浙江大学20_13_–20_14_学年_秋冬_学期

《 普通物理II 》课程期中考试试卷

I. Fill in the space underlined (40% in total)

1. A wire loop that encloses an area of 10 cm^2 has a resistance of 5 Ω. The loop is placed in a magnetic field of 0.5 T with its plane perpendicular to the field. The loop is suddenly removed from the field. How much charge flows past a given point in the wire? ____________________

$$\varepsilon = \frac{\Delta\Phi}{\Delta t} = \frac{\Delta(BA)}{\Delta t} = IR = \frac{\Delta q}{\Delta t}R$$

$$\Delta q = \frac{\Delta(BA)}{R} = \frac{0.5\times10\times10^{-4}}{5} = 1\times10^{-4}\,C$$

2. For the charge configuration of Fig. 1, the electric potential *V(r)* for points on the *x* axis, assuming *r>>d* is given by ______________. Set *V=0* at infinity.

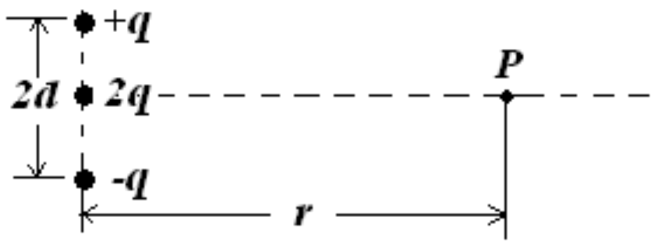


Figure 1

$$V = \frac{2q}{4\pi\varepsilon_0 r} + \frac{q}{4\pi\varepsilon_0\sqrt{r^2+d^2}} - \frac{q}{4\pi\varepsilon_0\sqrt{r^2+d^2}}$$

$$= \frac{q}{2\pi\varepsilon_0 r}$$

3. As shown in Fig. 2, a point charge +*q* is a distance *d/2* from a square surface of side *d* and is directly above the center of the square. The electric flux through the square is of _______.

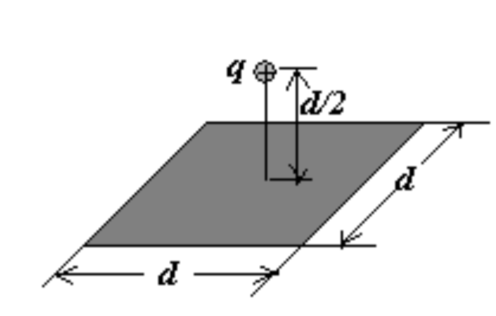


Figure 2

$$\Phi_E = \iint \vec{E}\bullet d\vec{A} = \frac{q}{\varepsilon_0}$$

$$\Phi = \frac{q}{6\varepsilon_0}$$

4. As shown in Fig. 3, a wire with a 3/4 circle is placed in a uniform magnetic field *B* which points out of the plane of the figure. If the wire carries a current *I*, the magnetic force acted on it is ______________________.

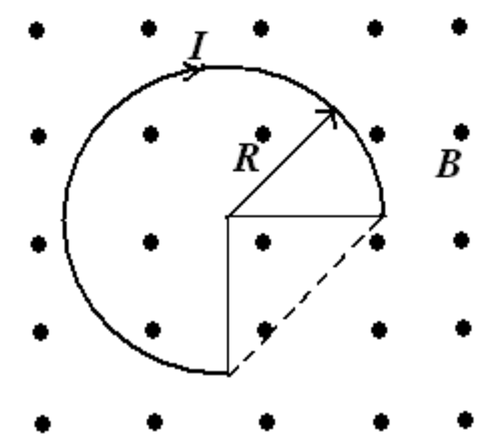


Figure 3

$$d\vec{F} = i d\vec{s} \times \vec{B}$$

$$\vec{F} = i\sqrt{2}RB = \sqrt{2}BIR$$

5. Figure 4 shows the cross section of a toroid, the total turn number of wire is *N*. If the electric current in the wire is *I*, how much is energy stored in the toroid?____________________.

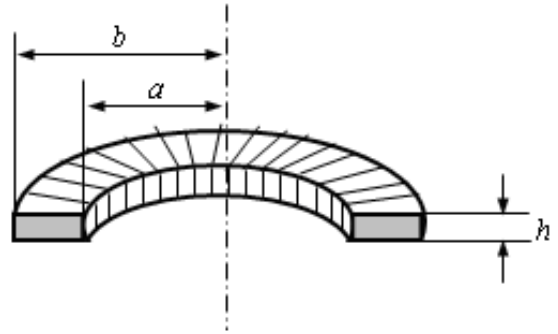


Figure 4

$$\int \vec{B} \bullet d\vec{l} = \mu_0 i$$

$$B \bullet 2\pi r = \mu_0 NI$$

$$B = \frac{\mu_0 NI}{2\pi r}$$

$$u_B = \frac{B^2}{2\mu_0}$$

$$U = \int_a^b u_B \bullet dv = \int_a^b \frac{(\frac{\mu_0 NI}{2\pi r})^2}{2\mu_0} \bullet d(2\pi rhdr)$$

$$= \frac{\mu_0 N^2 I^2 h}{4\pi} \ln \frac{b}{a}$$

6. As shown in Fig. 5, a parallel plate capacitor with capacitance C is charged to a potential difference V and is then disconnected from the charging source. The capacitor has an area A and a plate separation d. Assume that a glass plate of the same area A completely fills the space between the plates, and which has a dielectric constant κ_e. How much work is required to pull the glass plate out of the capacitor? ___________. Neglect fringe effects at the edges of the plates.

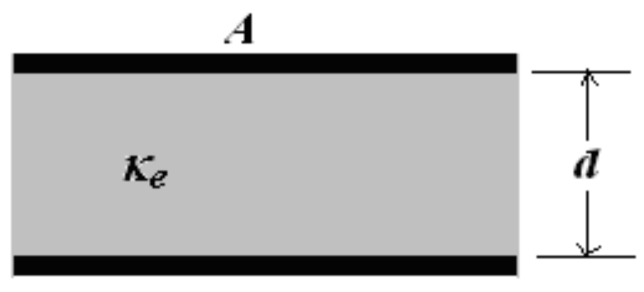


Figure 5

$$Q = CV, \text{(Constant)}$$

$$C = \frac{\varepsilon_0 A}{d}$$

$$W = \frac{1}{2} Q^2 \left(\frac{1}{C_f} - \frac{1}{C_i}\right)$$

$$= \frac{C^2 V^2}{2} \left(\frac{1}{\kappa_e C} - \frac{1}{C}\right)$$

$$= \frac{\varepsilon_0 A V^2}{2d} \left(\frac{1}{\kappa_e} - 1\right)$$

7. **As shown in Fig. 6, a long, straight conductor with a circular cross section of radius *R* carries a current *I*. There is a cylindrical hole inside the conductor, whose radius is of *a*, and whose axis is parallel to the axis of the conductor but offset a distance *b* from the axis of the conductor. The current *I* is uniformly distributed across the cross section of the conductor and is directed out of the page. The magnetic field at P point (*R>x>a+b*) at the *x* axis is ___________.**

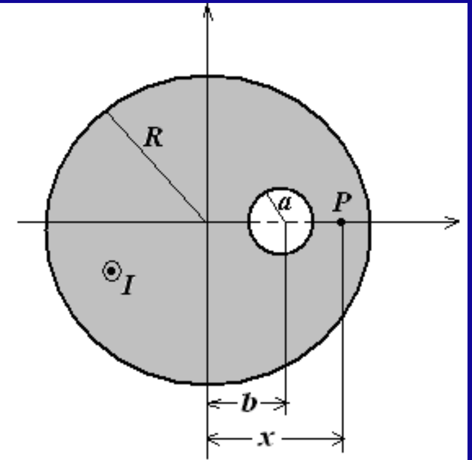


Figure 6

$$j = \frac{I}{\pi(R^2 - a^2)}$$

$$\int \vec{B} \bullet d\vec{l} = \mu_0 i$$

$$B_R \cdot 2\pi x = \mu_0 \frac{I}{\pi(R^2 - a^2)} \cdot \pi x^2$$

$$B_a \cdot 2\pi(x - b) = \mu_0 \frac{I}{\pi(R^2 - a^2)} \cdot \pi a^2$$

$$B = B_R - B_a = \frac{\mu_0 I}{2\pi(R^2 - a^2)}(x - \frac{a^2}{x - b})$$

8. Figure 7 shows three long wires in the *z* direction, each wire carries a current of *I* in the negative *z* direction. The separation distance between them is *d*. The magnetic field *B* in *y*-axis for *y* >0 is_____________________.

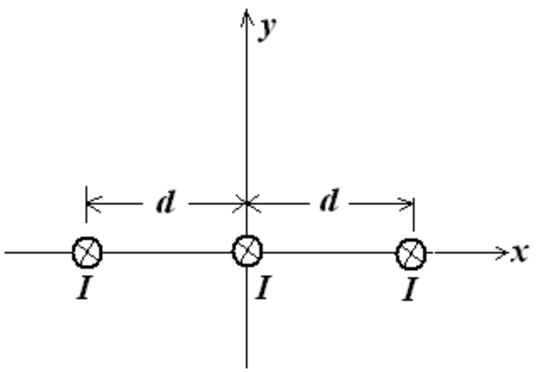


Figure 7

$$B = \frac{\mu_0 I}{2\pi r}$$

$$\vec{B} = \vec{B}_1 + \vec{B}_2 + \vec{B}_3$$

$$B_x = \frac{\mu_0 I}{2\pi y} + \frac{\mu_0 I}{2\pi\sqrt{y^2+d^2}} \cdot \frac{y}{\sqrt{y^2+d^2}} + \frac{\mu_0 I}{2\pi\sqrt{y^2+d^2}} \cdot \frac{y}{\sqrt{y^2+d^2}}$$

$$= \frac{\mu_0 I}{2\pi y} + \frac{\mu_0 I y}{\pi(y^2+d^2)} = \frac{\mu_0 I}{\pi}\left(\frac{1}{y} + \frac{y}{y^2+d^2}\right)$$

II. Problems (present the necessary equations in solution) (60%)

1. (12%) Please calculate the magnetic moment for a uniformly charged, rotating sphere, as shown in Fig.7, which has a radius *R*, and carried a charge *Q*, and is rotating with a angular speed of *ω*.

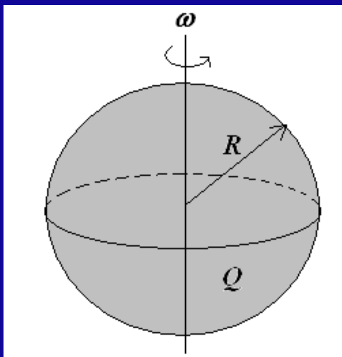
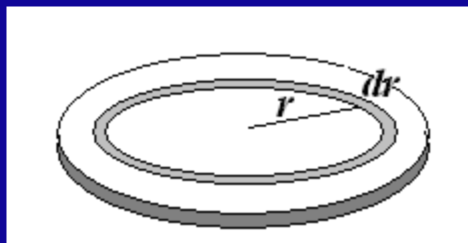


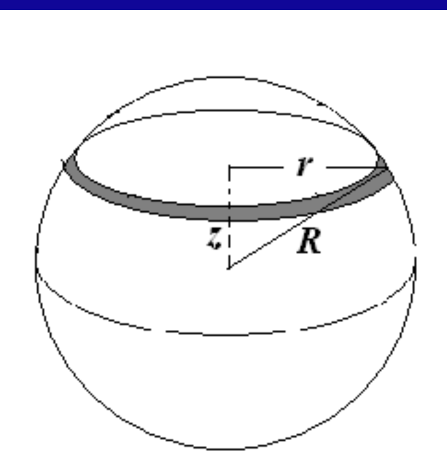
Figure 8

Solution: At first, we consider a disk with radius of *R*, charge of *q*.



$$d\mu = \pi r^2 di = \pi r^2 \frac{dq}{T} = \pi r^2 \frac{\omega}{2\pi} \cdot \frac{q}{\pi R^2} 2\pi r dr$$

$$\mu_d = \int_0^R \frac{\omega q}{R^2} r^3 dr = \frac{1}{4}\omega q R^2$$



$$d\mu_d = \frac{1}{4}\omega r^2 dq = \frac{1}{4}\omega (R^2 - z^2) \cdot \frac{Q}{\frac{4}{3}\pi R^3} \pi r^2 dz = \frac{3\omega (R^2 - z^2)^2 Q}{16 R^3} dz$$

$$\mu = 2\int_0^R \frac{3\omega (R^2 - z^2)^2 Q}{16 R^3} dz = \frac{3\omega Q}{8R^3} \int_0^R (R^4 - 2R^2 z^2 + z^4) dz = \frac{1}{5}\omega Q R^2$$

2. (13%) A static charge distribution produces a spherically radial electric field: $\vec{E} = A\dfrac{\exp(-br)}{r^2}\hat{r}$

, where *A* and *b* > 0 are the constants.

(a). What is the charge density *ρ(r)*?

(b). What is the total charge *Q*?

Solution:

$$\iint \vec{E}\bullet d\vec{A} = \frac{q}{\varepsilon_0}$$

$$\iiint (\nabla\bullet\vec{E})dv = \iiint \frac{\rho_e}{\varepsilon_0}dv$$

$$\nabla\bullet\vec{E} = \frac{\rho_e}{\varepsilon_0}$$

$$\text{(a)}\quad \rho_e = \varepsilon_0\nabla\cdot\vec{E} = \varepsilon_0\frac{1}{r^2}\frac{\partial}{\partial r}r^2\left(A\frac{e^{-br}}{r^2}\right)$$

$$= \varepsilon_0 A(-b)\frac{e^{-br}}{r^2}$$

$$= -\frac{\varepsilon_0 Abe^{-br}}{r^2}$$

$$\text{(b).}\quad Q = \varepsilon_0\iint \vec{E}\cdot d\vec{A}$$

$$= \varepsilon_0\lim_{r\to\infty}A\frac{e^{-br}}{r^2}4\pi r^2 = \lim_{r\to\infty}(\varepsilon_0 Ae^{-br}) = 0$$

3. (15%) As shown in Fig. 9, a loop of wire of resistance R and a coil of self-inductance L encloses an area A. A spatially uniform magnetic field is applied perpendicular to the plane of the loop with the following time dependence:

$$B = \begin{cases} 0, \text{ for } t < 0 \\ kt, \text{ for } 0 < t < t_0 \\ kt_0, \text{ (constant) for } t > t_0 \end{cases}$$

(a) Calculate the current I in the loop for all times $t > 0$, given that $I = 0$ for $t = 0$.

(b) Make simple sketches of the current vs. time for $t_0 < L/R$ and $t_0 >> L/R$.

Solution:

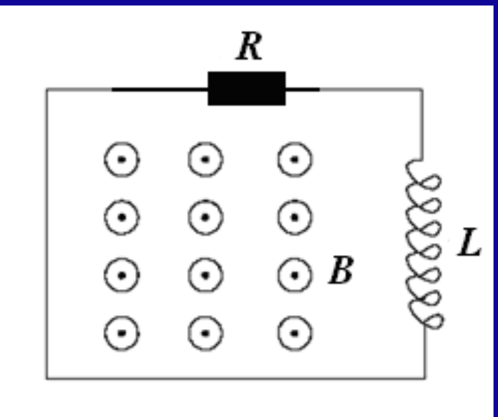


Figure 9

$$\Phi = BA$$

$$\varepsilon = \left| -\frac{d\Phi}{dt} \right| = \begin{cases} 0 & t = 0 \\ kA & 0 < t < t_0 \\ 0 & t > t_0 \end{cases}$$

$$(a).\quad \varepsilon - L\frac{dI}{dt} = IR$$

$$L\frac{dI}{dt} = kA - IR$$

$$\frac{dI}{I - \frac{kA}{R}} = -\frac{R}{L}dt$$

$$\ln(I - \frac{kA}{R}) = -\frac{R}{L}t + C'$$

$$I - \frac{kA}{R} = Ce^{-\frac{R}{L}t}$$

$$t = 0 \quad I = 0 \quad \therefore C = -\frac{kA}{R}$$

$$I = \frac{kA}{R}(1 - e^{-\frac{R}{L}t})$$

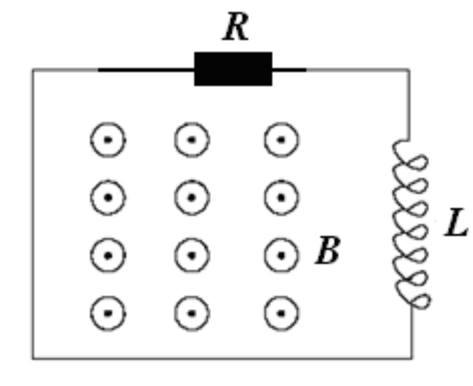


Figure 9

$$\Phi = BA$$

$$\varepsilon = \left|-\frac{d\Phi}{dt}\right| = \begin{cases} 0 & t=0 \\ kA & 0<t<t_0 \\ 0 & t>t_0 \end{cases}$$

$$0<t<t_0$$

$$I = \frac{kA}{R}\left(1-e^{-\frac{R}{L}t}\right)$$

$$t>t_0 \quad \varepsilon = 0$$

$$0 - L\frac{dI}{dt} = IR$$

$$\frac{dI}{I} = -\frac{R}{L}dt, \quad \ln I = -\frac{R}{L}t + C''$$

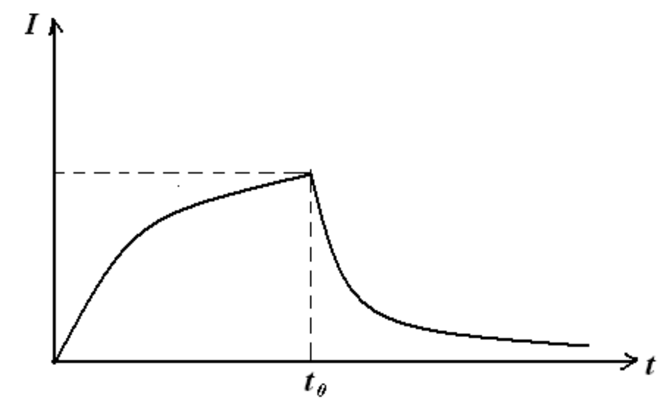
$$I = C'''e^{-\frac{R}{L}t}$$

$$I(t_0) = \frac{kA}{R}\left(1-e^{-\frac{R}{L}t_0}\right) = C'''e^{-\frac{R}{L}t_0}$$

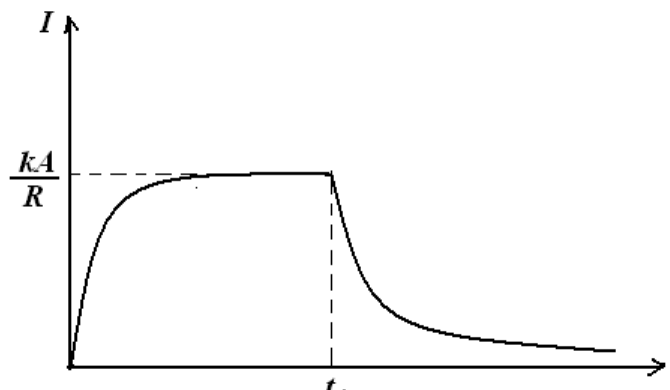
$$C''' = \frac{kA}{R}\left(e^{\frac{R}{L}t_0}-1\right)$$

$$I = \frac{kA}{R}\left(e^{\frac{R}{L}t_0}-1\right)e^{-\frac{R}{L}t}$$

(b): as $t_0<L/R$



as $t_0>>L/R$



4. (20%) Figure 10 shows a cross section of a spherical capacitor, in which the inner conductor is a solid sphere of radius *a* with a charge Q_0, and the outer conductor is a hollow spherical shell of inner radius *b*. The space between them is filled by the non-uniform dielectrics with a dielectric constant:

$$\kappa_e = \frac{\kappa_{e0}}{1+ar}$$

where κ_{e0} and α are the constants, *r* is the distance for the points inside dielectrics. Please calculate:

(a). The electric displacement vector for the region of $a<r<b$.

(b). The capacitance for this system *C*.

(c). The volume density of the polarization charge, $\rho_e'(r)$, in the region of $a<r<b$.

(d). The surface density of the polarization charge σ_e(a) and σ_e(b), at the $r=a$ and $r=b$ surfaces, respectively.

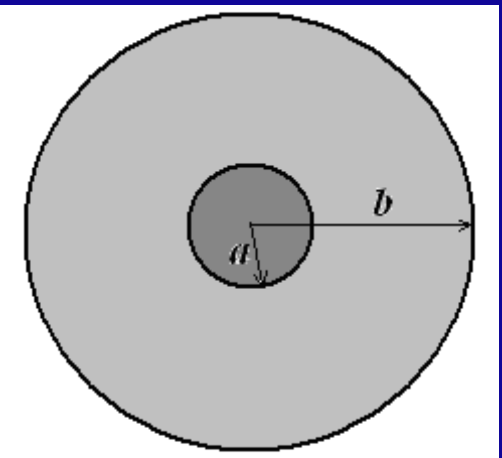


Figure 10

$$(a).\quad \iint \vec{D}\cdot d\vec{A} = Q_0$$

$$D\cdot 4\pi r^2 = Q_0$$

$$D = \frac{Q_0}{4\pi r^2}$$

$$(b).\quad D = \frac{Q_0}{4\pi r^2} = \kappa_e \varepsilon_0 E$$

$$E = \frac{Q_0}{4\pi\kappa_e\varepsilon_0 r^2} = \frac{Q_0}{4\pi\varepsilon_0 r^2}\cdot\frac{1+\alpha r}{\kappa_{e0}}$$

$$V = \int_a^b \frac{Q_0}{4\pi\varepsilon_0 r^2}\cdot\frac{1+\alpha r}{\kappa_{e0}}\,dr = \frac{Q_0}{4\pi\varepsilon_0\kappa_{e0}}\left[\frac{1}{a}-\frac{1}{b}+\alpha\ln\frac{b}{a}\right]$$

$$C = \frac{Q_0}{V} = 4\pi\varepsilon_0\kappa_{e0}\,\frac{1}{\frac{1}{a}-\frac{1}{b}+\alpha\ln\frac{b}{a}}$$

$$(c).\quad D = \frac{Q_0}{4\pi r^2} = \kappa_e \varepsilon_0 E$$

$$E = \frac{Q_0}{4\pi \kappa_e \varepsilon_0 r^2} = \frac{Q_0}{4\pi \varepsilon_0 r^2} \cdot \frac{1+\alpha r}{\kappa_{e0}}$$

$$P = \chi_e \varepsilon_0 E = (\kappa_e - 1)\varepsilon_0 E = \frac{Q_0}{4\pi r^2} \frac{\kappa_e - 1}{\kappa_e} = \frac{Q_0}{4\pi r^2}(1 - \frac{1+\alpha r}{\kappa_{e0}})$$

$$\rho_e' = -\nabla \bullet \vec{P} = -\frac{1}{r^2}\frac{d}{dr} r^2 [\frac{Q_0}{4\pi r^2}(1 - \frac{1+\alpha r}{\kappa_{e0}})] = \frac{Q_0}{4\pi \kappa_{e0}} \frac{\alpha}{r^2}$$

$$(d).\ P = \chi_e \varepsilon_0 E = \frac{Q_0}{4\pi r^2}(1 - \frac{1+\alpha r}{\kappa_{e0}}), \qquad \sigma_e' = \vec{P} \cdot \vec{n} = P_n$$

$$\therefore\ \sigma_e'(a) = -P(a) = -\frac{Q_0}{4\pi a^2}(1 - \frac{1+\alpha a}{\kappa_{e0}})$$

$$\sigma_e'(b) = P(b) = \frac{Q_0}{4\pi b^2}(1 - \frac{1+\alpha b}{\kappa_{e0}})$$

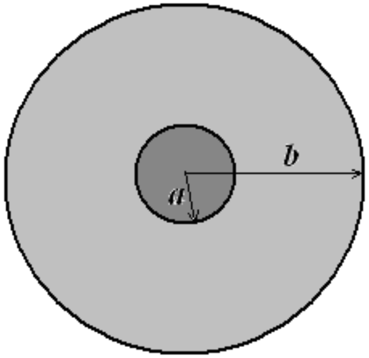


Figure 10