

$$(3) L(u, v) = -\frac{1}{2} u^T u - \frac{1}{2} v^T v - \frac{1}{2\sigma^2} \sum_{(i,j) \in \Omega} (\mu_i^T v_j v_j^T v_i - \mu_i^T v_j^T E_2(\phi_{ij})) + \text{const}$$

$$\nabla L(u) = 0 \Rightarrow \mu_i = \arg \max_{\mu_i} L(u_i)$$

$$\Rightarrow \mu_i = \left(\frac{1}{2} I + \sum_{j=1}^M v_j v_j^T / \sigma^2 \right)^{-1} \left(\sum_{j=1}^M v_j E_2(\phi_{ij}) / \sigma^2 \right)$$

$$\nabla L(v) = 0 \Rightarrow v_j = \arg \max_{v_j} L(v_j)$$

$$\Rightarrow v_j = \left(\frac{1}{2} I + \sum_{i=1}^N \mu_i \mu_i^T / \sigma^2 \right)^{-1} \left(\sum_{i=1}^N \mu_i E_2(\phi_{ij}) / \sigma^2 \right)$$

$$(u, v) = (\{\mu_i\}, \{v_j\})$$

(4)

1. Initialize μ_0, v_0

2. For iteration $t = 1, \dots, T$

(a) E-step: Calculate $\vec{E}_{\mu_t}(\phi) = \{E_{\mu_t}(\phi_{ij})\}_{(i,j) \in \Omega}$

$$E_{\mu_t}(\phi_{ij}) = \begin{cases} \mu_{t-1}^T v_{t-1} + \sigma \times \frac{\Phi'(-\mu_{t-1}^T v_{t-1} / \sigma)}{1 - \Phi(-\mu_{t-1}^T v_{t-1} / \sigma)} & \text{if } v_{ij} = 1 \\ \mu_{t-1}^T v_{t-1} + \sigma \times \frac{-\Phi'(-\mu_{t-1}^T v_{t-1} / \sigma)}{\Phi(-\mu_{t-1}^T v_{t-1} / \sigma)} & \text{if } v_{ij} = -1 \end{cases}$$

(b) M-step: update μ_t and v_t using following equation:

$$\begin{cases} \mu_t = \left(\frac{1}{2} I + \sum_{j=1}^M v_j v_j^T / \sigma^2 \right)^{-1} \left(\sum_{j=1}^M v_j E_{\mu_t}(\phi_{ij}) / \sigma^2 \right) \\ v_t = \left(\frac{1}{2} I + \sum_{i=1}^N \mu_i \mu_i^T / \sigma^2 \right)^{-1} \left(\sum_{i=1}^N \mu_i E_{\mu_t}(\phi_{ij}) / \sigma^2 \right) \end{cases}$$

(c). Calculate $\ln p(R, u, v)$ using the following equation:

$$\ln p(R, u, v) = -\frac{1}{2} u^T u - \frac{1}{2} v^T v - \frac{1}{2\sigma^2} \sum_{(i,j) \in \Omega} \left(\right.$$

$$\ln p(R, u, v) = \frac{d}{2\pi} \ln \left(\frac{1}{2\pi} \right) - \frac{1}{2} u^T u - \frac{1}{2} v^T v + \sum_{(i,j) \in \Omega} r_{ij} \ln \Phi \left(\frac{\mu_i^T v_j}{\sigma} \right) \\ + \sum_{(i,j) \in \Omega} \left(-\frac{r_{ij}}{2} \right) \ln \left(1 - \Phi \left(\frac{\mu_i^T v_j}{\sigma} \right) \right)$$



$$(3). L(u, v) = -\frac{t}{2} u^T u - \frac{t}{2} v^T v - \frac{1}{2\delta} \sum_{(i,j) \in \mathcal{R}} (\mu_i^T v_j v_j^T \mu_i \rightarrow \mu_i^T v E_{\Phi}(\phi_{ij})) + \text{const}$$

$$\nabla L(\mu_i) = 0 \Rightarrow \mu_i = \underset{\mu_i}{\operatorname{argmax}} L(\mu_i)$$

$$\Rightarrow \mu_i = (\frac{1}{t} I + \sum_{j=1}^M v_j v_j^T / \delta^2)^{-1} (\sum_{j=1}^M v_j E_{\Phi}(\phi_{ij}) / \delta^2)$$

$$\nabla L(v_j) = 0 \Rightarrow v_j = \underset{v_j}{\operatorname{argmax}} L(v_j)$$

$$\Rightarrow v_j = (\frac{1}{t} I + \sum_{i=1}^N \mu_i \mu_i^T / \delta^2)^{-1} (\sum_{i=1}^N \mu_i E_{\Phi}(\phi_{ij}) / \delta^2)$$

$$(u, v) = (\{\mu_i\}, \{v_j\})$$

(4)

1. Initialize μ_0, v_0

2. For iteration $t = 1, \dots, T$

(a). E-step: Calculate $\bar{E}_{\Phi_t}(\phi) = \{ \Phi E_{\Phi_t}(\phi_{ij}) \}_{(i,j) \in \mathcal{R}}$

$$E_{\Phi_t}(\phi_{ij}) = \begin{cases} \mu_{t-1}^T v_{t-1} + \delta \times \frac{\Phi'(-\mu_{t-1}^T v_{t-1} / \delta)}{1 - \Phi(-\mu_{t-1}^T v_{t-1} / \delta)} & \text{if } v_{ij} = 1 \\ \mu_{t-1}^T v_{t-1} + \delta \times \frac{-\Phi'(-\mu_{t-1}^T v_{t-1} / \delta)}{\Phi(-\mu_{t-1}^T v_{t-1} / \delta)} & \text{if } v_{ij} = -1 \end{cases}$$

(b) M-step: Update μ_t and v_t using following equation:

$$\begin{cases} \mu_t = (\frac{1}{t} I + \sum_{j=1}^M v_j v_j^T / \delta^2)^{-1} (\sum_{j=1}^M v_j \bar{E}_{\Phi_t}(\phi_{ij}) / \delta^2) \\ v_t = (\frac{1}{t} I + \sum_{i=1}^N \mu_i \mu_i^T / \delta^2)^{-1} (\sum_{i=1}^N \mu_i \bar{E}_{\Phi_t}(\phi_{ij}) / \delta^2) \end{cases}$$

(c). Calculate $\ln p(R, u, v)$ using the following equation:

$$\ln p(R, u, v) = -\frac{t}{2} u^T u - \frac{t}{2} v^T v - \frac{1}{2\delta} \sum_{(i,j) \in \mathcal{R}} (-$$

$$\ln p(R, u, v) = \frac{d}{2\pi} \ln(\frac{t}{2\pi}) - \frac{t}{2} u^T u - \frac{t}{2} v^T v + \sum_{(i,j) \in \mathcal{R}} r_{ij} \ln \Phi(\frac{\mu_i^T v_j}{\delta}) + \sum_{(i,j) \in \mathcal{R}} (-r_{ij}) \ln(1 - \Phi(\frac{\mu_i^T v_j}{\delta}))$$

