit 2 is $\frac{1}{2}$; of 1 to 3 is $\frac{1}{4}$; of 2 to 3 is $\frac{1}{2}$; because a transposition at the junction of two sections, each transposed at its centre, has almost no beneficial effect. The exposure of 1 to 5 is $\frac{1}{8}$; of 2 to 6 and 3 to 7, $\frac{1}{16}$; of 2 to 8 and 2 to 9, $\frac{1}{16}$; and so on. The tabulated exposures are given in Fig. 12, in terms of the length l of a

NO. OF TRANS.	TYPE NO. DERIVATION
1	X
2	X X
3	X X X
4	X X X X
5	X X X X X
6	X X X X X X
7	X X X X X X X
8	X X X X X X X X
9	X X X X X X X X X
10	X X X X X X X X X X
11	X X X X X X X X X X X
12	X X X X X X X X X X X X
13	X X X X X X X X X X X X X
14	X X X X X X X X X X X X X X
15	X X X X X X X X X X X X X X X X

Diagram below the table shows a transposition section of length l divided into n equal segments of length $\frac{l}{n}$. The first three segments are labeled $\frac{1}{10}l$, $\frac{1}{10}l$, and $\frac{1}{10}l$. The fourth segment is labeled $\frac{1}{10}l$. The fifth segment is labeled $\frac{1}{10}l$. The sixth segment is labeled $\frac{1}{10}l$. The seventh segment is labeled $\frac{1}{10}l$. The eighth segment is labeled $\frac{1}{10}l$. The ninth segment is labeled $\frac{1}{10}l$. The tenth segment is labeled $\frac{1}{10}l$. The eleventh segment is labeled $\frac{1}{10}l$. The twelfth segment is labeled $\frac{1}{10}l$. The thirteenth segment is labeled $\frac{1}{10}l$. The fourteenth segment is labeled $\frac{1}{10}l$. The fifteenth segment is labeled $\frac{1}{10}l$.

FIG. 11.

transposition section. The derivation of the types is simple; the first three are obvious. The fourth is obtained by doubling the number of divisions of the transposition section from two to four. The fifth is obtained by superposing the first type on the fourth type, the sixth by superposing the second on the fourth, etc. The composition of the complicated types is shown in Fig. 11. So far as the writer is aware this synthetical system of deriving types of dissimilarly transposed circuits is due to Mr. John A. Barrett. The method may be extended as far as desired, but 15 types are usually sufficient.

It is very common for a telephone line to be run along the same route as a power line, possibly for a few miles only or, in a few cases, for many miles. In the case of a communication line which is the property of a power company, this line may be run on the same towers as the power line. Interference with such communication circuits may be due to both electromagnetic and electrostatic action, the former producing currents which are superposed on the true speech currents, thereby setting up distortion, and the latter raising the potential of the communication circuit as a whole.

<http://www.myinsulators.com/acw/bookref/interference/>

The disturbances can be kept down by means of a thorough transposition of the conductors of both the power line and the telephone line. This transposition has the effect of splitting the induced E.M.F. into a series of mutually opposing E.M.F.s. In the case of a telephone line running parallel to a single-circuit power line, if the power line has no branch lines then a single transposition of the conductors of the telephone line is theoretically sufficient, but with both circuits run on the same towers it may be necessary to transpose the power conductors every three or four miles, and the telephone conductors about every 500 feet. ... A possible scheme is shown in Fig. 9.4, from which it will be seen that the scheme of transposition is a regular one for each individual circuit, and that it is arranged that not more than one transposition will take place at any one point in the line.

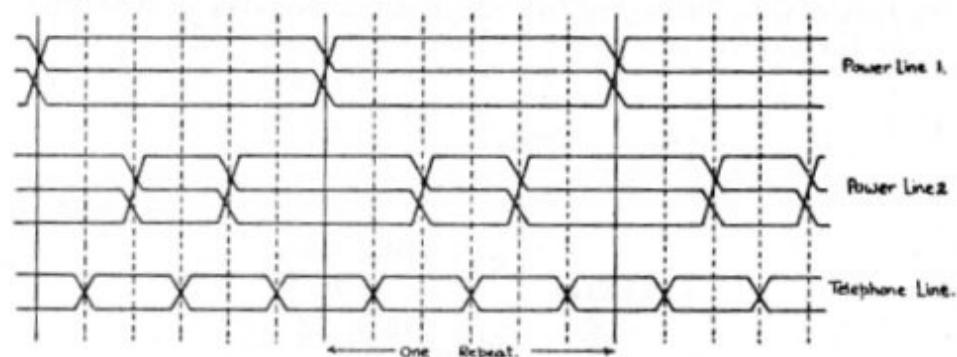


FIG. 9.4.—TRANSPOSITION SCHEME FOR TWO POWER LINES AND A TELEPHONE LINE.

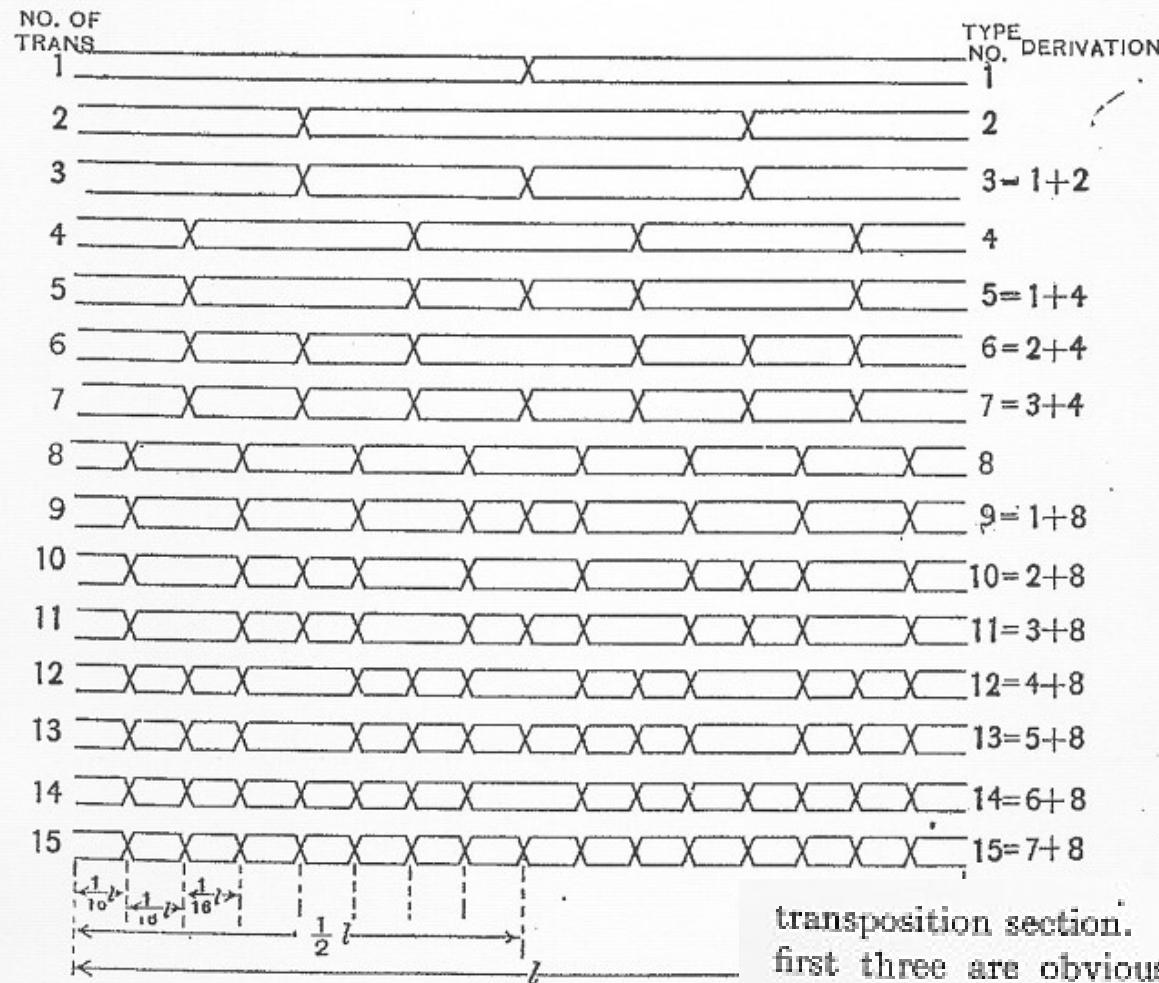


FIG. 11.

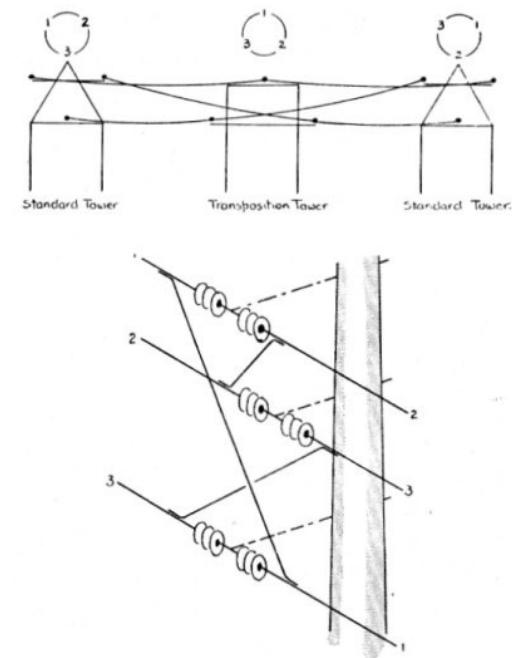


FIG. 9.5.—TWO METHODS OF TRANPOSITION.

transposition section. The derivation of the types is simple; the first three are obvious. The fourth is obtained by doubling the number of divisions of the transposition section from two to four. The fifth is obtained by superposing the first type on the fourth type, the sixth by superposing the second on the fourth, etc. The composition of the complicated types is shown in Fig. 11. So far as the writer is aware this synthetical system of deriving types of dissimilarly transposed circuits is due to Mr. John A. Barrett. The method may be extended as far as desired, but 15 types are usually sufficient.

2 Rademacher- und Walsh-Funktionen

2.1 Definition, grundlegende Eigenschaften

Definition 2.1: Die Funktion $r_k: \mathbb{R}_0^+ \rightarrow [-1,1], x \mapsto (-1)^{\pi_{k+1}(x)}, k \in \mathbb{Z}$ heißt k -te Rademacher-Funktion.

Bemerkung: Leider nicht so häufig werden Rademacher-Funktionen definiert als $\tilde{r}_k(x) := (-1)^{\pi_k(x)}$ (vgl. [Ta]). Wiewohl offenkundig äquivalent zur hier gegebenen Definition, haben die so definierten Funktionen einige schreibtechnische Vorteile. Da zumeist die hiesige Definition verwendet wird, habe ich mich für die etwas umständlichere Notation entschieden.

Beobachtungen:

- $r_{k+m}(x) = r_k(2^m x) \quad \forall k, m \in \mathbb{Z}$, die Rademacher-Funktionen gehen bei Dehnung um den Faktor 2 ineinander über.
- r_k ist reell-periodisch mit primitiver Periode 2^{-k} , \mathbb{K}_k ist ein Fundamentalbereich von r_k .
- Rademacher-Funktionen sind auf dyadischen Intervallen konstant: $r_k(\mathbb{K}_{k+1}(x)) = r_k(x)$.

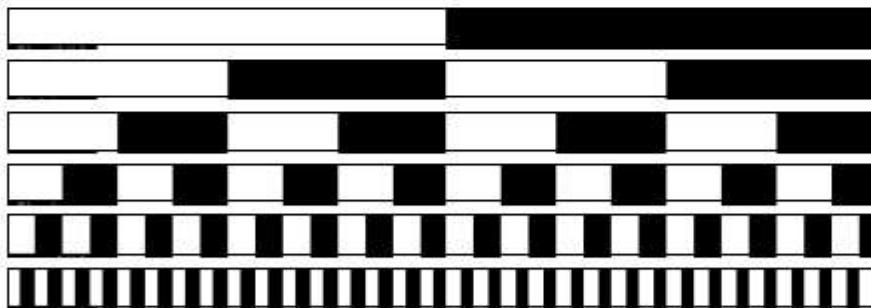
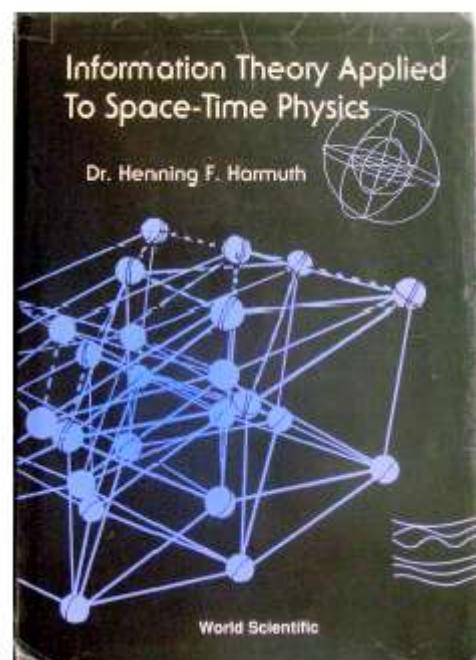
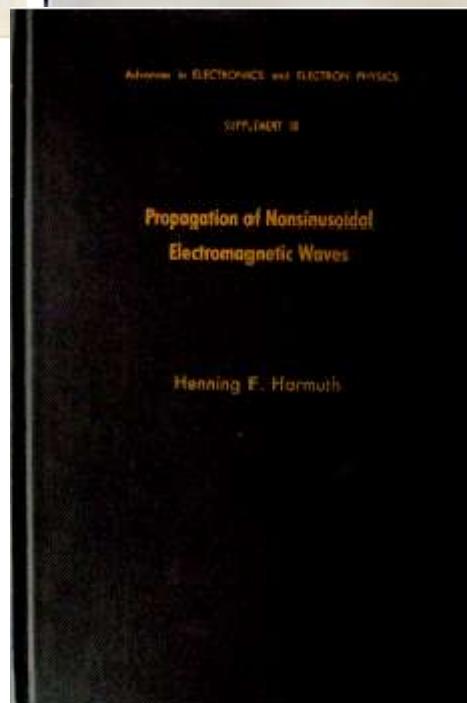
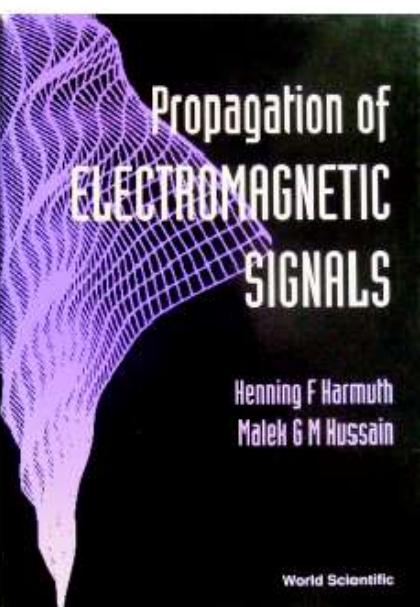
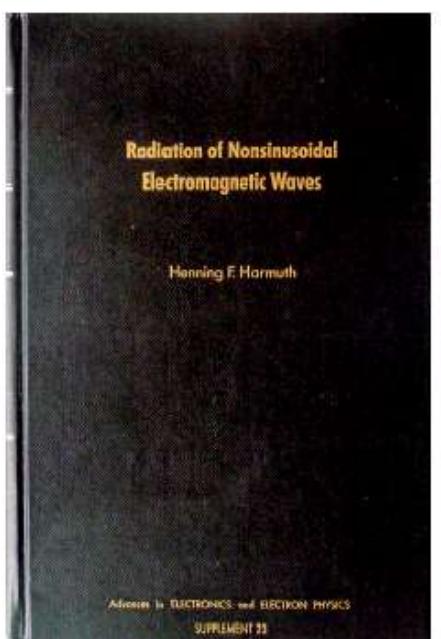
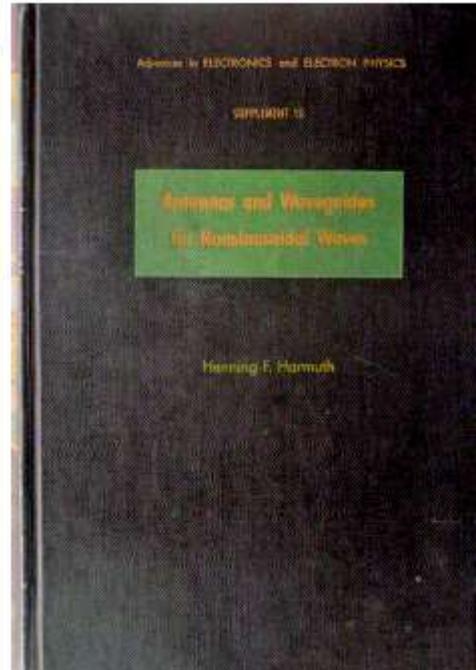
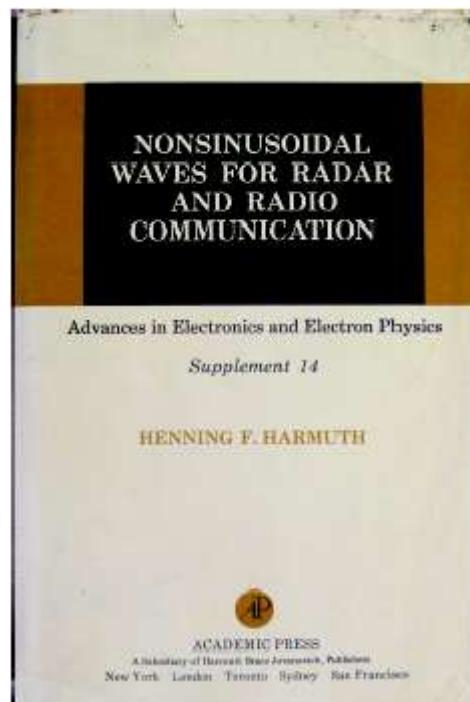
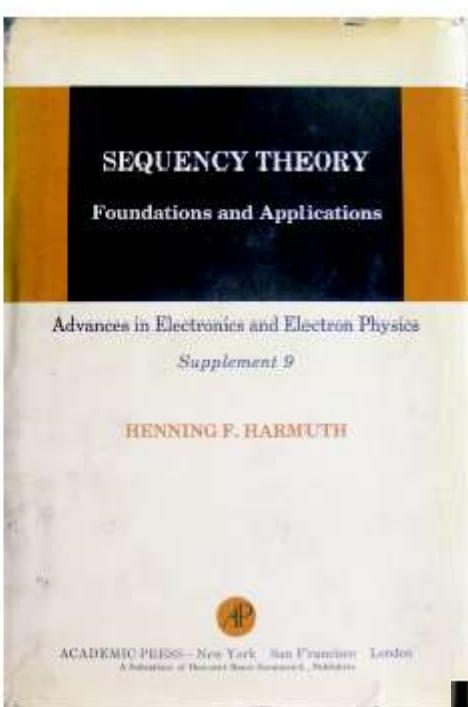
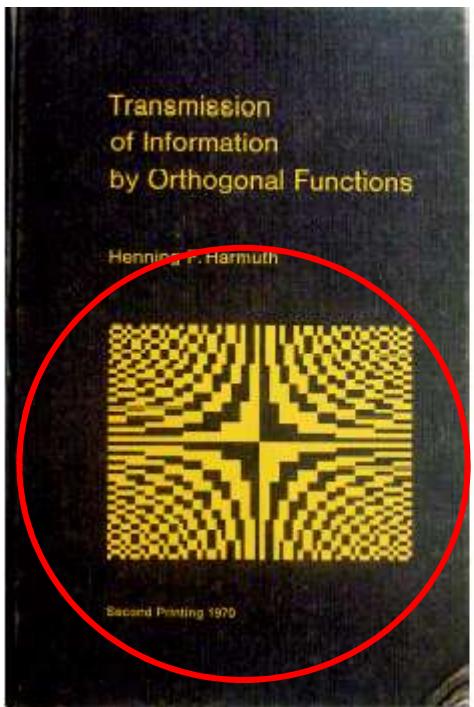
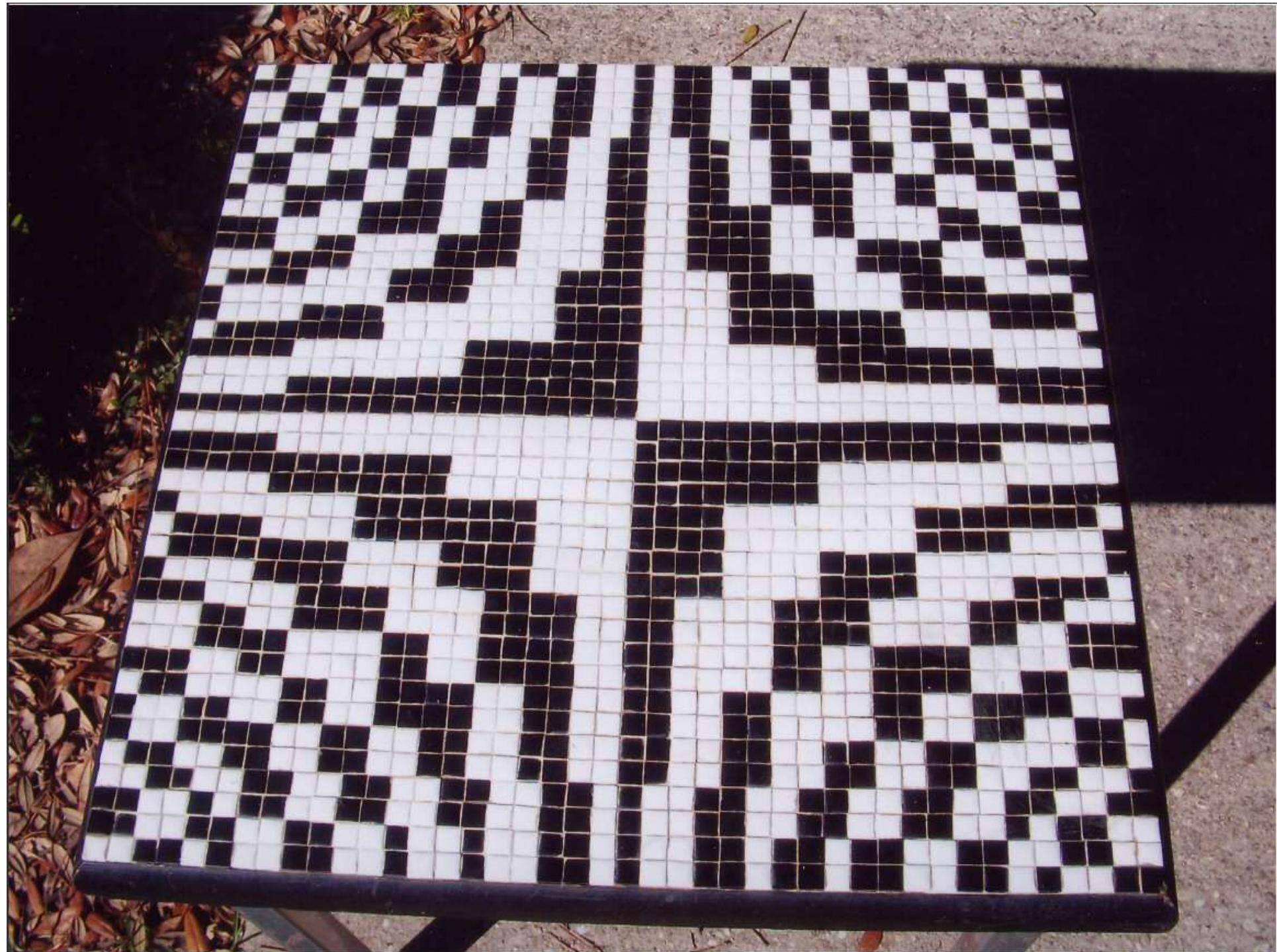


Abbildung 1: Die Rademacher-Funktionen r_0 bis r_5 auf $[0;1]$; weiß=1, schwarz=-1

Die Rademacher-Funktionen $(r_k)_{k \geq 0}$ sind bezüglich des Standard-Skalarprodukts auf $L^2[0;1]$ orthogonal, aber nicht vollständig: $\langle r_i, r_j \rangle = 0 \quad \forall i \in \mathbb{Z}$, aber $r_2 r_3 \neq 0$. Die Menge der Rademacher-Funktionen muß also geeignet ergänzt bzw. modifiziert werden, um „genügend“ Funktionen (also etwa alle stetigen oder quadratintegrablen) „darstellen“ (als Summe oder Reihe oder Integral der Basisfunktionen) zu können. Zwei Möglichkeiten bieten sich an:





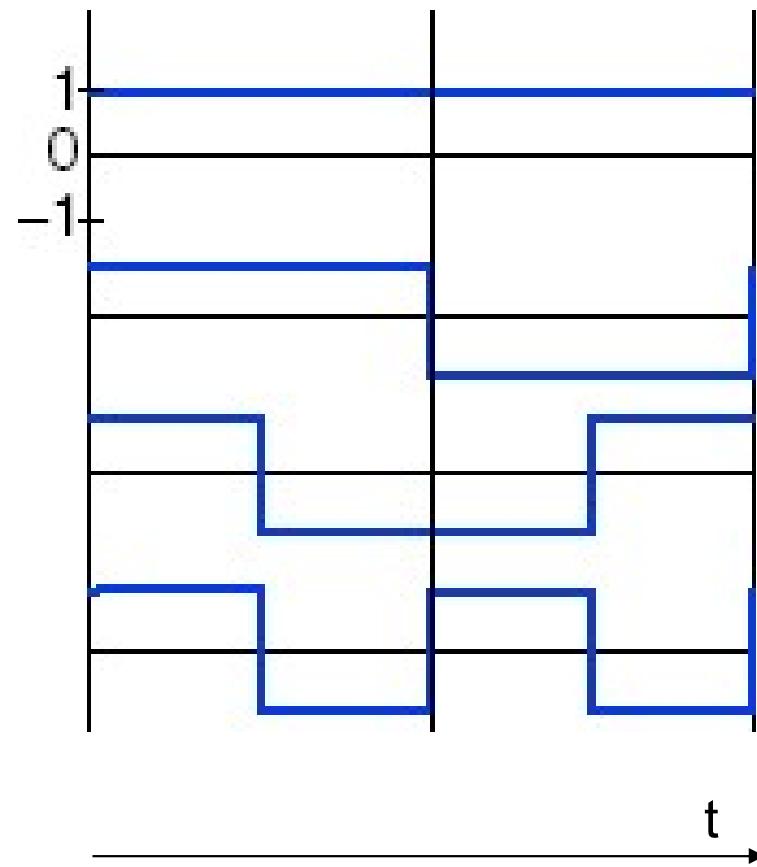


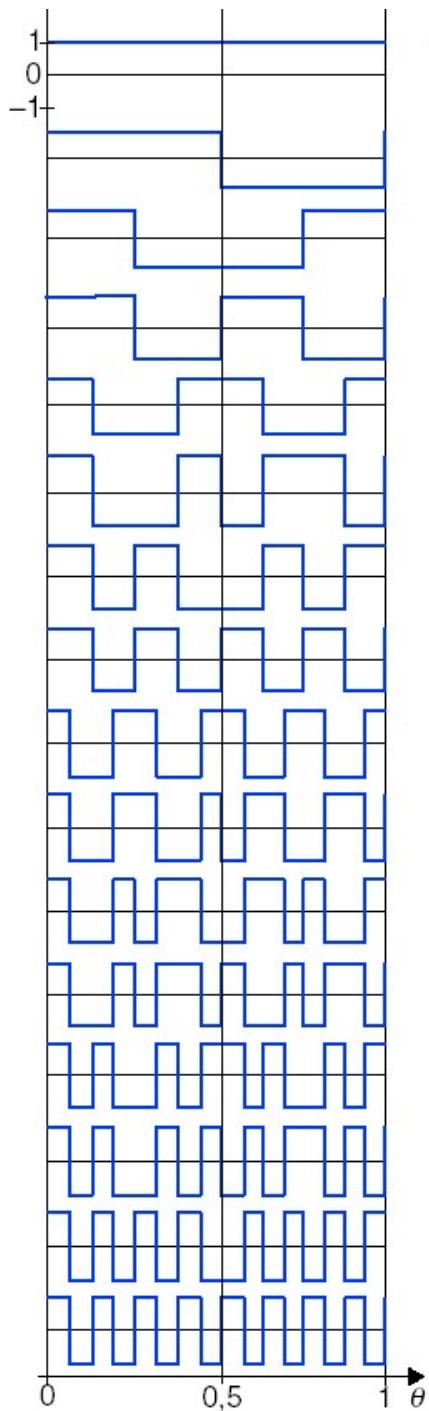


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Walshfunktionen: orthonormal, rechteckförmig, zweiwertig,
nichtäquidistante Nulldurchgänge





$s \text{ wal}(i, \theta)$	$b \text{ wal}(i, \theta)$	$k \text{ wal}(i, \theta)$
0	0	0
1	1	8
2	3	12
3	2	4
4	6	6
5	7	14
6	5	10
7	4	2
8	12	3
9	13	11
10	15	15
11	14	7
12	10	5
13	11	13
14	9	9
15	8	1

$S = \{ \text{wal}(\mu, \theta) \}$

$\text{cal}(s_i, \theta)$

$\text{sal}(s_i, \theta)$

s_i normierte Sequenz

entspricht der halben
Anzahl der Vorzeichen-
wechsel im halboffenen
Intervall

Walsh Function -- from Wolfram MathWorld - Mozilla Firefox

Datei Bearbeiten Ansicht Chronik Lesezeichen Extras Hilfe

Walsh Function -- from Wolfram MathWorld

mathworld.wolfram.com/WalshFunction.html

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The Walsh functions consist of trains of square pulses (with the allowed states being -1 and 1) such that transitions may only occur at fixed intervals of a unit time step, the initial state is always $+1$, and the functions satisfy certain other orthogonality relations. In particular, the 2^n Walsh functions of order n are given by the rows of the Hadamard matrix H_{2^n} when arranged in so-called "sequency" order (Thompson *et al.* 1986, p. 204; Wolfram 2002, p. 1073). There are 2^n Walsh functions of length 2^n , illustrated above for $n = 1, 2$, and 3 .

Sequenzordnung:

$$\text{swal}(0, \theta) = \begin{cases} 1 & \text{für } -\frac{1}{2} \leq \theta < \frac{1}{2} \\ 0 & \text{sonst} \end{cases}$$

$$\text{swal}(2j + p, \theta) = (-1)^{\left[\frac{j}{2}\right] + p}$$

$$\left(\text{swal} \left[j, 2 \left(\theta + \frac{1}{4} \right) \right] + (-1)^{j+p} \text{swal} \left[j, 2 \left(\theta - \frac{1}{4} \right) \right] \right)$$

mit $p \in \{0, 1\}$, $j = 0, 1, 2, \dots$ und $\left\lfloor \frac{j}{2} \right\rfloor$ als größte ganze Zahl $\leq \frac{j}{2}$.

Beispiel:

Graphische Ermittlung der Walshfunktionen **wal** (1, θ)

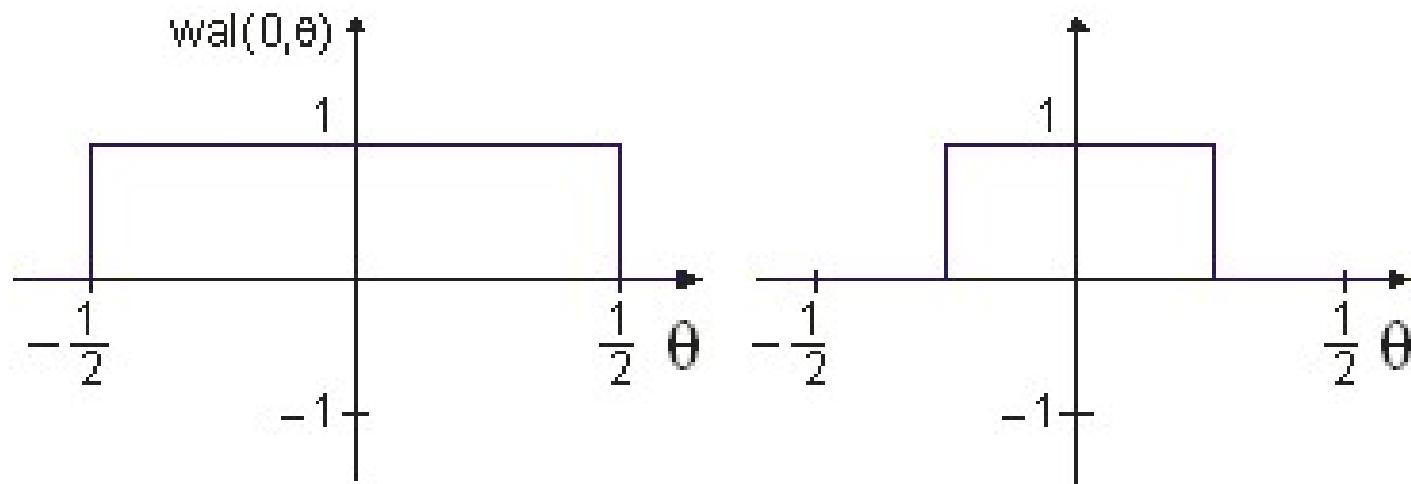
$$j=0, p=1 \quad j/2=0$$

$$\text{swal}(2j + p, \theta) =$$

$$(-1)^{\left[\frac{j}{2}\right]+p} \left(\text{swal} \left[j, 2 \left(\theta + \frac{1}{4} \right) \right] + (-1)^{j+p} \text{swal} \left[j, 2 \left(\theta - \frac{1}{4} \right) \right] \right)$$

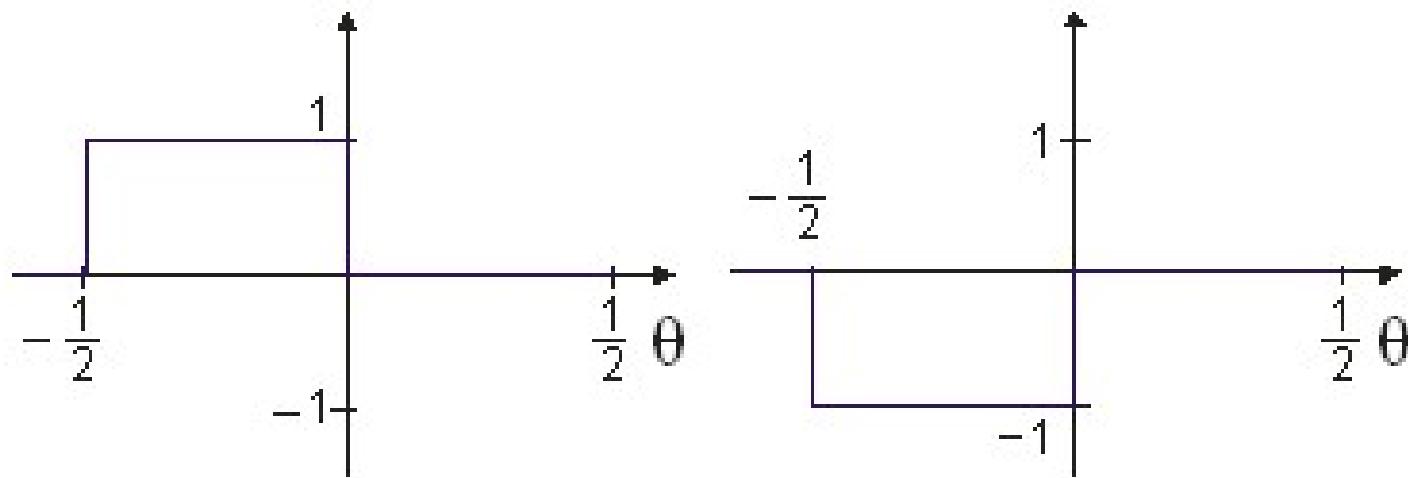
$$\text{swal}(1, \theta) = (-1)^{0+1} \{ \text{swal} [0, 2(\theta + \frac{1}{4})] + (-1)^{0+1} \text{swal} [0, 2(\theta - \frac{1}{4})] \}$$

1. Schritt: wal (0, θ) auf die Hälfte komprimieren



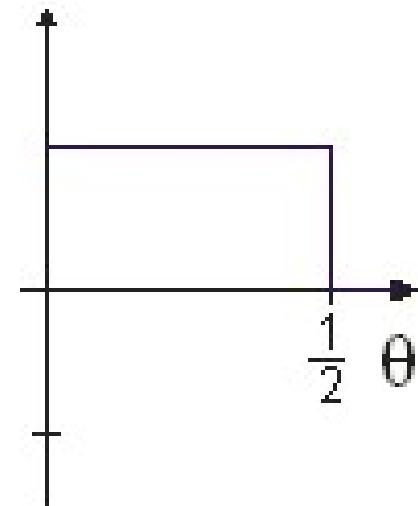
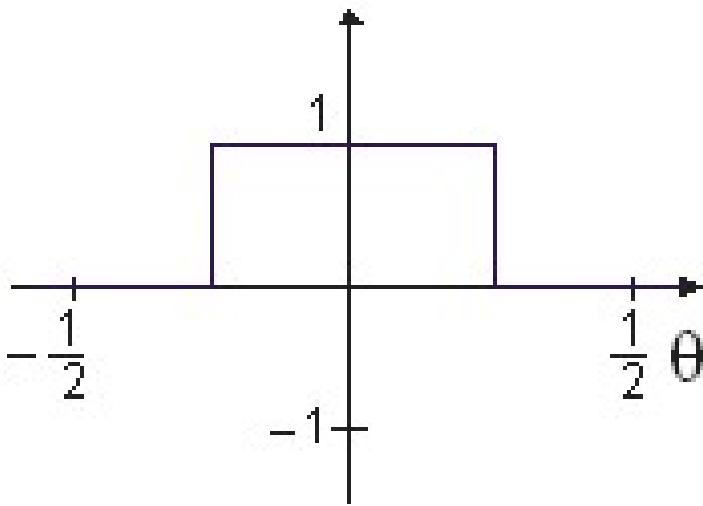
$$swal(1, \theta) = (-1)^{0+1} \{ swal [0, 2(\theta + \frac{1}{4})] + (-1)^{0+1} swal [0, 2(\theta - \frac{1}{4})] \}$$

2. Schritt: Funktion um $1/4$ nach links verschieben und mit -1 multiplizieren



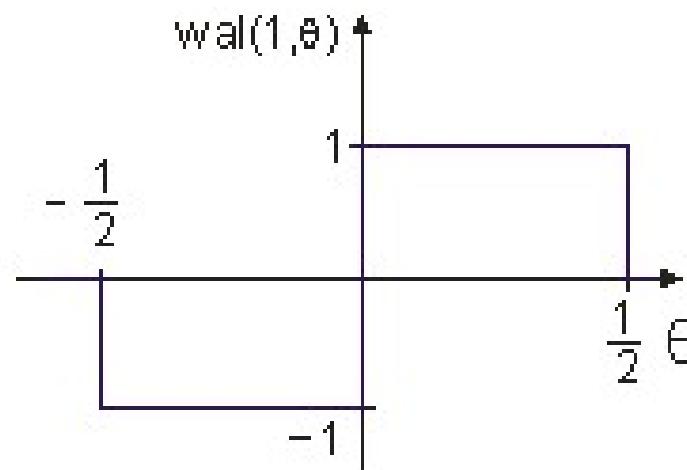
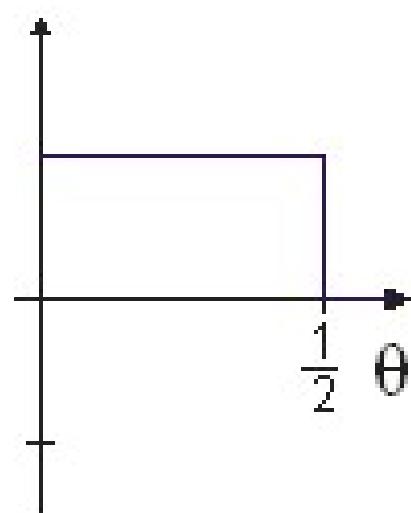
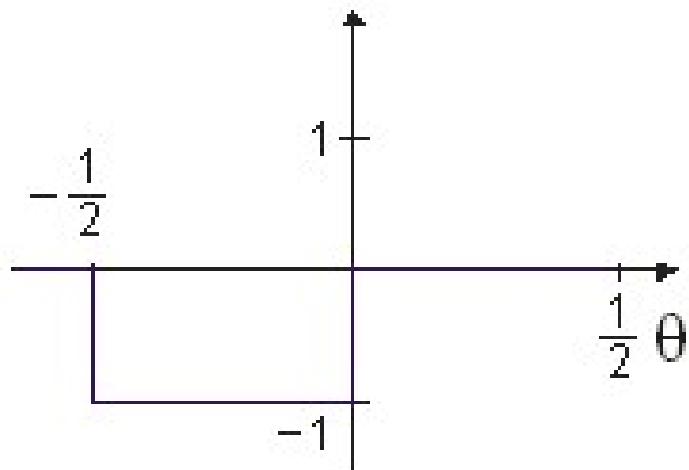
$$\text{swal}(1, \theta) = (-1)^{0+1} \{ \text{swal}[0, 2(\theta + \frac{1}{4})] + (-1)^{0+1} \text{swal}[0, 2(\theta - \frac{1}{4})] \}$$

3. Schritt: komprimierte Funktion um $1/4$ nach rechts verschieben und mit $+1$ multiplizieren



$$\text{swal}(1, \theta) = (-1)^{0+1} \{ \text{swal}[0, 2(\theta + \frac{1}{4})] + (-1)^{0+1} \text{swal}[0, 2(\theta - \frac{1}{4})] \}$$

4. Schritt: beide Teilfunktionen addieren:



$$\text{swal}(1, \theta) = (-1)^{0+1} \{ \text{swal}[0, 2(\theta + \frac{1}{4})] + (-1)^{0+1} \text{swal}[0, 2(\theta - \frac{1}{4})] \}$$

Beispiel:

Graphische Ermittlung der Walshfunktionen **wal (5, θ)**

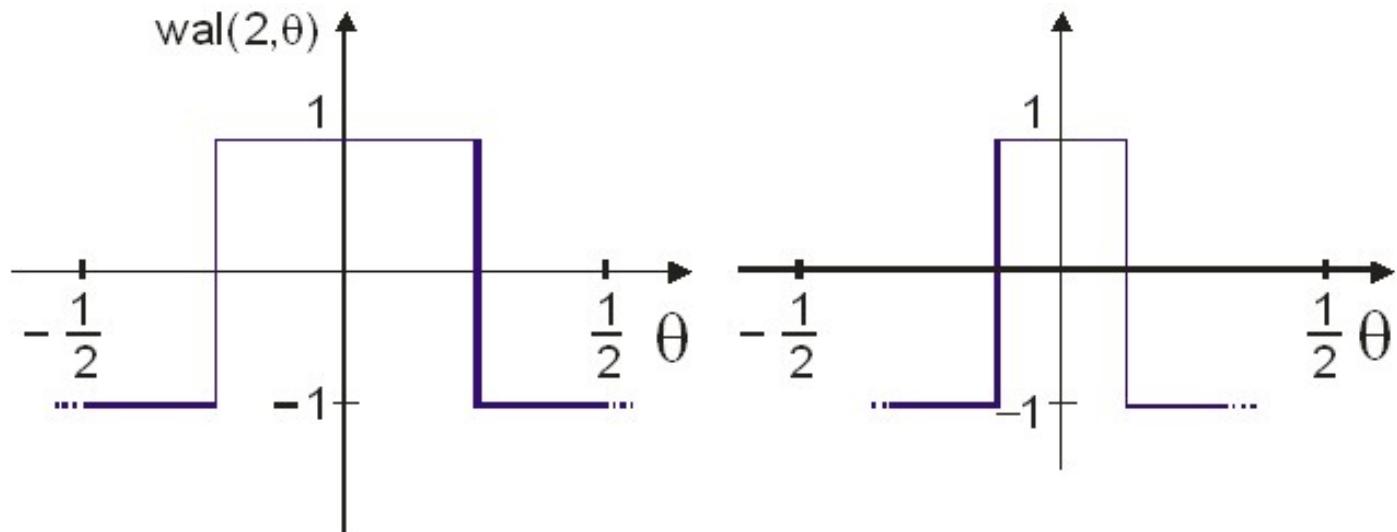
$$j=2, p=1 \quad j/2=1$$

$$\text{swal}(2j + p, \theta) =$$

$$(-1)^{\left[\frac{j}{2}\right]+p} \left(\text{swal} \left[j, 2 \left(\theta + \frac{1}{4} \right) \right] + (-1)^{j+p} \text{swal} \left[j, 2 \left(\theta - \frac{1}{4} \right) \right] \right)$$

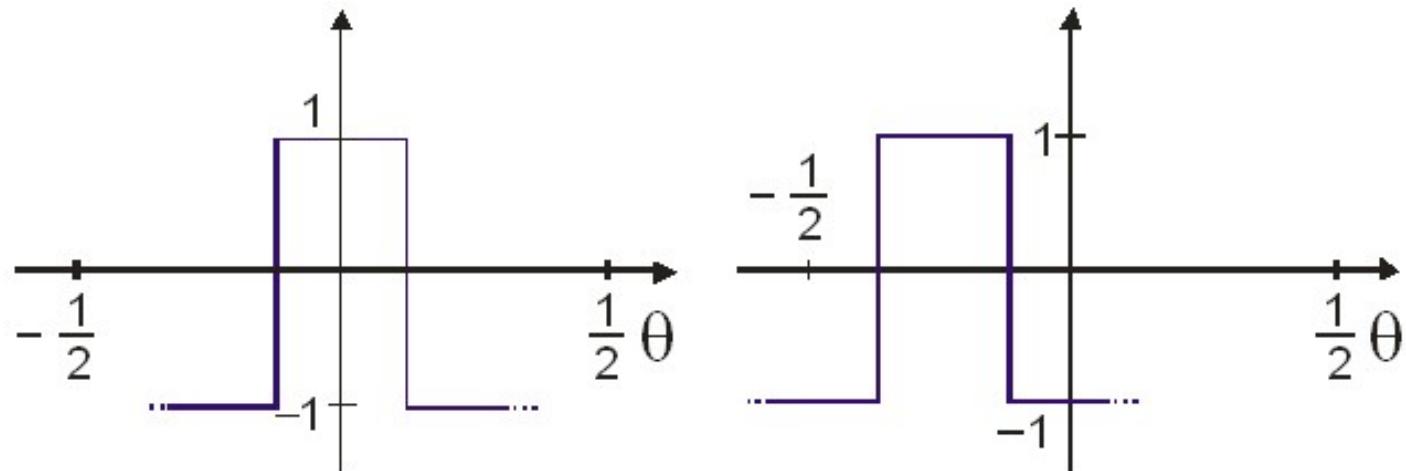
$$\text{swal}(5, \theta) = (-1)^{1+1} \{ \text{swal} [2, 2(\theta + \frac{1}{4})] + (-1)^{2+1} \text{swal} [2, 2(\theta - \frac{1}{4})] \}$$

1. Schritt: wal(2, θ) auf die Hälfte komprimieren



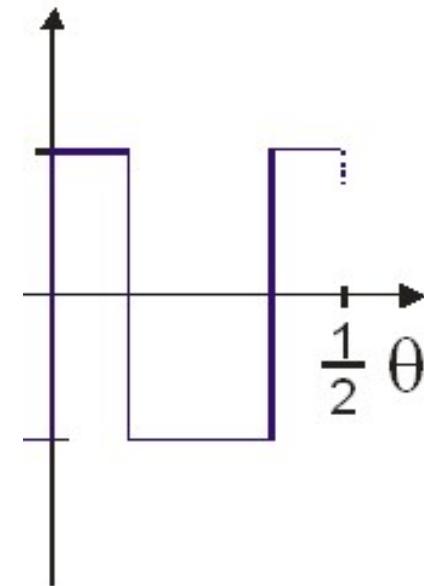
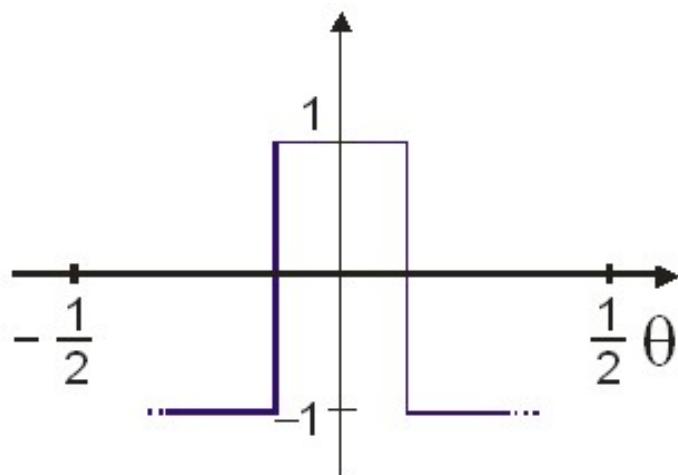
$$swal(5, \theta) = (-1)^{1+1} \{ swal [2, 2(\theta + \frac{1}{4})] + (-1)^{2+1} swal [2, 2(\theta - \frac{1}{4})] \}$$

2. Schritt: Funktion um $1/4$ nach links verschieben und mit $+1$ multiplizieren



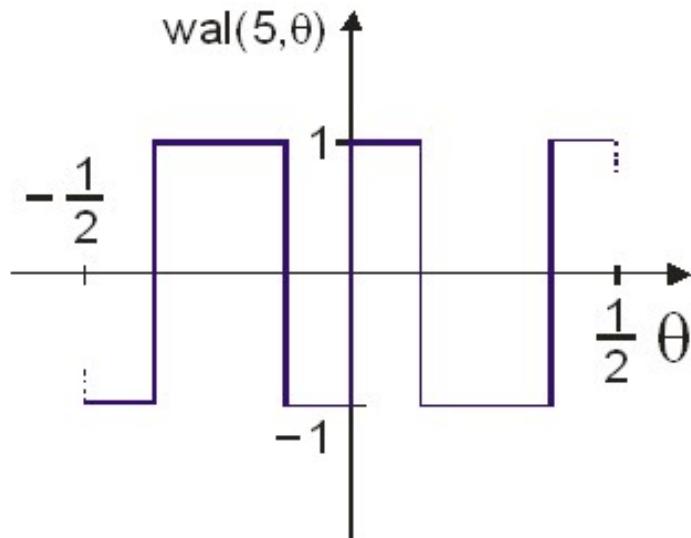
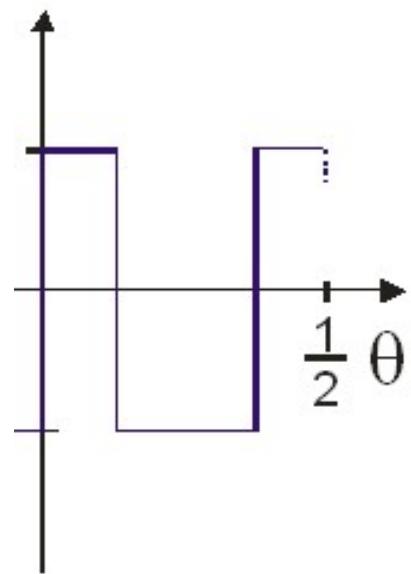
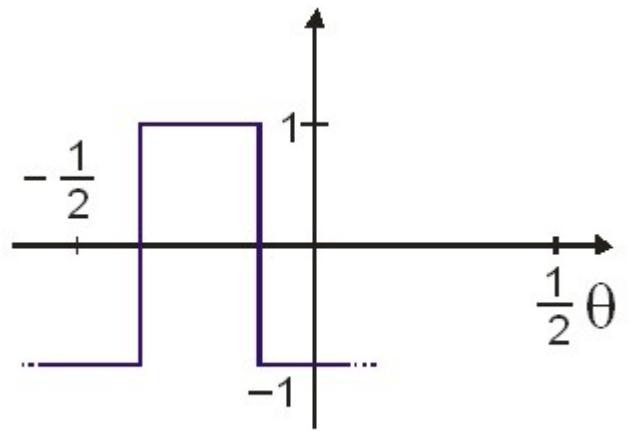
$$\text{swal}(5, \theta) = (-1)^{1+1} \{ \text{swal}[2, 2(\theta + \frac{1}{4})] + (-1)^{2+1} \text{swal}[2, 2(\theta - \frac{1}{4})] \}$$

3. Schritt: komprimierte Funktion um $1/4$ nach rechts verschieben und mit -1 multiplizieren



$$\text{swal}(5, \theta) = (-1)^{1+1} \{ \text{swal}[2, 2(\theta + \frac{1}{4})] + (-1)^{2+1} \text{swal}[2, 2(\theta - \frac{1}{4})] \}$$

4. Schritt: beide Teilfunktionen addieren



$$swal(5, \theta) = (-1)^{1+1} \{ swal[2, 2(\theta + \frac{1}{4})] + (-1)^{2+1} swal[2, 2(\theta - \frac{1}{4})] \}$$

The functions $\text{wal}(j, \theta)$ may be defined by the difference equation^{1,2}

$$\text{wal}(2j + p, \theta) = (-1)^{\lfloor j/2 \rfloor + p} \{ \text{wal}[j, 2(\theta + \frac{1}{4})] + (-1)^{j+p} \text{wal}[j, 2(\theta - \frac{1}{4})] \},$$

$p = 0 \quad \text{or} \quad 1, \quad j = 0, 1, 2, \dots$

$$\text{wal}(0, \theta) = \begin{cases} 1 & \text{for } -\frac{1}{2} \leq \theta < \frac{1}{2} \\ 0 & \text{for } \theta < -\frac{1}{2}, \quad \theta \geq +\frac{1}{2} \end{cases} \quad (2)$$

As an example, consider the cases $j = 0, p = 1$, and $j = 2, p = 1$. Using the values $[0/2] = 0$ and $[2/2] = 1$ one obtains

$$\text{wal}(1, \theta) = (-1)^{0+1} \{ \text{wal}[0, 2(\theta + \frac{1}{4})] + (-1)^{0+1} \text{wal}[0, 2(\theta - \frac{1}{4})] \} \quad j=0, p=1$$

$$\text{wal}(5, \theta) = (-1)^{1+1} \{ \text{wal}[2, 2(\theta + \frac{1}{4})] + (-1)^{2+1} \text{wal}[2, 2(\theta - \frac{1}{4})] \} \quad j=2, p=1$$

It may be verified from Fig. 111-1 that $\text{wal}(1, \theta) = \text{sal}(1, \theta)$ is obtained from $\text{wal}(0, \theta)$ by squeezing it to half its width, multiplying the function that is shifted to the left by -1 and the function that is shifted to the right by $+1$. $\text{wal}(5, \theta) = \text{sal}(3, \theta)$ is obtained by squeezing $\text{wal}(2, \theta) = \text{cal}(1, \theta)$ to half its width, multiplying the function that is shifted to the left by $+1$ and the function that is shifted to the right by -1 .

Die Walshfunktionen sollen nach der nichtrekursiven Definition von Ross und Kelly berechnet werden. [Diss. B Me S. 15.]

Die Funktionen heissen $swal(i, \theta)$ mit

i Ordnungszahl

θ normierte Zeit

n Anzahl der Bits in der Binärdarstellung von i .

Notwendig ist (erst einmal) die Eingabe der Sequenz i in Binärdarstellung (hilfsweise als Vektor). Um Widersprüche zu vermeiden, wird die Ordnungszahl in der Dezimaldarstellung mit j bezeichnet.

$$n := 4$$

$$\theta := 0, \frac{1}{1000} \dots 1$$

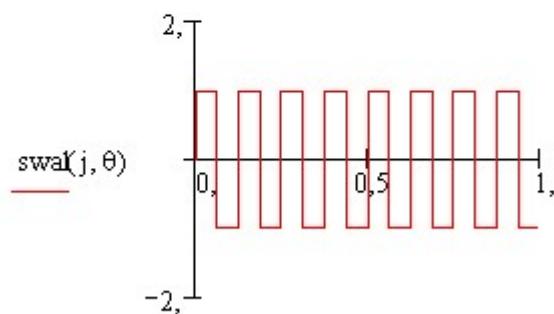
$$j := 15$$

$$i := \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} \quad \mu := 0 \dots n - 1$$

$$i_1 = 1$$

Es folgt die Formel in Originalversion von Me:

$$swal(j, \theta) := \text{sign} \left[(\sin(2 \cdot \pi \cdot \theta))^{i_0} \cdot \prod_{\mu=1}^{n-1} [\cos(2^\mu \cdot \pi \cdot \theta)]^{i_\mu} \right]$$



$$swal(j, \theta) := \text{sign} \left[(\sin(2 \cdot \pi \cdot \theta))^{i_0} \cdot \prod_{\mu=1}^{n-1} \left[(\cos(2^\mu \cdot \pi \cdot \theta))^{i_\mu} \right] \right]$$

Bildungsgesetz nach Ross und Kelly [119]

Das Bildungsgesetz definiert die kontinuierlichen Walshfunktionen nichtrekursiv über das Vorzeichen des Produktes der Kreisfunktionen:

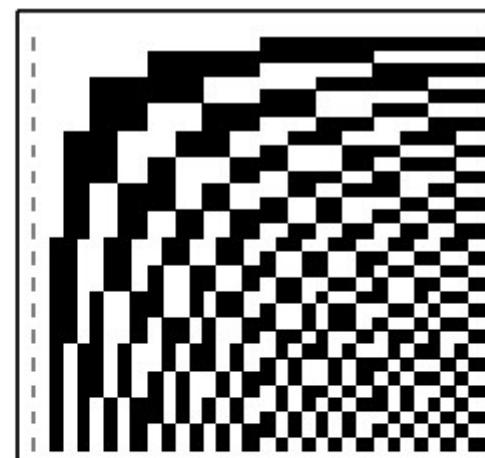
$$n := 5 \quad s := 0, 1..100 \quad z := 0..2^n - 1$$

$$\text{bit}(x, y) := \text{mod}\left(\text{trunc}\left(\frac{y}{2^x}\right), 2\right)$$

$$\text{swal}(i, \theta) := \text{sign} \left[(\sin(2\pi\theta))^{\text{bit}(0,i)} \cdot \prod_{\mu=1}^{n-1} (\cos(2^\mu\pi\theta))^{\text{bit}(\mu,i)} \right]$$

$$M_{z,s} := \text{swal}\left(z, \frac{s}{100}\right)$$

Kontinuierliche Walshfunktionen



Binärordnung:

Bit-komb.	reiner Binärc.	BCD- oder 8-4-2-1-Code	Aiken oder 2-4-2-1-Code	Gray-Code	3-Exzeß-Code
0000	0	0	0	0	#
0001	1	1	1	1	#
0010	2	2	2	3	#
0011	3	3	3	2	0
0100	4	4	4	7	1
0101	5	5	#	6	2
0110	6	6	#	4	3
0111	7	7	#	5	4
1000	8	8	#	#	5
1001	9	9	#	#	6
1010	10	#	#	#	7
1011	11	#	5	#	8
1100	12	#	6	8	9
1101	13	#	7	9	#
1110	14	#	8	#	#
1111	15	#	9	#	#

Gray-Code

	D	C	B	A
0				
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				
11				
12				
13				
14				
15				

Erweiterung

Unterscheidung von Codes:

einschrittig nicht einschrittig

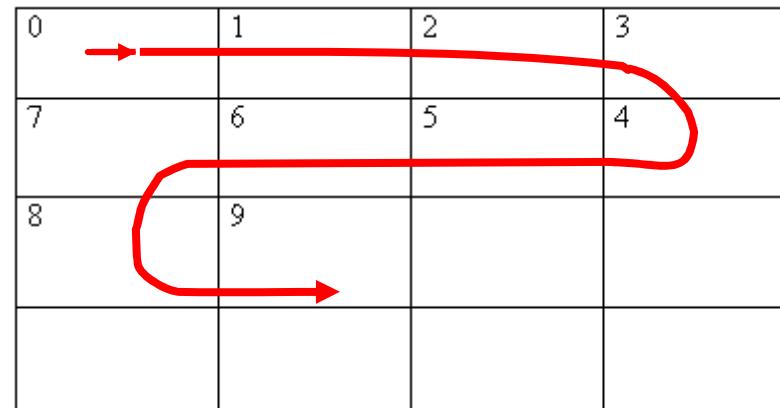
zyklisch nicht zyklisch

dekadisch nicht dekadisch

darstellbar in Karnaugh-Veitch-Diagramm

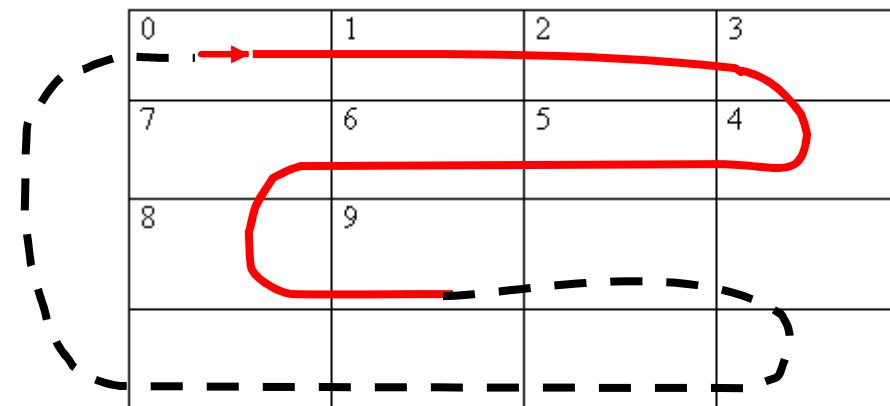
Gray-Code:

- einschrittig
- nicht zyklisch
- dekadisch



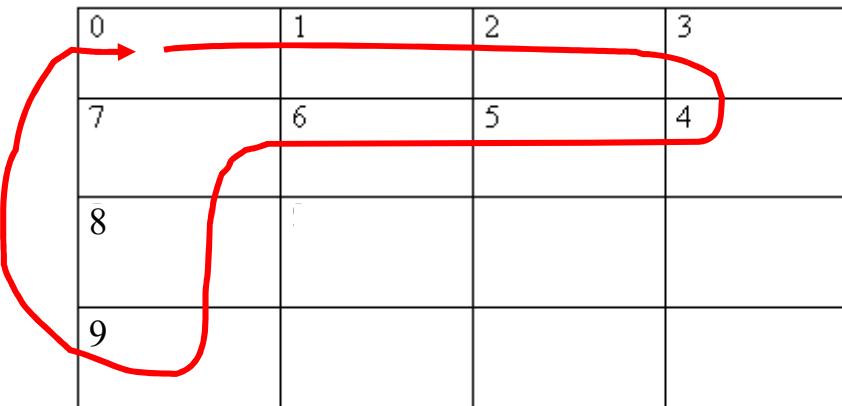
? Code:

- einschrittig
- zyklisch
- nicht dekadisch



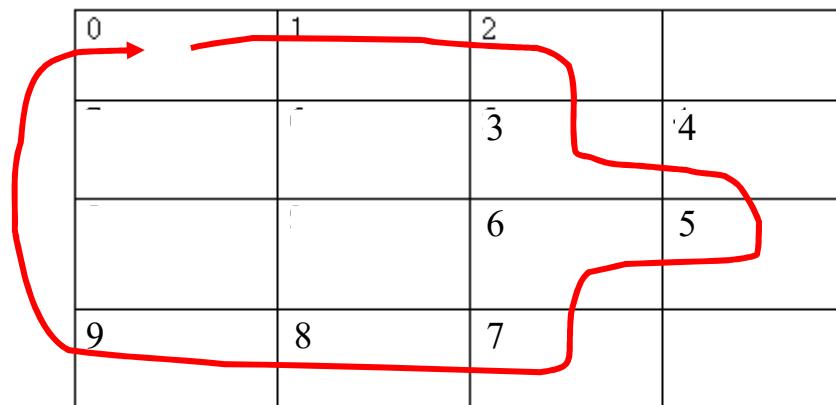
Glixon-Code:

- einschrittig
- zyklisch
- dekadisch



O'Brien-Code:

- einschrittig
- zyklisch
- dekadisch



Kroneckerproduktoperator

Ist A eine $m \times n$ -Matrix und B eine $p \times r$ -Matrix, so ist das Kronecker-Produkt $C = A \otimes B$ definiert als

$$C = (a_{ij} \cdot B) = \begin{pmatrix} a_{11}B & \cdots & a_{1n}B \\ \vdots & \ddots & \vdots \\ a_{m1}B & \cdots & a_{mn}B \end{pmatrix}$$

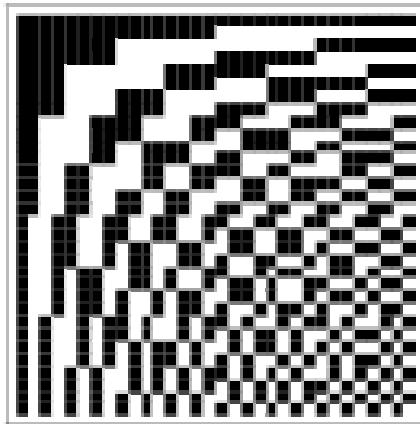
Das heißt jedes Element der Matrix A wird mit der Matrix B multipliziert. Das Ergebnis ist also wieder eine Matrix, allerdings von der Dimension $mp \times nr$.

1. Beispiel [Bearbeiten]

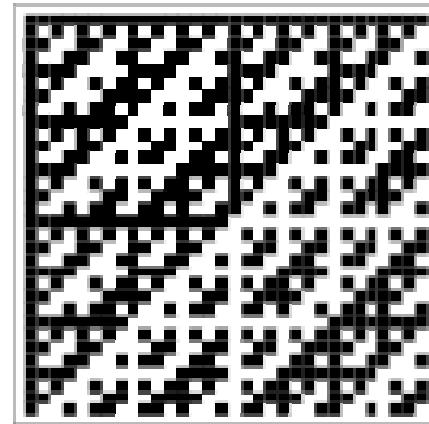
$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \otimes \begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix} = \begin{pmatrix} 1 \cdot \begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix} & 2 \cdot \begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix} \\ 3 \cdot \begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix} & 4 \cdot \begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix} \end{pmatrix} = \begin{pmatrix} 5 & 6 & 10 & 12 \\ 7 & 8 & 14 & 16 \\ 15 & 18 & 20 & 24 \\ 21 & 24 & 28 & 32 \end{pmatrix}$$

<http://de.wikipedia.org/wiki/Kronecker-Produkt>

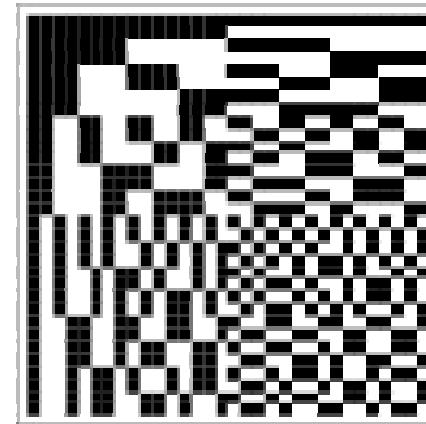
sequency



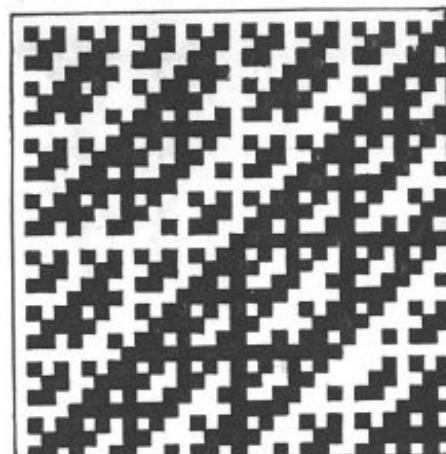
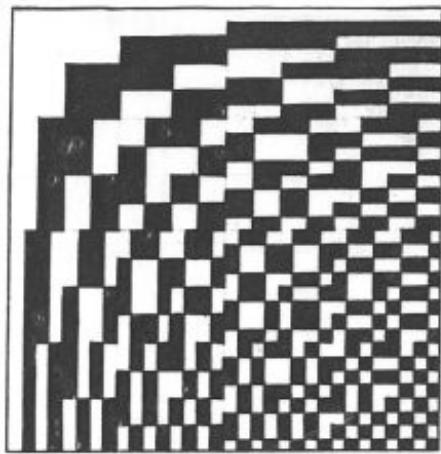
natural or Hadamard



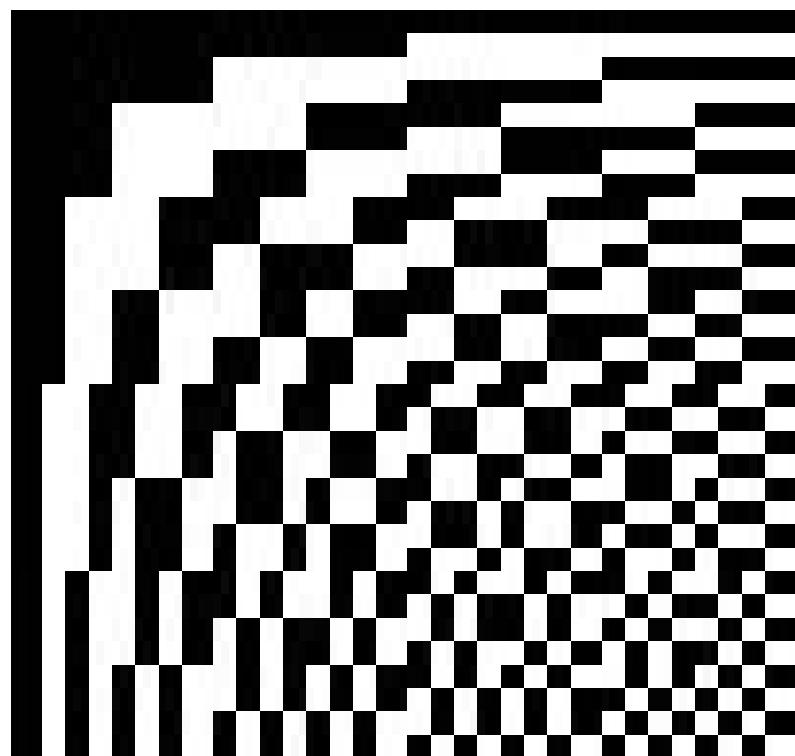
dyadic or Paley



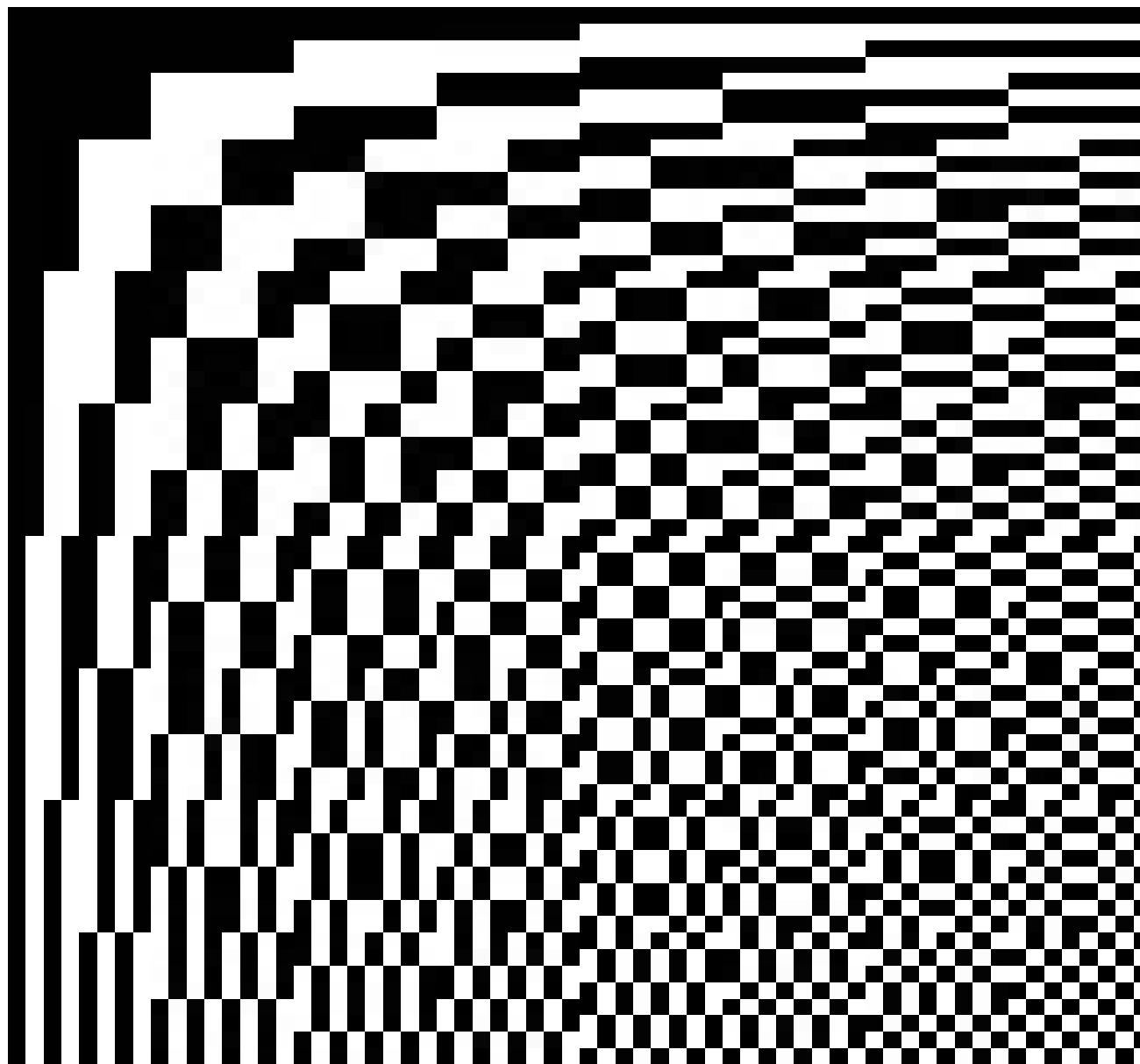
adapted from Wolfram, S. *A New Kind of Science*.
Wolfram Media, p. 1075, 2002.



Sequenz- und Hadamardordnung



M



Matrix

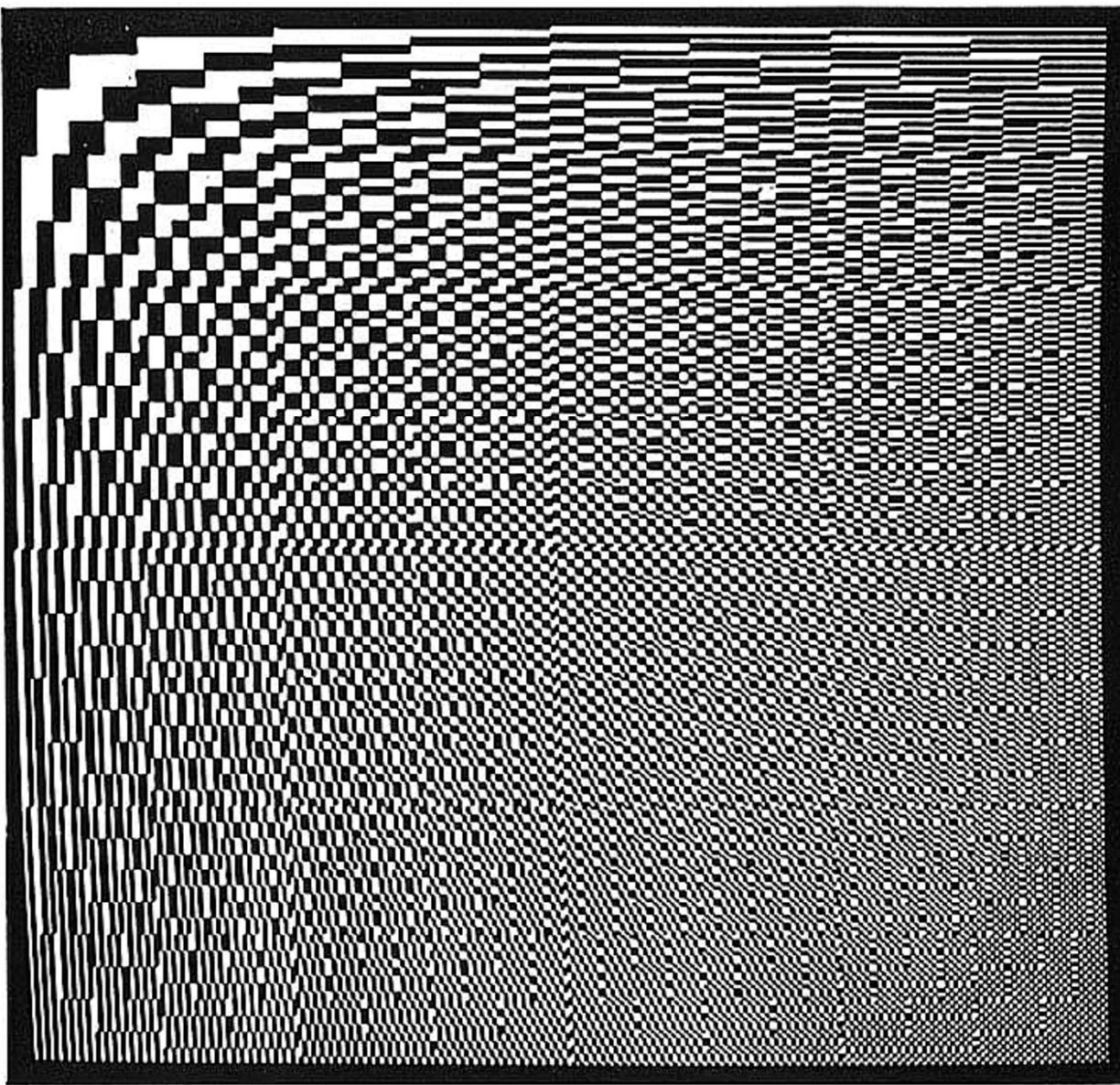
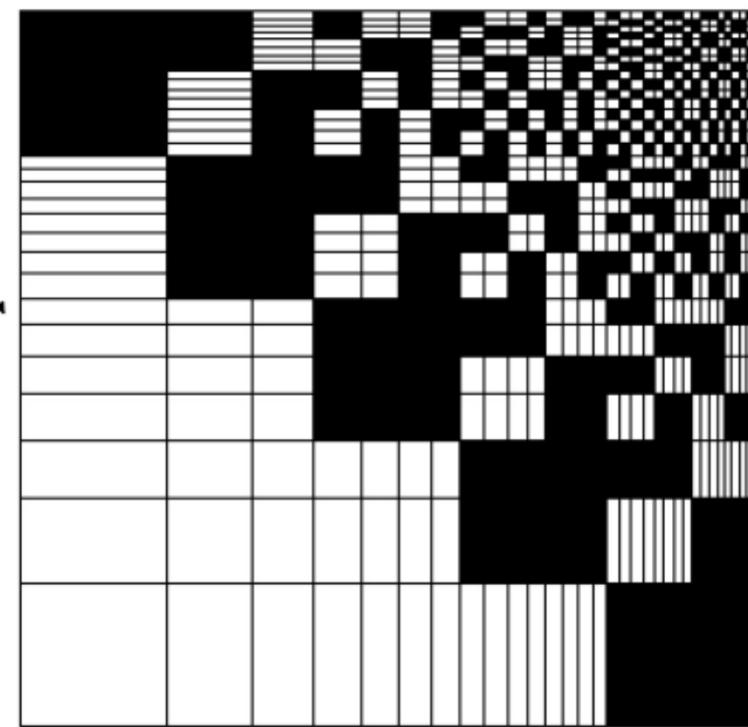
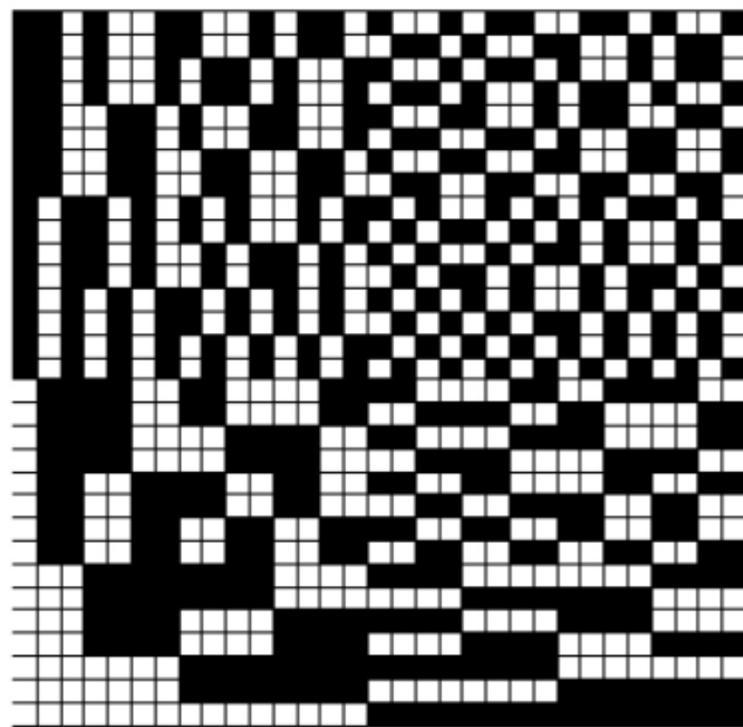


FIG. 114-7. The functions $wal(j, \theta)$ for $j = 0, \dots, 255$ in the interval $0 < \theta < 1$. (Courtesy A. H. Jones, British Broadcasting Corp., Research Department, Kingswood Warren, Tadworth, Surrey.)

[Harmonics : SVO-3]



Liebe Beate,

im Jahre 1977 wurde alles photographisch gemacht. Das gewuenschte Bild wurde mit einer Kupferplatte gedruckt, die eine Aufloesung von mindestens 1200 dpi hatte, vielleicht aber auch doppelt so viel. Ich suchte nach dem Original oder dem ersten Photo-Negativ, konnte aber nichts finden. Fuer eine Abtastung mit 600 dpi ist das Bild im Buch mehr als gut genug. Natuerlich kannst Du es kopieren und in einem neuen Buch verwenden.

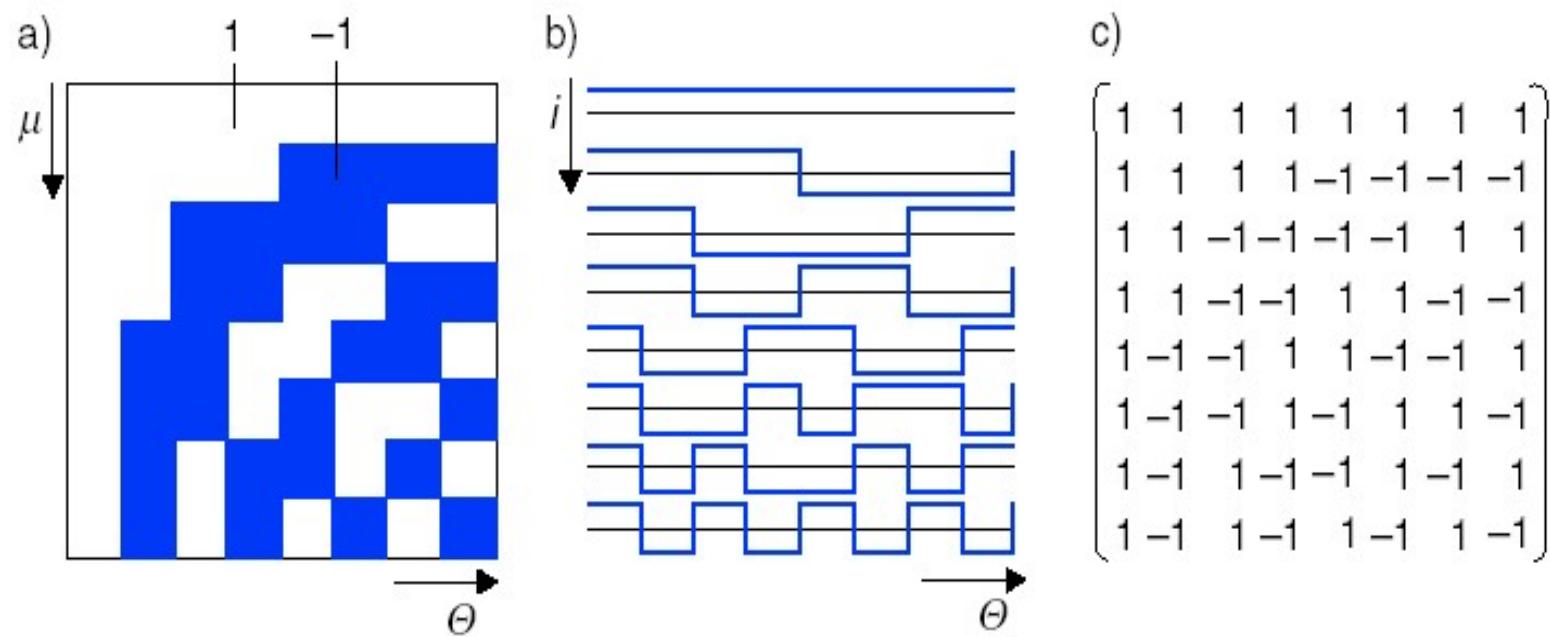
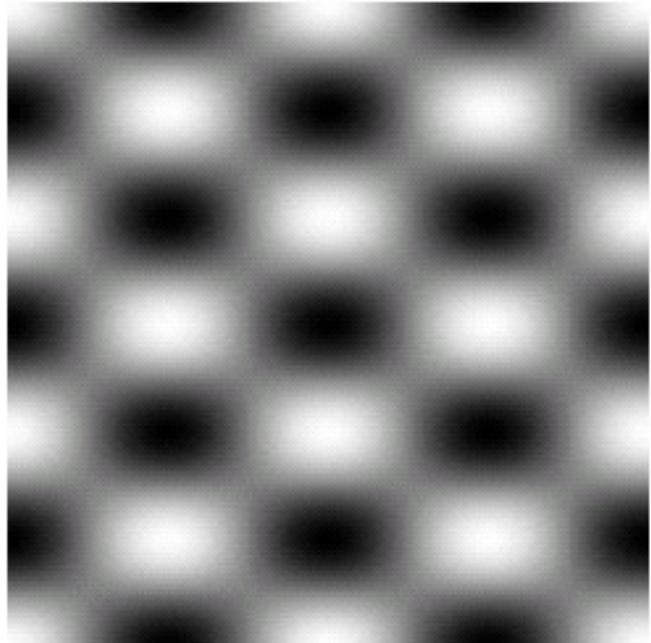


Abbildung 3.49: Diskretisierung der Walshfunktionen nach [37]

- a) kontinuierliche Werte für die Variablen μ und θ
- b) diskretes $\mu = i$ und kontinuierliches θ
- c) μ und θ diskret

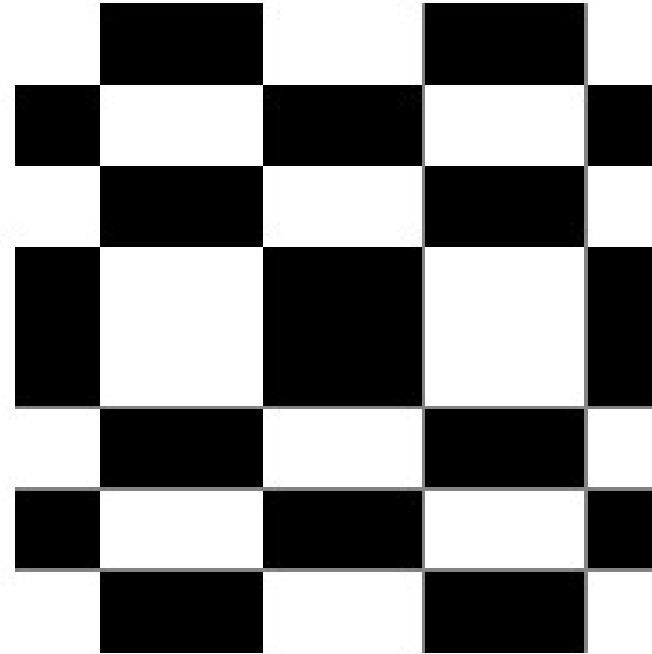
$\cos(3x)$



$\cos(2y)$

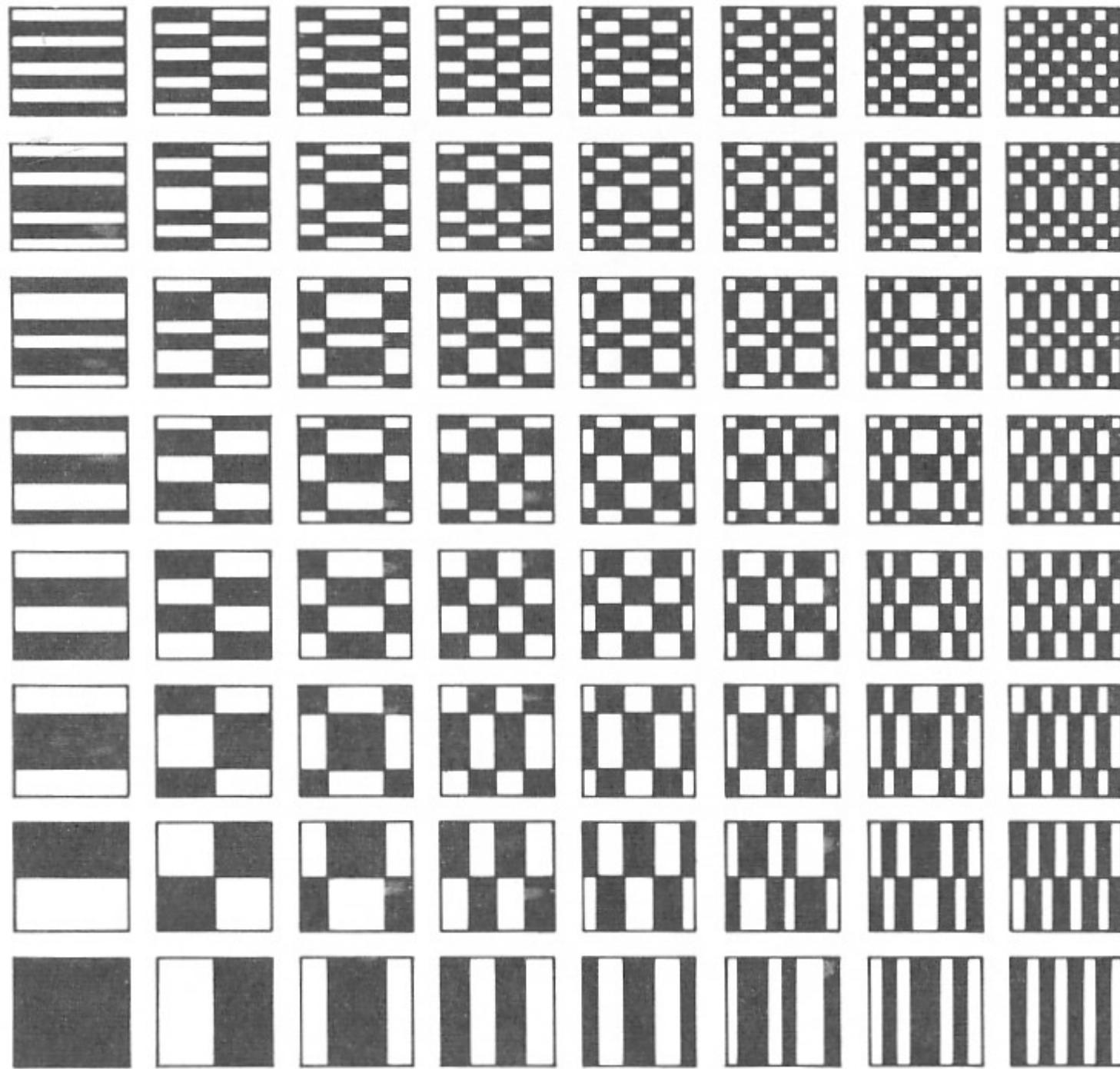
$\cos(3x) \cos(2y)$

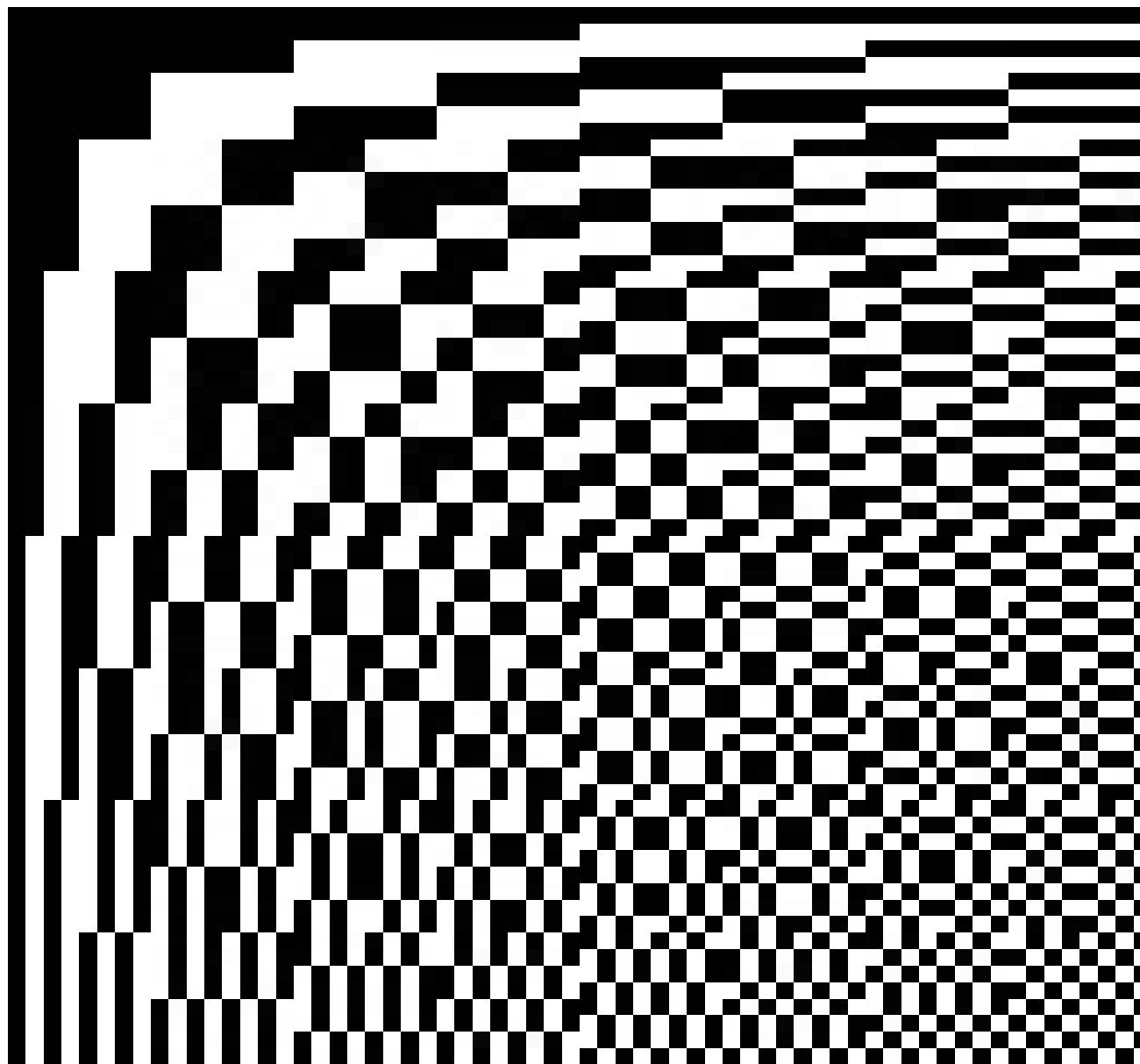
$\text{cal}(3,x)$



$\text{cal}(2,y)$

$\text{cal}(3x) \text{cal}(2y)$





Matrix

4 Orthogonalität und orthogonale Funktionen

4.1	Orthogonalität und Näherung
4.2	Harmonische Funktionen
4.2.1	Eindimensionale Funktionen
4.2.2	Zweidimensionale Funktionen
4.3	Nichtharmonische Funktionen
4.3.1	System der Blockpulse
4.3.2	System der Walshfunktionen
4.3.3	Haar-Funktionen
4.3.4	Slant-Funktionen

1909
Alfréd Haar



geb. 1885 Budapest
gest. 1933 in Szeged

- entdeckt Basisfunktionen, die heute als erste Wavelets eingeordnet werden
- gut für die Lehre geeignet
- wegen ihrer Rechteckform weniger gut für die Praxis
- rekonstruiertes Bild weist Blockstrukturen auf

- 1903 Haar gewinnt den Eötvös-Wettbewerb in Mathematik
- 1904 Haar geht nach Göttingen zu Hilbert
- 1909 Abschluss der Dissertation
"Zur Theorie der orthogonalen Funktionensysteme,"

Lehre

in Göttingen, Kolozsvár (Cluj), Budapest und Szeged (mit Riesz)

Arbeitsschwerpunkte

Analysis, Orthogonalsysteme, partielle Differentialgleichungen,
Tschebyscheff -Approximationen und Gruppentheorie

===== 1928. junius 8. =====



Riesz F.
Kürochák

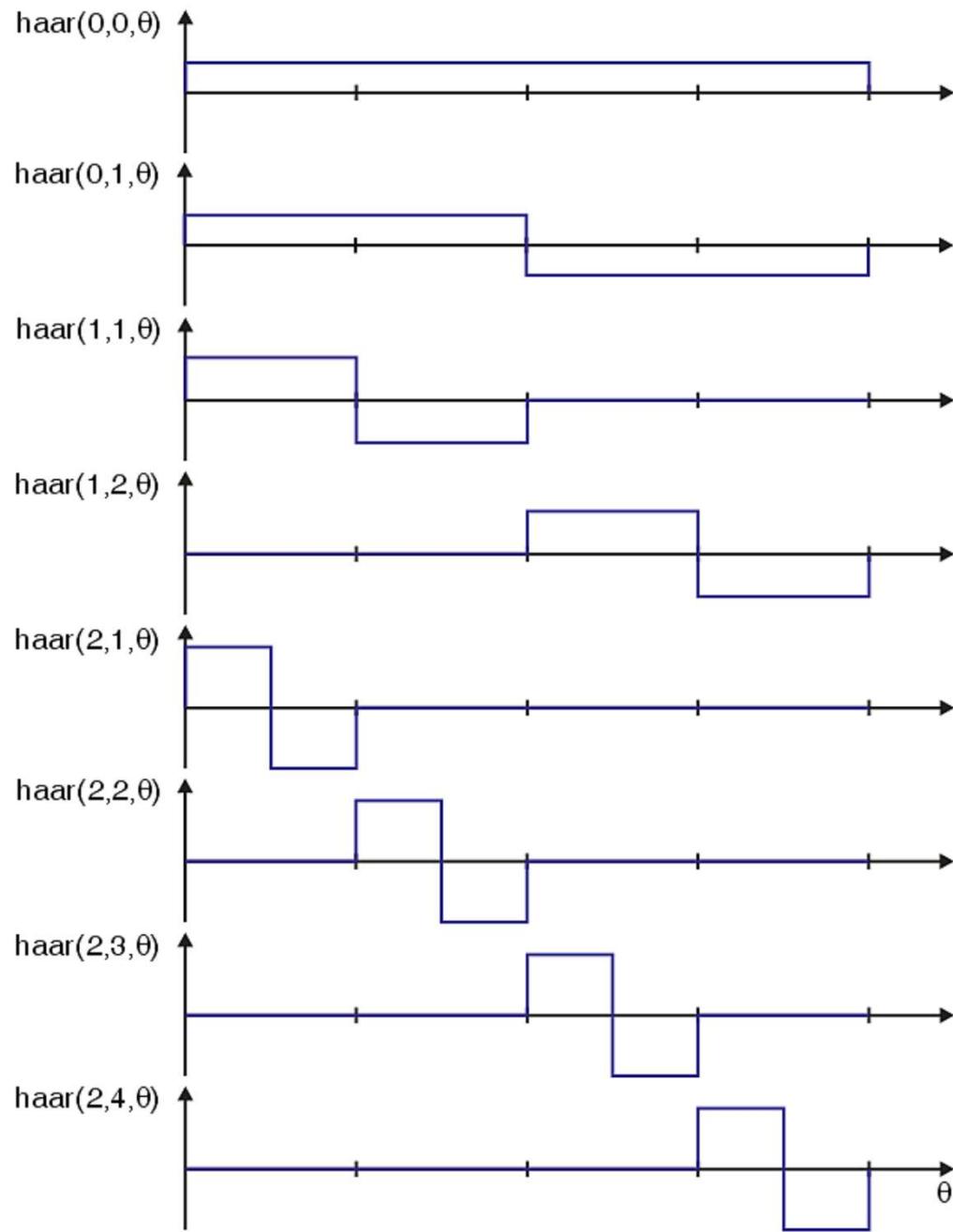
Kerékjártó
Birkhoff
Rado

Haar
Lipka

König D.
Kellogg

Ortvay
Tejér
Szász P.

Kalmár



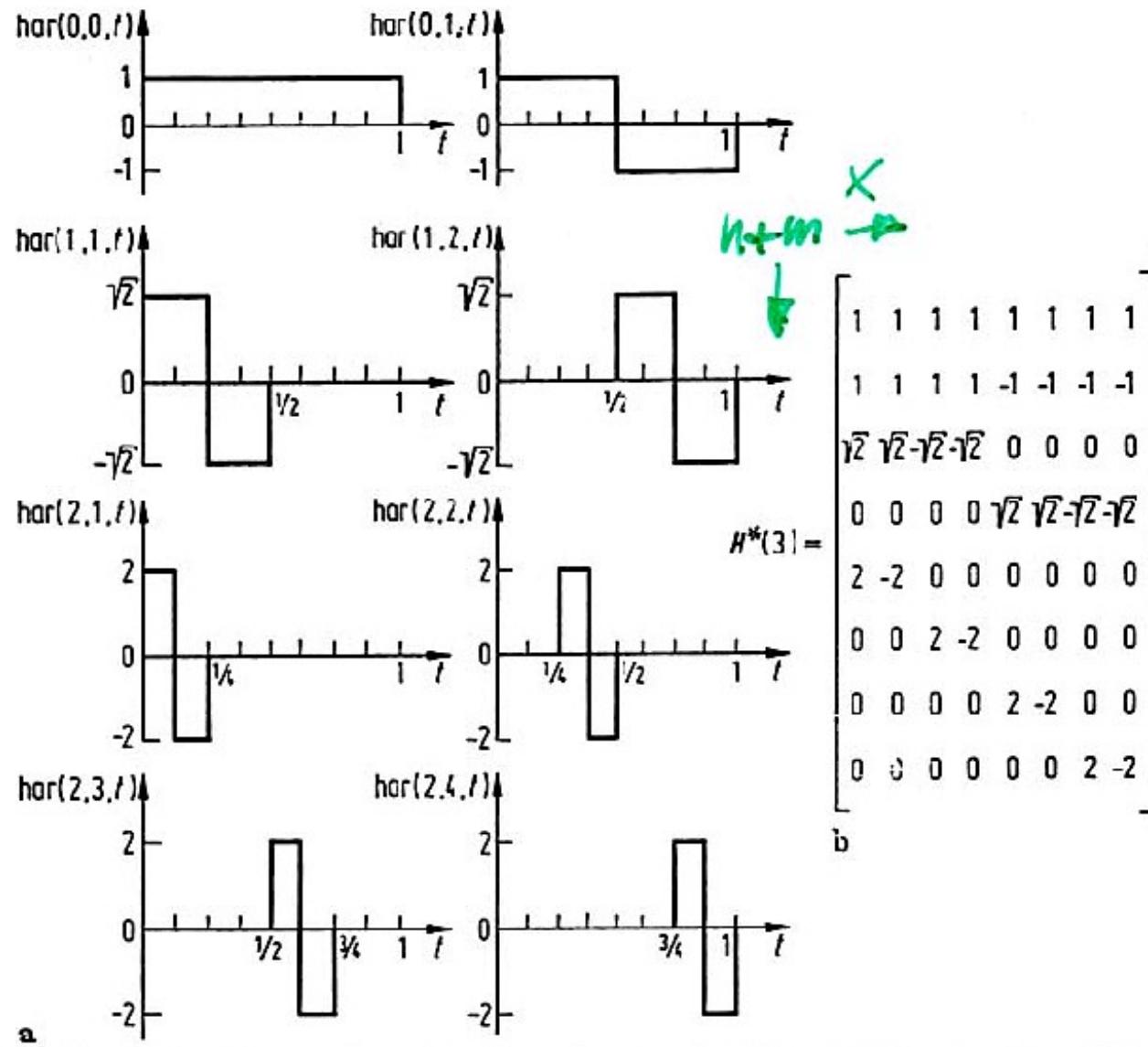
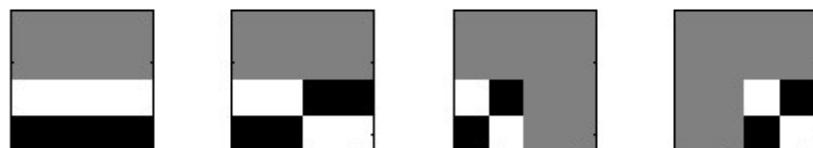
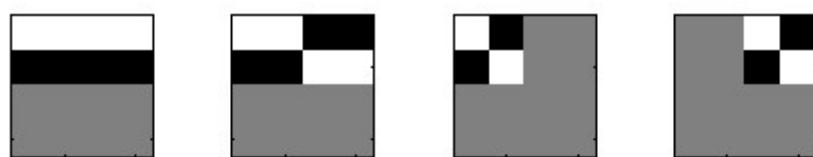
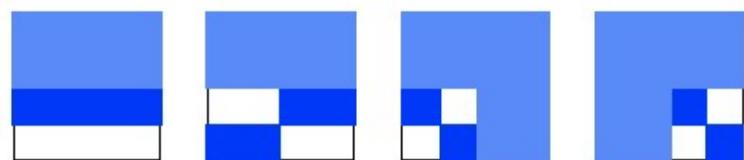
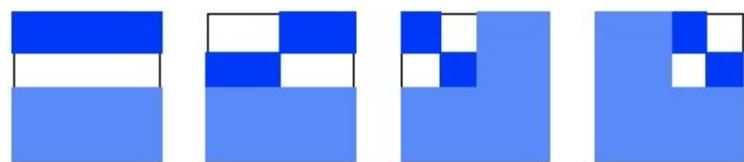
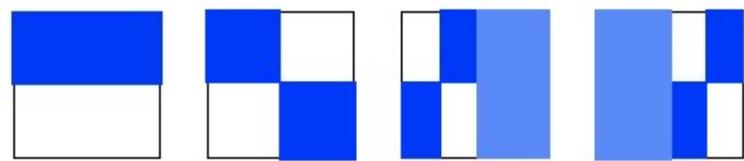
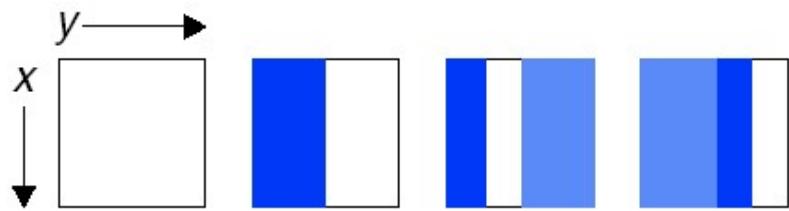


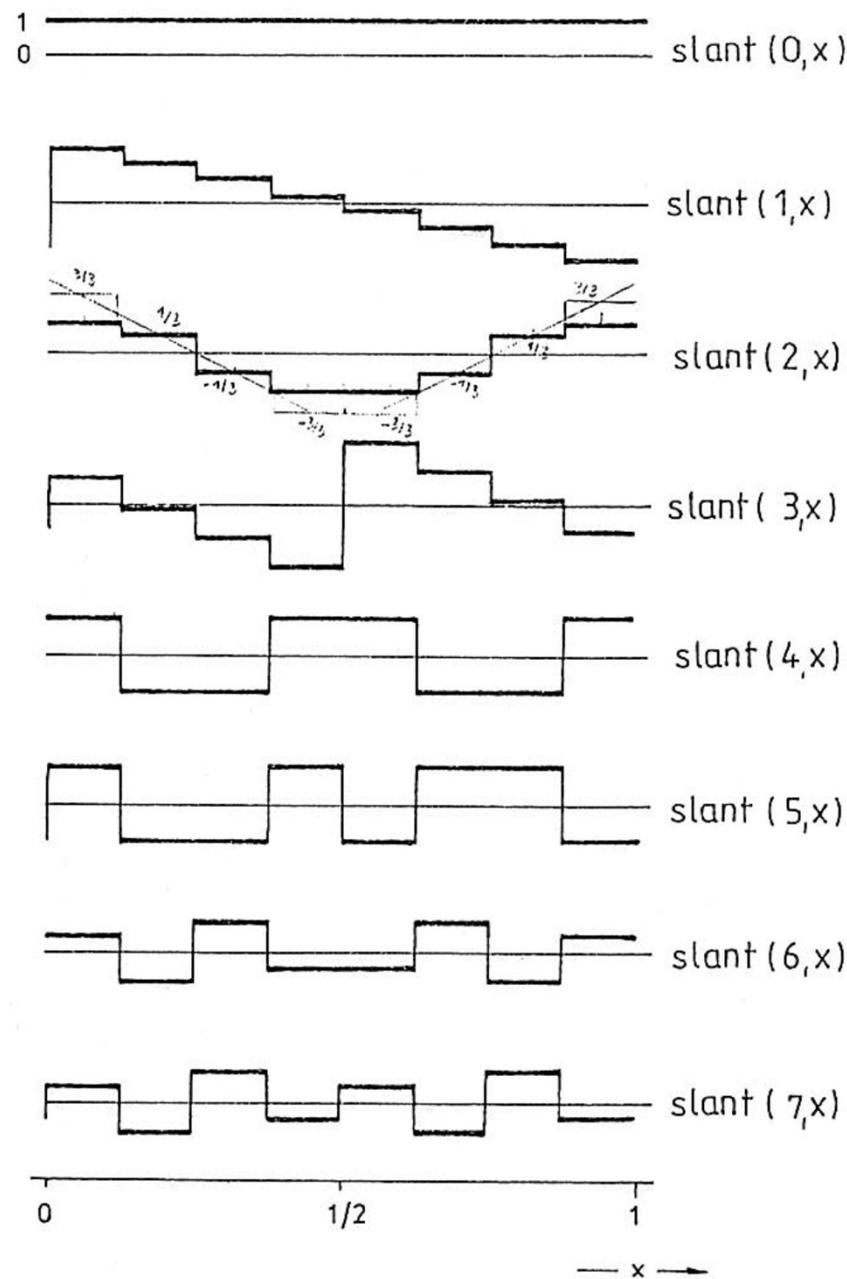
Fig. 5.4 a Continuous Haar functions, $N = 8$; b Discrete Haar functions, $N = 8$



Legende: $\square > 0$ $\blacksquare 0$ $\blacksquare < 0$

4 Orthogonalität und orthogonale Funktionen

4.1	Orthogonalität und Näherung
4.2	Harmonische Funktionen
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4.3.3	<u>Haar-Funktionen</u>
4.3.4	Slant-Funktionen



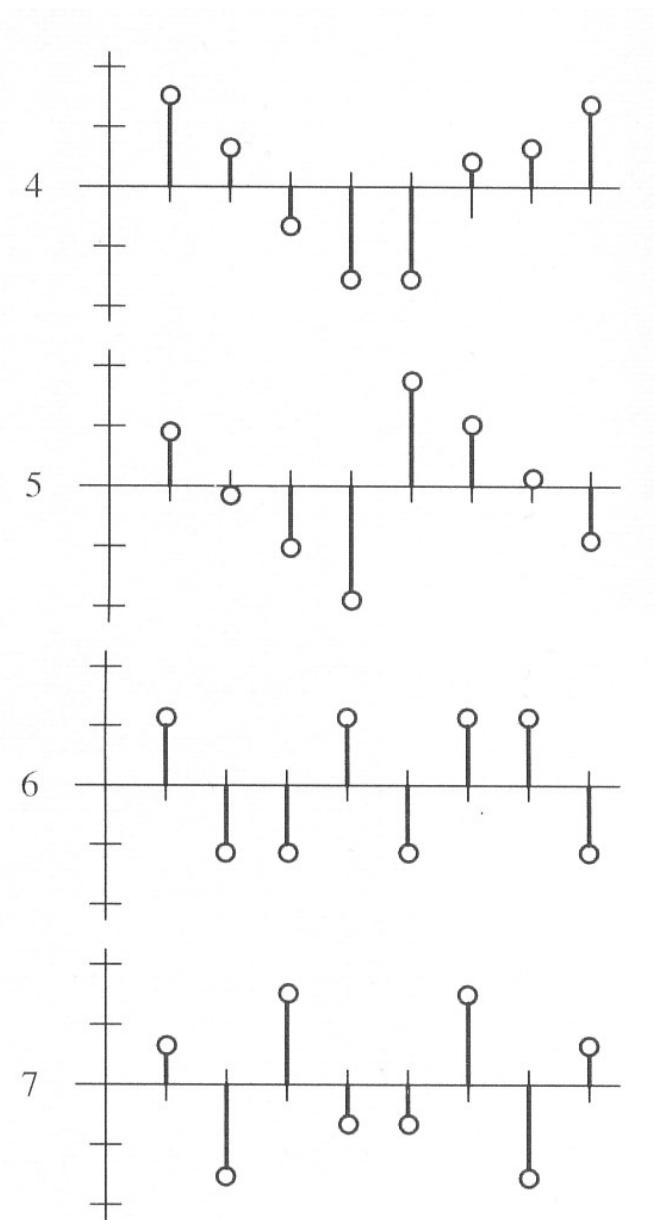
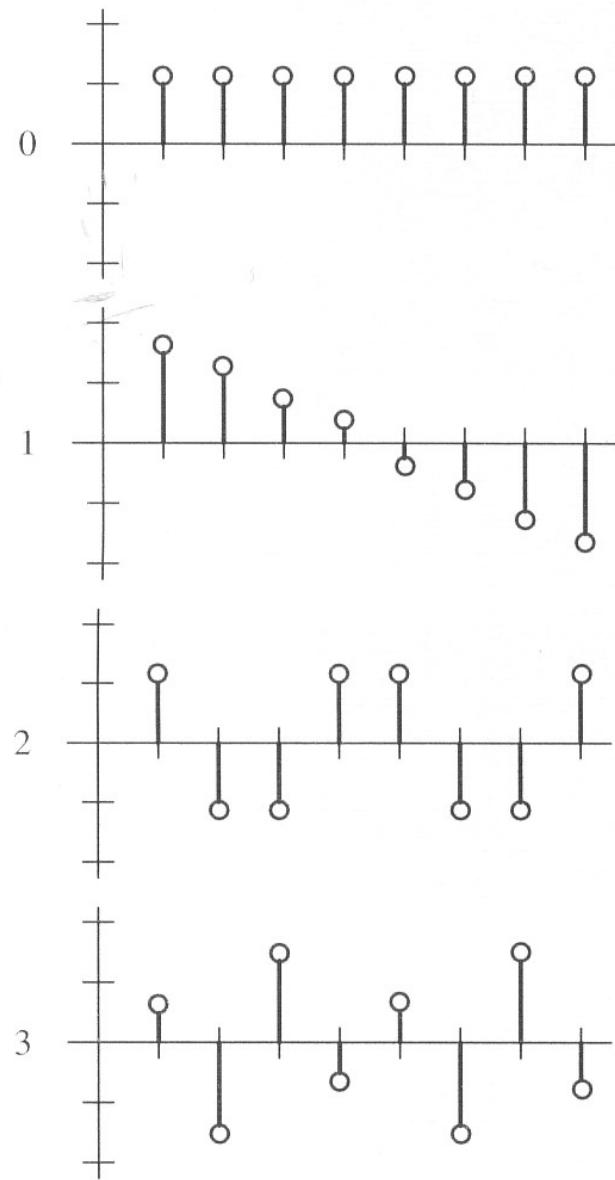


Figure 13–4 The slant transform basis functions for $N = 8$

$$S_N = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 & | & \mathbf{0} & | & 1 & 0 & | & \mathbf{0} \\ -a_N & b_N & | & \mathbf{0} & | & -a_N & b_N & | & \mathbf{0} \\ - & - & \mathbf{0} & | & \mathbf{I} & | & - & - & \mathbf{I} \\ - & - & 1 & | & \mathbf{0} & | & 0 & -1 & \mathbf{0} \\ 0 & 1 & | & \mathbf{0} & | & 0 & -1 & | & \mathbf{0} \\ -b_N & a_N & | & \mathbf{0} & | & b_N & a_N & | & \mathbf{0} \\ - & - & \mathbf{0} & | & \mathbf{I} & | & 0 & - & -\mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{S}_{N/2} & | & \mathbf{0} \\ \mathbf{0} & | & \mathbf{S}_{N/2} \end{bmatrix}$$

$$a_{2N} = \sqrt{\frac{3N^2}{4N^2 - 1}} \quad \text{and} \quad b_{2N} = \sqrt{\frac{N^2 - 1}{4N^2 - 1}}$$