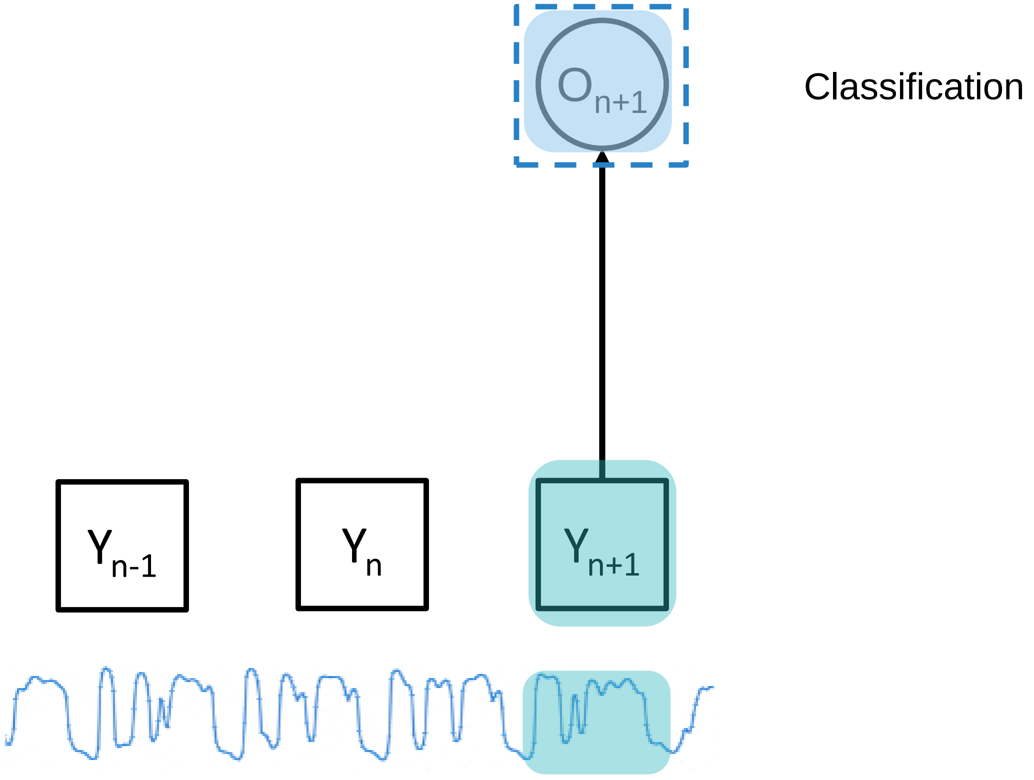
Neural Network Architectures for Time-Series: Recurrent Neural Network (RNN)

Prof. Seungchul Lee
Industrial AI Lab.

Most slides from CMU 11-785 Intro. to Deep Learning by Prof. Bhiksha Raj

Recurrent NN (RNN)

- Hidden state extraction and transformation



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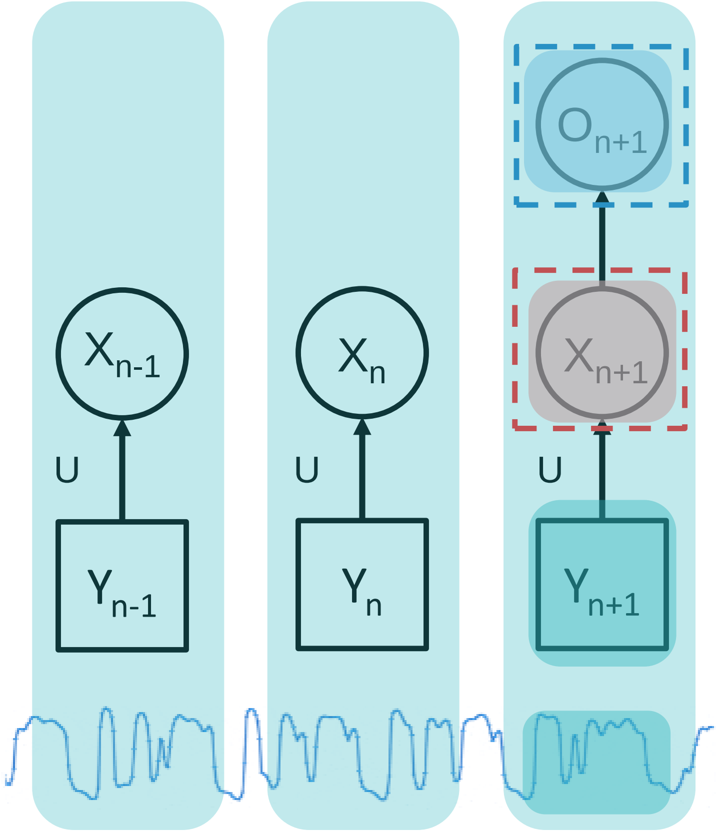
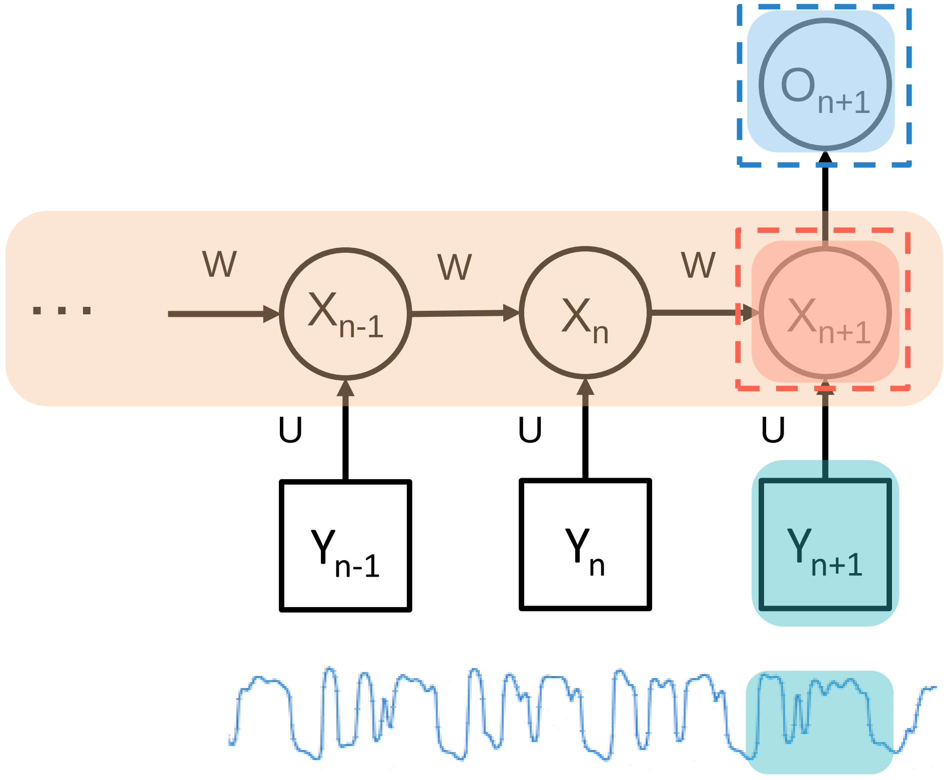


Classification based on states

Learned latent state

Recurrent NN (RNN)

- Hidden state extraction and transformation
- Good for sequential data (dynamic behavior)



Classification based on states

Learned latent state and its dynamics

Recurrent NN

- Recurrence
  - Consider the classical form of a dynamical system:

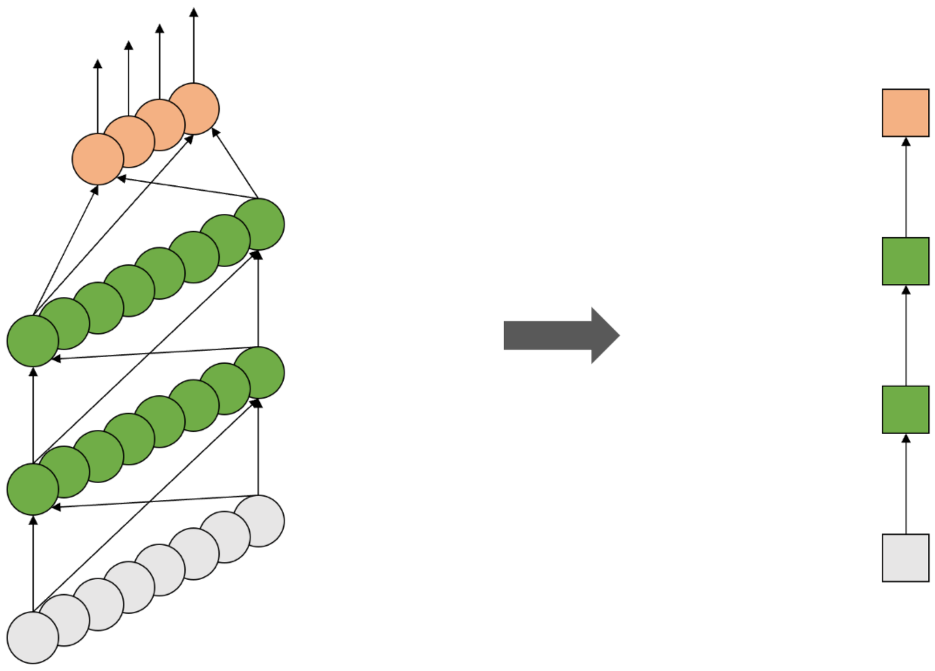
$$s^{(t)} = f(s^{(t-1)}; \theta)$$

  - This is recurrent because the definition of s at time t refers back to the same definition at time $t-1$

- Hidden state representation

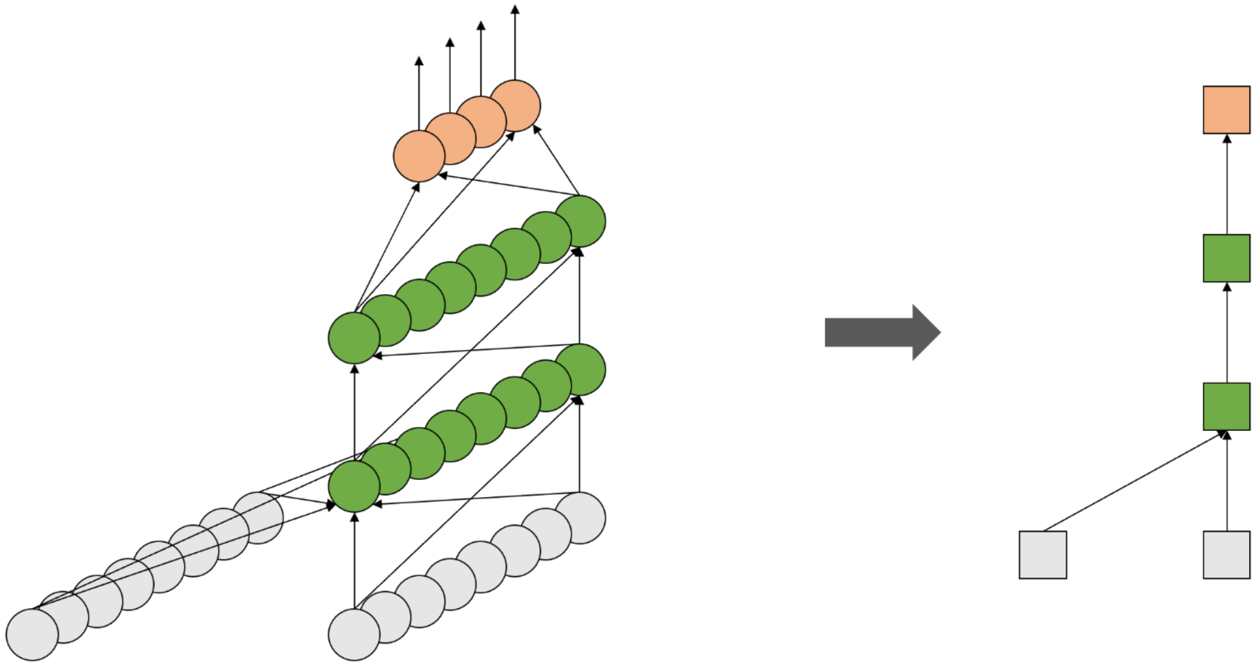
- Learn both from sequential data

Representation Shortcut



- Input at each time is a vector
- Each layer has many neurons
  - Output layer too may have many neurons
- But will represent everything simple boxes
  - Each box actually represents an entire layer with many units

Representation Shortcut



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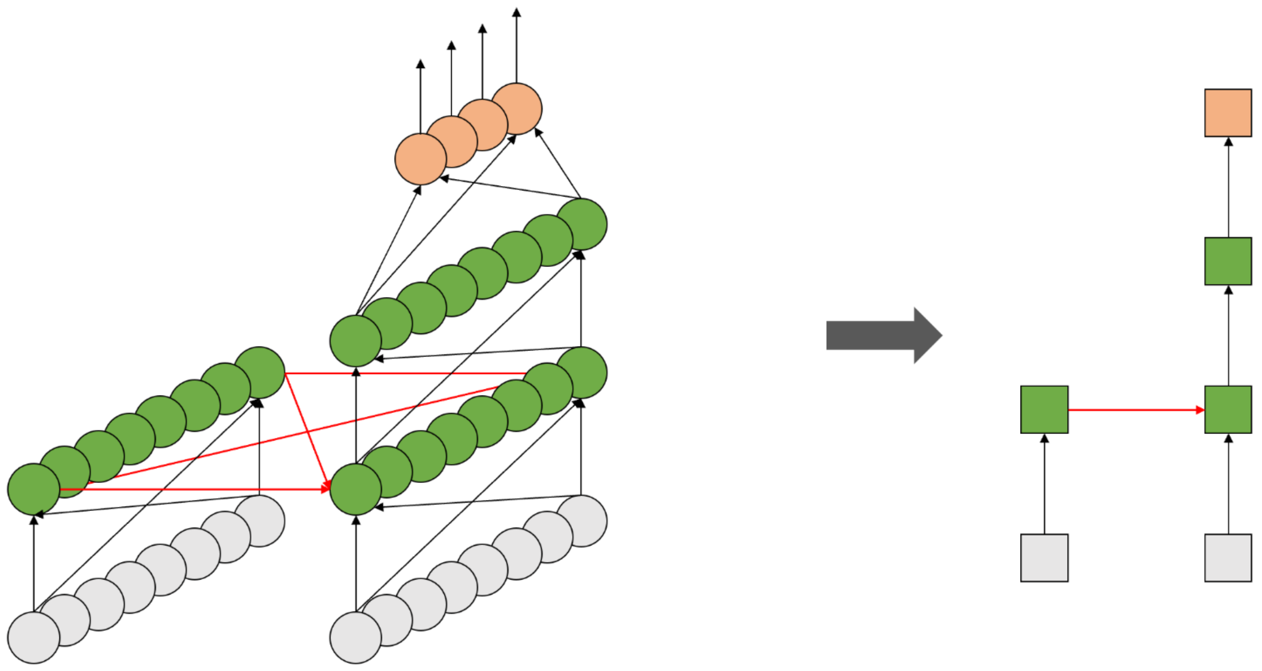

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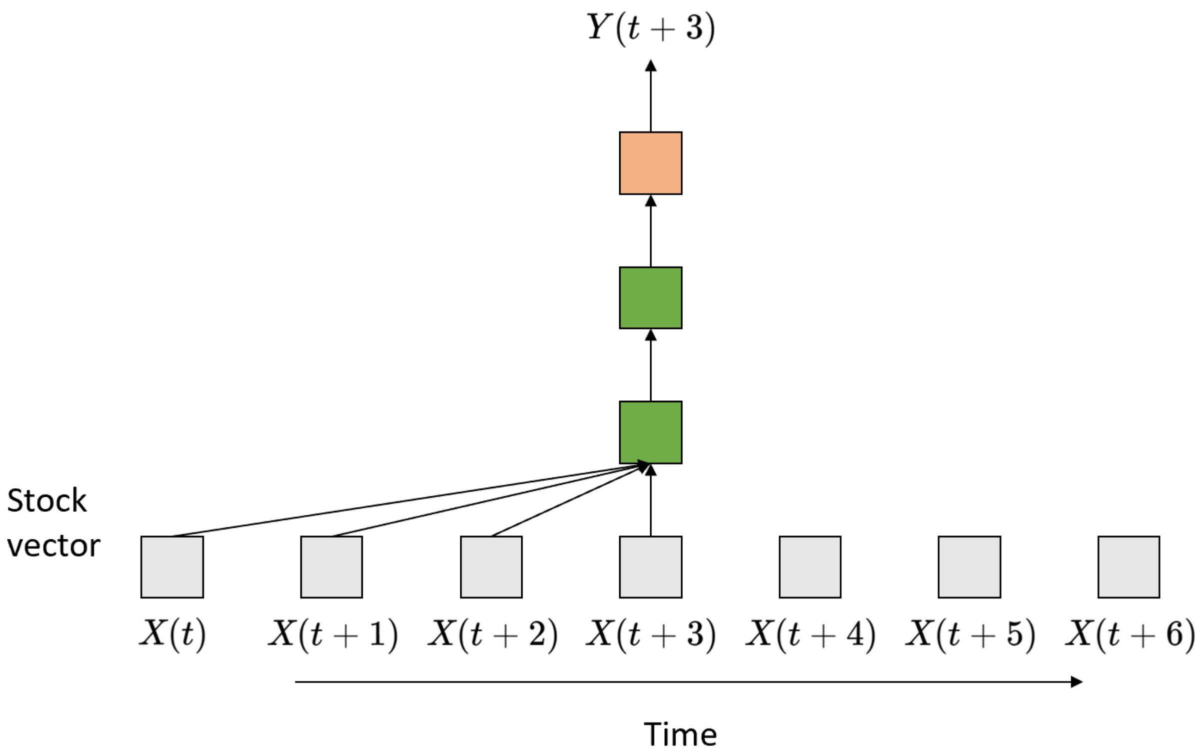
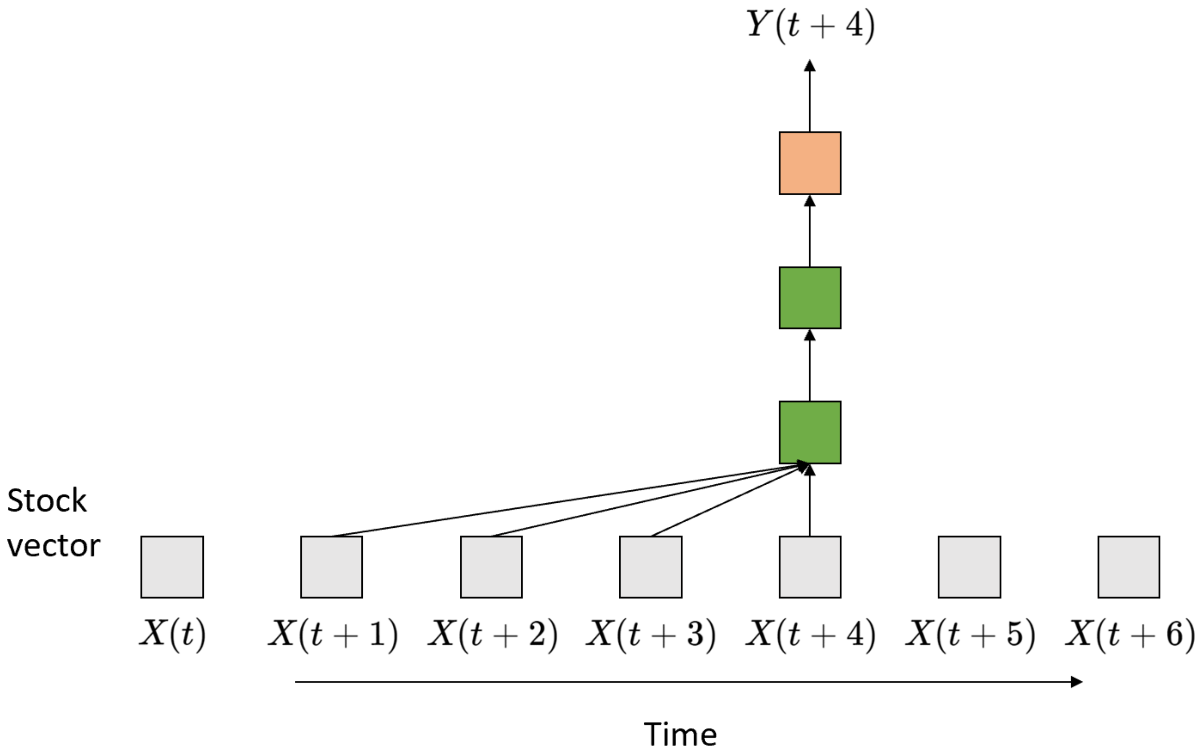
Sliding Predictor



- The sliding predictor
  - Look at the last few days
  - This is just a convolutional neural net applied to sequential data

Sliding Predictor



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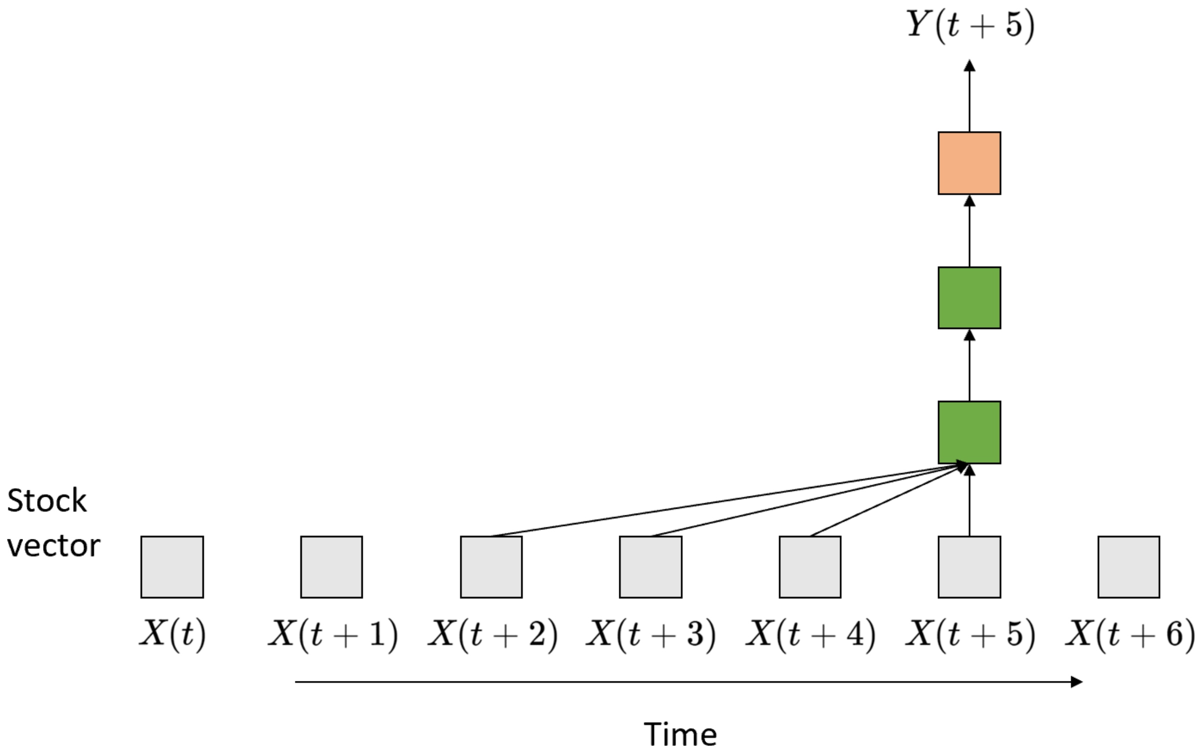

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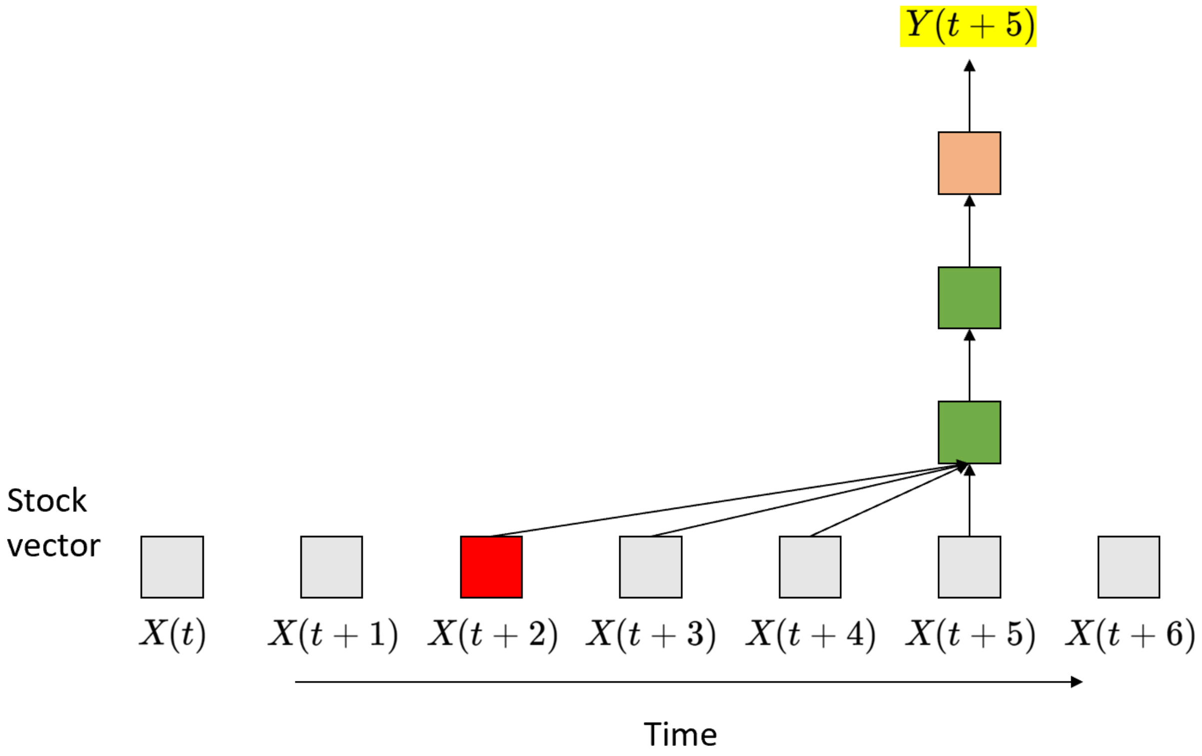

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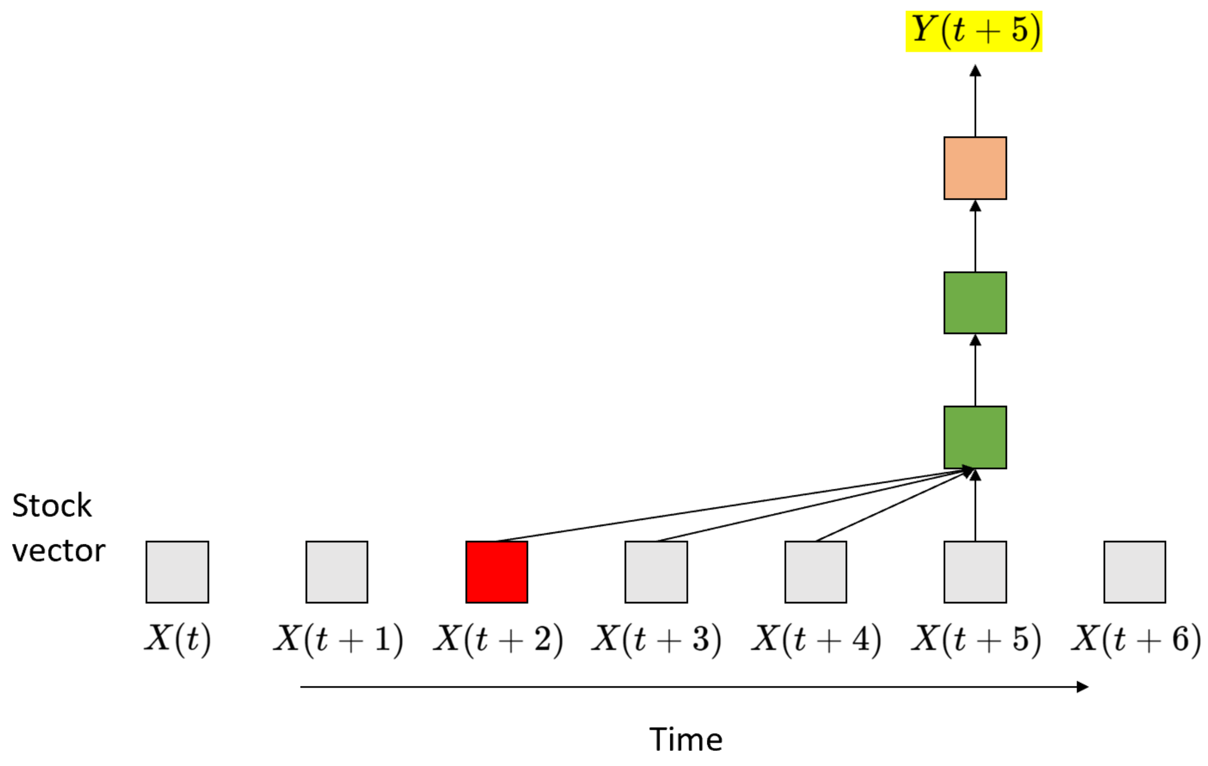
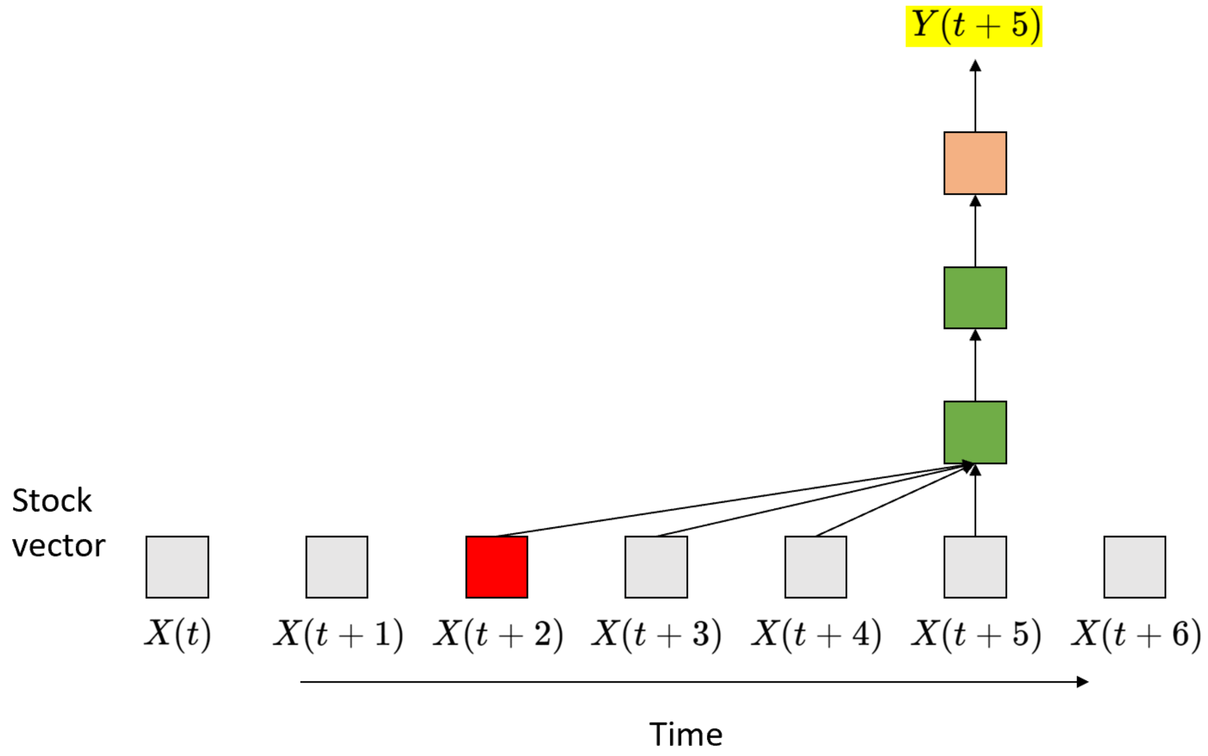
Finite-Response Model



- This is a finite response system
  - Something that happens today only affects the output of the system for N days into the future
  - N is the width of the system

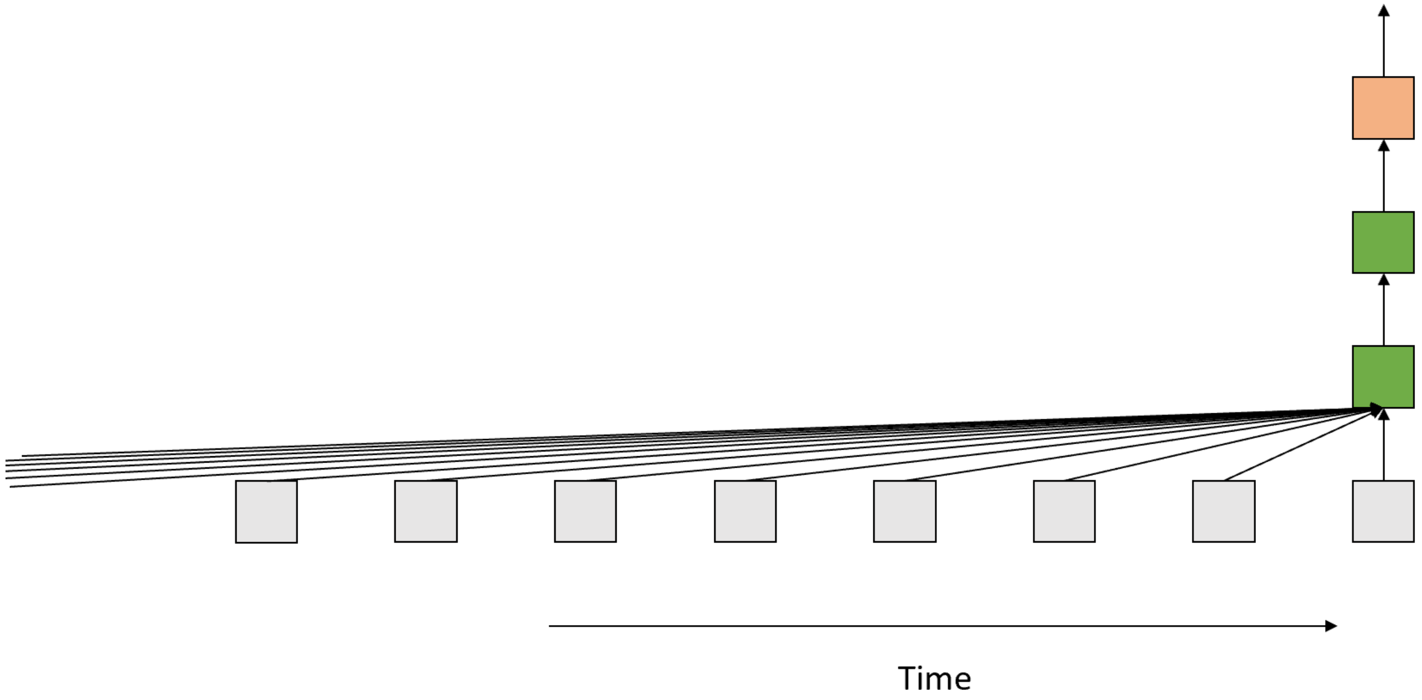
Finite-Response Model



- Problem: Increasing the "history" makes the network more complex

$$Y_t = f\left(X_t, X_{t-1}, \cdots, X_{t-N}\right)$$

In Theory, We Want Infinite Memory



- Required: Infinite response systems
  - What happens today can continue to affect the output forever
  - Possibly with weaker and weaker influence

$$Y_t = f\left(X_t, X_{t-1}, \cdots, X_{t-\infty}\right)$$

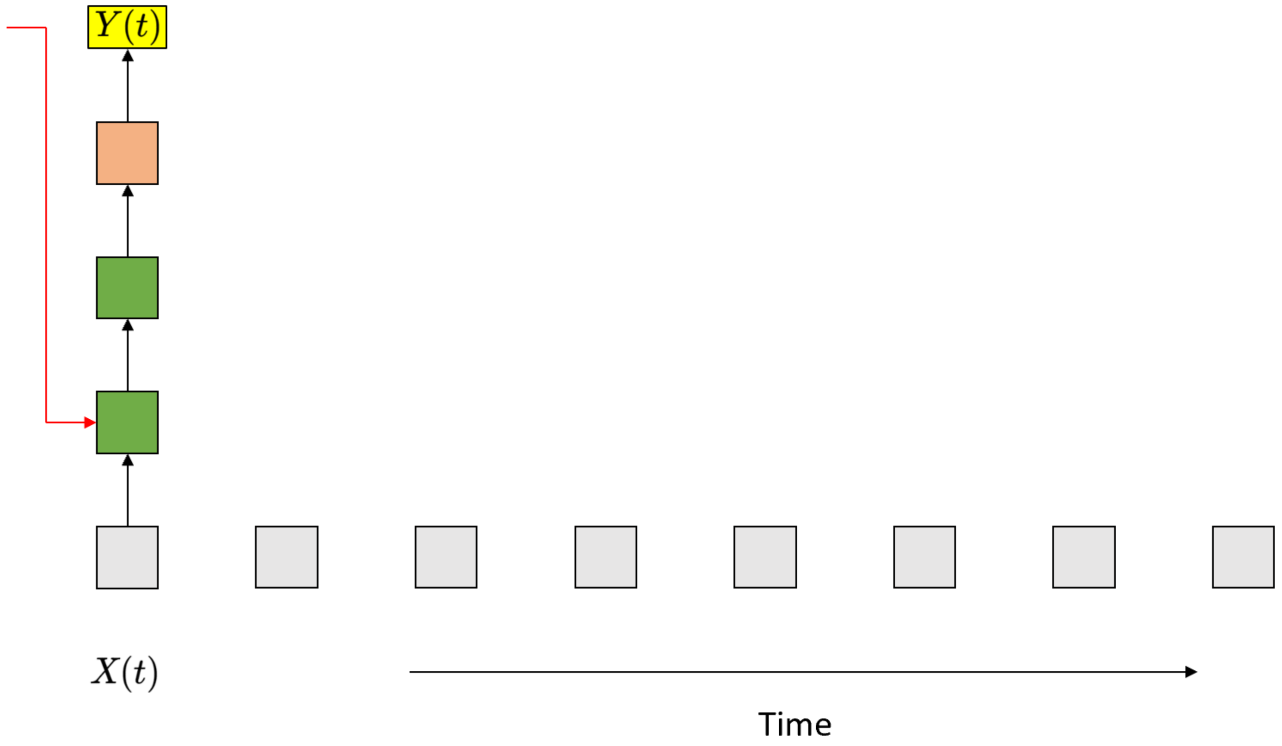
Infinite Response Systems

$$Y_t = f\left(X_t, X_{t-1}, \cdots, X_{t-\infty}\right) \quad \Longrightarrow \quad Y_t = f\left(X_t, Y_{t-1}\right)$$

- Recursive
  - Required: Define initial output: Y_{t-1} for $t = 0$
  - An input at X_0 at $t = 0$ produces Y_0
  - Y_0 produces Y_1 which produces Y_2 and so on until Y_∞ even if $X_1, \cdots, X_\infty$ are 0
  - Nonlinear autoregressive

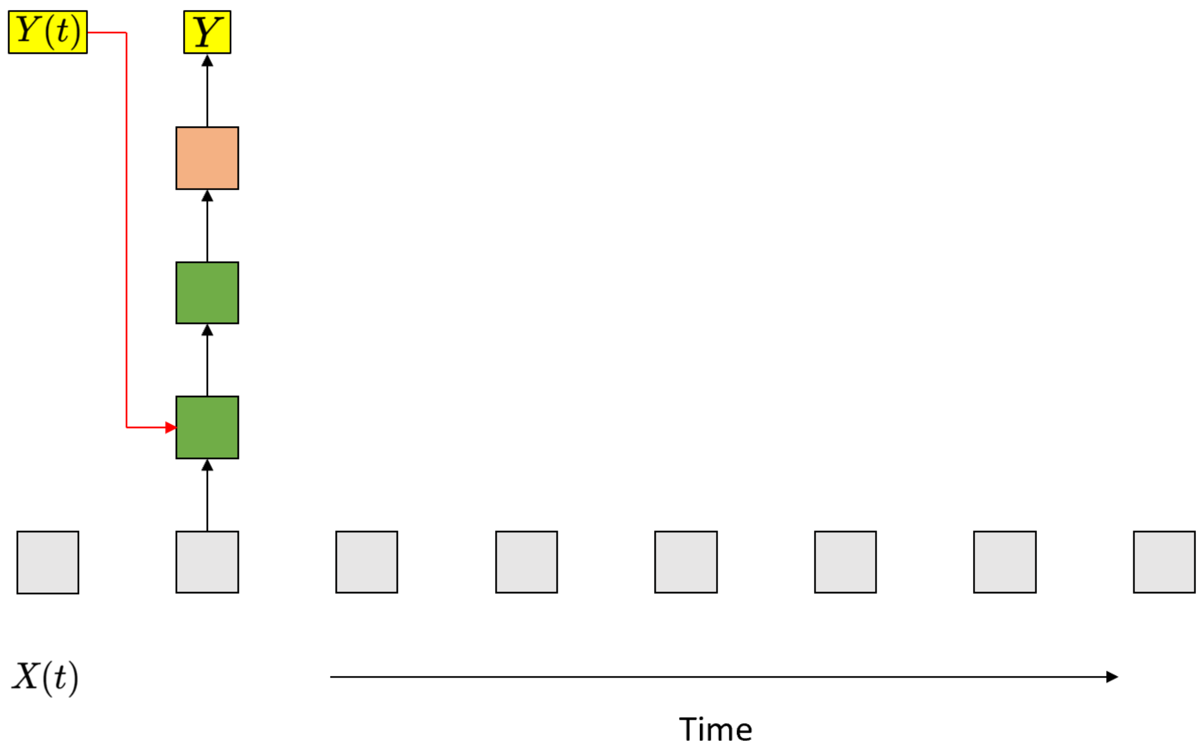
- Output contains information about the entire past

Autoregression



- An autoregressive net with recursion from the output

Autoregression



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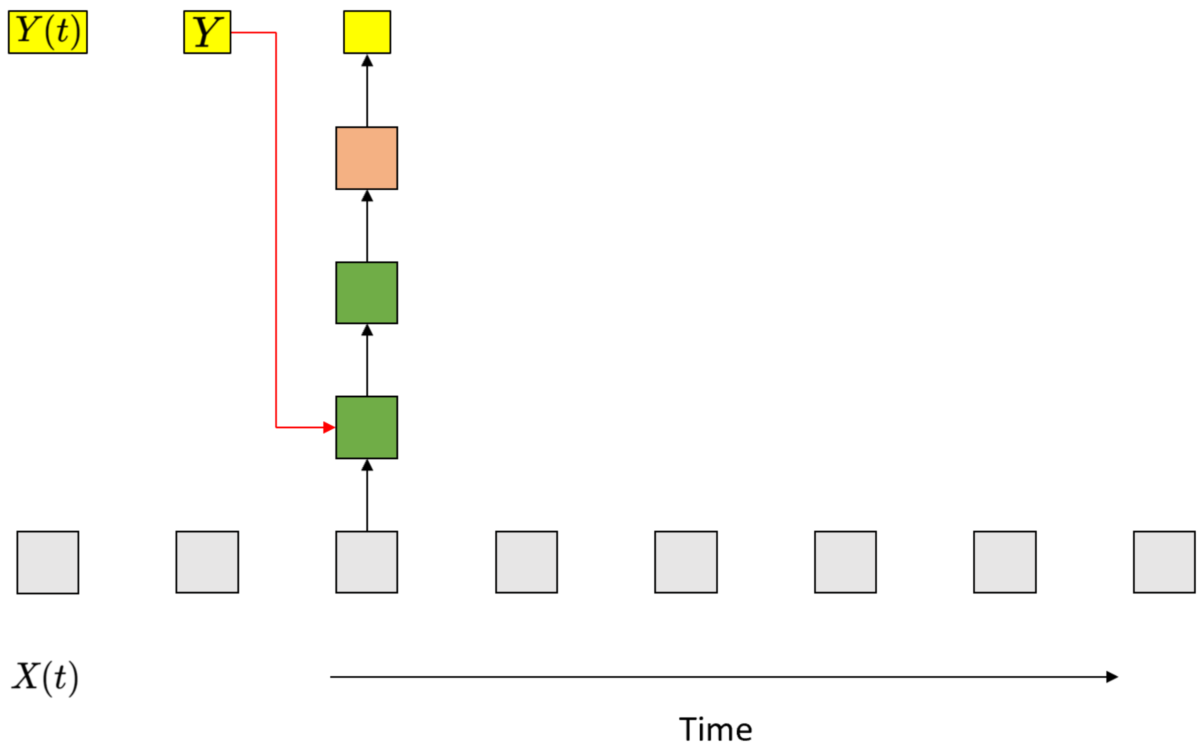

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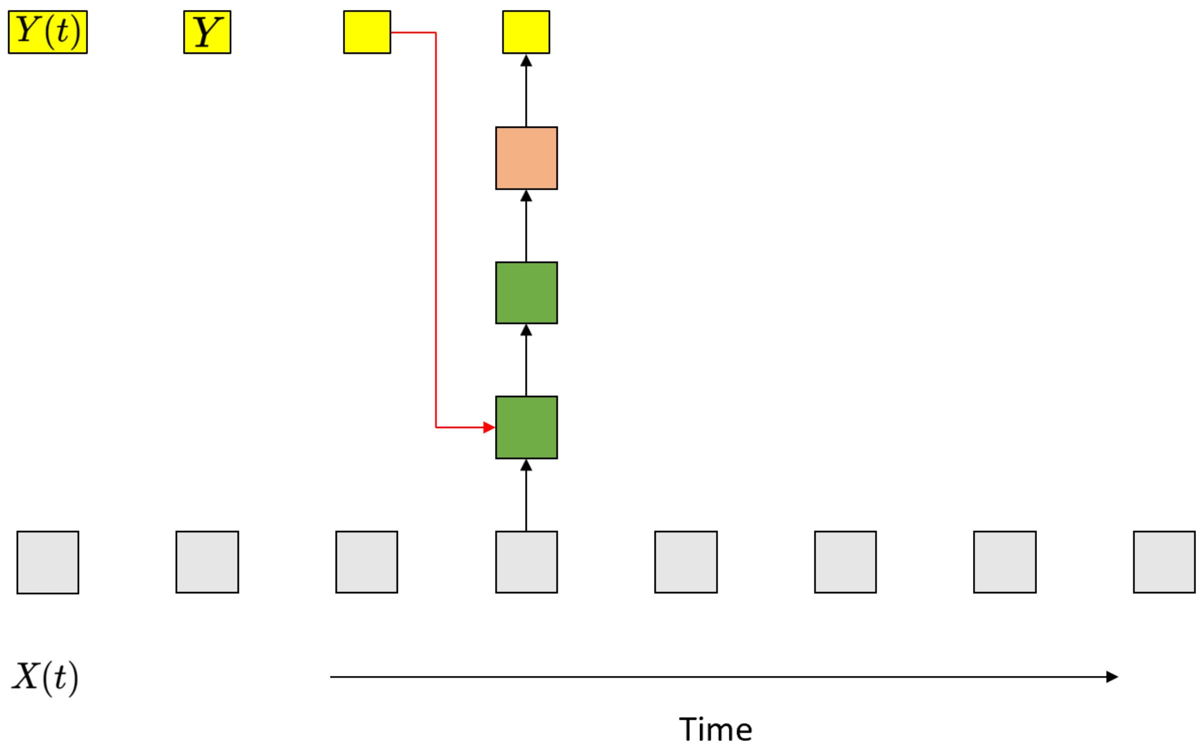

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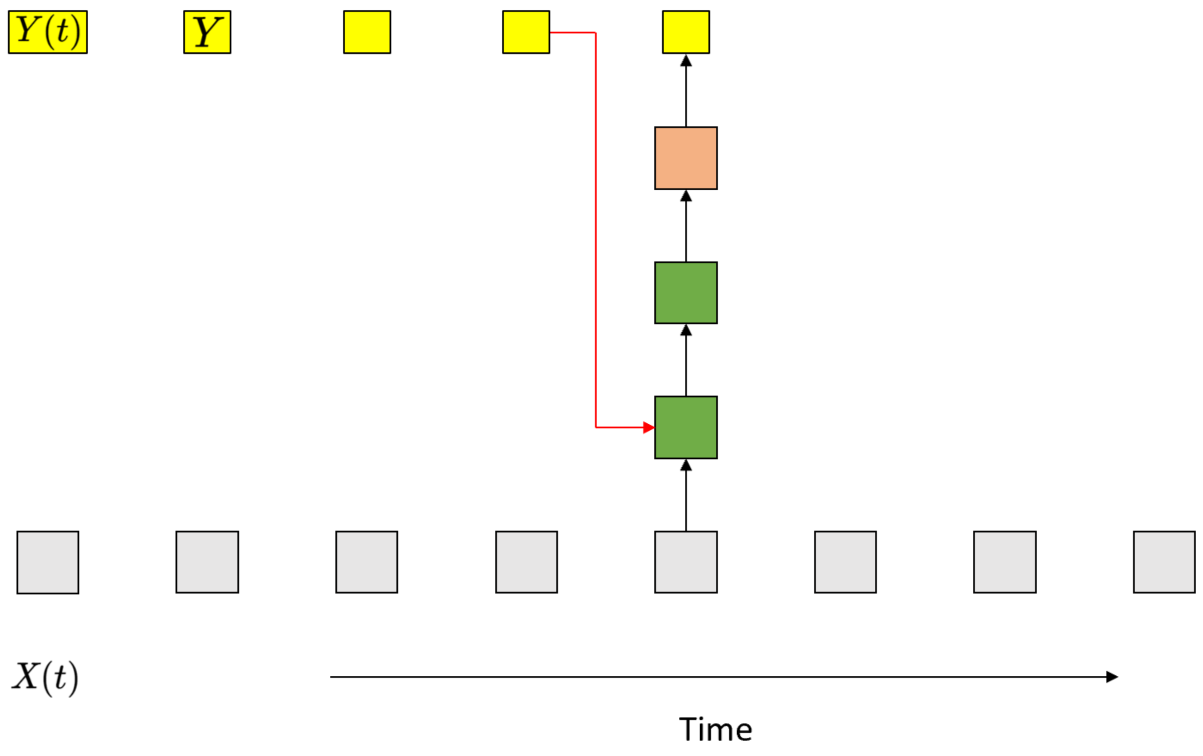

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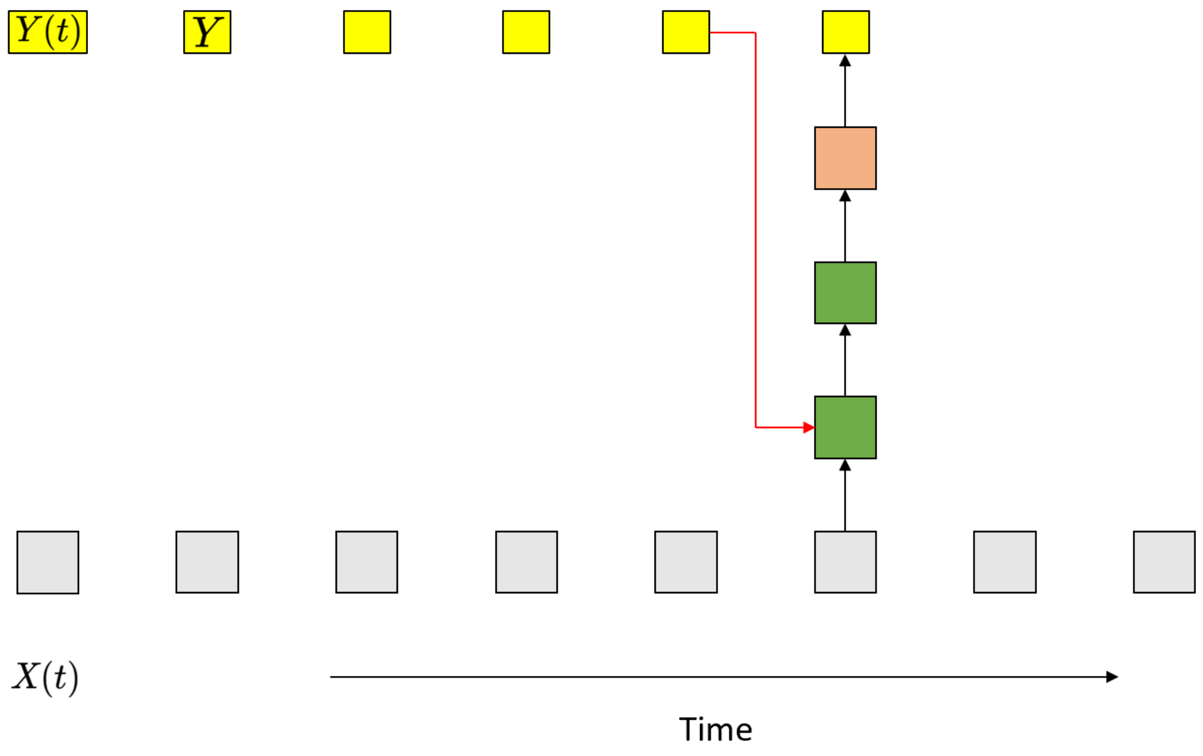

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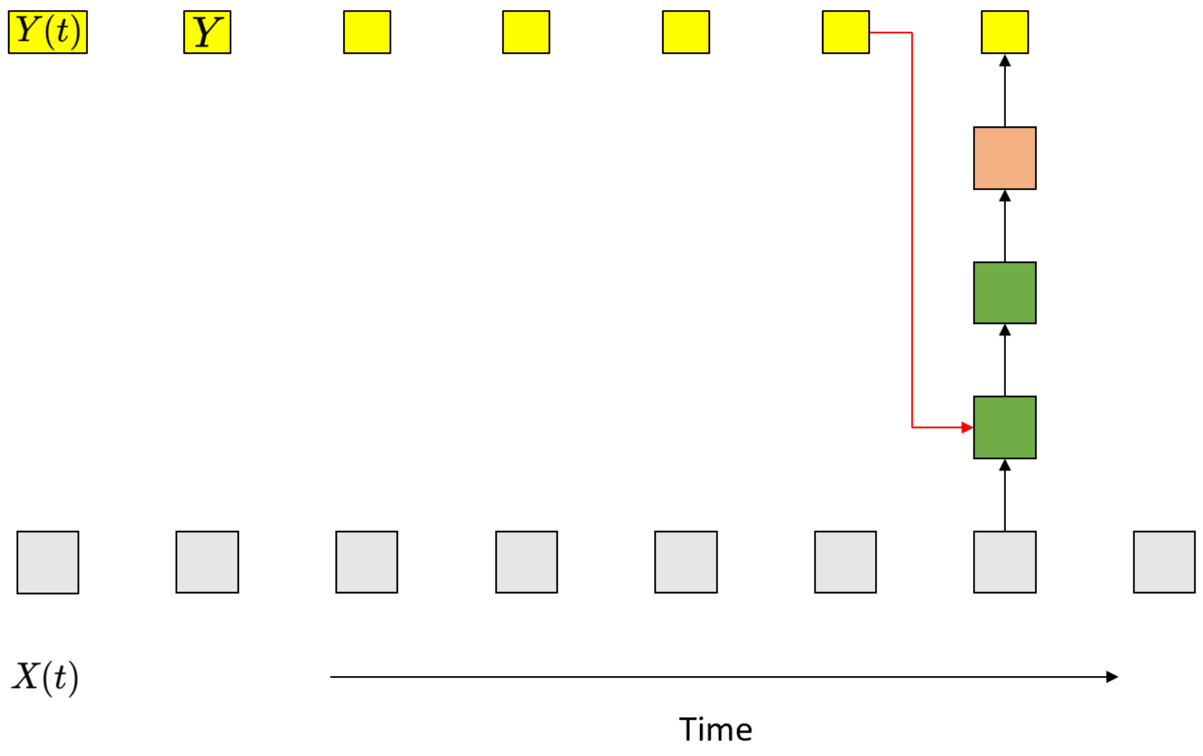

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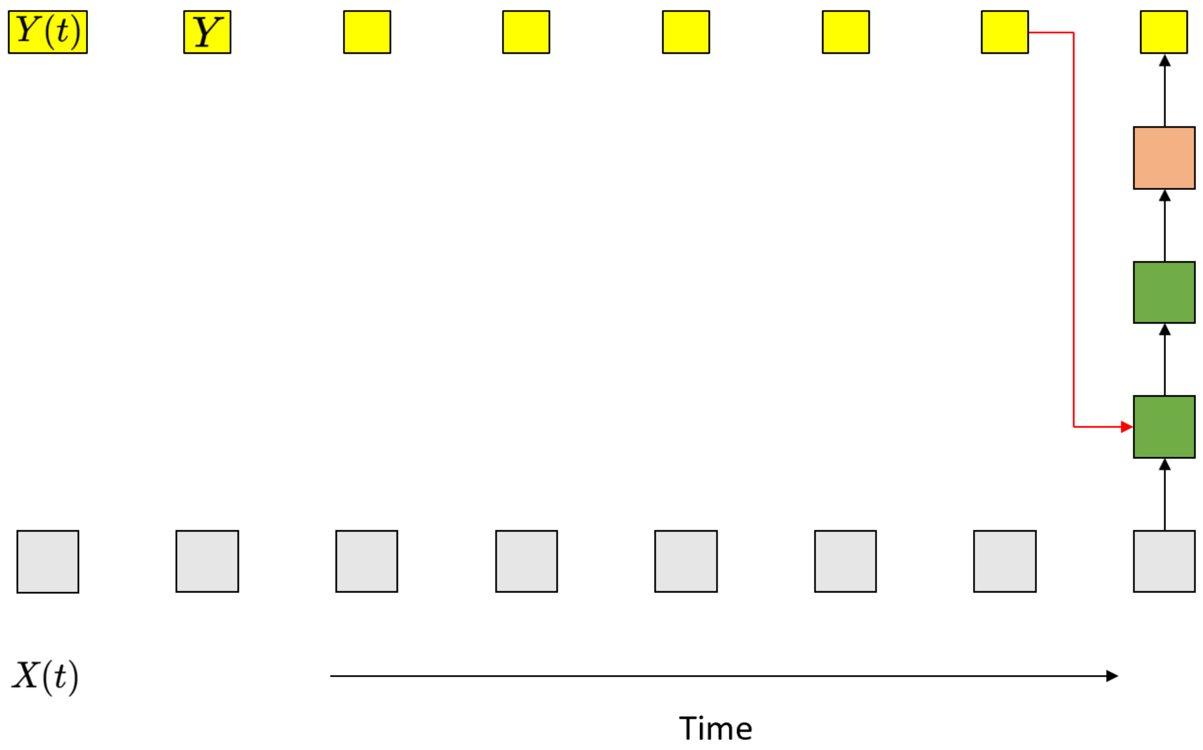
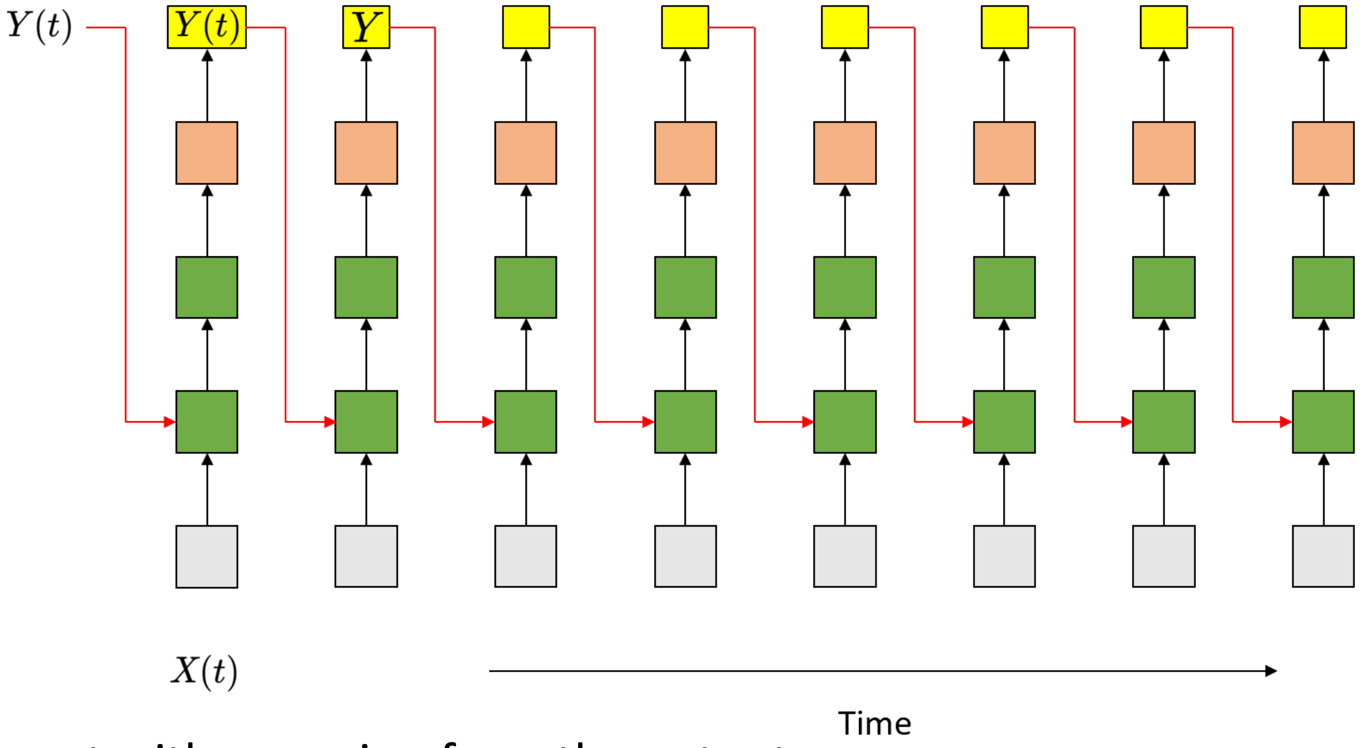


- An autoregressive net with recursion from the output

More Complete Representation



- An autoregressive net with recursion from the output
- Showing all computations
- All columns are identical
- An input at $t = 0$ affects outputs forever

An Alternate Model for Infinite Response Systems

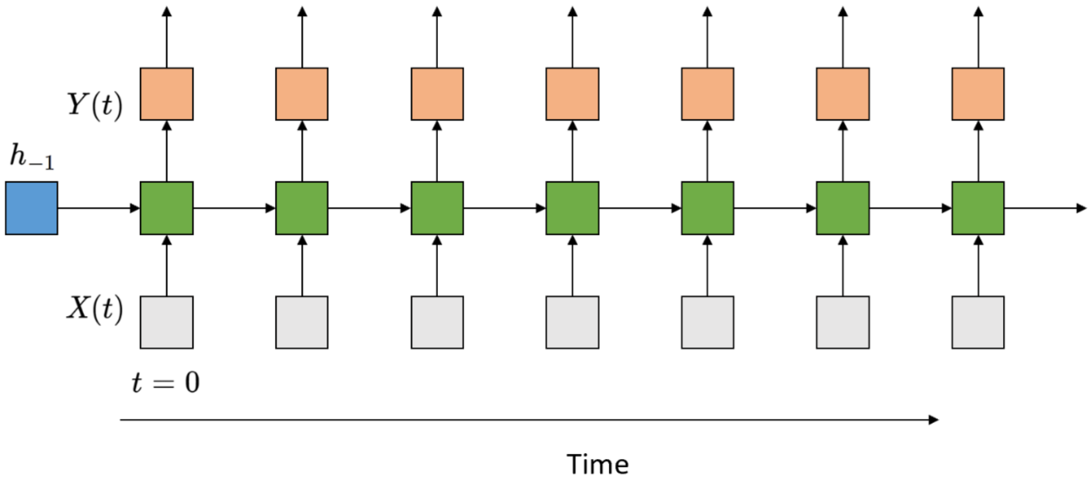
- the state-space model

$$h_t = f(x_t, h_{t-1})$$
$$y_t = g(h_t)$$

- h_t is the state of the network
- Need to define initial state h_{-1}

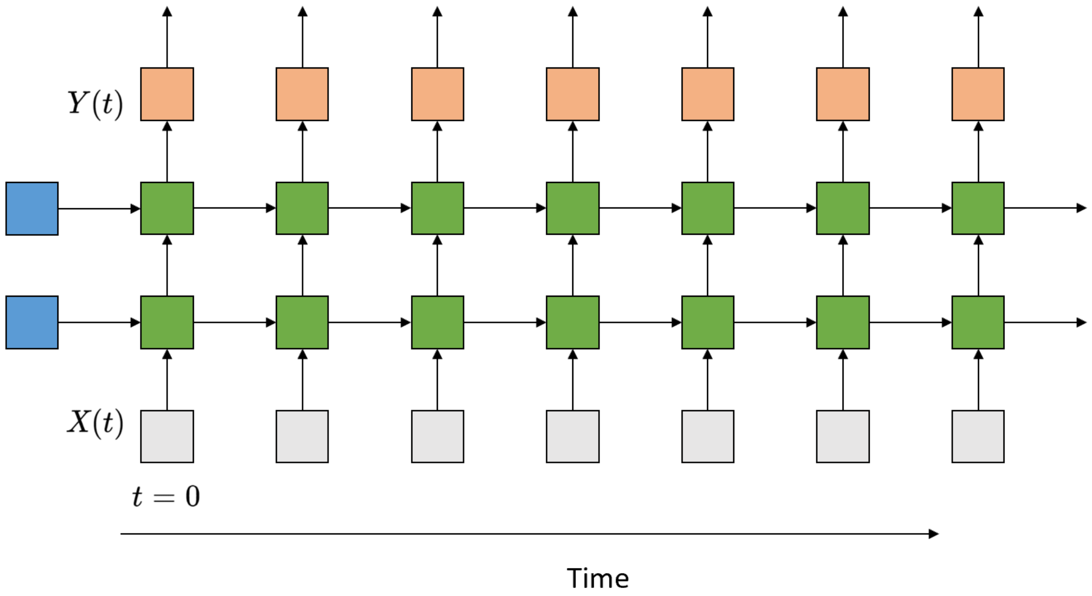
- This is a recurrent neural network
- State summarizes information about the entire past

Single Hidden Layer RNN (Simplest State-Space Model)



- The state (green) at any time is determined by the input at that time, and the state at the previous time
- All columns are identical
- An input at $t = 0$ affects outputs forever
- Also known as a recurrent neural net

Multiple Recurrent Layer RNN



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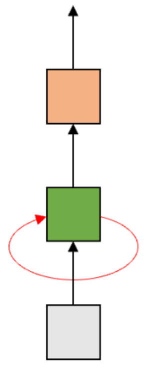
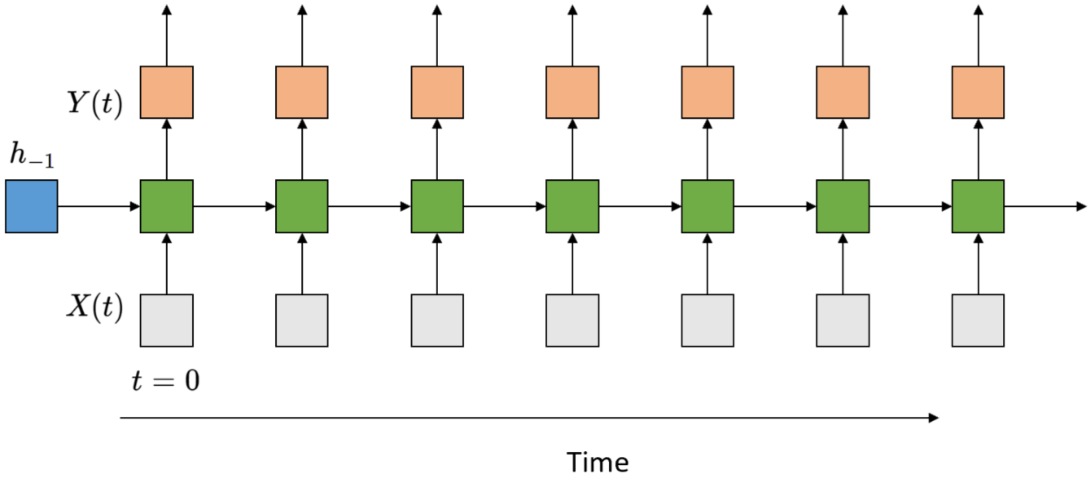
The Folded Version of RNN





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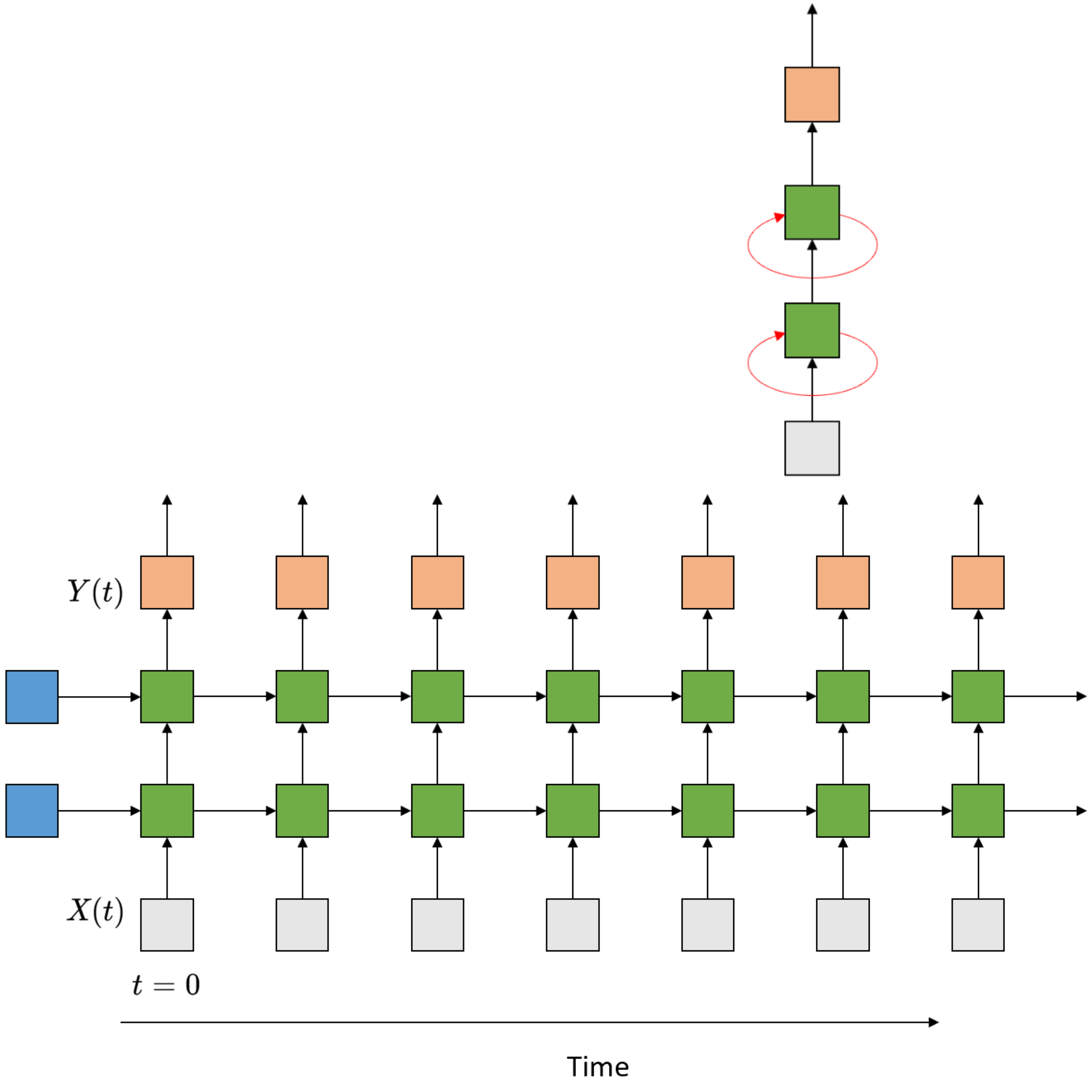
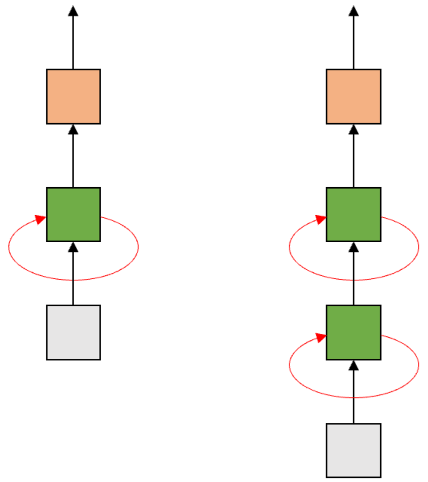


Recurrent Neural Network

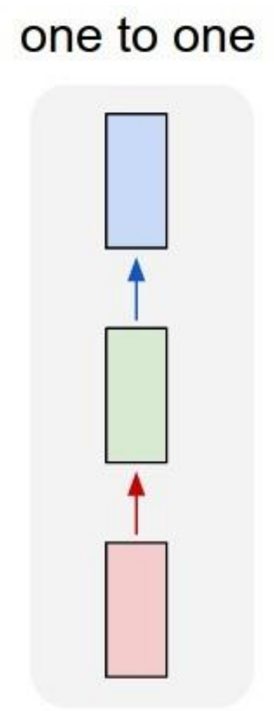


- Simplified models often drawn
- The loops imply recurrence

RNN Applications

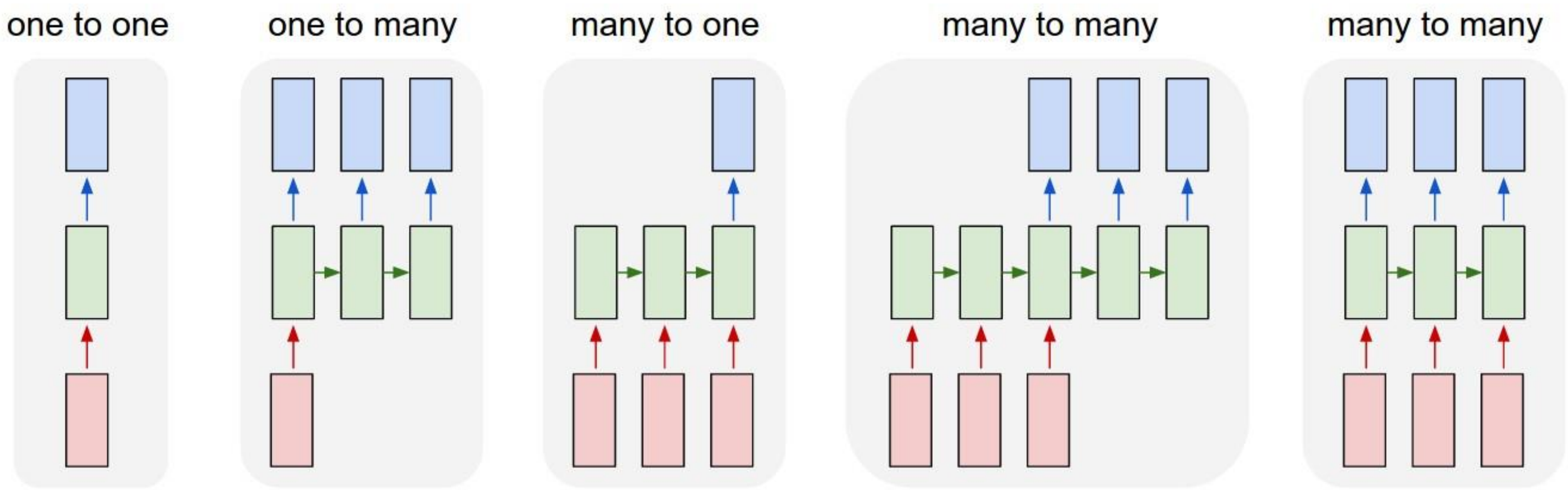
- Machine translation
- Speech recognition
- Text-to-speech
- Image captioning
- Video analysis/understanding

"Vanilla" Neural Network



http://karpathy.github.io/2015/05/21/rnn-effectiveness/

Recurrent Neural Network: Process Sequences



e.g. Image Captioning
image → sequence of words

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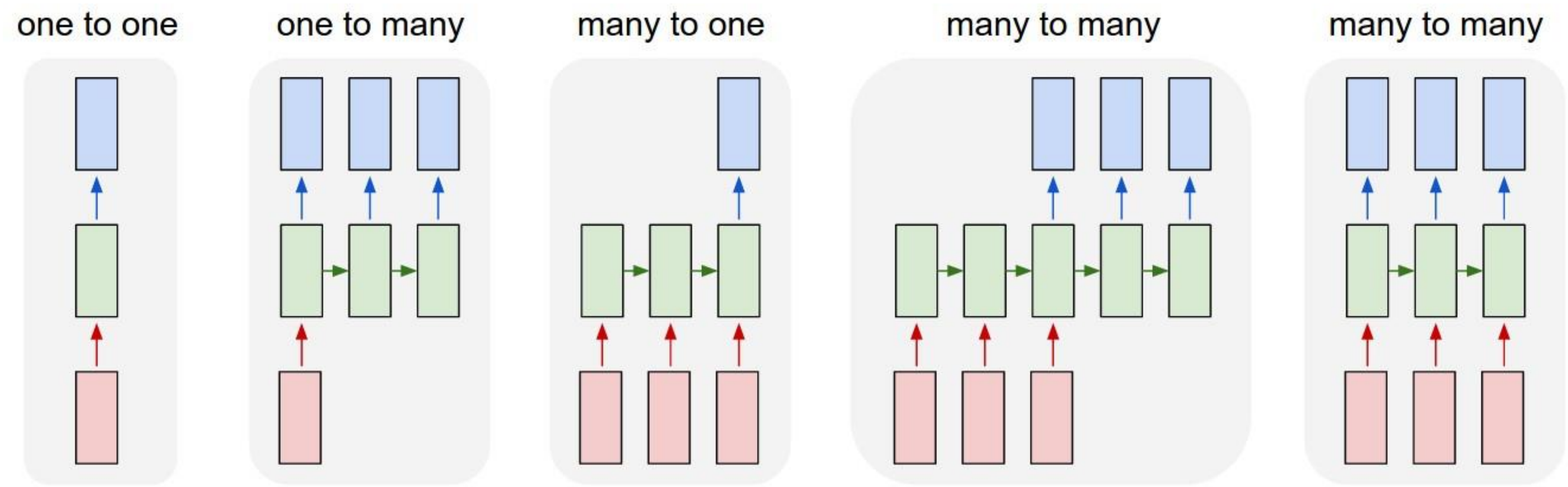
Recurrent Neural Network: Process Sequences



e.g. Sentiment Classification
sequence of words → sentiment

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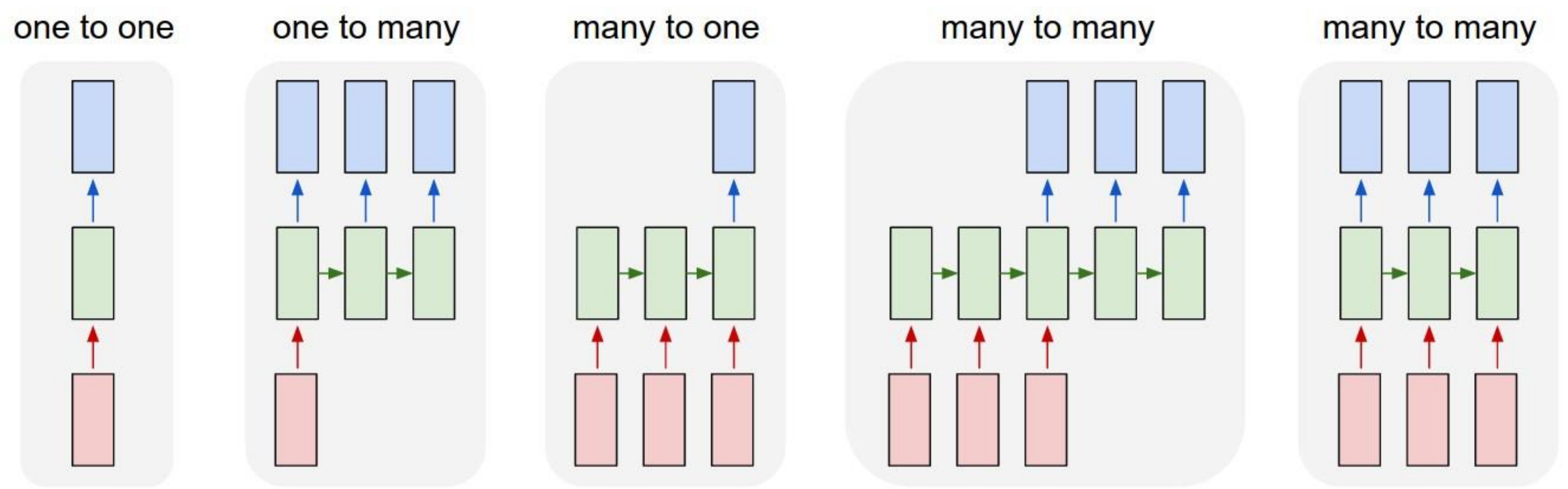
Recurrent Neural Network: Process Sequences



e.g. Machine Translation
Seq. of words → seq. of words

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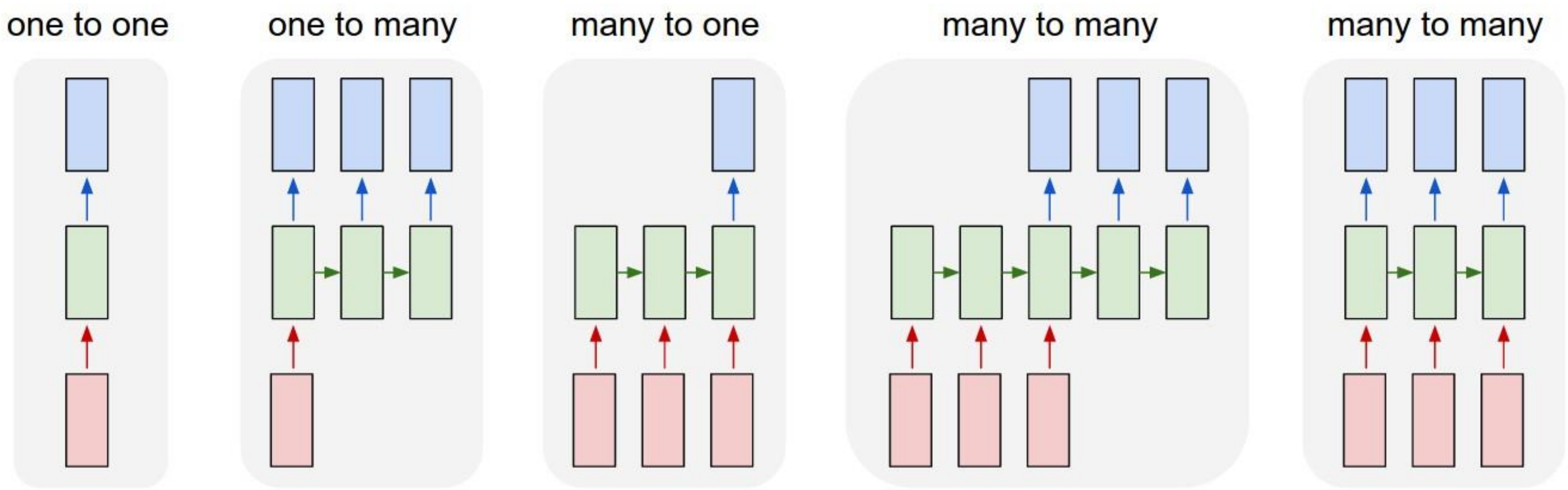
Recurrent Neural Network: Process Sequences



e.g. Video classification on frame level

http://karpathy.github.io/2015/05/21/rnn-effectiveness/

Recurrent Neural Networks

Unrolling the Recurrence

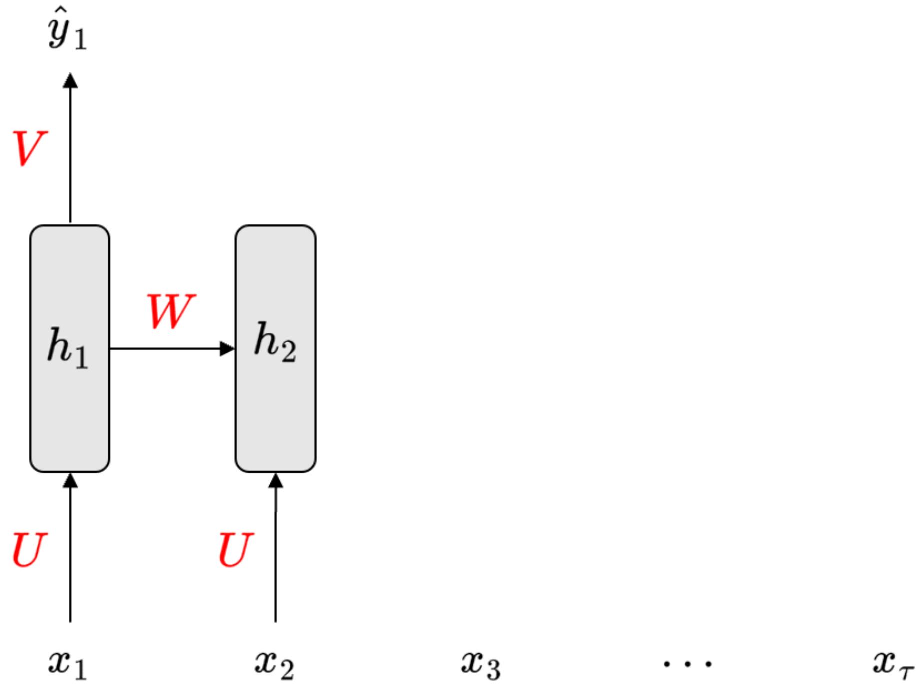
$x_1 \qquad x_2 \qquad x_3 \qquad \cdots \qquad x_\tau$

Unrolling the Recurrence

Unrolling the Recurrence

POSTECH

Unrolling the Recurrence



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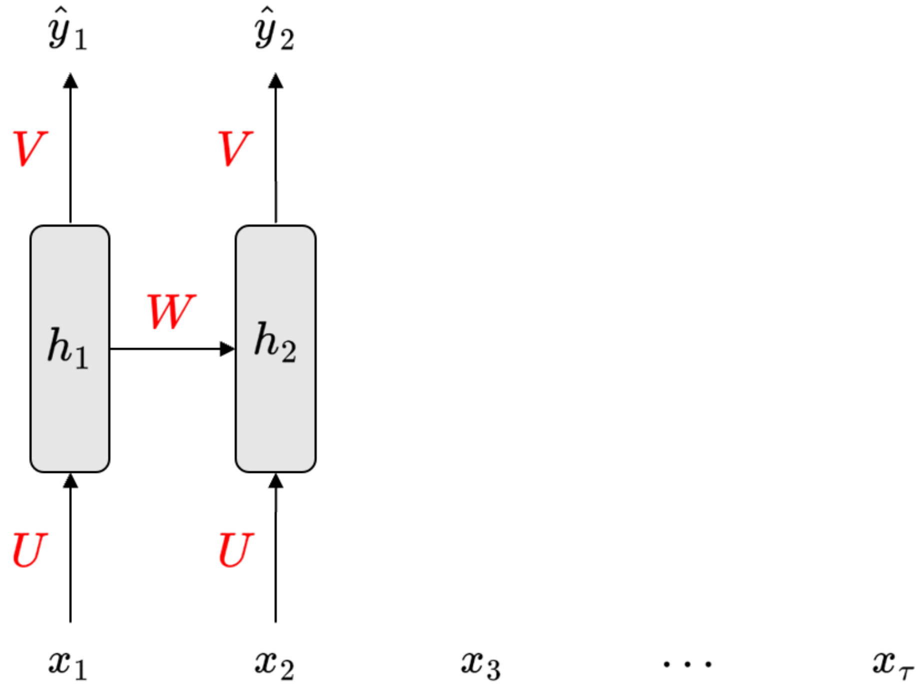

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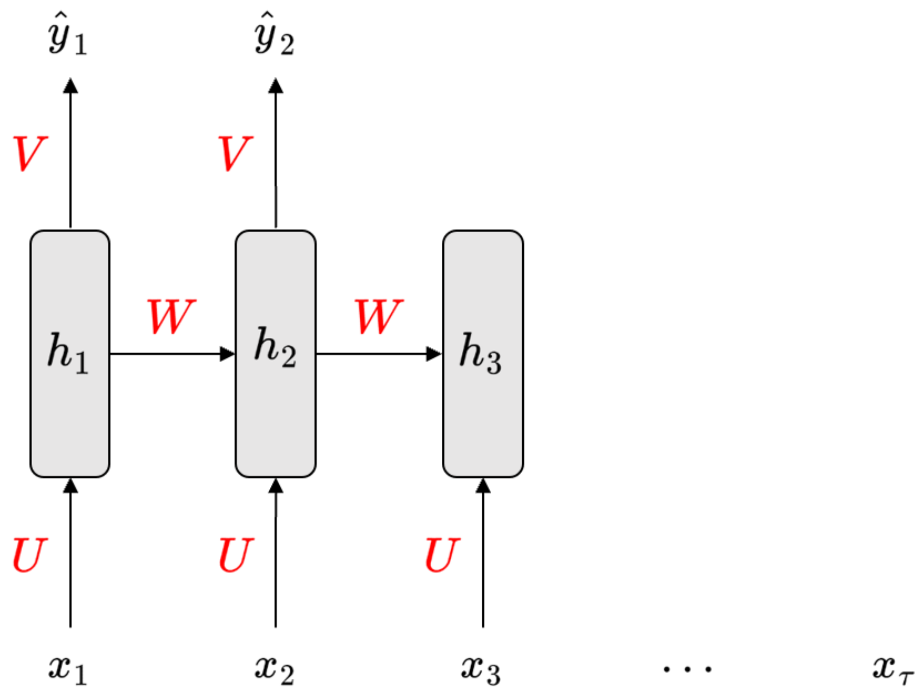

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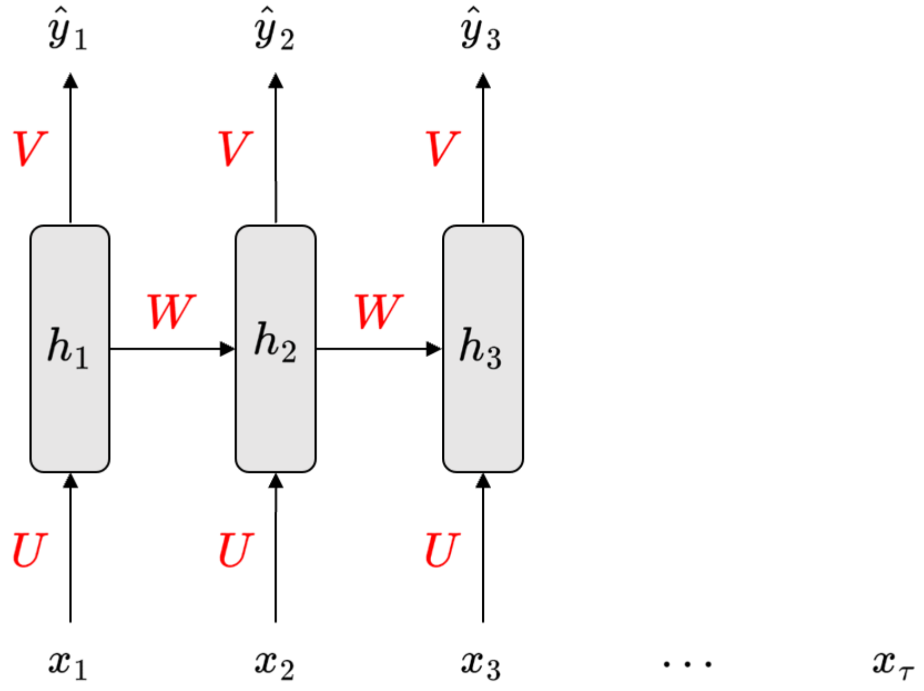

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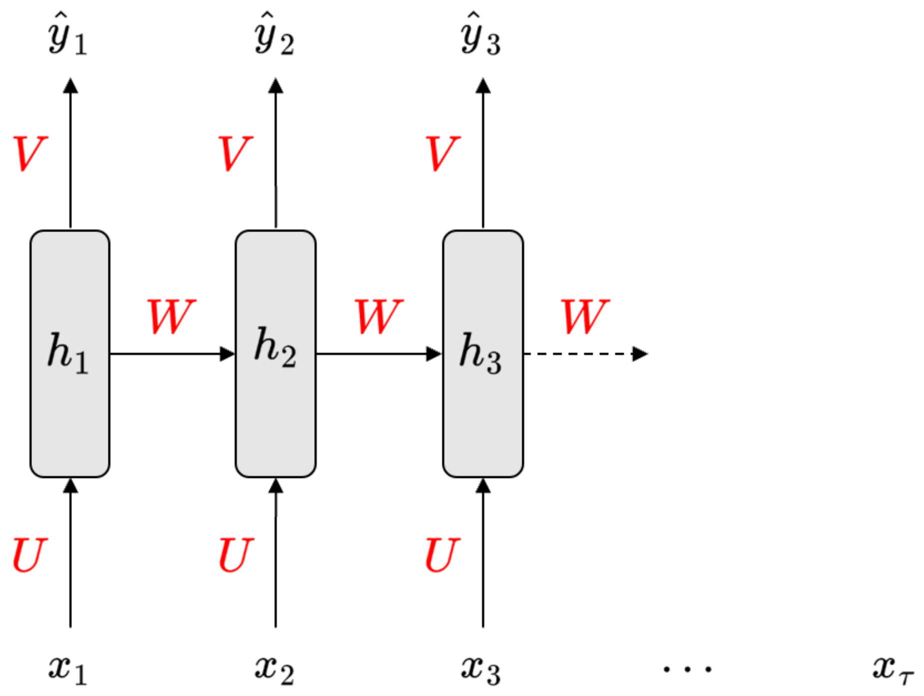

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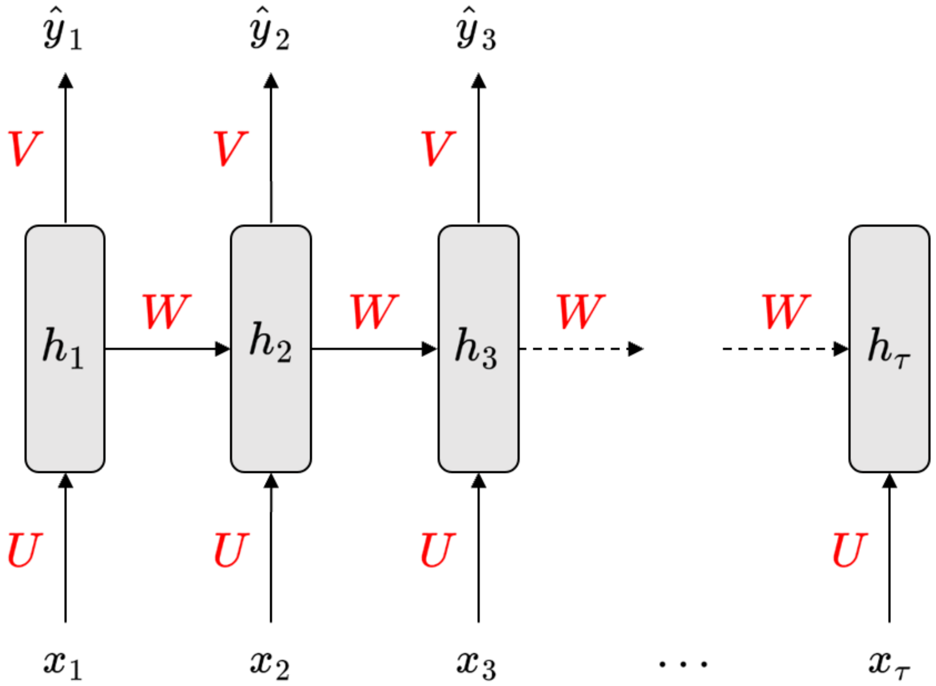

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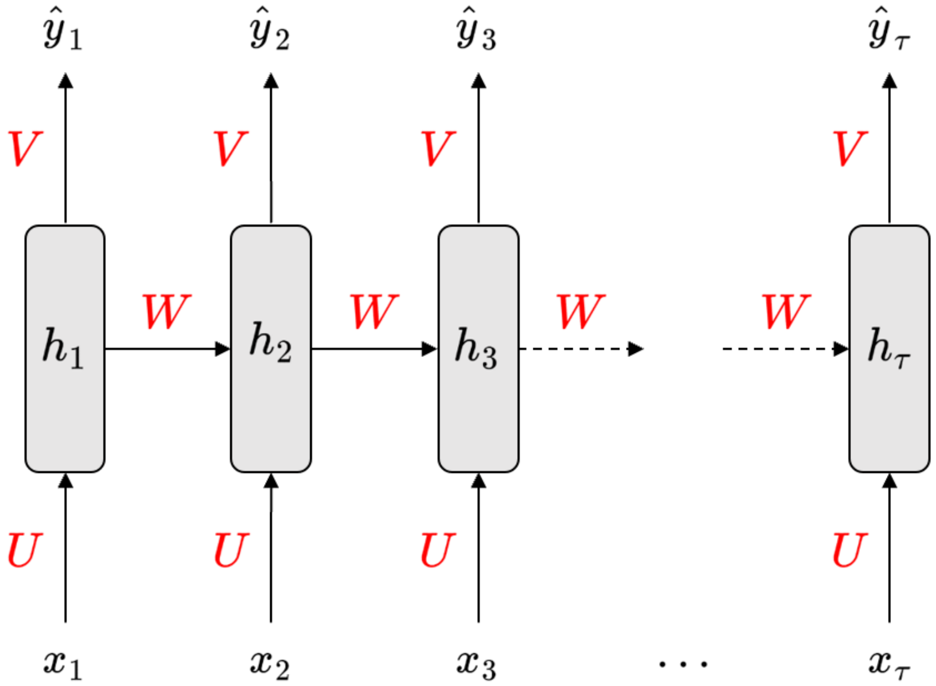
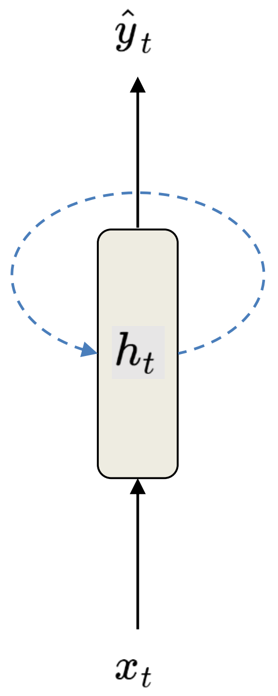


Recurrent Connections



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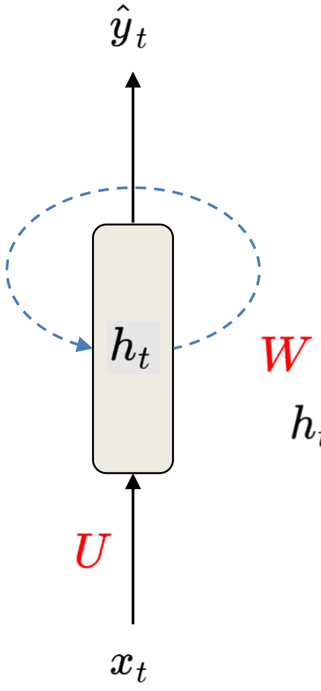
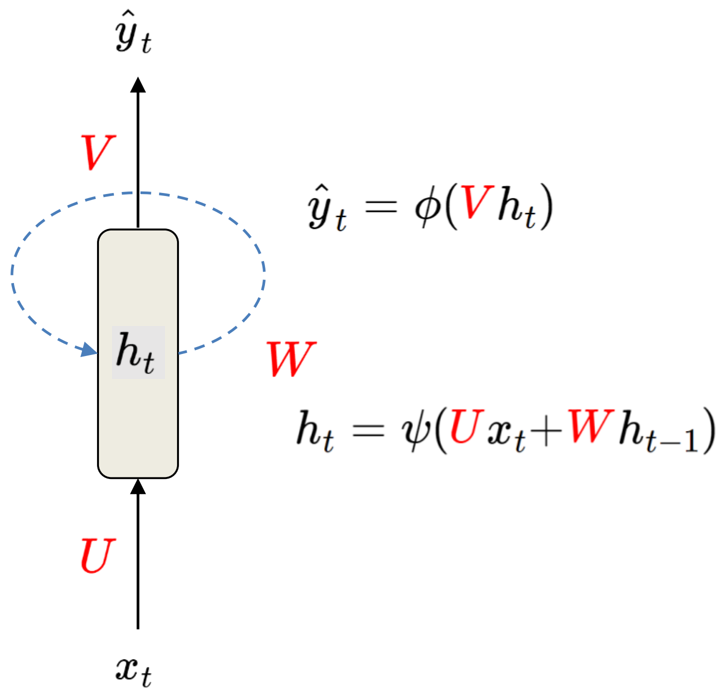


$$h_t = \psi(Ux_t + Wh_{t-1})$$

Feedforward Propagation

- This is a RNN where the input and output sequences are of the same length
- Feedforward operation proceeds from left to right
- Update Equations:

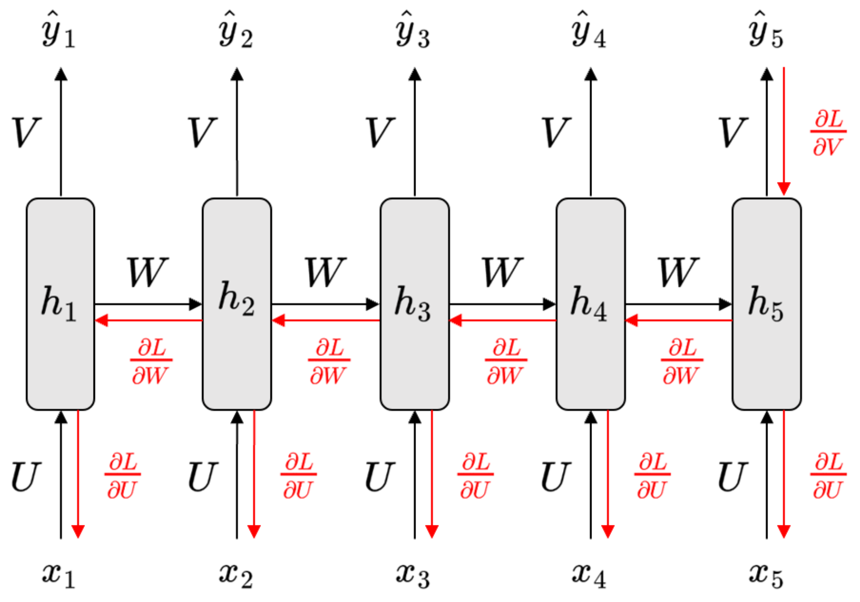
$$\mathbf{a}_t = b + W\mathbf{h}_{t-1} + U\mathbf{x}_t$$
$$\mathbf{h}_t = \tanh \mathbf{a}_t$$
$$\mathbf{o}_t = c + V\mathbf{h}_t$$
$$\hat{\mathbf{y}}_t = \text{softmax}(\mathbf{o}_t)$$



How to Train RNN

Backward Propagation

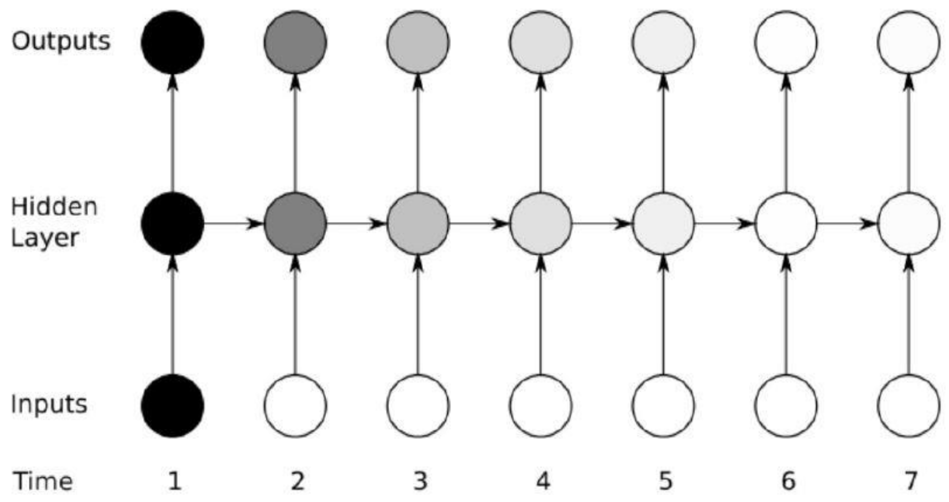
- Loss would just be the sum of losses over time steps
- Treat the recurrent network as a usual multilayer network and apply backpropagation on the unrolled network
- This is called Backpropagation through time (BPTT)



Long Short-Term Memory (LSTM)

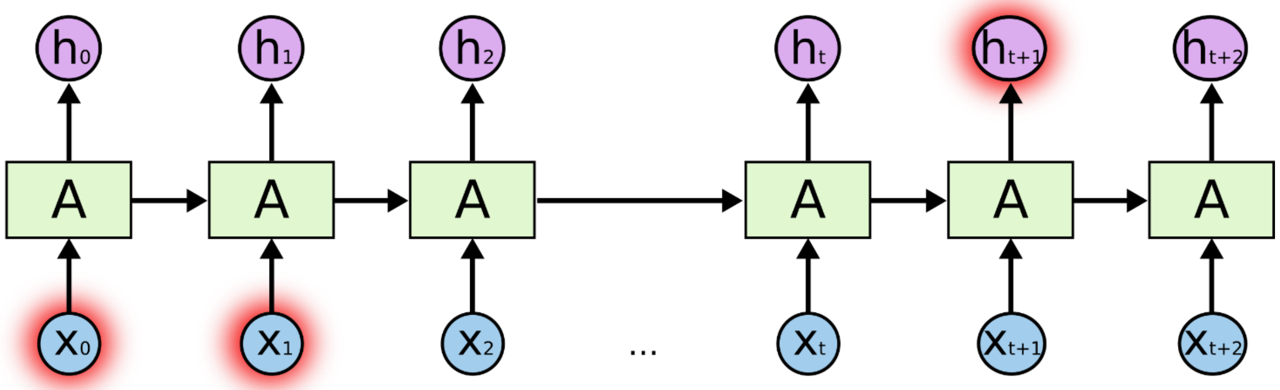
Long Short-Term Memory (LSTM)

- Long-Term Dependencies
  - Gradients propagated over many stages tend to either **vanish** or **explode**
  - Difficulty with long-term dependencies arises from the exponentially smaller weights given to long-term interactions
  - Introduce a memory state that runs through only linear operators
  - Use gating units to control the updates of the state



Example

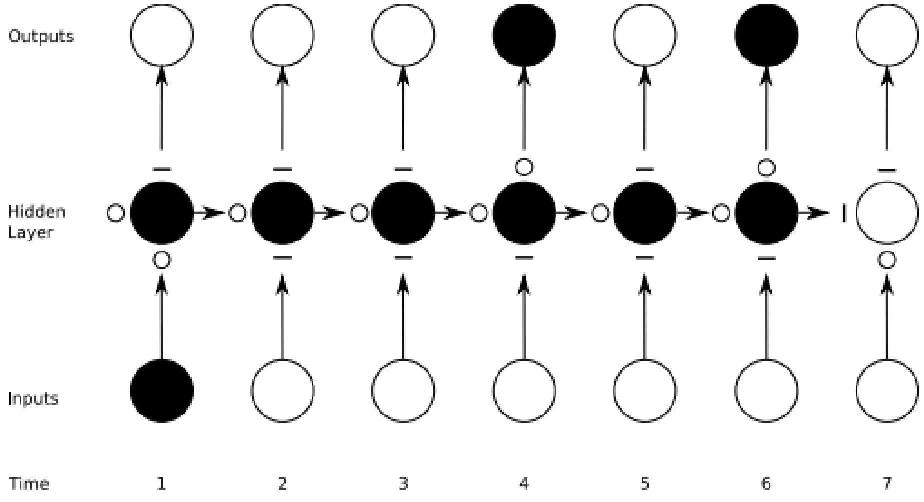
- "I grew up in France… I speak fluent *French*."



https://colah.github.io/posts/2015-08-Understanding-LSTMs/

Long Short-Term Memory (LSTM)

- Consists of a memory cell and a set of gating units
  - Memory cell is the context that carries over
  - Forget gate controls erase operation
  - Input gate controls write operation
  - Output gate controls the read operation



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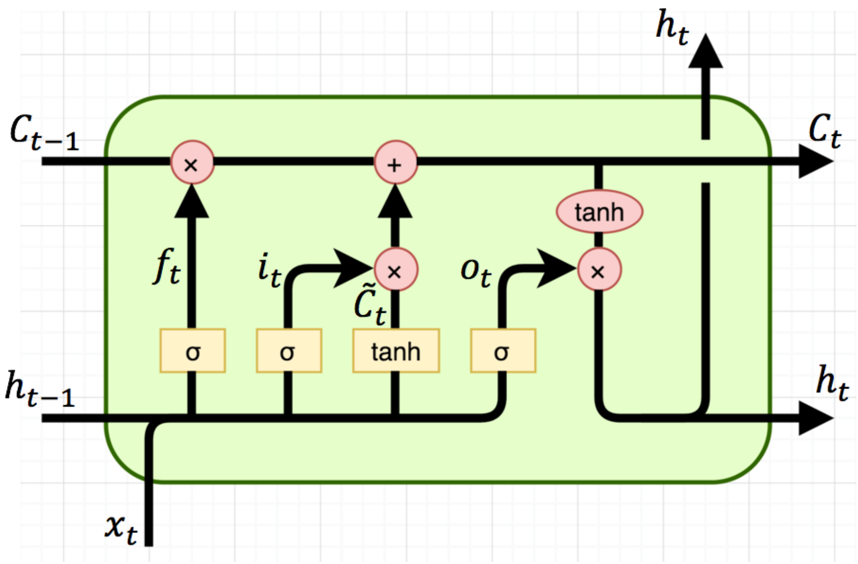

$$i_t = \sigma\left(x_t U^i + h_{t-1} W^i\right)$$
$$f_t = \sigma\left(x_t U^f + h_{t-1} W^f\right)$$
$$o_t = \sigma\left(x_t U^o + h_{t-1} W^o\right)$$
$$\tilde{C}_t = \tanh\left(x_t U^g + h_{t-1} W^g\right)$$
$$C_t = \sigma\left(f_t * C_{t-1} + i_t * \tilde{C}_t\right)$$
$$h_t = \tanh(C_t) * o_t$$

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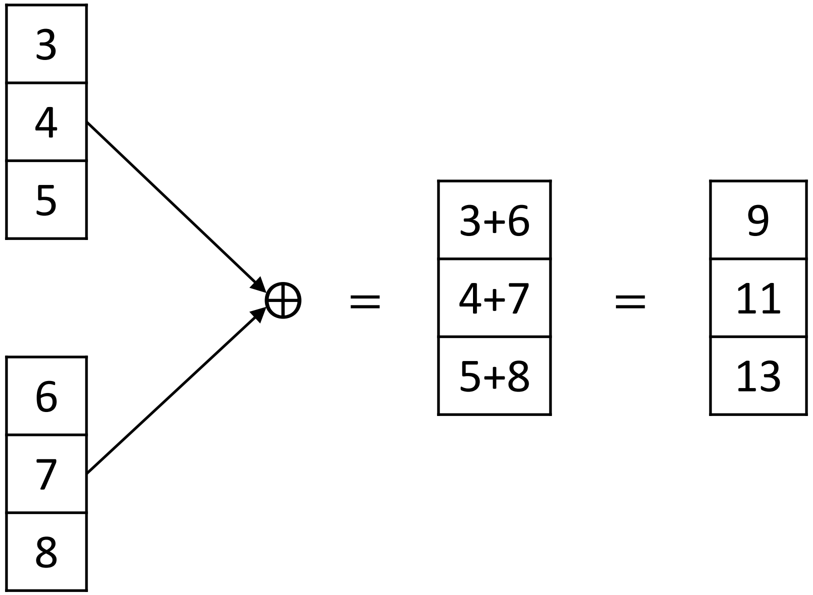
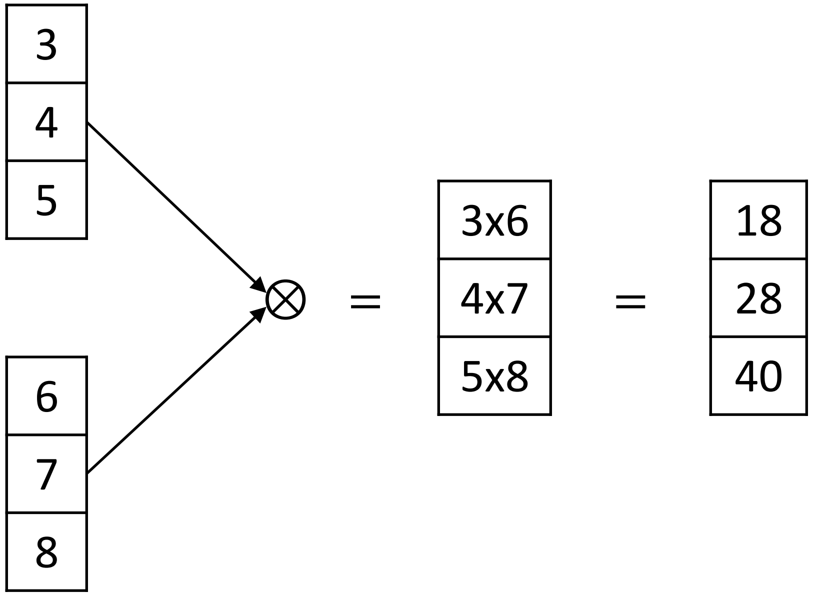
Element-by-Element

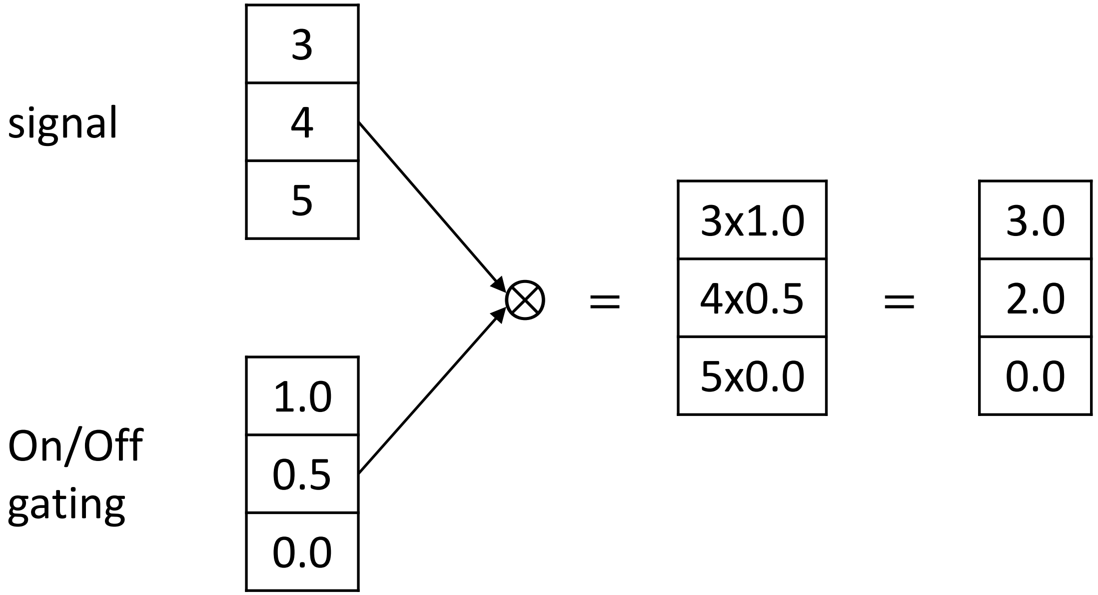
Element-by-Element Addition



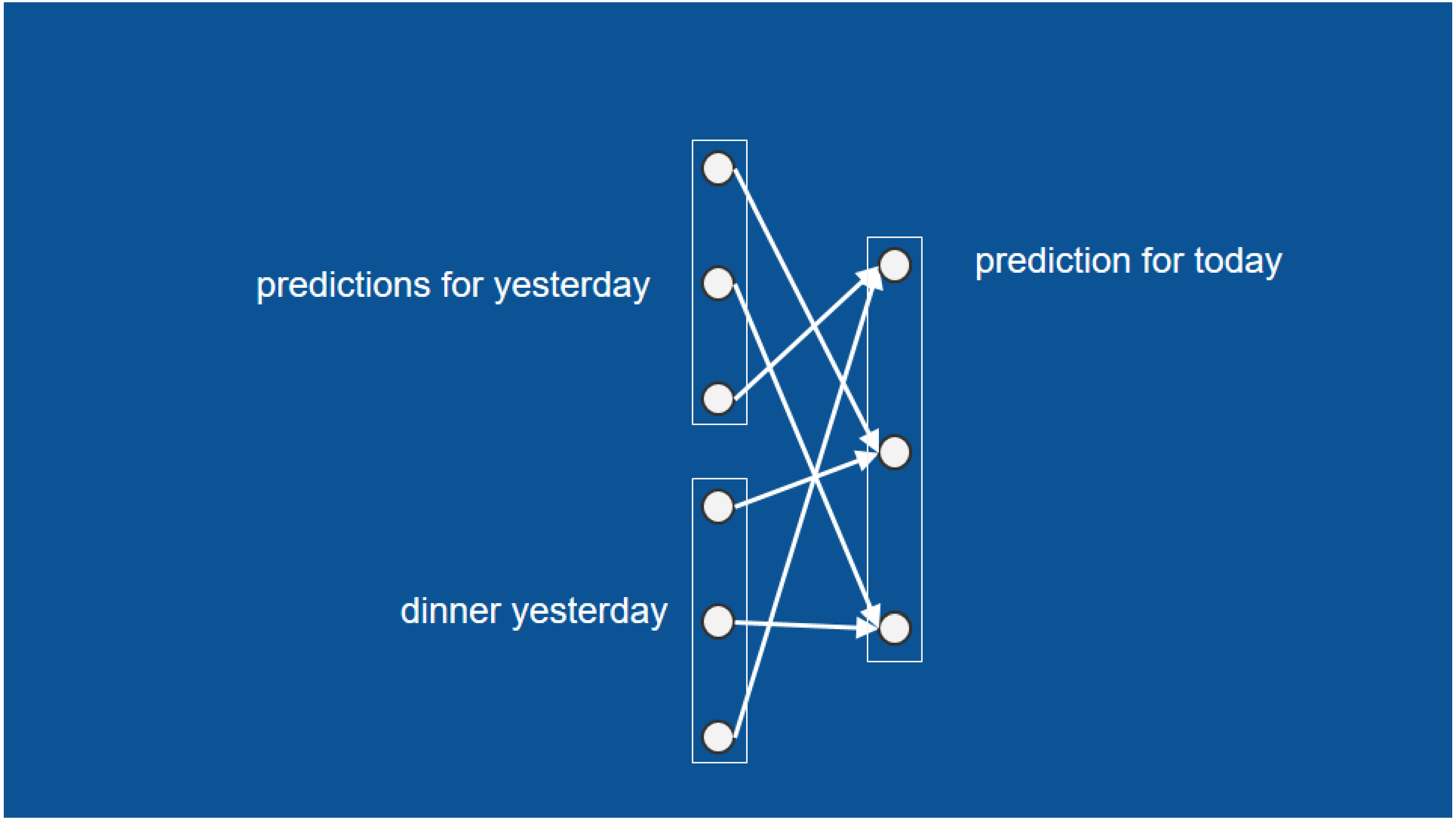
Element-by-Element Multiplication



Gating



Long Short-Term Memory (LSTM)



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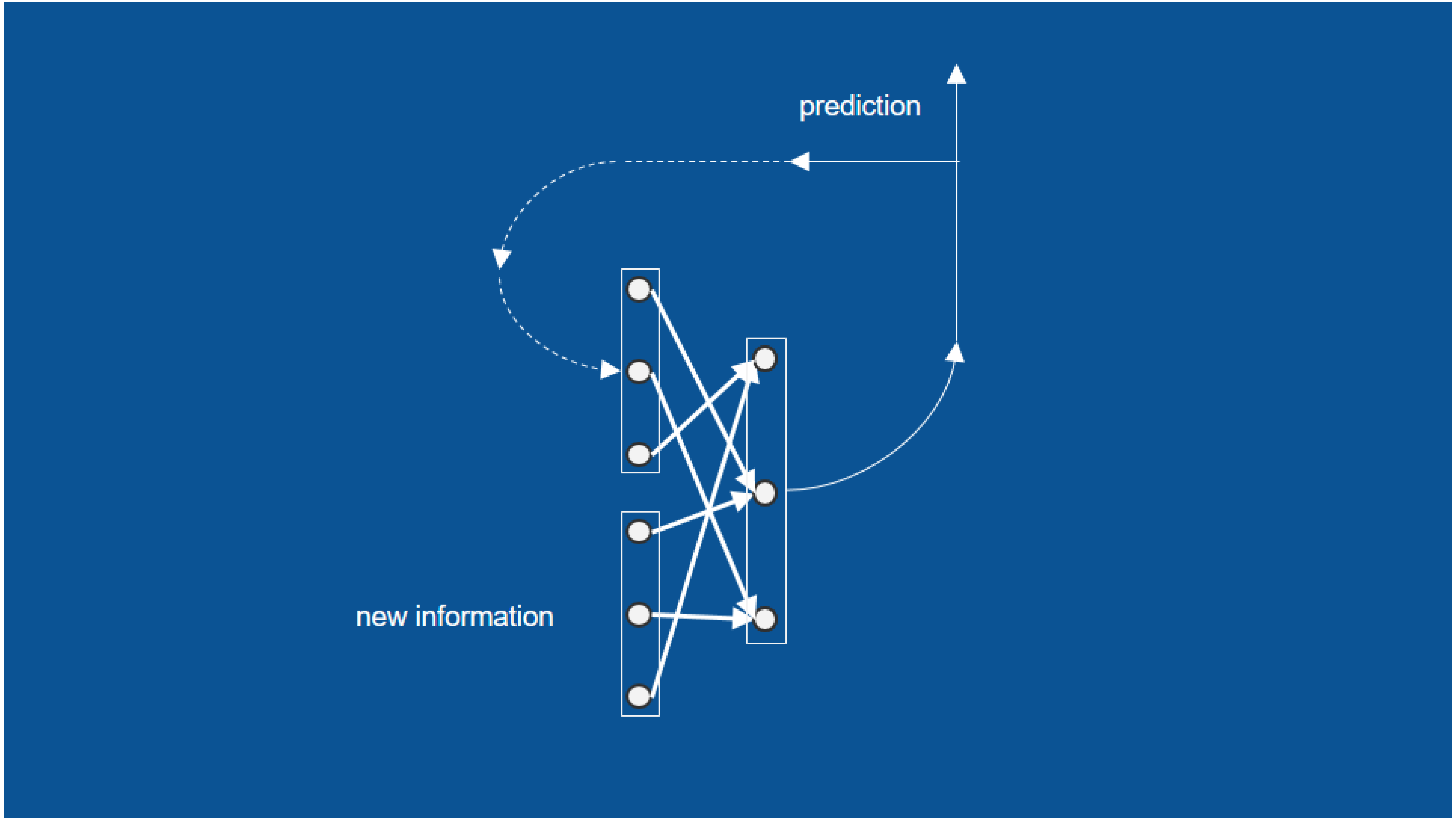

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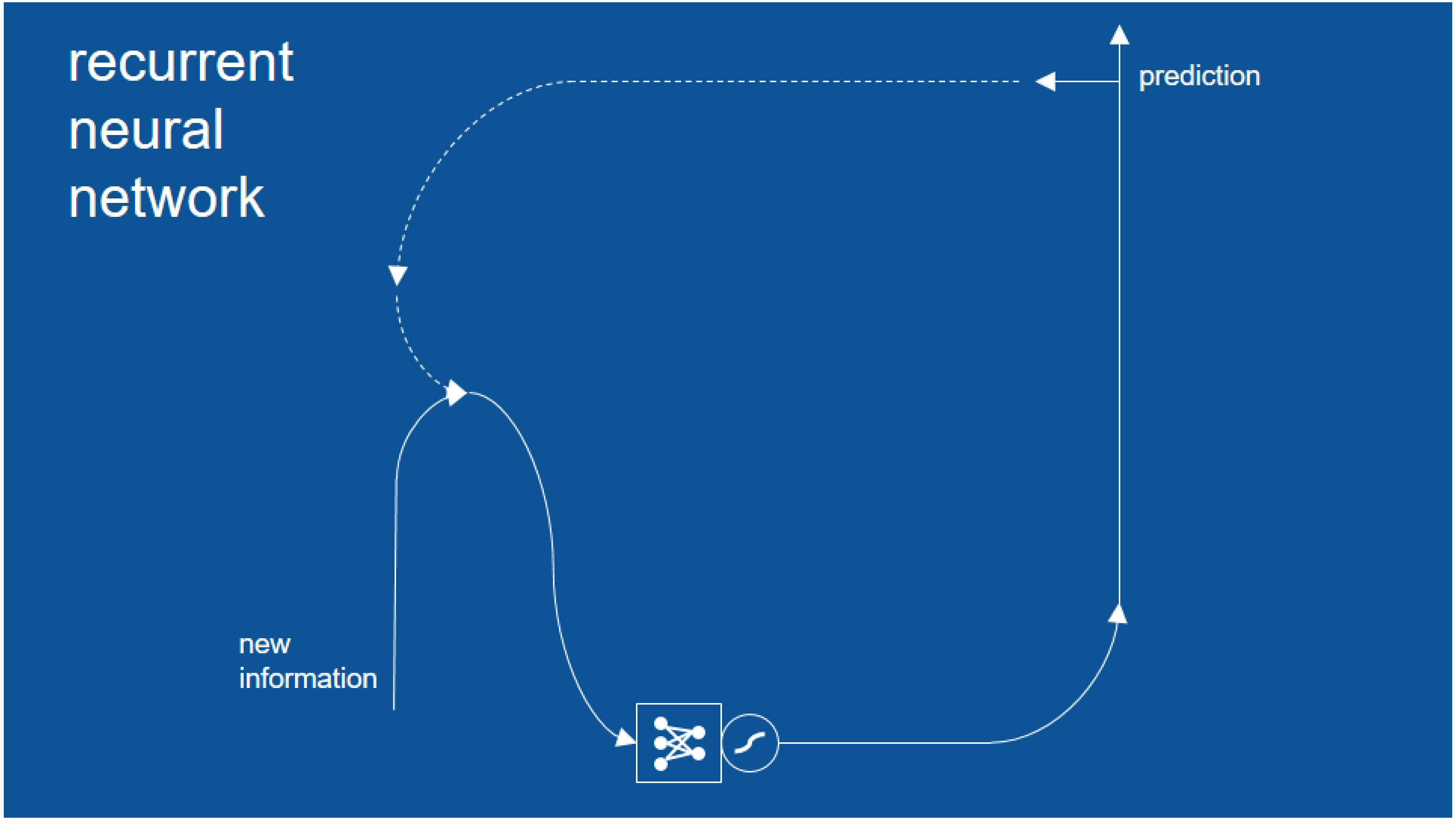

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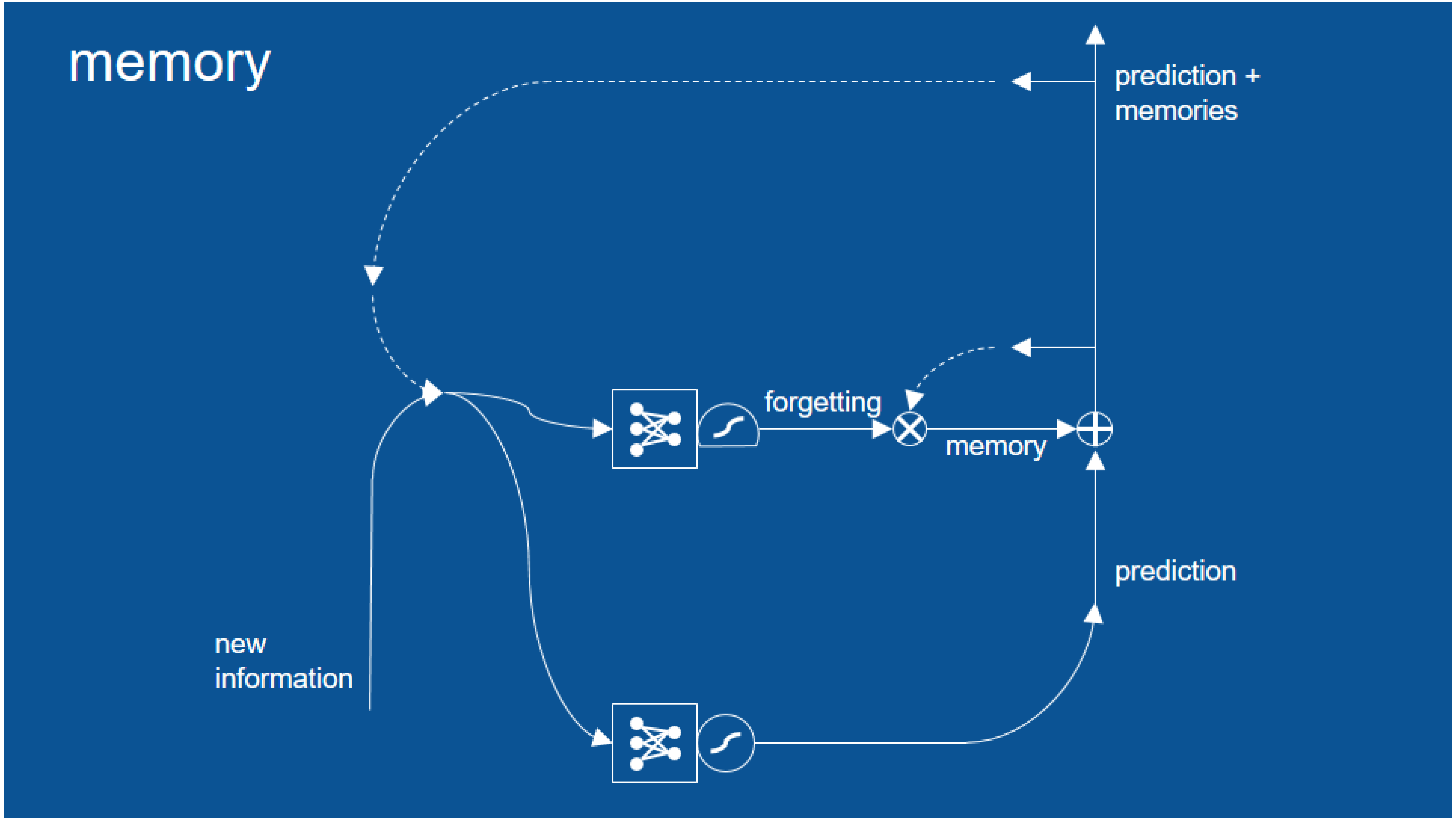

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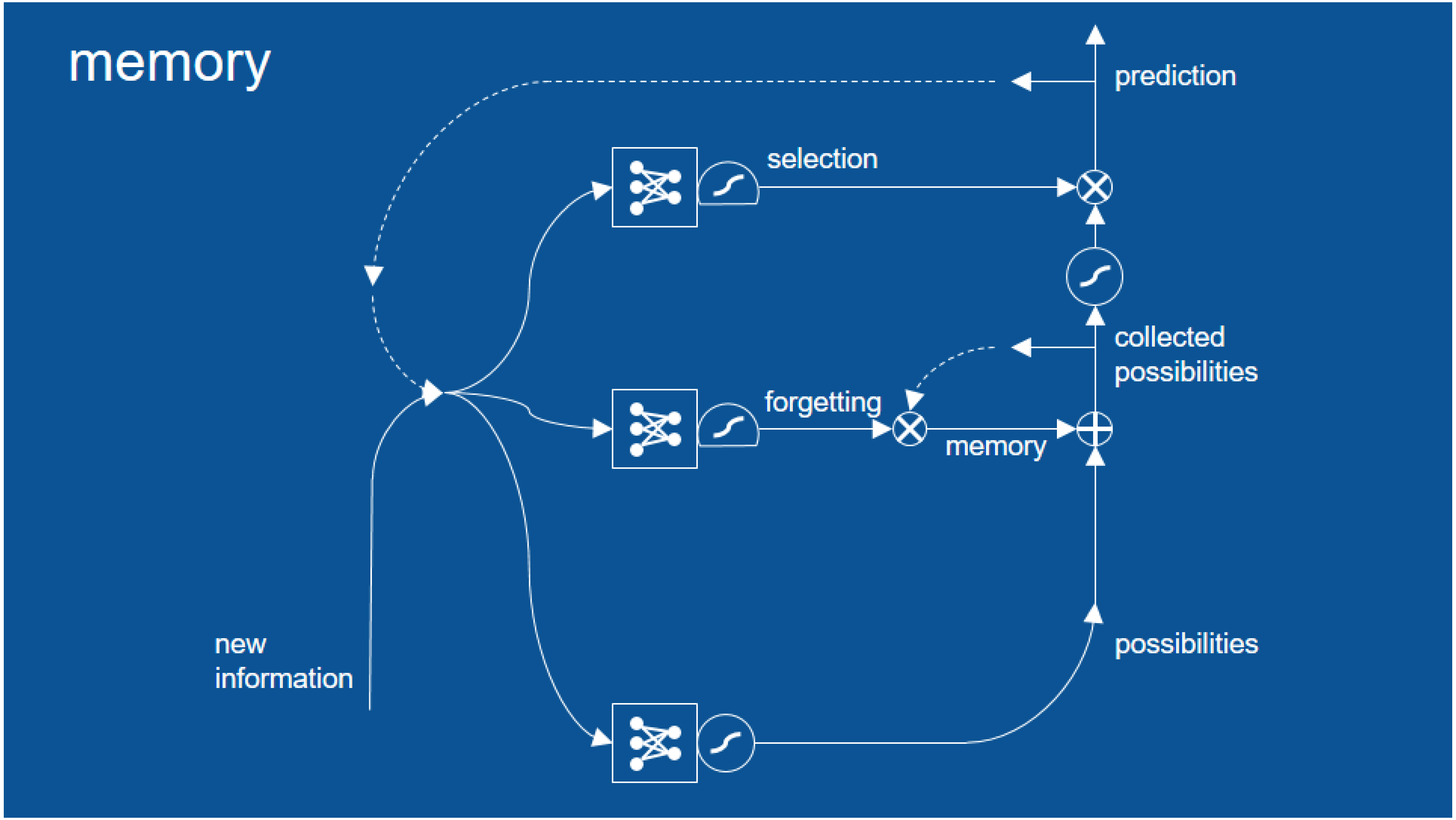

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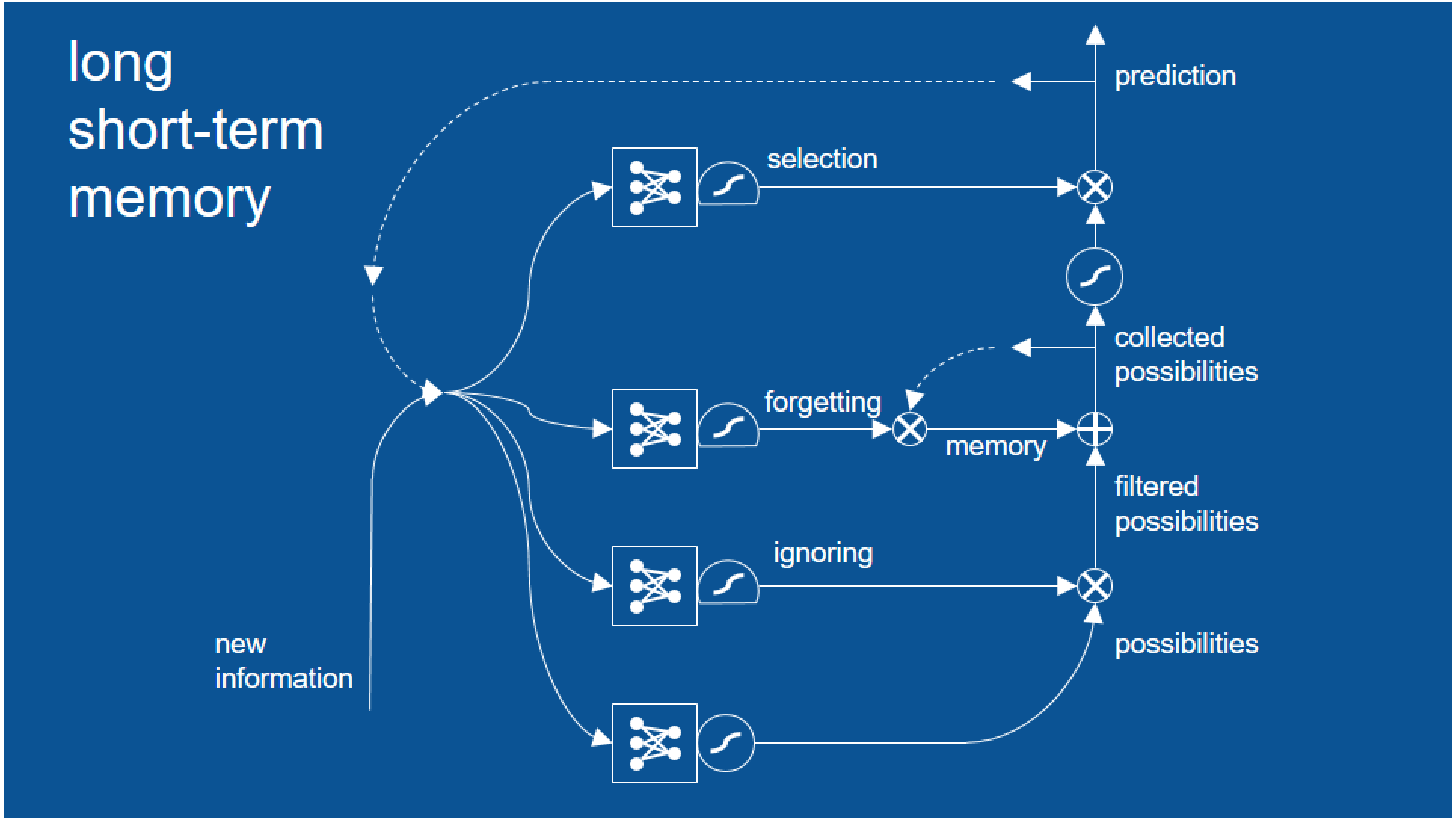
Long Short-Term Memory (LSTM)



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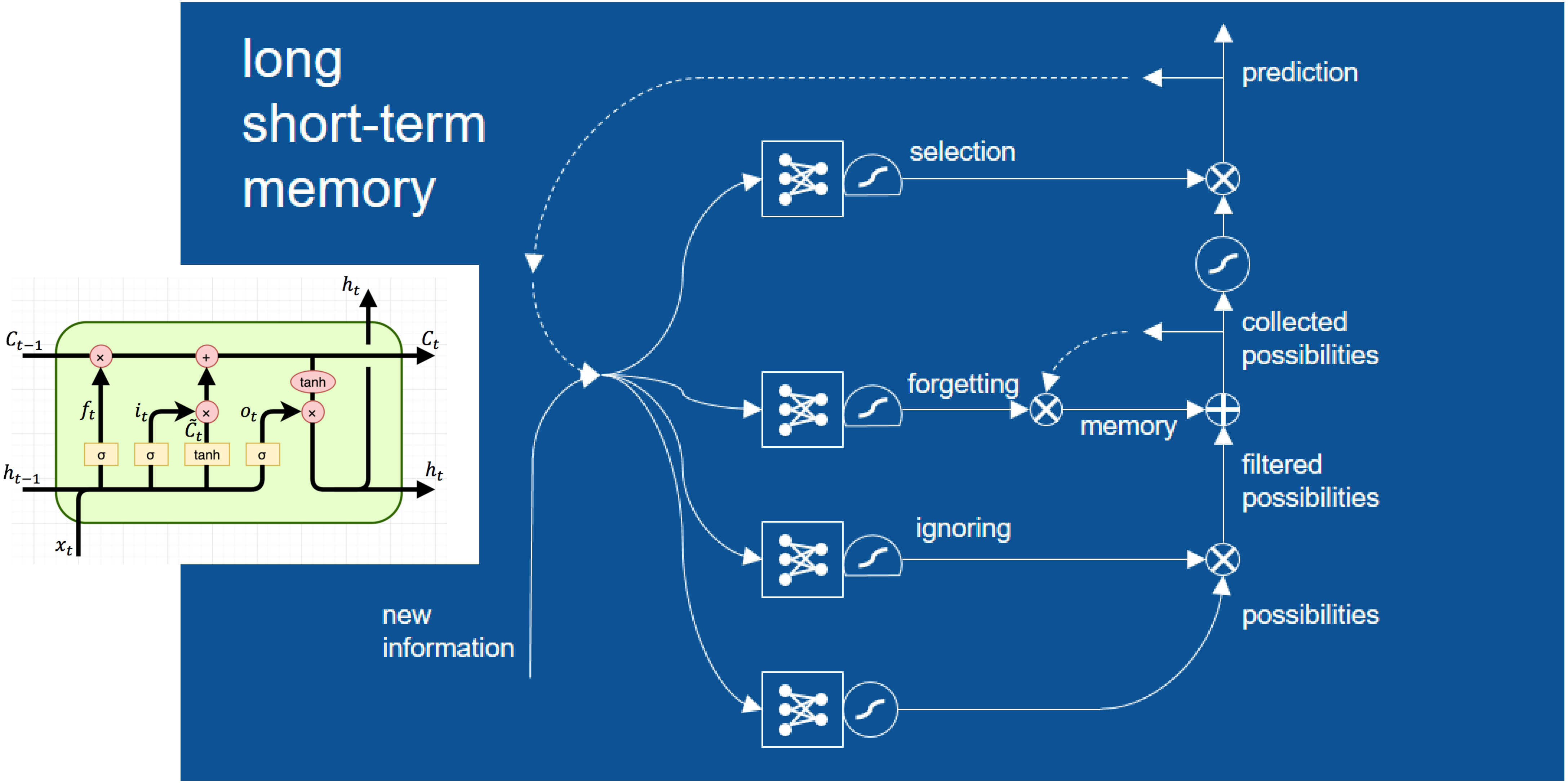
Long Short-Term Memory (LSTM)



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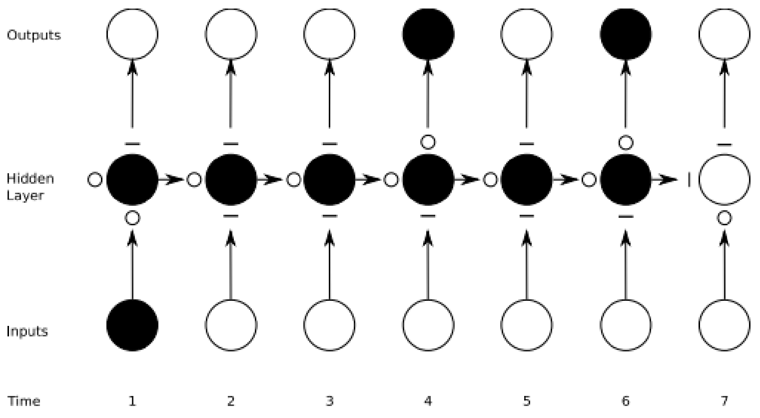
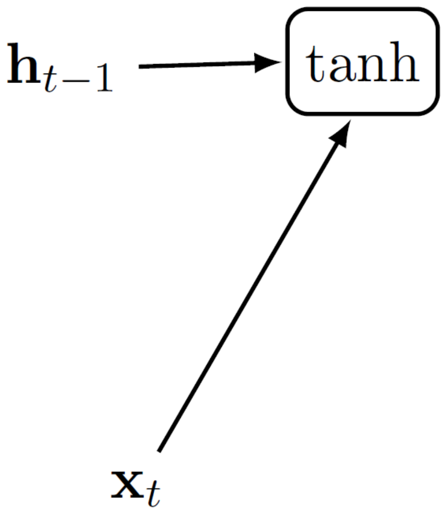
Long Short-Term Memory

- Forget gate controls erase operation
- Input gate controls write operation
- Output gate controls the read operation





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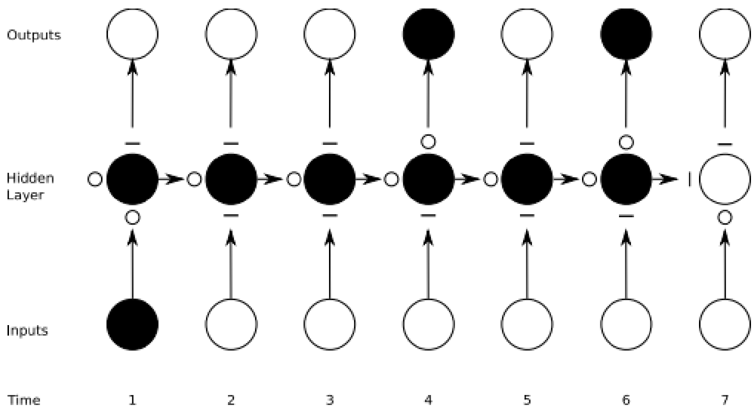

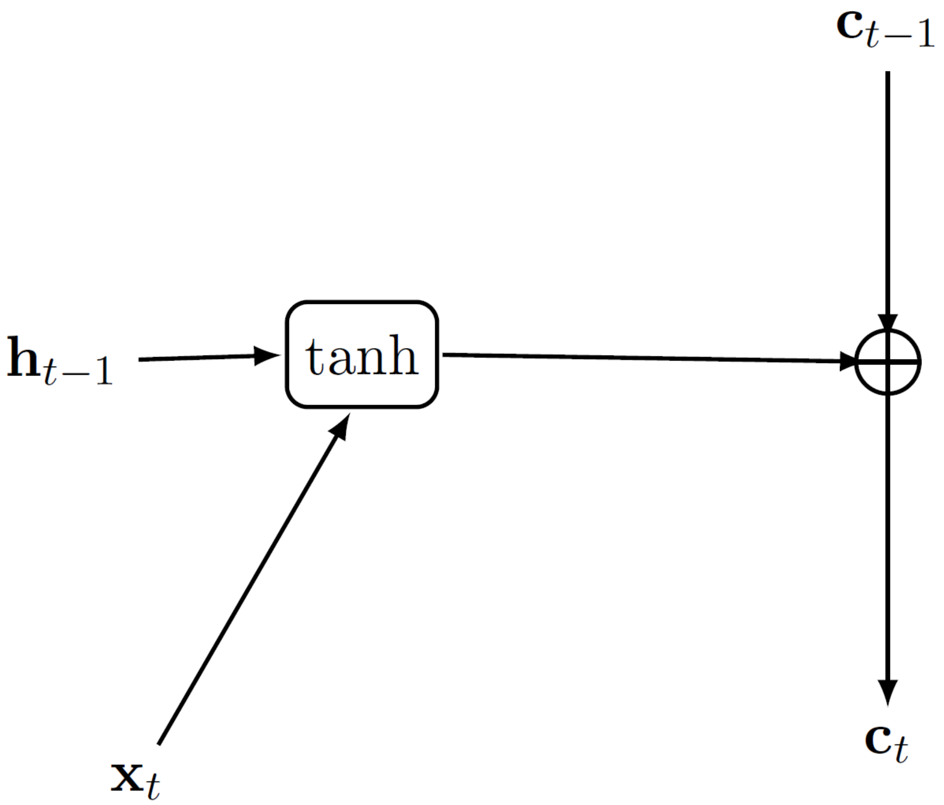

$$\tilde{\mathbf{c}}_t = \tanh(W\mathbf{h}_{t-1} + U\mathbf{x}_t)$$
$$\mathbf{c}_t = \mathbf{c}_{t-1} + \tilde{c}_t$$

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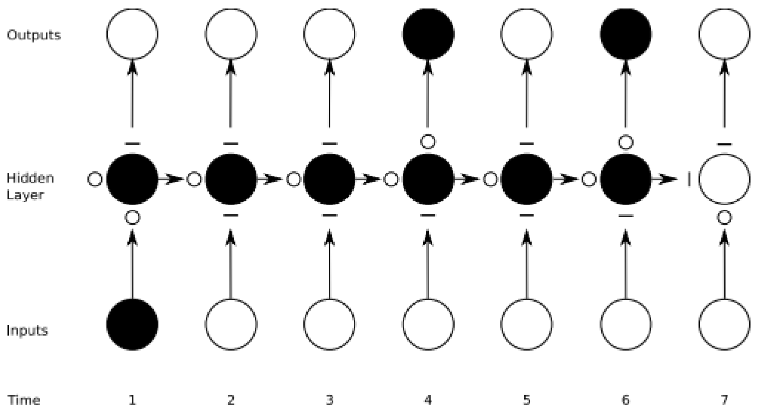

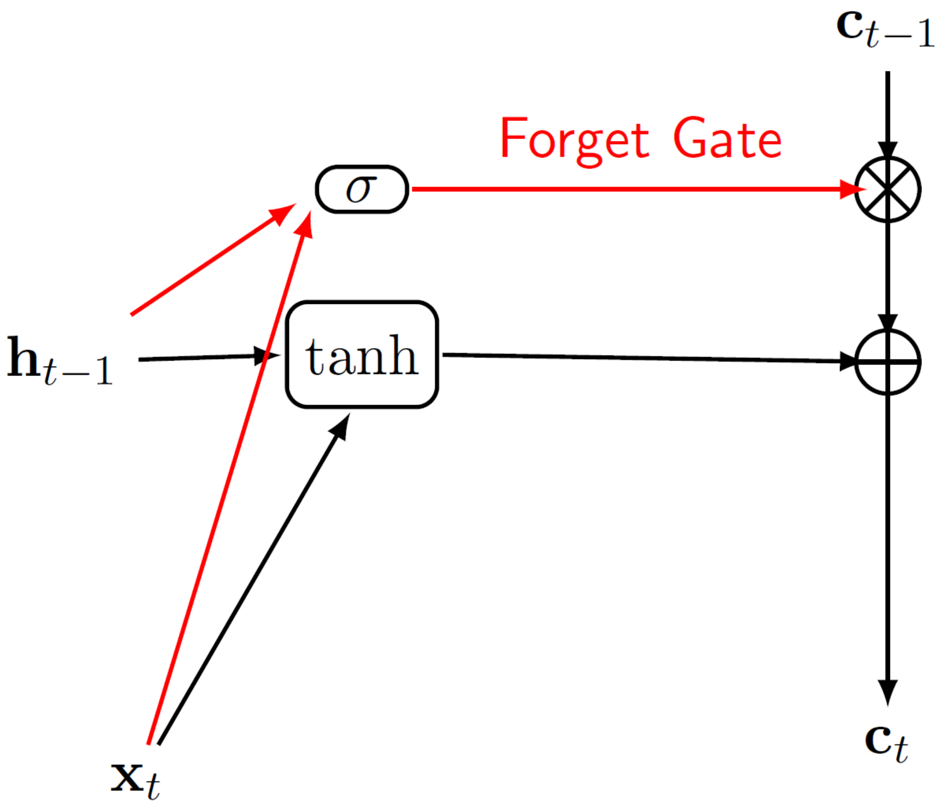

$$f_t = \sigma(W_f \mathbf{h}_{t-1} + U_f \mathbf{x}_t)$$

$$\tilde{\mathbf{c}}_t = \tanh(W \mathbf{h}_{t-1} + U \mathbf{x}_t)$$
$$\mathbf{c}_t = f_t \odot \mathbf{c}_{t-1} + \tilde{c}_t$$

Long Short-Term Memory

- Forget gate controls erase operation
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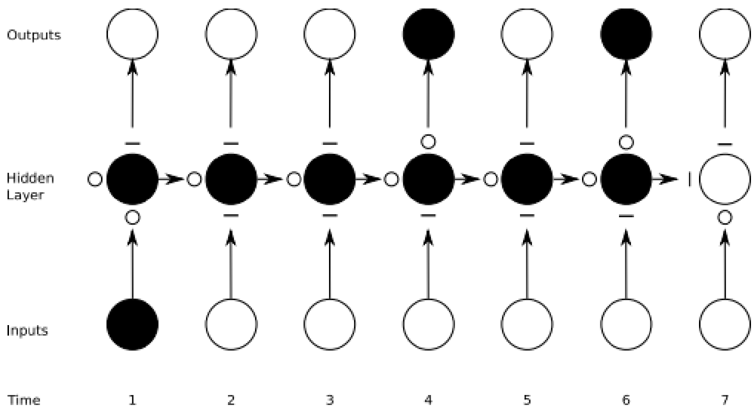

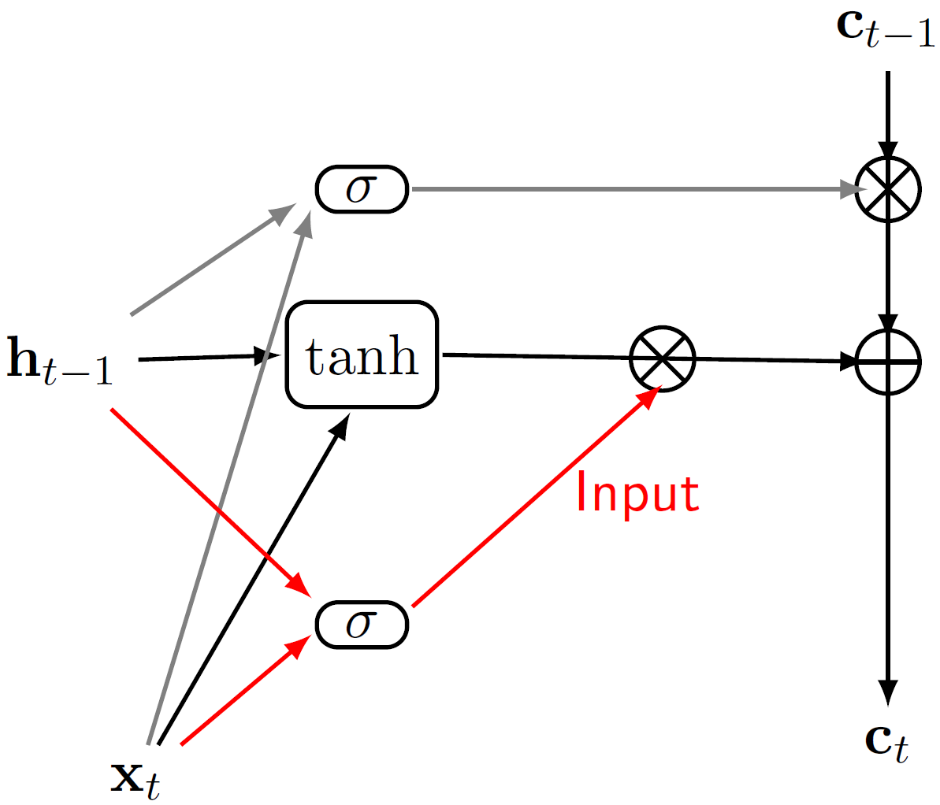

$$f_t = \sigma(W_f \mathbf{h}_{t-1} + U_f \mathbf{x}_t)$$
$$i_t = \sigma(W_i \mathbf{h}_{t-1} + U_i \mathbf{x}_t)$$

$$\tilde{\mathbf{c}}_t = \tanh(W \mathbf{h}_{t-1} + U \mathbf{x}_t)$$
$$\mathbf{c}_t = f_t \odot \mathbf{c}_{t-1} + i_t \odot \tilde{c}_t$$

Long Short-Term Memory

- Forget gate controls erase operation
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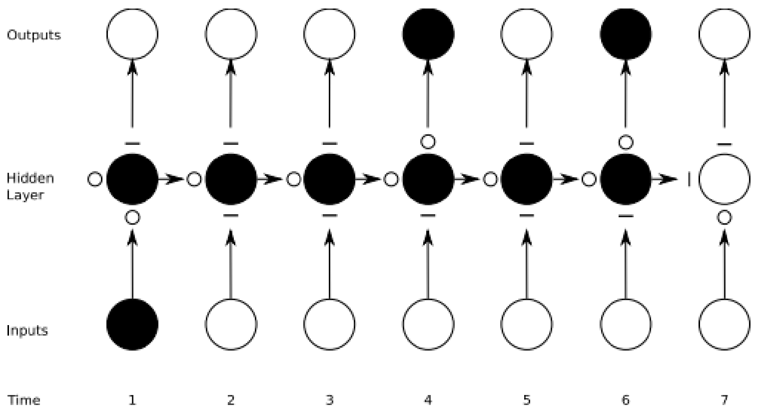
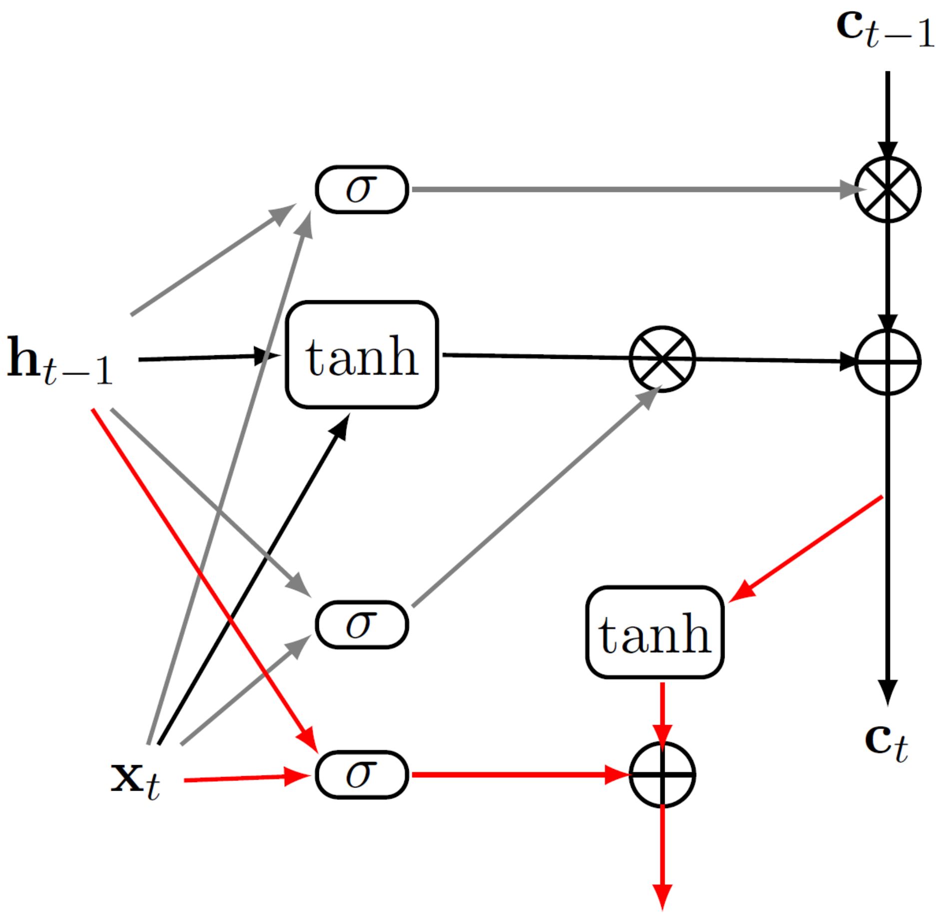


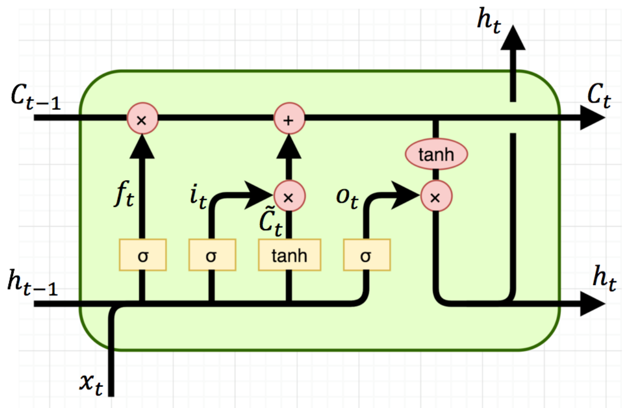


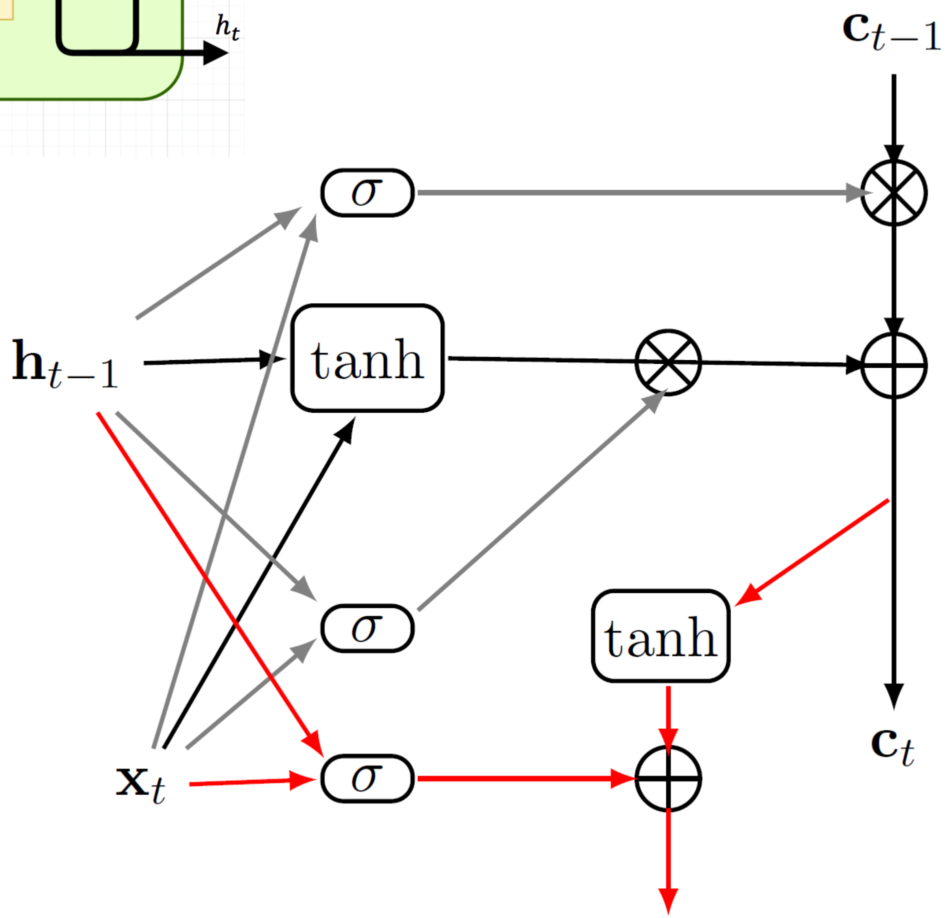
$$f_t = \sigma(W_f \mathbf{h}_{t-1} + U_f \mathbf{x}_t)$$
$$i_t = \sigma(W_i \mathbf{h}_{t-1} + U_i \mathbf{x}_t)$$
$$o_t = \sigma(W_o \mathbf{h}_{t-1} + U_o \mathbf{x}_t)$$

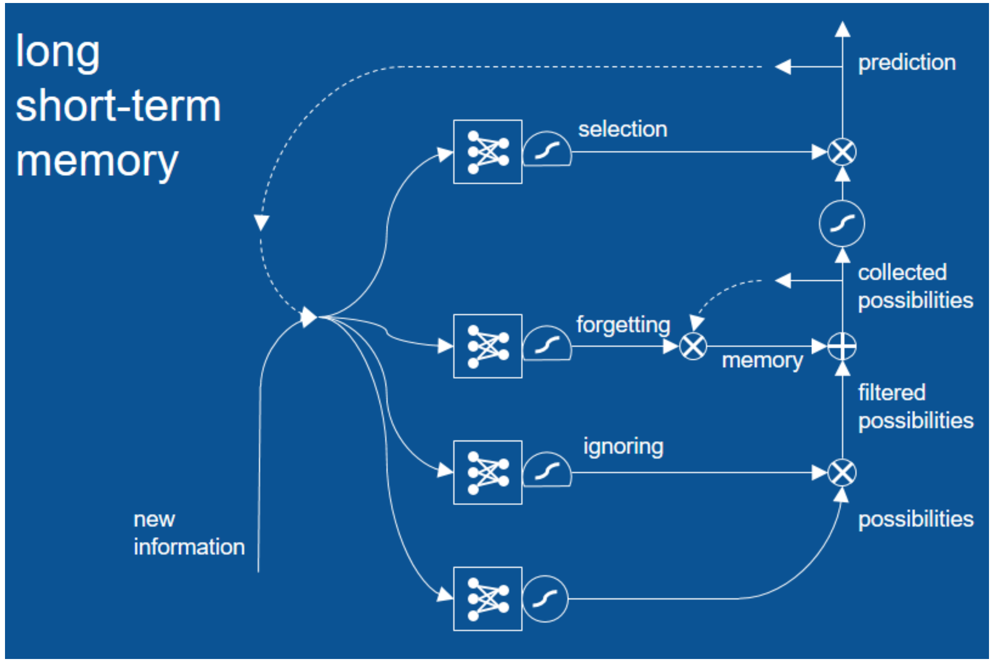
$$\tilde{\mathbf{c}}_t = \tanh(W \mathbf{h}_{t-1} + U \mathbf{x}_t)$$
$$\mathbf{c}_t = f_t \odot \mathbf{c}_{t-1} + i_t \odot \tilde{c}_t$$

$$\mathbf{h}_t = \mathbf{o}_t \odot \tanh(\mathbf{c}_t)$$

Long Short-Term Memory







Weakness of RNN and LSTM

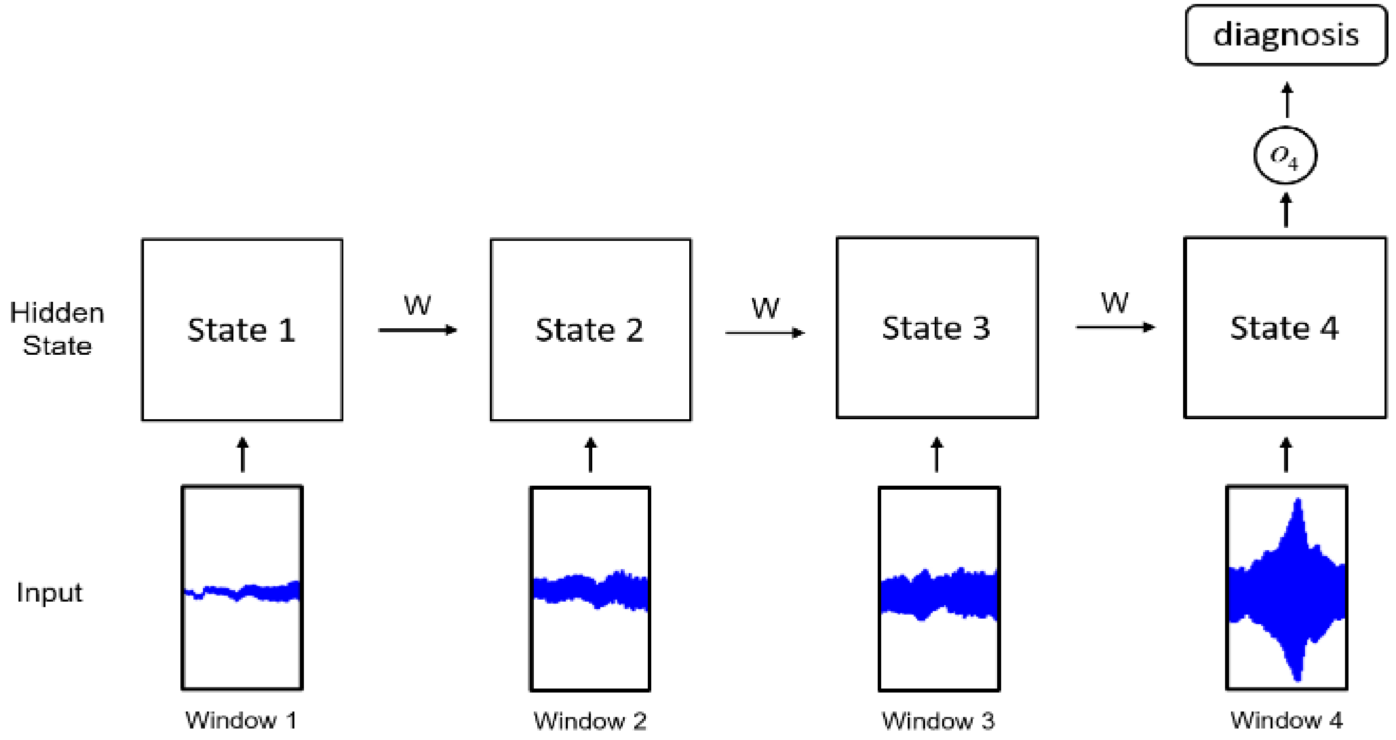
- Sequential computation is slow

- Vanishing and exploding gradients are still problematic

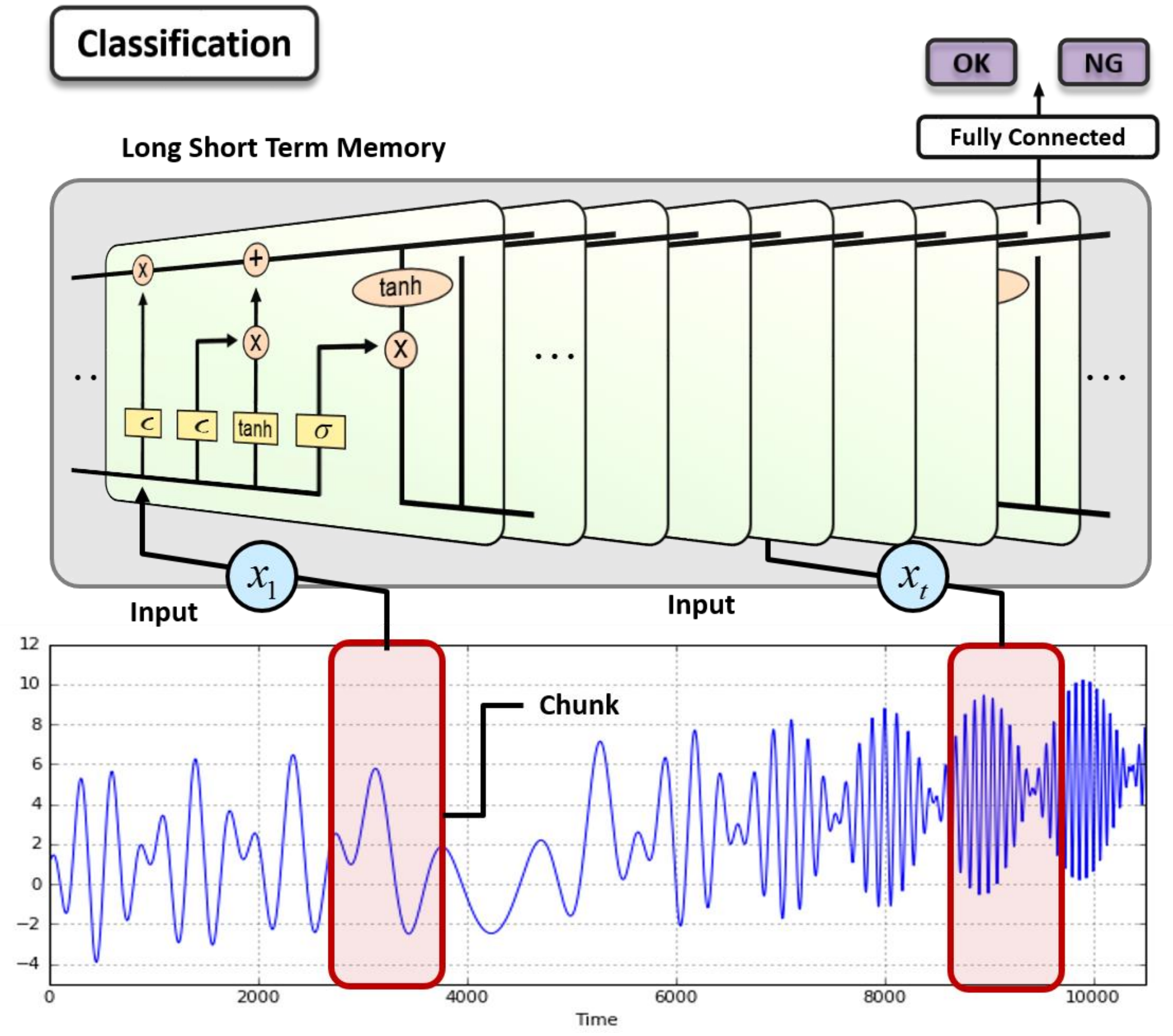
- Long-term credit assignment is difficult

LSTM Implementation

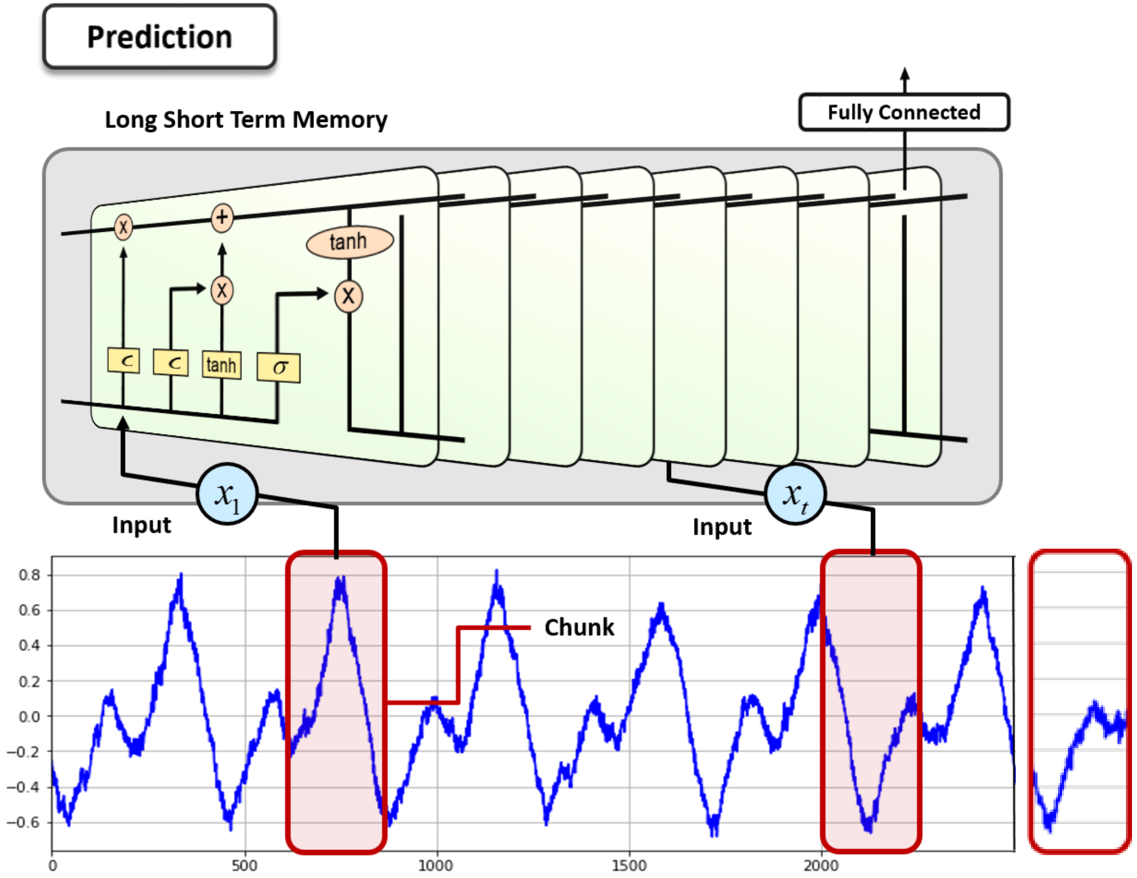
Time Series Data and LSTM



LSTM for Classification



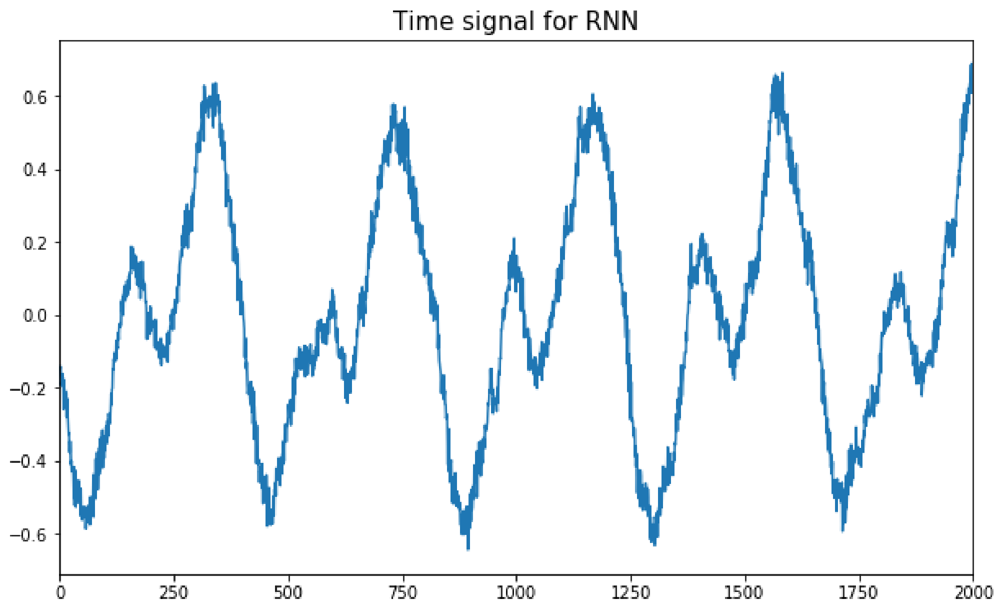
LSTM for Prediction



POSTECH

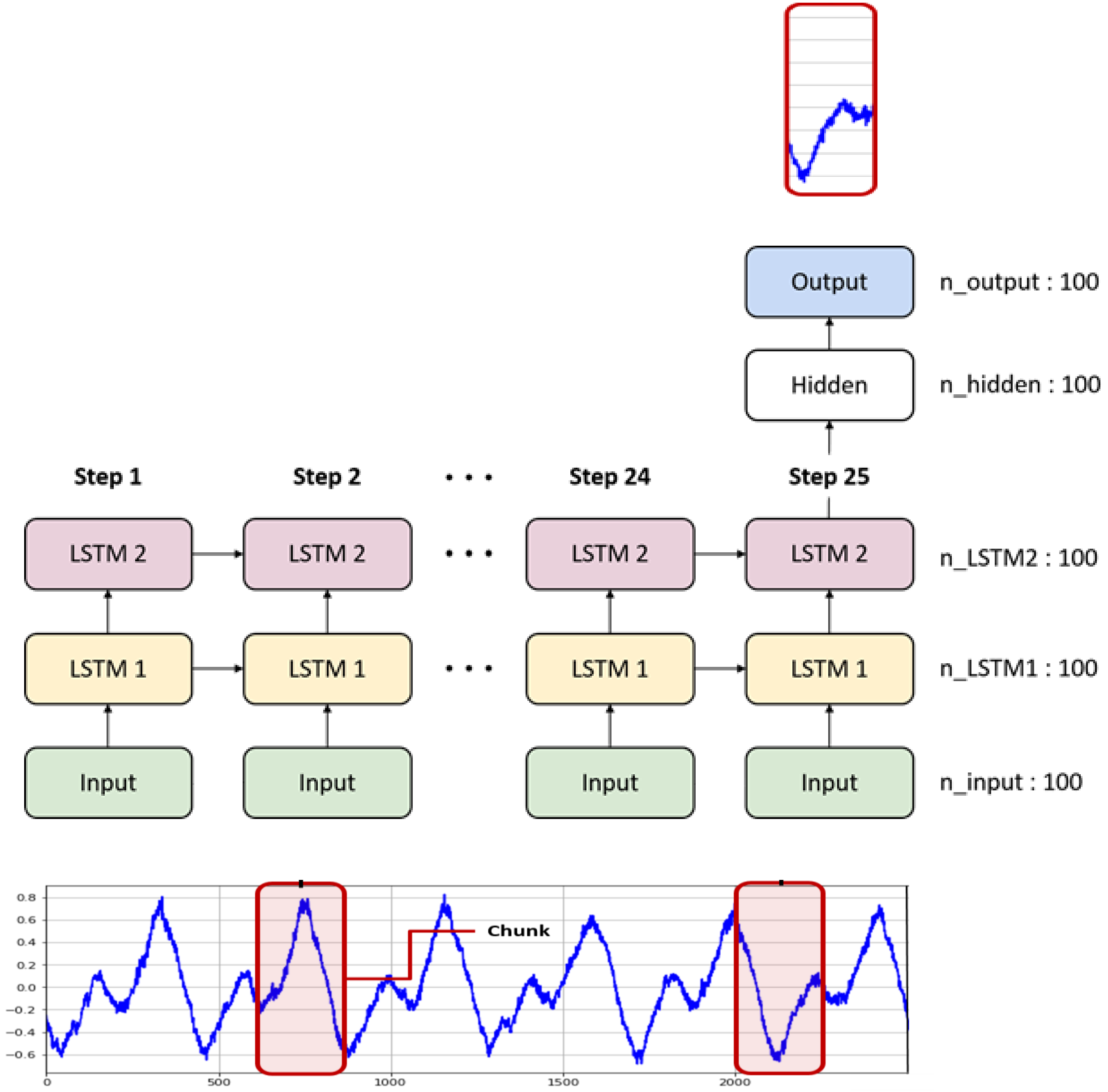
LSTM with TensorFlow

- An example for predicting a next piece of an acceleration signal
- Regression problem



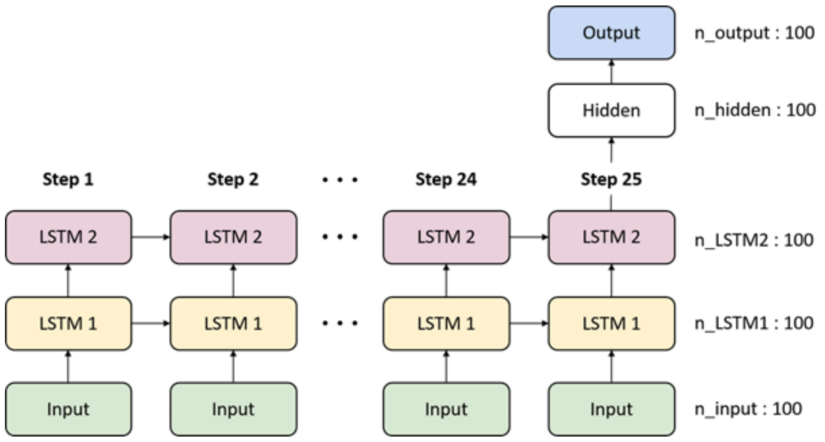
LSTM Structure

```
n_step = 25
n_input = 100

## LSTM shape
n_lstm1 = 100
n_lstm2 = 100

## Fully connected
n_hidden = 100
n_output = 100
```



LSTM, Weights and Biases

- LSTM Cell
  - Do not need to define weights and biases of LSTM cells

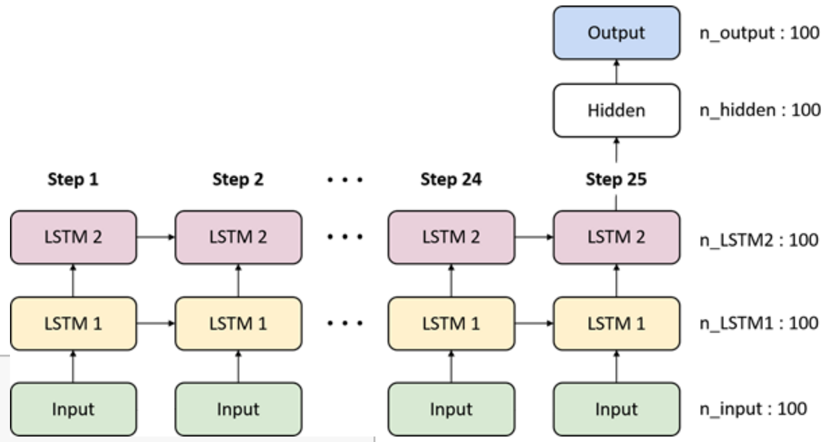


- Fully connected
  - Define parameters based on the predefined layer size
  - Initialize with a normal distribution with $\mu = 0$ and $\sigma = 0.01$

```
weights = {
    'hidden' : tf.Variable(tf.random_normal([n_lstm2, n_hidden], stddev = 0.01)),
    'output' : tf.Variable(tf.random_normal([n_hidden, n_output], stddev = 0.01))
}

biases = {
    'hidden' : tf.Variable(tf.random_normal([n_hidden], stddev = 0.01)),
    'output' : tf.Variable(tf.random_normal([n_output], stddev = 0.01))
}

x = tf.placeholder(tf.float32, [None, n_step, n_input])
y = tf.placeholder(tf.float32, [None, n_output])
```

Build a Model

- First, define the LSTM cells
- Second, compute hidden state (h) and LSTM cell (c) with the predefined LSTM cell and input




```python
def build_model(x, weights, biases):
    with tf.variable_scope('rnn'):
        # Build RNN network
        with tf.variable_scope('lstm1'):
            lstm1 = tf.nn.rnn_cell.LSTMCell(n_lstm1)
            h1, c1 = tf.nn.dynamic_rnn(lstm1, x, dtype = tf.float32)
        with tf.variable_scope('lstm2'):
            lstm2 = tf.nn.rnn_cell.LSTMCell(n_lstm2)
            h2, c2 = tf.nn.dynamic_rnn(lstm2, h1, dtype = tf.float32)

        # Build classifier
        hidden = tf.add(tf.matmul(h2[:,-1,:], weights['hidden']), biases['hidden'])
        hidden = tf.nn.relu(hidden)
        output = tf.add(tf.matmul(hidden, weights['output']), biases['output'])
        return output
```

Cost, Initializer and Optimizer

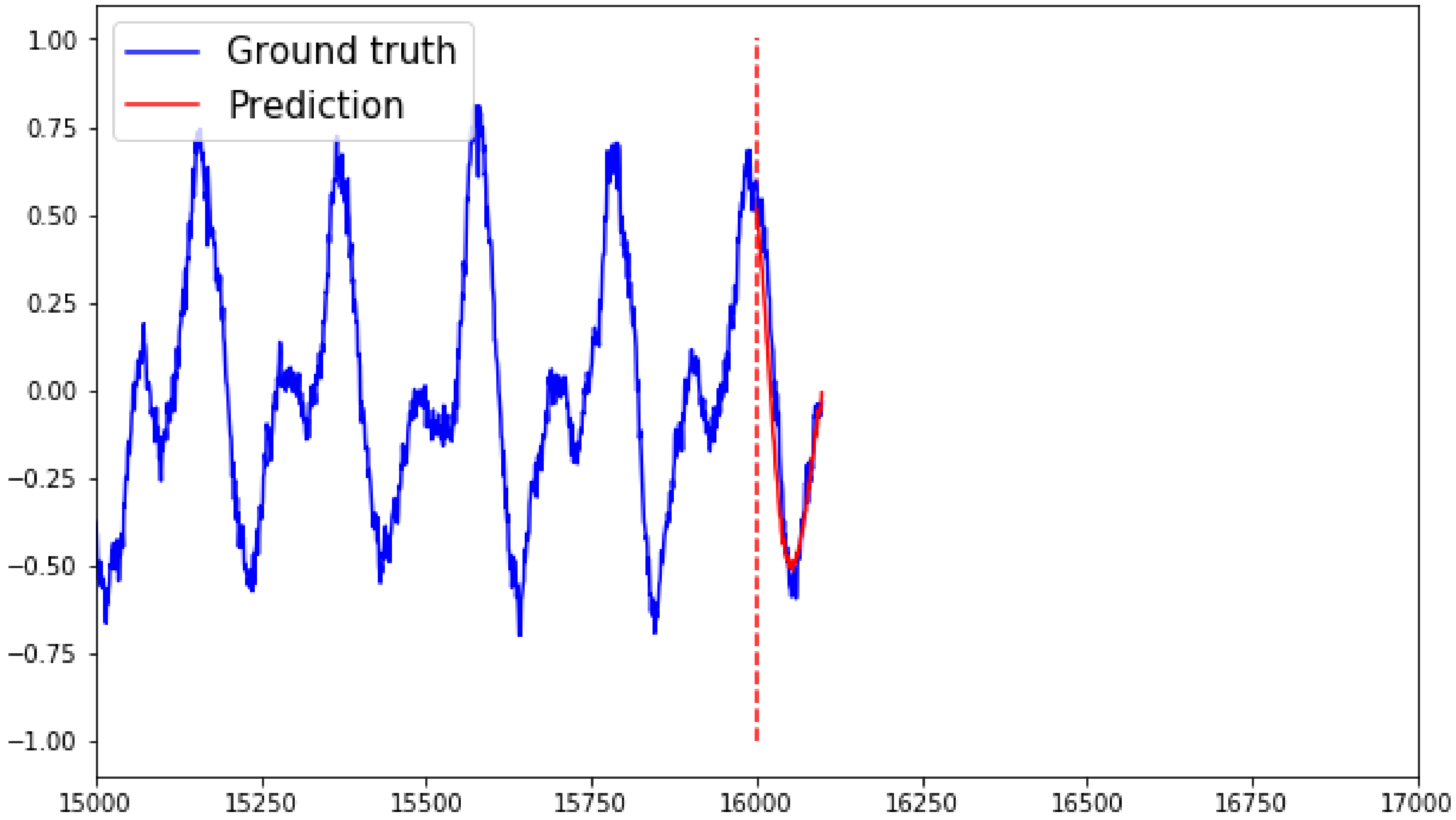
- Loss
  - Regression: Squared loss

$$\frac{1}{N}\sum_{i=1}^{N}(\hat{y}^{(i)} - y^{(i)})^2$$

- Initializer
  - Initialize all the empty variables
- Optimizer
  - AdamOptimizer: the most popular optimize

```
LR = 0.0001

pred = build_model(x, weights, biases)
loss = tf.square(tf.subtract(y, pred))
loss = tf.reduce_mean(loss)

optm = tf.train.AdamOptimizer(LR).minimize(loss)
init = tf.global_variables_initializer()

sess = tf.Session()
```

Prediction Example



Prediction Example