On A337559

True_hOREP

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1 Definition

Let $a(n)$ denote the number of length four $1..n$ vectors $v = (a, b, c)$ that contain their harmonic mean

$$h = \frac{3}{\frac{1}{a} + \frac{1}{b} + \frac{1}{c}}. \tag{1.1}$$

It can be shown that h must satisfy

$$h(ab + ac + bc) = 3abc. \tag{1.2}$$

2 Modular

$$a(n) - a(n-1) = 1 \pmod 6 \tag{2.1}$$

Vectors of the form (a, a, c) cannot contain their harmonic mean, and all the other forms other than the trivial solution (n, n, n) have 6 permutations (including themselves). Either all the elements in v are the same, or they are all different.

Proof. We show that vectors of the form (a, a, c) with $a \neq c$ are invalid. We have

$$h = \frac{3a^2c}{a^2 + 2ac} = \frac{3ac}{a + 2c}. \tag{2.2}$$

Suppose $h = a$. Then $3ac = a^2 + 2ac$, that is $ac = a^2$, thus $c = a$. Now suppose that $h = c$. Then $3ac = ac + 2c^2$, so $2ac = 2c^2$, from which $a = c$ follows. $\square$

3 Generation

Seeking new vectors in $a(n)$ and not $a(n-1)$, the vector must contain n otherwise it would have already been counted. If $h = n$, then $v = (n, n, n)$. Now assume that $h = b$. Then

$$b(ab + an + bn) = 3abn \tag{3.1}$$
$$ab + bn = 2an \tag{3.2}$$

and thus

$$b = \frac{2an}{a + n}. \tag{3.3}$$

We can then write

$$v = \left(a, \frac{2an}{a + n}, n\right) \tag{3.4}$$

for whenever $\frac{2an}{a+n} \in \mathbb{Z}^+$ with $1 \leq a \leq n$.