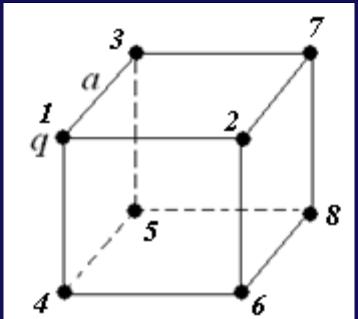
期中考试卷

I. Fill in the space underlined (50% in total)

1. As shown in Fig. 1, a cube of edge *a* carriers a point charge *q* at each corner. The resultant electric force on any one of the charges is given by _____+5_______, and its direction is along __+4____.



$$\vec{F}_{15} = -\frac{1}{4\pi\varepsilon_0}\cdot\frac{q^2}{2a^2}\frac{1}{\sqrt{2}}(\vec{j}+\vec{k})$$

$$\vec{F}_{16} = -\frac{1}{4\pi\varepsilon_0}\cdot\frac{q^2}{2a^2}\frac{1}{\sqrt{2}}(\vec{k}+\vec{i})$$

$$\vec{F}_{17} = -\frac{1}{4\pi\varepsilon_0}\cdot\frac{q^2}{2a^2}\frac{1}{\sqrt{2}}(\vec{i}+\vec{j})$$

$$\vec{F}_{18} = -\frac{1}{4\pi\varepsilon_0}\cdot\frac{q^2}{3a^2}\frac{1}{\sqrt{3}}(\vec{i}+\vec{j}+\vec{k})$$

$$\vec{F}_{12} = -\frac{1}{4\pi\varepsilon_0}\cdot\frac{q^2}{a^2}\vec{i}$$

$$\vec{F}_{13} = -\frac{1}{4\pi\varepsilon_0}\cdot\frac{q^2}{a^2}\vec{j}$$

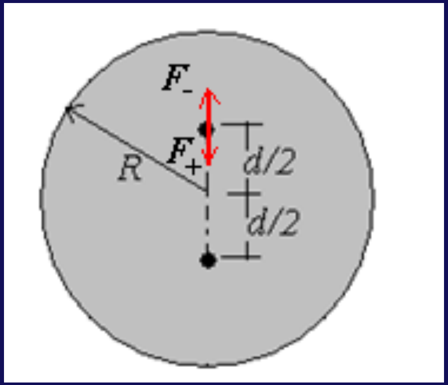
$$\vec{F}_{14} = -\frac{1}{4\pi\varepsilon_0}\cdot\frac{q^2}{a^2}\vec{k}$$

$$\vec{F} = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14} + \ldots..$$

$$= -\frac{q^2}{4\pi\varepsilon_0 a^2}\cdot\left(1+\frac{1}{\sqrt{2}}+\frac{1}{3\sqrt{3}}\right)(\vec{i}+\vec{j}+\vec{k})$$

$$F_{net} = -\frac{q^2}{4\pi\varepsilon_0 a^2}\times\sqrt{3}\times 1.90 = -0.262\frac{q^2}{\varepsilon_0 a^2}$$

2. Figure 2 shows a Thomson atom model of helium (Z=2). Two electrons, at rest, are embedded inside a uniform sphere of positive charge *2e*. Find the distance *d* between the electrons so that the configuration is in static equilibrium. +5



$$\oiint \vec{E} \bullet d\vec{A} = \frac{q}{\varepsilon_0}$$

$$E \cdot 4\pi\left(\frac{d}{2}\right)^2 = \frac{1}{\varepsilon_0}\frac{2e}{\frac{4}{3}\pi R^3}\frac{4}{3}\pi\left(\frac{d}{2}\right)^3$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{de}{R^3}$$

$$\frac{1}{4\pi\varepsilon_0}\frac{de^2}{R^3} = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{d^2}$$

$$d = R$$

3. A large, flat, nonconducting surface carriers a uniform charge density σ. A small circular hole of radius R has been cut in the middle of the sheet, as shown in Fig. 3. Ignore fringing of the field lines around all edges, please calculate the electric field E at the point P, a distance z from the center of the hole along its axis. +5

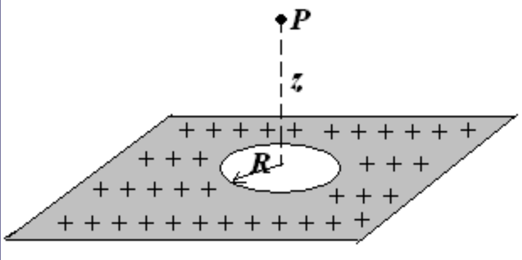


Figure 3

$$E_p = E_+ - E_-$$

$$= \frac{\sigma}{2\varepsilon_0} - \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{1}{\sqrt{1+\frac{R^2}{z^2}}}\right)$$

$$= \frac{\sigma}{2\varepsilon_0}\frac{1}{\sqrt{1+\frac{R^2}{z^2}}}$$

4. A slab of copper of thickness *b* is thrust into a parallel-plate capacitor as shown in Fig. 4.
(a) What is the capacitance after the slab is introduced?___+4_____.
(b) If a charge *q* is maintained on the plates, how much work is done on the slab as it is inserted? __+4___.

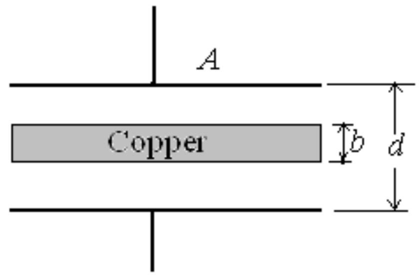


Figure 4

The energy stored before insert Copper :

$$U_1 = \frac{1}{2}\frac{q^2}{C_1}, C_1 = \frac{\varepsilon_0 A}{d}$$

The energy stored before insert Copper :

$$U_2 = \frac{1}{2}\frac{q^2}{C_2}, C_2 = \frac{\varepsilon_0 A}{d-b}$$

$$W = U_1 - U_2 = \frac{q^2}{2}\left(\frac{d}{\varepsilon_0 A} - \frac{d-b}{\varepsilon_0 A}\right)$$

$$= \frac{q^2 b}{2\varepsilon_0 A}$$

5. There is a thin disk of radius *a*, with total charge *Q* uniformly distributed on its surface, rotating at a constant angular speed *ω*, as shown in Fig. 5. What is the magnetic dipole moment (磁偶极矩)?

__+5__

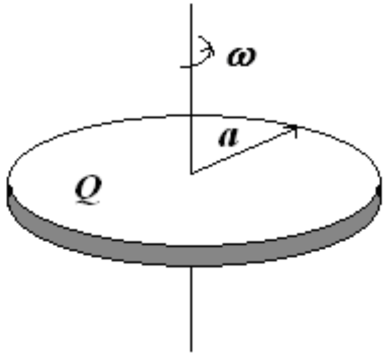


Figure 5

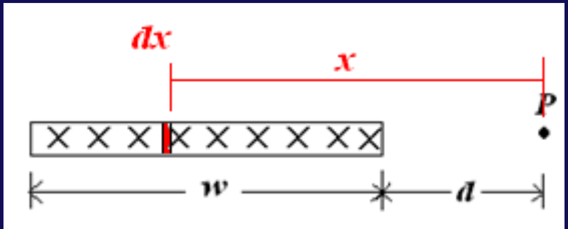
$$dq = \frac{Q}{\pi a^2} \cdot 2\pi r dr = \frac{2Q}{a^2} r dr$$

$$di = \frac{dq}{T} = \frac{\omega dq}{2\pi} = \frac{Q\omega}{\pi a^2} r dr$$

$$d\mu = \pi r^2 di = \frac{Q\omega}{a^2} r^3 dr$$

$$\mu = \int_0^a \frac{Q\omega}{a^2} r^3 dr = \frac{1}{4} Q\omega a^2$$

6. Figure 6 shows a cross section of a long, thin ribbon (薄带) of width w that is carrying a uniformly distributed total current I into the paper. Calculate the magnitude ______+4________and the direction ______+4______ of the magnetic field B at a point P in the plane of the ribbon at a distance d from its edge.



$$dB = \frac{\mu_0 di}{2\pi x} = \frac{\mu_0 I}{2\pi w}\frac{dx}{x}$$

$$B = \int_d^{d+w} \frac{\mu_0 I}{2\pi w}\frac{dx}{x} = \frac{\mu_0 I}{2\pi w}[\ln(d+w) - \ln d]$$

Direction : Point Down

7. Consider the circuit of Fig. 7. The curved segments are arcs of circles of radii *a* and *b*. The straight segments are along the radii. The magnetic field *B* at *P* is ____+5______, assuming a current *i* in the circuit.

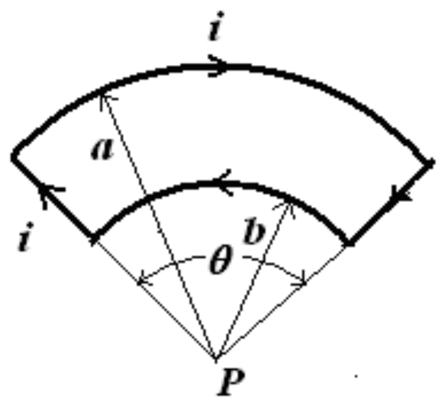


Figure 7

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{id\vec{s}\times\hat{r}}{r^2}$$

$$dB = \frac{\mu_0 ird\theta}{4\pi r^2} = \frac{\mu_0 id\theta}{4\pi r}$$

$$B_1 = \frac{\mu_0 i\theta}{4\pi a} \qquad B_2 = \frac{\mu_0 i\theta}{4\pi b}$$

$$B = B_2 - B_2 = \frac{\mu_0 i\theta}{4\pi}\left(\frac{1}{b} - \frac{1}{a}\right)$$

8. A very long, straight conductor with a circular cross section of radius R carries a current I. Inside the conductor is a cylindrical hole of radius a whose axis is parallel to the axis of the conductor but offset a distance b from the axis of the conductor. The current I is uniformly distributed across the cross section of the conductor and is directed out of the page. Find the magnetic field everywhere outside the conductor. __+5____.

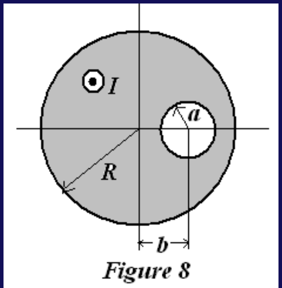


Figure 8

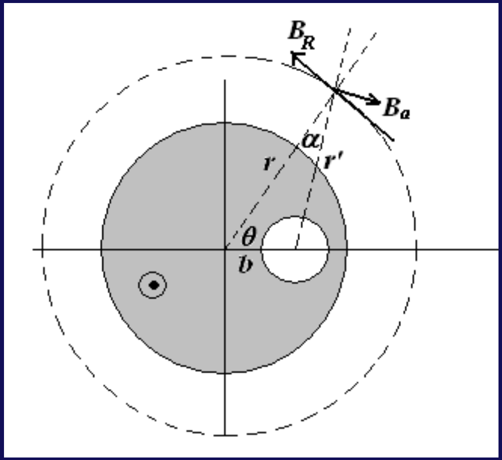
Current density : $j = \dfrac{I}{\pi(R^2 - a^2)}$

$$r' = \frac{\sin\theta}{\sin\alpha} b$$

$$\alpha = \tan^{-1}\left(\frac{\sin\theta}{\frac{r}{b} - \cos\theta}\right)$$

$$B_R = \frac{\mu_0 I_R}{2\pi r} = \frac{\mu_0 I}{2\pi r}\frac{R^2}{R^2 - a^2}$$

$$B_a = \frac{\mu_0 I_a}{2\pi r'} = \frac{\mu_0 I}{2\pi r'}\frac{a^2}{R^2 - a^2}$$



$$\vec{B}_R = \frac{\mu_0 I_R}{2\pi r}\vec{e}_\theta = \frac{\mu_0 I}{2\pi r}\frac{R^2}{R^2 - a^2}\vec{e}_\theta$$

$$\vec{B}_a = B_a \sin\alpha\, \vec{e}_r - B_a \cos\alpha\, \vec{e}_\theta$$

II. Problems (Present the necessary equations in solution) (50%)

1. (10%)In a spherical space, as shown in Fig.9, if we know the electrical potential distribution as

$$V(r,\theta,\varphi) = Ar^2 \sin\theta \cos\varphi$$

Please find the distributions of the electric field *E*(*r,θ,φ*), and volume charge density $\rho_e(r,\theta,\varphi)$.

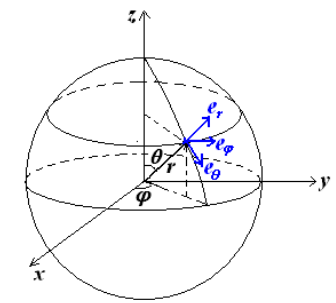


Figure 9

$$V(r,\theta,\varphi) = Ar^2 \sin\theta \cos\varphi, \quad \vec{E} = -\nabla V$$

$$E_r = -\frac{\partial V}{\partial r} = -2Ar\sin\theta\cos\varphi$$

$$E_\theta = -\frac{\partial V}{r\partial\theta} = -Ar\cos\theta\cos\varphi$$

$$E_\varphi = -\frac{\partial V}{r\sin\theta\partial\varphi} = Ar\sin\varphi$$

$$\vec{E} = -2Ar\sin\theta\cos\varphi\,\hat{e}_r - Ar\cos\theta\cos\varphi\,\hat{e}_\theta + Ar\sin\varphi\,\hat{e}_\varphi$$

$$\rho_e = \varepsilon_0 \nabla \bullet \vec{E}$$

$$= \varepsilon_0\left[\frac{\partial E_r}{\partial r} + \frac{\partial E_\theta}{r\partial\theta} + \frac{\partial E_\varphi}{r\sin\theta\partial\varphi}\right]$$

$$= \varepsilon_0\left[-2A\sin\theta\cos\varphi + A\sin\theta\cos\varphi + A\frac{\cos\varphi}{\sin\theta}\right]$$

$$= \varepsilon_0 A\cos\varphi\frac{\cos^2\theta}{\sin\theta}$$

2. (13%) As shown in Fig. 10, a pair of parallel plate electrodes with a width d and a distance s apart at a voltage difference V_0 is dipped into a dielectric fluid with dielectric constant κ_e. The fluid has a mass density ρ_m and gravity acts downward. How high does the liquid rise between the plates?

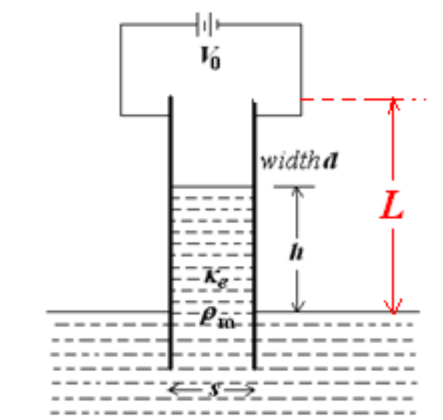


Figure 10

$$C = C_1 + C_2 = \frac{\varepsilon_0 d(L-h)}{s} + \frac{\kappa_e \varepsilon_0 dh}{s}$$

$$= \frac{\varepsilon_0 d}{s}[(L-h) + \kappa_e h]$$

$$U = \frac{1}{2}CV^2 = \frac{\varepsilon_0 dV^2}{2s}[(L-h) + \kappa_e h]$$

$$F = -\frac{\partial U}{\partial h} = -\frac{\varepsilon_0 dV^2}{2s}(\kappa_e - 1)$$

$$\rho_m gh = \frac{\frac{\varepsilon_0 dV^2}{2s}(\kappa_e - 1)}{s \cdot d}$$

$$h = \frac{\varepsilon_0 V^2(\kappa_e - 1)}{2s^2 \rho_m g}$$

3. (12%) The two rails of a superconducting track are separated by a distance d. A conductor can slide along the track. The conductor, initially at rest, is pulled to the right by a constant force F. The friction between the conductor and the track is directly proportional to its velocity with a proportionality constant α. The portion of the conductor between the rails has a resistance of R. The entire setup is in a uniform magnetic field, B, as shown in Fig. 11. The field B points into the page.

(a). What is the direction of the induced current in the conductor?

(b). Determine the magnitude of the velocity of the conductor as a function of time.

(c). Determine the magnitude of the induced current as a function of time.

(d). Determine the terminal velocity of the conductor.

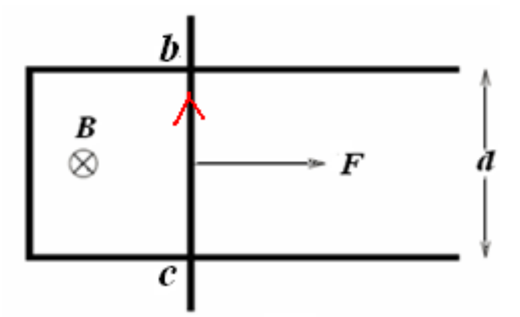


Figure 11

(a). Current is from c to b.

$$(b).\ F - \alpha v - B\frac{Bdv}{R}d = ma$$

$$F - \left(\alpha + \frac{B^2 d^2}{R}\right)v = m\frac{dv}{dt}$$

$$\frac{\alpha + \frac{B^2 d^2}{R}}{m}dt = \frac{dv}{F/\left(\alpha + \frac{B^2 d^2}{R}\right) - v}$$

$$t = 0,\ v = 0$$

$$v = \frac{F}{\alpha + \frac{B^2 d^2}{R}}\left[1 - e^{-\frac{\alpha + \frac{B^2 d^2}{R}}{m}t}\right]$$

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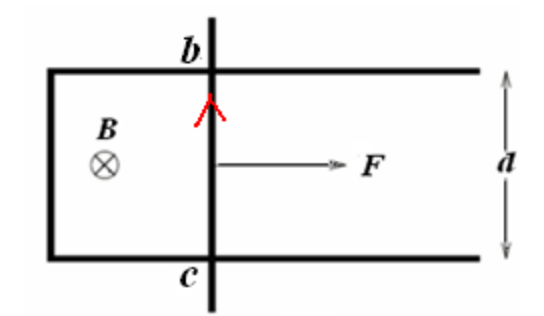

Figure 11

$$(c).\quad v=\frac{F}{\alpha+\frac{B^2d^2}{R}}\left[1-e^{-\frac{\alpha+\frac{B^2d^2}{R}}{m}t}\right]$$

$$I=\frac{Bdv}{R}=\frac{BdF}{R\alpha+B^2d^2}\left[1-e^{-\frac{\alpha+\frac{B^2d^2}{R}}{m}t}\right]$$

$(d).\ t\to\infty,$

$$v=\frac{F}{\alpha+\frac{B^2d^2}{R}}=\frac{RF}{\alpha R+B^2d^2}$$

4. (15%) As shown in Fig. 12, a uniformly charged dielectric sphere with dielectric constant κ_{e1}, radius R_1 and Q, is placed in a dielectric sphere (电介质球) with a radius R_2 and dielectric constant κ_{e2}.

(a). Please calculate the electric field E, displacement vector D, and polarization vector P for the three regions: $0 < r < R_1$, $R_1 < r < R_2$ and $r > R_2$;

(b). The surfaces induced charge density σ_e at both $r = R_1$ and $r = R_2$ surfaces.

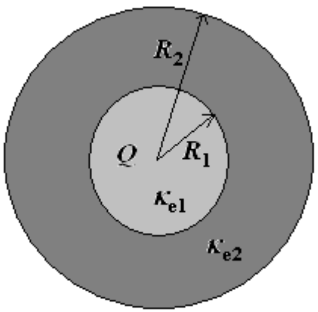

Figure 12

$$(a).\,0 < r < R_1,\ \oiint \vec{D}\bullet d\vec{A} = q_0$$

$$D\cdot 4\pi r^2 = \frac{Q}{\frac{4}{3}\pi R_1^3}\cdot\frac{4}{3}\pi r^3,\ D = \frac{Q}{4\pi R_1^3}r$$

$$\vec{D} = \kappa_e\varepsilon_0\vec{E},\quad E_1 = \frac{Q}{4\pi\kappa_{e1}\varepsilon_0 R_1^3}r$$

$$\vec{P} = \chi_e\varepsilon_0\vec{E} = (\kappa_e - 1)\varepsilon_0\vec{E},\quad P_1 = \frac{(\kappa_{e1}-1)Q}{4\pi\kappa_{e1}R_1^3}r$$

$$R_1 < r < R_2,\ \oiint \vec{D}\bullet d\vec{A} = q_0$$

$$D\cdot 4\pi r^2 = Q,\ D = \frac{Q}{4\pi r^2}$$

$$\vec{D} = \kappa_e\varepsilon_0\vec{E},\quad E_2 = \frac{Q}{4\pi\kappa_{e2}\varepsilon_0 r^2}$$

$$\vec{P} = \chi_e\varepsilon_0\vec{E} = (\kappa_e - 1)\varepsilon_0\vec{E},\quad P_2 = (\kappa_{e2}-1)\varepsilon_0 E_2 = \frac{(\kappa_{e2}-1)Q}{4\pi\kappa_{e2}r^2}$$

$$r > R_2,\ \oiint \vec{D}\bullet d\vec{A} = q_0$$

$$D\cdot 4\pi r^2 = Q,\ D = \frac{Q}{4\pi r^2}$$

$$\vec{D} = \varepsilon_0\vec{E},\quad E_3 = \frac{Q}{4\pi\varepsilon_0 r^2}$$

$$P_3 = 0$$

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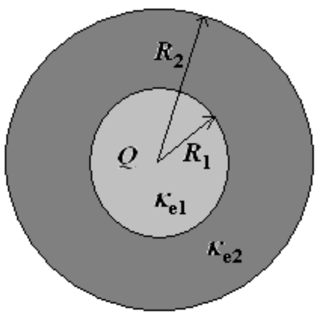

Figure 12

$$r = R_2,\ \sigma_{e2} = \vec{P}_2 \cdot \vec{n}_2 - \vec{P}_3 \cdot \vec{n}_3$$

$$= \frac{(\kappa_{e2} - 1)Q}{4\pi\kappa_{e2}R_2^2}$$

$$= \frac{Q}{4\pi R_2^2}\frac{\kappa_{e2} - 1}{\kappa_{e2}})$$

$$(b).\quad r = R_1,\ \sigma_{e1} = \vec{P}_1 \cdot \vec{n}_1 - \vec{P}_2 \cdot \vec{n}_2$$

$$= \frac{(\kappa_{e1} - 1)Q}{4\pi\kappa_{e1}R_1^2} - \frac{(\kappa_{e2} - 1)Q}{4\pi\kappa_{e2}R_1^2}$$

$$= \frac{Q}{4\pi R_1^2}\left(\frac{\kappa_{e1} - 1}{\kappa_{e1}} - \frac{\kappa_{e2} - 1}{\kappa_{e2}}\right)$$

$$= \frac{Q}{4\pi R_1^2}\frac{\kappa_{e1} - \kappa_{e2}}{\kappa_{e1}\kappa_{e2}}$$