Output Feedback

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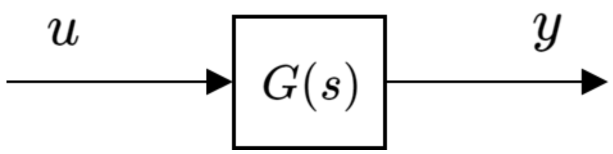
Most slides from Control Bootcamp by Prof. Steve Brunton and Control of Mobile Robots by Prof. Magnus Egerstedt

Plant G (or System)

- Consider the plant G
  - Input $u(t)$
  - Output $y(t)$

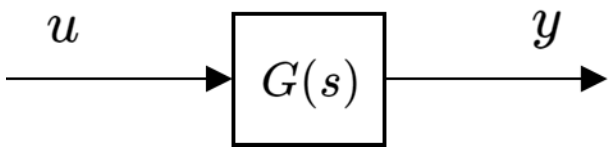
- So far, we have learnt about dynamics of plant G

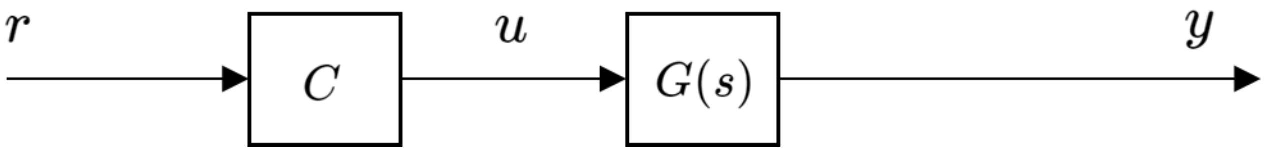


- Control problem
  - The output of this plant track a desired reference trajectory $r(t)$

Open Loop Control

- The simplest solution to the tracking problem is to use a pre-compensator G
  - $C = \frac{1}{G}$
  - Then $y(t) = r(t)$



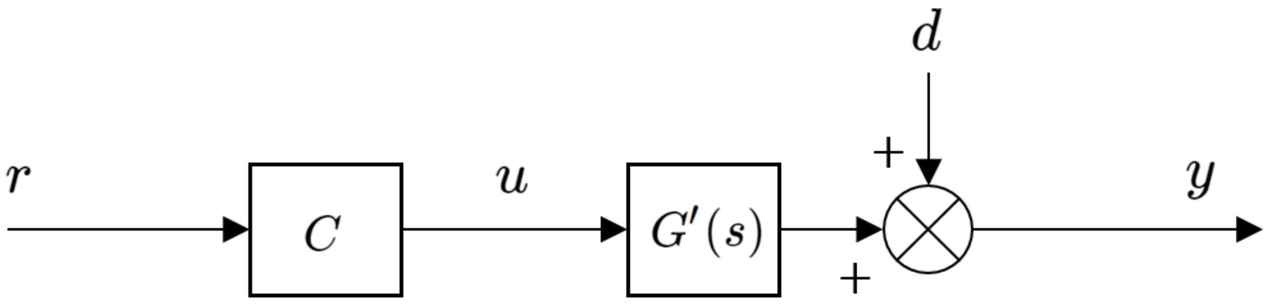


- Do you see any problems of this solution?
  - Model uncertainty
  - Disturbance

Open Loop Control

- Consider the important practical issue of model uncertainty and disturbance.
  - It is always the case that the true system we wish to control will deviate from the nominal model used in control design

- Suppose uncertain plant G' and disturbance $d(t)$

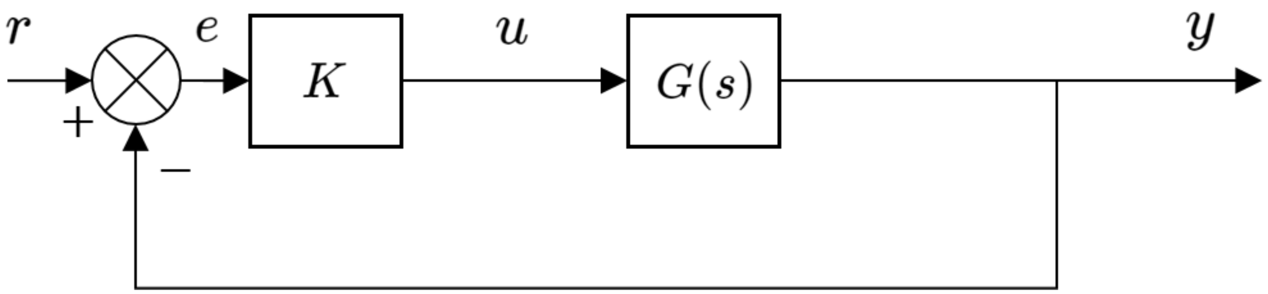


- Open loop controller $C = \frac{1}{G}$

$$y(t) = \frac{G'}{G} r(t) + d(t)$$

Closed Loop Control (= Negative Feedback Control)

- An alternative solution is to purchase a sensor and use feedback control
  - Use a constant gain compensator K, that multiplies the measured tracking error $e(t) = r(t) - y(t)$



$$
\begin{aligned}
u &= Ke \\
e &= r - y \\
y &= Gu
\end{aligned}
$$

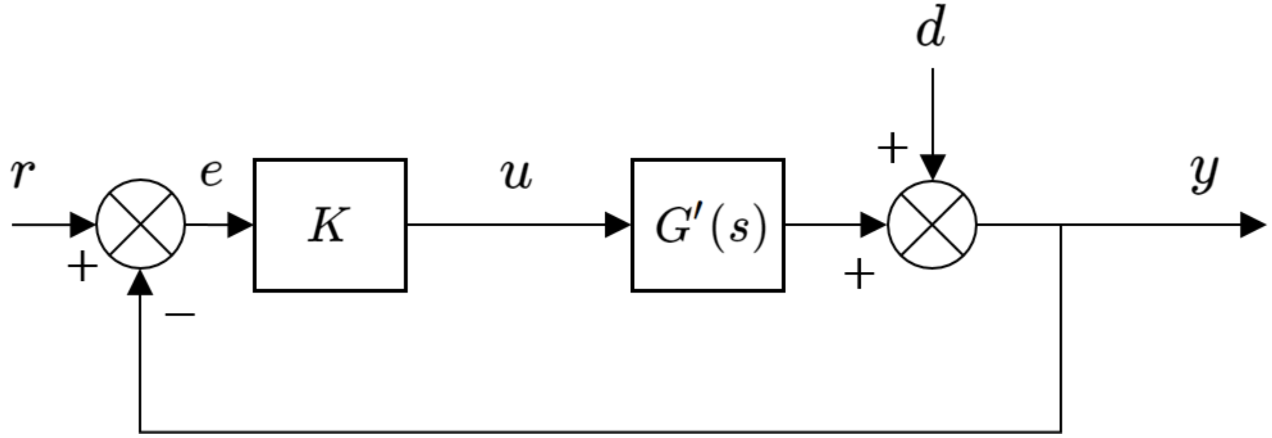
$$e = \frac{1}{1+GK} r$$

$$y = \frac{GK}{1+GK} r$$

As $|GK| \to \infty,\ e(t) \to 0$ and $y(t) \to r(t)$

Closed Loop Control

- Feedback controller
  - Use of feedback with sufficiently high gain provides an approximate solution to the tracking problem even in the presence of system uncertainty and disturbance

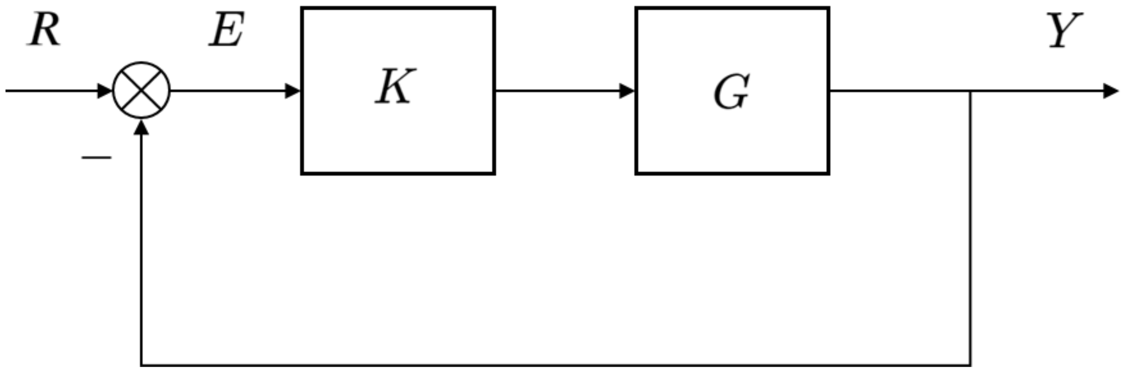


$$e = \frac{1}{1+G'K}r - \frac{1}{1+G'K}d$$

$$y = \frac{G'K}{1+G'K}r + \frac{1}{1+G'K}d$$

Transfer Function for Closed Loop System

- Feedback changes the system transfer function
  - Change system dynamics
  - Change poles and zeros
  - Might change system stability



$$E = R - Y$$
$$Y = KGE$$

$$\begin{aligned} Y &= KG(R - Y) \\ &= KGR - KGY \end{aligned}$$

$$(1 + KG)Y = KGR$$

$$H = \frac{Y}{R} = \frac{KG}{1 + KG}$$

Transfer Function for Closed Loop System

- Feedback changes the system transfer function
  - Change system dynamics
  - Change poles and zeros
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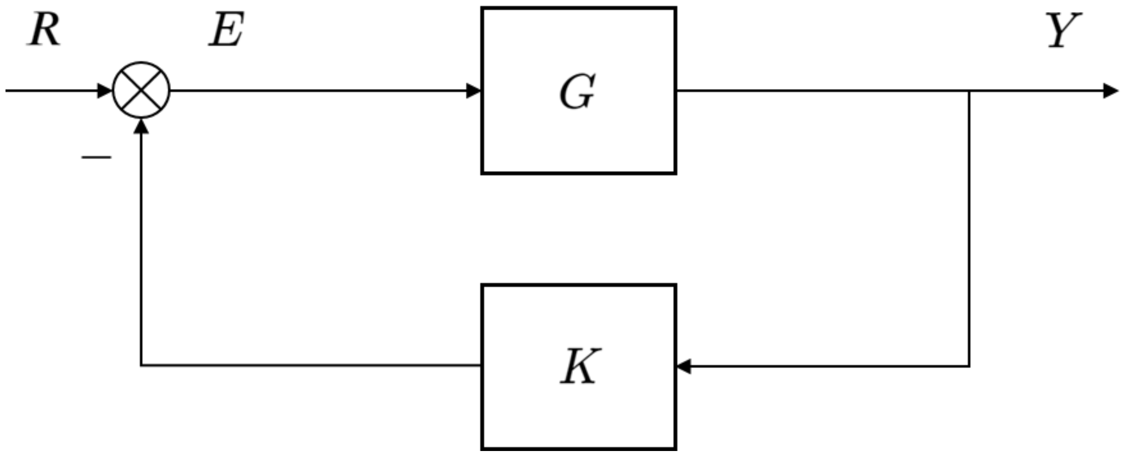


$$E = R - KY$$
$$Y = GE$$

$$\begin{aligned} Y &= G(R - KY) \\ &= GR - KGY \end{aligned}$$

$$(1 + KG)Y = GR$$

$$H = \frac{Y}{R} = \frac{G}{1 + KG}$$

Transfer Function for Closed Loop System

- Example 1

$$G(s) = \frac{1}{s+1}, \quad \text{pole at } -1$$

$$H(s) = \frac{KG}{1+KG} = \frac{\frac{K}{s+1}}{1+\frac{K}{s+1}} = \frac{K}{s+1+K}, \quad \text{new pole at } \ -(1+K)$$

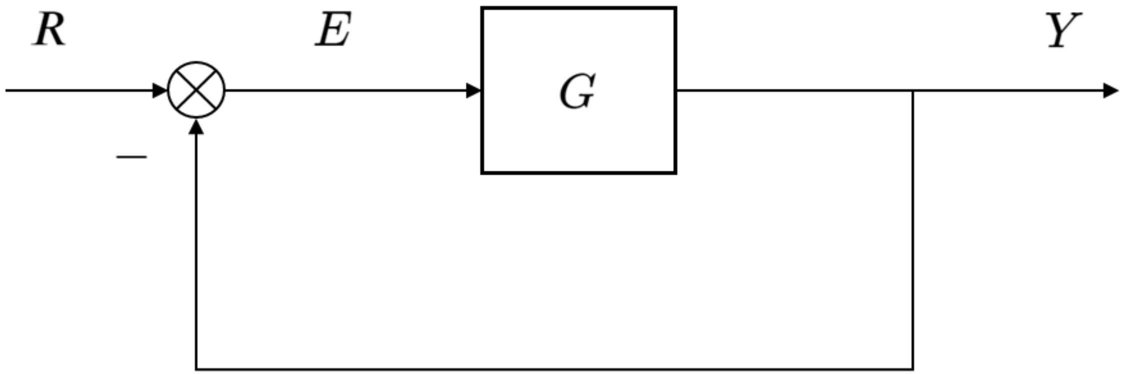
- Example 2

$$G(s) = \frac{1}{s-1}, \quad \text{pole at } +1, \text{unstable}$$

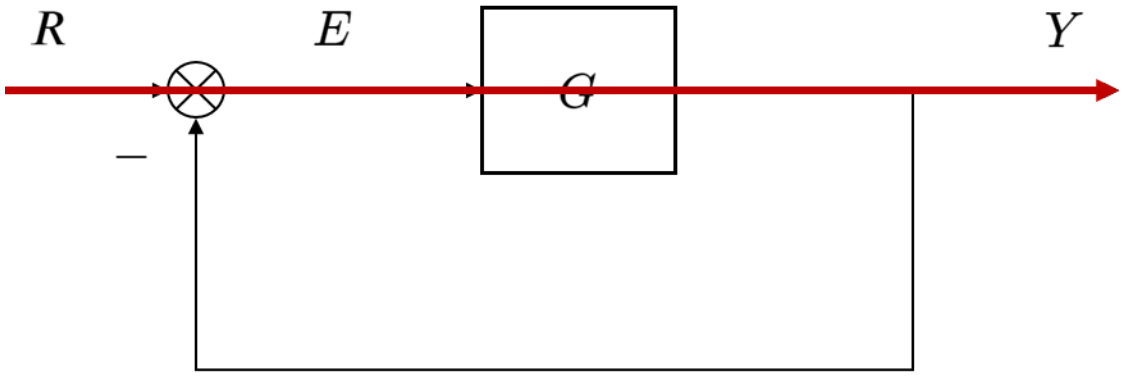
$$H(s) = \frac{KG}{1+KG} = \frac{\frac{K}{s-1}}{1+\frac{K}{s-1}} = \frac{K}{s-1+K}, \quad \text{new pole at } (1-K)$$

if $k > 1$, the closed loop system $H(s)$ becomes stable

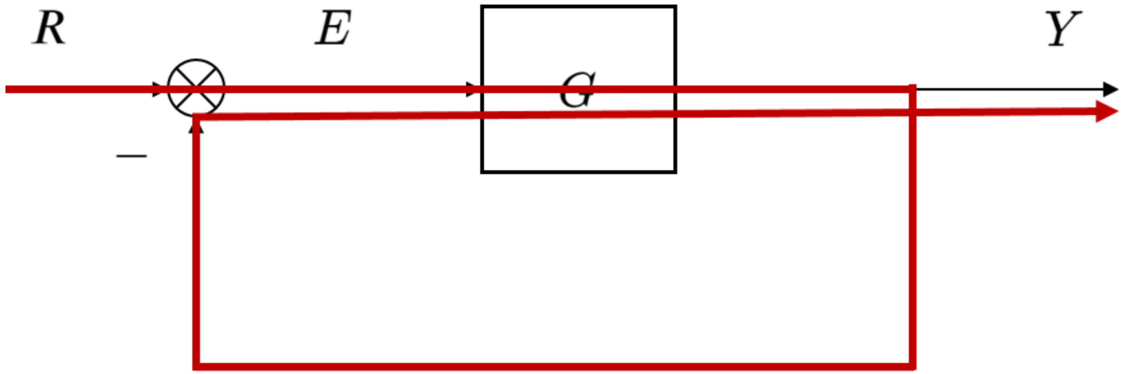
Visualize Feedback



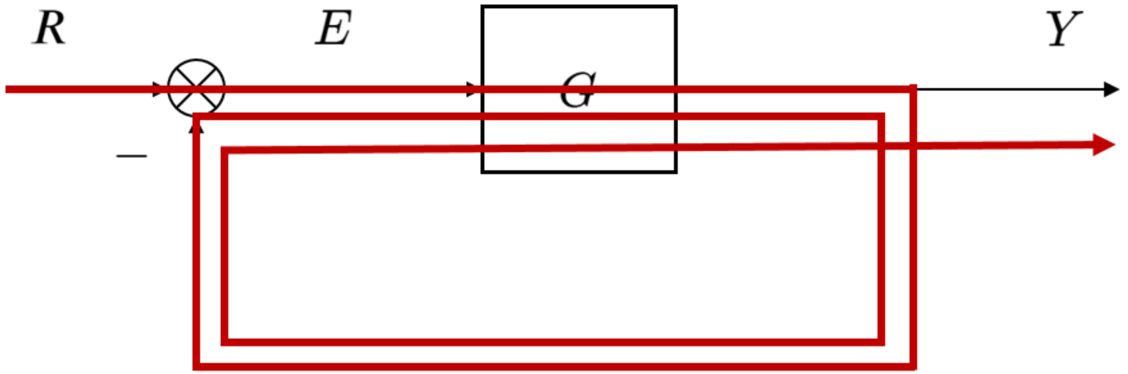
Visualize Feedback



Visualize Feedback

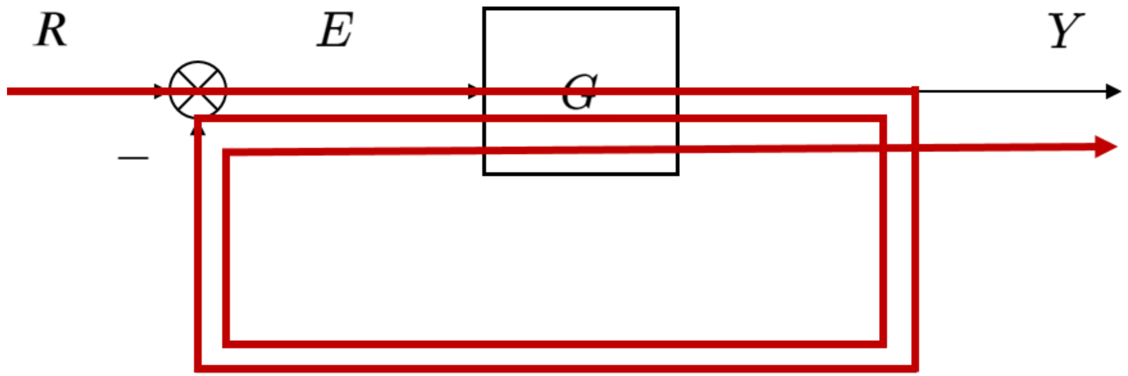


Visualize Feedback



Visualize Feedback

- Feedback = autoregressive = infinite length response



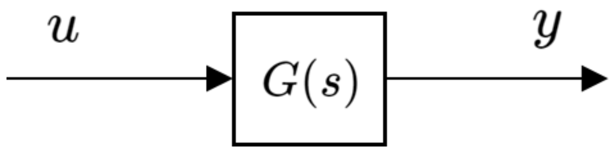
- If $G = \frac{1}{s}$

$$H = \frac{Y}{R} = \frac{1}{1+s} = 1 - s + s^2 - s^3 + \cdots$$

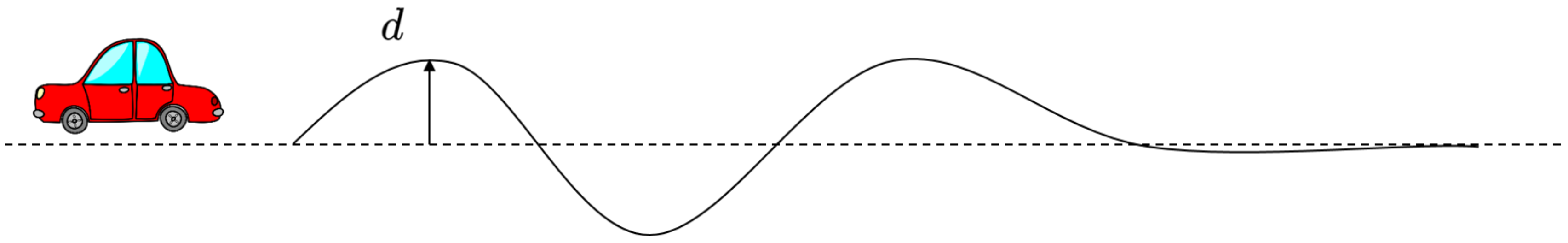
Open Loop vs. Closed Loop

Open Loop vs. Closed Loop

- Suppose that there is a car
  - input: force $u(t)$
  - output: velocity $y(t)$
  - transfer function $G(s)$

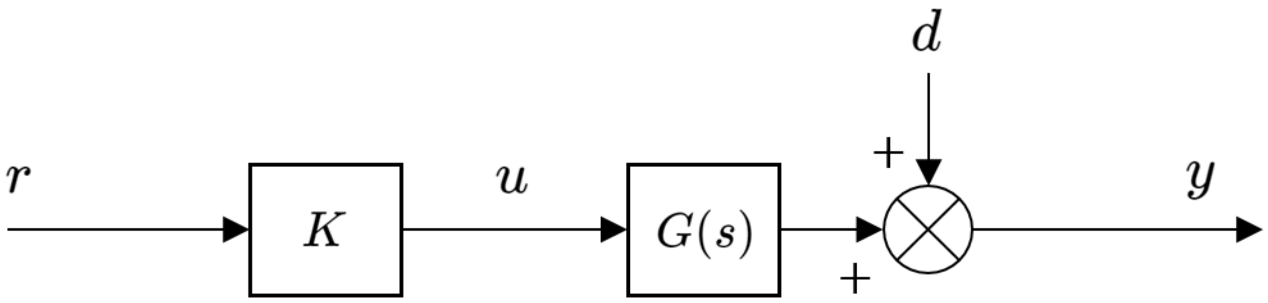


- Suppose that there is a car driving with a road wave disturbance d



Open Loop

- Reference velocity $r(t)$ that is desired by a driver
- Assume the force produced by a engine is $u = Kr$



- The system input u

$$u = Kr$$

- Calculating y

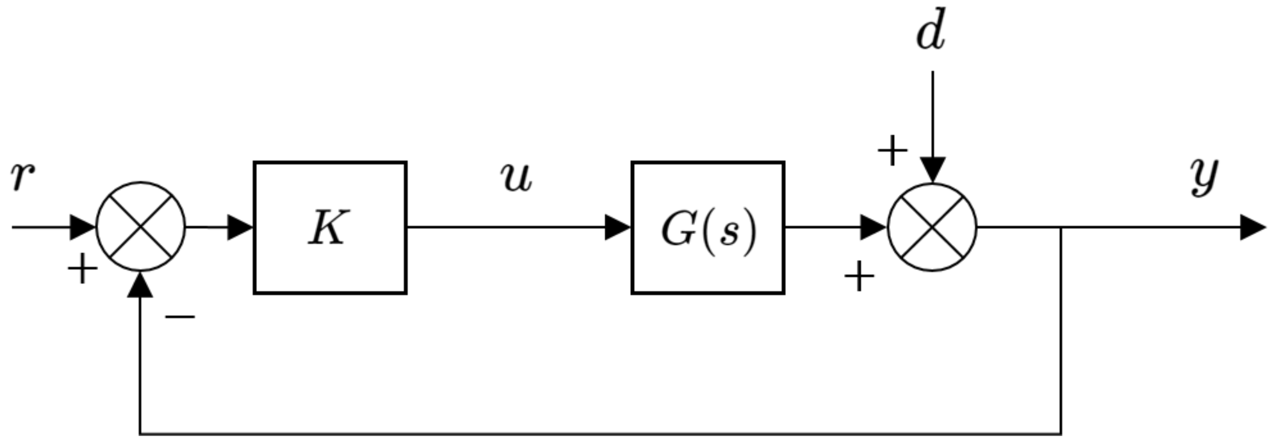
$$y = Gu + d = GKr + d$$

- Error

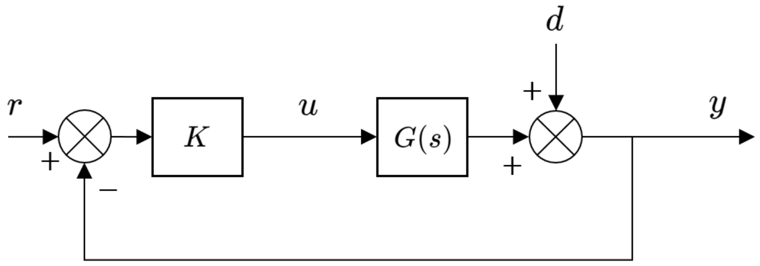
$$e = r - y = r - GKr - d$$
$$= r(1 - GK) - d$$

Closed Loop

- Think about how we drive
  - We step on the gas or put the brake on based on the desired speed and the current one
  - We care about the difference $r - y$
  - The term of “Negative feedback” is coming from $-y$



Closed Loop



- Now assume K is a controller
- The system input u

$$u = K(r - y)$$

- Calculating y

$$y = GK(r - y) + d$$

$$= KGr - KGy + d$$

$$(1 + KG)y = KGr + d$$

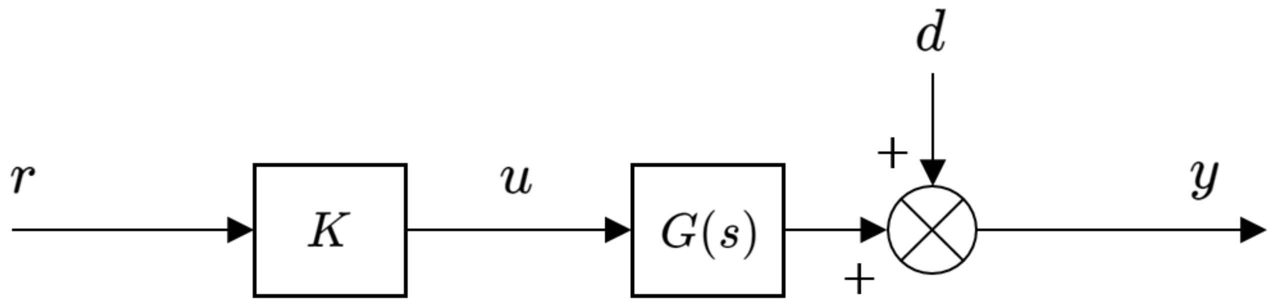
$$\therefore y = \frac{KGr + d}{1 + KG} = \frac{KG}{1 + KG}r + \frac{1}{1 + KG}d$$

- Error
  - When K is large, the error becomes small

$$e = r - y = r - \frac{KGr + d}{1 + KG} = \frac{r - d}{1 + KG}$$

Model Uncertainty of Open Loop

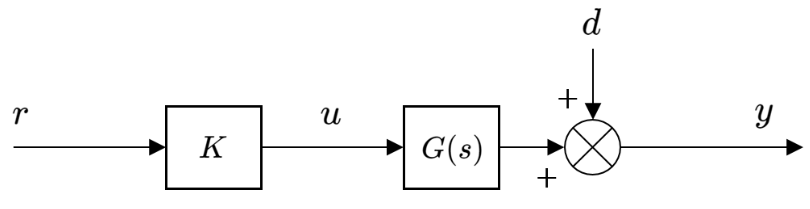
- Let us suppose that the predicted model is $G(s) = 2$, and actually $G(s) = 1$
- Let's design the K value when the desired output speed is $100\ km/h$



- In the open loop model, y is

$$y_{\text{model}} = G_{\text{model}} K r + d = 2Kr + d$$

Model Uncertainty of Open Loop



- In the open loop model, y is

$$y_{\text{model}} = G_{\text{model}} K r + d = 2Kr + d$$

- In order for y to approach 100 reference input

$$K = 0.5$$

- Then the actual y is

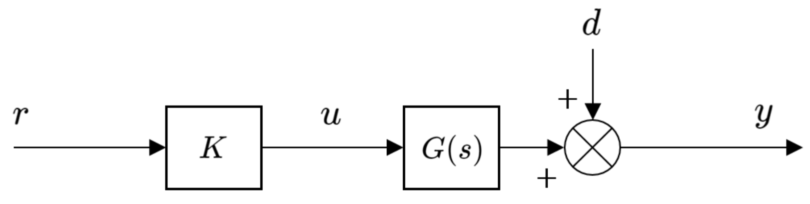
$$y_{\text{true}} = G_{\text{true}} K r + d = Kr + d$$

- The model and true errors are

$$e_{\text{model}} = r - y_{\text{model}} = r(1 - G_{\text{model}} K) - d = r(1 - 2K) - d$$

$$e_{\text{true}} = r - y_{\text{true}} = r(1 - G_{\text{true}} K) - d = r(1 - K) - d$$

Model Uncertainty of Open Loop

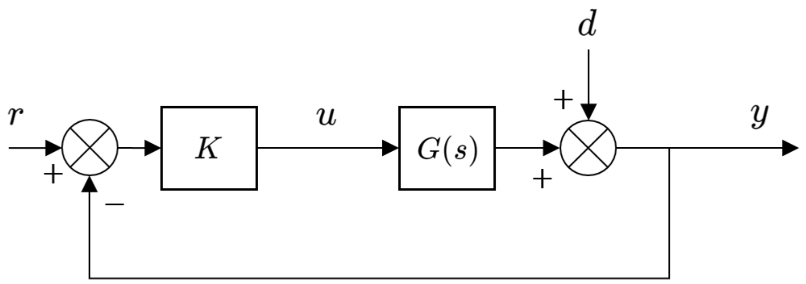


- Discrepancy from model uncertainty

$$y_{\text{model}} - y_{\text{true}} = Kr$$

- Stability
  - The open loop system cannot change the poles of system
- Uncertainty, Low Robustness
  - Predicting models incorrectly has a critical impact on speed
- Disturbance rejection
  - Disturbance directly affects the system

Model Uncertainty of Closed Loop



- In the closed loop model, y is

$$y_{\text{model}} = \frac{KG_{\text{model}}r + d}{1 + KG_{\text{model}}} = \frac{2Kr + d}{1 + 2K} = \frac{2K}{1 + 2K}r + \frac{1}{1 + 2K}d$$

- In order for y to approach 100 reference input, the larger K is better

$$K = 100$$

- Then the actual y is

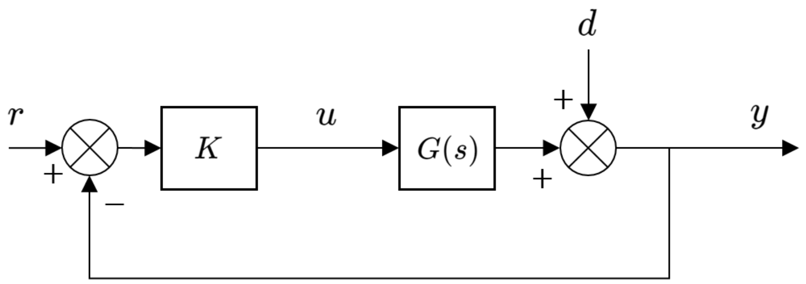
$$y_{\text{true}} = \frac{KG_{\text{true}}r + d}{1 + KG_{\text{true}}} = \frac{Kr + d}{1 + K} = \frac{K}{1 + K}r + \frac{1}{1 + K}d$$

- The model and true errors are

$$e_{\text{model}} = r - y_{\text{model}} = \frac{r - d}{1 + KG_{\text{model}}} = \frac{r - d}{1 + 2K} = \frac{1}{1 + 2K}r - \frac{1}{1 + 2K}d$$

$$e_{\text{true}} = r - y_{\text{true}} = \frac{r - d}{1 + KG_{\text{true}}} = \frac{r - d}{1 + K} = \frac{1}{1 + K}r - \frac{1}{1 + K}d$$

Model Uncertainty of Closed Loop



- Discrepancy from model uncertainty (assume $d = 0$) is

$$y_{\text{model}} - y_{\text{true}} = \frac{K}{2K^2 + 3K + 1} r \approx 0$$

- Stability
  - The closed loop system can change the poles of system

- Uncertainty, Robustness
  - Model uncertainty has a reduced impact on speed

- Disturbance rejection
  - Disturbance little affects the system

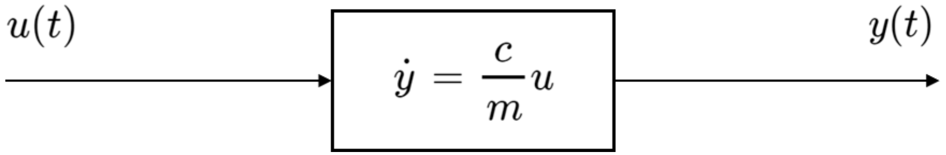
PID Control

Car Model

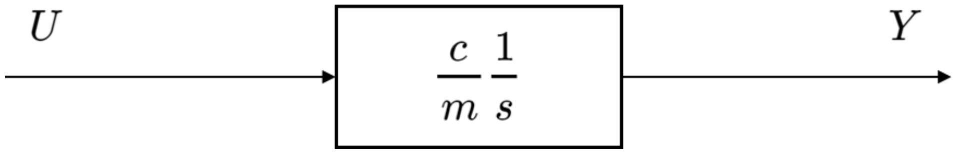
- For the car model
  - velocity y
  - input force u

$$\dot{y} = \frac{c}{m}u$$

- In a block diagram



- In a Laplace transform



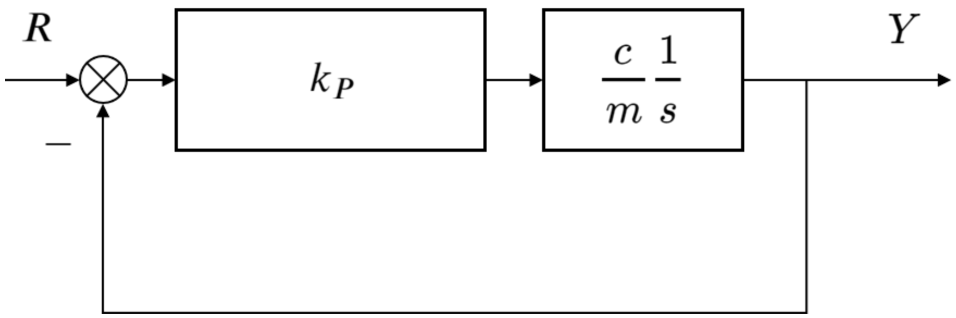
- We want to achieve

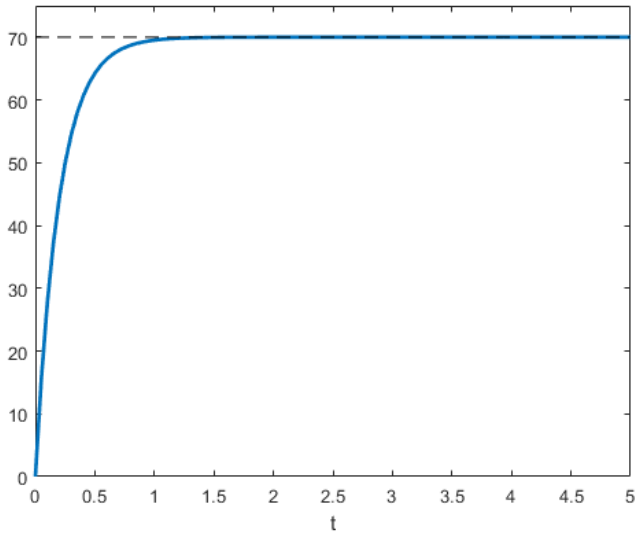
$$y \to r \quad \text{as} \quad t \to \infty \ (e = r - y \to 0)$$

P Control

- The proportional term produces an output value that is proportional to the current error value.
  - Small error yields small control signals
  - Nice and smooth
  - So-called proportional regulation (P regulator)

$$u = k_P e$$



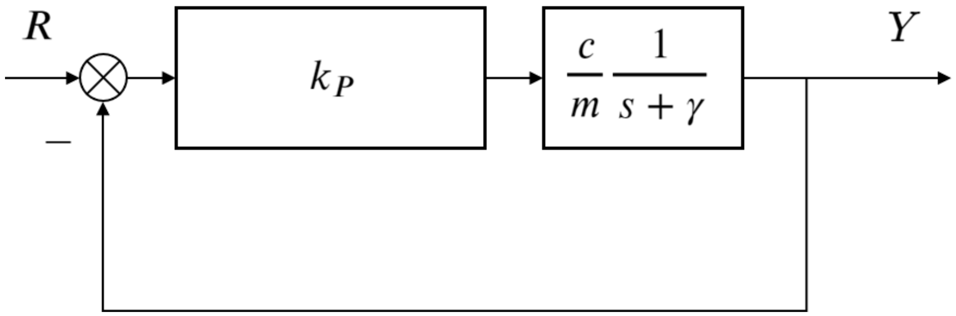


Model Uncertainty

- Caveat: the "real" model is augmented to include a wind resistance term:

$$\dot{y} = \frac{c}{m}u - \gamma y$$

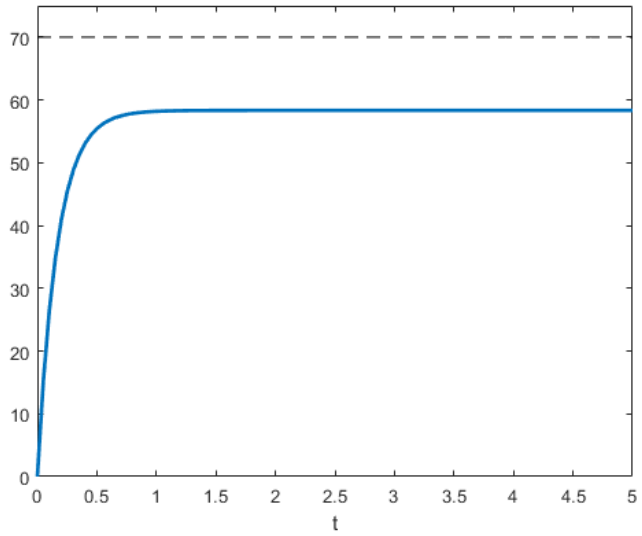
$$u = k_P e = k_P(r - y)$$



- At steady-state

$$\dot{y} = 0 = \frac{c}{m}u - \gamma y = \frac{c}{m}k_P(r - y) - \gamma y$$

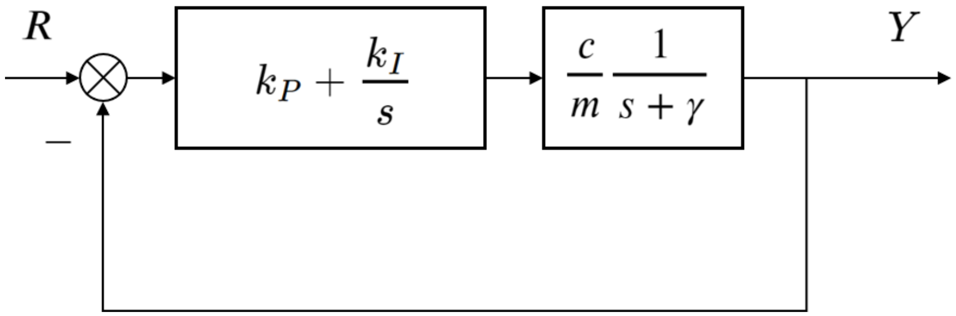
$$\implies y = \frac{ck_P}{ck_P + m\gamma}r$$

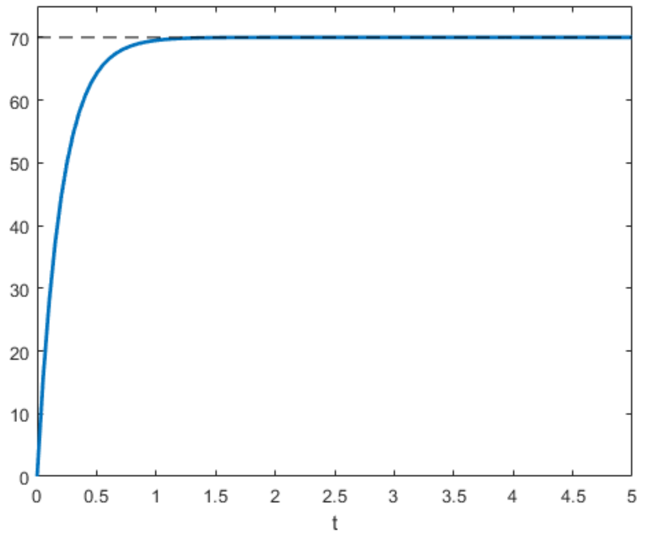


PI Control

- The integral controller is the sum of the instantaneous error over time and gives the accumulated offset that should have been corrected previously
  - Stability
  - Tracking
  - Robustness

$$u(t) = k_P\, e(t) + k_I \int_0^t e(\tau) d\tau$$

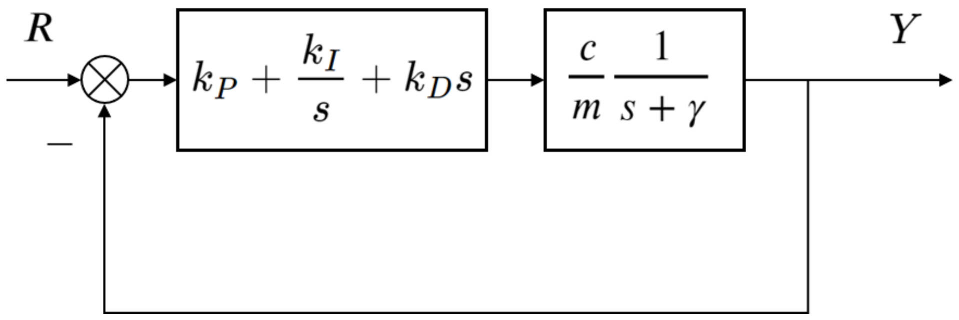




PID Control

- PID: by far the most used low-level controller
  - P: contributes to stability, medium-rate responsiveness
  - I: tracking and disturbance rejection, slow-rate responsive, may cause oscillations
  - D: fast-rate responsiveness, sensitive to noise

$$u(t) = k_P\, e(t) + k_I \int_0^t e(\tau) d\tau + k_D \frac{de(t)}{dt}$$



- Feedback has a remarkable ability to fight uncertainty in model parameters !

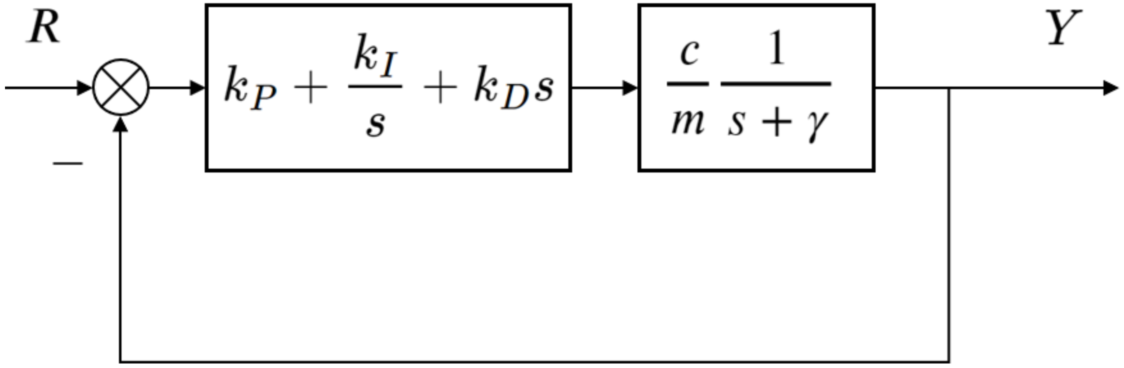
PID Control

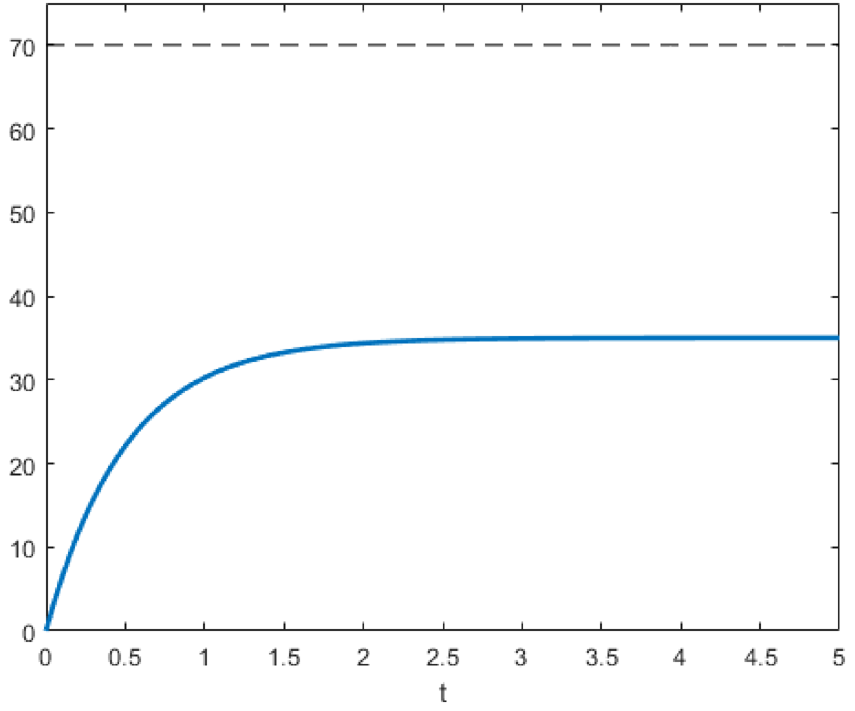
- Feedback has a remarkable ability to fight uncertainty in model parameters !

- The goal of this problem is to show how each of the term, k_P, k_I and k_D contributes to obtaining the common goals of:
  - Fast rise time
  - Minimal overshot
  - Zero steady-state error

PID Gains

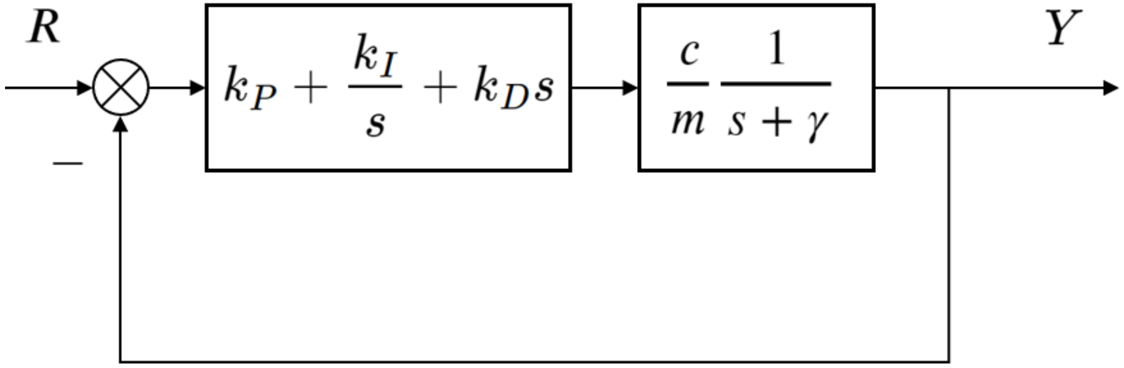
$$k_P = 1, k_I = 0, k_D = 0$$

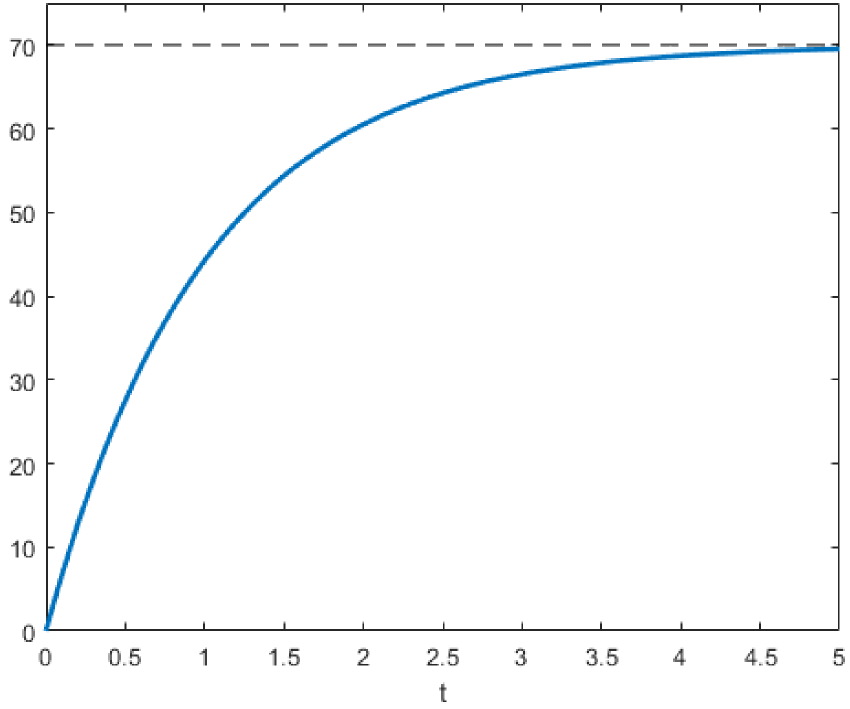




PID Gains

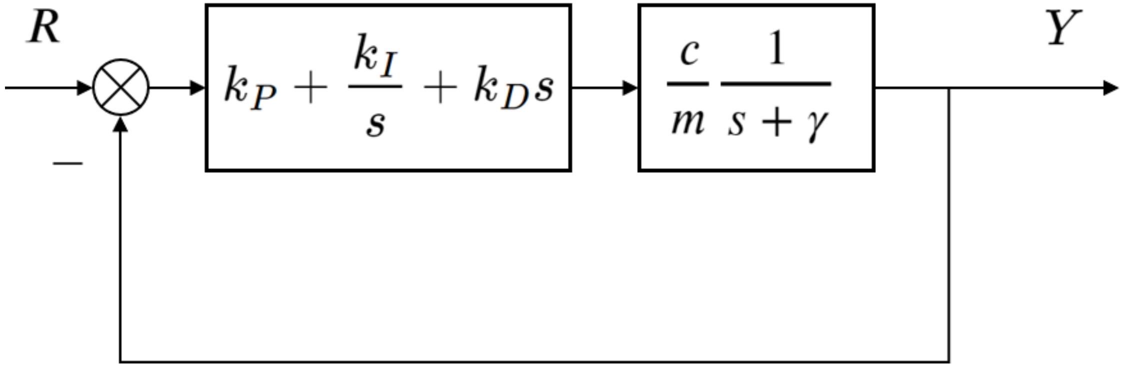
$k_P = 1, k_I = 1, k_D = 0$

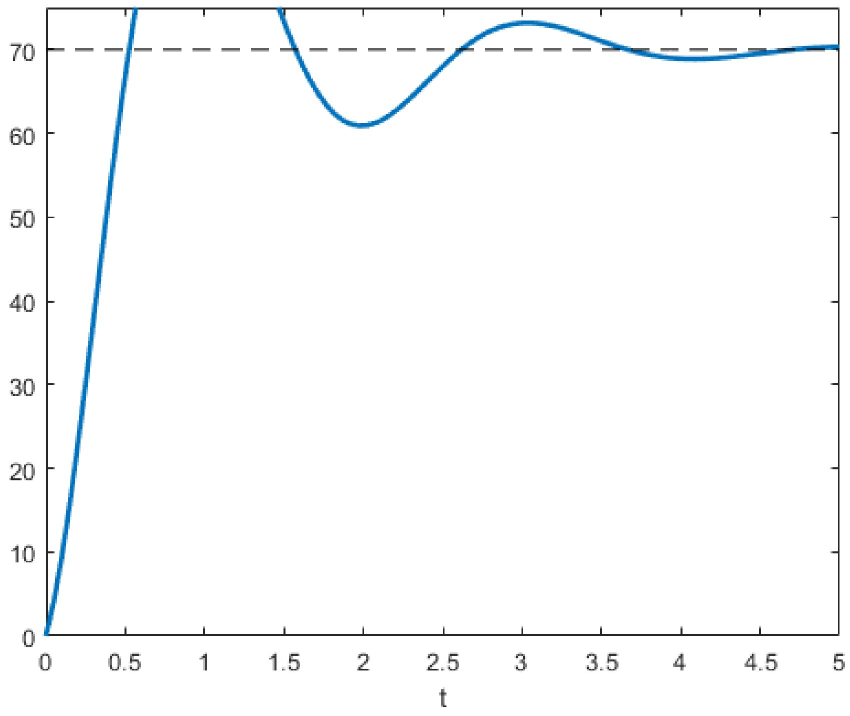




PID Gains

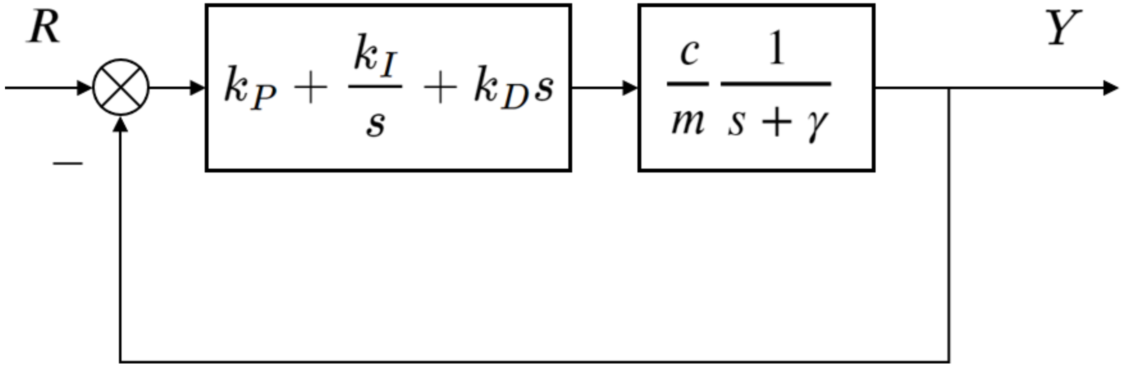
$k_P = 1, k_I = 10, k_D = 0$

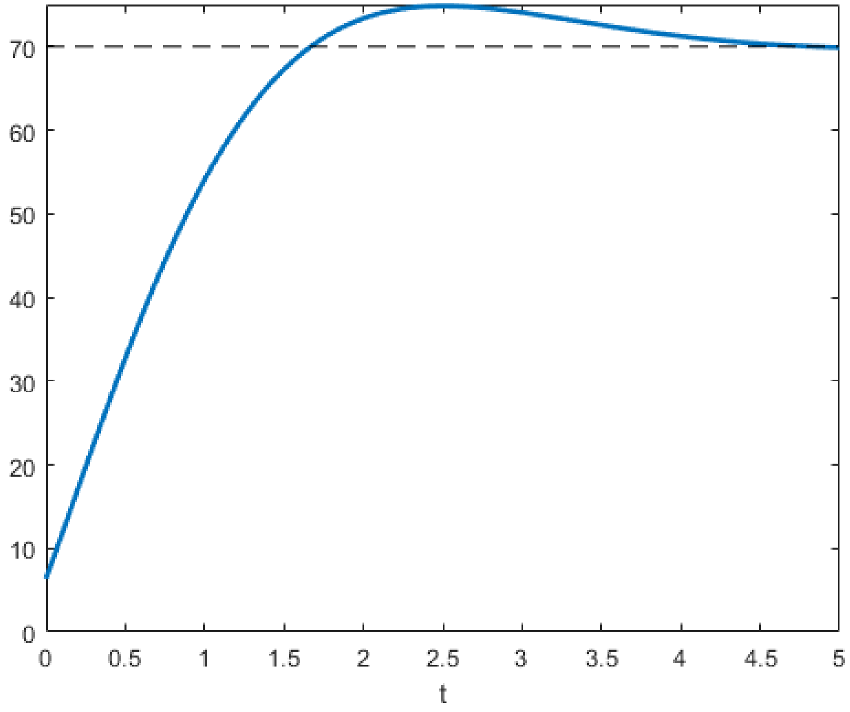




PID Gains

$$k_P = 1, k_I = 2, k_D = 0.1$$





General Tips for Designing a PID Controller

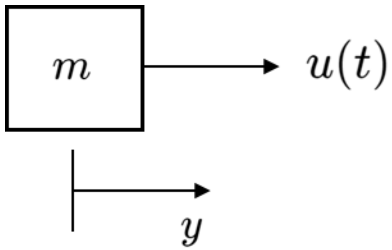
- When you are designing a PID controller for a given system, follow the steps shown below to obtain a desired response.
  - Obtain an open-loop response and determine what needs to be improved
  - Add a proportional control to improve the rise time
  - Add a derivative control to reduce the overshoot
  - Add an integral control to reduce the steady-state error
  - Adjust each of the gains k_P, k_I and k_D until you obtain a desired overall response.

- Lastly, please keep in mind that you do not need to implement all three controllers (proportional, derivative, and integral) into a single system, if not necessary. For example, if a PI controller meets the given requirements (like the above example), then you don't need to implement a derivative controller on the system. Keep the controller as simple as possible.

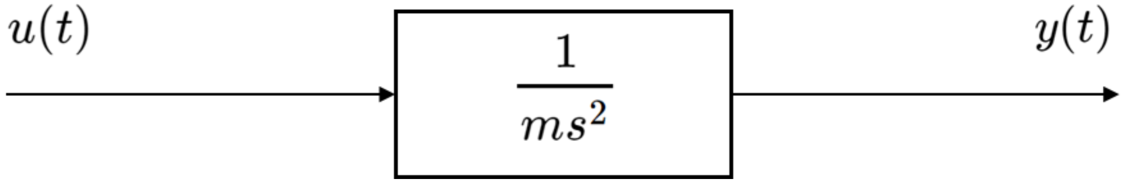
MKC System as Closed Loop

MKC System as Closed Loop

- Mass

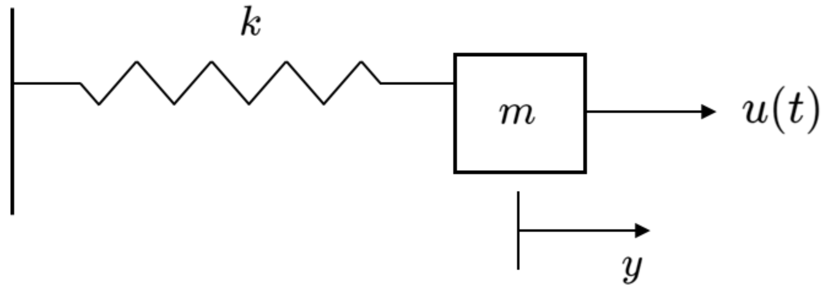


$$m\ddot{y} = u$$

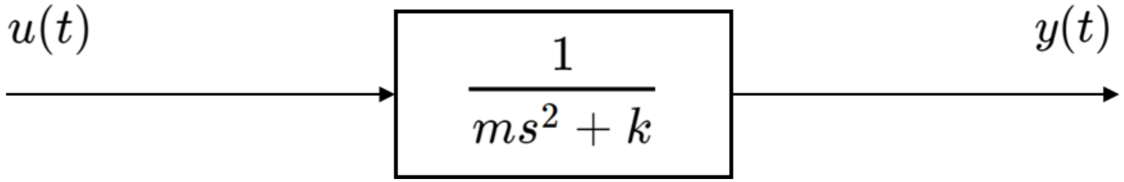


MKC System as Closed Loop

- Mass and spring

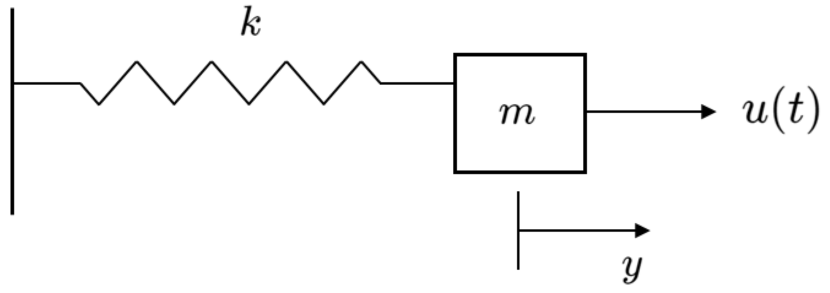


$$m\ddot{y} + ky = u$$



MKC System as Closed Loop

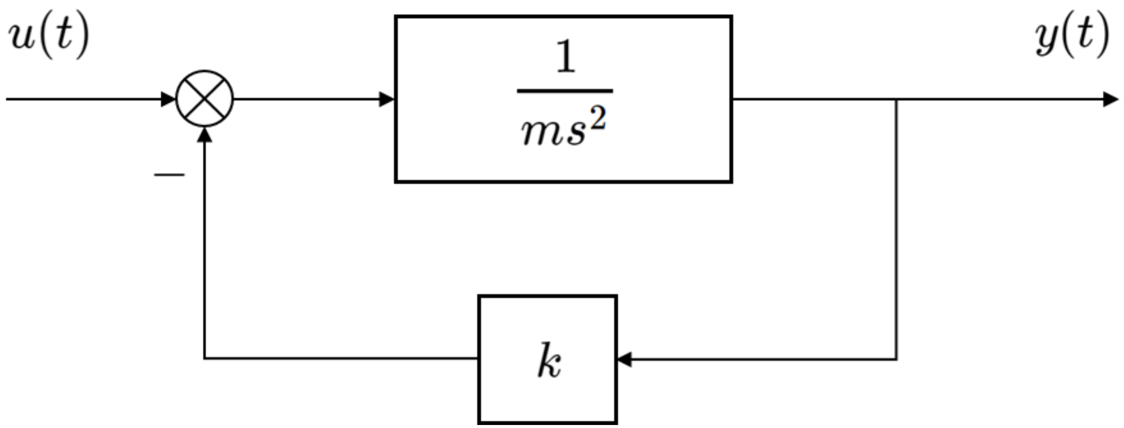
- Mass and spring



$$m\ddot{y} + ky = u$$

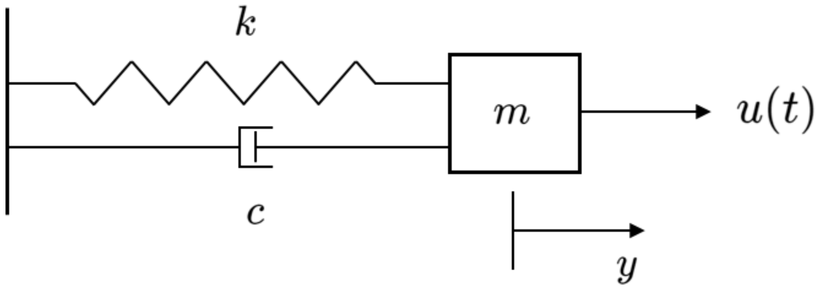
$$\Downarrow$$

$$m\ddot{y} = u - ky$$

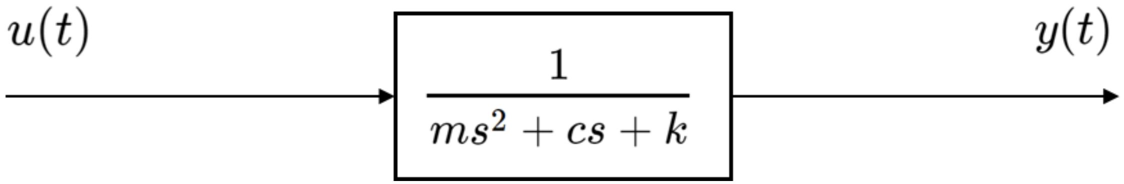


MKC System as Closed Loop

- Mass, spring and damper



$$m\ddot{y} + c\dot{y} + ky = u$$



MKC System as Closed Loop

- Mass, spring and damper

$$m\ddot{y} + c\dot{y} + ky = u \quad \Rightarrow \quad m\ddot{y} = u - ky - c\dot{y}$$