

Zadanie 1.

$$\left[\begin{array}{ccc|c} -1 & 1 & -4 & 0 \\ 2 & 2 & 0 & 1 \\ 3 & 3 & 2 & \frac{1}{2} \end{array} \right] \cdot (2) \sim \left[\begin{array}{ccc|c} -1 & 1 & -4 & 0 \\ 0 & 4 & -8 & 1 \\ 3 & 3 & 2 & \frac{1}{2} \end{array} \right] \cdot (3) \sim \left[\begin{array}{ccc|c} -1 & 1 & -4 & 0 \\ 0 & 4 & -8 & 1 \\ 0 & 6 & -10 & \frac{1}{2} \end{array} \right] \cdot (-\frac{3}{2})$$

$$\sim \left[\begin{array}{ccc|c} -1 & 1 & -4 & 0 \\ 0 & 4 & -8 & 1 \\ 0 & 0 & 2 & -1 \end{array} \right]$$

$$-x_1 + x_2 - 4x_3 = 0$$

$$\Rightarrow x_3 = -\frac{1}{2}$$

$$4x_2 - 8x_3 = 1$$

$$2x_3 = -1$$

$$-4x_3 = 1 \quad 4x_2 = 1 + (-4)$$

$$4x_2 = -3$$

$$x_2 = -\frac{3}{4}$$

$$-x_1 = -\frac{3}{4} + 4 \cdot -\frac{1}{2}$$

$$-x_1 = -\frac{3}{4} - 2$$

$$x_1 = \frac{5}{4}$$

$$\text{Odp. } x = \begin{pmatrix} \frac{5}{4} \\ -\frac{3}{4} \\ -\frac{1}{2} \end{pmatrix}$$

Zadanie 2.

$$L = \begin{bmatrix} 1 & 0 & 0 \\ \frac{3}{2} & 1 & 0 \\ \frac{1}{2} & \frac{11}{13} & 1 \end{bmatrix}, \quad U = \begin{bmatrix} 2 & -3 & -1 \\ 0 & \frac{13}{2} & -\frac{7}{2} \\ 0 & 0 & \frac{32}{13} \end{bmatrix}, \quad b = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$$

$$L \cdot U \cdot x = b$$

$$1) \quad L \cdot y = b$$

$$U \cdot x = y$$

$$L \cdot y = b$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ \frac{3}{2} & 1 & 0 & -1 \\ \frac{1}{2} & \frac{11}{13} & 1 & 2 \end{array} \right] \xrightarrow{\cdot \begin{pmatrix} -\frac{3}{2} \\ -\frac{1}{2} \end{pmatrix}} \sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -\frac{5}{2} \\ \frac{1}{2} & \frac{11}{13} & 1 & 2 \end{array} \right] \xrightarrow{\cdot \begin{pmatrix} \frac{1}{2} \end{pmatrix}}$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -\frac{5}{2} \\ 0 & \frac{11}{13} & 1 & \frac{3}{2} \end{array} \right] \xrightarrow{\cdot \begin{pmatrix} -\frac{11}{13} \end{pmatrix}} \sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -\frac{5}{2} \\ 0 & 0 & 1 & \frac{47}{13} \end{array} \right]$$

$$y = \begin{cases} x_1 = 1 \\ x_2 = -\frac{5}{2} \\ x_3 = \frac{47}{13} \end{cases}$$

$$2) \quad U \cdot x = y$$

$$2x_1 - 3x_2 - x_3 = 1$$

$$\frac{13}{2}x_2 - \frac{7}{2}x_3 = -\frac{3}{2}$$

$$\frac{32}{13}x_3 = \frac{47}{13} \quad | \cdot 13 \quad : 32$$

$$x_3 = \frac{47}{32}$$

$$\frac{13}{2}x_2 - \frac{7}{2} \cdot \frac{47}{32} = -\frac{3}{2}$$

$$\frac{13}{2}x_2 - \frac{329}{64} = -\frac{3}{2} \quad | \cdot 2$$

$$13x_2 - \frac{329}{32} = -3 \quad | \cdot 32$$

$$416x_2 - 329 = -96$$

$$416x_2 = 233 \quad | : 416$$

$$x_2 = \frac{233}{416}$$

$$2x_1 - 3 \cdot \frac{233}{416} - \frac{47}{32} = 1$$

$$2x_1 - \frac{699}{416} - \frac{47}{32} = 1$$

$$2x_1 - \frac{11}{32} = 1$$

$$2x_1 = \frac{41}{32}$$

$$x_1 = \frac{41}{64}$$

$$2x_1 - \frac{655}{208} = 1$$

$$2x_1 = \frac{863}{208}$$

$$x_1 = \frac{863}{416}$$

Ans: $x = \begin{pmatrix} \frac{863}{416} \\ \frac{233}{416} \\ \frac{47}{32} \end{pmatrix}$