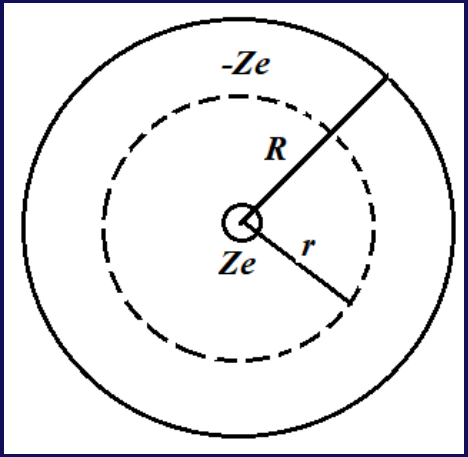
2019年期中考试卷答案

I. Fill in the space underlined (40% in total)

1. In a 1911 paper, Ernest Rutherford said: In order to form some idea of the forces required to deflect an α particle through a large angle, consider an atom containing a point positive charge ***Ze*** at its center and surrounded by a distribution of negative electricity ***-Ze*** uniformly distributed within a sphere of radius ***R***. The electric field ***E*** for a point inside an atom at a distance ***r*** from the center is ______________.



$$\oint \vec{E}\bullet d\vec{A} = \frac{q}{\varepsilon_0}$$

$$q = Ze - \frac{4}{3}\pi r^3 \cdot \frac{Ze}{\frac{4}{3}\pi R^3} = Ze\left(1 - \frac{r^3}{R^3}\right)$$

$$E \cdot 4\pi r^2 = \frac{Ze}{\varepsilon_0}\left(1 - \frac{r^3}{R^3}\right)$$

$$E = \frac{Ze}{4\pi\varepsilon_0}\left[\frac{1}{r^2} - \frac{r}{R^3}\right]$$

2. As shown in Fig. 1, two long wires a distance ***b*** apart carry equal antiparallel currents ***i***. The magnetic induction strength ***B*** at the point P, which is equidistant from the wires, is given by

____________ .

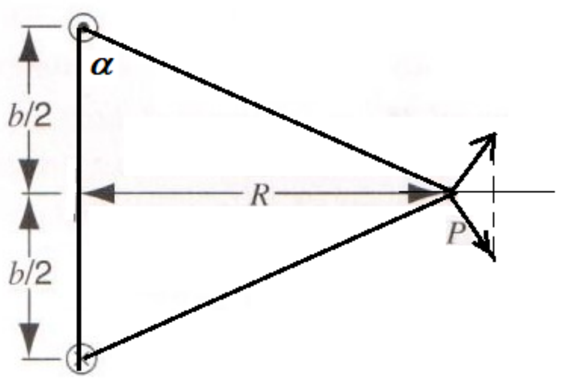


Figure 1

$$B = \frac{\mu_0 i}{2\pi r}$$

$$B_x = \frac{\mu_0 i}{2\pi r} \cdot 2\cos\alpha$$

$$= \frac{\mu_0 i}{2\pi\sqrt{R^2 + \frac{b^2}{4}}} \cdot \frac{2 \cdot \frac{b}{2}}{\sqrt{R^2 + \frac{b^2}{4}}}$$

$$= \frac{\mu_0 i b}{2\pi(R^2 + \frac{b^2}{4})} = \frac{2\mu_0 i b}{\pi(4R^2 + b^2)}$$

3. As shown in Fig. 2, a parallel plate capacitor with capacitance *C* is charged to a potential difference *V* and is then disconnected from the charging source. The capacitor has an area *A* and a plate separation *d*. Assume that a glass plate of the same area *A* completely fills the space between the plates, and which has a dielectric constant κ_e. How much work is required to pull the glass plate out of the capacitor? ____________________________. Neglect fringe effects.

Figure 2

$$C_0 = \frac{\varepsilon_0 A}{d}, \quad C = \kappa_e \frac{\varepsilon_0 A}{d},$$

$$Q = CV = \kappa_e \frac{\varepsilon_0 A}{d} V$$

$$W = U' - U = \frac{1}{2} Q^2 \left(\frac{1}{C_0} - \frac{1}{C}\right)$$

$$= \frac{1}{2}(CV)^2 \left(\frac{C}{C_0} - 1\right)\frac{1}{C}$$

$$= \frac{1}{2}\kappa_e \frac{\varepsilon_0 A}{d} V^2 (\kappa_e - 1)$$

4. The capacitance of the capacitor in Fig. 3 is___.

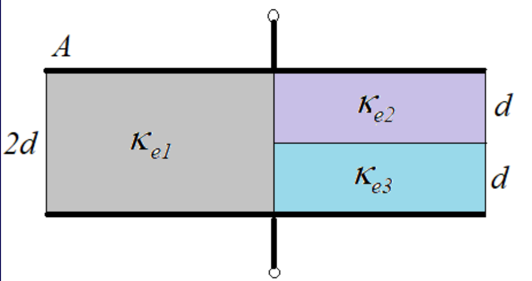


Figure 3

$$C_1 = \kappa_{e1}\frac{\varepsilon_0 A/2}{2d} = \frac{\kappa_{e1}\varepsilon_0 A}{4d},$$

$$C_2 = \kappa_{e2}\frac{\varepsilon_0 A/2}{d} = \frac{\kappa_{e2}\varepsilon_0 A}{2d}$$

$$C_3 = \kappa_{e3}\frac{\varepsilon_0 A/2}{d} = \frac{\kappa_{e3}\varepsilon_0 A}{2d}$$

$$C = C_1 + \frac{C_2 C_3}{C_2 + C_3} = \frac{\kappa_{e1}\varepsilon_0 A}{4d}[\kappa_{e1} + \frac{2\kappa_{e2}\kappa_{e3}}{\kappa_{e2}+\kappa_{e3}}]$$

5. Figure 5 shows a long wire carrying a current i_1. The rectangular loop carries a current i_2. Assume that a = 1.10 cm, b = 9.20 cm, L = 32.3 cm, i_1 = 28.6 A, and i_2 = 21.8 A. The resultant force acting on the loop is ______________.

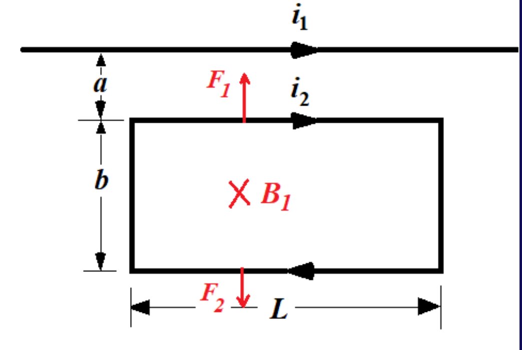


Figure 4

$$B_1 = \frac{\mu_0 i_1}{2\pi r}$$

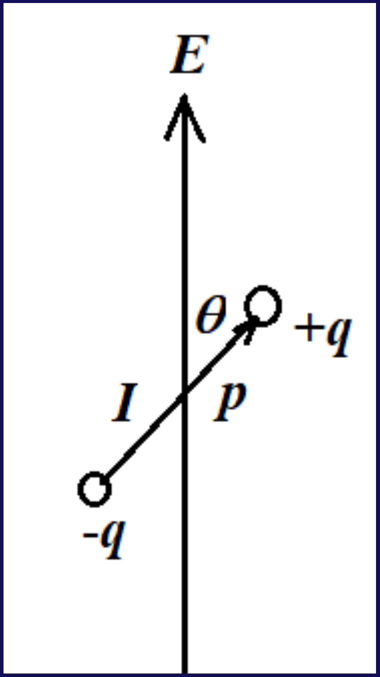
$$F = F_1 - F_2 = \frac{\mu_0 i_1}{2\pi a} i_2 L - \frac{\mu_0 i_1}{2\pi(a+b)} i_2 L$$

$$= \frac{\mu_0 i_1 i_2 L}{2\pi}\left[\frac{1}{a} - \frac{1}{a+b}\right]$$

$$= 2\times10^{-7}\times28.6\times21.8\times0.323\left[\frac{100}{1.1} - \frac{100}{10.3}\right]$$

$$= 3.27\times10^{-3}\,N$$

6. If an electric dipole with moment *p* (电偶极矩) and rotational inertia *I* (转动惯量) is placed in a uniform electric field *E*, the frequency for small amplitudes of oscillation about its equilibrium position is _____________.



$$\vec{\tau} = \vec{p} \times \vec{E}$$

$$-pE\sin\theta = I\frac{d^2\theta}{dt^2}$$

$$\frac{d^2\theta}{dt^2} + \frac{pE}{I}\theta = 0$$

$$\omega = \sqrt{\frac{pE}{I}}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{pE}{I}}$$

7. As shown in Fig. 5, there is a coaxial cable (同轴电缆) with a current density J_0 along z axis (out paper) in the region of $0 \le r \le a$, and a current density J_0 along $-z$ axis (inside paper) in the region of $a \le r \le b$. The magnetic induction strength $B(r)$ in the region of $a \le r \le b$ is ________.

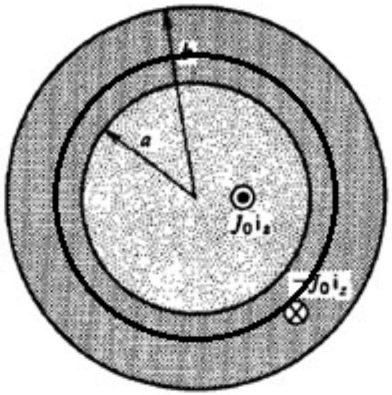


Figure 5

$$\int \vec{B} \cdot d\vec{l} = \mu_0 I$$

$$B \cdot 2\pi r = \mu_0 [J_0 \cdot \pi a^2 - J_0 \cdot \pi (r^2 - a^2)]$$

$$B = \frac{\mu_0}{2} J_0 (\frac{2a^2}{r} - r)$$

8. As shown in Fig. 6, there is a very long rectangular current loop, considered as two infinitely parallel line currents, a distance *D* apart, carrying the same current *I* in opposite directions. Another small rectangular loop with a size of *a* × *b* is located at the place with a distance *d* away from the left line current. The mutual inductance (互感系数) between them is ____________.

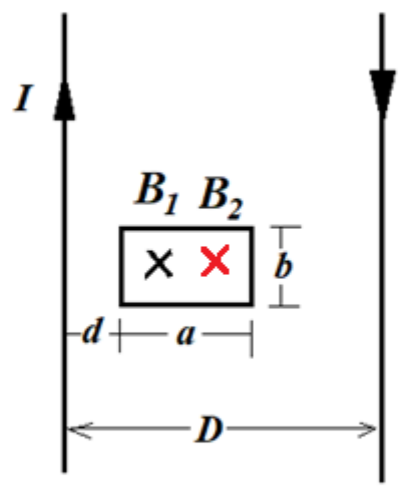


Figure 6

$$B=\frac{\mu_0 I}{2\pi r}$$

$$\Phi_1=\frac{\mu_0 I}{2\pi}\int_d^{d+a}\frac{bdr}{r}=\frac{\mu_0 Ib}{2\pi}\ln\frac{a+d}{d}$$

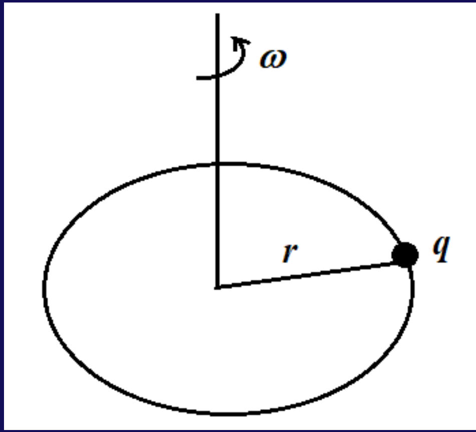
$$\Phi_2=\frac{\mu_0 I}{2\pi}\int_{D-(d+a)}^{D-d}\frac{bdr}{r}=\frac{\mu_0 Ib}{2\pi}\ln\frac{D-d}{D-(d+a)}$$

$$\Phi=\Phi_1+\Phi_2=\frac{\mu_0 Ib}{2\pi}[\ln\frac{a+d}{d}+\ln\frac{D-d}{D-(d+a)}]$$

$$=\frac{\mu_0 Ib}{2\pi}\ln\frac{(a+d)(D-d)}{d[D-(d+a)]}$$

$$M=\frac{\Phi}{I}=\frac{\mu_0 b}{2\pi}\ln\frac{(a+d)(D-d)}{d[D-(d+a)]}$$

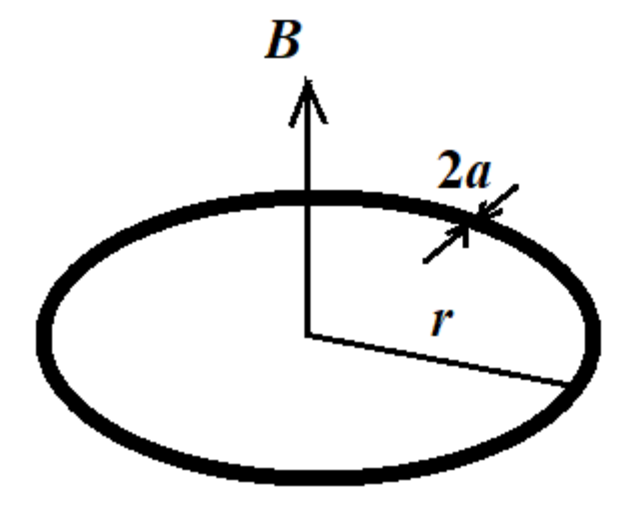
9. A charge q is distributed uniformly around a thin ring of radius r. The ring is rotating about an axis through its center and perpendicular to its plane at an angular speed ω. The magnetic moment due to the rotating charge is __________.



$$i = \frac{q}{T} = \frac{q\omega}{2\pi}$$

$$\mu = iA = \frac{q\omega}{2\pi} \cdot \pi r^2 = \frac{1}{2} q\omega r^2$$

10. A uniform magnetic field *B* is changing in magnitude a constant rate *dB/dt*. You are given a mass *m* of copper, which has the resistivity (电阻率) ρ and the density (密度) δ, that is to be drawn a circular loop (圆环). The induced current *i*, assuming *B* perpendicular to the loop, is given by



$$\varepsilon = \int \vec{E}\cdot d\vec{l} = -\frac{d\Phi}{dt} = -\pi r^2 \frac{dB}{dt}$$

$$i = \frac{\varepsilon}{R} = \frac{\pi r^2 \dfrac{dB}{dt}}{\rho\cdot\dfrac{2\pi r}{\pi a^2}} = \frac{\pi a^2 r}{2\rho}\frac{dB}{dt}$$

$$m = \delta\cdot 2\pi r\cdot \pi a^2$$

$$\pi a^2 r = \frac{m}{2\pi\delta}$$

$$i = \frac{\pi a^2 r}{2\rho}\frac{dB}{dt} = \frac{m}{4\pi\delta\rho}\frac{dB}{dt}$$

II. Problems (Present the necessary equations in solution) (60%)

1. (10%) As shown in Fig. 7, an infinite slab of thickness d carries a uniform current density $J_0 i_z$ except within a cylindrical hole of radius a centered within the slab. Please calculate the magnetic field inside and outside slab.

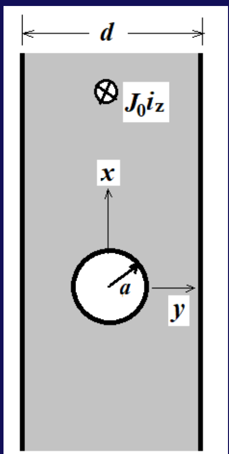
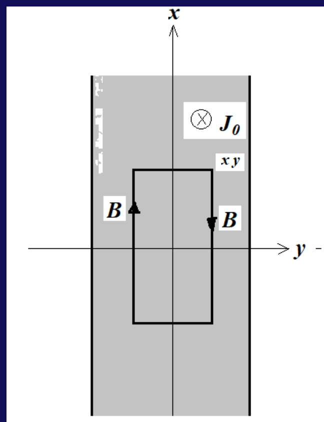


Figure 7

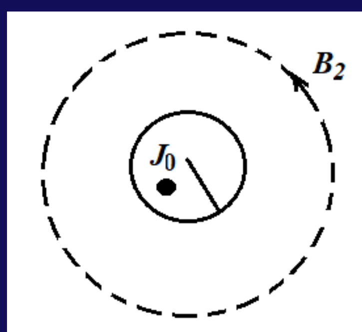


(1) 无限平板产生的磁场，填满空园柱

$$\int \vec{B}\cdot d\vec{l} = \mu_0 i$$

板内部：$\left(y \le \dfrac{d}{2}\right)$：　$B_1 \cdot 4x = \mu_0 J_0 \cdot 2x \cdot 2y$，$B_1 = \mu_0 J_0 y$

板外部：$\left(y > \dfrac{d}{2}\right)$：　$B_1 \cdot 4x = \mu_0 J_0 \cdot 2x \cdot d$，$B_1 = \dfrac{1}{2}\mu_0 J_0 d$

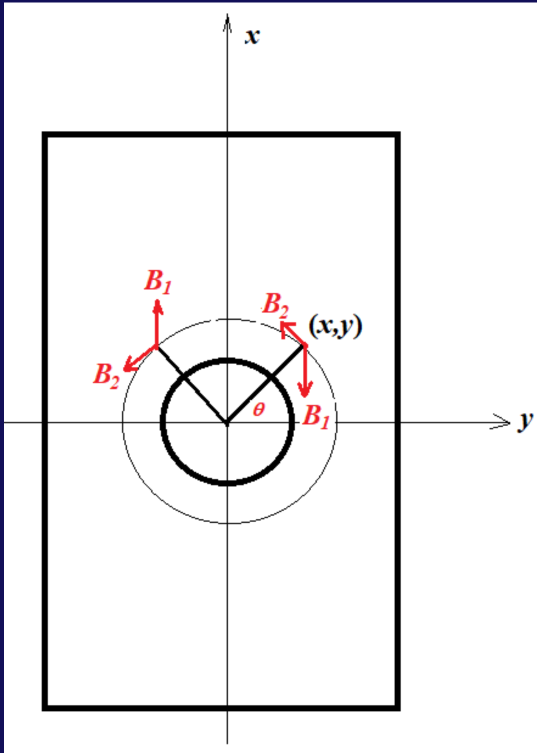


(2) 带反向电流的空园柱产生的磁场

$$\int \vec{B}\cdot d\vec{l} = \mu_0 i$$

$$2\pi r B_2 = \mu_0 \pi a^2 J_0$$

$$B_2 = \frac{1}{2}\mu_0 J_0 \frac{a^2}{r}$$



$$B_1 = \begin{cases} \mu_0 J_0 y & y \le \dfrac{d}{2} \\ \dfrac{1}{2}\mu_0 J_0 d & y > \dfrac{d}{2} \end{cases}$$

$$B_2 = \frac{1}{2}\mu_0 J_0 \frac{a^2}{r}$$

(4) 无限平板内部磁场

$$B_x = B_2 \cos\theta - B_1 = \frac{1}{2}\mu_0 J_0 \frac{a^2}{r}\cdot\frac{y}{r} - \mu_0 J_0 y$$

$$= \mu_0 J_0 y[\frac{a^2}{x^2+y^2} - 1]$$

$$B_y = -B_2 \sin\theta = -\frac{1}{2}\mu_0 J_0 \frac{a^2}{r}\cdot\frac{x}{r}$$

$$= -\frac{1}{2}\mu_0 J_0 x \frac{a^2}{x^2+y^2}$$

(5) 无限平板外部磁场

$$B_x = B_2 \cos\theta - B_1 = \frac{1}{2}\mu_0 J_0 \frac{a^2}{r}\cdot\frac{y}{r} - \frac{1}{2}\mu_0 J_0 d$$

$$= \frac{1}{2}\mu_0 J_0 [\frac{a^2}{x^2+y^2} y - d]$$

$$B_y = -B_2 \sin\theta = -\frac{1}{2}\mu_0 J_0 \frac{a^2}{r}\cdot\frac{x}{r}$$

$$= -\frac{1}{2}\mu_0 J_0 x \frac{a^2}{x^2+y^2}$$

2. (15%) As shown in Fig. 8, a hemisphere of radius R_0 has a uniformly distributed surface charge with total charge Q.

(a) What is the electric potential at any position along the z axis due to the entire hemisphere of surface charge?

(b) What is the electric field at any position alone the z axis?

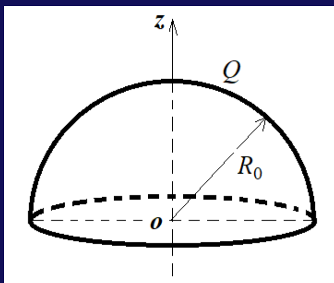


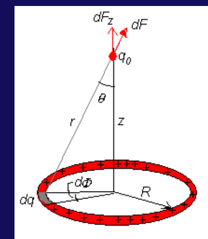
Figure 8

(1) 半球壳表面电荷密度

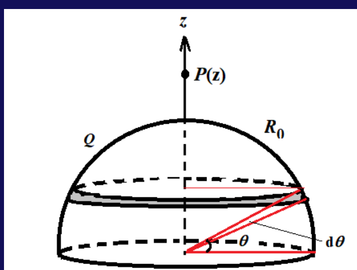
$$\sigma = \frac{Q}{2\pi R_0^2}$$

$$dq = \sigma \cdot 2\pi \cdot R_0 \cos\theta \cdot R_0 d\theta$$

$$= \sigma \cdot 2\pi R_0^2 \cos\theta d\theta = Q\cos\theta d\theta$$



$$V = \frac{q}{4\pi\varepsilon_0\sqrt{z^2 + R^2}}$$



(2) P点的电势

$$dV = \frac{dq}{4\pi\varepsilon_0\sqrt{(z - R_0\sin\theta)^2 + (R_0\cos\theta)^2}}$$

$$= \frac{Q\cos\theta d\theta}{4\pi\varepsilon_0\sqrt{z^2 - 2zR_0\sin\theta + R_0^2}}$$

$$V(z) = \int_0^{\frac{\pi}{2}} \frac{Q\cos\theta d\theta}{4\pi\varepsilon_0\sqrt{z^2 - 2zR_0\sin\theta + R_0^2}}$$

$$= \int_0^{\frac{\pi}{2}} \frac{Qd\sin\theta}{4\pi\varepsilon_0\sqrt{z^2 - 2zR_0\sin\theta + R_0^2}}$$

$$= \int_0^1 \frac{Qdx}{4\pi\varepsilon_0\sqrt{z^2 - 2zR_0 x + R_0^2}}$$

$$V(z) = \int_0^1 \frac{Qdx}{4\pi\varepsilon_0\sqrt{z^2 - 2zR_0 x + R_0^2}}$$

$$V(z) = \int_0^1 \frac{Qd(2zR_0 x)}{8\pi\varepsilon_0 zR_0\sqrt{z^2 - 2zR_0 x + R_0^2}}$$

$$= \frac{Q}{8\pi\varepsilon_0 zR_0}\left[\frac{-1}{-\frac{1}{2}+1}\sqrt{z^2 - 2zR_0 x + R_0^2}\right]_0^1$$

$$= \frac{Q}{4\pi\varepsilon_0 zR_0}\left[\sqrt{z^2 + R_0^2} - |z - R_0|\right]$$

$$= \begin{cases} \dfrac{Q}{4\pi\varepsilon_0 zR_0}\left[\sqrt{z^2 + R_0^2} - z + R_0\right] & z \ge R_0 \\ \dfrac{Q}{4\pi\varepsilon_0 zR_0}\left[\sqrt{z^2 + R_0^2} - R_0 + z\right] & z < R_0 \end{cases}$$

2. (15%) As shown in Fig. 8, a hemisphere of radius R_0 has a uniformly distributed surface charge with total charge Q.

(a) What is the electric potential at any position along the z axis due to the entire hemisphere of surface charge?

(b) What is the electric field at any position alone the z axis?

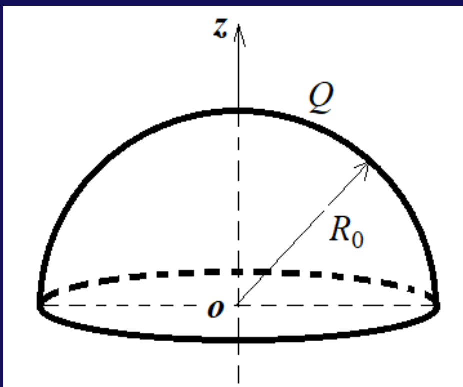
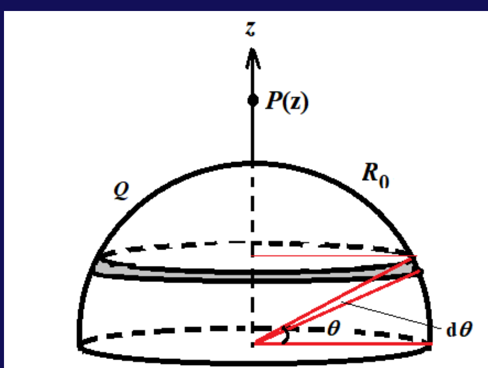


Figure 8



$$V(z)=\begin{cases}\dfrac{Q}{4\pi\varepsilon_0 z R_0}[\sqrt{z^2+R_0^2}\text{-}z+R_0] & z\geq R_0\\[2ex] \dfrac{Q}{4\pi\varepsilon_0 z R_0}[\sqrt{z^2+R_0^2}\text{-}R_0+z] & z<R_0\end{cases}$$

$$E_z=-\frac{\partial V}{\partial z}=\begin{cases}\dfrac{Q}{4\pi\varepsilon_0}[\dfrac{R_0}{z^2\sqrt{z^2+R_0^2}}+\dfrac{1}{z^2}] & z>R_0\\[2ex] \dfrac{Q}{4\pi\varepsilon_0}[\dfrac{R_0}{z^2\sqrt{z^2+R_0^2}}-\dfrac{1}{z^2}] & z<R_0\end{cases}$$

3. (15%) As shown in Fig. 9, parallel plate electrodes with a width of l and a length of D at voltage difference V_0 enclose an elastic dielectric with dielectric constant κ_e. ***The electric force of attraction between the electrodes is balanced by the elastic force of the dielectric.***

***(a) When the electrode spacing is d*, what is the free surface charge density on the upper electrode?**

(b) What is the electric force per unit area that the electrode exerts on the dielectric interface?

(c) The elastic restoring force per unit area is given by the relation $F_A = Y\ln\frac{d}{d_0}$ **, where Y is the modulus of elasticity and d_0 is the unstressed ($V_0 = 0$) thickness of the dielectric. Write a transcendental expression (超越方程表达式) for the equilibrium thickness of the dielectric.**

(d) What is the minimum equilibrium dielectric thickness and at what voltage does it occur? If a larger voltage is applied there is no equilibrium and the dielectric fractures as the electric stress overcomes the elastic restoring force (electromechanical breakdown).

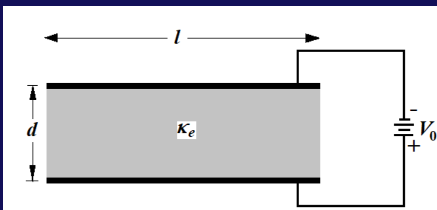


Figure 9

$$(a)\quad E=\frac{V_0}{d},\quad D=\kappa_e\varepsilon_0 E=\kappa_e\varepsilon_0\frac{V_0}{d}$$

$$\iint \vec{D}\cdot d\vec{A}=Q_0$$

$$\sigma_{e0}=D=\kappa_e\varepsilon_0\frac{V_0}{d}$$

$$\text{(b)}\quad C=\kappa_e\frac{\varepsilon_0 lD}{d}$$

$$U=\frac{1}{2}CV^2=\frac{1}{2}\kappa_e\frac{\varepsilon_0 lD}{d}V_0^2$$

$$F=-\frac{\partial U}{\partial d}=\frac{1}{2}\kappa_e\frac{\varepsilon_0 lD}{d^2}V_0^2$$

$$f=\frac{F}{lD}=\frac{1}{2}\kappa_e\varepsilon_0(\frac{V_0}{d})^2$$

$$\text{(c)}\quad -Y\ln\frac{d}{d_0}=\frac{1}{2}\kappa_e\varepsilon_0(\frac{V_0}{d})^2$$

(d)当d和V_0可变时

$$-Y\ln\frac{d}{d_0}=\frac{1}{2}\kappa_e\varepsilon_0(\frac{V}{d})^2$$

$$Yd^2\ln\frac{d_0}{d}=\frac{1}{2}\kappa_e\varepsilon_0 V^2$$

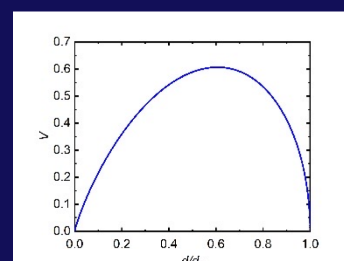
$$(Y2d\ln\frac{d_0}{d}\text{-}Yd^2\frac{1}{d})dd=\kappa_e\varepsilon_0 VdV$$

$$\frac{dV}{dd}=\frac{Y2d\ln\frac{d_0}{d}\text{-}Yd}{\kappa_e\varepsilon_0 V}=\frac{Yd(2\ln\frac{d_0}{d}\text{-}1)}{\kappa_e\varepsilon_0 V}=0$$

$$2\ln\frac{d_0}{d}\text{-}1=0$$

$$d=d_0e^{-0.5}$$

V存在一极大值



4. (20%) As shown in Fig. 9, a thin block with Ohmic conductivity σ and thickness δ moves with constant velocity $v\mathbf{i}_x$ between short circuited superconducting parallel plates. An initial surface current K_0 (the current density, the current per width) is imposed at $t = 0$ when $x = x_0$, but the source is then removed.

(a). The surface current on the plates K(t) will vary with time. What is the magnetic field in term of K(t)? Neglect fringing effects.

(b). Because the moving block is so thin, the current is uniformly distributed over the thickness δ. Please find K(t) as a function of time.

(c). What value of velocity will just keep the magnetic field constant with time until the moving block reaches the end?

(d). What happens to the magnetic field for larger and smaller velocities?

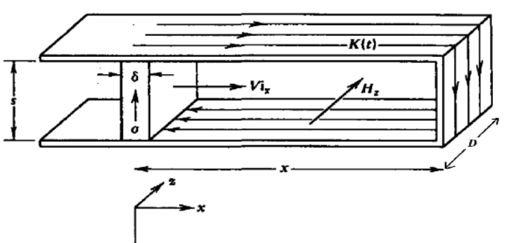


Figure 10

(a) 在任意时刻，薄板位置：$x = x_0 - vt$

$$\int \vec{H}\cdot d\vec{l} = I_0$$
$$H_z D = K(t)D$$
$$H_z = K(t)$$
$$B_z = \mu_0 H_z = \mu_0 K(t)$$

(b) 薄板电阻：$R = \rho\dfrac{l}{A} = \dfrac{1}{\sigma}\cdot\dfrac{s}{D\delta} = \dfrac{s}{\sigma D\delta}$

$$\Phi = B_z x(t)s = \mu_0 K(t)x(t)s$$
$$\varepsilon = -\frac{d\Phi}{dt} = -\mu_0 s\frac{d[K(t)x(t)]}{dt}$$
$$= -\mu_0 s[\frac{dK(t)}{dt}x(t) + K(t)\frac{dx(t)}{dt}]$$
$$= -\mu_0 s[x(t)\frac{dK(t)}{dt} - vK(t)]$$

$$\varepsilon = IR$$
$$\mu_0 s[x(t)\frac{dK(t)}{dt} - vK(t)] + K(t)DR = 0$$
$$\mu_0 s[x(t)\frac{dK(t)}{dt} - vK(t)] + \frac{s}{\sigma\delta}K(t) = 0$$
$$\mu_0(x_0 - vt)\frac{dK(t)}{dt} = [\mu_0 v - \frac{1}{\sigma\delta}]K(t)$$

$$\frac{dK(t)}{K(t)} = \frac{\mu_0 v - \dfrac{1}{\sigma\delta}}{\mu_0 v(\dfrac{x_0}{v} - t)}dt$$

$$\ln K(t) = -\frac{\mu_0 v - \dfrac{1}{\sigma\delta}}{\mu_0 v}\ln\left|\frac{x_0}{v} - t\right| + c'$$

$$K(t) = K'(\frac{x_0}{v} - t)^{(1-\frac{1}{\mu_0 v\sigma\delta})}$$

$$t = 0,\quad K(0) = K_0$$

$$K' = \frac{K_0}{(\dfrac{x_0}{v})^{(1-\frac{1}{\mu_0 v\sigma\delta})}} = K_0(\frac{x_0}{v})^{(\frac{1}{\mu_0 v\sigma\delta}-1)}$$

$$K(t) = K_0(1 - \frac{v}{x_0}t)^{(\frac{1}{\mu_0 v\sigma\delta}-1)}$$

(c) $B_z = \mu_0 K(t) = \text{constant}$

$$K(t) = K_0(1 - \frac{v}{x_0}t)^{(\frac{1}{\mu_0 v\sigma\delta}-1)} = \text{constant}$$

$$\frac{1}{\mu_0 v\sigma\delta} - 1 = 0$$

$$v = \frac{1}{\mu_0\sigma\delta}$$

(d) 当$v > v_c$

$$\frac{1}{\mu_0 v\sigma\delta} - 1 < 0,\quad K(t) = K_0(1 - \frac{v}{x_0}t)^{(\frac{1}{\mu_0 v\sigma\delta}-1)}\uparrow,\ B_z\uparrow$$

当$v < v_c$

$$\frac{1}{\mu_0 v\sigma\delta} - 1 > 0,\quad K(t) = K_0(1 - \frac{v}{x_0}t)^{(\frac{1}{\mu_0 v\sigma\delta}-1)}\downarrow,\ B_z\downarrow$$