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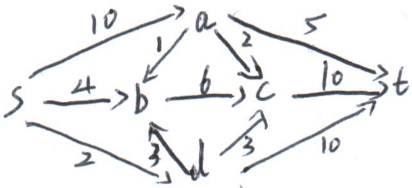
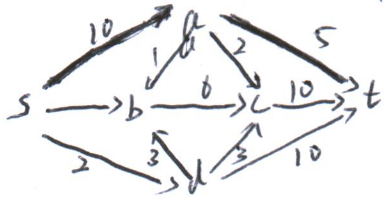
1.

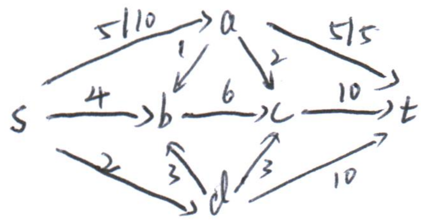
Residual Graph

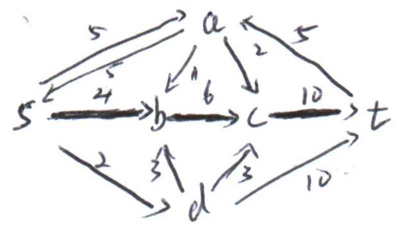




P: $s \to a \to t$

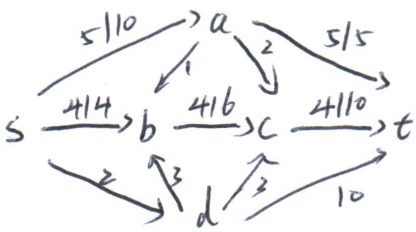
$c(P) = 5$

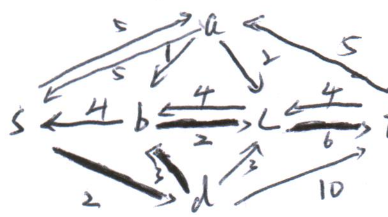




P: $s \to b \to c \to t$

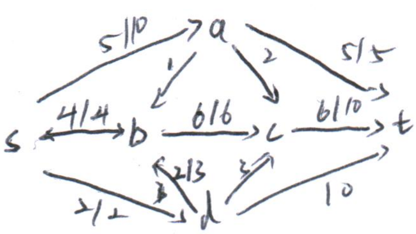
$c(P) = 4$

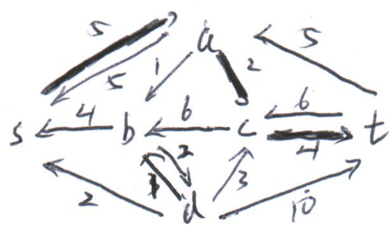




P: $s \to d \to b \to c \to t$

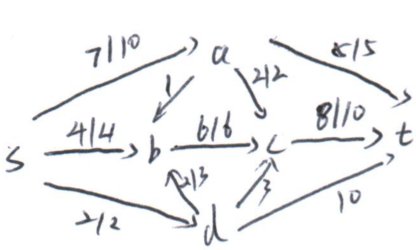
$c(P) = 2$

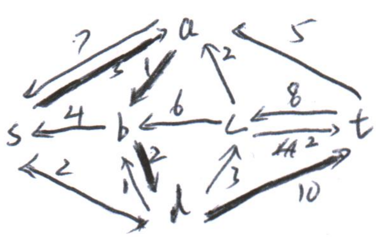




P: $s \to a \to c \to t$

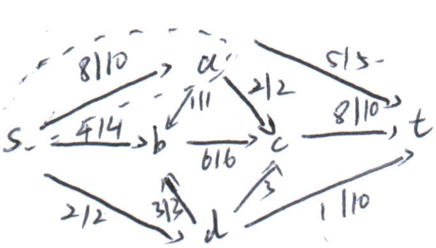
$c(P) = 2$

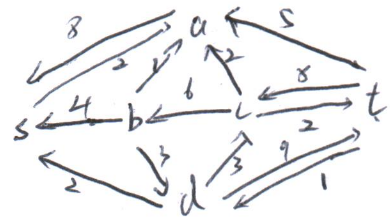




P: $s \to a \to b \to d \to t$

$c(P) = 1$

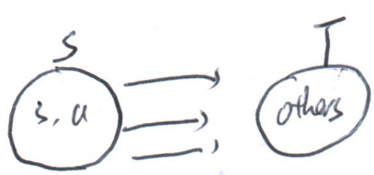




No more P available

Maximum flow: ~~8~~ $1+2+5+4+2=14$

Minimum cut:



2. Answer:

(1). $d_v > 0 \Rightarrow$ sink $\qquad 0 \le f(e) \le c_e$

$d_v < 0 \Rightarrow$ source $\qquad d_v = f^{in}(v) - f^{out}(v)$

$\therefore \sum_v d_v = \sum_v f^{in}(v) - \sum_v f^{out}(v)$.

the sum of "out" flow

the sum of "in" flow

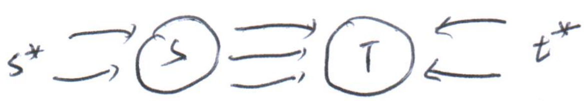
for all the vertices has been summed ~~tice, twice~~ in circulation, $\sum_v f^{in}(v) = \sum_v f^{out}(v)$.

$\therefore \sum_v d_v = 0$. $\longrightarrow$ feasible circulation.

(2) Let S be the set of sources ($d_v < 0$);

Let T be the set of sinks ($d_v > 0$).

We add new source s^* that connect to every source in S, and add new sink t^* that connect to every sink in T.



capacity: $c(s^* \to s) = -d_s$; $\qquad c(t \to t^*) = d_t$;

for $c(s^* \to s)$ will limit the supply for every source,

and every sink require d_t flow. from $c(t \to t^*)$.

In this case, we formulate ~~a~~ maximum flow problem.

We just need to run ford-fulkerson to compute the maximum flow, as well as the flow in each edge. Then we remove the s^* and t^*, ~~so are~~ and their corresponding edges. And we can calculate the net flow for each vertex. ~~It~~ If it satisfies the demand, we find a feasible circulation.

3. Answer.

(1). If we add an edge from sink to source with cost -1, and the other edge's cost is 0. Then the minimum cost flow will try to send as many units as possiblam of flows from sink to source. (because this is the only edge with negative cost.) Then ue other edges, the source will try to send as much flow to the sink. This is a maximum flow problem.

Note: the capacity for the edge from sink to source can be infinity.

(2). $\min a(v,w) f(v,w)$

subject to $f(v,w) \le c(v,w)$. $\forall (v,w) \in E$

$\sum f(v,w) - \sum f(w,v) = s(v) \quad \forall v \in V$

$f(v,w) \ge 0 \quad \forall (v,w) \in E$.

This is a linear program.

4. Answer:

Assumed that there is a $\vec{v}$ that represents the number of ways to divide 3m roll.

such as:

v_1 : 7×42 cm

v_2 : $4 \times 42 + 93$ cm

v_3 : $4 \times 42 + 108$ cm

v_4 : $3 \times 42 + 135$ cm

v_5 : $2 \times 42 + 2 \times 93$ cm

v_6 : $2 \times 42 + 93 + 108$ cm

v_7 : $2 \times 42 + 2 \times 108$ cm

v_8 : $42 + 93 + 135$ cm

v_9 : $42 + 108 + 135$ cm

v_{10} : 3×93 cm

v_{11} : $2 \times 93 + 108$ cm

v_{12} : 2×135 cm

$$\min_{v_i \geq 0} \sum_{i=1}^{12} v_i .$$

subject to :
$$7v_1 + 4v_2 + 4v_3 + 3v_4 + 2v_5 + 2v_6 + 2v_7 + v_8 + v_9 \geq 211$$
$$v_2 + 2v_5 + v_6 + v_8 + 3v_{10} + 2v_{11} \geq 395$$
$$v_3 + v_6 + 2v_7 + v_9 + v_{10} \geq 610$$
$$v_4 + v_8 + v_9 + 2v_{12} \geq 97$$

v_i can only be integers.