# Assignment 2

**Q1:** Find the arc length of $y = Sinx$ from x=0 to x=π. Consider h= π/6

**Q2:** Solve the boundary value problem using finite difference method.

$$\boldsymbol{y'' = x + y} \qquad 0 \leq x \leq 1$$

With the boundary conditions $\boldsymbol{y(0) = 0}$ , $\boldsymbol{y(1) = 0.\ Take\ h = 0.25}$

**Q3:** Determine the L2 Norm of the matrix.

$$\begin{bmatrix} \mathbf{3} & \mathbf{-1} & \mathbf{0} \\ \mathbf{-1} & \mathbf{2} & \mathbf{-1} \\ \mathbf{0} & \mathbf{-1} & \mathbf{3} \end{bmatrix}$$

**Q4:** Consider $y = f(x) = Cosx$. Use four nodes $x_0$=0, $x_1$=0.4, $x_2$=0.8 and $x_3$=1.2 to construct a Lagrange polynomial $P_3(x)$. Then calculate $P_3(\pi/4)$.

**Q 5:** Let $f(x, y) = arctan(\frac{y}{x})$

Calculate approximations to $\mathbf{f_x}$ (3,4) and $\mathbf{f_y}$(3,4) of accuracy $O(h_2)$.

With h=0.1

**Q6**: Consider a general interval [a, b]. Show that Simpson's rule produces exact results for the functions f (x) = $x^3$

$$\int_a^b x^3\,dx = \frac{b^4}{4} - \frac{a^4}{4}$$

**Q7**: Integrate the Lagrange interpolation polynomial

$$P_1(x) = f_0 \frac{x - x_1}{x_0 - x_1} + f_1 \frac{x - x_0}{x_1 - x_0}$$

Over the interval $[x_0, x_1]$ to establish the trapezoidal rule.

**Q 8**: The accompanying table gives the speeds, in miles per second, at various times for a test rocket that was fired upward from the surface of the Earth. Use these values to approximate the number of miles traveled during the first 180s. Round your answer to the nearest tenth of a mile.

| TIME $t$ (s) | SPEED $v$ (mi/s) |
|---|---|
| 0 | 0.00 |
| 30 | 0.03 |
| 60 | 0.08 |
| 90 | 0.16 |
| 120 | 0.27 |
| 150 | 0.42 |
| 180 | 0.65 |

**Q9:** Solve the boundary value problem using finite difference method.

$$\boldsymbol{y'' + 5y' + 4y = 1}\,, \qquad 0 \le x \le 1$$

With the boundary conditions $\boldsymbol{y(0) = 0}\,,\ \boldsymbol{y(1) = 0}$ $\boldsymbol{.\,Take\ h = 0.25}$

b) Write down the Newton's interpolation polynomial with your data generated.
c) Find f(0.65)

**Q10: Determine the L2 Norm of the matrix.**

$$\begin{bmatrix} \mathbf{1} & \mathbf{1} & \mathbf{0} \\ \mathbf{1} & \mathbf{2} & \mathbf{1} \\ \mathbf{-1} & \mathbf{1} & \mathbf{2} \end{bmatrix}$$

**Q11 : Use Lagrange interpolation to find the unique polynomial $P_3(x)$, of degree 3 or less, that agrees with the following data:**

| $i$ | $x_i$ | $y_i$ |
|---|---|---|
| 0 | $-1$ | 3 |
| 1 | 0 | $-4$ |
| 2 | 1 | 5 |
| 3 | 2 | $-6$ |

**Q12: Let** $f(x,y) = \frac{xy}{(x+y)}$

Calculate approximations to $\mathbf{f_x}$ (2,3) and $\mathbf{f_y}$(2,3) of accuracy O($h_2$). With h=0.1,0.01 and 0.001

**Q13**: A car laps a race track in 84 seconds. The speed of the car at each 6 seconds interval is determined by using a radar gun and is give from the beginning of the lap, in feet/second, by the entries in the following table

| Time | 0 | 6 | 12 | 18 | 24 | 30 | 36 | 42 | 48 | 54 | 60 | 66 | 72 | 78 | 84 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Speed | 124 | 134 | 148 | 156 | 147 | 133 | 121 | 109 | 99 | 85 | 78 | 89 | 104 | 116 | 123 |

How long is the track?

**Q14.** Evaluate the total arc length of the ellipse which is represented by the following integral. Take n=5

$$4\int_0^{\pi/2} \sqrt{1 + 3\sin^2 t}\, dt$$

**Q15.** Find a function $g(x)$ of the form $g(x) = Ax^2 + Bx + C$ whose graph contains the points $(m - \Delta x, f(m - \Delta x))$, $(m, f(m))$, and $(m + \Delta x, f(m + \Delta x))$, for the given function $f(x)$ and the given values of $m$ and $\Delta x$. Then verify Formula

$$\int_{m-\Delta x}^{m+\Delta x} g(x)\, dx = \frac{\Delta x}{3}[Y_0 + 4Y_1 + Y_2]$$

where $Y_0 = f(m - \Delta x)$, $Y_1 = f(m)$, and $Y_2 = f(m + \Delta x)$.

Given that : $f(x) = \frac{1}{x}$; $m = 3$, $\Delta x = 1$