CS:GO Bomb Damage Integral Method

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ABSTRACT: This document describes an integral based method for calculating the average bomb damage received by a player in the non-death zone from an area of a (or the entire) bombsite.

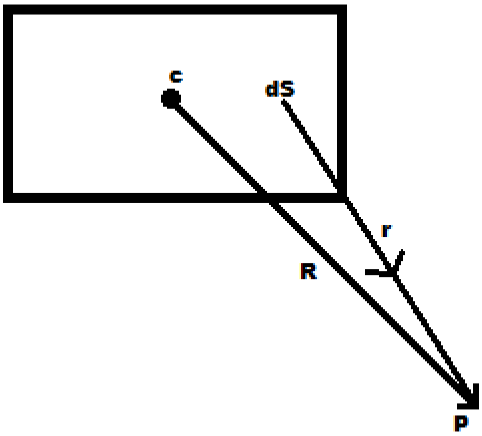

Figure 1. Model of bombsite to player point

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1 Setup

Suppose we have a bombsite $\Sigma \subset \mathbb{R}^3$, with a centre (average position) $C : (\hat{x}_c, \hat{y}_c, \hat{z}_c)$. Additionally suppose there is a player at point $P : (\hat{X}_p, \hat{Y}_p, \hat{Z}_p)$. We seek the average damage received at point P from all possible planting positions dS in the bombsite.

1.1 Change of coordinates

We first make a change of coordinates so the centre of the bombsite lies at the origin $(0,0,0)$, so let $x = \hat{x} - \hat{x}_c, y = \hat{y} - \hat{y}_c$ and $z = \hat{z} - \hat{z}_c$. Then $C : (\hat{x}_c, \hat{y}_c, \hat{z}_c) \to (0,0,0)$, and $P : (\hat{X}_p, \hat{Y}_p, \hat{Z}_p) \to (X_p, Y_p, Z_p)$. The distance [1] from C to P can then be written as

$$R = \sqrt{X_p^2 + Y_p^2 + Z_p^2}. \tag{1.1}$$

The distance from an element $dS : (x, y, z)$ of the bombsite to P can be written as

$$r = \sqrt{(x - X_p)^2 + (y - Y_p)^2 + (z - Z_p)^2}. \tag{1.2}$$

1.2 General Solution

Now, we model (we typically assume that $z = z(x, y)$ and is approximately a step function) a sample contribution of damage at P, $d\Psi_P$, as

$$d\Psi_P = ae^{-\left(\frac{r-b}{c}\right)^2} dS \tag{1.3}$$

where $a, b, c \in \mathbb{R}$ are constants that depend on whether or not the player is armoured. This is a good approximation when $r > b$ units of the bombsite, and okay when $0 \leq r \leq b$. This means we can write the average contribution [2] as

$$\Psi_P = \frac{a}{A} \iint_\Sigma e^{-\left(\frac{r-b}{c}\right)^2} dS \tag{1.4}$$

where A is the total surface area of the bombsite, given by

$$A = \iint_\Sigma dS. \tag{1.5}$$

Thus

$$\begin{aligned}
\Psi_P &= \frac{a}{A} \iint_\Sigma e^{-\left(\frac{\sqrt{(x-X_p)^2+(y-Y_p)^2+(z-Z_p)^2}-b}{c}\right)^2} dS &(1.6)\\
&= \frac{a}{A} \iint_\Sigma e^{-\frac{(x-X_p)^2+(y-Y_p)^2+(z-Z_p)^2-2b\sqrt{(x-X_p)^2+(y-Y_p)^2+(z-Z_p)^2}+b^2}{c^2}} dS &(1.7)\\
&= \frac{a}{A} e^{-\frac{b^2}{c^2}} \iint_\Sigma e^{-\frac{x^2-2xX_p+X_p^2+y^2-2yY_p+Y_p^2+z^2-2zZ_p+Z_p^2-2b\sqrt{x^2-2xX_p+X_p^2+y^2-2yY_p+Y_p^2+z^2-2zZ_p+Z_p^2}}{c^2}} dS &(1.8)\\
&= \frac{a}{A} e^{-\frac{b^2}{c^2}} \iint_\Sigma e^{-\frac{R^2+x^2+y^2+z^2-2(xX_p+yY_p+zZ_p)-2b\sqrt{R^2+x^2+y^2+z^2-2(xX_p+yY_p+zZ_p)}}{c^2}} dS &(1.9)\\
&= \frac{a}{A} e^{-\frac{R^2+b^2}{c^2}} \iint_\Sigma e^{-\frac{x^2+y^2+z^2-2(xX_p+yY_p+zZ_p)-2b\sqrt{R^2+x^2+y^2+z^2-2(xX_p+yY_p+zZ_p)}}{c^2}} dS. &(1.10)
\end{aligned}$$

2 Approximate Solution

2.1 Derivation

Using the Binomial Theorem [3] we can approximate the square root term as with

$$\sqrt{R^2+x^2+y^2+z^2-2(xX_p+yY_p+zZ_p)} = R\sqrt{1+\frac{x^2+y^2+z^2}{R^2}-\frac{2(xX_p+yY_p+zZ_p)}{R^2}} \tag{2.1}$$

$$\approx R\left[1+\frac{x^2+y^2+z^2}{2R^2}-\frac{xX_p+yY_p+zZ_p}{R^2}\right] \tag{2.2}$$

$$= R+\frac{x^2+y^2+z^2}{2R}-\frac{xX_p+yY_p+zZ_p}{R}. \tag{2.3}$$

Then, after some simplification, we have

$$\Psi_P \approx \frac{a}{A}e^{-\frac{(R-b)^2}{c^2}} \iint_\Sigma e^{-\frac{\left(\frac{b}{R}+1\right)(x^2+y^2+z^2)+2\left(\frac{b}{R}-1\right)(xX_p+yY_p+zZ_p)}{c^2}}\,dS. \tag{2.4}$$

Letting $\sigma = \frac{b}{R}+1$ and $\gamma = \frac{b}{R}-1$ this can be written as

$$\Psi_P \approx \frac{a}{A}e^{-\frac{(R-b)^2}{c^2}} \iint_\Sigma e^{-\frac{\sigma(x^2+y^2+z^2)+2\gamma(xX_p+yY_p+zZ_p)}{c^2}}\,dS. \tag{2.5}$$

Now, if we assume that $z(x,y)$ is completely flat at a height of z_0 then we have

$$\Psi_P(X_p,Y_p,Z_p) = \frac{a}{A}e^{-\frac{(R-b)^2}{c^2}} \iint_\Sigma e^{-\frac{\sigma x^2+\sigma y^2+\sigma z_0^2+2\gamma(xX_p+yY_p+z_0Z_p)}{c^2}}\,dS \tag{2.6}$$

$$= \frac{a}{A}e^{-\frac{(R-b)^2+\sigma z_0^2+2\gamma z_0 Z_p}{c^2}} \iint_\Sigma e^{-\frac{\sigma x^2+\sigma y^2+2\gamma(xX_p+yY_p)}{c^2}}\,dS. \tag{2.7}$$

Applying Fubini's theorem [5],

$$\iint_\Sigma e^{-\frac{\sigma x^2+\sigma y^2+2\gamma(xX_p+yY_p)}{c^2}}\,dS = \int_{x_0}^{x_1} e^{-\frac{\sigma x^2+2\gamma X_p x}{c^2}}\,dx \int_{y_0}^{y_1} e^{-\frac{\sigma y^2+2\gamma Y_p y}{c^2}}\,dy \tag{2.8}$$

with closed form [6] [7]

$$\int_{x_0}^{x_1} e^{-\frac{\sigma x^2+2\gamma X_p x}{c^2}}\,dx = \frac{\sqrt{\pi}c}{2\sqrt{\sigma}}e^{\frac{\gamma^2X_p^2}{\sigma c^2}}\left(\mathrm{erf}\left(\frac{\sigma x_1+\gamma X_p}{c\sqrt{\sigma}}\right)-\mathrm{erf}\left(\frac{\sigma x_0+\gamma X_p}{c\sqrt{\sigma}}\right)\right). \tag{2.9}$$

We then have

$$\Psi_P = \frac{a}{A}e^{-\frac{(R-b)^2+\sigma z_0^2+2\gamma z_0 Z_p}{c^2}}\frac{\sqrt{\pi}c}{2\sqrt{\sigma}}e^{\frac{\gamma^2X_p^2}{\sigma c^2}}\left(\mathrm{erf}\left(\frac{\sigma x_1+\gamma X_p}{c\sqrt{\sigma}}\right)-\mathrm{erf}\left(\frac{\sigma x_0+\gamma X_p}{c\sqrt{\sigma}}\right)\right) \tag{2.10}$$

$$\frac{\sqrt{\pi}c}{2\sqrt{\sigma}}e^{\frac{\gamma^2Y_p^2}{\sigma c^2}}\left(\mathrm{erf}\left(\frac{\sigma y_1+\gamma Y_p}{c\sqrt{\sigma}}\right)-\mathrm{erf}\left(\frac{\sigma y_0+\gamma Y_p}{c\sqrt{\sigma}}\right)\right) \tag{2.11}$$

$$= \frac{\pi a c^2}{4\sigma A}e^{-\frac{(R-b)^2+\sigma z_0^2+2\gamma z_0 Z_p}{c^2}+\frac{\gamma^2X_p^2+\gamma^2Y_p^2}{\sigma c^2}}\left(\mathrm{erf}\left(\frac{\sigma x_1+\gamma X_p}{c\sqrt{\sigma}}\right)-\mathrm{erf}\left(\frac{\sigma x_0+\gamma X_p}{c\sqrt{\sigma}}\right)\right) \tag{2.12}$$

$$\left(\mathrm{erf}\left(\frac{\sigma y_1+\gamma Y_p}{c\sqrt{\sigma}}\right)-\mathrm{erf}\left(\frac{\sigma y_0+\gamma Y_p}{c\sqrt{\sigma}}\right)\right). \tag{2.13}$$

3 Summary and Formal Computation

3.1 Important Definitions

Write

$$\mathcal{M}(Z_p) = \frac{\pi a c^2}{4\sigma} \exp\left(\frac{\gamma^2 R^2 - \sigma(R-b)^2 - (\sigma z_0 + \gamma Z_p)^2}{\sigma c^2}\right) \tag{3.1}$$

and

$$\Phi(x_0, x_1; X_p) = \operatorname{erf}\left(\frac{\sigma x_1 + \gamma X_p}{c\sqrt{\sigma}}\right) - \operatorname{erf}\left(\frac{\sigma x_0 + \gamma X_p}{c\sqrt{\sigma}}\right). \tag{3.2}$$

We then define the close-field fundamental solution as

$$\Theta(x_0, y_0, x_1, y_1, z_0; X_p, Y_p, Z_p) = \mathcal{M}(Z_p)\Phi(x_0, x_1; X_p)\Phi(y_0, y_1; Y_p). \tag{3.3}$$

Using the linearity of the integral and the principle of superposition, we can combine rectangular sections of bombsites to produce a field for the entire bombsite using these fundamental solutions.

3.2 Centroid (mean position)

Suppose we are able to cut up our bombsite into $n \in \mathbb{Z}^+$ flat rectangular pieces, with the i^{th} ($i = 1, 2, \ldots, n-1$) piece having bottom left coordinate (x_{i_0}, y_{i_0}) and top right coordinate (x_{i_1}, y_{i_1}) with height z_i. Then compute the area of each piece with

$$A_i = (x_{i_1} - x_{i_0})(y_{i_1} - y_{i_0}). \tag{3.4}$$

The total area is given by

$$\mathcal{A} = \sum_{i=1}^{n} A_i. \tag{3.5}$$

The coordinates of the centroid [8] are given by

$$\hat{x}_c = \frac{1}{2\mathcal{A}} \sum_{i=1}^{n} A_i \left(x_{i_1} + x_{i_0}\right), \tag{3.6}$$

$$\hat{y}_c = \frac{1}{2\mathcal{A}} \sum_{i=1}^{n} A_i \left(y_{i_1} + y_{i_0}\right), \tag{3.7}$$

$$\hat{z}_c = \frac{1}{\mathcal{A}} \sum_{i=1}^{n} A_i z_i. \tag{3.8}$$

3.3 Distance constants

The distance, R, from position $\left(\hat{X}_p, \hat{Y}_p, \hat{Z}_p\right)$ to the centroid is given by

$$R = \sqrt{\left(\hat{X}_p - \hat{x}_c\right)^2 + \left(\hat{Y}_p - \hat{y}_c\right)^2 + \left(\hat{Z}_p - \hat{z}_c\right)^2}. \tag{3.9}$$

Then

$$\gamma = \frac{b}{R} - 1, \tag{3.10}$$

$$\sigma = \gamma + 2. \tag{3.11}$$

3.4 Fundamental Solutions

The fundamental solution associated with the i^{th} piece is given by

$$\Theta_i = \mathcal{M}\left(\hat{Z}_p - \hat{z}_c\right)\Phi\left(x_{i_0}, x_{i_1}; \hat{X}_p - \hat{x}_c\right)\Phi\left(y_{i_0}, y_{i_1}; \hat{Y}_p - \hat{y}_c\right). \tag{3.12}$$

3.5 Summed solution

The solution we are seeking is then

$$\Psi_P = \frac{1}{\mathcal{A}}\sum_{i=0}^{n}\Theta_i. \tag{3.13}$$

3.6 Constants

The constants [4] for unarmoured players are

$$a_1 = 450.7 \text{ HP}, \tag{3.14}$$

$$b_1 = 75.68 \text{ units}, \tag{3.15}$$

$$c_1 = 789.2 \text{ units}. \tag{3.16}$$

For armoured players they are

$$a_2 = 200.2 \text{ HP}, \tag{3.17}$$

$$b_2 = 162.7 \text{ units}, \tag{3.18}$$

$$c_2 = 747.0 \text{ units}. \tag{3.19}$$

4 Testing

4.1 Simple Example Test - Armoured

Suppose we have a bombsite that runs over $(x_0, y_0) = (-32, -32) \to (x_1, y_1) = (32, 32)$ at a height of $z_0 = 0$

Using the texture grid points as a reference with position and angle set by "setpos 1056 0; setang 0 180 0", these results of damage were gathered.

43	43	43	43	43	43
46	46	46	45	45	45
48	48	48	48	48	48
50	50	50	50	50	50
53	53	53	53	53	53

Table 1.

This has an average of 47.6 damage. Applying our simple model, we have $x_0 = y_0 = 32$ units. This gives an area of $A = 4096$ units. We approximate the eyepos as $(X_p, Y_p, Z_p) = (1056, 0, 64)$.

Applying our model from above, we get very good agreement with the measured average

$$\Psi_P = \Theta(-32, -32, 32, 32, 0; 1056, 0, 64)/4096 = 47.626387295986966. \tag{4.1}$$

4.2 Example 2

Suppose we have a bombsite similar to the first example, except that the region inside $(-32, 0) \to (0, 32)$ is raised by $z_0 = 500$ units. The first issue we must take care of is that it is not possible to plant directly next to the wall, as the player takes up $32 \cdot 32 = 1024$ units^2 of space. This means that any walls within a bombsite remove a 16 unit thick contour around the wall. This is shown in Figure 2.

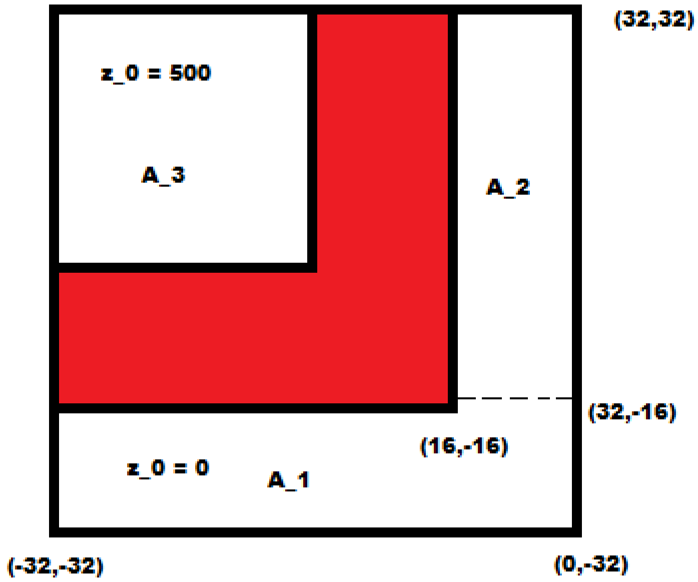

Figure 2. Simple Bombsite 2

We first find the areas.

$$
\begin{aligned}
A_1 &= (32 - -32) \cdot (-16 - -32) = 1024, &(4.2)\\
A_2 &= (32 - 16) \cdot (32 - -16) = 768, &(4.3)\\
A_3 &= (0 - -32) \cdot (32 - -0) = 1024, &(4.4)\\
\implies \mathcal{A} &= A_1 + A_2 + A_3 = 2816. &(4.5)
\end{aligned}
$$

The centre of the bombsite is given by

$$
\begin{aligned}
\hat{x}_c &= \frac{8}{11} &(4.6)\\
\hat{y}_c &= \frac{-8}{11} &(4.7)\\
\hat{z}_c &= \frac{2000}{11}. &(4.8)
\end{aligned}
$$

The predicted value is

$$\Psi_p \approx 39.01379338747492\cdots . \tag{4.9}$$

References

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[5] Aarts, Ronald M. and Weisstein, Eric W. "Fubini Theorem." From MathWorld–A Wolfram Web Resource. https://mathworld.wolfram.com/FubiniTheorem.html

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[8] Weisstein, Eric W. "Geometric Centroid." From MathWorld–A Wolfram Web Resource. https://mathworld.wolfram.com/GeometricCentroid.html