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THE PHYSICS AND ASTRONOMY CLUB
IIT MADRAS

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Week - 2
Module - 2

ORBITAL MECHANICS

16.07.21

VARUN KUMAR S

Overview

- History
- Kepler's Laws of planetary motion
- Two body problem
- Types of Orbits
- Orbital Maneuvers
- Animating Hohmann transfer

History

- Heliocentric model was first proposed by Nicolaus Copernicus in 16th century.
- Johannes Kepler introduced laws for the orbits in the following century.
- Galileo supported Keplerian laws through his observations.
- Galileo's observation received opposition from Catholic church.


Galileo galilei trying to explain that the earth orbits the sun
the church

- Newton published more general laws related to celestial motion.
- Until the rise of space travel there was little distinction between orbital and celestial mechanics.
- During the launch of Sputnik the field was called space dynamics.
- Astrodynamics was developed in 1930s by Samuel Herrick.
- Astrodynamics combined with powerful computers made the first moon landing possible in 1969.

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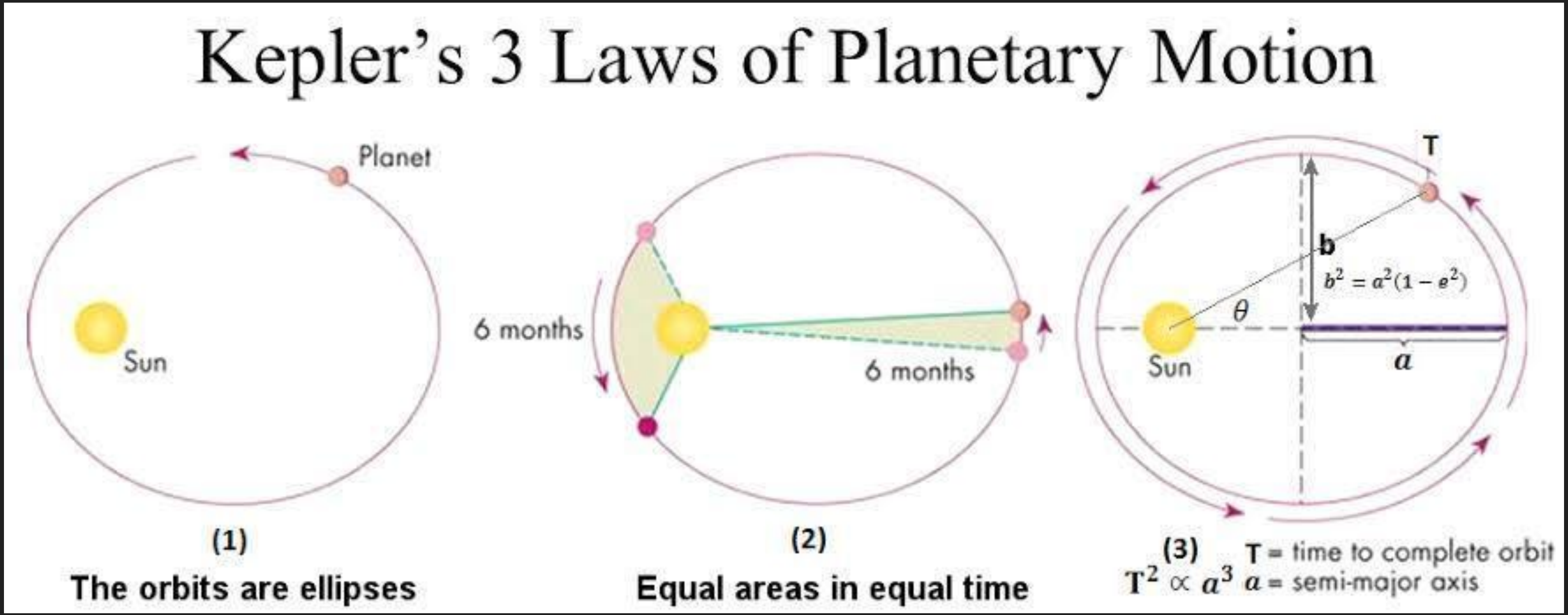
Kepler’s Laws of planetary motion

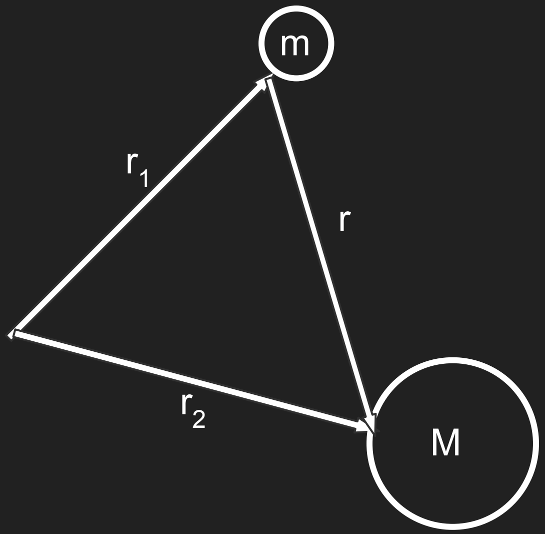
- All planets move about the Sun in elliptical orbits, having the Sun as one of the foci.
- A radius vector joining any planet to the Sun sweeps out equal areas in equal lengths of time.
- The square of the time period of revolution of a planet about the Sun is proportional to the cube of the major axis of the ellipse.


Kepler's 3 Laws of Planetary Motion
Planet
Sun
(1)
The orbits are ellipses
6 months
6 months
(2)
Equal areas in equal time
T
b
$b^2 = a^2(1 - e^2)$
θ
Sun
a
(3)
T = time to complete orbit
$T^2 \propto a^3$
a = semi-major axis

Two body problem

Newton's universal law of gravitation:

$$F = -\frac{GMm}{|\vec{r}|^3}\vec{r}$$



After some simplifications,

$$\ddot{\vec{r}} + \frac{\mu}{|\vec{r}|^3}\vec{r} = 0$$ where, $\mu = G(M+m)$

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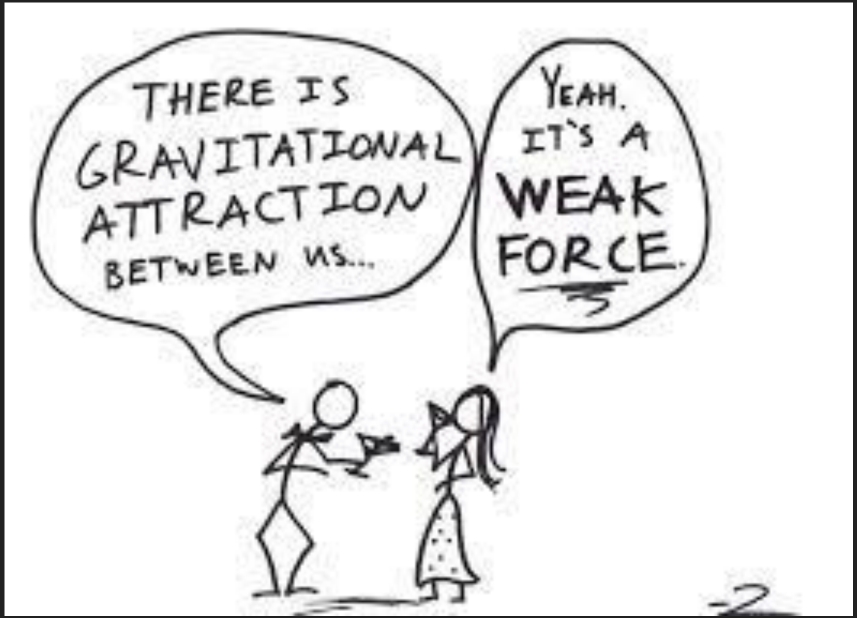

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Converting to polar coordinates

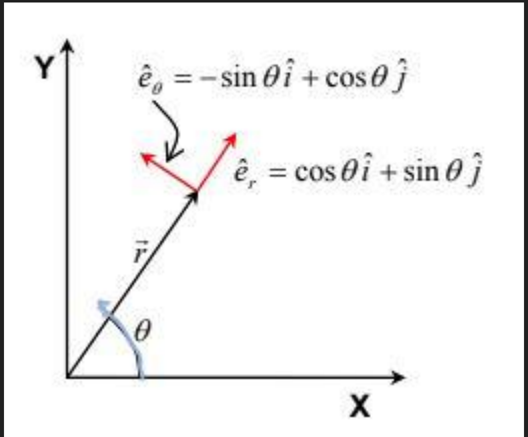
$$\dot{\hat{e}}_r = \dot{\theta}\hat{e}_\theta$$

$$\dot{\hat{e}}_\theta = -\dot{\theta}\hat{e}_r$$

$$\vec{r} = r\hat{e}_r$$

$$\dot{\vec{r}} = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$$

$$\ddot{\vec{r}} = (\ddot{r} - r\dot{\theta}^2)\hat{e}_r + (2\dot{r}\dot{\theta} + r\ddot{\theta})\hat{e}_\theta$$



Equation of motion in polar coordinates

$$(\ddot{r} - r\dot{\theta}^2 + \frac{\mu}{r^2})\hat{e}_r + (2\dot{r}\dot{\theta} + r\ddot{\theta})\hat{e}_\theta = 0$$

Radial component: $\ddot{r} - r\dot{\theta}^2 + \frac{\mu}{r^2} \longrightarrow$ Energy = E

Circumferential component: $2\dot{r}\dot{\theta} + r\ddot{\theta} \longrightarrow$ Angular momentum = h

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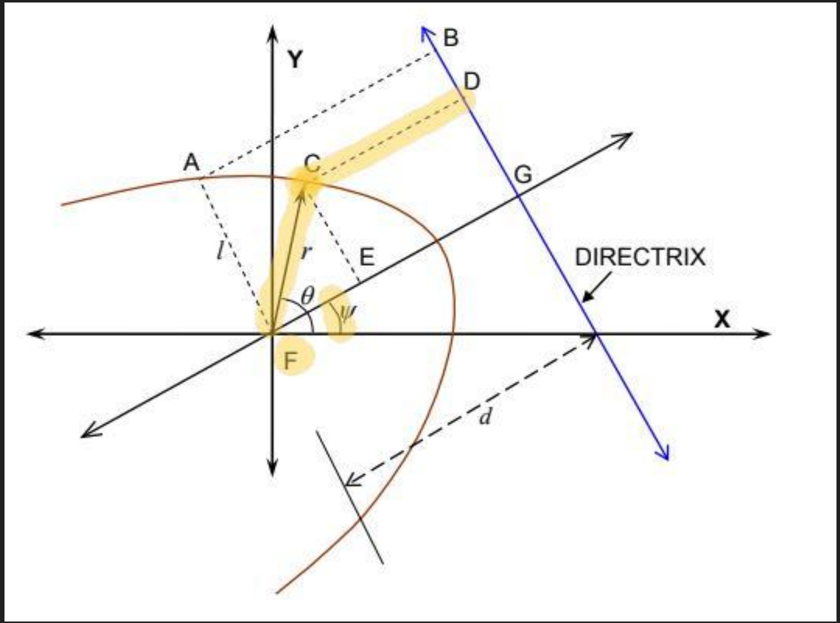
Equation of trajectory

$$\ddot{r} - r\dot{\theta}^2 + \frac{\mu}{r^2} = 0$$

$$r = \frac{h^2/\mu}{(1 + e\cos(\theta - \psi))}$$

H and Ψ are constants



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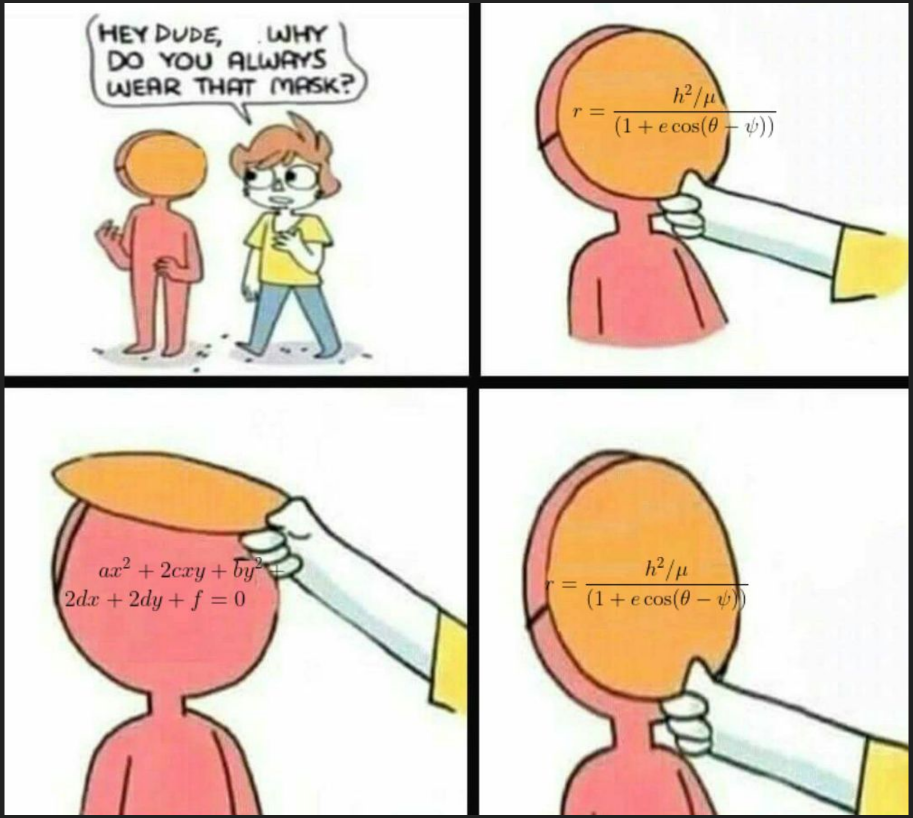

Types of Orbits

Energy equation

$$E_T = \frac{1}{2}\dot{r}^2 + \frac{1}{2}r\dot{\theta}^2 - \frac{\mu}{r}$$

Also, $$E_T = -\frac{\mu^2(1-e^2)}{2h^2}$$

- Elliptic Orbit: e<1 then E_T<0 , P.E > K.E
- Parabolic trajectory: e=1 then E_T=0, P.E = K.E
- Hyperbolic trajectory: e>1 then E_T>0, P.E < K.E

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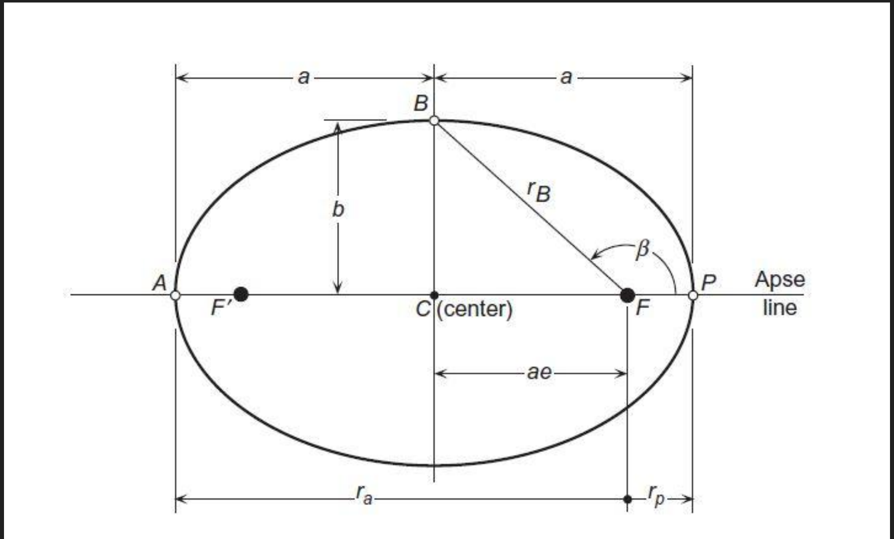
Elliptic Orbits

$$r = \frac{h^2/\mu}{1 + e\cos\theta}$$

Perigee: $$r_p = \frac{h^2/\mu}{1+e} = a(1-e)$$

Apogee: $$r_a = \frac{h^2/\mu}{1-e} = a(1+e)$$

$$\frac{h^2}{\mu} = a(1-e^2) = \frac{b^2}{a}$$



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Orbital Energy:

$$E_T = -\frac{\mu^2(1-e^2)}{2h^2} = -\frac{\mu}{2a}$$

$$E_T = \frac{V^2}{2} - \frac{\mu}{r} = -\frac{\mu}{2a}$$

Velocity at any arbitrary point on the ellipse:

$$V = \sqrt{\mu\left(\frac{2}{r} - \frac{1}{a}\right)} = \sqrt{\frac{\mu}{a}\left(\frac{2a}{r} - 1\right)}$$

Velocity at Periapsis: $$V_P = \sqrt{\frac{\mu}{a}\left(\frac{1+e}{1-e}\right)}$$

Velocity at Apoapsis: $$V_A = \sqrt{\frac{\mu}{a}\left(\frac{1-e}{1+e}\right)}$$

Circular Orbit (e = 0)

Velocity: $$V = \sqrt{\frac{\mu}{r}}$$

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Parabolic Trajectory

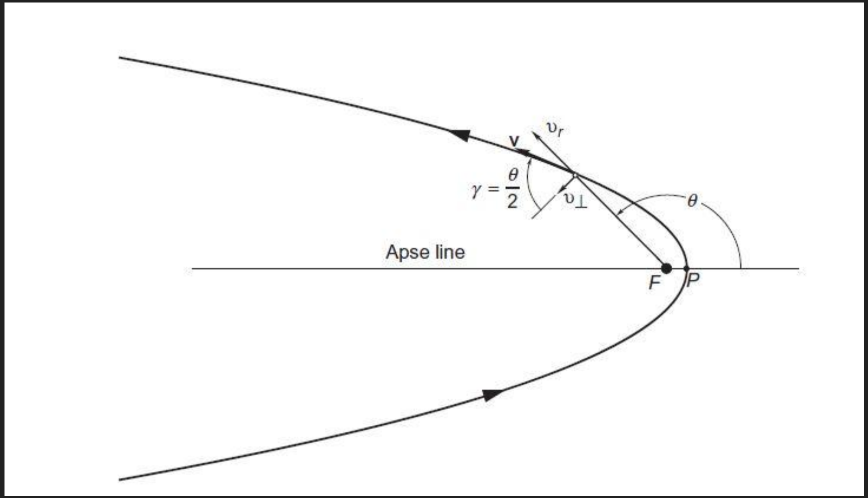
$$r = \frac{h^2/\mu}{1+\cos\theta}$$

$$r_P = \frac{h^2}{2\mu}$$

Velocity: $V = \sqrt{\frac{2\mu}{r}}$

$$V_{max} = \sqrt{\frac{2\mu}{r_P}} = \frac{2\mu}{h}$$

$$V_{min} = 0$$



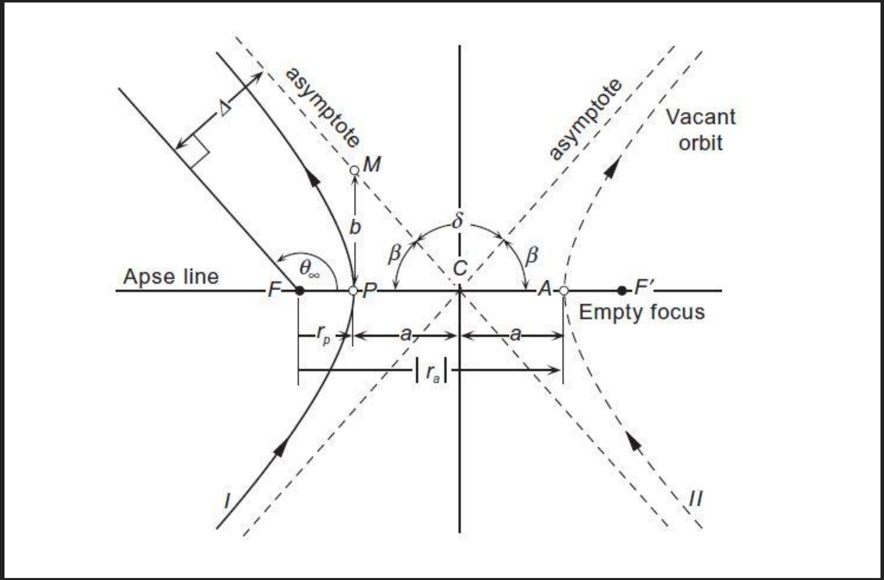
Hyperbolic Trajectory

Orbital Energy: $$E_T = -\frac{\mu^2(1-e^2)}{2h^2} = \frac{\mu}{2a}$$

Velocity: $$V = \sqrt{\frac{\mu}{a}\left(\frac{2a}{r}+1\right)}$$

Velocity at Periapsis: $$V_P = \sqrt{\frac{\mu}{a}\left(\frac{e+1}{e-1}\right)}$$

Velocity at infinity: $$V_{r\to\infty} = \sqrt{\frac{\mu}{a}}$$



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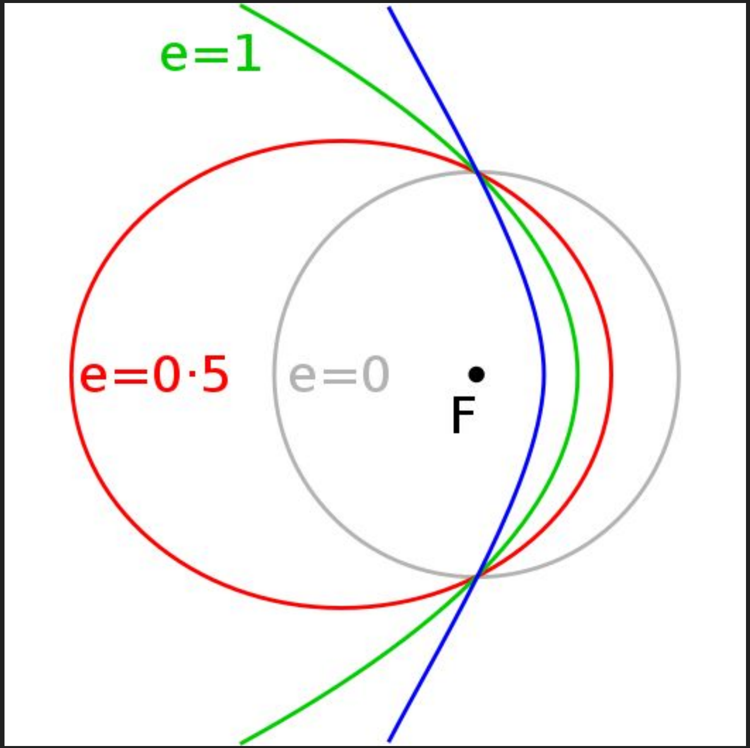
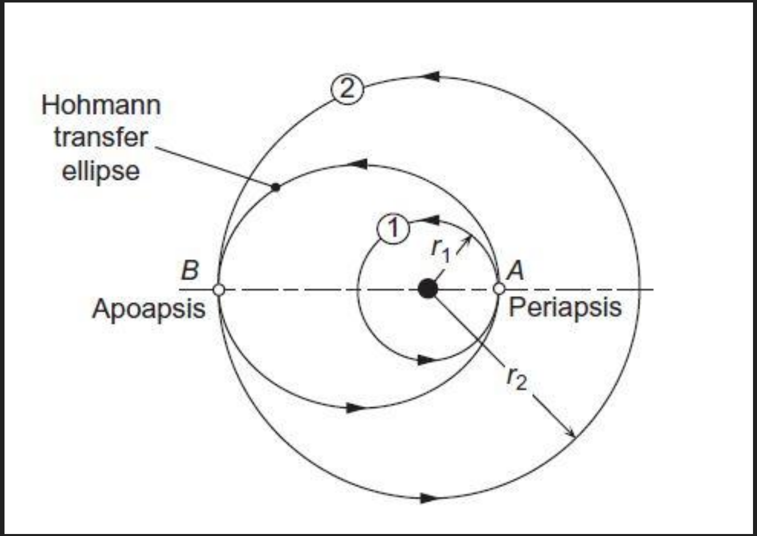


Orbital Maneuvers

Hohmann transfer

- Most energy efficient two impulse maneuver for orbit transfer between coplanar circular orbits with common focus.
- Periapsis and Apoapsis are the radii of inner and outer circular orbit respectively.
- This transfer can occur in either direction, from inside to outside or vice-versa.



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Velocities for circular orbit:

$$V_{r_1} = \sqrt{\frac{\mu}{r_1}} \qquad V_{r_2} = \sqrt{\frac{\mu}{r_2}}$$

Velocities for elliptical orbit:

$$V_P = \sqrt{\frac{2\mu r_2}{r_1(r_1 + r_2)}} \qquad V_A = \sqrt{\frac{2\mu r_1}{r_2(r_1 + r_2)}}$$

$$\Delta V_1 = V_P - V_{r_1} = \sqrt{\frac{2\mu r_2}{r_1(r_1 + r_2)}} - \sqrt{\frac{\mu}{r_1}}$$

$$\Delta V_2 = V_A - V_{r_2} = \sqrt{\frac{2\mu r_1}{r_2(r_1 + r_2)}} - \sqrt{\frac{\mu}{r_2}}$$

$$\Delta V_{Total} = \Delta V_1 + \Delta V_2$$

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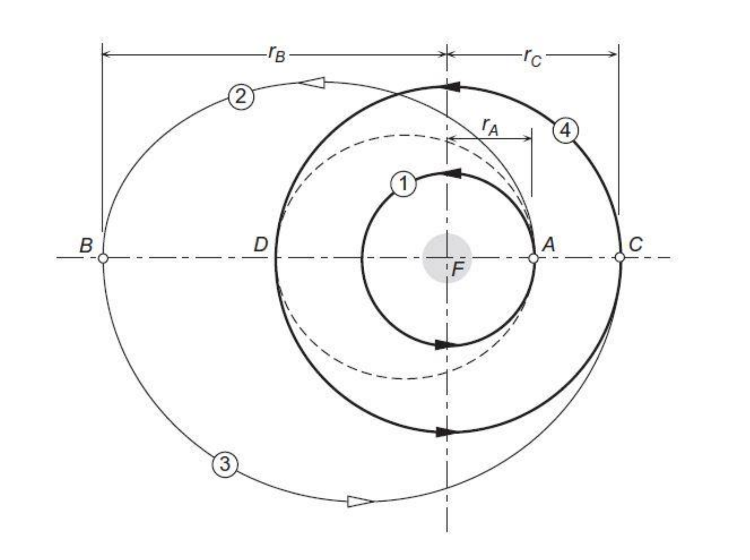
Bi-elliptic transfer

- Two half elliptic orbits instead of one.
- Some bi-elliptic transfer requires lower amount of ΔV than Hohmann transfer.
- r_C/r_A <11.94 then Hohmann transfer is always better.
- 11.94 < r_C/r_A < 15.58 then the best case depends upon r_B .
- r_C/r_A > 15.58 then any bi-elliptic transfer is better.



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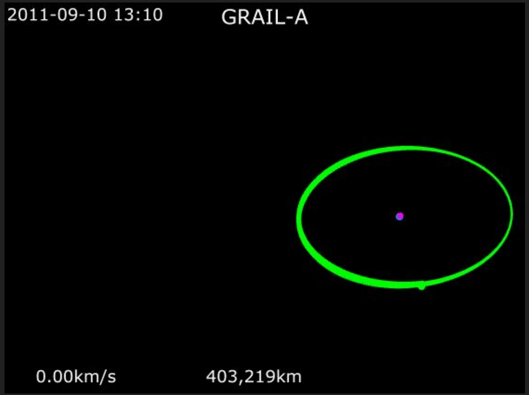
Low-energy transfer

- Change orbits using very little fuel.
- It takes much longer than other maneuvers.
- Also known as weak stability boundary trajectories or ballistic capture trajectories.
- It follows special pathway in space which is referred as Interplanetary transport network.



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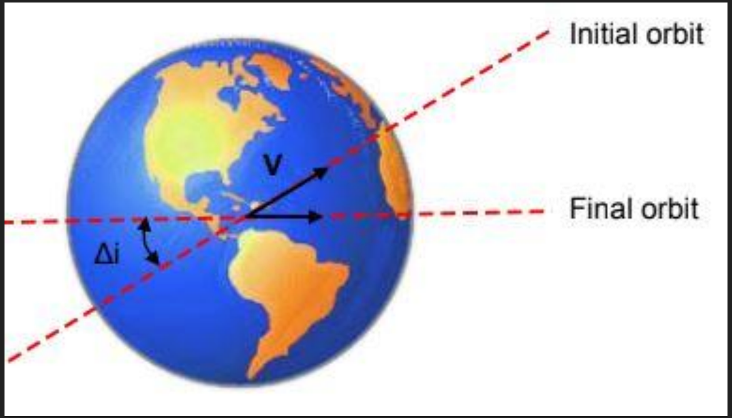
Inclination change maneuver

- Aimed to change inclination of the orbit.
- Also known as orbital plane change.
- ΔV is applied at the point of intersection of both the planes.
- For a circular orbit:

$$\Delta v_i = 2v\sin\left(\frac{\Delta i}{2}\right)$$



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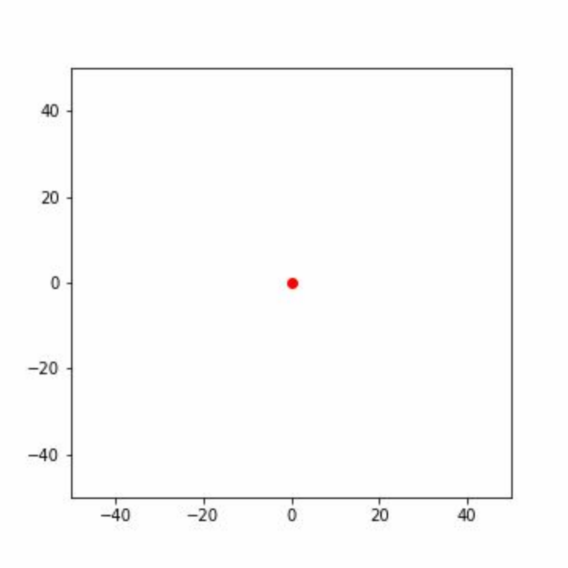
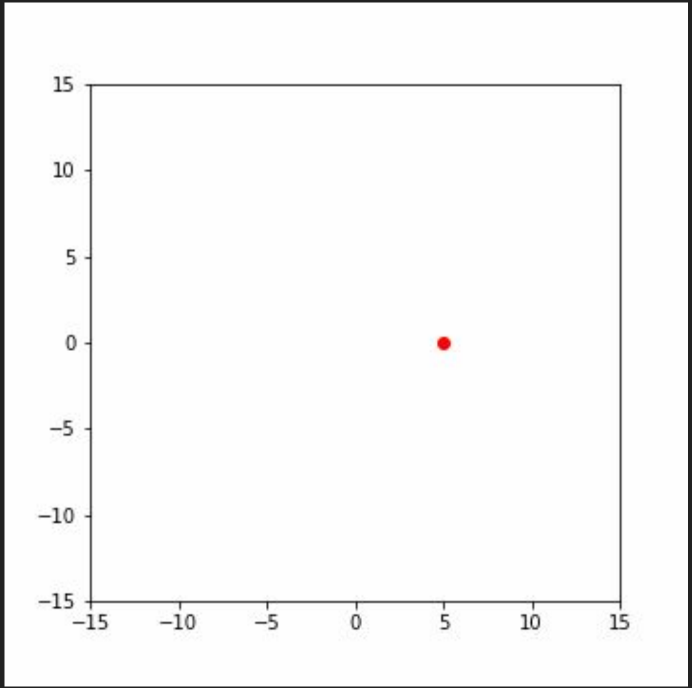
Animating Hohmann transfer

matplotlib.animation




When you learn
Orbital Mechanics
But then there
is a task based
on Orbital Mechanics

Hohmann transfer



Useful Links:

- Python animation - https://www.youtube.com/watch?v=GtZxk8Wa3Jw
- Matplotlib documentation for animation - https://matplotlib.org/stable/api/animation_api.html
- matplotlib.animation.FuncAnimation documentation - https://matplotlib.org/stable/api/_as_gen/matplotlib.animation.FuncAnimation.html
- Some animation examples - https://www.geeksforgeeks.org/using-matplotlib-for-animations/