I. Fill in the space underlined (40%)

1. (4%)As shown in Figure 1, a point charge $+q_0$ is a distance $d/2$ from a square surface of side d and is just above the center of the square. The electric flux through the square is of _______.

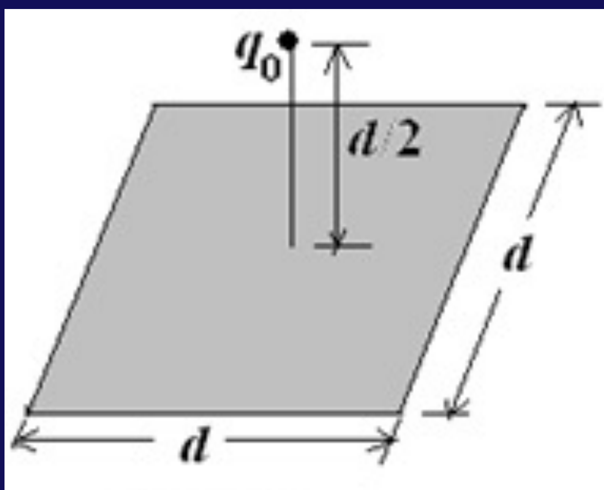


Figure 1

$$\iint \vec{E} \bullet d\vec{A} = \frac{q_0}{\varepsilon_0}$$

$$\Phi = \frac{q_0}{6\varepsilon_0}$$

2. (5%) As shown in Figure 2, the rod has a uniform positive charge density λ on the top half of the rod and a uniform charge density -λ on the bottom half of the rod. The net force on the point charge q_0 is ________________.

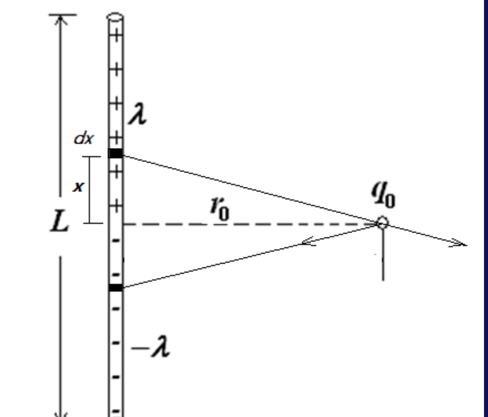


Figure 2

$$dF = \frac{1}{4\pi\varepsilon_0}\frac{q_0\lambda dx}{r_0^2 + x^2}$$

$$dF_y = \frac{1}{4\pi\varepsilon_0}\frac{q_0\lambda dx}{r_0^2 + x^2}\sin\alpha = \frac{1}{4\pi\varepsilon_0}\frac{q_0\lambda dx}{r_0^2 + x^2}\frac{x}{\sqrt{r_0^2 + x^2}}$$

$$F_y = -\int_0^{L/2}\frac{q_0\lambda}{4\pi\varepsilon_0}\frac{xdx}{(r_0^2 + x^2)^{3/2}} = -\frac{q_0\lambda}{8\pi\varepsilon_0}\frac{1}{-\frac{3}{2}+1}\frac{1}{(r_0^2 + x^2)^{1/2}}\Bigg|_0^{L/2}$$

$$= -\frac{q_0\lambda}{4\pi\varepsilon_0}\left(\frac{1}{\sqrt{r_0^2 + L^2/4}} - \frac{1}{r_0}\right)$$

$$F = 2F_y = \frac{q_0\lambda}{2\pi\varepsilon_0}\left(\frac{1}{r_0} - \frac{1}{\sqrt{r_0^2 + L^2/4}}\right)$$

3. (5%) A square wire loop with 2.3m sides is perpendicular to a uniform magnetic field, with half the area of the loop in the field, as shown in Fig. 3. The loop contains a 2.0V battery with negligible internal resistance (内阻). If the magnitude of the field varies with time according to B = (0.042 T – (0.87 T/s)t, the total emf(电动势) in the circuit is ____________________.

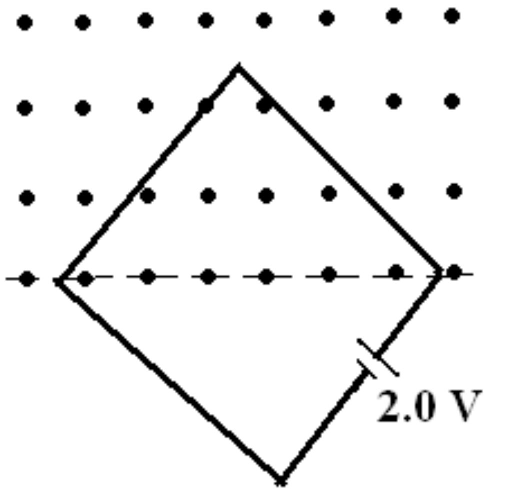


Figure 3

$$B = 0.042 - 0.87t$$

$$\varepsilon_L = -\frac{dB}{dt} \bullet A = \frac{1}{2} \times 0.87 \times 2.3 \times 2.3 = 2.3V$$

$$\varepsilon = \varepsilon_0 + \varepsilon_L = 2.0 + 2.3 = 4.3V$$

4. (5%) In Bohr's theory of hydrogen atom, the electron can be thought of as moving in a circular orbit of radius *r* about the proton (质子). Suppose that such an atom is placed in a magnetic field *B*, with the plane of the orbit at right angles to *B*, as shown in Fig. 4. If the electron is circulating counterclockwise (反时针), as viewed by an observer sighting along *B*, and assume the orbit radius does not change. the change in the frequency of revolution caused by the magnetic field is given approximately by __________________________. Such frequency shifts were observed by Zeeman in 1896.

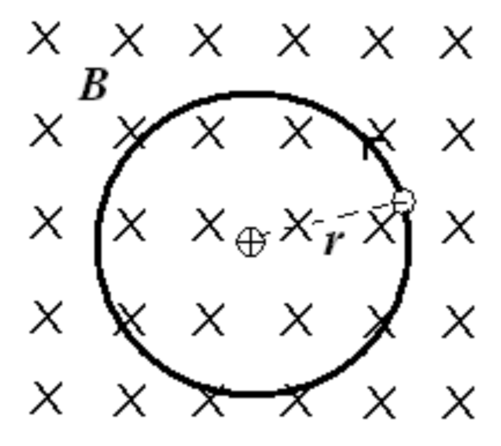


Figure 4

$$\frac{e^2}{4\pi\varepsilon_0 r^2} - e\omega r B = m\omega^2 r$$

$$\omega = \omega_0 - \Delta\omega$$

$$\Delta\omega = \frac{eB}{2m}$$

5. (5%) A parallel-plate capacitor is filled with three dielectrics as in Fig. 5, the capacitance is given by ______________________.

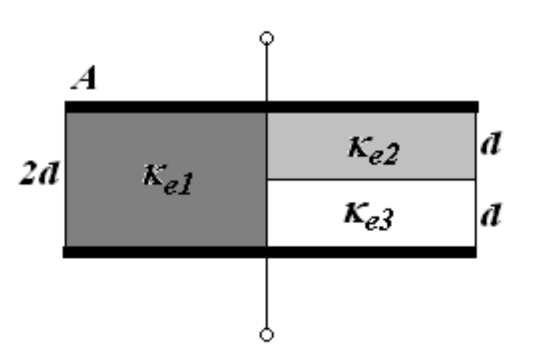
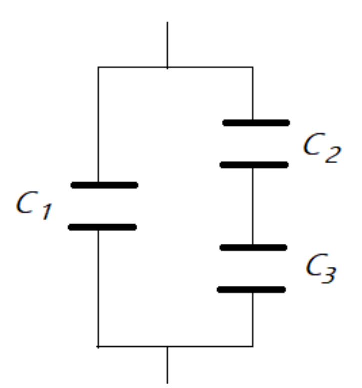


Figure 5



$$C_1 = \kappa_{e1} \frac{\varepsilon_0 A/2}{2d}, \quad C_2 = \kappa_{e2} \frac{\varepsilon_0 A/2}{d}, \quad C_3 = \kappa_{e3} \frac{\varepsilon_0 A/2}{d}$$

$$\frac{1}{C_{2,3}} = \frac{1}{C_2} + \frac{1}{C_3}, \quad C_{2,3} = \frac{C_2 C_3}{C_2 + C_3} = \frac{\kappa_{e2} \cdot \kappa_{e3}}{\kappa_{e2} + \kappa_{e3}} \cdot \frac{\varepsilon_0 A}{2d}$$

$$C = C_{2,3} + C_1 = [\frac{\kappa_{e1}}{2} + \frac{\kappa_{e2} \cdot \kappa_{e3}}{\kappa_{e2} + \kappa_{e3}}] \bullet \frac{\varepsilon_0 A}{2d}$$

6. (5%) As shown in Fig. 6, a long, straight conducting wire with circular cross section of radius R carries a current I. There is a cylindrical hole inside the conductor, whose radius is of a, and whose axis is parallel to the axis of the conductor but offset a distance b from the axis of the wire. The current I is uniformly distributed and is directed out of the page. The magnetic field at P point ($R > x > a+b$) at the x axis is ___________.

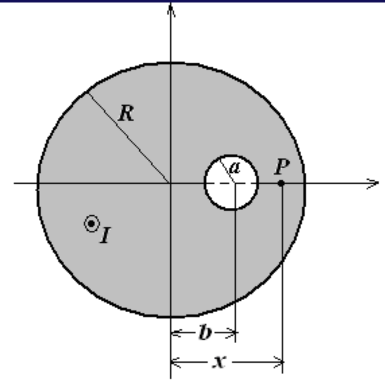


Figure 6

$$\int \vec{B} \bullet d\vec{l} = \mu_0 \frac{\pi x^2}{\pi(R^2 - a^2)} I$$

$$B \cdot 2\pi x = \mu_0 \frac{x^2}{R^2 - a^2} I$$

$$B_R = \frac{\mu_0 I x}{2\pi(R^2 - a^2)}$$

$$\int \vec{B} \bullet d\vec{l} = \mu_0 \frac{\pi a^2}{\pi(R^2 - a^2)} I$$

$$B \cdot 2\pi(x - b) = \mu_0 \frac{a^2}{R^2 - a^2} I$$

$$B_a = \frac{\mu_0 I a^2}{2\pi(x - b)(R^2 - a^2)}$$

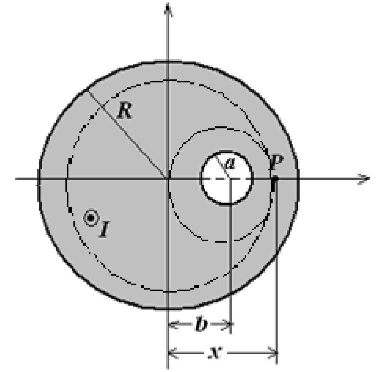


Figure 6

$$B = B_R - B_a = \frac{\mu_0 I}{2\pi(R^2 - a^2)}\left(x - \frac{a^2}{x - b}\right)$$

7. (5%) A long, circular pipe, with an outside radius of *R*, carries a (uniformly distributed) current of i_0 (into the paper as shown in Fig. 7). A wire runs parallel to the pipe at a distance *3R* from the center to center. What are the magnitude ____________________ and direction ____________________ of the current in the wire that would cause the resultant magnetic field at the point *P* to be zero.

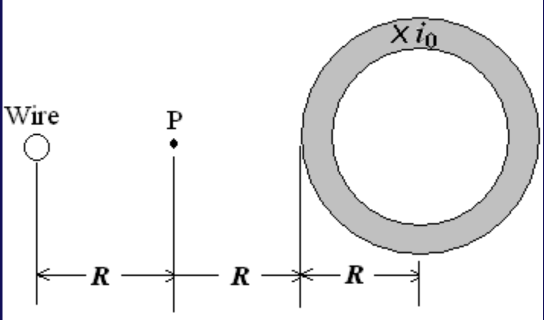
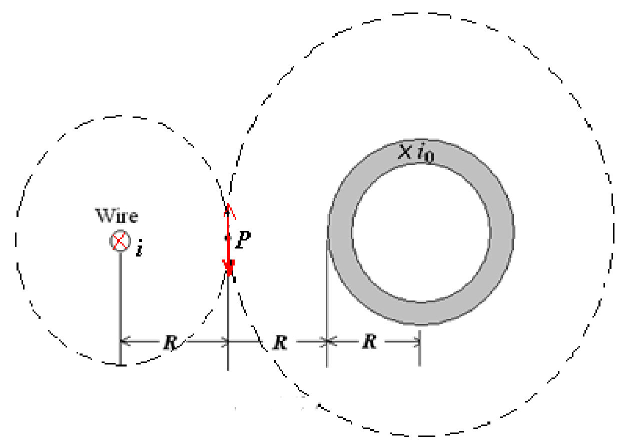


Figure 7

$$\int \vec{B}\bullet d\vec{l} = \mu_0 i_0$$
$$B \cdot 2\pi \cdot 2R = \mu_0 i_0$$
$$B_1 = \frac{\mu_0 i_0}{4\pi R}$$

$$\int \vec{B}\bullet d\vec{l} = \mu_0 i$$
$$B \cdot 2\pi R = \mu_0 i$$
$$B_2 = \frac{\mu_0 i}{2\pi R}$$



$$B_1 = B_2, \quad i = \frac{i_0}{2}$$

8. (6%) As shown in Fig. 8, an infinity slab (无限大平板) of thickness d has a uniform volume charge density ρ. Find the magnitude of the electric field in region I __________, II__________, and III___________, in terms of x, the distance measured from the median plane of the slab.

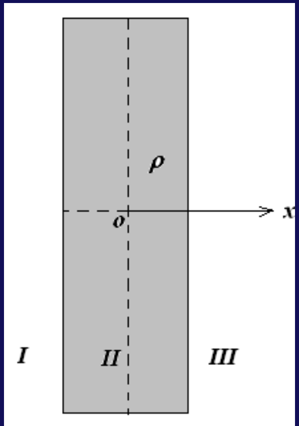


Figure 8

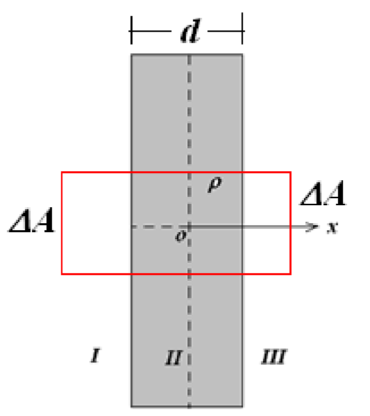


Figure 8

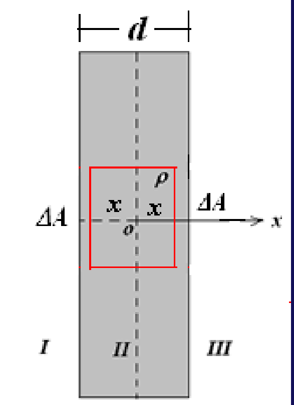


Figure 8

$$\iint \vec{E}\cdot d\vec{A} = \frac{q}{\varepsilon_0}$$

$$2E\cdot\Delta A = \frac{1}{\varepsilon_0}\rho\cdot d\Delta A,\quad E = \frac{\rho d}{2\varepsilon_0},\qquad \text{I: } E_I = -\frac{\rho d}{2\varepsilon_0},\quad \text{III: } E_{III} = \frac{\rho d}{2\varepsilon_0}$$

$$2E\cdot\Delta A = \frac{1}{\varepsilon_0}\rho\cdot 2x\Delta A,\quad E = \frac{\rho x}{\varepsilon_0},\qquad \text{II: } E_{II} = \frac{\rho x}{\varepsilon_0}$$

II. Problems (Present the necessary equations in solution) (60%)

1. (12%)As shown in Fig. 9, assume that the electron is not a point, but a sphere of radius *R* having a uniform volume charge distribution (均匀体电荷分布). Please calculate the total electrostatic energy for such electron.

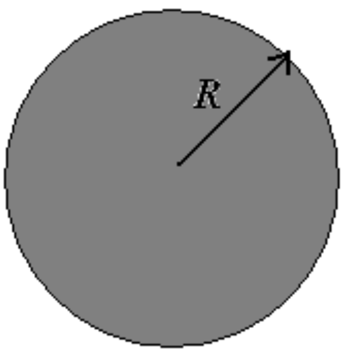


Figure 9

$$\oiint \vec{E}\bullet d\vec{A} = \frac{q_0}{\varepsilon_0}, \quad E = \begin{cases} \dfrac{e}{4\pi\varepsilon_0 r^2}, & (r > R) \\ \dfrac{er}{4\pi\varepsilon_0 R^3}, & (r \le R) \end{cases}$$

$$U = \int_0^R \frac{1}{2}\varepsilon_0 E^2 \cdot 4\pi r^2 dr + \int_R^\infty \frac{1}{2}\varepsilon_0 E^2 \cdot 4\pi r^2 dr$$

$$= \frac{e^2}{8\pi\varepsilon_0}[\int_0^R \frac{r^4}{R^6} dr + \int_R^\infty \frac{dr}{r}]$$

$$= \frac{e^2}{8\pi\varepsilon_0}[\frac{1}{5R} + \frac{1}{R}] = \frac{3e^2}{20\pi\varepsilon_0 R}$$

2. (13%) As shown in Figure 10, a wireframe (线框) with current i_2 is composed of an arc (圆弧) with the flare angle (张角) of 2(π-φ) and the radius of *a*, and a subtense (弦) connecting two ends of the arc. Meanwhile, there is a long straight conducting wire with current i_1, which is located at the center of the arc and perpendicular to the plane of the wireframe. Please calculate the torque (力矩) τ acted on the wireframe.

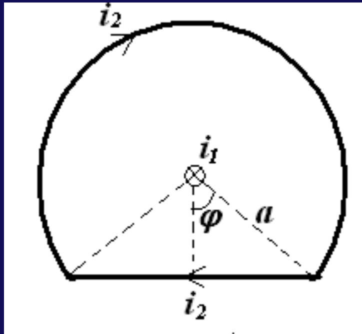
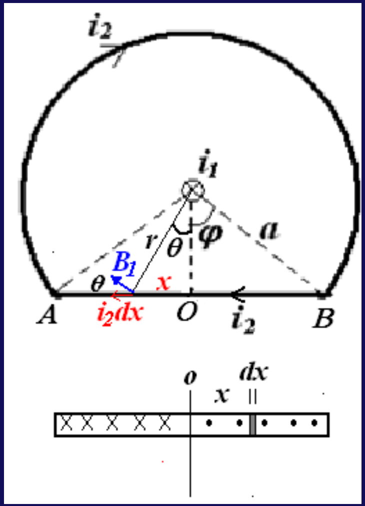


Figure 10

The magnetic field induced by i_1: $B = \dfrac{\mu_0 i}{2\pi r}$

The torque on the AB arc: $\tau = 0$

The torque on the AB substence: $\tau \neq 0$



$$d\vec{F} = id\vec{s} \times \vec{B}, \qquad dF = i_2 dx \cdot B \sin\theta = \frac{\mu_0 i_1 i_2 dx}{2\pi r} \sin\theta$$

$$d\tau = x \cdot dF = \frac{\mu_0 i_1 i_2 \sin\theta}{2\pi r} x dx$$

$$x = a\cos\varphi \cdot tg\theta, \quad dx = a\cos\varphi \cdot \frac{d\theta}{\cos^2\theta}$$

$$r = \frac{a\cos\varphi}{\cos\theta}$$

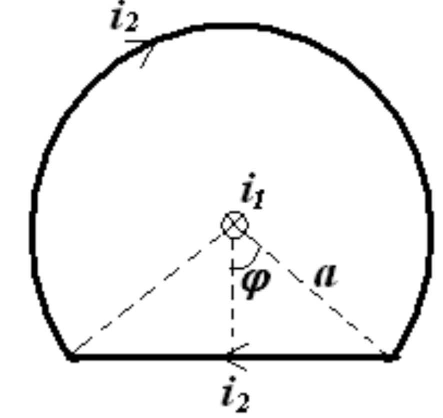


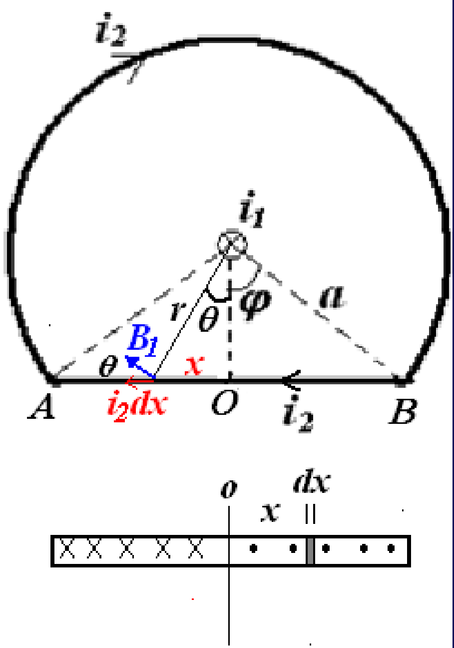
Figure 10

$$d\vec{F} = i d\vec{s} \times \vec{B}, \qquad dF = i_2 dx \cdot B \sin\theta = \frac{\mu_0 i_1 i_2 dx}{2\pi r} \sin\theta$$

$$d\tau = x \cdot dF = \frac{\mu_0 i_1 i_2 \sin\theta}{2\pi r} x dx$$

$$x = a\cos\varphi \cdot tg\theta, \quad dx = a\cos\varphi \cdot \frac{d\theta}{\cos^2\theta}$$

$$r = \frac{a\cos\varphi}{\cos\theta}$$



$$d\tau = x \cdot dF = \frac{\mu_0 i_1 i_2 \sin\theta \cdot \cos\theta}{2\pi a\cos\varphi} a\cos\varphi \cdot tg\theta \cdot \frac{a\cos\varphi}{\cos^2\theta} d\theta$$

$$= \frac{\mu_0 i_1 i_2 a\cos\varphi}{2\pi} tg^2\theta d\theta$$

$$\tau = \int_{-\varphi}^{\varphi} \frac{\mu_0 i_1 i_2 a\cos\varphi}{2\pi} tg^2\theta d\theta$$

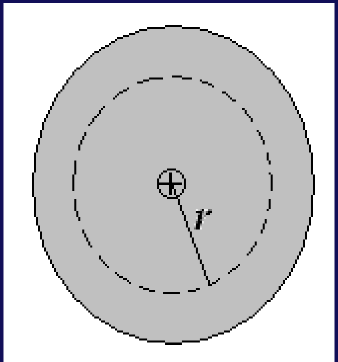
$$= \frac{\mu_0 i_1 i_2 a\cos\varphi}{2\pi} \int_{-\varphi}^{\varphi} [\frac{1}{\cos^2\theta} - 1] d\theta$$

$$= \frac{\mu_0 i_1 i_2 a}{\pi} (\sin\varphi - \varphi\cos\varphi)$$

3. (15%)According to the quantum mechanics, H atom is composed of a positive charge q_e (considered as a point charge) in nucleus, and the electron cloud. At the normal condition (*s* orbit), the charge volume density of electron cloud is as:

$$\boxed{\rho = -\frac{q_e}{\pi a_0^3} e^{-\frac{2r}{a_0}}}$$

where a_0 is a constant, *i.e.* Bohr radius. Please calculate the electric field distribution E(r).



$$\iint \vec{E} \cdot d\vec{A} = \frac{q}{\varepsilon_0}$$

$$E \cdot 4\pi r^2 = \frac{1}{\varepsilon_0}[q_e - \int_0^r \frac{q_e}{\pi a_0^3} e^{-\frac{2r}{a_0}} 4\pi r^2 dr]$$

$$E = \frac{q_e}{4\pi\varepsilon_0 r^2}[1 - \frac{4}{a_0^3}\int_0^r r^2 e^{-\frac{2r}{a_0}} dr] = \frac{q_e}{4\pi\varepsilon_0 r^2}[1 - \frac{4}{a_0^3}\int_0^r r^2 e^{-\frac{2r}{a_0}} dr]$$

$$= \frac{q_e}{4\pi\varepsilon_0 r^2}[1 + \frac{2r}{a_0} + \frac{2r^2}{a_0^2}]e^{-\frac{2r}{a_0}}$$

4. (20%)As shown in Fig. 11, a parallel plate capacitor is composed of two plates with length of $2l_1$ and width of l_2, separated by a distance of a. It is filled with a non-uniform dielectrics (非均匀电介质), its dielectric constant as a function of position x is:

$$\kappa_e = 1 + A(x/l_1)^2$$

where A is a constant. The capacitor is connected with a battery, thus the potential difference between plates is V. Please calculate:

(a) The area charge density distribution, $\sigma_e(x)$, on the plates.

(b) The total charge on the plates, Q.

(c) The capacitance, C.

(d) The polarization $P(x)$ distribution in dielectrics.

(e) The induced charge density distribution, $\sigma'_e(x)$, on the surface of dielectrics.

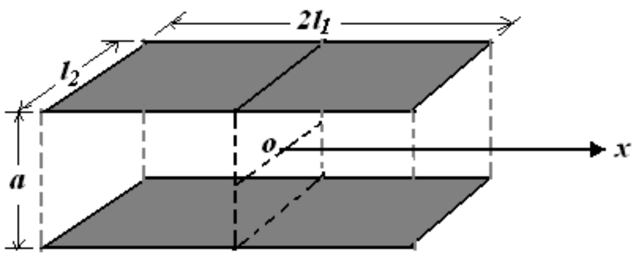


Figure 11

$$\oiint \vec{E} \cdot d\vec{A} = \frac{q}{\varepsilon_0}$$

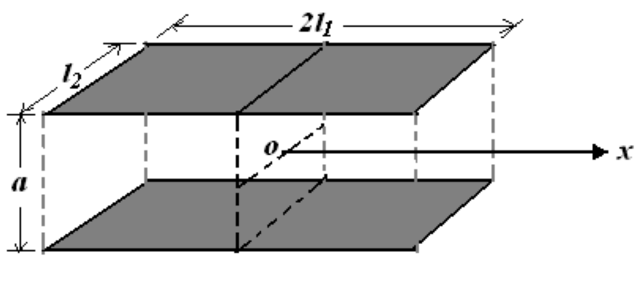
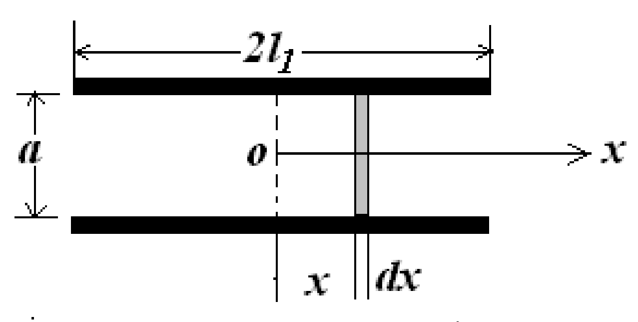


Figure 11

$$\kappa_e = 1 + A(x/l_1)^2$$

$$dc = \frac{\kappa_e \varepsilon_0 dA}{a} = \frac{\kappa_e \varepsilon_0 l_2 dx}{a}$$



$$(a).\ \sigma_e = \frac{dQ}{dA} = \frac{Vdc}{dA} = \frac{V\kappa_e \varepsilon_0 l_2 dx}{al_2 dx}$$

$$= \frac{\kappa_e \varepsilon_0 V}{a} = \frac{\varepsilon_0 V}{a}[1 + A(\frac{x}{l_1})^2]$$

$$(b).\ Q = \int_{-l_1}^{l_1} \frac{\varepsilon_0 V}{a}[1 + A(\frac{x}{l_1})^2] l_2 dx = \frac{\varepsilon_0 V l_2}{a}[2l_1 + \frac{A}{l_1^2}\frac{2}{3}l_1^3]$$

$$= \frac{2\varepsilon_0 V l_2 l_1}{a}[1 + \frac{1}{3}A]$$

$$(c).\ C = \frac{Q}{V} = \frac{2\varepsilon_0 l_2 l_1}{a}[1 + \frac{1}{3}A]$$

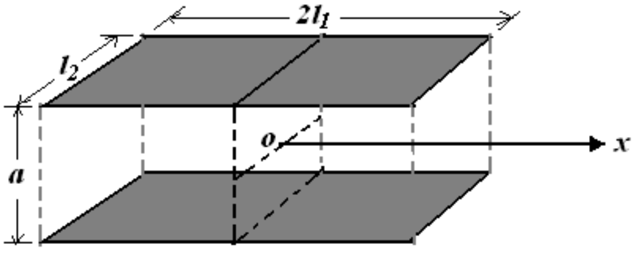
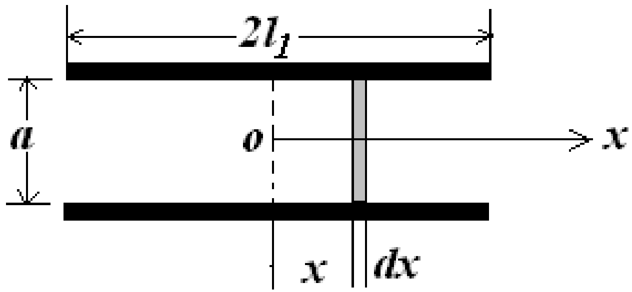


Figure 11



$$\kappa_e = 1 + A(x/l_1)^2$$

$$(d).\ E = \frac{V}{a},\quad \vec{P} = \chi_e \varepsilon_0 \vec{E} = (\kappa_e - 1)\varepsilon_0 \vec{E}$$

$$P = A\left(\frac{x}{l_1}\right)^2 \frac{\varepsilon_0 V}{a}$$

$$(e).\ \sigma_e'(x) = \vec{P} \bullet \hat{n}$$

The upper surface of dielectrics:

$$\sigma_e'(x) = \vec{P} \bullet \hat{n} = -A\left(\frac{x}{l_1}\right)^2 \frac{\varepsilon_0 V}{a}$$

The lower surface of dielectrics:

$$\sigma_e'(x) = \vec{P} \bullet \hat{n} = A\left(\frac{x}{l_1}\right)^2 \frac{\varepsilon_0 V}{a}$$