2016年期中考试卷答案

I. Fill in the space underlined (40% in total)

1. **A clock face (钟面) has negative charges –*q*, -2*q*, -3*q*,…..-12*q* fixed at the positions of the corresponding numerals. The clock hands do not perturb the field. At what time does the hour hand point in the same direction as the electric field at the center of the dial? ___9:30____.**

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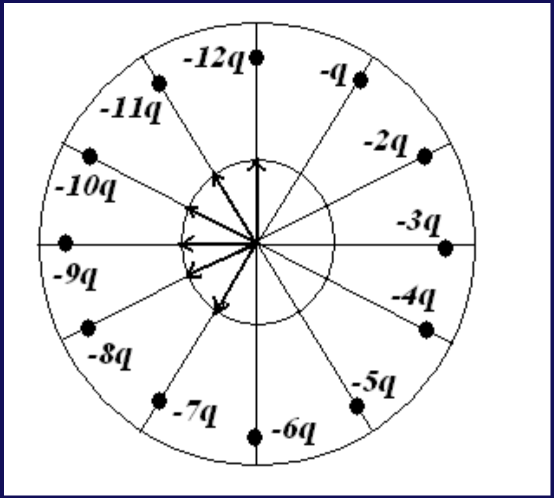


$$E = \frac{-6q}{4\pi\varepsilon_0 r^2}$$

2. A spherical drop of water carrying a charge of 32.0 *p*C has a potential of 512 V at its surface. Set *V* = 0 at infinity.
(*a*). What is the radius of the drop? ______________________.
(*b*). If two such drops of the same charge and radius combine to form a single spherical drop, what is the potential at the surface of the new drop? ______________________.

$$(a).\quad q = 32.0\times 10^{-12}C,\ V = \frac{q}{4\pi\varepsilon_0 R_0}$$

$$R_0 = \frac{q}{4\pi\varepsilon_0 V} = \frac{32\times 10^{-12}}{4\pi\cdot 8.85\times 10^{-12}\times 512} = 5.62\times 10^{-4}m$$

$$(b).\quad q' = 2q = 64.0\times 10^{-12}C,$$

$$2\times\frac{4}{3}\pi R_0^3 = \frac{4}{3}\pi R^3,\ R = \sqrt[3]{2}R_0$$

$$V = \frac{q'}{4\pi\varepsilon_0 R} = \frac{2q}{4\pi\varepsilon_0\times\sqrt[3]{2}\times R_0} = \sqrt[3]{4}\times V \approx 1.6\times 512 = 819.2V$$

3. As shown in Fig. 1, a parallel-plate capacitor has plates of area A and separation d and is charged to a potential difference ΔV. The charging battery is then disconnected and the plates are pulled apart until their separation is $2d$. The work required to separate the plates is ____

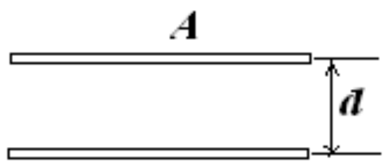


Figure 1

$$C=\frac{\varepsilon_0 A}{d},\quad q=C\Delta V=\frac{\varepsilon_0 A\Delta V}{d},\quad U=\frac{1}{2}\frac{q^2}{C}$$

$$C'=\frac{\varepsilon_0 A}{2d},\quad U'=\frac{1}{2}\frac{q^2}{C'}$$

$$W=U'-U=\frac{1}{2}q^2\left(\frac{1}{C'}-\frac{1}{C}\right)$$

$$=\frac{1}{2}\left(\frac{\varepsilon_0 A\Delta V}{d}\right)^2\frac{d}{\varepsilon_0 A}$$

$$=\frac{1}{2}\frac{\varepsilon_0 A}{d}(\Delta V)^2$$

4. As shown in Fig. 2, a plane slab of thickness d has a uniform volume charge density ρ. Find the magnitude of the electric field in region I __________, II_________, and III___________, in terms of x, the distance measured from the median plane of the slab.

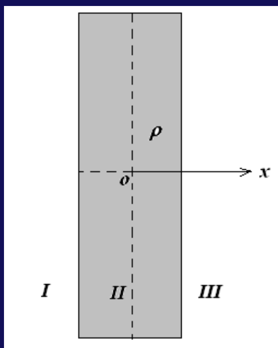


Figure 2

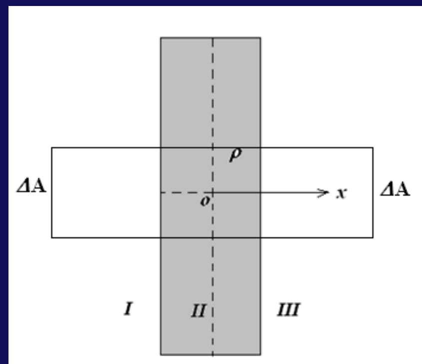


Figure 2

$$\oiint \vec{E}\bullet d\vec{A}=\frac{q}{\varepsilon_0}$$

$$E\bullet\Delta A+E\bullet\Delta A=\frac{1}{\varepsilon_0}\rho\cdot d\cdot\Delta A$$

$$E=\frac{\rho d}{2\varepsilon_0}$$

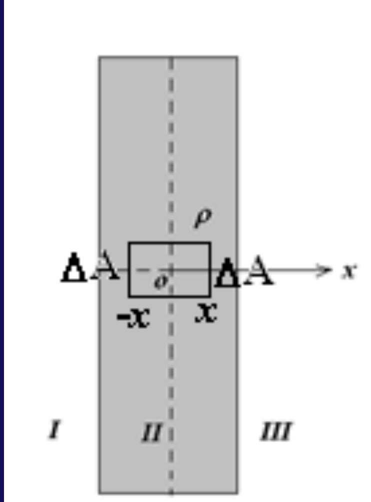


Figure 2

$$\oiint \vec{E}\bullet d\vec{A}=\frac{q}{\varepsilon_0}$$

$$E\cdot\Delta A+E\cdot\Delta A=\frac{1}{\varepsilon_0}(\rho\Delta A\cdot 2x)$$

$$E=\frac{\rho x}{\varepsilon_0}$$

$$\text{I region}:\quad E=-\frac{\rho d}{2\varepsilon_0},$$

$$\text{III region}: E=\frac{\rho d}{2\varepsilon_0}$$

$$\text{II region}:\quad E=\frac{\rho x}{\varepsilon_0}$$

5. There is a thin disk of radius *a*, with total charge *Q* uniformly distributed on its surface, rotating at a constant angular speed *ω*, as shown in Fig. 5. What is the magnetic dipole moment (磁偶极矩)?

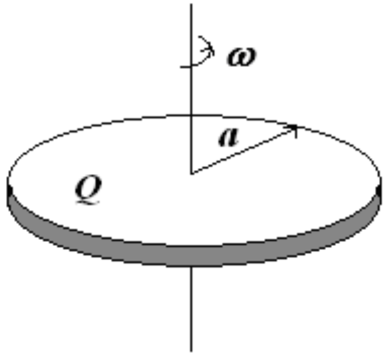


Figure 5

$$dq = \frac{Q}{\pi a^2} \cdot 2\pi r dr = \frac{2Q}{a^2} r dr$$

$$di = \frac{dq}{T} = \frac{\omega dq}{2\pi} = \frac{Q\omega}{\pi a^2} r dr$$

$$d\mu = \pi r^2 di = \frac{Q\omega}{a^2} r^3 dr$$

$$\mu = \int_0^a \frac{Q\omega}{a^2} r^3 dr = \frac{1}{4} Q\omega a^2$$

6. A long, circular pipe, with an outside radius of R, carries a (uniformly distributed) current of i_0 (into the paper as shown in Fig. 6). A wire runs parallel to the pipe at a distance $3R$ from the center to center. What are the magnitude ________________ and direction ________________ of the current in the wire that would cause the resultant magnetic field at the point P to have the same magnitude, but the opposite direction.

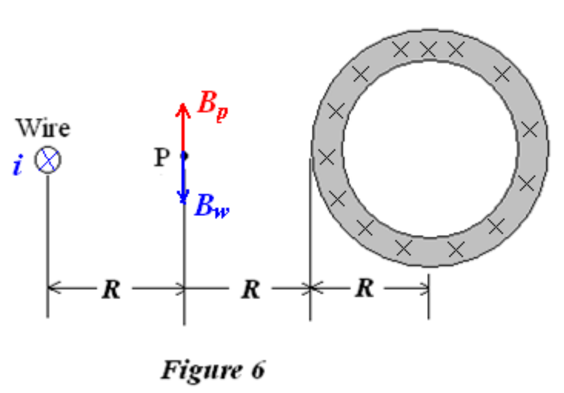


Figure 6

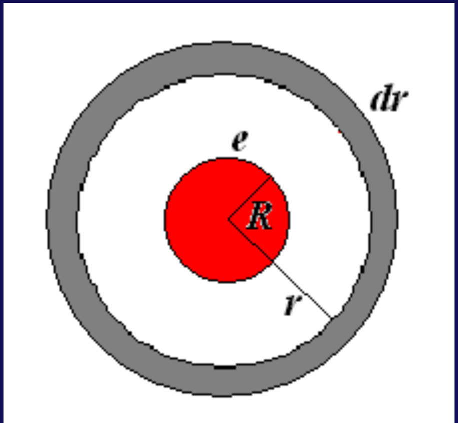
$$\oint \vec{B} \cdot d\vec{l} = \mu_0 i, \; B_p \cdot 2\pi \cdot 2R = \mu_0 i_0$$

$$B_p = \frac{\mu_0 i_0}{4\pi R}, \; B_w = B_p,$$

$$\frac{\mu_0 i}{2\pi R} = \frac{\mu_0 i_0}{4\pi R}$$

$$\therefore \; i = \frac{1}{2} i_0$$

7. Assume that the electron is not a point but a sphere of radius *R* over whose surface the electron charges is uniformly distributed. The energy associated with the external electric field in vacuum of the electron is ____________.



$$E = \frac{e}{4\pi\varepsilon_0 r^2}$$

$$U = \int_R^\infty \frac{1}{2}\varepsilon_0 E^2 \cdot 4\pi r^2 dr$$

$$= \int_R^\infty \frac{1}{2}\varepsilon_0 \left(\frac{e}{4\pi\varepsilon_0 r^2}\right)^2 \cdot 4\pi r^2 dr$$

$$= \frac{e^2}{8\pi\varepsilon_0}\int_R^\infty \frac{1}{r^2}dr = \frac{e^2}{8\pi\varepsilon_0 R}$$

8. As shown in Fig. 5, there is a square with side length 2.0 cm. A magnetic field points out of the page, its magnitude is changing with time by $B = (4\ \mathrm{T/m\cdot s^2})t^2y$. The emf (电动势) around the square at t =2.5s is ________, and its direction is________.

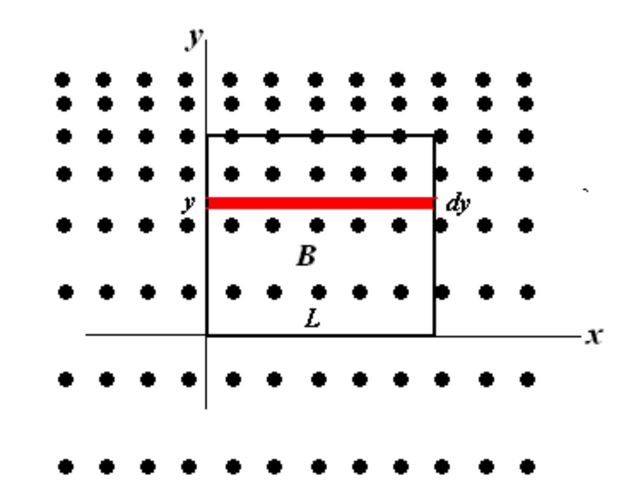


Figure 5

$$L = 2cm = 2\times10^{-2}m$$

$$\Phi_B = \iint \vec{B}\cdot d\vec{A}$$

$$= \int_0^{0.02} 4t^2 y\cdot L dy$$

$$= 8\times10^{-2}t^2\int_0^{0.02} ydy$$

$$= 8\times10^{-2}t^2\frac{1}{2}(0.02)^2$$

$$= 1.6\times10^{-5}t^2$$

$$\varepsilon = -\frac{d\Phi_B}{dt} = -3.2\times10^{-5}t$$

$$t = 2.5s$$

$$\varepsilon = -8\times10^{-5}V = -80\mu V$$

Direction: Clockwise

II. Problems (Present the necessary equations in solution) (50%)

1. (15%) As shown in Fig. 6, a (non-conducting) solid sphere of radius R carries a charge density $\rho(r) = k\,r^2$ (where k is a constant). Find the electric field at a distance r such that:

(a) $r \geq R$,

(b) $0 \leq r \leq R$;

(c) Calculate the electric potential for all r. Let $V = 0$ at $r = \infty$.

(d) Find the work required to assemble this charge distribution.

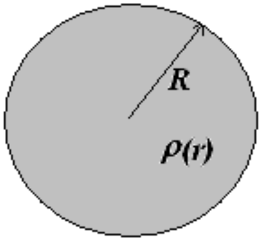
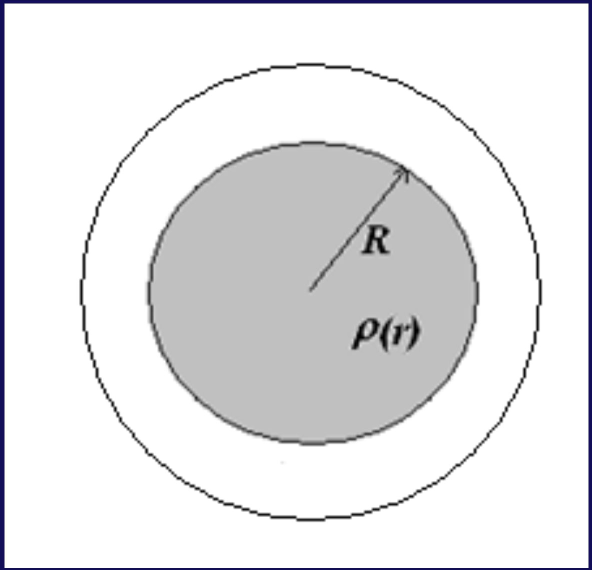


Figure 6



$(a).\ r \geq R,$

$$\oiint \vec{E} \cdot d\vec{A} = \frac{q}{\varepsilon_0}$$

$$E \cdot 4\pi r^2 = \frac{1}{\varepsilon_0}\int_0^R kr^2 \cdot 4\pi r^2 dr$$

$$E = \frac{1}{4\pi\varepsilon_0 r^2} \cdot 4\pi k \frac{1}{5} R^5 = \frac{kR^5}{5\varepsilon_0 r^2}$$

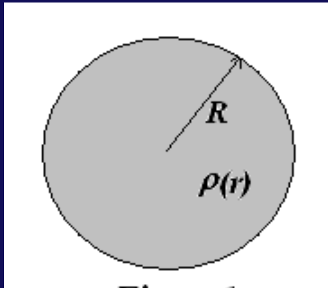
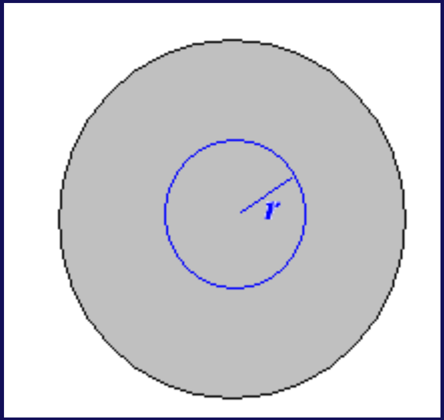


Figure 6



$$(b).\ 0 \le r \le R,$$

$$\oiint \vec{E}\cdot d\vec{A} = \frac{q}{\varepsilon_0}$$

$$E \cdot 4\pi r^2 = \frac{1}{\varepsilon_0}\int_0^r kr^2 \cdot 4\pi r^2 dr$$

$$E = \frac{1}{4\pi\varepsilon_0 r^2}\cdot 4\pi k \frac{1}{5} r^5 = \frac{kr^3}{5\varepsilon_0}$$

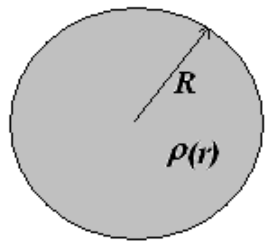


Figure 6

$$(c).\ r \geq R,\ \ E = \frac{kR^5}{5\varepsilon_0 r^2}$$

$$V(r) = \int_r^\infty E\,dr = \int_r^\infty \frac{kR^5}{5\varepsilon_0 r^2}\,dr = \frac{kR^5}{5\varepsilon_0 r}$$

$$0 \leq r \leq R,\ \ E = \frac{kr^3}{5\varepsilon_0}$$

$$V(r) = \int_r^R \frac{kr^3}{5\varepsilon_0}\,dr + \int_R^\infty \frac{kR^5}{5\varepsilon_0 r^2}\,dr$$

$$= \frac{k}{5\varepsilon_0}[\frac{1}{4}(R^4 - r^4) + R^4]$$

$$= \frac{k}{20\varepsilon_0}(5R^4 - r^4)$$

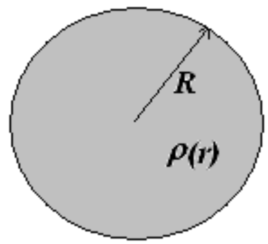


Figure 6

$$(d).\quad 0 \le r \le R,\ E = \frac{kr^3}{5\varepsilon_0}$$

$$r \ge R,\ E = \frac{kR^5}{5\varepsilon_0 r^2}$$

$$U = \int_0^R \frac{1}{2}\varepsilon_0 E^2 \cdot 4\pi r^2 dr + \int_R^\infty \frac{1}{2}\varepsilon_0 E^2 \cdot 4\pi r^2 dr$$

$$= 2\pi\varepsilon_0 [\int_0^R (\frac{kr^3}{5\varepsilon_0})^2 r^2 dr + \int_R^\infty (\frac{kR^5}{5\varepsilon_0 r^2})^2 r^2 dr]$$

$$= \frac{2\pi\varepsilon_0 k^2}{25\varepsilon_0^2}[\int_0^R r^8 dr + \int_R^\infty \frac{R^{10}}{r^2} dr]$$

$$= \frac{2\pi k^2}{25\varepsilon_0}[\frac{1}{9}R^9 + R^9] = \frac{4\pi k^2 R^9}{45\varepsilon_0}$$

2. (10%) A wire carrying current i has the configuration shown in Fig. 7. Two semi-infinite straight sections, each tangent to the same circle, are connected by a circular arc, of angle θ, along the circumference of the circle, with all sections lying in the same plane. What must θ be in order to for B to be zero at the center of the circle?

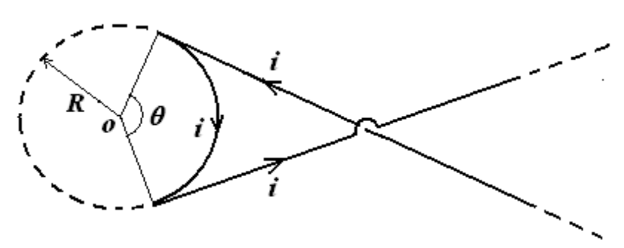
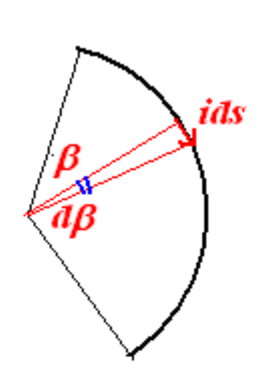


Figure 7

$$B_1 = B_3 = \frac{1}{2} \cdot \frac{\mu_0 i}{2\pi R}$$

$$B_{out} = B_1 + B_3 = \frac{\mu_0 i}{2\pi R}$$



$$B_{in} = B_2 = \frac{\mu_0}{4\pi} \int \frac{i d\vec{s} \times \hat{r}}{r^2}$$

$$= \frac{\mu_0}{4\pi} \int_0^\theta \frac{iR d\beta}{R^2} = \frac{\mu_0 i}{4\pi R} \theta$$

$$B_{in} = B_{out}$$

$$\frac{\mu_0 i}{2\pi R} = \frac{\mu_0 i}{4\pi R} \theta$$

$$\theta = 2, \ (\frac{2}{2\pi} = \frac{1}{\pi} \text{圆弧})$$

3. (15%) As shown in Fig. 8, a long cylindrical capacitor (圆柱形电容器), with radius *a* and *b*, length *L*, is filled by two kind dielectrics, whose dielectric constants are κ_{e1}, κ_{e2}, respectively, and are separated at a radius of *R*. If the inner conductor (with the radius of *a*) is charged with a line charge density λ, please

(*a*). Calculate the electric field *E*, displacement vector *D*, and polarization vector *P* for both $a < r < R$ and $R < r < b$ regions.

(*b*). The surface induced charge density σ_e' at $r = a$, $r = R$ and $r = b$ surfaces.

(*c*). The capacitance (电容) for this cylindrical capacitor.

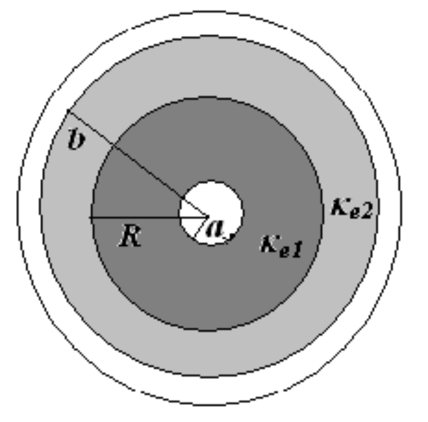


Figure 8

(*a*). Calculate $\vec{D}$, $\vec{E}$, $\vec{P}$

$$\oiint \vec{D}\cdot d\vec{A} = q_0, \quad D\cdot 2\pi r L = \lambda L, \quad D_1 = D_2 = D = \frac{\lambda}{2\pi r}$$

$$D = \kappa_e \varepsilon_0 E$$

$$a < r < R, \quad E_1 = \frac{D_1}{\kappa_{e1}\varepsilon_0} = \frac{\lambda}{2\pi\kappa_{e1}\varepsilon_0 r},$$

$$P_1 = \chi_{e1}\varepsilon_0 E_1 = (\kappa_{e1} - 1)\varepsilon_0 E_1 = \frac{(\kappa_{e1} - 1)\lambda}{2\pi\kappa_{e1} r}$$

$$R < r < b, \quad E_2 = \frac{D_2}{\kappa_{e2}\varepsilon_0} = \frac{\lambda}{2\pi\kappa_{e2}\varepsilon_0 r},$$

$$P_2 = \chi_{e2}\varepsilon_0 E_2 = (\kappa_{e2} - 1)\varepsilon_0 E_2 = \frac{(\kappa_{e2} - 1)\lambda}{2\pi\kappa_{e2} r}$$

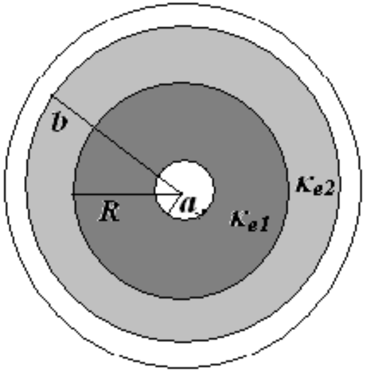


Figure 8

(b). Calculate σ_e'

$$\text{in } \kappa_{e1},\ P_1 = \frac{(\kappa_{e1}-1)\lambda}{2\pi\kappa_{e1} r}$$

$$\text{in } \kappa_{e2},\ P_2 = \frac{(\kappa_{e2}-1)\lambda}{2\pi\kappa_{e2} r}$$

$$\sigma_e'(r=a) = \vec{P}_1 \bullet \hat{n} = -\frac{(\kappa_{e1}-1)\lambda}{2\pi\kappa_{e1} a}$$

$$\sigma_e'(r=R) = \vec{P}_1 \bullet \hat{n}_1 + \vec{P}_2 \bullet \hat{n}_2$$

$$= \frac{(\kappa_{e1}-1)\lambda}{2\pi\kappa_{e1} R} - \frac{(\kappa_{e2}-1)\lambda}{2\pi\kappa_{e2} R}$$

$$= \frac{\lambda}{2\pi R}\frac{\kappa_{e1}-\kappa_{e2}}{\kappa_{e2}\kappa_{e1}}$$

$$\sigma_e'(r=b) = \vec{P}_2 \bullet \hat{n} = \frac{(\kappa_{e2}-1)\lambda}{2\pi\kappa_{e2} b}$$

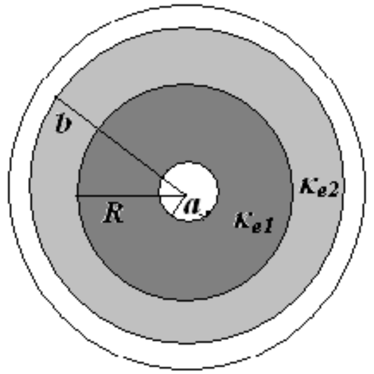


Figure 8

(c). Calculate C

$$a < r < R,\ E_1 = \frac{\lambda}{2\pi\kappa_{e1}\varepsilon_0 r},$$

$$R < r < b,\ E_2 = \frac{\lambda}{2\pi\kappa_{e2}\varepsilon_0 r},$$

$$\Delta V = \int_a^R E_1 dr + \int_R^b E_2 dr = \frac{\lambda}{2\pi\varepsilon_0}\left[\frac{1}{\kappa_{e1}}\ln\frac{R}{a} + \frac{1}{\kappa_{e2}}\ln\frac{b}{R}\right]$$

$$C = \frac{\lambda L}{\Delta V} = 2\pi\varepsilon_0 L \frac{1}{\frac{1}{\kappa_{e1}}\ln\frac{R}{a} + \frac{1}{\kappa_{e2}}\ln\frac{b}{R}}$$

Another way :

$$C_1 = \frac{2\pi\varepsilon_0\kappa_{e1}L}{\ln\frac{R}{a}},\ C_2 = \frac{2\pi\varepsilon_0\kappa_{e2}L}{\ln\frac{b}{R}}$$

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2},\ C = \frac{C_1C_2}{C_1 + C_2} = 2\pi\varepsilon_0 L \frac{1}{\frac{1}{\kappa_{e1}}\ln\frac{R}{a} + \frac{1}{\kappa_{e2}}\ln\frac{b}{R}}$$

4. (20%) As shown in Fig. 9, a point charge $-Q_1$ of mass m travels in a circular orbit of radius R about a charge of opposite sign Q_2.

(*a*). What is the equilibrium angular speed (平衡角速度) of the charge $-Q_1$?

(*b*). This problem describes Bohr's one electron model of the atom if the charge $-Q_1$ is that of an electron and $Q_2 = Ze$ is the nuclear charge, where Z is the number of protons. According to the postulates of quantum mechanics the angular momentum (角动量) L of the electron must be quantized,

$$L = mvR = nh/2\pi \quad n = 1, 2, 3 \ldots$$

Where $h = 6.63\times10^{-34}$ J·s is Planck's constant. What are the allowed values of R?

(*c*). For the hydrogen atom ($Z = 1$) what is the radius of the smallest allowed orbit and what is the electron's orbital velocity?

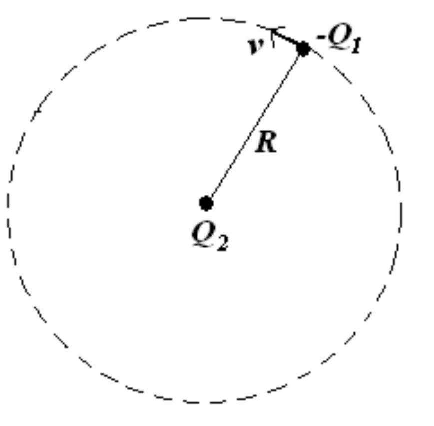


Figure 9

$$(a). \quad \frac{Q_1 Q_2}{4\pi\varepsilon_0 R^2} = m\omega^2 R$$

$$\omega = \left[\frac{Q_1 Q_2}{4\pi\varepsilon_0 R^3 m}\right]^{1/2}$$

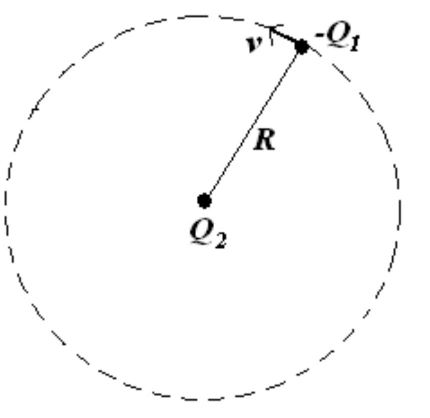


Figure 9

$$(b).\quad Q_1 = e,\ Q_2 = Ze$$

$$\frac{Ze^2}{4\pi\varepsilon_0 R^2} = m\frac{v^2}{R},\quad v = \left(\frac{Ze^2}{4\pi\varepsilon_0 mR}\right)^{1/2}$$

$$L = mvR = \left[\frac{Ze^2 mR}{4\pi\varepsilon_0}\right]^{1/2} = n\hbar$$

$$R = \frac{4\pi\varepsilon_0 n^2\hbar^2}{Ze^2 m}\quad (n = 1, 2, 3....)$$

Energy of electon :

$$\frac{Q_1 Q_2}{4\pi\varepsilon_0 R^2} = m\frac{v^2}{R}$$

$$E = \frac{1}{2}mv^2 + U = \frac{1}{2}\frac{Ze^2}{4\pi\varepsilon_0 R} - \frac{Ze^2}{4\pi\varepsilon_0 R} = -\frac{Ze^2}{8\pi\varepsilon_0 R}$$

$$= -\frac{Ze^2}{8\pi\varepsilon_0}\frac{Ze^2 m}{4\pi\varepsilon_0 n^2\hbar^2} = -\frac{Z^2 e^4 m}{32\pi^2\varepsilon_0^2\hbar^2}\cdot\frac{1}{n^2} = -Z^2\frac{13.6\text{ev}}{n^2}$$

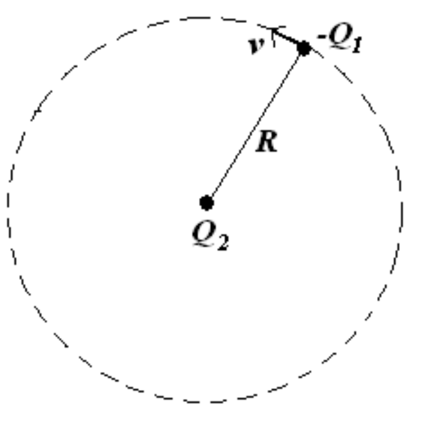


Figure 9

$$(c).\quad Z=1,\ n=1$$

$$R_1=\frac{4\pi\varepsilon_0 n^2\hbar^2}{Ze^2m}=\frac{4\pi\varepsilon_0\hbar^2}{e^2m}$$

$$=\frac{4\times3.14\times8.85\times10^{-12}\times(\frac{6.63}{6.28}\times10^{-34})^2}{(1.6\times10^{-19})^2\times9.11\times10^{-31}}$$

$$=\frac{123.89\times10^{-80}}{23.32\times10^{-69}}=5.31\times10^{-11}m=0.531A^\circ$$

$$Q_1=e,\ Q_2=Ze$$

$$L=mvR=n\hbar\quad(n=1)$$

$$v_1=\frac{\hbar}{mR_1}=\frac{\frac{6.63}{2\pi}\times10^{-34}}{9.11\times10^{-31}\times5.31\times10^{-11}}$$

$$=\frac{1.05\times10^{-34}}{48.37\times10^{-42}}=2.17\times10^6 m/s$$