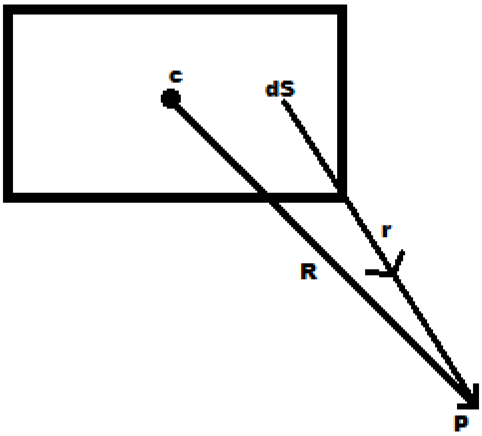

Figure 1. Model of bombsite to player point

CS:GO Bomb Damage Integral Method

True_hOREP

1 Setup

Suppose we have a bombsite $\Sigma \subset \mathbb{R}^3$, with a centre (average position, or a convenient point such as the middle of the lowest plane) $C : (\hat{x}_c, \hat{y}_c, \hat{z}_c)$. Additionally suppose there is a player at point $P : (\hat{X}_p, \hat{Y}_p, \hat{Z}_p)$. We seek the average damage received at point P from all possible planting positions dS in the bombsite.

1.1

We first make a change of coordinates so the centre of the bombsite lies at the origin $(0, 0, 0)$, so let $x = \hat{x} - \hat{x}_c, y = \hat{y} - \hat{y}_c$ and $z = \hat{z} - \hat{z}_c$. Then $C : (\hat{x}_c, \hat{y}_c, \hat{z}_c) \to (0, 0, 0)$, and $P : (\hat{X}_p, \hat{Y}_p, \hat{Z}_p) \to (X_p, Y_p, Z_p)$. The distance from C to P can then be written as

$$R = \sqrt{X_p^2 + Y_p^2 + Z_p^2}. \tag{1.1}$$

The distance from an element $dS : (x, y, z)$ of the bombsite to P can be written as

$$r = \sqrt{(x - X_p)^2 + (y - Y_p)^2 + (z - Z_p)^2}. \tag{1.2}$$

1.2

Now, we model (we assume that $z = z(x, y)$ and is approximately a step function) a sample contribution of damage at P, $d\Psi_P$, as

$$d\Psi_P = ae^{-\left(\frac{r-b}{c}\right)^2}\, dS \tag{1.3}$$

where $a, b, c \in \mathbb{R}$ are constants that depend on whether or not the player is armoured. This is a good approximation when $r > b$ units of the bombsite, and okay when $0 \le r \le b$. This means we can write the average contribution as

$$\Psi_P = \frac{a}{A}\iint_\Sigma e^{-\left(\frac{r-b}{c}\right)^2}\, dS \tag{1.4}$$

where A is the total surface area of the bombsite, given by

$$A = \iint_\Sigma dS. \tag{1.5}$$

Thus

$$\begin{aligned}
\Psi_P &= \frac{a}{A}\iint_\Sigma e^{-\left(\frac{\sqrt{(x-X_p)^2+(y-Y_p)^2+(z-Z_p)^2}-b}{c}\right)^2}\, dS && (1.6)\\
&= \frac{a}{A}\iint_\Sigma e^{-\frac{(x-X_p)^2+(y-Y_p)^2+(z-Z_p)^2-2b\sqrt{(x-X_p)^2+(y-Y_p)^2+(z-Z_p)^2}+b^2}{c^2}}\, dS && (1.7)\\
&= \frac{a}{A}e^{-\frac{b^2}{c^2}}\iint_\Sigma e^{-\frac{x^2-2xX_p+X_p^2+y^2-2yY_p+Y_p^2+z^2-2zZ_p+Z_p^2-2b\sqrt{x^2-2xX_p+X_p^2+y^2-2yY_p+Y_p^2+z^2-2zZ_p+Z_p^2}}{c^2}}\, dS && (1.8)\\
&= \frac{a}{A}e^{-\frac{b^2}{c^2}}\iint_\Sigma e^{-\frac{R^2+x^2+y^2+z^2-2(xX_p+yY_p+zZ_p)-2b\sqrt{R^2+x^2+y^2+z^2-2(xX_p+yY_p+zZ_p)}}{c^2}}\, dS && (1.9)\\
&= \frac{a}{A}e^{-\frac{R^2+b^2}{c^2}}\iint_\Sigma e^{-\frac{x^2+y^2+z^2-2(xX_p+yY_p+zZ_p)-2b\sqrt{R^2+x^2+y^2+z^2-2(xX_p+yY_p+zZ_p)}}{c^2}}\, dS. && (1.10)
\end{aligned}$$

If we seek an approximate solution, then assuming the dimensions of the bombsite are small relative to the distance of P we have

$$\begin{aligned}
\sqrt{R^2+x^2+y^2+z^2-2(xX_p+yY_p+zZ_p)} &= R\sqrt{1+\frac{x^2+y^2+z^2}{R^2}-\frac{2(xX_p+yY_p+zZ_p)}{R^2}} && (1.11)\\
&\approx R\sqrt{1-\frac{2(xX_p+yY_p+zZ_p)}{R^2}} && (1.12)\\
&\approx R\left[1-\frac{xX_p+yY_p+zZ_p}{R^2}\right]. && (1.13)
\end{aligned}$$

We can then write

$$\begin{aligned}
\Psi_P &\approx \frac{a}{A}e^{-\frac{R^2+b^2}{c^2}}\iint_\Sigma e^{-\frac{x^2+y^2+z^2-2(xX_p+yY_p+zZ_p)-2bR\left[1-\frac{xX_p+yY_p+zZ_p}{R^2}\right]}{c^2}}\, dS && (1.14)\\
&= \frac{a}{A}e^{-\frac{R^2+b^2}{c^2}}\iint_\Sigma e^{-\frac{x^2+y^2+z^2-2(xX_p+yY_p+zZ_p)-2bR+2b\frac{xX_p+yY_p+zZ_p}{R}}{c^2}}\, dS && (1.15)\\
&= \frac{a}{A}e^{-\frac{(R-b)^2}{c^2}}\iint_\Sigma e^{-\frac{x^2+y^2+z^2-2(xX_p+yY_p+zZ_p)+\frac{2b}{R}(xX_p+yY_p+zZ_p)}{c^2}}\, dS && (1.16)\\
&= \frac{a}{A}e^{-\frac{(R-b)^2}{c^2}}\iint_\Sigma e^{-\frac{x^2+y^2+z^2+2\left(\frac{b}{R}-1\right)(xX_p+yY_p+zZ_p)}{c^2}}\, dS. && (1.17)
\end{aligned}$$

Finally, let $\gamma = \frac{b}{R} - 1$. Then

$$\Psi_P = \frac{a}{A} e^{-\frac{(R-b)^2}{c^2}} \iint_\Sigma e^{-\frac{x^2+y^2+z^2+2\gamma(xX_p+yY_p+zZ_p)}{c^2}}\, dS \tag{1.18}$$

1.3 Constants

The constants for unarmoured players are

$$a_1 = 450.7 \text{ HP}, \tag{1.19}$$
$$b_1 = 75.68 \text{ units}, \tag{1.20}$$
$$c_1 = 789.2 \text{ units}. \tag{1.21}$$

For armoured players they are

$$a_2 = 200.2 \text{ HP}, \tag{1.22}$$
$$b_2 = 162.7 \text{ units}, \tag{1.23}$$
$$c_2 = 747.0 \text{ units}. \tag{1.24}$$

1.4 Symmetry

If the bombsite has symmetry, then a simple approximation to the average of the entire site is the evaluation of the damage model from the mean centre.

1.5 Fundamental Solution

Let's suppose we have a rectangular piece of a bombsite at $z = z_0$ everywhere, that runs from $x = x_0$ to $x = x_1$, and $y = y_0$ to $y = y_1$ with the bombsite centre at the origin. We require that $x_0 < x_1$ and $y_0 < y_1$. Our far-field approximate solution is then

$$\Psi_P(X_p, Y_p, Z_p) = \frac{a}{A} e^{-\frac{(R-b)^2}{c^2}} \iint_\Sigma e^{-\frac{x^2+y^2+z_0^2+2\gamma(xX_p+yY_p+z_0Z_p)}{c^2}}\, dS \tag{1.25}$$

$$= \frac{a}{A} e^{-\frac{(R-b)^2+z_0^2+2\gamma z_0 Z_p}{c^2}} \iint_\Sigma e^{-\frac{x^2+y^2+2\gamma(xX_p+yY_p)}{c^2}}\, dS. \tag{1.26}$$

Applying Fubini's theorem we can split this into a product of two integrals and get

$$\iint_\Sigma e^{-\frac{x^2+y^2+2\gamma(xX_p+yY_p)}{c^2}}\, dS = \int_{x_0}^{x_1} e^{-\frac{x^2+2\gamma X_p x}{c^2}}\, dx \int_{y_0}^{y_1} e^{-\frac{y^2+2\gamma Y_p y}{c^2}}\, dy. \tag{1.27}$$

These have a closed form,

$$\int_{x_0}^{x_1} e^{-\frac{x^2+2\gamma X_p x}{c^2}}\, dx = \frac{1}{2}\sqrt{\pi} c e^{\frac{\gamma^2 X_p^2}{c^2}} \left(\operatorname{erf}\left(\frac{\gamma X_p + x_1}{c}\right) - \operatorname{erf}\left(\frac{\gamma X_p + x_0}{c}\right)\right) \tag{1.28}$$

so we can write

$$\Psi_P(X_p, Y_p, Z_p) = \frac{a}{A} e^{-\frac{(R-b)^2+z_0^2+2\gamma z_0 Z_p}{c^2}} \frac{1}{2}\sqrt{\pi} c e^{\frac{\gamma^2 X_p^2}{c^2}} \left(\text{erf}\left(\frac{\gamma X_p + x_1}{c}\right) - \text{erf}\left(\frac{\gamma X_p + x_0}{c}\right)\right) \tag{1.29}$$

$$\cdot \frac{1}{2}\sqrt{\pi} c e^{\frac{\gamma^2 Y_p^2}{c^2}} \left(\text{erf}\left(\frac{\gamma Y_p + y_1}{c}\right) - \text{erf}\left(\frac{\gamma Y_p + y_0}{c}\right)\right) \tag{1.30}$$

$$= \frac{\pi a c^2}{4A} e^{-\frac{(R-b)^2+z_0^2+2\gamma z_0 Z_p - \gamma^2 X_p^2 - \gamma^2 Y_p^2}{c^2}} \left(\text{erf}\left(\frac{\gamma X_p + x_1}{c}\right) - \text{erf}\left(\frac{\gamma X_p + x_0}{c}\right)\right) \tag{1.31}$$

$$\left(\text{erf}\left(\frac{\gamma Y_p + y_1}{c}\right) - \text{erf}\left(\frac{\gamma Y_p + y_0}{c}\right)\right). \tag{1.32}$$

Observe that

$$z_0^2 + 2\gamma z_0 Z_p - \gamma^2 X_p^2 - \gamma^2 Y_p^2 = (z_0 + \gamma Z_p)^2 - \gamma^2 X_p^2 - \gamma^2 Y_p^2 - \gamma^2 Z_p^2 \tag{1.33}$$

$$= (z_0 + \gamma Z_p)^2 - \gamma^2 R^2. \tag{1.34}$$

It follows that

$$e^{-\frac{(R-b)^2+z_0^2+2\gamma z_0 Z_p - \gamma^2 X_p^2 - \gamma^2 Y_p^2}{c^2}} = e^{-\frac{(R-b)^2 + (z_0+\gamma Z_p)^2 - \gamma^2 R^2}{c^2}}. \tag{1.35}$$

Let

$$\mathcal{N} \equiv \frac{\pi a c^2}{4} \exp\left(\frac{\gamma^2 R^2 - (R-b)^2 - (z_0 + \gamma Z_p)^2}{c^2}\right) \tag{1.36}$$

and let

$$\phi(x_0, x_1; X_p) \equiv \text{erf}\left(\frac{\gamma X_p + x_1}{c}\right) - \text{erf}\left(\frac{\gamma X_p + x_0}{c}\right). \tag{1.37}$$

Then we can write our "fundamental solution" as

$$\vartheta(x_0, y_0, x_1, y_1, z_0; X_p, Y_p, Z_p) = \mathcal{N}\phi(x_0, x_1; X_p)\phi(y_0, y_1; Y_p). \tag{1.38}$$

Complicated bombsites can be modelled with sums of multiple ϑ functions by considering the different contributions from sections of the bombsite. All of these solutions must use the same centre. This method simplifies the issue to finding the geometry of the bombsite. That is,

$$\Psi_P(X_p, Y_p, Z_p) = \frac{1}{A} \sum \vartheta(X_p, Y_p, Z_p). \tag{1.39}$$

The total area is given (informally) by

$$A = \sum (x_1 - x_0)(y_1 - y_0). \tag{1.40}$$

1.6 Simple Example

Consider a flat rectangular bombsite ($z_0 = 0$) centred at the origin that ranges from $x = -x_0$ to $x = x_0$ and $y = -y_0$ to $y = y_0$. Then the solution can be given by just one fundamental solution as

$$\vartheta(-x_0, -y_0, x_0, y_0, 0; X_p, Y_p, Z_p) = \mathcal{N}\phi(-x_0, x_0; X_p)\phi(-y_0, y_0; Y_p). \tag{1.41}$$

The area is

$$A = (2x_0) \cdot (2y_0) = 4x_0 y_0. \tag{1.42}$$

1.7 Simple Example Test - Armoured

Using the texture grid points as a reference with position and angle set by "setpos 1056 0; setang 0 180 0", these results of damage were gathered.

43	43	43	43	43	43
46	46	46	45	45	45
48	48	48	48	48	48
50	50	50	50	50	50
53	53	53	53	53	53

Table 1.

This has an average of 47.6 damage. Applying our simple model, we have $x_0 = y_0 = 32$ units. This gives an area of $A = 4096$ units. We approximate the eyepos as $(X_p, Y_p, Z_p) = (1056, 0, 64)$. Then

$$R = \sqrt{1056^2 + 64^2} = 32\sqrt{1093}, \tag{1.43}$$

$$\gamma = \frac{162.7}{32\sqrt{1093}} - 1 = -0.8462102136, \tag{1.44}$$

$$\phi(-y_0 = -32, y_0 = 32; Y_p = 0) = 2\mathrm{erf}\left(\frac{32}{747.0}\right) = 0.096615952359360713, \tag{1.45}$$

$$\phi(-x_0 = -32, x_0 = 32; X_p = 1056) = \mathrm{erf}\left(\frac{1056\gamma + 32}{747.0}\right) - \mathrm{erf}\left(\frac{1056\gamma - 32}{747.0}\right) = 0.023138120118125438, \tag{1.46}$$

$$\mathcal{N} = \frac{\pi a c^2}{4} \exp\left(\frac{1119232\gamma^2 - (32\sqrt{1093} - b)^2 - (64\gamma)^2}{c^2}\right) \tag{1.47}$$

$$= 87279530.555288792. \tag{1.48}$$

Then we finally get

$$\vartheta(-32, -32, 32, 32, 0; 1056, 0, 64)/A = \frac{\mathcal{N}}{4096} \cdot \phi(-32, 32; 1056)\phi(-32, 32; 64) \tag{1.49}$$

$$= 47.635350398582275. \tag{1.50}$$

This has an error of about 0.1 HP.

1.8 Example 2

Suppose we have a bombsite similar to the first example, except that the region inside $(-32, 0) \to (0, 32)$ is raised by $z_0 = 500$ units. The first issue we must take care of is that it is not possible to plant directly next to the wall, as the player takes up $32 \cdot 32 = 1024$ units^2 of space. This means that any walls within a bombsite remove a 16 unit thick contour around the wall. This is shown in Figure 2.

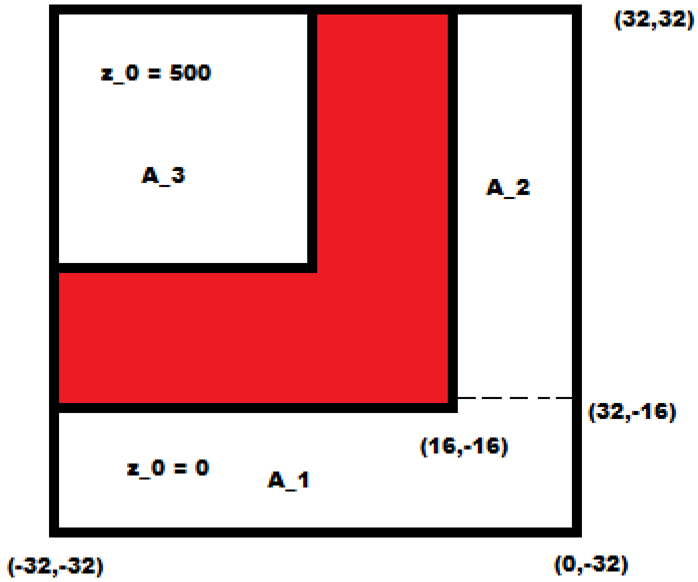

Figure 2. Simple Bombsite 2

We first find the areas.

$$A_1 = (32 - -32) \cdot (-16 - -32) = 1024, \tag{1.51}$$
$$A_2 = (32 - 16) \cdot (32 - -16) = 768, \tag{1.52}$$
$$A_3 = (0 - -32) \cdot (32 - -0) = 1024, \tag{1.53}$$
$$\implies A = A_1 + A_2 + A_3 = 2816. \tag{1.54}$$

Now, strictly speaking we can place our bombsite centre anywhere, but this approximation method works better if it is placed at the mean of the bombsite. Using the areas calculated we can find this. Since we're already centred relative to x and y, we can find the average z position as

$$\hat{Z}_c = \frac{1024 \cdot 500 + 1024 \cdot 0 + 768 \cdot 0}{2816} = \frac{2000}{11}. \tag{1.55}$$

Then, applying our transformation that we stated at the beginning we transform to Figure 3.
Now, for the calculations. We simply sum over the three areas. For each area we get

$$A_1 : \vartheta_1(-32, -32, 32, -16, -2000/11; 1056, 0, -1296/11) = 48316.354393862166, \tag{1.56}$$
$$A_2 : \vartheta_2(16, -16, 32, 32, -2000/11; 1056, 0, -1296/11) = 39076.478821471494, \tag{1.57}$$
$$A_3 : \vartheta_3(-32, 0, 0, 32, 3500/11; 1056, 0, -1296/11) = 33941.816322584353. \tag{1.58}$$

Then the average damage is given by

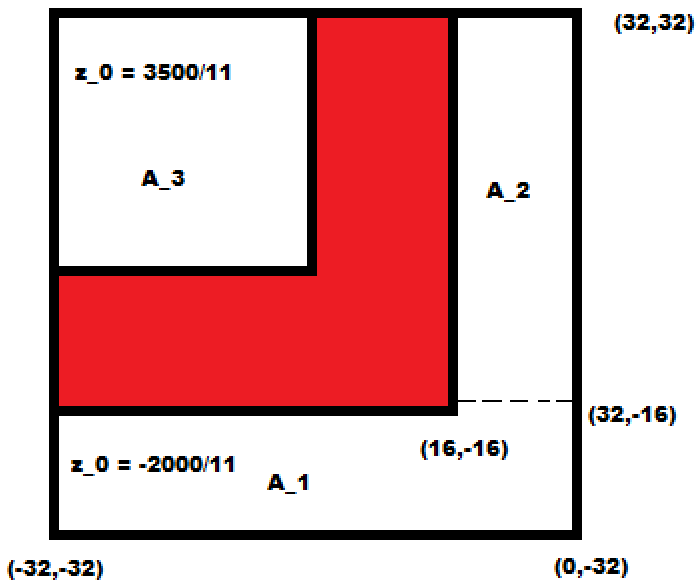

Figure 3. Centred Simple Bombsite 2

$$\frac{\vartheta_1 + \vartheta_2 + \vartheta_3}{A_1 + A_2 + A_3} = 43.087588614317475 \text{ HP}. \tag{1.59}$$

43	43	43	43	43	43
46	46	46	45	45	45
48	48	48	48	48	48
50	50	50	50	50	50
53	53	53	53	53	53

Table 2. Not Completed

1.9 When far-field approximation fails

If the bombsite is large enough compared to the distance from the bombsite centre, two problems can occur. Firstly, the damage model can fail, and to correct this would require a new model. The second problem is that

$$\frac{x^2 + y^2 + z^2}{R^2} \tag{1.60}$$

can become large enough that the approximation made earlier can fail.

$$\sqrt{R^2 + x^2 + y^2 + z^2 - 2(xX_p + yY_p + zZ_p)} = R\sqrt{1 + \frac{x^2 + y^2 + z^2}{R^2} - \frac{2(xX_p + yY_p + zZ_p)}{R^2}} \tag{1.61}$$

$$\approx R\left[1 + \frac{x^2 + y^2 + z^2}{2R^2} - \frac{xX_p + yY_p + zZ_p}{R^2}\right] \tag{1.62}$$

$$= R + \frac{x^2 + y^2 + z^2}{2R} - \frac{xX_p + yY_p + zZ_p}{R}. \tag{1.63}$$

Then, after some simplification we have

$$\Psi_P \approx \frac{a}{A} e^{-\frac{(R-b)^2}{c^2}} \iint_\Sigma e^{-\frac{\left(\frac{b}{R}+1\right)(x^2+y^2+z^2)+2\left(\frac{b}{R}-1\right)(xX_p+yY_p+zZ_p)}{c^2}} dS. \tag{1.64}$$

Letting $\sigma = \frac{b}{R} + 1 \,(= \gamma + 2)$ this can be written as

$$\Psi_P \approx \frac{a}{A} e^{-\frac{(R-b)^2}{c^2}} \iint_\Sigma e^{-\frac{\sigma(x^2+y^2+z^2)+2\gamma(xX_p+yY_p+zZ_p)}{c^2}} dS. \tag{1.65}$$

Now, if we use the same approach as earlier to develop the fundamental solutions we have

$$\Psi_P(X_p, Y_p, Z_p) = \frac{a}{A} e^{-\frac{(R-b)^2}{c^2}} \iint_\Sigma e^{-\frac{\sigma x^2+\sigma y^2+\sigma z_0^2+2\gamma(xX_p+yY_p+z_0 Z_p)}{c^2}} dS \tag{1.66}$$

$$= \frac{a}{A} e^{-\frac{(R-b)^2+\sigma z_0^2+2\gamma z_0 Z_p}{c^2}} \iint_\Sigma e^{-\frac{\sigma x^2+\sigma y^2+2\gamma(xX_p+yY_p)}{c^2}} dS. \tag{1.67}$$

Applying Fubini's theorem,

$$\iint_\Sigma e^{-\frac{\sigma x^2+\sigma y^2+2\gamma(xX_p+yY_p)}{c^2}} dS = \int_{x_0}^{x_1} e^{-\frac{\sigma x^2+2\gamma X_p x}{c^2}} dx \int_{y_0}^{y_1} e^{-\frac{\sigma y^2+2\gamma Y_p y}{c^2}} dy \tag{1.68}$$

with closed form

$$\int_{x_0}^{x_1} e^{-\frac{\sigma x^2+2\gamma X_p x}{c^2}} dx = \frac{\sqrt{\pi} c}{2\sqrt{\sigma}} e^{\frac{\gamma^2 X_p^2}{\sigma c^2}} \left(\operatorname{erf}\left(\frac{\sigma x_1 + \gamma X_p}{c\sqrt{\sigma}}\right) - \operatorname{erf}\left(\frac{\sigma x_0 + \gamma X_p}{c\sqrt{\sigma}}\right)\right). \tag{1.69}$$

We then have

$$\Psi_P = \frac{a}{A} e^{-\frac{(R-b)^2+\sigma z_0^2+2\gamma z_0 Z_p}{c^2}} \frac{\sqrt{\pi} c}{2\sqrt{\sigma}} e^{\frac{\gamma^2 X_p^2}{\sigma c^2}} \left(\operatorname{erf}\left(\frac{\sigma x_1 + \gamma X_p}{c\sqrt{\sigma}}\right) - \operatorname{erf}\left(\frac{\sigma x_0 + \gamma X_p}{c\sqrt{\sigma}}\right)\right) \tag{1.70}$$

$$\frac{\sqrt{\pi} c}{2\sqrt{\sigma}} e^{\frac{\gamma^2 Y_p^2}{\sigma c^2}} \left(\operatorname{erf}\left(\frac{\sigma y_1 + \gamma Y_p}{c\sqrt{\sigma}}\right) - \operatorname{erf}\left(\frac{\sigma y_0 + \gamma Y_p}{c\sqrt{\sigma}}\right)\right) \tag{1.71}$$

$$= \frac{\pi a c^2}{4\sigma A} e^{-\frac{(R-b)^2+\sigma z_0^2+2\gamma z_0 Z_p}{c^2} + \frac{\gamma^2 X_p^2+\gamma^2 Y_p^2}{\sigma c^2}} \left(\operatorname{erf}\left(\frac{\sigma x_1 + \gamma X_p}{c\sqrt{\sigma}}\right) - \operatorname{erf}\left(\frac{\sigma x_0 + \gamma X_p}{c\sqrt{\sigma}}\right)\right) \tag{1.72}$$

$$\left(\operatorname{erf}\left(\frac{\sigma y_1 + \gamma Y_p}{c\sqrt{\sigma}}\right) - \operatorname{erf}\left(\frac{\sigma y_0 + \gamma Y_p}{c\sqrt{\sigma}}\right)\right). \tag{1.73}$$

Then, write

$$\mathcal{M} = \frac{\pi a c^2}{4\sigma} e^{\frac{\gamma^2 R^2 - \sigma(R-b)^2 - (\sigma z_0 + \gamma Z_p)^2}{\sigma c^2}} \tag{1.74}$$

and

$$\Phi(x_0, x_1; X_p) = \operatorname{erf}\left(\frac{\sigma x_1 + \gamma X_p}{c\sqrt{\sigma}}\right) - \operatorname{erf}\left(\frac{\sigma x_0 + \gamma X_p}{c\sqrt{\sigma}}\right). \tag{1.75}$$

We then define the close-field fundamental solution as

$$\Theta(x_0, y_0, x_1, y_1, z_0; X_p, Y_p, Z_p) = \mathcal{M}\Phi(x_0, x_1; X_p)\Phi(y_0, y_1; Y_p). \tag{1.76}$$

Trying the simple example from earlier, we get agreement,

$$\Psi_P = \Theta(-32, -32, 32, 32, 0; 1056, 0, 64)/4096 = 47.626387295986966. \tag{1.77}$$

For the second example, we get

$$A_1 : \Theta_1(-32, -32, 32, -16, -2000/11; 1056, 0, -1296/11) = 47867.709129173651, \tag{1.78}$$

$$A_2 : \Theta_2(16, -16, 32, 32, -2000/11; 1056, 0, -1296/11) = 38714.507568581248, \tag{1.79}$$

$$A_3 : \Theta_3(-32, 0, 0, 32, 3500/11; 1056, 0, -1296/11) = 33005.757134970576. \tag{1.80}$$

The more accurate average is

$$\frac{\Theta_1 + \Theta_2 + \Theta_3}{A_1 + A_2 + A_3} = 42.46732025309853. \tag{1.81}$$

This is significantly different to the previous value, by about 1 HP.