# 浙江大学 2018-2019 学年 秋冬 学期

## 《离散数学》课程期末考试试卷

课程号：21120401　开课学院：计算机学院

考试试卷：☑ A卷　☐ B卷

考试形式：☑ 闭卷　☐ 开卷，允许带 ________入场

考试日期: 2019 年 1 月 24 日，考试时间: 120 分钟

**诚信考试，沉着应考，杜绝违纪**

考生姓名________________ 学号________________ 所属院系________________

| 题序 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 总分 |
|---|---|---|---|---|---|---|---|---|---|
| 得分 | | | | | | | | | |
| 评卷人 | | | | | | | | | |

Zhejiang University
Discrete Mathematics, Fall-Winter 2018
Final Exam

1. **(20 pts) Determine whether the following statements are true or false. If it is true fill a $\surd$ otherwise a $\times$ in the bracket before the statement.**

   (a) (　) There are two sets $A$ and $B$, such that both $A \subseteq B$ and $A \in B$.

   (b) (　) Let $A, B$ be two sets. If $2^A \subseteq 2^B$, then $A \subseteq B$, where $2^X$ is the power set of $X$.

   (c) (　) Let $\varphi$ be a proposition with three variables $p$, $q$, and $r$, if exactly one of the assignments $000, 011$, 100, and 111 make $\varphi$ true, then $\varphi$ can be converted in full disjunctive normal form $\Sigma(1, 2, 5, 6)$.

   (d) (　) Let $\mathrm{P} = \{f | f$ is a propositional formula with variables $p, q, r\}$ and R be the equivalent relation on P by $\varphi \mathrm{R} \psi$ iff $\varphi \Leftrightarrow \psi$, for $\varphi, \psi \in \mathrm{P}$, then the number of equivalence classes of P on R is 256.

   (e) (　) Let $P(x), Q(x)$ be two predicates, then $\forall x P(x) \to \exists y Q(y) \Leftrightarrow \forall x \exists y (P(x) \to Q(y))$.

   (f) (　) There is a binary relation $R$ on set $A$ such that $R$ is both an equivalence relation and partial order on $A$.

   (g) (　) Suppose $(A, \preceq)$ is a finite nonempty poset. Then $A$ has a minimal element.

   (h) (　) There is no known algorithm to decide if a graph has a Hamilton circuit.

   (i) (　) Every connected bipartite graph contains a circuit of even length.

   (j) (　) A binary tree with height $h$ has at most $2^h$ leaves.

2. **(12 pts)** Construct a logical argument using rules of inference to show that the following sentences imply the conclusion: *It rained.*
*If it does not rain or if it is not foggy, then the sailing race will be held and the life-saving demonstration will go on. If the sailing race is held, then the trophy will be awarded. The trophy was not awarded.*

Justify each step by indicating the rule you applied.

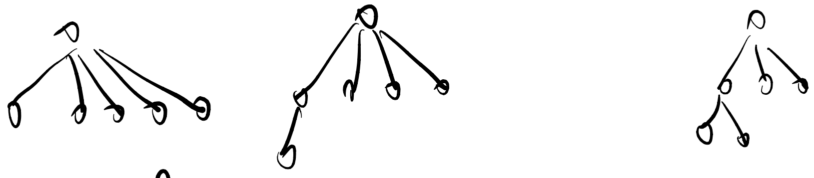
3. **(12%)** Prove that the two sets of real numbers $(0, 1)$ and $[a, b) = \{x \in \mathbb{R} | a \leq x < b\}$ have the same cardinality(基数).

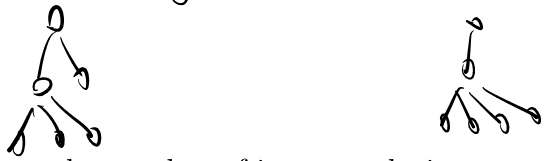
4. **(10 pts)** Let $S \subseteq \{1, 2, 3, 4, 5, 6, \cdots, 98, 99, 100\}$ where $|S| = 62$. Prove that there exist $x, y \in S$ such that $x - y = 23$.

5. **(12 pts)** Define a binary relation $\mathcal{R}$ on $\mathbb{R} \times \mathbb{R}$ by $(x, y)\mathcal{R}(a, b)$ if and only if $y - b = 5(x - a)$.
   (a) Show that $\mathcal{R}$ is an equivalence relation on $\mathbb{R} \times \mathbb{R}$.
   (b) Find the equivalence classes of $(0, 0)$ and $(1, -2)$.
   (c) Describe the geometric meaning the equivalence class $[(a, b)]$.

6. **(12 pts)** Let $G = (V, E)$ be a simple graph with $n$ vertices, where $n \geq 3$, and $d(u) + d(v) \geq n$, for every $u, v \in V$. Prove
   (a) $G$ is connected.
   (b) $G$ is a Hamilton graph.

7. **(10 pts)** Find all non-isomorphism trees with exactly 6 vertices and four or more leaves.





8. **(12 pts)**

(a) Let $a_n$ denote the number of integer solutions to

$$5x_1 + 4x_2 + x_3 + x_4 = n, 0 \leq x_1, 0 \leq x_2, 0 \leq x_3 \leq 4, 0 \leq x_4 \leq 3$$

(i) Find the generating function $G(x) = \sum_{n=o}^{\infty} a_n x^n$. (ii) Obtain a formula for $a_n$, explicitly.

(i) $(1+x^5+x^{10}+\dots)(1+x^4+x^8+\dots)(1+\dots+x^4)(1+\dots+x^3)$

$= \frac{1-x^4}{1-x} \frac{1-x^5}{1-x} \cdot \frac{1}{1-x^4} \frac{1}{1-x^5} = \frac{1}{(1-x)^2}$, (ii): $a_n = n+1$

(b) For all integers $n$, let $b_n$ be the number of binary strings of length $n$ that contain the substring 000. (i) Give $b_4, b_5, b_6$. (ii) Write a recurrence relation for $b_n$. (iii) Solve the recurrence relation for $b_n$.

$b_1 = b_2 = 0$, $b_3 = 1$, $b_4 = 3$, $b_5 = 8$, $b_6 = 20$

$b_n = (2^{n-4} - b_{n-4}) + 2b_{n-1}$ $(n \geq 5)$

方程: $r^4 - 2r^3 + 1. = r^4 - r^3 - r^3 + r^2 - r^2 + r - r + 1$

$= (r-1)(r^3 - r^2 - r - 1)$

$= (r-1)(r-i)$

7. **(10 pts)** Find all non-isomorphism trees with exactly 6 vertices and four or more leaves.

8. **(12 pts)**

(a) Let $a_n$ denote the number of integer solutions to

$$5x_1 + 4x_2 + x_3 + x_4 = n, 0 \leq x_1, 0 \leq x_2, 0 \leq x_3 \leq 4, 0 \leq x_4 \leq 3$$

(i) Find the generating function $G(x) = \sum_{n=o}^{\infty} a_n x^n$. (ii) Obtain a formula for $a_n$, explicitly.

(b) For all integers $n$, let $b_n$ be the number of binary strings of length $n$ that contain the substring 000. (i) Give $b_4, b_5, b_6$. (ii) Write a recurrence relation for $b_n$. (iii) Solve the recurrence relation for $b_n$.

ENJOY YOUR SPRING FESTIVAL!

7. **(10 pts)** Find all non-isomorphism trees with exactly 6 vertices and four or more leaves.

8. **(12 pts)**

(a) Let $a_n$ denote the number of integer solutions to

$$5x_1 + 4x_2 + x_3 + x_4 = n, 0 \le x_1, 0 \le x_2, 0 \le x_3 \le 4, 0 \le x_4 \le 3$$

(i) Find the generating function $G(x) = \sum_{n=o}^{\infty} a_n x^n$. (ii) Obtain a formula for $a_n$, explicitly.

(b) For all integers $n$, let $b_n$ be the number of binary strings of length $n$ that contain the substring 000. (i) Give $b_4, b_5, b_6$. (ii) Write a recurrence relation for $b_n$. (iii) Solve the recurrence relation for $b_n$.

Enjoy Your Spring Festival!