A Model for Storage Facility Design with Energy Costs

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Abstract— **The design of a warehouse or distribution center (DC) has become an essential tool for the optimization of the supply chain in most industries. Energy consumption in this type of facilities is an important issue which has not received much attention in the scientific community which has focused instead in two-dimensional routing optimization. In this paper the problem of designing a rectangular warehouse in a fully or partially automated factory or distribution center is addressed. In these facilities the movement is performed either by an AGV (Automated Guided Vehicle) or by automated robotics equipment of Cartesian movement attached to the storage racks. In the literature there exist works with formulas to design shelves in two and three dimensions buy they do not adequately consider the problem of movement in the Z-axis, thereby giving inefficient results with very high shelves and ignoring energy consumption and costs. This occurs because the known approaches consider the movement only on the X-Y plane as expensive as in height movement, which in general does not hold in actual facilities. The problem is exacerbated for heavy materials due to greater energy waste. In this paper the above problem is solved by a model with an extra component for the cost of movement in height. With the model and derived formulas, lower shelves with less energy consumption are generated. The formulas developed are optimal with respect to travel distances and they are obtained from solving a nonlinear optimization problem with linear constraints through a Lagrange transformation. The paper contribution is to optimize warehouse design for reduced energy consumption and pollution both relevant aspects in sustainable engineering systems. A practical application in a distributor of MRO items is presented along with measured energy cost impacts.**

Index Terms-Energy optimization, integrated facility design, green supply chains.

I. INTRODUCTION

The design of a warehouse or a distribution center (DC) has become a fundamental decision for the optimization of a supply chain. Even though there exist several articles in the academic literature in which efficient DC layouts have been studied, it has not been formulated a simple mathematical model in order to obtain the basic variables for the DC construction in three dimensions (3D). The formulas found on the literature mainly address the problem in two dimensions; for example [1] considered a rectangular warehouse and racks parallel or perpendicular to the wall. They discussed also the optimal location of the warehouse door, and the optimal design when the storage area is divided into different zones. All of their work is developed on the horizontal plane; they fix a priori a given number of vertical racks and that value is used as a parameter.

Some formulas for three dimensions that have been developed, consider implicit equations whose resolution must be made with an iterative method, which makes it difficult to use, as for example in [2]. In this paper it is considered that supplies are received from several suppliers and dispatched to many customers; consequently the study is complemented with the design of a multi-level warehouse (according to product flow) by using an ABC methodology. Baker and Canessa [3] state that a structured approach for the design of a storage system in the company is not currently available and make a compilation of different approaches to finalize with the design of their own methodology. Another classic work is Gu et al. in [4] in which a detailed revision of the warehouse design research is presented by considering practical case studies and computational support tools. Moreover, they present a framework for a systematic classification. Gu et al., [4] also conclude that even though there are a large number of papers focusing on details of the different systems, there are few addressing the decision making concerning storage systems.

In this paper, easy-to-use explicit mathematical formulas for designing a DC are developed; these assume that a rectangular space is available, with a unique main entrance door located in front at the center of the longest wall and an exit door similarly located and the back wall. The formulas yield the number of double racks to be utilized, the number of pallets in each rack and the height also measured in pallets, so that the total cost of goods movement is optimized at the DC.

II. ASSUMPTIONS

Some assumptions to be used in developing the layout of the Distribution Center are:

• Goods are stored in double racks, except for the rack adjacent the wall which has only one side.

• Goods enter through a door located in one of walls of the store and leave through an opposite to the front door.

• The shape of the store is rectangular (see Figure 1).

• There should be wide aisles between the racks, and along the walls the width of these aisles should be the same.

The first study [4] conducted on the design of a Distribution Center provides two possible distributions of the shelves. In this paper, only the distribution shown in Figure 1 is analyzed, where the shelves are located in the sense of the narrowest direction (*v*).

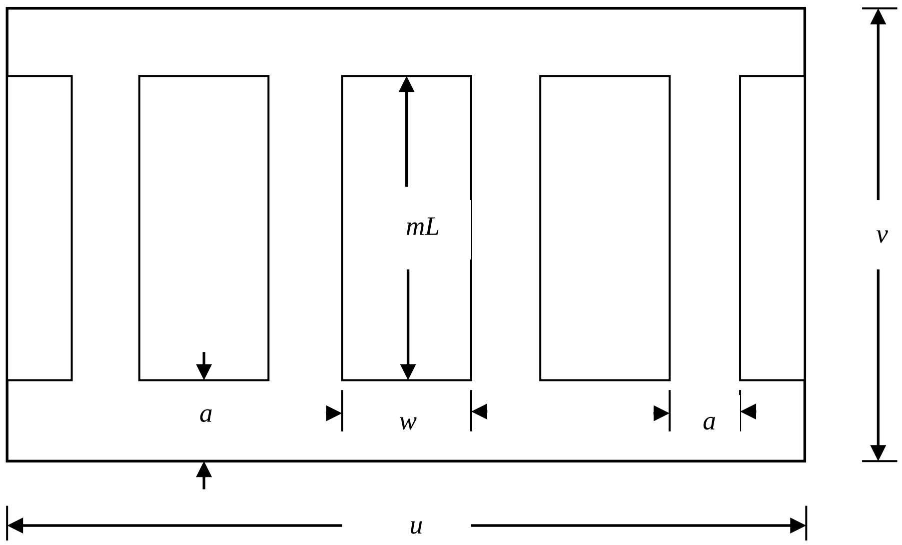

Figure 1: Layout of the Rectangular Warehouse

The notation used in this study is as follows:

• w = width of double shelf.
• L = length of a storage unit in the shelf.
• m = total number of storage spaces along a shelf.
• h = number of storage spaces vertically.
• n = number of double shelves.
• K = total storage capacity in storage spaces.
• a = width of an aisle, where it is assumed that all the aisles have the same width.
• u = length of the store.
• v = width of the store.
• d = annual movement (demand) of the warehouse, in storage units. It is assumed that a unit of stored article occupies a unit of space (in items / year).
• C_h = material handling cost of moving a unit of article per unit of length (in $/m).
• T_v = average travel distance on the vertical axis.
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• T_u = average travel distance on the horizontal axis.
• T_h = average travel distance in height

Cost function

This paper considers only the cost of moving and handling of goods inside the warehouse. For this we use Figure 1 as a reference and hence the measures considered. First the length and width of the distribution center will be determined, which are determined by:

$$Width \Rightarrow u = n(w+a) \qquad (1)$$

$$Lenght \Rightarrow v = 2a + mL \qquad (2)$$

Then the area of the distribution center, its dimensions are as follows:

$$Area \quad \Rightarrow uv = n(w+a)(2a+mL) \quad (3)$$

In order to calculate the cost of moving goods, the average travel distance of each stored unit is required. It will be assumed that the doors are located at the centers of the walls and all goods are equally likely to be used. To determine the cost function of the DC, we must first set the probabilities of carrying a good in the horizontal plane and in height. In the horizontal plane, two axes of movement, one of lateral movement (direction u) and move towards the bottom of the warehouse (*v* direction) are defined; these letters are defined in relation to Figure 1. Thus, we will have T_v, T_u and T_h representing the distances in each direction. As the probability of taking a good to the left or right from the door of the CD store is the same, the average travel distance on the horizontal axis is (4):

$$T_u = \frac{u}{4} \quad (4)$$

For the distance in the direction *v*, the average travel distance depends on whether a good has category A, B or C. Let P_a , P_b and P_c be the likelihood that the good to be moved has category A, B and C, respectively. In addition, m_a , m_b and m_c correspond to the number of spaces in the direction *v* reserved for products of categories A,B,C, respectively. With this formulation the average travel distance in the direction *v* is given by:

$$T_v = a + P_a\left(\frac{m_aL}{2}\right) + P_b\left(m_aL + \frac{m_bL}{2}\right) + P_c\left(m_aL + m_bL + \frac{m_bL}{2}\right) \quad (5)$$

By property of probabilities, the following holds:

$$P_a + P_b + P_c = 1 \quad (6)$$

By simplifying equation (5), we have:

$$T_v = a + L\left[(m_a+m_b)\left(1-\frac{P_a}{2}\right) + (m_c+m_b)\left(\frac{P_c}{2}\right) - \frac{m_b}{2}\right] \quad (7)$$

The total capacity having category A, B and C within the DC to store goods can be expressed as follows:

$$N_a - 2m_anh = 0 \quad (8)$$

$$N_b - 2m_bnh = 0 \quad (9)$$

$$N_c - 2m_cnh = 0 \quad (10)$$

Combining equations (8), (9) and (10), the average travel distance in the vertical

$$T_v = a + \frac{m}{K}\,L\,\left[(N_a+N_b)\left(1-\frac{P_a}{2}\right) + (N_c+N_b)\left(\frac{P_c}{2}\right) - \frac{N_b}{2}\right] \quad (11)$$

Finally, the average travel distance in the height axis, is given by the following equation:

$$T_h = A\,\frac{h}{2} \quad (12)$$

Where A is called the *height loading factor*, which is a factor expressing how many times more complex is to move load in the vertical axis than in the horizontal plane. So the cost equation for a DC with ABC method is:

$$C = 4dC_h\left[a + \frac{m}{K}L\left[(N_a+N_b)\left(1-\frac{P_a}{2}\right) + (N_c+N_b)\left(\frac{P_c}{2}\right) - \frac{N_b}{2}\right] + \frac{u}{4} + A\,\frac{h}{2}\right] \quad (13)$$

The term :

$$(N_a+N_b)\left(1-\frac{P_a}{2}\right) + (N_c+N_b)\left(\frac{P_c}{2}\right) - \frac{N_b}{2}$$

depends only on the dimensions associated with the categories A, B and C. To simplify the expression of the cost formula, let us make the following definition:

$$Cte = (N_a+N_b)\left(1-\frac{P_a}{2}\right) + (N_c+N_b)\left(\frac{P_c}{2}\right) - \frac{N_b}{2} \quad (14)$$

The system is

$$Min\ C = 4dC_h\left[a + \frac{m}{K}L\,Cte + \frac{n(w+a)}{4} + A\,\frac{h}{2}\right] \quad (15)$$

Subject to $N = 2mnh$

By transforming the system into a Lagrangian function, we have:

$$\frac{\partial C}{\partial m} = 4dC_h * \frac{L\,cte}{K} - 2\,\lambda\,n\,h = 0 \quad (16)$$

$$\frac{\partial C}{\partial n} = 4dC_h * \frac{(w+a)}{K} - 2\,\lambda\,m\,h = 0 \quad (17)$$

$$\frac{\partial C}{\partial h} = 4dC_h * A - 2\,\lambda\,m\,n = 0 \quad (18)$$

$$\frac{\partial C}{\partial \lambda} = K - 2\ m\,n\,h = 0 \quad (19)$$

By arranging, it holds that:

$$m = K\sqrt[3]{\frac{(w+a)A}{8L^2cte^2}} \quad (20)$$

$$h = \sqrt[3]{\frac{Lcte(w+a)}{8A^2}} \quad (21)$$

$$n = \sqrt[3]{\frac{8ALcte}{(w+a)^2}} \quad (22)$$

III. EXPERIMENTING WITH A TEST PROBLEM

In order to validate the developed formulas, several warehouse layout calculations were performed using data from the reference paper [2] in which an application is made to a warehouse of toilet and cleaning products holding categories ABC. Note that this is the only reference found in the literature in which formulas to the case in 3 dimensions are presented. In the cited paper, the developed formulas are implicit and the problem was solved with the Particles Swarm Optimization (PSO) method. The warehouse used was scheduled to handle six families of products, which include personal cleansing products, house and kitchen cleaning items, maintenance products, chemical raw materials, electronics and ceramic objects, including 10, 14, 62, 11 and 7 types of items, respectively.

Before applying the formulas developed in this paper, a procedure by which it is determined what kind of product belongs to each group according to the ABC method, is applied. So, class A products are located closer to the door, while class B at a mid-range distance and class C to greater distance. Once this process is complete, 14 items in class A, 24 items in class B and 12 items in class C, were found. The probabilities of belonging to each class were taken as 0.6; 0.3; 0.1, respectively.

The total flow of the warehouse is 120,000 pallets of products per year and the capacity of the warehouse is 6,000 pallets. The capacity of the warehouse is divided into size classes 3,000 (Na), 2000 (Nb) and 1000 (Nc). With respect to the dimensions of the warehouse, there are available racks with loading by both sides of a total width (*w*) of 2.2 mt., a length of 0.9 m wide and 1 m. each pallet high. The width of the aisles is 2 mt. and the width of doors 4 mt. The cost factors considered are the cost of materials handling and is obtained by adding the cost of workers using cranes and fuel and depreciation costs and the energy cost generated by the movement in 3D, whereby a cost of US $ 1.13 * 10-3 is obtained per meter transported.

With this information, the summary data are:

- w = 2.2 meters (width of double shelf)
- L = 0.9 meters (length of storage unit)
- K = 6000 (total storage capacity)
- a = 2.0 meters (width of a corridor).
- d = 120,000 (annual demand in pallets)
- C_h = 0.00113 (material handling cost, $ / mt.)
- A = 4 (load factor in height)
- N_a = 3000 (number of pallets in category A)
- N_b = 2000 (number of pallets in category B)
- N_c = 1000 (number of pallets in category C)
- P_a = 0.6 (probability that a product is Category A)
- P_b = 0.3 (probability that a product is Category B)
- P_c = 0.1 (probability that a product is Category C)

A good approximation for the load factor is to assume that $A \cong e^{\sqrt{h/2}}$; with the values obtained for $h = 5$ it can be adjusted the height load factor to $A \cong 4.86$, with this value it can be recalculated m, n and h.

The rounded values are:

m: 36 (number of pallets in the direction v)
n: 20 (number of shelves in the direction u)
h: 5 (number of pallets in height)

With the above, the value of the total cost of moving goods per year is C = US $ 30,489, which is a cost of about 4% less than the value presented in the paper by [4]. Increasing the value of the height load factor produces greater savings. Table 1 shows these values and the savings with respect to the value obtained by the heuristic solution of [2] .

Table 1: Summary of savings for different h values.

A	h	n	m	Costo US$	Ahorro %
1	13	12	21	18108	-2,00%
2	8	15	27	22534	-3,00%
3	6	17	31	25484	1,10%
4	5	19	34	28118	5,70%
5	5	20	36	30678	9,50%
6	4	22	39	32497	14,40%
7	4	23	41	34786	17,20%
8	4	24	42	36758	20,20%
9	3	25	44	36606	27,00%

Note that, as the factor A increases also the cost increases. This is due to the movement of goods in height, which is more expensive. By considering a value of $A = 5$ which is close to using the exponential expression in relation to the average height variable, a savings of 14.4% is obtained. This benefit, in addition to having direct formulas, it reflects a contribution in the knowledge in the discipline.

IV. CONCLUSIONS

In this paper a new approach for the design of distribution centers was presented; in particular for a rectangular layout with movement at height. Explicit formulas for calculating the number of pallets, the number of double shelves, the number of unit spaces for each shelf and the number of unit spaces in height, were developed. The obtained formulas are simple to use; they were obtained by posing a Lagrangian function obtained from a quadratic minimization problem with equality constraints. In the literature there are not explicit formulas in three dimensions by which this paper is a valuable contribution to the design of distribution centers in three dimensions.

V. REFERENCES

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