浙江大学20 17 –20 18学年 秋冬 学期
《 普通物理II 》课程期中考试试卷

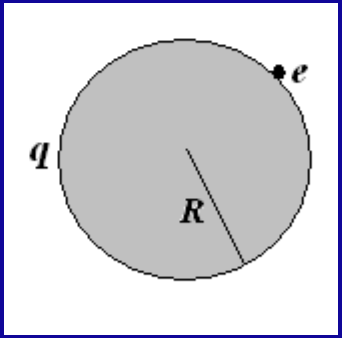
I. Fill in the space underlined (40% in total)

1. A wire loop that encloses an area of 10 cm^2 has a resistance of 5 Ω. The loop is placed in a magnetic field of 0.5 T with its plane perpendicular to the field. The loop is suddenly removed from the field. How much charge flows past a given point in the wire? ____________________

$$\varepsilon = \frac{\Delta\Phi}{\Delta t} = \frac{\Delta(BA)}{\Delta t} = IR = \frac{\Delta q}{\Delta t}R$$

$$\Delta q = \frac{\Delta(BA)}{R} = \frac{0.5\times 10\times 10^{-4}}{5} = 1\times 10^{-4}C$$

2. The escape speed (逃逸速度) for an electron from the surface of a uniformly charged sphere of radius 1.22 cm and total charge 1.70×10^{-15} C is of __________. Neglect gravitational forces.



$$U=\frac{qe}{4\pi\varepsilon_0 R}=\frac{1}{2}m_e v_e^2$$

$$v_e^2=\frac{qe}{2\pi\varepsilon_0 m_e R}$$

$$v_e=\sqrt{\frac{1.70\times10^{-15}\times1.60\times10^{-19}}{2\pi\times8.85\times10^{-12}\times9.11\times10^{-31}\times1.22\times10^{-2}}}=2.13\times10^{4}\,m/s$$

3. In the Bohr model of the hydrogen atom, the electron circulates around the nucleus in a circular path of radius 5.29×10^{-11} m at a frequency f of 6.60×10^{15} Hz. What value of the equivalent magnetic dipole moment (等效磁偶极矩) is _______________.

$$\mu_B = i\bullet A = ef\bullet A = 1.60\times10^{-19}\times6.63\times10^{15}\times\pi\times(5.29\times10^{-11})^2$$
$$=0.923\times10^{-23}\,A/m^2$$

4. As shown in Fig. 1, a hollow, cylindrical conductor of radii a and b, carries a uniformly distributed current i. The magnetic induction strength B(r) for the range $a < r < b$ is given by: ____.

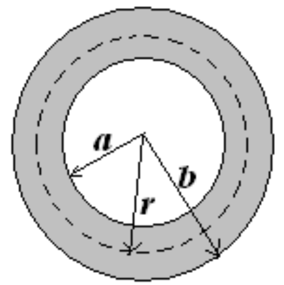


Figure 1

$$\int \vec{B}\bullet d\vec{l} = \mu_0 I$$

$$B\cdot 2\pi r = \mu_0 \frac{i}{\pi(b^2-a^2)}\cdot\pi(r^2-a^2)$$

$$B = \frac{\mu_0 i}{2\pi r}\cdot\frac{r^2-a^2}{b^2-a^2}$$

5. As shown in Fig. 2, a long, straight conductor with a circular cross section of radius R carries a current I. There is a cylindrical hole inside the conductor, whose radius is of a, and whose axis is parallel to the axis of the conductor but offset a distance b from the axis of the conductor. The current I is uniformly distributed across the cross section of the conductor and is directed out of the page. The magnetic field at P point ($R>x>a+b$) at the x axis is ___________.

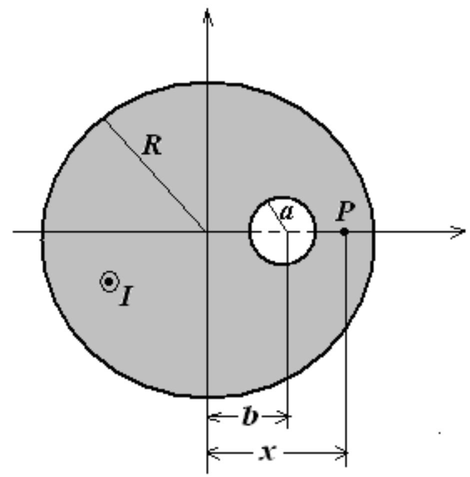


Figure 2

$$j = \frac{I}{\pi(R^2 - a^2)}$$

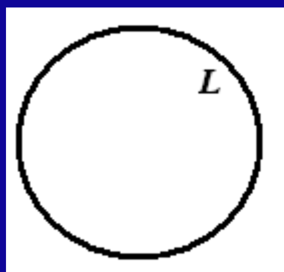
$$\int \vec{B} \bullet d\vec{l} = \mu_0 i$$

$$B_R \cdot 2\pi x = \mu_0 \frac{I}{\pi(R^2 - a^2)} \cdot \pi x^2$$

$$B_a \cdot 2\pi(x-b) = \mu_0 \frac{I}{\pi(R^2 - a^2)} \cdot \pi a^2$$

$$B = B_R - B_a = \frac{\mu_0 I}{2\pi(R^2 - a^2)}\left(x - \frac{a^2}{x-b}\right)$$

6. A length L of wire carries a current i. If the wire is formed into a circular coil with one turn (一匝) only, the maximum torque (力矩) in a given magnetic field B has a magnitude ________.



$$2\pi r = L \qquad r = \frac{L}{2\pi}, \qquad \mu = iA = i\pi r^2 = \frac{L^2}{4\pi} i$$

$$\vec{\tau} = \vec{\mu} \times \vec{B}, \qquad \tau_{\max} = \frac{L^2 B}{4\pi} i$$

7. As shown in Fig. 3, a wire with a 3/4 circle is placed in a uniform magnetic field B which points out of the plane of the figure. If the wire carries a current I, the magnetic force acted on it is ______.

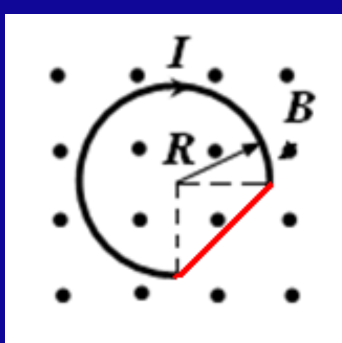


Figure 3

$$d\vec{F} = i d\vec{s} \times \vec{B}$$

$$F = i \cdot \sqrt{2} R \cdot B = \sqrt{2} RIB$$

8. As shown in Fig. 4, a parallel plate capacitor with capacitance C is charged to a potential difference V and is then disconnected from the charging source. The capacitor has an area A and a plate separation d. Assume that a glass plate of the same area A completely fills the space between the plates, and which has a dielectric constant κ_e. How much work is required to pull the glass plate out of the capacitor? ______. Neglect fringe effects at the edges of the plates.

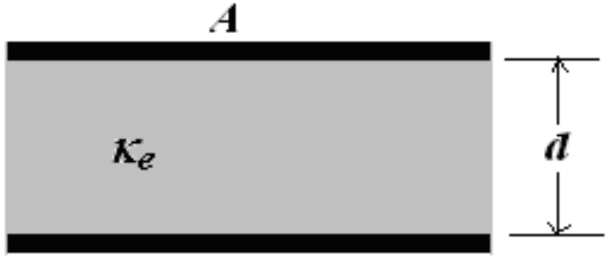


Figure 4

$$Q = CV \quad C = \frac{\varepsilon_0 A}{d}$$

$$W = \frac{1}{2}Q\left(\frac{1}{C_f} - \frac{1}{C_i}\right) = \frac{1}{2}C^2V^2\left(\frac{1}{\kappa_e C} - \frac{1}{C}\right)$$

$$= \frac{\varepsilon_0 A V^2}{2d}\left(\frac{1}{\kappa_e} - 1\right) = \frac{1}{2}CV^2\left(\frac{1}{\kappa_e} - 1\right)$$

9. Figure 5 shows a rod of length L caused to move at constant speed v along horizontal conducting rails. In this case the magnetic field in which the rod moves is not uniform but is provided by a current i in a long, parallel wire. Assume the v=4.86 m/s, a =10.2 mm, L=9.83 cm, and i =110 A. The emf induced in the rod is of ______.

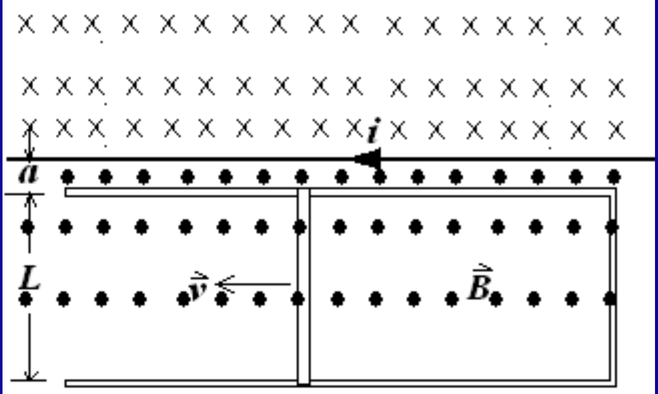


Figure 5

$$B = \frac{\mu_0 i}{2\pi r}$$

$$\varepsilon = \int (\vec{v} \times \vec{B}) \bullet d\vec{l} = -\int_a^{L+a} vB dr = -\int_a^{L+a} v \frac{\mu_0 i}{2\pi r} dr$$

$$= -\frac{\mu_0 i}{2\pi} v \ln \frac{L+a}{a} = \frac{\mu_0 i}{2\pi} v \ln \frac{a}{L+a}$$

10. Figure 6 shows a uniform magnetic field B confined to a cylindrical volume of radius R. B is decreasing in magnitude at a constant rate of 10.7 mT/s. What is the instantaneous acceleration (direction and magnitude) experienced by an electron placed at A point?__________.

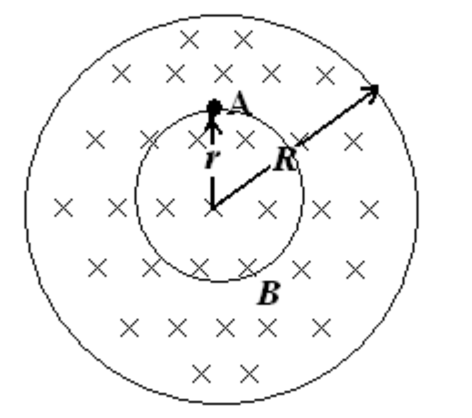


Figure 6

$$\oint \vec{E}\cdot d\vec{l} = -\iint \frac{\partial \vec{B}}{\partial t}\cdot d\vec{A}$$

$$E\cdot 2\pi r = -\frac{dB}{dt}\cdot \pi r^2$$

$$E = -\frac{1}{2}\frac{dB}{dt}r$$

$$F = eE = ma$$

$$a = \frac{eE}{m} = -\frac{er}{2m}\frac{dB}{dt} = -\frac{1.60\times 10^{-19}\times 10.7\times 10^{-3}}{2\times 9.11\times 10^{-31}}r$$

$$= -9.4\times 10^{8} r$$

Direction: Left

II. Problems (present the necessary equations in solution) (60%)

1. **(13%) A static charge distribution produces a spherically radial electric field:** $\vec{E} = A\dfrac{e^{-br}}{r^2}\hat{r}$ **, where A and $b > 0$ are the constants.**

(a). What is the charge density $\rho(r)$?

(b). What is the total charge Q?

Solution:

$$\iint \vec{E}\bullet d\vec{A} = \frac{q}{\varepsilon_0}$$

$$\iiint (\nabla\bullet\vec{E})dv = \iiint \frac{\rho_e}{\varepsilon_0}dv$$

$$\nabla\bullet\vec{E} = \frac{\rho_e}{\varepsilon_0}$$

$$\text{(a)}\quad \rho = \varepsilon_0\nabla\cdot\vec{E} = \varepsilon_0\frac{1}{r^2}\frac{\partial}{\partial r}r^2\left(A\frac{e^{-br}}{r^2}\right)$$

$$= \varepsilon_0 A(-b)\frac{e^{-br}}{r^2} = -\frac{\varepsilon_0 Abe^{-br}}{r^2}$$

$$\text{(b).}\quad Q = \iiint \rho dv$$

$$= \int_0^r \rho 4\pi r^2 dr = \int_0^r \left(-\varepsilon_0 Ab\frac{e^{-br}}{r^2}\right)4\pi r^2 dr = \int_0^r (-4\pi\varepsilon_0 Ab)e^{-br}dr$$

$$= 4\pi\varepsilon_0 A[e^{-br} - 1]$$

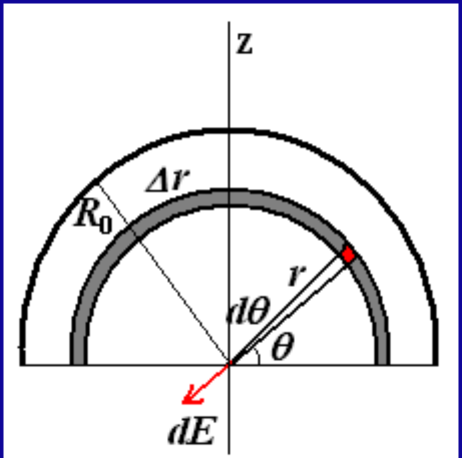
$$r \to \infty,\ Q = -4\pi\varepsilon_0 A$$

$$\text{(b).}\quad Q(r) = \varepsilon_0\iint \vec{E}\cdot d\vec{A} = \varepsilon_0 A\frac{e^{-br}}{r^2}\cdot 4\pi r^2 = 4\pi\varepsilon_0 Ae^{-br}$$

It is not the total charge.

2. (12%) As shown in Fig.7, an infinitely long semi-cylinder (半圆柱形) insulator of radius R_0 carries a uniform volume charge distribution ρ_0. Please calculate the electric field along the axis of the cylinder.

Solution:



$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

$$d\lambda = \frac{1}{L} \cdot r d\theta \cdot \Delta r \cdot L \cdot \rho_0 = \rho_0 r \Delta r d\theta$$

$$dE = \frac{\rho_0 r \Delta r}{2\pi\varepsilon_0 r} d\theta$$

$$dE_z = \frac{\rho_0 r \Delta r}{2\pi\varepsilon_0 r} \sin\theta d\theta = \frac{\rho_0 \Delta r}{2\pi\varepsilon_0} \sin\theta d\theta$$

$$E_z = \frac{\rho_0 \Delta r}{2\pi\varepsilon_0} \int_0^{\pi} \sin\theta d\theta = \frac{\rho_0 \Delta r}{\pi\varepsilon_0}$$

$$E_z = \frac{\rho_0 \Delta r}{\pi\varepsilon_0}$$

$$E = \int_0^R \frac{\rho_0 dr}{\pi\varepsilon_0} = \frac{\rho_0 R}{\pi\varepsilon_0}$$

3. (15%) As shown in Fig.8, a pendulum with a weightless string of length l has on its end a small sphere with charge q and mass m. A distance D away on either side of the pendulum mass are two fixed spheres each carrying a charge Q. The three spheres are of sufficiently small size that they can be considered as point charges and masses.

(a). Assuming the pendulum displacement ξ to be small ($\xi \ll D$), and at $t = 0$ the pendulum is released from rest with $\xi = \xi_o$. What is the subsequent pendulum motion? Please write out the $\xi(t)$ function.

(b). For what values of qQ is the motion unbounded with time?

Solution:

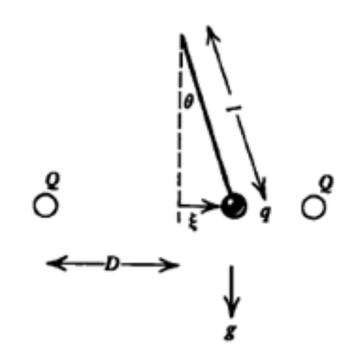
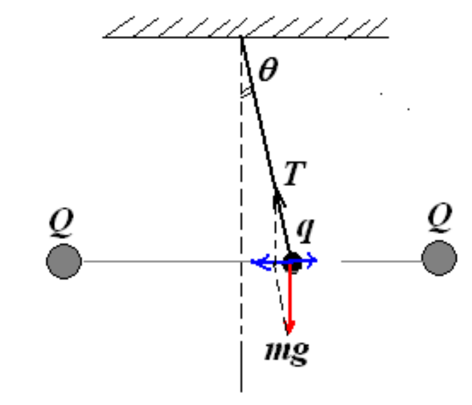


Figure 8



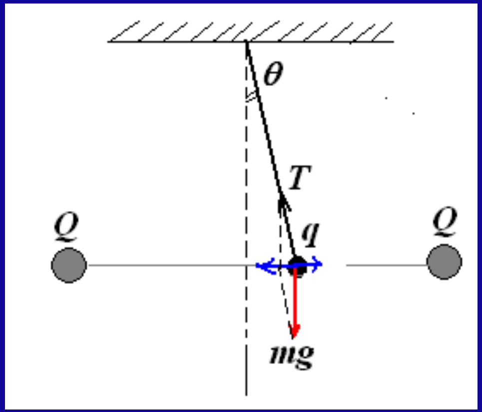
$$F = -[mgtg\theta + \frac{qQ}{4\pi\varepsilon_0 (D-\xi)^2} - \frac{qQ}{4\pi\varepsilon_0 (D+\xi)^2}]$$

$$= -mgtg\theta - \frac{qQ}{4\pi\varepsilon_0} \frac{4D\xi}{(D^2-\xi^2)^2}$$

$$\approx -mg\frac{\xi}{l} - \frac{qQ\xi}{\pi\varepsilon_0 D^3}$$

$$= -(\frac{mg}{l} + \frac{qQ}{\pi\varepsilon_0 D^3})\xi$$

$$= -k\xi$$



$$F = -\left(\frac{mg}{l} + \frac{qQ}{\pi\varepsilon_0 D^3}\right)\xi = -k\xi,$$

$$k = \frac{mg}{l} + \frac{qQ}{\pi\varepsilon_0 D^3}$$

$$m\frac{d^2\xi}{dt^2} = -k\xi$$

$$\frac{d^2\xi}{dt^2} + \frac{k}{m}\xi = 0$$

$$\frac{d^2\xi}{dt^2} + \omega^2\xi = 0$$

$$\omega = \sqrt{\frac{g}{l} + \frac{qQ}{\pi\varepsilon_0 m D^3}}$$

$$(a) \quad \frac{d^2\xi}{dt^2} + \omega^2\xi = 0$$

$$\xi(t) = \xi_0 \cos(\omega t + \varphi)$$

$$\omega = \sqrt{\frac{g}{l} + \frac{qQ}{\pi\varepsilon_0 m D^3}}$$

$$\because t = 0, \quad \xi(0) = \xi_0, \quad \therefore \varphi = 0$$

Simple harmonic vibration!

简谐振动！

$$(b) \quad \text{if } \frac{g}{l} + \frac{qQ}{\pi\varepsilon_0 m D^3} = 0$$

$$F = 0, \quad \textit{the motion unbounded with time!}$$

$$qQ = -\frac{g\pi\varepsilon_0 m D^3}{l}$$

4. (20%) Figure 9 shows a cross section of a cylinder capacitor (圆柱形电容器), in which the inner conductor radius is of a, and the outer conductor is a hollow cylinder shell with radius of b. The space between them is filled by the non-uniform dielectrics (非均匀电介质) with a dielectric constant (介电常数), $\kappa_e = \frac{\kappa_{e0}}{1+ar}$ where κ_{e0} and α are the constants, r is the distance for the points inside dielectrics from the axis. Please calculate:

(a) The electric displacement vector for the region of $a < r < b$ as the inner conductor is charged with a line density of λ.

(b) The capacitance (电容) for this system C.

(c) The volume density of the polarization charge, $\rho_e'(r)$, in the region of $a<r<b$.

(d). The surface density of the polarization charge σ_e(a) and σ_e(b), at the $r = a$ and $r = b$ surfaces, respectively.

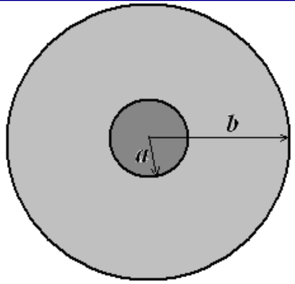


Figure 9

$$(a).\quad \iint \vec{D}\cdot d\vec{A} = Q_0$$

$$D\cdot 2\pi r\cdot L = Q_0$$

$$D = \frac{\lambda}{2\pi r}$$

$$(b).\quad D = \kappa_e \varepsilon_0 E$$

$$E = \frac{D}{\kappa_e \varepsilon_0} = \frac{1+\alpha r}{\kappa_{e0}\varepsilon_0}\frac{\lambda}{2\pi r}$$

$$\Delta V = \int_a^b E dr = \frac{\lambda}{2\pi\kappa_{e0}\varepsilon_0}\int_a^b \frac{1+\alpha r}{r}dr = \frac{\lambda}{2\pi\kappa_{e0}\varepsilon_0}[\ln\frac{b}{a} + \alpha(b-a)]$$

$$C = \frac{Q}{V} = 2\pi\varepsilon_0\kappa_{e0}\frac{L}{\ln\frac{b}{a} + \alpha(b-a)}$$

$$(c).\quad D=\kappa_e\varepsilon_0 E,\quad \kappa_e=\frac{\kappa_{e0}}{1+\alpha r}$$

$$E=\frac{D}{\kappa_e\varepsilon_0}=\frac{1+\alpha r}{\kappa_{e0}\varepsilon_0}\frac{\lambda}{2\pi r}$$

$$\vec{P}=\chi_e\varepsilon_0 E=(\kappa_e-1)\varepsilon_0 E$$

$$=(\frac{\kappa_{e0}}{1+\alpha r}-1)\frac{1+\alpha r}{\kappa_{e0}}\frac{\lambda}{2\pi r}$$

$$=(1-\frac{1+\alpha r}{\kappa_{e0}})\frac{\lambda}{2\pi r}$$

$$\rho_e'=-\nabla\bullet\vec{P}=-\frac{1}{r}\frac{d}{dr}(rP_r)$$

$$=-\frac{1}{r}\frac{d}{dr}[\frac{\lambda}{2\pi}(1-\frac{1+\alpha r}{\kappa_{e0}})]$$

$$=\frac{1}{r}\frac{d}{dr}\frac{\lambda\alpha r}{2\pi\kappa_{e0}}$$

$$=\frac{\alpha\lambda}{2\pi\kappa_{e0}r}$$

$$(d).\quad P=\chi_e\varepsilon_0 E\qquad \sigma_e'=\vec{P}\cdot\vec{n}=P_n$$

$$\therefore\quad \sigma_e'(a)=-P(a)=-\frac{\lambda}{2\pi a}(1-\frac{1+\alpha a}{\kappa_{e0}})=\frac{\lambda}{2\pi a}(\frac{1+\alpha a}{\kappa_{e0}}-1)$$

$$\sigma_e'(b)=P(b)=\frac{\lambda}{2\pi b}(1-\frac{1+\alpha b}{\kappa_{e0}})$$

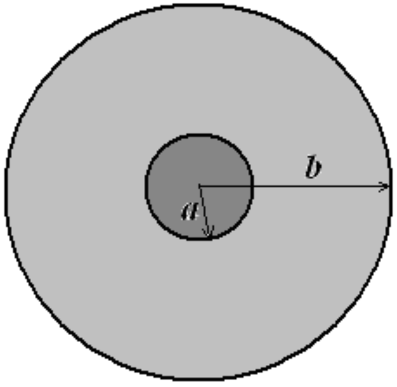


Figure 9