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1. Answer:

(a). ① $z = a_n$

② $z = a_n x + a_{n-1}$

③ $z = a_n x^2 + a_{n-1} x + a_{n-2}$

⋮

(n+1) $z = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$

It solves the problem, and $z = p(x) = a_0 + a_1 x + \cdots + a_n x^n$

(b) Additions: n Multiplications: n.

result $= a_0$
for i from 1 to n:
  result $= a_0 + a_i \cdot \text{pow}(x, i)$.

2. Answer:

Assume $H_k \cdot v = \begin{bmatrix} H_{k-1} & H_{k-1} \\ H_{k-1} & -H_{k-1} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$.

$= \begin{bmatrix} H_{k-1} \cdot v_1 + H_{k-1} \cdot v_2 \\ H_{k-1} \cdot v_1 - H_{k-1} \cdot v_2 \end{bmatrix}$ → $O(\frac{n}{2})$, → $O(\frac{n}{2})$

If $k == 0$:
  return $(H_k \cdot v)$
else:
  return $\begin{bmatrix} \text{function}(H_{k-1}, v_1 + v_2) \\ \text{function}(H_{k-1}, v_1 - v_2) \end{bmatrix}$

∴ The problem can be divide into two subproblem:

$\begin{cases} H_{k-1}(v_1 + v_2) \\ H_{k-1}(v_1 - v_2) \end{cases}$ ⟹ recursively computing. $O(\log n)$.

∴ Total time: $O(n \log n)$

3. Answer

(a) Assume there are N items.

The probability that the k th item is selected is:

$$P_k = \cancel{\frac{1}{k} \sum_{n=k+1}^{N} (1 - \frac{1}{n})} \frac{1}{k} \prod_{n=k+1}^{N} (1 - \frac{1}{n})$$

$$= \frac{1}{k} \left(\frac{k}{k+1} \cdot \frac{k+1}{k+2} \cdot \ldots \cdot \frac{N-1}{N} \right)$$

$$= \frac{1}{N}$$

So it is uniformly distributed.

(b). Assume there are N items, P_k — probability that ~~k the~~ kth item is selected.

$$P_k = \frac{1}{2} \prod_{n=k+1}^{N} (1-\frac{1}{2})$$

$$= \frac{1}{2} \cdot (\frac{1}{2})^{N-k} = (\frac{1}{2})^{N-k+1}.$$

4. Answer:

(a)

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for i = 1 to n:
    for j = 1 to n:
        curr_num = 0
        for k = 1 to n:
            curr_num = curr_num + A[i][k] * B[k][j]
        If curr_num != C[i][j]:
            return False
return True.
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(b). $M \neq 0$, $Mx = 0 \Rightarrow \sum_{j=1}^{n} m_{xj} x_j = 0$

Assume there is an $m_{xy} \neq 0$ that $x_y = -\sum_{j \neq y} \frac{m_{xj} x_j}{m_{xy}}$

$Pr(x_y = -\sum_{j\neq y} \frac{m_{xj} x_j}{m_{xy}}) = \sum_X Pr((x_y = -\sum_{j \neq y} \frac{m_{xy} x_j}{m_{xy}}), X)$ (X is the whole set of $\vec{x}$, the num of X is 2^n).

$= \sum_X Pr((x_y = -\sum_{j\neq y} \frac{m_{xy} x_j}{m_{xy}} \mid X) Pr(S)$ (x_y can either be 0 or 1, so the probability must less than $\frac{1}{2}$)

$\leq \sum_X \frac{1}{2} \cdot Pr(S)$ ($Pr(S) = \frac{1}{2^n}$).

$= \frac{1}{2}$

$\therefore Pr(Mx) \leq \frac{1}{2}$.

(c). Let x be an n-dimensional vector with entries randomly and indepently chosen to 0 or 1.

1), If $ABx = Cx$, we can conclude $AB = C$; otherwise $AB \neq C$.

As we shown in (b), $M = AB - C$.

If $AB \neq C$, $Pr(ABx = Cx) = Pr((AB-C)x = 0) = Pr(Mx = 0) \leq \frac{1}{2}$. (It's when the mistake come)

2) Run time: Compute $ABx \longrightarrow$ First compute Bx, then $A(Bx)$, So $O(n^2)$

Compute $Cx \longrightarrow O(n^2) \longrightarrow O(n^2)$.

3) The probability that it returns right answer is 50%

To improve success probability, We can Repeat Running k times, the ~~re~~ correctness $\geq 1 - \frac{1}{2^k}$