2014年期中考试答卷

I. Fill in the space underlined. (50% in total)

1. The four point charges are placed on the vertices of a square with sides of length ***a*** in the *xy* plane centered at the origin, with $q_1 = q_3 = q$ and $q_2 = q_4 = -q$, as shown in Figure 1. The electric field along the *z* axis at **P** point is ______________*E=0*____________.

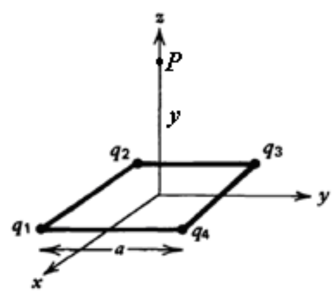


Figure 1

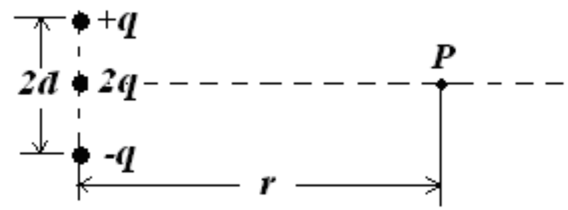


Figure 2

2. For the charge configuration of Fig. 2, the electric potential ***V(r)*** for points on the ***x*** axis, assuming ***r>>d*** is given by _____________. Set ***V=0*** at infinity.

$$V = \frac{q}{4\pi\varepsilon_0\sqrt{r^2+d^2}} + \frac{-q}{4\pi\varepsilon_0\sqrt{r^2+d^2}} + \frac{2q}{4\pi\varepsilon_0 r} = \frac{q}{2\pi\varepsilon_0 r}$$

3. A charge per unit length λ is distributed uniformly along a thin rod of length L. The potential (chosen to be zero at infinity) at a point **P** a distance y from one end of the rod and in line with it is V_P=________________。 The component of the electric field at **P** in the y direction (along the rod) is E_y=________________________. The component of the electric field at **P** in a direction perpendicular to the rod is E_x= ________________________________.

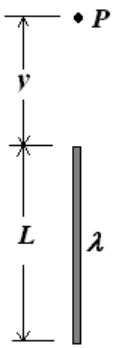


Figure 3

$$dV_p = \frac{\lambda dl}{4\pi\varepsilon_0 (l+y)}$$

$$V_P = \int_0^L \frac{\lambda dl}{4\pi\varepsilon_0 (l+y)} = \frac{\lambda}{4\pi\varepsilon_0} \ln(\frac{L+y}{y})$$

$$E_y = -\frac{\partial V_P}{\partial y} = \frac{\lambda}{4\pi\varepsilon_0} (\frac{1}{y} - \frac{1}{L+y})$$

$$E_y = -\frac{\partial V_P}{\partial x} = 0$$

4. There is a thin disk of radius ***a***, with total charge ***Q*** uniformly distributed on its surface, rotating at a constant angular speed ***ω***, as shown in Fig. 4. What is the magnetic dipole moment (磁偶极矩)? ________________________

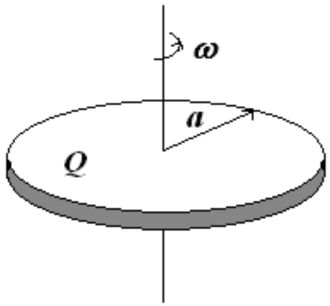


Figure 4

$$d\mu = \pi r^2 di = \pi r^2 \cdot \frac{Q}{\pi R^2} \cdot \frac{2\pi r dr}{2\pi / \omega} = \frac{\omega Q}{R^2} r^3 dr$$

$$\mu = \int_0^R \frac{\omega Q}{R^2} r^3 dr = \frac{1}{4} \omega Q R^2$$

5. A long hairpin is formed by bending a piece of wire as shown in Fig. 5. If the wire carries a current ***i*=11.5 A,** Which is the direction Out of Page and what is magnitude ________________ of magnetic induction strength ***B*** at point ***a***, taking ***R*=5.20** mm.

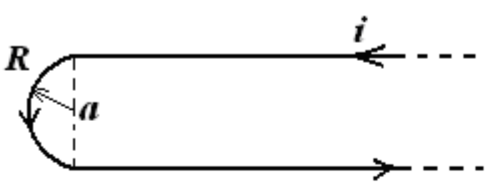


Figure 5

$$B = \frac{1}{2}\left(\frac{\mu_0 i}{2R} + \frac{\mu_0 i}{2\pi R} \cdot 2\right)$$

$$= \frac{4\pi \times 10^{-7} \times 11.5}{4(5.20 \times 10^{-3})} \cdot \left(\frac{2}{\pi} + 1\right) = 1.14 \times 10^{-3} T$$

6. Figure 6 shows a cross section of a long, thin ribbon (薄帯) of width ***w*** that is carrying a uniformly distributed total current ***I*** into the paper. Calculate the magnitude ________________________and the direction __________________ of the magnetic field ***B*** at a point **P** in the plane of the ribbon at a distance ***d*** from its edge.

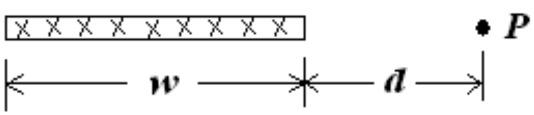


Figure 6

$$dB = \frac{\mu_0 \frac{I}{w} dl}{2\pi(d+l)} = \frac{\mu_0 I dl}{2\pi w(d+l)}$$

$$B = \int_0^w \frac{\mu_0 I dl}{2\pi w(d+l)} = \frac{\mu_0 I}{2\pi w} \ln(\frac{d+w}{w})$$

7. Two long parallel line currents of mass per unit length ***m*** in a gravity field ***g*** each carry a current ***I*** in opposite directions. They are suspended by cords (细线) of length ***l***. What is the angle ***θ*** between the cords? ______________________

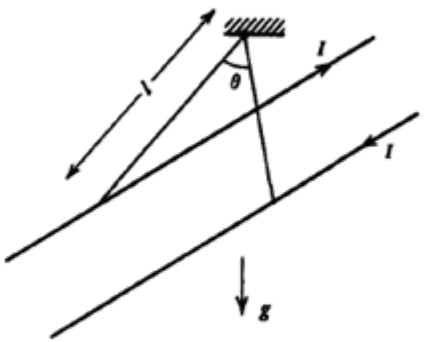
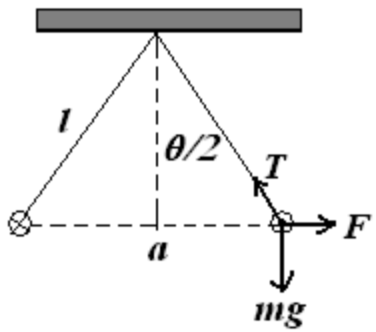


Figure 7



$$B=\frac{\mu_0 I}{2\pi a},\quad d\vec{f}=id\vec{s}\times\vec{B}$$

$$F=\frac{df}{ds}=\frac{\mu_0 I^2}{2\pi a}=\frac{\mu_0 I^2}{4\pi l\sin(\theta/2)}$$

$$\frac{F}{mg}=tg(\theta/2)$$

$$\frac{\mu_0 I^2}{4\pi lmg\sin(\theta/2)}=tg(\theta/2)$$

$$\frac{\mu_0 I^2}{4\pi lmg}=\frac{\sin^2(\theta/2)}{\cos(\theta/2)}=\frac{1-\cos^2(\theta/2)}{\cos(\theta/2)}$$

$$\cos^2(\theta/2)+\frac{\mu_0 I^2}{4\pi lmg}\cos(\theta/2)-1=0$$

$$\cos\frac{\theta}{2}=-\frac{\mu_0 I^2}{8\pi lmg}+\sqrt{\frac{\mu_0^2 I^4}{54\pi^2 l^2 m^2 g^2}+1}$$

$$\theta=2\arccos\left(-\frac{\mu_0 I^2}{8\pi lmg}+\sqrt{\frac{\mu_0^2 I^4}{54\pi^2 l^2 m^2 g^2}+1}\right)$$

8. Two straight, conducting rails form an angle θ where their ends are joined. A conducting bar in contact with the rails and forming an isosceles triangle (等腰三角形) with them starts at the vertex at time t =0 and moves with constant velocity v to the right, as shown in Fig. 8. A magnetic field B points out of the paper. The emf (电动势) induced as a function of time is ___________________________

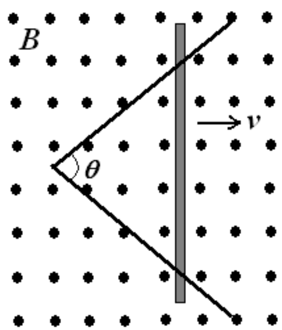


Figure 8

$$A = \frac{1}{2} \cdot vt \cdot (2 \cdot vt \cdot tg\frac{\theta}{2}) = v^2 t^2 tg\frac{\theta}{2}$$

$$\Phi_B = Bv^2 t^2 tg\frac{\theta}{2}$$

$$\varepsilon = -\frac{d\Phi_B}{dt} = -2Bv^2 t \cdot tg\frac{\theta}{2}$$

II. Problems (Present the necessary equations in solution) (50%)

1. (12%) As shown in Fig. 9, **a**n infinitely long line of radius ***a*** with a line charge density ***λ*** is placed at the center of a dielectric cylinder (圆柱形电介质) of radius ***R*** and dielectric constant κ_e. Please calculate the electric field ***E***, displacement vector ***D***, and polarization vector ***P*** for both ***a*** < ***r*** <***R*** and ***r*** ≥ ***R*** regions and the surface induced charge density σ_e at ***r*** = ***a*** and ***r*** = ***R*** surfaces.

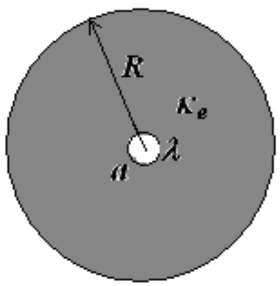


Figure 9

$a < r < R:$

$$(1)\ D = \frac{\lambda}{2\pi r}$$

$$D = \kappa_e \varepsilon_0 E, \quad E = \frac{D}{\kappa_e \varepsilon_0} = \frac{\lambda}{2\pi \kappa_e \varepsilon_0 r}$$

$$P = \chi_e \varepsilon_0 E = (\kappa_e - 1)\varepsilon_0 E = \frac{(\kappa_e - 1)\varepsilon_0 \lambda}{2\pi \kappa_e \varepsilon_0 r} = \frac{(\kappa_e - 1)\lambda}{2\pi \kappa_e r}$$

$$\oiint \vec{D} \bullet d\vec{A} = q_0$$

$$D \cdot 2\pi r \cdot L = \lambda \cdot L$$

$$D = \frac{\lambda}{2\pi r}$$

$r > R:$

$$D = \frac{\lambda}{2\pi r}$$

$$D = \varepsilon_0 E,$$

$$E = \frac{D}{\varepsilon_0} = \frac{\lambda}{2\pi \varepsilon_0 r}$$

$$P = 0$$

$r = a$

$$P = \frac{(\kappa_e - 1)\lambda}{2\pi \kappa_e a}$$

$$\sigma_e'(a) = \vec{P} \bullet \vec{n} = -\frac{(\kappa_e - 1)\lambda}{2\pi \kappa_e a}$$

$r = R$

$$P = \frac{(\kappa_e - 1)\lambda}{2\pi \kappa_e R}$$

$$\sigma_e'(R) = \vec{P} \bullet \vec{n} = \frac{(\kappa_e - 1)\lambda}{2\pi \kappa_e R}$$

2. (**18%**) Lorentz calculated the local electric field acting on a dipole due to a surrounding uniformly polarized medium stressed by a macroscopic field $\boldsymbol{E_0 i_z}$, by encircling the dipole with a small spherical free space cavity of radius $\boldsymbol{R}$, as shown in Fig. 10.

(a). If the medium outside the cavity has polarization $\boldsymbol{P_o i_z}$, what is the induced surface charge on the spherical interface?

(b) Break this surface induced charge distribution into hoop line charge elements of thickness $\boldsymbol{d\theta}$. What is the total charge on a particular shell at angle $\boldsymbol{\theta}$?

(c) What is the electric field due to this shell at the center of the sphere where the dipole is?

(d). By integrating over all shells, find the total electric field acting on the dipole. This is called the Lorentz field.

$$\sigma_e' = \vec{P} \bullet \vec{n} = -P\cos\theta$$

$$(a).\ q' = \int_0^\pi (-P_0\cos\theta)\cdot(2\pi R\sin\theta)\cdot R d\theta = -2\pi R^2 P_0 \int_0^\pi \sin\theta\cos\theta d\theta = 0$$

$$(b).\ dq' = \sigma_e' dA = -P_0\cos\theta\cdot(2\pi R\sin\theta)\cdot R d\theta$$

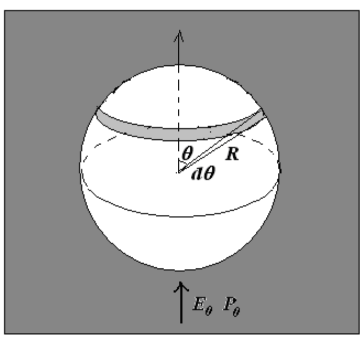


Figure 10

$$(c).\ dE_z = -\frac{R\cos\theta\cdot dq'}{4\pi\varepsilon_0 R^3} = \frac{2\pi R^3 P_0\sin\theta\cos^2\theta d\theta}{4\pi\varepsilon_0 R^3} = \frac{P_0\sin\theta\cos^2\theta d\theta}{2\varepsilon_0}$$

$$(d).\ E_z = \frac{P_0}{2\varepsilon_0}\int_0^\pi \sin\theta\cos^2\theta d\theta = \frac{P_0}{3\varepsilon_0}$$

3. As shown in Fig.11 (a), a thin circular disk of radius ***a***, thickness ***d***, and conductivity (电导率) σ is placed in a uniform time varying magnetic field ***B(t)***.

(a). Neglecting the magnetic field of the eddy currents, what is the current induced in a thin circular filament at radius ***r*** of thickness ***dr***.

(b). What power is dissipated (损耗) in this thin current loop?

(c). How much power is dissipated in the whole disk?

(d). If the disk is cut up into ***N*** smaller circular disks with negligible wastage (缝隙), as shown in Fig. 11(b), what is the approximate radius of each smaller disk?

(e). If these ***N*** smaller disks are laminated (叠压) together to form a thin disk of closely packed cylindrical wires, what is the power dissipated?

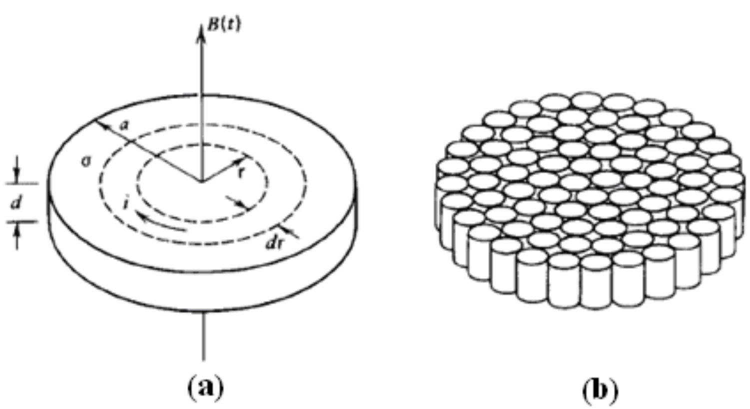


Figure 11

$$(a).\ \oint \vec{E} \bullet d\vec{l} = -\iint \frac{\partial \vec{B}}{\partial t} \bullet d\vec{A}$$

$$E \cdot 2\pi r = -\frac{dB}{dt} \cdot \pi r^2,\ \ E = -\frac{1}{2} r \frac{dB}{dt}$$

$$j = \sigma E = \frac{1}{2} \sigma r \frac{dB}{dt}$$

$$di = j \cdot d \cdot dr = \frac{1}{2} r \sigma d \frac{dB}{dt} dr$$

$$(b).\quad dP = \varepsilon di = E \cdot 2\pi r di = \frac{1}{2} r \frac{dB}{dt} \cdot 2\pi r \cdot \frac{1}{2} r \sigma d \frac{dB}{dt} dr$$

$$= \frac{1}{2} \pi \sigma d (\frac{dB}{dt})^2 r^3 dr$$

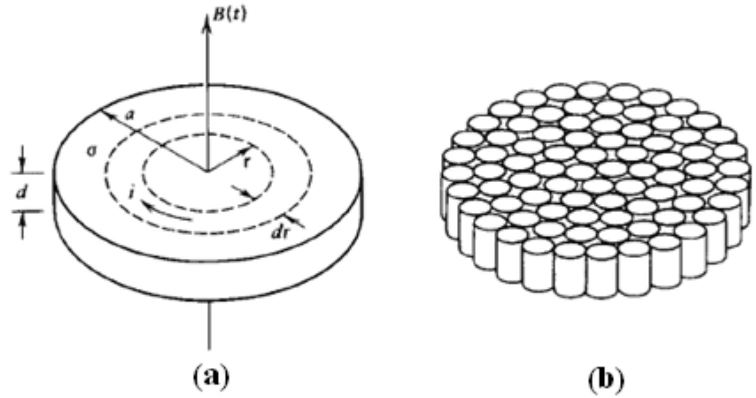


Figure 11

$$(c).\quad P=\int_0^{a}\frac{1}{2}\pi\sigma d\left(\frac{dB}{dt}\right)^2 r^3 dr=\frac{1}{8}\pi\sigma d\left(\frac{dB}{dt}\right)^2 a^4$$

$$(d).\quad \pi a^2=N(\pi R^2)\quad R=\sqrt{\frac{1}{N}}a$$

$$(e).\quad P_{small}=\frac{1}{8}\pi\sigma d\left(\frac{dB}{dt}\right)^2\cdot\frac{1}{N^2}a^4$$

$$P_{total}=N\cdot P_{small}=\frac{1}{N}P$$