1: Assume at first she picked ①.

switch

①	②	③	
✓	X	X	lose
X	✓	X	win
X	X	✓	win

stay

①	②	③	
✓	X	X	win
X	✓	X	lose
X	X	✓	lose

If she switches, she has $\frac{2}{3}$ probability to win, otherwise $\frac{1}{3}$.

So SWITCH!

2. prior: Dirichlet Distribution.

$\vec{\pi} \sim Dir(\partial_1, \cdots \partial_K)$; $x_1, \cdots, x_K \sim Mult(\pi_1, \cdots, \pi_K)$

$$p(\vec{\partial} \mid \vec{x}, \vec{\pi}) = \frac{p(\vec{\pi} \mid \vec{x}, \vec{\partial}) \cdot p(\vec{\partial})}{\int_{R_D} p(\vec{\pi} \mid \vec{x}, \vec{\partial}) \cdot p(\vec{\partial}) \, d\partial}$$

$$\propto p(\vec{\pi} \mid \vec{\partial}) \cdot \prod_{x_i \in D} p(x_i \mid \vec{\pi})$$

$$\propto \prod_{j=1}^{K} \pi_j^{\partial_j - 1} \prod_{x_i \in D} \prod_{j=1}^{K} \pi_j^{x_i^{(j)}} = \prod_{j=1}^{K} \pi_j^{\partial_j - 1 + \sum_{x_i \in D} x_i^{(j)}}.$$

$\therefore p(\vec{\partial} \mid \vec{x}, \vec{\pi}) \sim Dir(\partial_1', \cdots, \partial_K')$ $\quad (\partial_j' = \partial_j + \sum_{x_i \in D} x_i^{(j)})$.

3. $x_i \sim \text{Poisson}(\lambda) \qquad \lambda \sim \text{Gamma}(a, b)$.

(1). $P(x_1, \cdots, x_N \mid \lambda) = \prod_{i=1}^{N} P(x_i \mid \lambda)$

$$P(\lambda \mid \vec{x}) = \frac{P(\vec{x} \mid \lambda) \cdot P(\lambda)}{\int_{R_\lambda} P(\vec{x} \mid \lambda) P(\lambda) d\lambda} \propto P(\vec{x} \mid \lambda) \cdot P(\lambda).$$

$$\propto \prod_{i=1}^{N} P(x_i \mid \lambda) \cdot P(\lambda) = \left[\prod_{i=1}^{N} \text{Possion}(x_i \mid \lambda)\right] \cdot \text{Gamma}(\lambda \mid a, b).$$

$$= \left(\prod_{i=1}^{N} \frac{\lambda^{x_i}}{x_i!} e^{-\lambda}\right) \cdot \frac{b^a}{\Gamma(a)} \lambda^{a-1} e^{-b\lambda}$$

$$\propto \lambda^{(a + \sum_{i=1}^{N} x_i - 1)} e^{-(b+N)\lambda}$$

$\therefore a^* = a + \sum_{i=1}^{N} x_i \quad ; \quad b^* = b + N$

~~$P(\vec{x})$~~ $P(\lambda \mid \vec{x}) \sim \text{Gamma}(a + \sum_{i=1}^{N} x_i, \ b + N)$.

(2). $P(\lambda \mid \vec{x}) \sim \text{Gamma}(a^*, b^*) \qquad (a^* = a + \sum_{i=1}^{N} x_i, \ b^* = b + N)$

$$P(x^* \mid \vec{x}) = \int_0^\infty P(x^* \mid \lambda) \cdot P(\lambda \mid \vec{x}) d\lambda = \int_0^\infty \frac{\lambda^{x^*}}{x^*!} e^{-\lambda} \cdot \frac{b^{*a^*}}{\Gamma(a^*)} \lambda^{a^*-1} e^{-b^*\lambda} d\lambda$$

$$= \frac{b^{*a^*}}{\Gamma(a^*) x^*!} \int_0^\infty \lambda^{a^* + x^* - 1} e^{-(b^*+1)\lambda} d\lambda$$

Assume $x = (b^* + 1)\lambda \ ; \ dx = (b^* + 1) d\lambda$.

$$\int_0^\infty \left(\frac{x}{b^*+1}\right)^{a^* + x^* - 1} e^{-x} \frac{dx}{b^*+1} = \left(\frac{1}{b^*+1}\right)^{a^* + x^*} \int_0^\infty x^{a^* + x^* - 1} e^{-x} dx$$

$$= \frac{\Gamma(a^* + x^*)}{(b^*+1)^{a^* + x^*}}$$

$$P(x^* \mid \vec{x}) = \frac{b^{*a^*} \Gamma(a^* + x^*)}{(b^*+1)^{a^* + x^*} \Gamma(a^*) x^*!} \qquad (a^* = a + \sum_{i=1}^{N} x_i, \ b^* = b + N).$$

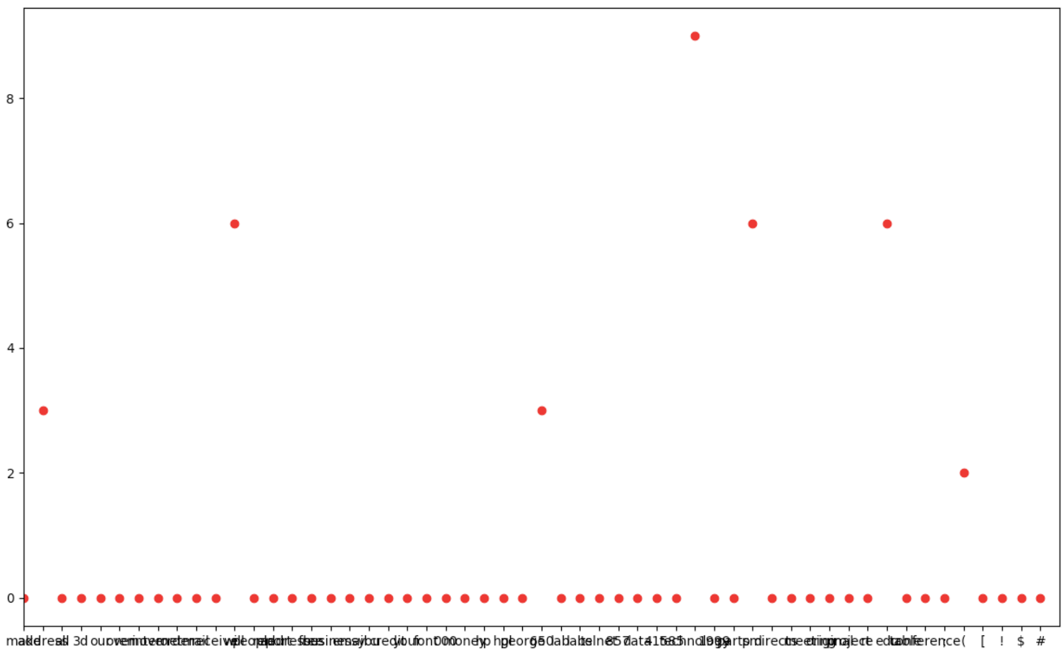
Problem4

(b)

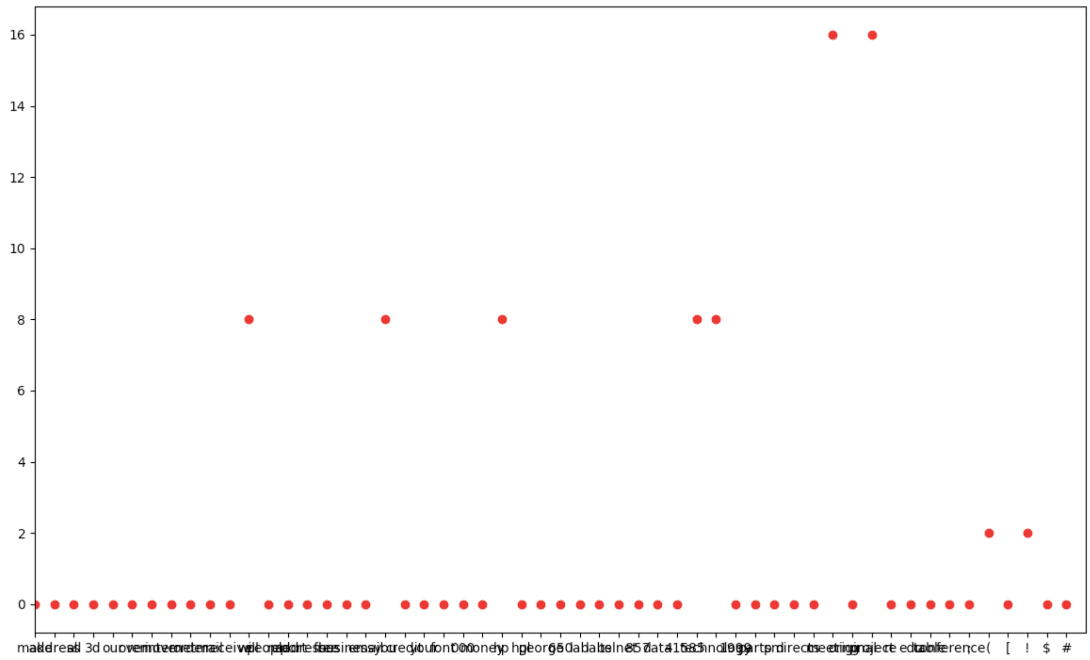
	0	1
Train_result	279	180
Test_result	182	281

(c)

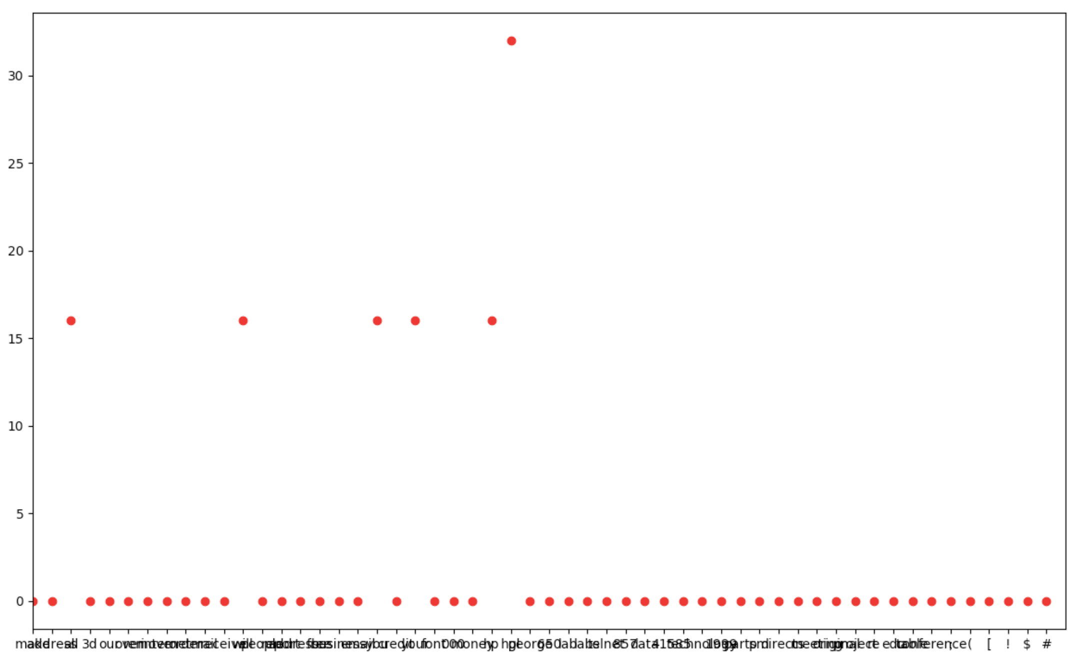
x1 = [0, 3, 0, 0, 0, 0, 0, 0, 0, 0, 0, 6, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 3, 0, 0, 0, 0, 0, 0, 0, 9, 0, 0, 6, 0, 0, 0, 0, 0, 0, 6, 0, 0, 0, 2, 0, 0, 0, 0]



x2 = [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 8, 0, 0, 0, 0, 0, 0, 8, 0, 0, 0, 0, 0, 8, 0, 0, 0, 0, 0, 0, 0, 0, 0, 8, 8, 0, 0, 0, 0, 0, 16, 0, 16, 0, 0, 0, 0, 0, 2, 0, 2, 0, 0]

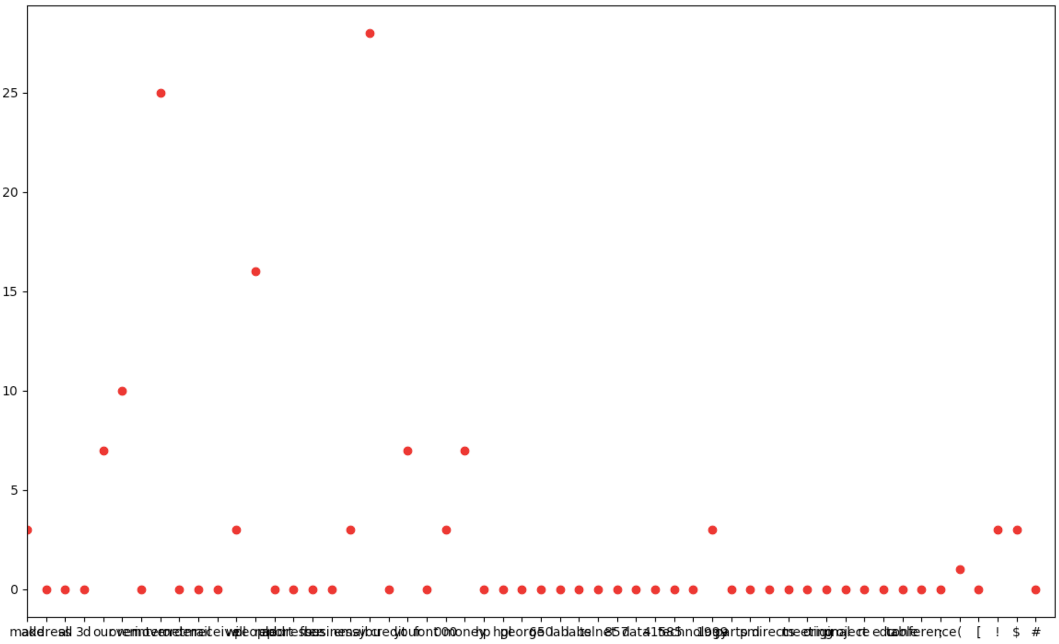


x3 = [0, 0, 16, 0, 0, 0, 0, 0, 0, 0, 0, 16, 0, 0, 0, 0, 0, 0, 16, 0, 16, 0, 0, 0, 16, 32, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]

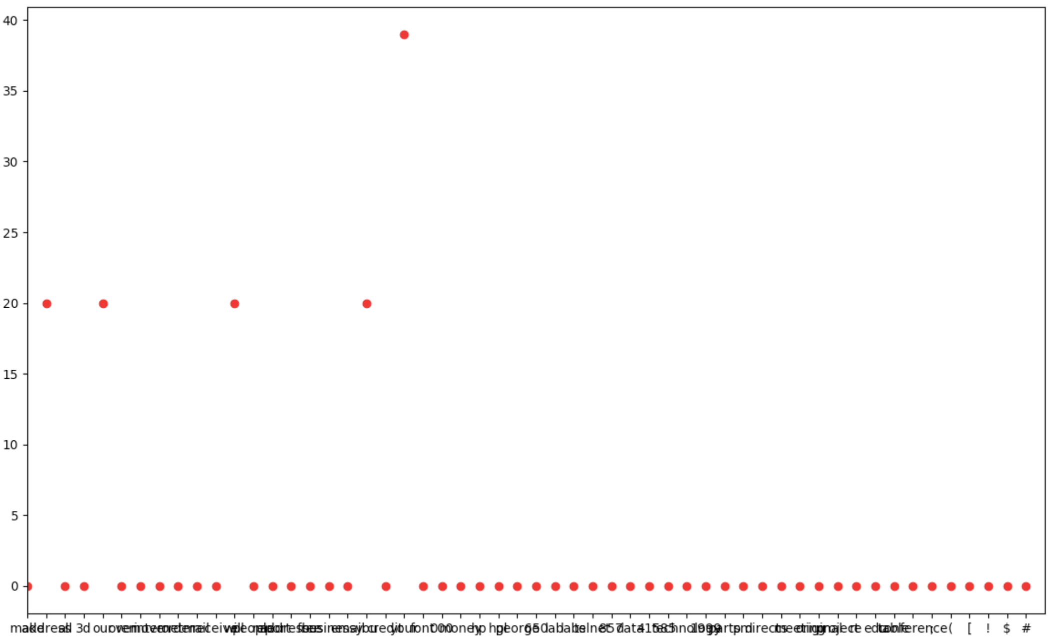


(d)

x1 = [3, 0, 0, 0, 7, 10, 0, 25, 0, 0, 0, 3, 16, 0, 0, 0, 0, 3, 28, 0, 7, 0, 3, 7, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 3, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 3, 3, 0]



x2 = [0, 20, 0, 0, 20, 0, 0, 0, 0, 0, 0, 20, 0, 0, 0, 0, 0, 0, 20, 0, 39, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]



x3 = [0, 6, 0, 0, 0, 0, 6, 0, 0, 6, 0, 6, 0, 0, 0, 0, 0, 12, 18, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 3, 0, 0, 0, 0]