Flashbang Optimisation Problem

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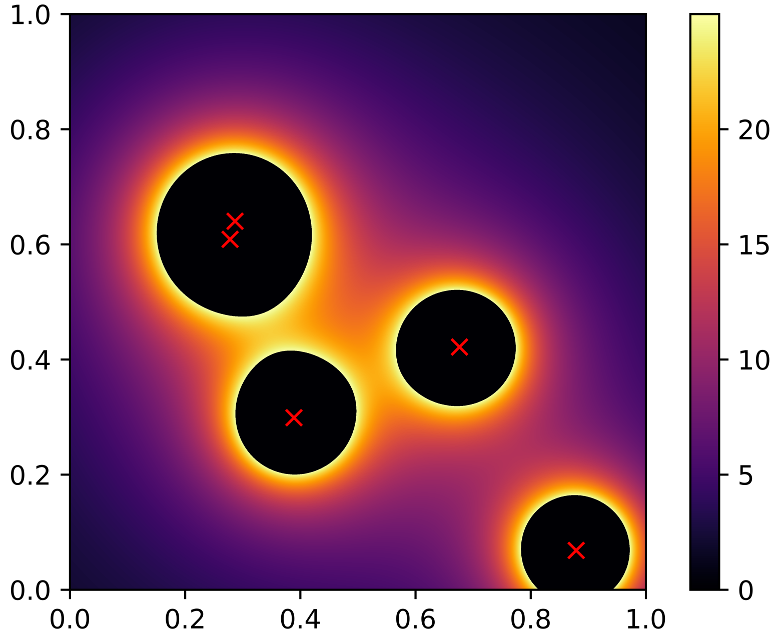

Figure 1. Example with 5 terrorists, and $\beta = 0.2 = \alpha$.

1 Introduction

Suppose there are n terrorists in a space in the $2d$ box $[0,1]^2$, labelled (x_k, y_k) for $k = 1 \ldots n$. What position (x, y) should you throw a flashbang to blind them all as much as possible? The light drops off with the inverse square law, $1/r^2$ where r is the distance between the terrorist and the flashbang. We wish to maximise the objective function for $r_k = \sqrt{(x - x_k)^2 + (y - y_k)^2}$

$$h(x, y) = \alpha \sum_{k=1}^{n} \frac{1}{r_k^2} \tag{1.1}$$

where α is a parameter to be varied for dimensions. We add the constraint that if the light is placed within β of the terrorist then h returns 0. Without this, simply placing the light directly inside of a terrorist is possible, which will return an infinite value. An example using this objective function is shown in Figure 1, where the red x indicates a T.

An alternative to this is to use the objective function

$$g(x, y) = \alpha \frac{a^2 e^2}{4} \sum_{k=1}^{n} \frac{\exp(-a/r_k)}{r_k^2} \tag{1.2}$$

with a parameter $a > 0$ that determines the strength of the falloff near the terrorist, and the prefactor simply normalises the exponential so that changing a doesn't change the maximum of a single contribution. We will use this instead, as it is smoother. To find where g is a maximum, we must search the boundary and find where $\partial g/\partial x, \partial g/\partial y$ are zero. We need only find the partial derivatives of

$$f(x,y) = \frac{\exp\left(\frac{-a}{\sqrt{(x-x_k)^2+(y-y_k)^2}}\right)}{(x-x_k)^2+(y-y_k)^2} \tag{1.3}$$

for some k, and then sum them up to get the derivatives of g. Computing,

$$\frac{\partial f}{\partial x} = \frac{\partial}{\partial x}\left(\frac{\exp\left(\frac{-a}{\sqrt{(x-x_k)^2+(y-y_k)^2}}\right)}{(x-x_k)^2+(y-y_k)^2}\right) \tag{1.4}$$

$$= \frac{(x_k-x)\exp\left(\frac{-a}{\sqrt{(x_k-x)^2+(y_k-y)^2}}\right)\left(2\sqrt{(x_k-x)^2+(y_k-y)^2}-a\right)}{((x_k-x)^2+(y_k-y)^2)^{5/2}}. \tag{1.5}$$

We can then deduce via symmetry that the partial derivative with respect to y is the above with the x and $y's$ replaced. That is,

$$\frac{\partial f}{\partial y} = \frac{(y_k-y)\exp\left(\frac{-a}{\sqrt{(x_k-x)^2+(y_k-y)^2}}\right)\left(2\sqrt{(x_k-x)^2+(y_k-y)^2}-a\right)}{((x_k-x)^2+(y_k-y)^2)^{5/2}} \tag{1.6}$$

The condition for having a stationary point inside the region is

$$\frac{\partial g}{\partial x} = 0, \quad \frac{\partial g}{\partial y} = 0 \tag{1.7}$$

and using the above, we have (dividing out the constants in front)

$$\sum_{k=1}^{n} \frac{(x_k-x)\exp\left(\frac{-a}{\sqrt{(x_k-x)^2+(y_k-y)^2}}\right)\left(2\sqrt{(x_k-x)^2+(y_k-y)^2}-a\right)}{((x_k-x)^2+(y_k-y)^2)^{5/2}} = 0, \tag{1.8}$$

$$\sum_{k=1}^{n} \frac{(y_k-y)\exp\left(\frac{-a}{\sqrt{(x_k-x)^2+(y_k-y)^2}}\right)\left(2\sqrt{(x_k-x)^2+(y_k-y)^2}-a\right)}{((x_k-x)^2+(y_k-y)^2)^{5/2}} = 0. \tag{1.9}$$

1.1 One T

In the case $n = 1$, the condition for a stationary point becomes

$$(x_1-x)^2+(y_1-y)^2 = \frac{a^2}{4}. \tag{1.10}$$

$$\tag{1.11}$$

Note the disappearance of the factors $(x_1 - x)$ and $(y_1 - y)$, as these are non-zero, and similarly the denominator can be safely multiplied through. The exponential term also vanishes, as it is always non-zero. Then equation (1.10) is simply the equation of a circle of radius $a/2$, with center (x_1, y_1). A plot of this is shown in Figure 2, with $a = 0.4$.

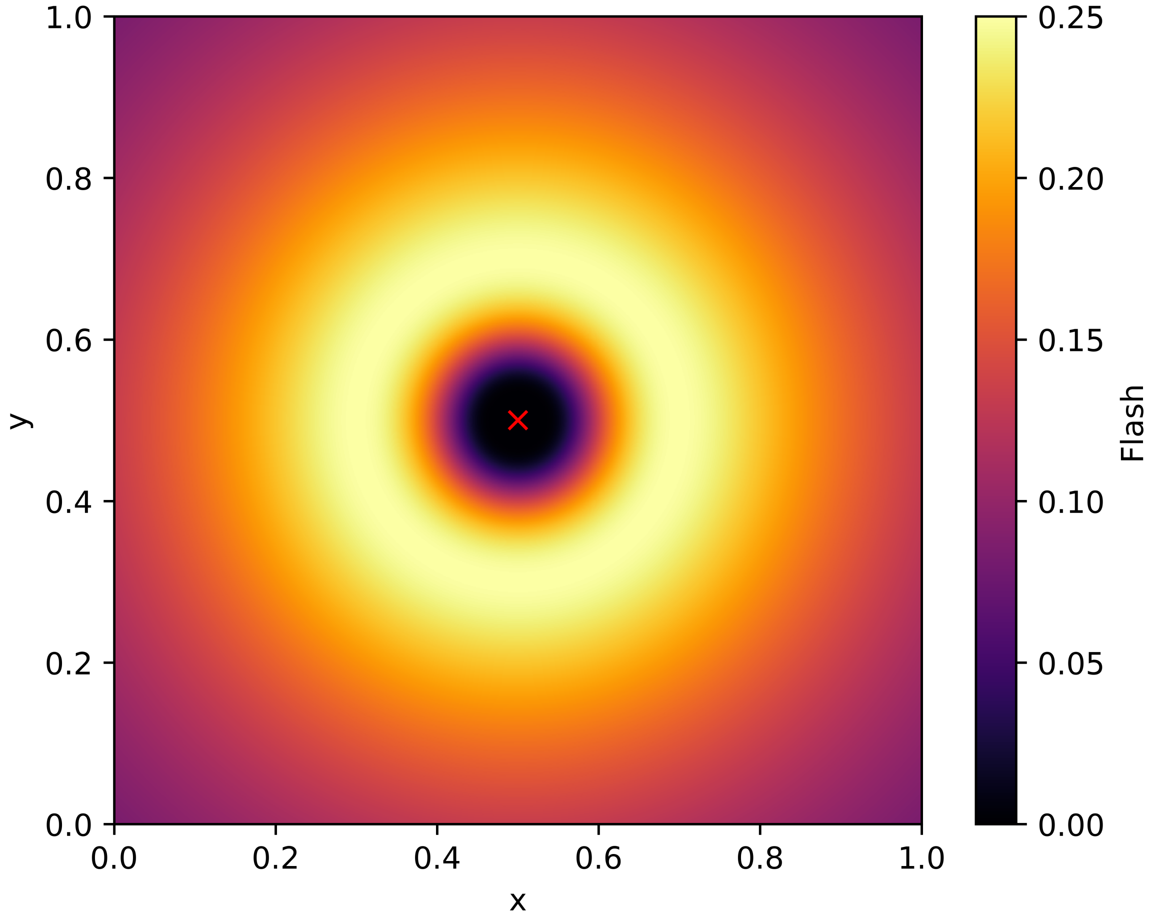

Figure 2. With $a = 0.4$. The exponential objective function with one T.

1.2 Two T's

When $n = 2$, g takes the form

$$g(x, y) = \frac{\alpha a^2 e^2}{4} \left(\frac{\exp(-a/r_1)}{r_1^2} + \frac{\exp(-a/r_2)}{r_2^2} \right). \tag{1.12}$$

Graphing this with $(x_1, y_1) = (0.35, 0.5), (x_2, y_2) = (0.65, 0.5)$ get Figure 3. Based upon our previous work, if the maximum circles intersect, then the maximum of the sum will occur at those points. Assuming they intersect (and they may not if a is small), we deduce the maximum occurs at (x, y) such that

$$(x_1 - x)^2 + (y_1 - y)^2 = (x_2 - x)^2 + (y_2 - y)^2. \tag{1.13}$$

Multiplying out, we get

$$x_1^2 - 2x_1x + x^2 + y_1^2 - 2y_1y + y^2 = x_2^2 - 2x_2x + x^2 + y_2^2 - 2y_2y + y^2. \tag{1.14}$$

Simplifying, we have

$$2x(x_2 - x_1) + 2y(y_2 - y_1) = x_2^2 - x_1^2 + y_2^2 - y_1^2. \tag{1.15}$$

The actual coordinates can then be determined by solving for x or y and substituting back into one of the circular equations.

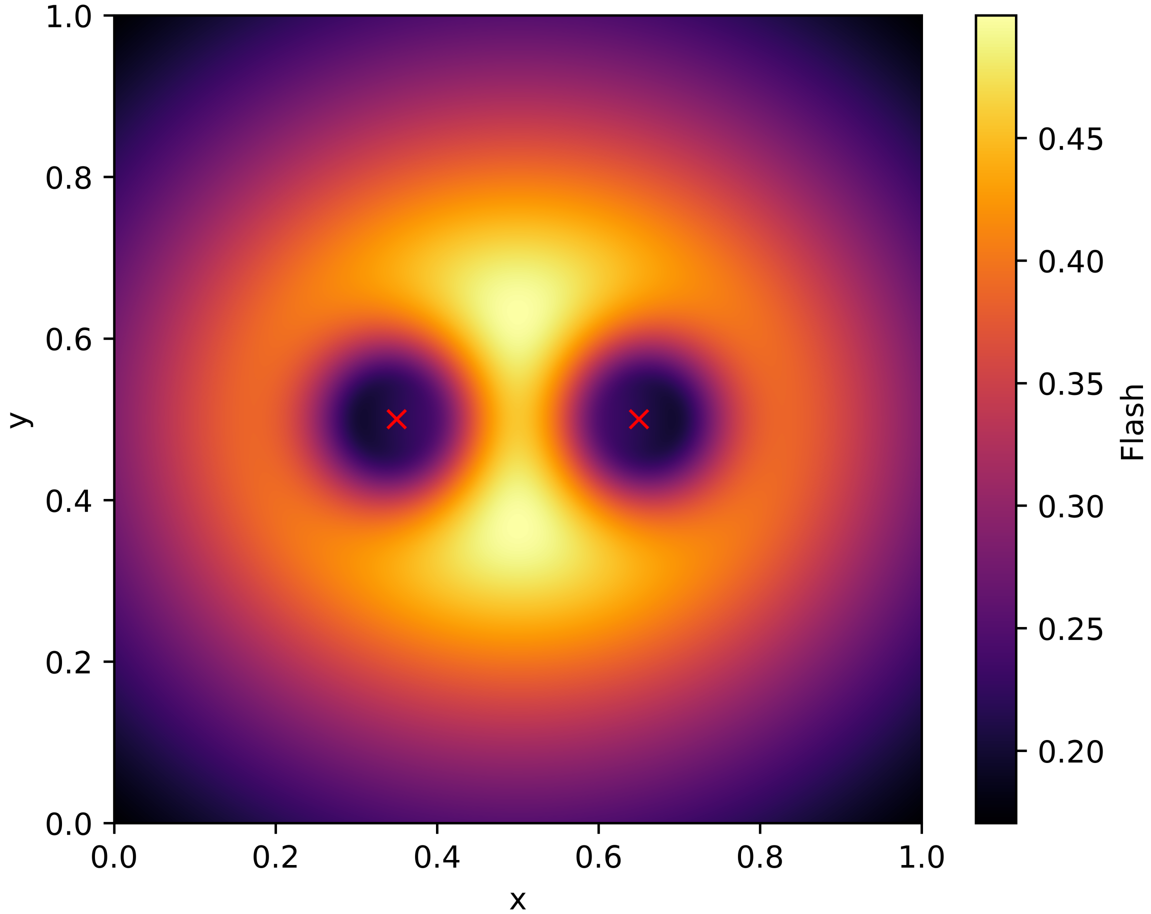

Figure 3. With $a = 0.4$. The exponential objective function with two T's. Clearly the flash should land between them for maximum effectiveness.