Opinion Evolution of a Social Group with Extreme Opinion Leaders

Yiyi Zhao, Gang Kou
School of Business Administration
Southwest University of Finance and Economics of China
Chengdu 611130, China
zyisarah@163.com

***Abstract*—Opinion dynamics is a collective decision making and focuses on the study of evolution and formation of opinions in a social group. Bounded confidence rule is one of intrinsical interaction principles in the human behavior dynamics. When there is no opinion leader in a social group, for a bounded confidence based opinion dynamics, the evolution of the collective opinion of the group generally depends on the initial opinions, the confidence levels, and the group size. In this paper, a new opinion dynamics model is built, on the basis of the bounded confidence rule, to consider the opinion formation in a community with extreme opinion leaders. The leaders are divided into two classes: positive leaders and negative leaders. All the agents are assumed to have heterogeneous confidence levels. Then the impacts of the opinion leaders on the final opinions of the social agents are analyzed for different trust degree and bounded confidence levels. Finally, some computer simulation results are presented to demonstrate the formation of the collective opinion.**

***Index Terms*—Opinion dynamics; bounded confidence; decision-making; opinion leaders; collective opinion pattern**

I. Introduction

In a social group, almost all the interactions of individuals are determined by beliefs and opinions. The opinions of social agents interact each other and form a complex social network. The opinion dynamics focuses on the study of the collective opinions' change as a function of a network's structure. Recent years have witnessed that opinion dynamics is becoming an important research issue in social systems and management science [1], [2], [3].

Many models so far have been developed to investigate the mechanisms in the collective opinion formation process. In sociology and economics, various "threshold" or "critical mass" models have been developed to explore the underlying mechanisms of individuals' attitudes and behaviors in collective decision-making, for example, in references [4], [5], which shows that individuals tend to obey a majority rule in the group. A majority rule model was built for opinion dynamics in [6]. From the sociological and social psychological perspectives, some social influence models, such as Frenchs social power model and Latanes social impact model, were proposed to treat the underlying mechanisms of opinion spreading [7], [8]. In the decision-making science, DeGroot model was proposed to investigate the consensus mechanism in opinion dynamics [9]. Recently, a bounded confidence situation was noticed in opinion dynamics by some references [10], [11], where agents communicate (namely, have discussions) each other only if they have sufficiently close opinions. The Deffuant-Weisbuch (DW) model and the Hegselmann-Krause (HK) model are two representative bounded confidence models.

Recently, some extension works have been developed for opinion dynamics. For example, a two-step flow model was proposed in [12] by introducing opinion leaders, which were called influentials in [13], [14], [15] and had occupied a central place in the literatures of the marketing, diffusion of innovations, and communication research. Opinion leader is referred to an individual having prominent influence on the decision-making of other individuals in the opinion communication process [16], [17], [18]. In the original HK model, all agents are homogeneous and have an identical confidence level. However, in a real society, different agents should have different confidence levels due to diverse individual characteristics. Lorenz et al. proposed the agent-based and density-based bounded confidence model with heterogeneous confidence levels in [1]. Very recently, a heterogeneous HK opinion dynamics was proposed to analyze the impacts of confidence levels on the the collective opinion evolution for social agents with multi-level confidence levels in [2], [19].

This paper will investigate opinion dynamics in the framework of HK model. The aim is to analyze the evolution of the collective opinion for a social group with a variant of HK model, which includes positive opinion leaders and negative opinion leaders. The motivation for the consideration is twofold. First, it is closer to real social phenomena. Second, it can be easily implemented as soon as a leadership is available. A strong evidence can be demonstrated in Fukushima disaster. In March 2011, a panic buying of iodized salt was triggered by some rumors in many countries and regions in the Fukushima nuclear accident. The natural emergency easily gives illegal sellers the opportunity to spread rumors and undermine the stability of market. At the same time, some governmental organizations release the related news timely and propagate nuclear radiation science knowledge. Herein, the illegal sellers plays the role of negative opinion leaders, while the governmental organizations are responsible for positive leaders. The contribution of the paper is the quantitative investigation of opinion formation systems with extreme opinion leaders. A mathematical model is built, based on a bounded confidence rule, for the collective opinion evolution under different scenarios. Then, the impacts of the leaders and the confidence levels on the opinion evolution is analyzed through computer simulations.

II. Opinion dynamics with extreme leaders

In the field of opinion dynamics, an opinion leader is referred to an individual who has a significant influence on the ordinary agents (i.e., opinion followers) in the opinion communication. In the context of bounded confidence opinion dynamics, a pair of agents communicate only if their opinion difference is less than a given threshold, which is called the confidence level. Opinion leaders generally have much smaller confidence levels than those of the opinion followers.

In this paper, we assume that a whole social network consists of three subnetworks: opinion follower subnetwork, positive opinion leader subnetwork and negative opinion leader subnetwork. It is assumed that opinion leaders have very low confidence levels and only communicate with the peers and the opinion truth. Under such a group differentiation, the opinion leaders exchange their opinions with their peers by neglecting the interference of opinion followers. On the contrary, opinion followers communicate the opinions with all agents, whose opinions are close with them, in the opinion update.

Consider a social network with N agents, where there are N_1 opinion followers, N_2 positive opinion leaders, and N_3 negative opinion leaders. Obviously, $N = N_1 + N_2 + N_3$. At any discrete time t, the opinion of agent i is denoted by $x_i(t)$ for $i = 1, \cdots, N$, which is a variable in the opinion interval $[-1, 1]$. When $x_i(t) = 1$, agent i has a completely positive opinion to support the opinion object. On the contrary, when $x_i(t) = -1$, agent i holds a negative opinion to oppose the opinion object. The collective opinion vector is denoted by $x(t) = \mathrm{col}(x_1(t), \cdots, x_N(t)) \in \mathbb{R}^N$ and thus the initial state is denoted by $x(0)$ at the initial time $t = 0$.

The opinion update scheme of opinion follower i is described as follows:

$$\begin{aligned} x_i(t+1) &= (1-\alpha_i-\beta_i)\tfrac{1}{N_i^S(t)}\sum_{j=1}^{N_1} a_{ij}(t)x_j(t) \\ &+\alpha_i \tfrac{1}{N_i^P(t)}\sum_{j=N_1+1}^{N_1+N_2} a_{ij}(t)x_j(t) \\ &+\beta_i \tfrac{1}{N_i^N(t)}\sum_{j=N_1+N_2+1}^{N} a_{ij}(t)x_j(t), \end{aligned} \tag{1}$$

where $i = 1, \cdots, N_1$, the weights $a_{ij}(t) = 1$ if agent j is the opinion neighbor of agent i at time t, i.e., $|x_i(t)-x_j(t)| \le \varepsilon_i$; and $a_{ij}(t) = 0$, otherwise. The parameter ε_i denotes the confidence level of agent i. The three sums $N_i^S(t) = \sum_{j=1}^{N_1} a_{ij}(t)$, $N_i^P(t) = \sum_{j=N_1+1}^{N_2} a_{ij}(t)$ and $N_i^N(t) = \sum_{j=N_2+1}^{N_3} a_{ij}(t)$ denote, respectively, the number of opinion neighbors in the follower subgroup, the positive leader subgroup and the negative leader subgroup. The parameters α_i and β_i denote the trust degrees of agent i on the positive leaders and the negative leaders. Generally, if the opinion follower has a more trust of the opinion leaders, he will have a lower trust of the peers. Therefore, it is proper to weight the three average opinions with the coefficients $1 - \alpha_i - \beta_i$, α_i and β_i, respectively.

The opinions of the leaders are effected by two important factors: the peers and the opinion truth. To this end, the opinion update law of the positive opinion leader i is given by

$$\begin{aligned} x_i(t+1) &= (1-w_i)\tfrac{1}{N_i^{PL}(t)}\sum_{j=N_1+1}^{N_2} a_{ij}(t)x_j(t) \\ &+w_i d,\ i = N_1+1, \cdots, N_1+N_2, \end{aligned} \tag{2}$$

where $a_{ij}(t) = 1$, if $|x_i(t) - x_j(t)| \le \varepsilon_i$; and $a_{ij}(t) = 0$, otherwise. ε_i is the confidence level. The sum $N_i^{PL}(t) = \sum_{j=N_1+1}^{N_1+N_2} a_{ij}(t)$ denotes the number of neighbors in the positive leader subgroup. The constant $d = 1$ denotes the positive target opinion and w_i denotes the influence weight of the target opinion on the positive opinion leaders. Similarly, the opinion of the negative leader i evolves according to the following rule:

$$\begin{aligned} x_i(t+1) &= (1-z_i)\tfrac{1}{N_i^{NL}(t)}\sum_{j=N_1+N_2+1}^{N} a_{ij}(t)x_j(t) \\ &+z_i g,\ i = N_1+N_2+1, \cdots, N, \end{aligned} \tag{3}$$

where $a_{ij}(t) = 1$, if $|x_i(t) - x_j(t)| \le \varepsilon_i$; and $a_{ij}(t) = 0$, otherwise. ε_i is the confidence level. The sum $N_i^{NL}(t) = \sum_{j=N_1+N_2+1}^{N} a_{ij}(t)$ denotes the number of neighbors in the negative leader subgroup. The constant $g = -1$ denotes the negative target opinion and z_i denotes the influence weight of the negative target opinion on the opinion leaders.

III. Computer simulations

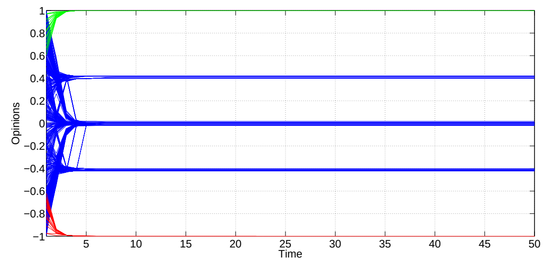
In this section, computer simulation studies are given to the proposed opinion formation model (1), (2) and (3). We focus mainly on the investigation of the impacts of opinion leaders on the collective opinion evolution in the different cases with respect to trust degrees and confidence levels.

The size of the considered social network is selected as $N = 500$ with $N_1 = 460$, $N_2 = 20$, and $N_3 = 20$. In the subsequent experiments, the initial opinion of each agent obeys a uniform distribution in the opinion interval $[-1, 1]$. In the following experiments, we investigate the impact of the trust degree on the collective opinion evolution of the opinion followers. Assume the trust degrees satisfy $\alpha_i + \beta_i = 0.8$ and $w_i = 0.8, z_i = 0.8$.

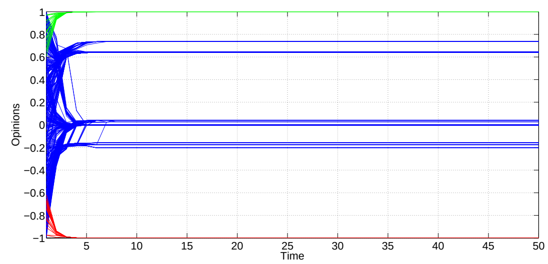
A. Case I: Trust Degree

We assume that the confidence levels of all agents are heterogenous and fixed. The confidence levels of opinion followers are uniformly distributed in the unit-length interval $[0, 1]$. Generally, the opinion leaders have smaller confidence levels and thus their confidence levels are uniformly distributed in the interval $[0, 0.25]$.

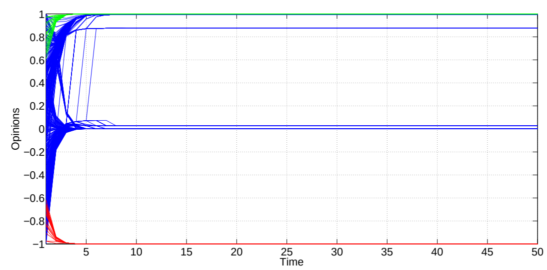
Since there are two kinds of opinion leaders in the collective opinion evolution, we may just as well focus on the influence of the positive leaders on the opinion followers. Thus, it is proper to observe how the opinions of opinion followers change as they have higher trust degrees on the positive leaders.



(a) $\alpha_i = 0.4, \beta_i = 0.4$



(b) $\alpha_i = 0.6, \beta_i = 0.2$



(c) $\alpha_i = 0.8, \beta_i = 0$

Fig. 1. Opinion evolution of social agents with different trust degrees

In Figure 1, the green, blue and red lines, respectively, denote the opinion evolutions of the positive leaders, opinion followers, and the negative leaders. From Figure 1, the positive leaders and the negative leaders are approaching to the target opinions 1 and -1, respectively. More interestingly, when the trust degrees α_i increase from 0.4 to 0.8, more and more opinion followers close to the positive leaders. As shown in Figure 1(c), when the trust degree α_i increases to 0.8, the opinion follower do not have trust on the negative leaders, then there are no opinions in the proximity of the negative target opinion -1 for all the followers.

Figure 2 shows the size distributions with different trust degrees. The x-axis denotes the order of the opinion sections. The right endpoint 20 corresponds to the 20'th opinion, i.e., the subinterval $[0.9, 1]$. In these five cases, the curves show that, as the followers have larger trust degrees on the positive leaders, the number, close to the positive leaders, increases obviously.

The simulation results show that, the leaders can have influence on the opinion update of the followers. The trust degrees of the followers on the leaders are larger, the influence power becomes more evidently. Therefore, it is important to improve the public credibility of a governmental organization in the field of public management.

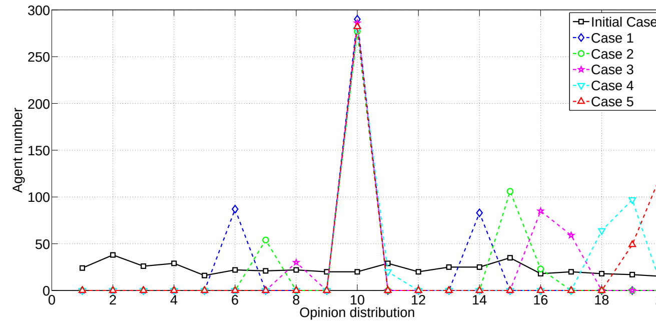


Fig. 2. The relationship between the agent number and the opinion distribution with different trust degrees. Case 1: $\alpha_i = 0.4, \beta_i = 0.4$; Case 2: $\alpha_i = 0.5, \beta_i = 0.3$; Case 3: $\alpha_i = 0.6, \beta_i = 0.2$; Case 4: $\alpha_i = 0.7, \beta_i = 0.1$; Case 5: $\alpha_i = 0.8, \beta_i = 0$.

B. *Case II: Bounded Confidence Levels*

In this subsection, we select $\alpha_i = 0.6$ and $\beta_i = 0.2$. Different from the Subsection A, the confidence levels of the followers are now changing. However, the confidence levels of the leaders are still assumed be within the interval $[0, 0.25]$. We increase gradually the confidence levels of the followers and study how the influence of the leaders varies.

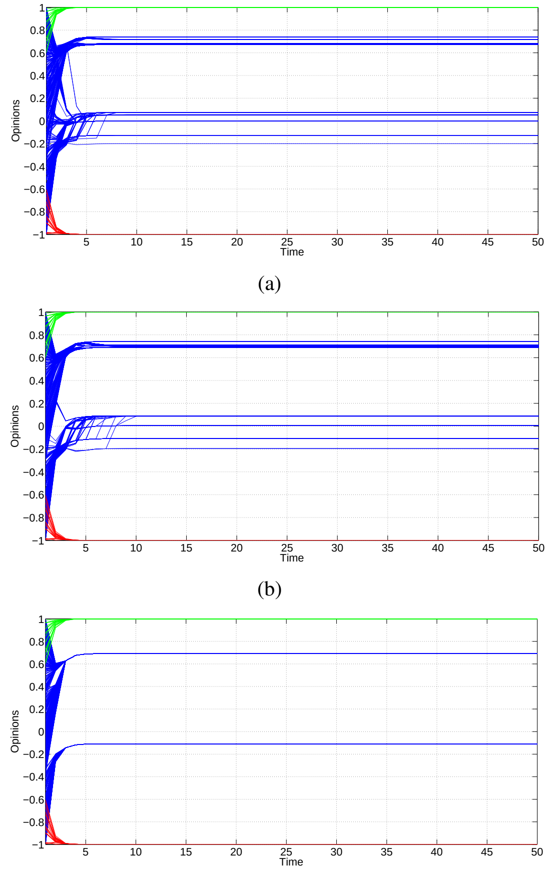
In Figure 3, we consider three cases for the confidence levels: (a) ε_i is distributed uniformly in $[0.3, 1]$; (b) ε_i is distributed uniformly in $[0.6, 1]$; (b) ε_i is distributed uniformly in $[0.9, 1]$. Combined with the result in Figure 1(b), we can find that, as the confidence levels of the confidences of the followers increase, the influence of the leaders is also increasing. Simultaneously, the followers reach consensus more easily, even though there is an opinion fragmentation in the final stage.

Figure 4 shows that, when the followers have larger confidence levels, the followers are split gradually into two parties. The one with larger size is approaching to the positive leaders, since the trust degrees $\alpha_i = 0.6$ of the followers. Another party moves to the negative leaders. When the confidence levels ε_i are distributed in $[0.9, 1]$, there is no more any agent insisting on the opinion around the opinion 0.

The simulation results show that, it is generally difficult to reach consensus in a social group with small confidence levels, and, at the same time, the roles of the leaders can also be constrained. Thus, some sagacious measures have to be adopted to balance the leader influence and the individual confidence levels for a harmonious social group.

IV. Conclusion

In this paper, a new opinion dynamics model has been developed by using the bounded confidence rule for a social group with extreme leaders. The whole group was divided into three fractions: opinion followers, positive opinion leaders

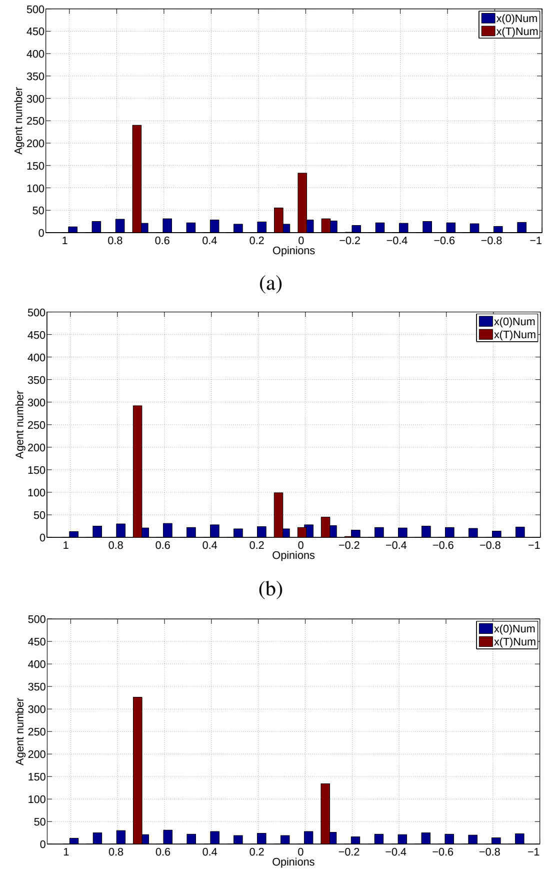


(a)

(b)

(c)

Fig. 3. Opinion evolution of social agents with different confidence levels



(a)

(b)

(c)

Fig. 4. Size distribution of social agents at initial and final stages.

and negative opinion leaders. Based on the proposed opinion dynamics model, some computer simulation results have been provided to validate the new model and investigate the impacts of the opinion leaders and the confidence levels on the opinion followers. The simulation results have showed that, as the trust degrees of the followers on the positive leaders increase, the positive leaders have more powerful influence on the followers. At the same time, when the confidence levels of the followers increase, the influence of the leaders becomes strengthened and the followers reach consensus more easily. Our analysis results have shown that the public credibility of a governmental organization plays very important in the public management and wise measures need to be designed to balance the leader influence and the individual confidence levels.

Acknowledgment

This research was partially supported by Grants 71222108, 71471149, 71433001, and 71503206 from the National Natural Science Foundation of China and Grant 2015M582564 from China Postdoctoral Science Foundation Funded Project.

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