# Optimization of thresholds in serial multimodal biometric systems


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***Abstract*—Multimodal biometric verification systems use information from several biometric modalities to verify an identity of a person. The false acceptance rate (FAR) and false rejection rate (FRR) are metrics generally used to measure the performance of such systems.**

**In this paper we propose a novel approach to determine the upper and lower acceptance thresholds in sequential multimodal biometric matching, in such a way that the expected values of FAR and FRR for the entire system are minimized. We linearize locally the score distributions of both genuine users and impostors using the least squares method, and derive formulas for the approximated FAR and FRR for each matcher. Further, we aim to minimize both probabilities for entire processing chain. In order to find the best compromise between them, we analyze the efficient solutions to the associated bi-objective programming problem.**

**The results of our experiments are also reported in the paper. They showed a good performance of the sequential multiple biometric matching system based on optimized thresholds comparing with the widely adopted parallel fusion multimodal biometric systems.**



***Index Terms*—multi-modal biometrics, sequential fusion, multi-criteria optimization**


## I. Introduction

Biometrics is the automated recognition of individuals based on their behavioral and biological characteristics [11]. Biometric recognition is used for many purposes including criminal identification, secure access control, forensics and so forth. It was intensively researched and widely applied in the last decade [17]. A number of biometric technologies have been developed and several of them are being used in a variety of applications [7]. The most commonly used modalities are fingerprints, face, iris, speech, and hand geometry. Due to their strengths and weaknesses, the choice of one or another modality is strictly dependent on the application requirements.

In his book [5], Kaklauskas presented different methods for analyzing the body language (movement, position, use of personal space, silences, pauses and tone, the eyes, pupil dilation or constriction, smiles, body temperature and the like) for better understanding people's needs and actions, including biometric data gathering and reading. Filip [3], briefly reviewed the book, and emphasized that it addressed two modern research domains: intelligent and integrated decision support systems and biometrics-based human-computer interface.

An analysis of a multimodal biometric system based on level of fusion was presented in [10]. The authors discussed the biometric systems, the limitations of individual biometric, and various fusion levels and methods of multimodal systems.

The parallel fusion mode was first introduced in 1998 [4]. Fingerprint and face modalities were simultaneously used for identification. Serial fusion of multiple matchers is a good trade-off between the widely adopted parallel fusion and the use of a mono-modal verification system [12]. An alternative to parallel fusion of biometric data is the use of serial fusion.

Kumar and Kumar [6] presented a new approach for the adaptive management of multimodal biometrics. They employed the ant colony optimization for the selection of the key parameters like decision threshold and fusion rule, to ensure the optimal performance in meeting varying security requirements during the deployment of multimodal biometrics systems.

Zhang et al. [17] proposed a novel framework for serial multimodal biometric systems based on semi-supervised learning techniques. They have promoted the discriminating power of the weaker but more user convenient traits over the use of the stronger but less user convenient traits. In this way, they proposed an alternative to other existing serial multimodal biometric systems that suggest optimized orderings of the traits deployed and parameterizations of the corresponding matchers but ignore the most important requirements of common applications. Their experiments on two prototype systems demonstrated the advantages of their methodology.

Marcialis et al. [8] proposed a theoretical framework for the assessment of performance of serial fusion multimodal systems, theoretically evaluated the benefits in terms of performance, and estimated the errors in the model parameters computation. They analyzed the model from the point of view of its pros and cons, and performed preliminary experiments on a benchmark found in the literature.

The importance of the use of multimodal biometrics in the area of secure person authentication is highlighted in a recent study [13]. That study provided a different perception on how to use biometrics on the highest level of the network security with the fusion of multiple biometric modalities.

Snelick et al. [14] studied the performance of the multimodal biometric authentication systems using the state-of-the-art commercial off-the-shelf (COTS) fingerprint and face biometric systems on a large-scale population. They also proposed new methods of normalization and fusion that improved the accuracy of the biometric systems.

The remainder of the paper is organized as follows. In Section 2 we formulate the problem that we solve. Our novel approach is introduced in Section 3. Some computational results are given in Section 4. Final remarks and directions for future work are inserted in Section 5.

## II. Problem formulation

Multimodal biometric verification systems use information from several biometric modalities to verify an identity of a person. The false acceptance rate (FAR) and false rejection rate (FRR) are metrics generally used to measure the performance of such systems. The FAR is the probability that the system incorrectly matches the input pattern to a non-matching template in the database. It measures the percent of invalid inputs which are incorrectly accepted. In case of similarity scale, if the person is an impostor in reality, but the matching score is higher than the threshold, then he is treated as genuine. The FAR depends on the threshold value: the FAR increases as the threshold decreases. The FRR is the probability that the system fails to detect a match between the input pattern and a matching template in the database. It measures the percent of valid inputs which are incorrectly rejected. It also depends on the threshold value: the FRR increases as the threshold value increases. The visual characterization of the trade-off between the FAR and FRR is generally given by a graphic representation of the genuine acceptance rate (GAR=1-FRR) with respect to the false acceptance rate.

In general, a matching algorithm performs a decision based on a threshold. The threshold determines how close to a template the input needs to be for it to be considered a match. If the threshold is reduced, there will be fewer false non-matches but more false accepts. Conversely, a higher threshold will reduce the FAR but increase the FRR. Our goal is to find the system's thresholds that assure a good compromise between the minimizations of the false acceptance rate and false rejection rate.

Pato and Millett, in their book [11], emphasized that the biometric recognition systems are inherently probabilistic. In their opinion, the biometric recognition involves matching, within a tolerance of approximation, of observed biometric traits against previously collected data for a subject. The approximate matching is required due to the variations in biological attributes and behaviors both within and between persons.

Let us assume that the multimodal biometric system consists of $N$ matchings. After any of the first $N-1$ matchings, one of the following three decisions will be made: accept the person as genuine, reject the person as impostor, or demand another matching. Naturally, after the last matching, only two decisions will be possible: to accept, or to reject the person.

For each modality in the system, we collect data from $n$ persons. For each person we take $m$ samples, and construct an $m \times n$ matrix $M$ of input information. For real-life databases the components of matrix M are vectors derived by using classic protocols, that extract biometric features from the real data collected (images, videos, speeches).

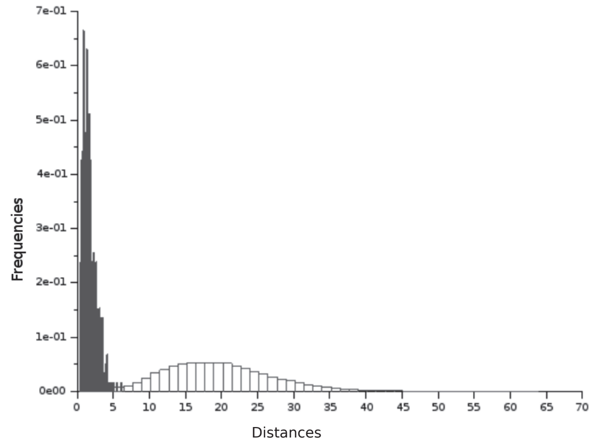


Fig. 1. An example of a pair of genuine and impostor distributions based on the same set of collected data. The genuine distribution is located on the left side of the representation.

In the beginning we restrict our attention to one of the first $N-1$ matchings. Using the matrix $M$ we compute the distances between each two collected samples, and derive the distributions for the genuine users and impostors. The distribution of the genuine users is constructed using the distances between each two components of the same column of the matrix $M$. The distribution of the impostors uses the distances between each component of matrix $M$ and each component of matrix $M$ that lies on a different column. Two samples obtained from the same person are highly expected to have a small distance between them. Thus, the genuine distribution will generally have a range of smaller values than the impostor distribution. A graphical representation of such distributions is presented in Figure 1 using normalized histograms.

Since the number of distances computed for the genuine distribution is significantly less then the number of distances computed for the impostor distribution, the shape of the impostor distribution is more likely a normal distribution, than the genuine one. Since the number of intervals used to construct the histograms is the same for both distributions, the height of the genuine distribution is significantly greater than the height of the impostor distribution.

Let $A$ and $B$ denote the minimal distance in the impostors distribution and the maximal distance in the genuine distribution, respectively. When one of the first $N-1$ matchings of the biometric system is used to verify a person, a decision is made according to two thresholds $x_1, x_2 \in [A, B]$, as follows: if the distance between the given sample and the sample in the database is less than $x_1$ the person is accepted as genuine; if it is greater than $x_2$ the person is rejected as impostor; but if the distance belongs to the uncertainty region $[x_1, x_2]$, the verification process demands another matching.

For the last matching in the sequence, the distributions of both genuine users and impostors are constructed in the same way; two values $C$ and $D$ are specified, with the same meaning as $A$ and $B$ from the first matching; but the decision is based on a single threshold, $x_3$, that lies between $C$ and $D$ as follows: if the biometric is less than $x_3$ the identity of the verified person is accepted as genuine, otherwise it is rejected as impostor.

The main problem is to find proper values for the thresholds involved in the given sequence of matchings. Our goal is to provide optimized values for these thresholds, in sense of minimizing both false acceptance and false rejection errors, FAR and FRR, respectively.

## III. Solving method

In this section, we restrict our attention to a bi-modal matching system. We first evaluate the false acceptance and false rejection errors with respect to the thresholds $x_1$, $x_2$ and $x_3$. Let $a(x_1)$ denote the area under the impostor distribution bounded to the right by the vertical line that passes through $x_1$; and let $b(x_2)$ denote the area under the genuine distribution bounded to the left by the vertical line that passes through $x_2$ – for the first matching. Similarly, let $c(x_3)$ denote the area under the impostor distribution bounded to the right by the vertical line that passes through $x_3$; and $d(x_3)$ denote the area under the genuine distribution bounded to the left by the vertical line that passes through $x_3$ – for the second matching.

The probability of a false match error based on the first matching is $a(x_1)$, and it is $c(x_3)$ for the second matching. Similarly, the probability of a false non-match error based on the first matching is $b(x_2)$, and it is $d(x_3)$ for the second matching.

Therefore, the probability of a false match error, and the probability of a false non-match error, based on both matchings are

$$\begin{array}{l} a(x_1) + (a(x_2) - a(x_1))\, c(x_3), \\ b(x_2) + (b(x_1) - b(x_2))\, d(x_3), \end{array} \tag{1}$$

respectively.

In order to find proper bounds to the uncertainty regions involved in the verification process, we minimize both probabilities of error. Since a low false match error means a high false non-match error and reverse, we have to search for a good compromise between the two errors. Such compromise is achieved by solving the bi-objective programming problem

$$\begin{array}{ll} \min & a(x_1) + (a(x_2) - a(x_1))\, c(x_3), \\ \min & b(x_2) + (b(x_1) - b(x_2))\, d(x_3), \\ \text{s.t.} & A \le x_1 \le x_2 \le B, \\ & C \le x_3 \le D. \end{array} \tag{2}$$

In order to compute the probabilities involved in Formulas (1), we linearize locally the score distributions of both genuine users and impostors using the least squares method. See Figure 2 for a graphical representation of both estimated distributions and their local linearizations for one matching.

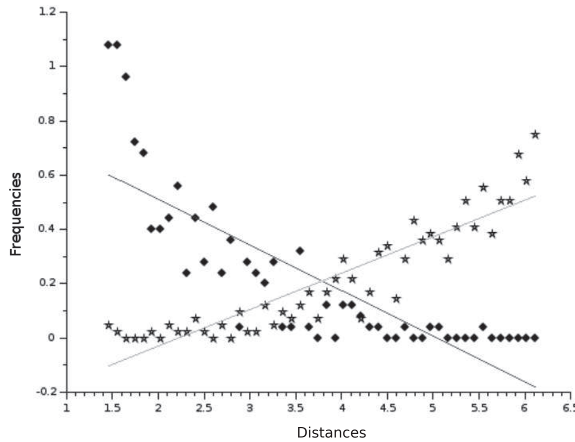


Fig. 2. Locally linearized distributions by the least square method

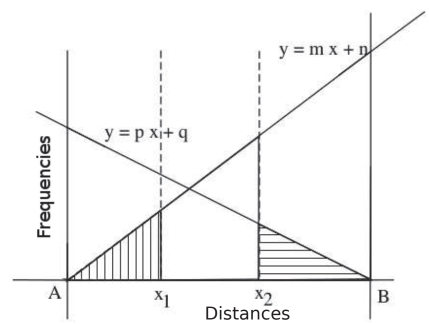


Fig. 3. The areas involved in computing the FAR (vertical shaded) and FRR (horizontal shaded) on the first $N-1$ match levels

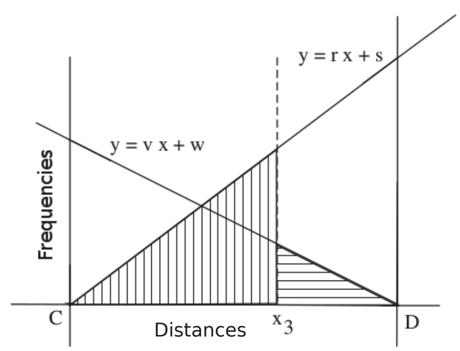


Fig. 4. The areas involved in computing the FAR (vertical shaded) and FRR (horizontal shaded) on the last match level

Figures 3 and 4 show how the lines obtained by the least square method bound the regions involved in the evaluation of the FAR and FRR for both matchings. The thresholds used in the decision process are also seen in these figures.

Let $f_1(x)$ and $g_1(x)$ denote the linear expressions $mx+n$ and $px+q$ obtained for the first match, as seen in Figure 3. Let $f_2(x)$ and $g_2(x)$ denote the linear expressions $rx+s$ and $vx+w$ obtained for the second match, as seen in Figure 4.

Intersecting the graphs of $f_i(x)$ and $g_i(x)$, $i = 1, 2$ with the horizontal axis we redefine $A = -n/m$, $B = -q/p$, $C = -s/r$, and $D = -w/v$; and derive the formulas for the approximated FAR and FRR for each matching as

$$
\begin{array}{ll}
a(x_1) = (mx_1 + n)^2/2m, & b(x_2) = -(px_2 + q)^2/2p, \\
c(x_3) = (rx_3 + s)^2/2r, & d(x_3) = -(vx_3 + w)^2/2v.
\end{array}
$$

These are quadratic expressions with respect to the thresholds values $x_1$, $x_2$ and $x_3$, respectively. Combining them we derive the polynomial expressions of degree 4, for the functions presented in (1). For a multimodal matching system, these expressions become polynomial expressions of degree $2N$.

By these transformations, the general bi-objective problem (2) becomes a bi-objective problem easy to solve using the classic optimizations packages for nonlinear programming. One way to solve the bi-objective problem (2) is to aggregate the two objective functions, and optimize the obtained function. We propose the weighted sum method, with the initial weights $(1, 1)$, to aggregate the objectives. By changing the set of weights, we favorize one or another type of error.

The advantage of finding optimized thresholds, to be used by the decision-maker in constructing the sequential multimodal system, resides in yielding the needed information in the a priori stage of the decision. Generally, the trade-off between two conflicting objectives, and particularly the trade-off between FAR and FRR, may be a subject of a wider discussion. The usefulness of a priori, a posteriori, and interactive methods in multiple objective optimization are highly emphasized in the literature (see for instance [1]). A visualization technique for accessing the solution pool in the interactive methods of multiple objective optimization can be found in [2].

## IV. Computation results

### A. *Experiments using random generated instances*

In order to test the performance of our method we organized the experiments as follows.

First, the input data, necessary to construct the score distributions, were randomly generated according to the rules that made them proper data for biometric tests. More precisely, we have generated a set of vectors with $l$ real positive components. Each of these vectors simulates the essential information that in real situations is extracted from the taken pictures of a person during a biometric measurement. The first sample of each user is generated as a vector of uniformly distributed random numbers. The mean and standard deviation were varied through instances. Each subsequent sample of the same user is generated as a vector of random numbers with normal distribution, keeping the same mean and variance as for its corresponding first sample. In this way we provided that the samples of one person are more similar to each other than when compared to samples of another person.

Then, we have computed the Euclidean distances between each two generated vectors; split them in two categories to be used for the construction of the genuine and impostor distributions; linearized the distributions, and computed $A$, $B$, $C$ and $D$. We use the linearized expressions to evaluate the errors of false acceptance and false rejection, and construct the bi-objective optimization model. In order to find the optimized thresholds we add the two error functions, and minimize the total error.

Each triple $(x_1, x_2, x_3)$, including the triple of optimized thresholds, defines a bi-modal biometric system, whose performance will be evaluated.

In order to estimate the FAR and FRR of each system using the same data as in constructing the genuine and impostor distributions, we successively collect the answers of the system, obtained when each person $i = 1, \ldots, n$ claims that he/she is the person $k = 1, \ldots, n$, and he/she is verified with all samples $j = 1, \ldots, m$ of the person $k$, according to the specific thresholds of the system. Each time when the system accepts a person $i$ as being the person $k$, the numerator of the ratio FAR increases with 1 unit. Similarly, each time when the system rejects a person $i$ when he/she claims that he/she is person $i$ the numerator of the ratio FRR increases with 1 unit. The nominator of FAR is $n(n-1)m^2$, while the nominator of FRR is $m(m-1)n/2$. Finally, we compute the genuine acceptance rate (GAR) as 1-FRR. When biometric samples of a control group are available, we collect the system's answers obtained by checking the control samples instead of the initial samples.

In what follows we present detailed computation results for one random generated instance with $n = 100$, $m = 5$, and $l = 3$. Starting from the score distributions of both genuine and impostors we computed

$$A = 10.49, \quad B = 934.62, \quad C = 15.69, \quad D = 963.22.$$

Linearizing the genuine and impostor distributions for each separately, we obtained the slopes and the values on the ordinates of the linear expressions that locally approximate the distribution functions

$$
\begin{array}{ll}
m = 1.17 \cdot 10^{-6}, & n = -9.25 \cdot 10^{-5}, \\
p = -2.73 \cdot 10^{-6}, & q = 2.37 \cdot 10^{-3}, \\
r = 1.16 \cdot 10^{-6}, & s = -6.47 \cdot 10^{-5}, \\
v = -2.67 \cdot 10^{-6}, & w = 2.36 \cdot 10^{-3}.
\end{array}
$$

The updated bounds, that have to be included in the optimization model, are

$$A = 78.79, \quad B = 868.9, \quad C = 55.58, \quad D = 882.7.$$

We further derived the formulas for the areas involved in the computation of the approximated FAR and FRR, and constructed the optimization model

$$
\begin{array}{l}
\min \\
(0.5x_3 - 27.7)^2 \left[(0.117x_2 - 9.25)^2 - (0.117x_1 - 9.25)^2\right] \\
+ (76.36x_1 - 6037.3)^2 + (101468.1 - 116.9x_2)^2 + \\
(436.7 - 0.49x_3)^2 \left[(237 - 0.273x_1)^2 - (237 - 0.273x_2)^2\right]
\end{array}
$$

subject to

$$
\begin{array}{l}
x_1 \leq x_2, \\
78.79 \leq x_1 \leq 868.9, \\
78.79 \leq x_2 \leq 868.9, \\
55.58 \leq x_3 < 882.7.
\end{array}
$$

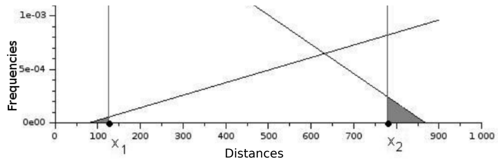


Fig. 5. The optimized thresholds of the first modality involved in the bi-modal biometric system

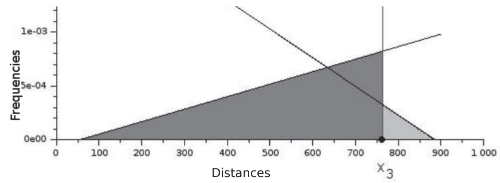


Fig. 6. The optimized thresholds of the second modality involved in the bi-modal biometric system

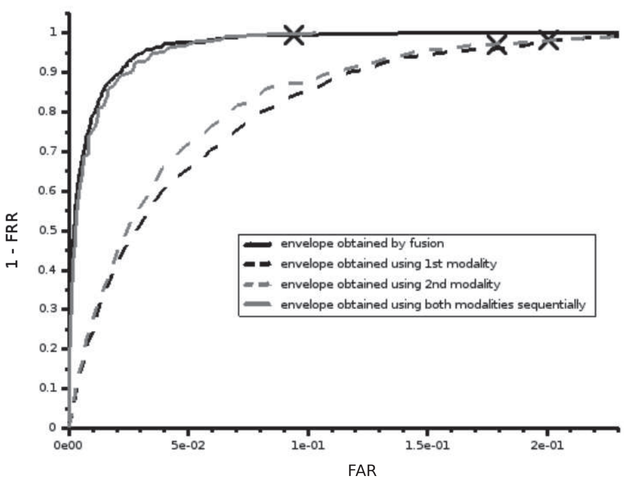


Fig. 7. The bi-modal system based on optimized thresholds versus the bi-modal systems based on fusion – a graphic comparison (ROC curves)

After optimization, we obtained

$$x_1 = 125.005, \quad x_2 = 779.137, \quad x_3 = 763.576,$$
$$\text{FAR} = 0.094, \quad \text{FRR} = 0.004, \quad \text{GAR} = 0.996.$$

A graphic representation of the locally linearized distributions and optimized thresholds for the bi-modal system can be found in Figures 5 and 6.

A graphic representation of the genuine acceptance and false acceptance rates of the bi-modal system with optimized thresholds on one side, and the bi-modal systems based on fusion [14], and 1282 distinct values fof the thresholds – obtained by exhaustive search – on the other side, are shown in Figure 7.

Figure 7 also includes the graphic representation of the same rates for the unimodal systems based separately on the same two modalities. In the representation, we highlighted the thresholds obtained by optimization.

TABLE I
THE FALSE ACCEPTANCE, GENUINE ACCEPTANCE, AND TOTAL ERROR RATES FOR CERTAIN INSTANCES WITH GIVEN CHARACTERISTICS

| Instances characteristics | FAR | GAR | TER |
|---|---|---|---|
| $n = 100, \quad m = 5, \quad l = 3$ | 9.4% | 99.6% | 9.8% |
| $n = 100, \quad m = 5, \quad l = 5$ | 2.2% | 100% | 2.2% |
| $n = 100, \quad m = 5, \quad l = 10, 8$ | 0.05% | 99.2% | 0.85% |
| $n = 200, \quad m = 5, \quad l = 10$ | 0.001% | 99.5% | 0.501% |

In the end we include summarized results for more instances (see Table I). We report the characteristics of the instances we generated and analyzed, the false acceptance, genuine acceptance, and total error rates.

For all our experiments, the pair (FAR, GAR) obtained by our method dominates at least one pair (FAR, GAR) in the set of the final results obtained by the fusion based method, thus the experiments showed a good performance of the sequential bi-modal biometric system based on optimized thresholds comparing with the widely adopted multimodal biometric systems based on parallel fusion.

### *B. Experiments using the NIST-BSSR matching score sets*

The NIST BSSR1 multimodal database contains scores from 517 users. For each user, the database contains one score set from the comparison of two right index fingerprints, one score set from the comparison of two left index fingerprints, and two score sets (from two separate matchers) from the comparison of two frontal faces. The score sets from the face matchings are referred as "C" and "G". The score sets from the left (right) indexes are referred as "Li" ("Ri"). Each matching set contains 517 genuine scores and $266,772$ (i.e. $516 \times 517$) impostor scores. We transformed the given scores into distances, i.e. a great (small) score representing a similarity (non-similarity) between two collected samples is transformed to a small (great) distance between the same two samples.

As a part of our experiments we derived the optimized thresholds for the bi-modal systems developed from the BSSR1 database; and considered all 12 possible 2-matcher combinations. See Table II, were the FAR, GAR, and TER obtained with optimized thresholds were reported.

The NIST BSSR2 multimodal database contains scores from 6000 users. For each user, the detabase contains one score set from the comparison of two right index fingerprints, and one score set from the comparison of two left index fingerprints. The score set from the left (right) indexes are referred as "Li" ("Ri"). Each matching set contains 6000 genuine scores and 35,994,000 (i.e $5999 \times 6000$ ) impostor scores. As for the BSSR1 dataset, we transformed the similarity scores into distances; derived the optimized thresholds; and considered both possible combinations. See Table III, were the FAR, GAR, and TER obtained with optimized thresholds were reported.

Many papers referred to the same matching score dataset (see for instance [6], [9], [15], and [16]). Searching for papers that report experimental results to compare with, we faced two

TABLE II

THE FALSE ACCEPTANCE, GENUINE ACCEPTANCE, AND TOTAL ERROR RATES FOR THE BI-MODAL SYSTEMS USING NIST-BSSR1 DATABASE

| $1^{st}$match | $2^{nd}$ match | FAR | GAR | TER |
|---|---|---|---|---|
| C | G | 1.89% | 91.88% | 10% |
| G | C | 0.21% | 85.48% | 14.7% |
| Li | Ri | 0.15% | 94.59% | 5.57% |
| Ri | Li | 0.11% | 93.82% | 6.3% |
| C | Li | 0.56% | 96.13% | 4.43% |
| Li | C | 0.20% | 97.3% | 2.90% |
| C | Ri | 0.5% | 97.68% | 2.82% |
| Ri | C | 0.14% | 97.88% | 2.28% |
| G | Li | 0.42% | 98.07% | 2.4% |
| Li | G | 0.5% | 98.26% | 2.24% |
| G | Ri | 0.27% | 96.65% | 1.62% |
| Ri | G | 0.26% | 98.65% | 1.61% |

TABLE III

THE FALSE ACCEPTANCE, GENUINE ACCEPTANCE, AND TOTAL ERROR RATES FOR THE BI-MODAL SYSTEMS USING NIST-BSSR2 DATABASE

| $1^{st}$match | $2^{nd}$ match | FAR | GAR | TER |
|---|---|---|---|---|
| Li | Ri | 0.96% | 95.87% | 5.09% |
| Ri | Li | 1.55% | 96.12% | 5.4% |

main issues. First issue is related to the fact that there is no consistent way to deal with this database. For example, some authors randomly selected the scores for the system training, and used the rest for evaluation, thus making impossible to repeat their experiments; and/or discarded some scores due to apparent template acquisition errors, but without explaining which scores were discarded [15]. The second one is related to the fact that we propose a set of thresholds to be used in the multimodal system; but we do not generate a ROC curve, thus the Equal Error Rate (EER) cannot be employed straitforward to validate our approach.

## V. Conclusions and Future Works

In this paper we proposed a novel approach to determine the upper and lower acceptance thresholds in sequential multimodal biometric matching systems. The obtained values of the thresholds offer a good compromise between the false acceptance rate and false rejected rate. We linearized locally the score distributions of both genuine users and impostors using the least squares method, and derived formulas for the approximated FAR and FRR for each matcher. Further, we minimized the sum of the probabilities for entire processing chain. We formalized the optimization model for a bi-modal biometric system. The extension to a general multimodal system will be part of our future work. The results of our experiments were reported in the paper.

The optimization of the thresholds for a serial multimodal system serves the a priori need of the decision maker when building a convenient multimodal biometric system. Our method that provides optimized thresholds for a multimodal biometric system is fast, and relatively simple to implement. For a biometric system that works in real time, the existence of multiple matchings, and the possibility that the unproblematic genuine users pass the system after the first match with a low false rejection rate, offers the advantage of an increased speed of the matching process. Moreover, if the first match is based on a face image, or a video record that may be taken even without user's will, then the process is even faster.

For the majority of our experiments on generated instances, the pair (FAR, GAR), obtained by our method, dominates at least one pair (FAR, GAR) from the set of the final results obtained by the fusion based method. Thus, the experiments showed a good performance of the sequential bi-modal biometric matching system based on optimized thresholds comparing with the widely adopted multimodal biometric systems based on parallel fusion.

Our experimental results on NIST-BSSR1 and BSSR2 datasets were also reported in the paper. We intend to extend our experiments to more benchmark data from the literature; and to multimodal biometric systems with more than two matchings.

In our future research we will also focus on refining our methodology by using an improved approximation in deriving the expressions for the FAR and FRR needed in the optimization step.

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