浙江大学 2019-2020 学年 秋冬 学期

《离散数学》课程期末考试试卷

课程号：21120401 开课学院：计算机学院

考试试卷：☑ A卷 □ B卷

考试形式：☑ 闭卷 □ 开卷，允许带 ________ 入场

考试日期：2020 年 1 月 14 日，考试时间：120 分钟

诚信考试，沉着应考，杜绝违纪

考生姓名________________ 学号________________ 所属院系________________

题序	1	2	3	4	5	6	7	8	总分
得分									
评卷人									

Zhejiang University
Discrete Mathematics, Fall-Winter 2019
Final Exam

1. **(20 pts) Determine whether the following statements are true or false. If it is true fill a $\surd$ otherwise a $\times$ in the bracket before the statement.**

   (a) (   ) Let A, B, C and D be arbitrary sets, then $(A \cup B) \times (C \cup D) = (A \times C) \cup (B \times D)$.

   (b) (   ) Let A, B be two sets. If $2^A \in 2^B$, then $A \in B$, where 2^X is the power set of X.

   (c) (   ) Let A, B and C be arbitrary sets. If $2^A \oplus 2^C = 2^B \oplus 2^C$, then $A = B$, where $\oplus$ denotes symmetric difference.

   (d) (   ) Let $P(x), Q(x)$ be two predicates, then $\exists x(P(x) \vee Q(x)) \Leftrightarrow \exists x P(x) \vee \exists y Q(y)$.

   (e) (   ) If exactly one of the assignments $000, 011$, 100, and 111 make the propositional formula φ false, then φ can be converted in full disjunctive normal form $\Sigma(0, 3, 4, 7)$.

   (f) (   ) The set of all functions from $\mathbb{N}$ to $\mathbb{N}$ is uncountable infinite.

   (g) (   ) Let R be a binary relation. If R is symmetric and transitive, then R is reflexive.

   (h) (   ) Let $(S, \preceq)$ be a partially ordered set, if there is unique minimal element e of S, then e is the least element of S.

   (i) (   ) If a graph contains an Hamilton circuit, then it does not have a cut-edge.

   (j) (   ) Let G be a simple planar graph with e edges, v vertices and r regions, then $r = e - v + 2$.

2. **(12 pts)** Construct arguments to prove that the following reasoning is valid.

   Hypothesis: $\forall x(\forall y(B(x,y) \to \neg A(y)) \to \neg C(x))$
   Conclusion: $\forall x(C(x) \to \exists y(A(y) \land B(x,y)))$

3. **(12%)** Prove that $\mathbb{N} \times \mathbb{N}$ is countable infinite.

4. **(10 pts)** There are 51 houses on a street. Each house has an address between 1000 and 1099, inclusive. Show that at least two houses have addresses that are consecutive integers.

5. **(12 pts)** Let F be the set of all propositional formulas involving variables $p_1, p_2, \cdots, p_n$. Define a binary relation $\mathcal{R}$ on F:

$$\forall A, B \in F, A\mathcal{R}B \text{ if and only if } A \leftrightarrow B \text{ is a tautology.}$$

(a) Prove that the binary relation $\mathcal{R}$ on F is an equivalence relation.

(b) Give a description of the equivalence classes.

(c) How many different equivalence classes of $\mathcal{R}$ are there?

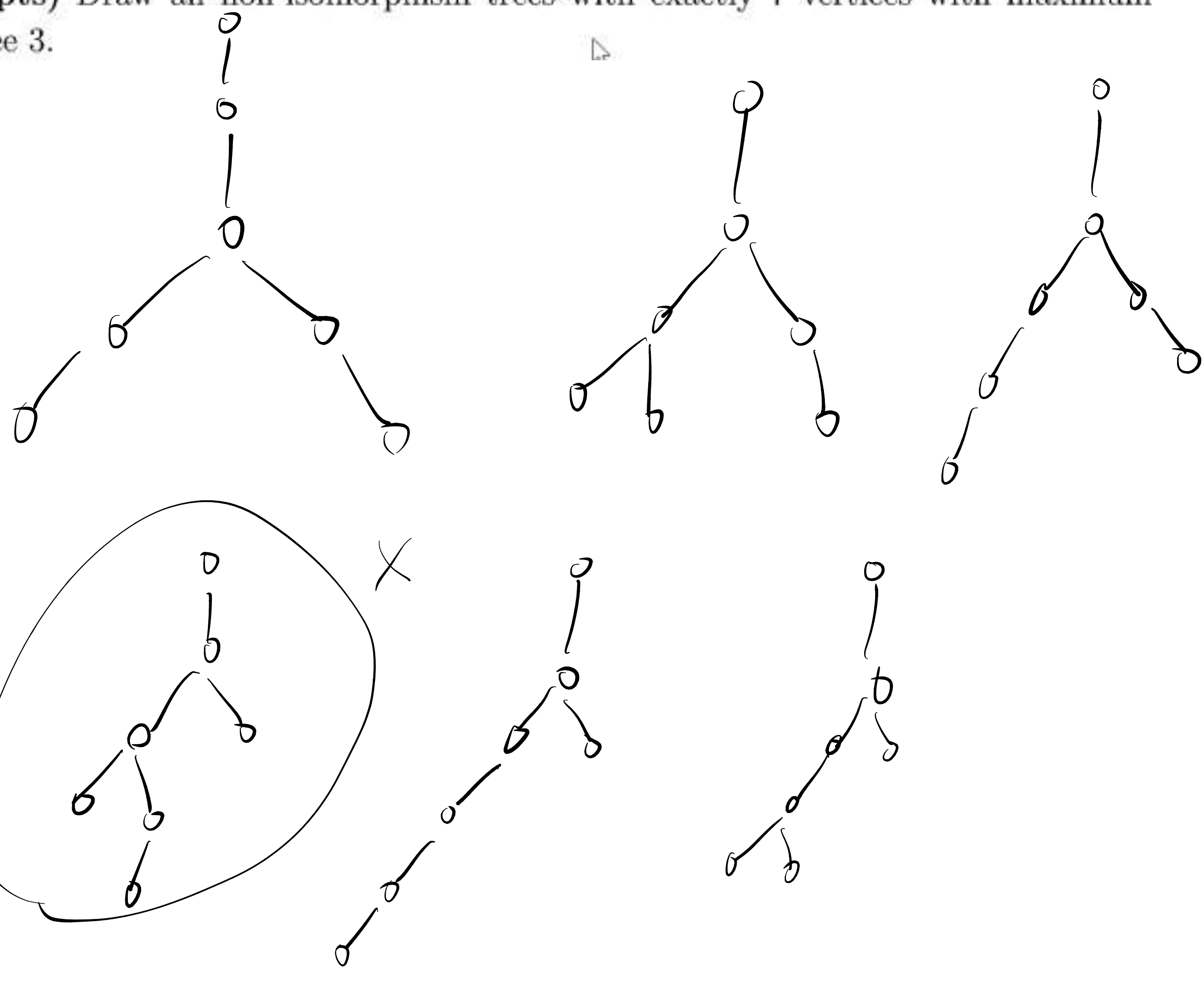
6. **(14 pts)** Let $G = (V, E)$ be a simple graph. Define its complement $\overline{G}$ as a graph on the vertex set V with an edge set $\overline{E}$ (the complement of E).

   (a) What is the degree sequence of $\overline{G}$ in terms of the degree sequence of G?

   (b) An automorphism of a graph G is a permutation of its vertices which preserves adjacency (i.e. $(u, v) \in E \Leftrightarrow (\varphi(u), \varphi(v)) \in E$). Let $Aut(G)$ be a set of automorphisms of G. Show that $Aut(G) = Aut(\overline{G})$.

   (c) Prove that at least one of G and $\overline{G}$ is connected.

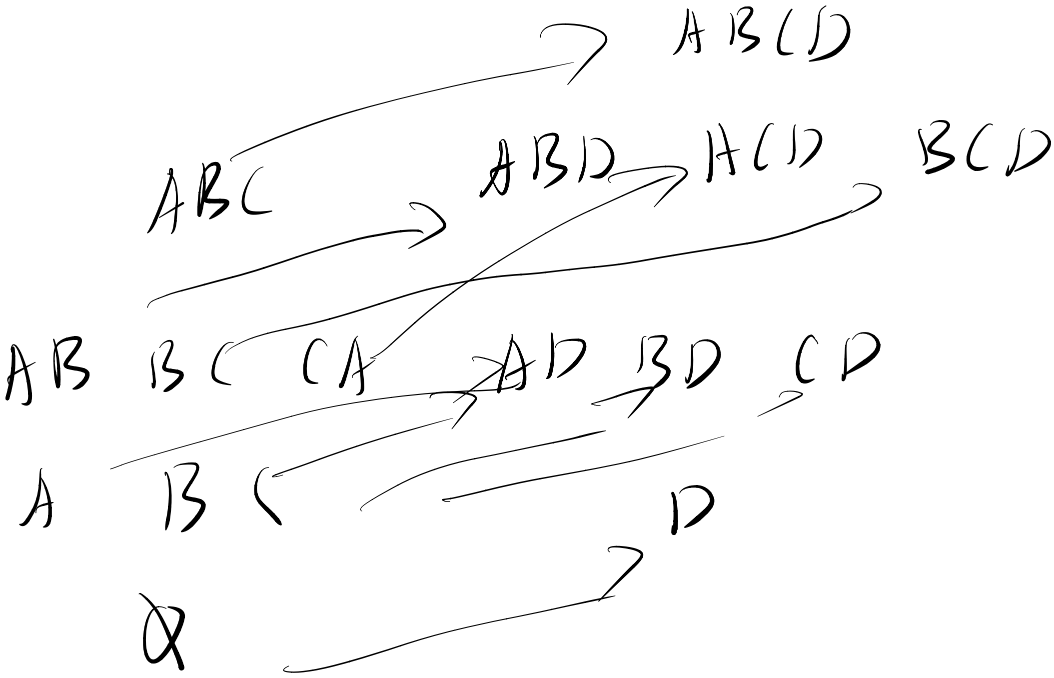
7. **(10 pts)** Draw all non-isomorphism trees with exactly 7 vertices with maximum degree 3.



8. **(10 pts)** Let S be a set having n elements. Let H be the Hasse diagram for the partial ordering $\{(A, B) \mid A \subseteq B\}$ on the power set of S. Let f_n denote the number of edges in a Hasse diagram representing a set with n element.

   (a) Compute f_0, f_1, f_2 and f_3.

   (b) Find a recurrence relation for f_n, and justify your answer.



$$a_n = 2a_{n-1} + 2^{n-1}$$