

The wonderful adventures of the mathematician in Logic-land: from Łukasiewicz-Moisil Logic to computers

(Invited paper)

Ioana I. Moisil

Professor, Senior Researcher
Universitatea “Lucian Blaga”, Sibiu, Romania
im25sibiu@gmail.com

Informatics restores not only the union of pure and applied mathematics, of concrete technique and abstract mathematics, but also the union of natural sciences with man and society. It re-establishes abstract and formal concepts, and brings piece between art and science, not only in the scientist's spirit, where they always are at piece, but also in their philosophy.

Grigore C. Moisil



Abstract. In 1996 Grigore C. Moisil received posthumously the IEEE Computer Pioneer Award for *the development of polyvalent logic switching circuits, the Romanian School of Computing, and support of the first Romanian computers*. Moisil, one of the greatest Romanian intellectuals, was first of all a gifted mathematician. His mentors were Bertrand Russell, Stefan Banach and Kazimierz Kuratowski, and the Romanian mathematicians Spiru Haret, Dimitrie Pompeiu and Gheorghe Țițeica, among others. He graduated mathematics at the University of Bucharest in 1926 and obtained the Ph.D. in 1929 with a thesis of applied mathematics “Analytical Mechanics of Continuous Systems”. His first works were devoted to functional analysis applied in the mechanics of deformable media and also to abstract algebra. But where we will focus on mostly is his visionary attitude embedded into a special sense

for new directions in mathematics and the encounter during his activity at the University of Iași, with the Polish school of logic, marking the beginning of the wonderful adventures in the field of multi-valued logic. My contribution is focused on emphasising the major milestones of the journey of the mathematician towards the magic world of computer science.

Keywords - Gr.C.Moisil, multi-valued logic, Łukasiewicz-Moisil algebras, switching circuits, Romanian School of Computing

I. HISTORICAL BACKGROUND

Grigore C. Moisil was born in 1906, in Tulcea, Romania. His father, Constantin C. Moisil, came from a family of priests and professors from Transylvania. He was the founder of the Romanian school of archives, a well known numismatist and a professor of history. Grigore C. Moisil's mother – Elena – was also a teacher; she was of Greek origin and she had a great influence on her son education. From an early age, Grigore C. Moisil was attracted by mathematics, but his education was a universal one, including literature, history, philosophy, music and arts, geography and biology. The family moved in 1910 to Bucharest. In 1926, Grigore C. Moisil graduated mathematics at the University of Bucharest. In the same time he followed some courses at the Polytechnic School of Bucharest. He also attended conferences given by the cultural personalities of the time in various fields, from literature to philosophy and law. His far away mentors were Bertrand Russell, Stefan Banach and Kazimierz Kuratowski. It is worth to note that at the beginning of the XX century Romania was shaping and adapting its scientific and cultural profile Romania has obtained its independence from the Ottoman Empire in 1877-78, after the Russo-Turkish War and was entering in a period of progress and stability, when culture and science reached their main level of international affirmation and were strongly connected to the European trends. In mathematics, Gr.C. Moisil was influenced by the great Romanian mathematicians of the time: Spiru Haret, David Emmanuel, Dimitrie Pompeiu, Gheorghe Țițeica, Anton

Davidoglu. They all were complete personalities, not only mathematicians; they all had studied in Romania and in France, at the Sorbonne, and their research had international recognition. Perhaps the most prominent figure of all was Spîru Haret (1851- 1912). He was a mathematician, an astronomer and a politician. Haret studied mathematics and physics in Bucharest and after graduation he went with a scholarship at the Sorbonne. There he obtained the diplomas in mathematics and physics and then the Ph.D., in 1878, with a thesis *Sur l'invariabilité des grandes axes des orbites planétaires* (*On the invariability of the major axis of planetary orbits*), defended in front of the commission led by Victor Puiseux. The thesis was published in Vol. XVIII of the *Annales de l'Observatoire de Paris*. Back in now independent Romania, he dedicated his work to the improvement of the educational system. As Minister of Education he conducted maybe the most important reform of the Romanian educational system, embedding what was suitable from the French and German educational systems. David Emmanuel (1854–1941) obtained the Ph.D. in mathematics from the Sorbonne in 1879 with a thesis on the *Study of abelian integrals of the third species*. Haret studies in mathematics and mechanics were continued by Dimitrie Pompeiu (1873-1954) who also obtained his PhD degree in mathematics at the Sorbonne University in Paris, in 1905, with a thesis *On the continuity of complex variable functions*, written under the guidance of Henri Poincaré. Pompeiu must be considered as one of the founders of the theory of hyperspaces [1]. He achieved remarkable results in complex analysis, but also in mechanics [2, 3]. Gheorghe Țițeica (1873-1939), publishing also as George or Georges Tzitzeica, is considered the founder of the Romanian school of differential geometry. Țițeica completed his studies at a preparatory school in Paris, being colleague with Henri Lebesgue and Paul Montel. In 1899 he defended his doctoral thesis *Sur les congruences cycliques et sur les systèmes triplement conjugués*, under the guidance of Gaston Darboux. He gave a series of lectures at the Faculty of Sciences in Sorbonne, at the University of Brussels (1926) and the University of Rome (1927). Carrying on the researches of the American geometer of German origin Ernest Wilczynski, Țițeica discovered a new category of surfaces and a new category of curves (Țițeica curves); his contributions represent the beginning of a new chapter in mathematics, namely the affine differential geometry [4]. Anton Davidoglu (1876-1958) was specialized in differential equations. He studied with Jacques Hadamard at the École Normale Supérieure in Paris, graduated with the thesis *Sur l'équation des vibrations transversales des verges élastique* (*On the equation of transversal vibrations of elastic string*, Sorbona, 1900) and became a professor at the University of Bucharest. His work on the theory of elasticity was cited by A. H. Love in his book *Mathematical theory of Elasticity* (Cambridge, 1934). Young Gr. C. Moisil, got inspired and motivated by these scientists to bring to mathematics the same remarkable contributions. As a student at the Faculty of Sciences at the University of

Bucharest he published 14 papers, graduated with success and in 1926, under the guidance of his spiritual model-Dimitrie Pompeiu, he obtained the Ph.D. at the University of Bucharest with the thesis *La mécanique analytique des systèmes continus* (The analytic mechanics of continuous systems). In his thesis, Moisil had applied the new methods of functional analysis to mechanics. An event that greatly influenced him was the publishing by Van der Waerden, in 1930, of the first version of *Moderne Algebra* [5]. The book was considered to have “a tremendous impact, and is widely considered to be the major text on algebra in the twentieth century” [6] for it was one of the first that used an axiomatic approach to groups, rings and fields. Moisil was fascinated by this new way of approaching algebraic concepts and this will be reflected in many of his works. Between 1930 and 1932, Moisil obtained fellowships to universities from Paris and Rome. In Paris he met, among others, with two of the greatest mathematicians of the world, Elie Cartan (1869-1951) and Jacques Hadamard (1865-1963). Cartan, with great pedagogical skills, was known for his works on group theory, systems of differential equations, and geometry, and studies on the theory of Lie groups. Hadamard, “the living legend of mathematics”, was a universal mathematician, with important results in various fields: number theory, analytical mechanics and geometry, calculus of variations and functionals, partial differential equations and elasticity and hydrodynamics. Moisil attended the seminars organized by Hadamard at the College de France and participated with a contribution to the Congress of the Scientific Societies held in 1931 at Clermont-Ferrand. In Rome, Moisil worked with Vito Volterra (1860-1940), mathematician and physicist, one of the founders of functional analysis, with contributions to integral equations, mathematical biology and the behaviour of ductile materials. The time spent in Paris and Rome was extremely rewarding for the young mathematician. In 1931 Gr. C. Moisil received the doctor degree with the paper: *Sur une classe de systèmes d'équations aux dérivées partielles* (On a class of systems of equations with partial derivatives). He also obtained a generalization of Volterra's conjugate functions and he generalized Hadamard's total geodesic varieties. In 1931 Gr.C. Moisil obtained a position at the Faculty of Sciences of the University “Alexandru Ioan Cuza” from Iași. Moisil remain in Iași for ten years. He continued to work in the field of functional analysis but in the same time devoted more time to structural mathematics. In 1934 Moisil published his first paper on non-associative algebra. At the University of Iași the atmosphere was always stimulating. Moisil attended the seminars of A. Myller and through the lectures of T. Kotarbinski became acquainted with the Lwów–Warsaw School of Logic and Warsaw school of mathematics, especially with the works of Łukasiewicz on multi-valued logics. In 1941, Moisil obtained a position at the University of Bucharest where he started to teach analysis, modern algebra and mathematical logic.

II. MULTI-VALUED LOGICS

It seems that the idea of multi-valued logic is very old. In the 4th century BC Aristotle was one of the first to question the law of the excluded middle. In the chapter IX of *De Interpretatione* the philosopher is discussing the future contingent sentences, in the frame of modal theory. Let's see what Aristotle say: "...this view leads to an impossible conclusion; for we see that both deliberation and action are causative with regard to the future, and that, to speak more generally, in those things which are not continuously actual there is potentiality in either direction. Such things may either be or not be; events also therefore may either take place or not take place. There are many obvious instances of this. It is possible that this coat may be cut in half, and yet it may not be cut in half, but wear out first. In the same way, it is possible that it should not be cut in half; unless this were so, it would not be possible that it should wear out first. So it is therefore with all other events which possess this kind of potentiality. It is therefore plain that it is not of necessity that everything is or takes place; but in some instances there are real alternatives, in which case the affirmation is no more true and no more false than the denial; while some exhibit a predisposition and general tendency in one direction or the other, and yet can issue in the opposite direction by exception." Aristotle was emphasising that these laws did not all apply to future events, but he didn't create a system of multi-valued logic. Instead he left for the future researchers to deal with the paradox of the sea battle. Aristotle was illustrating the paradox as follows : "A sea-battle must either take place to-morrow or not, but it is not necessary that it should take place to-morrow, neither is it necessary that it should not take place, yet it is necessary that it either should or should not take place to-morrow. Since propositions correspond with facts, it is evident that when in future events there is a real alternative, and a potentiality in contrary directions, the corresponding affirmation and denial have the same character." The fact that Aristotle considered that statements about the future are neither true nor false made his followers considering that the philosopher was thinking of a three-valued logic for future propositions. It is one of the reasons why the logics where the *tertium non datur* principle is not valid are called non-Chrisippian logics and not non-Aristotelian (classical logic, where the principle of excluded middle is valid, is also called Aristotelian logic).

The beginning of the 20th century brought back in the attention of logicians and philosophers the idea of multi-valued logic systems. In 1920, Jan Łukasiewicz (1878–1956) a Polish logician and philosopher introduced the three-valued logic [7]. The idea was simple: instead of two truth values: *true* and *false*, we can consider three values: *true*, *false* and *unknown*. Łukasiewicz three-value logic was built up by means of the truth-table method [8]. The matrix defining this logic is the following (fig.1):

C	0	$\frac{1}{2}$	1	N
0	1	1	1	1
$\frac{1}{2}$	$\frac{1}{2}$	1	1	$\frac{1}{2}$
1	0	$\frac{1}{2}$	1	0

Fig.1. Łukasiewicz three-value logic matrix

In [9] Moisil observed that classical mathematical logic is viewed in two different ways: as the algebra of logic represented by the study of Boolean algebras, and as calculus independent of mathematics. The point of view of Łukasiewicz is the one of the logician. He wanted to build a modal mathematical logic by considering a third truth value: "not-certain" or "unknown". Łukasiewicz and the Polish school had important contributions on the logics with three and n values. In 1921, E.L. Post introduced the algebraic study of systems with n values. In 1930, Botchvar, in order to avoid paradoxes, considers besides "true" and "false" a third value, "non-sense". Many other researchers developed the topic. The three-valued Łukasiewicz logic was axiomatised in 1931 by Wajsberg [10]. Łukasiewicz characteristic matrix can be considered as an algebraic structure. In 1940, Gr.C. Moisil, in his paper "Sur la logique à trois valeurs de Łukasiewicz" (On the third valued logic of Łukasiewicz) has introduced the notion of three-valued Łukasiewicz algebra as an attempt to give an algebraic approach to the three-valued propositional calculus considered by Łukasiewicz. In 1941, Moisil published *Contributions on the study of non-chrysippien logics. A new axiomatic system for the Łukasiewicz third valued logic*. Moisil demonstrated that the three-valued Łukasiewicz propositional calculus is an extension of the intuitionist one. His main project was to build for Łukasiewicz's logic of several values an algebraic framework in a way similar to the way George Boole has proposed in the XIXth century an algebraic model for Aristotle's logic (based on the principles of identity, non-contradiction and excluded middle). Moisil called this framework "Łukasiewicz algebras". Moisil has shown in [11] that three-valued Łukasiewicz algebras are Heyting algebras. Later on, Monteiro [12] showed that they are symmetrical Heyting algebras because of the existence of a De Morgan negation. Eventually these algebras received the more appropriate name of "Łukasiewicz-Moisil algebras" – LM_n algebras [14]. In 1956, A. Rose demonstrated that for $n \geq 5$ the LM_n algebra does not model the Łukasiewicz logic. In 1964, Moisil published a logic that models his algebra in the general case of $n \geq 5$, called today *Moisil logic* [13]. Łukasiewicz-Moisil algebras are widely presented and discussed in the Monograph by V.Boicescu, A. Filipoiu, G. Georgescu and S. Rudeanu [15].

A LM_n algebra is a De Morgan algebra (a notion also introduced by Moisil) with $n-1$ additional unary, "modal" operations: $\nabla_1, \dots, \nabla_{n-1}$, i.e. an algebra of signature $(A, \vee, \wedge, \neg, \nabla_{j \in J}, 0, 1)$ where $J = \{1, 2, \dots, n-1\}$. (Some sources denote the additional operators as $\nabla_{j \in J}^n$ to emphasize that they depend on the order n of the algebra.^[7]) The additional unary operators ∇_j must satisfy the following axioms for all $x, y \in A$ and $j, k \in J$:^[3]

1. $\nabla_j(x \vee y) = (\nabla_j x) \vee (\nabla_j y)$
2. $\nabla_j x \vee \neg \nabla_j x = 1$
3. $\nabla_j(\nabla_k x) = \nabla_k x$
4. $\nabla_j \neg x = \neg \nabla_{n-j} x$
5. $\nabla_1 x \leq \nabla_2 x \leq \dots \leq \nabla_{n-1} x$
6. if $\nabla_j x = \nabla_j y$ for all $j \in J$, then $x = y$.

Fig. 2 Definition of a LM_n algebra

In figure 2 there is the definition of a LM_n algebra as given in [14].

In the autumn of 1967, professor L. Zadeh visited Bucharest. With this occasion Gr.C.Moisil became acquainted with Zadeh's works about *fuzzy sets*. Moisil started to study *fuzzy sets* as a set theory in logic with a totally ordered set of logical values.

It is worth mentioning Gr.C.Moisil other contributions to mathematical logic: strict positive logic, modal logics, theory of pseudo-classes, intuitionist logic, logic of cuantic theory, contributions that eventually find applications in various fields of computer science.

III. APPLICATION OF ALGEBRA TO MODERN COMPUTERS

Since 1950 Moisil applied his ideas to the theory of switching circuits. At the University of Bucharest and at the Institute of mathematics, Gr. C. Moisil created a school that applied algebra to switching circuits and other fields that could be of interest for computer science (mathematical linguistics, automatic translation, and artificial intelligence). Since 1954 there is a special course on "The algebraic theory of switching theory" at the faculty of Mathematics and physics in Bucharest. The algebraic theory of switching circuits has been developed in 1938 by Claude Shannon in his master thesis *A Symbolic Analysis of Relay and Switching Circuits* [18] and by V.I.Shestakow in his dissertation at the University of Moscow. [16]. There were developments also in Japan. Nakashima and Hanzawa had studies (published in Japanese, in 1936, and in English in 1938) where "many laws of the algebra are developed which in fact coincide with familiar laws of Boolean algebra, but the authors do not state that the algebra is a Boolean algebra" [17]. The Romanian school of algebraic switching theory considered several methods: Boolean algebras, Galois complex fields and multi-valued logic.

In general, a switching circuit (with contacts or relays) has three kinds of elements: command elements, execution elements and intermediary elements. In the case of discrete switching circuits all compound elements can have a finite number of states and a finite number of positions. Shannon considered a class of switching circuits: *Series-Parallel Two-Terminal Circuits* (also named circuits II). Shannon

constructed a calculus based on a set of postulates which described basic switching ideas and showed that this calculus was equivalent to certain elementary parts of the calculus of propositions, which in turn was derived from the Boolean algebra of logic [18].

A closed algebraic system that contains a set \mathbf{K} of two or more elements, two binary operators " \cdot " logic conjunction **AND**, and " \vee " logic disjunction **OR**, a unique element **1** (one) that is the identity element for the logical conjunction, and a unique element **0** (zero) that is the identity element for the logical disjunction, is a Boolean algebra if for any $\mathbf{a}, \mathbf{b}, \mathbf{c}$ in \mathbf{K} the axioms in figure 3 are satisfied. There is also a unary operator " \neg " negation **NOT**.

Idempotence	$a \vee a = a, \quad a \cdot a = a$
Comutativity	$a \vee b = b \vee a, \quad a \cdot b = b \cdot a$
Associativity	$a \vee (b \vee c) = (a \vee b) \vee c,$ $a \cdot (b \cdot c) = (a \cdot b) \cdot c$
Absorption	$a \vee (a \cdot b) = a, \quad a \cdot (a \vee b) = a$
Distributivity	$a \vee (b \cdot c) = (a \vee b) \cdot (a \vee c)$ $a \cdot (b \vee c) = (a \cdot b) \vee (a \cdot c)$
Involutivity	$\overline{\overline{a}} = a$
Complement	$a \vee \overline{a} = 1, \quad a \cdot \overline{a} = 0$ $a \vee 0 = a, \quad a \cdot 1 = a$
Identity	$a \vee 1 = 1, \quad a \cdot 0 = 0$
De Morgan Laws	$\overline{(a \vee b)} = \overline{a} \cdot \overline{b}, \quad \overline{a \cdot b} = \overline{a} \vee \overline{b}$

Fig. 3 Axioms of Boolean algebra

A contact, for example, can be *on* or *off*. We will associate to the contact a variable x with two values: $x = 0$ if the contact is *on* and $x = 1$ if the contact is *off*. The main problem that engineers had to face in designing and building such circuits is a problem of cost optimization, i.e. to find an equivalent schema with fewer components. In fact there are four categories of problems: analysis, synthesis, simplification and classification. Shannon explained the analysis and synthesis method for circuits as follows: *Any circuit is represented by a set of equations, the terms of the equations corresponding to the various relays and switches in the circuit. A calculus is developed for manipulating these equations by simple mathematical processes, most of which are similar to ordinary algebraic algorithms. For the synthesis problem the desired characteristics are first written as a system of equations, and the equations are then manipulated into the form representing the simplest circuit.* [18].

In figure 4 there are represented the notations and symbols used by Shannon and Nakashima.

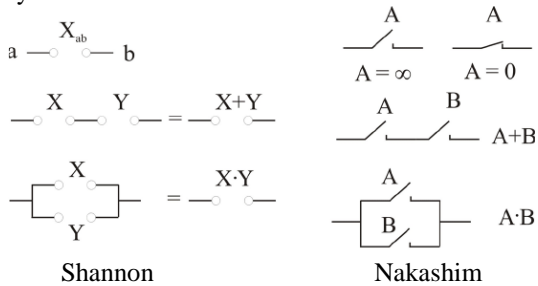


Fig. 4 Notations and symbols used by Shannon and Nakashima

Gr.C.Moisil published in 1953-54 his lectures on the algebraic theory of switching circuits. During 1954 he published four papers: *Using Galois complex fields on the theory of switching circuits. I. On rectifier circuits, The algebra of rectifier circuits diagrams, Using Galois complex fields on the theory of switching circuits. II. Circuits with two intermediary elements, Algebraic theory of the functioning of sequential switching circuits with two intermediary elements.* In the Addendum there is the bibliography of Romanian works published between 1950 and 1959. In figure 5 there are symbols and notations used by Moisil.

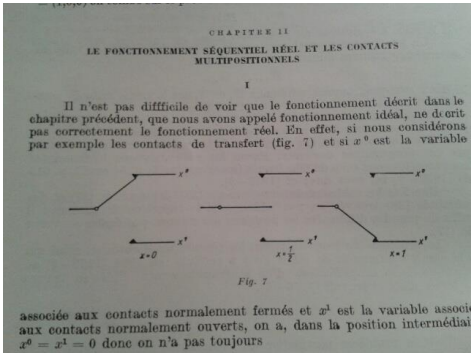


Fig. 5a Symbols and notations used by Moisil

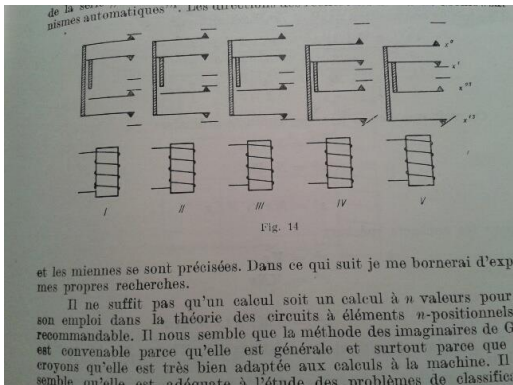


Fig. 5b Symbols and notations used by Moisil

Besides Boolean algebra and Galois finite fields theory, Moisil used in the study of switching circuits another very strong algebraic instrument – the algebra of multi-valued logics.

In 1959 Gr. C. Moisil published the book “*Teoria algebrică a mecanismelor automate*” (*The algebraic theory of switching circuits*). The book is about the research carried on by Moisil and his co-workers in the domain of algebraic theory of switching circuits, with special applications of the Galois finite fields theory. Edward Forrest Moore (1925-2003) the American professor of mathematics and computer science, inventor of the *Moore finite state machine*, and an early pioneer of artificial life, made a review of the Moisil’s book. E.F. Moore wrote: *The most distinctive characteristic of this approach is the very strong use that is made of Galois fields. Tables for multiplication, addition, and exponentiation, for GF(3³) and GF(5²) are inserts which fold out from the book, and similar tables for GF(pⁿ) whenever pⁿ < 25, are printed in the text of the book. The pⁿ states of a circuit having n relays, each of which has p positions, are represented by the elements of GF(pⁿ). Lagrangian interpolation polynomials are used in these finite fields, to permit finding algebraic formulas for functions which take on arbitrary values at each state of the circuit. Time is usually assumed to take on only discrete values. Extensive use is also made of n-valued logic.*

In 1958 Gr.C. Moisil explained himself: *As soon as I was acquainted with the works of V.I.Shestakow and M.A. Gavrilow on the application of logic to the study of electrical circuits, I wanted to use also the multi-valued logics. In the fall o 1955, during a voyage in Poland, I have the opportunity to discuss this topic with Henryk Greniewski, Marek Greniewski and R. Marczyński. We have published, Mark Greniewski and myself, the first two Notes of the series “Emploi des logiques à trois valeurs dans la théorie des mécanismes automatiques”.* Following these meetings, Moisil knew what he wanted to do. He had observed that, in order to be used in the theory of switching circuits with *n*-positions, it is not sufficient for a calculus to be a *n*-values calculus. In Moisil opinion, the method of Galois finite fields is suitable because it has a general character and it seems to be very well adapted to machine computations. The method is also adequate to the study of classification problems. The logics with three and five values are considered by Moisil extremely well adapted to the study of the real functioning of ordinary relays, and those with four values to the study of certain contacts mounted on a relay. He considered that a general theory of switching mechanisms can be developed. In [9], Moisil explained that there are five types of finite automata:

- Circuits with electromagnetic components: switching circuits, ordinary relays with ideal or real functioning, polarized relays, time-delay relays, selectors etc.

- Circuits with electronic components: diodes, triodes, pentodes, flip-flops, delay lines, cryotrons, magnetic rings, transistors.
- Neuronal systems.
- Mechanisms with pneumatics elements.
- Devices with continuous variation encoded by a discrete code.

For these five types, Moisil presented models of the general theory of finite automata [1958-4] [1959-3].

The results of the research carried on by Moisil and his co-workers were presented to the European researchers from Bari, Bologna, Bratislava, Brno, Dresden, Kiev, Milan, Moscow, Napoli, Prague, Rome, Sofia, Turin, Warsaw, Wrocław.

IV. CONTRIBUTIONS OF GR.C. MOISIL TO ROMANIAN SCHOOL OF COMPUTING AND THE DEVELOPMENT OF THE FIRST ROMANIAN COMPUTERS

The contributions of Gr.C. Moisil to Romanian School of Computing and the development of the first Romanian computers have been widely presented by Solomon Marcus [19], Gheorghe Paun [20], Dragos Vaida [21], Constantin Popovici, Vasile Baltac [22], George Georgescu, Leon Livovschi, Cristian Calude, Sergiu Rudeanu, Afrodita Iorgulesc, Ion Dincă, Alexandru Popovici [23], Ioan Văduva, Marin Vlada, and many others.



Gr.C. Moisil at the Computer Center of the University of Bucharest

Fig.5 Gr.C.Moisil and his students and co-workers at the Computer Centre of the University of Bucharest

Gr.C.Moisil was not only a gifted mathematician; he was a promoter of novelty in science and technology, a true catalyzer. And he was a methodical and perseverant personality. He knew that if you want to open the doors to computers and cybernetics, you have to build the infrastructure: to educate people to understand the new, to form tutors to teach the new, to organize research in the field. He started, in 1954, a series of lectures on the algebraic theory of finite automata (*Teoria algebrică a*

mecanismelor automate) at the Faculty of Mathematics and Physics in Bucharest. The lectures were attended not only by mathematicians but also by engineers. It was the birth of the Section of *Computing Machinery* at the University of Bucharest. In 1956 he was nominated as president of the *commission* for automation of the Romanian Academy and supported the creation of the *section of automatic control* at the Polytechnic Institute from Bucharest. In the '50, Gr.C. Moisil assisted the development of the first Romanian computers: CIFA (Calculatorul Institutului de Fizică Atomică) at the Institute for Atomic Physics in Bucharest, MECIPT, at the Polytechnic Institute from Timișoara, and DACICC from Computing Institute of the Romanian Academy from Cluj. In 1962 he organized in Bucharest the University Computing Center. In 1968 following the visit of professor Moisil and other Romanian personalities in USA, the Computing Center of the University of Bucharest received a IBM/360 computer. In the same year, Moisil organized in Bucharest, under the aegis of IFAC the international congress on *Hazard and Race Phenomena in Switching Circuits*. In 1968 Romania bought from the French Company CII-Compagnie Internationale d'Informatique the licence for IRIS 50 computers and started the manufacturing of the Felix computers family (Felix C-256, Felix C-32, Felix C-512, Felix C-515, Felix C-1024, Felix C-5000, Felix C-8000, Felix C-8010). In 1972, Gr.C.Moisil, with the support of the minister of Education – Mircea Malița - obtained the creation of a Section of Informatics in all Faculties of Mathematics of Romania. Moisil supported the organisation of computing centres in all main cities in the country, the establishing of two important institutions: the Institute for Computing Technique (ITC), in 1967, and in 1970, the Central Institute for Informatics (ICI Bucuresti). ITC was in charge with the development of hardware components, embedded equipments etc, and ICI, with the design and implementation of software systems, training for programmers, analysts, computer operators, software engineers. Gr.C. Moisil promoted the organisation of informatics summer schools and he lectured in many high schools in the country explaining what computer science is and why computer science needs mathematics. It is worth to mention the lectures given by Moisil, and the papers published, on the mathematical linguistics and automated translation, and on the application of mathematics in human sciences (history, psychology, sociology, anthropology, archeology etc.).

Gr.C. Moisil, member of the Romanian Academy, was also member of the Academy of Sciences from Bologna, the International Institute of Philosophy from Paris, the Polish Academy of Sciences, *Doctor Honoris Causa* of the Academy of Sciences from Bratislava, the Academy of Science of Messina, Italy, the Royal Society for Science in Liege, and in 1971 was elected vice-president of the International Union of History and Philosophy of Science.

V. IN STEAD OF CONCLUSIONS

In 1973 Gr.C. Moisil was invited for a series of lectures in USA and Canada. He died of a CVD on the 21st of May, in Ottawa.



Fig. 6 The medal “Computer Pioneer” – IEEE Computer Society

In 1996, the IEEE Computer Society granted him posthumously the Computer Pioneer Award, in recognition of his merits as founder of computer science, along with other thirty-two scientists among whom E.W.Dijkstra, H.H. Aitken, J.F.Forester, K.Zuse and D.A.Huffman.[24] Solomon Marcus (1925-2016), one of the outstanding personalities of the Romanian culture and mathematical world, wrote: *Moisil's heritage belongs to the Romanian culture and the new generations deserve to know this unusual personality.*

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