# 1 water value difference

Water from reservoirs 1 and 2 can be used for production of 3, and 1 and 2 are higher than 3. So water in the upper reservoirs has a higher water value.

# 2 cost function

$$min_{q,t}(wv_1 \cdot f(\sum_{t=1}^{4}(l_t \cdot q_{1,t}) + \sum_{t=1}^{4}(l_t \cdot q_{2,t})) + wv_2 \cdot \sum_{t=1}^{4}(l_t \cdot q_{3,t})) \tag{1}$$

$$1\frac{m^3}{s} = 1 \cdot 10^{-6}\frac{Mm^3}{s} = 3600 \cdot 10^{-6}\frac{Mm^3}{h} = 3.6 \cdot 10^{-3}\frac{Mm^3}{h} \tag{2}$$

$$f = 3.6 \cdot 10^{-3}\frac{Ms}{h} \tag{3}$$

# 3 balance equatinos

Generally the values can be calculated using following formulas.

$$r_{1,in} = f\int_{t_0}^{t}(Q_1 - s_1)dt \tag{4}$$

$$r_{2,in} = f\int_{t_0}^{t}(Q_2 - s_2)dt \tag{5}$$

$$r_{3,in} = \int_{t_0}^{t}(q_1 + s_1 + q_2 - s_2 + Q_3)dt \tag{6}$$

$$r_1 = r_{1,0} + f\int_{t_0}^{t}(r_{1,in} - r1, out)dt = r_{1,0} + f\int_{t_0}^{t}(Q_1 - s_1 - q_1)dt \tag{7}$$

$$r_2 = r_{2,0} + f\int_{t_0}^{t}(r_{2,in} - r2, out)dt = r_{2,0} + f\int_{t_0}^{t}(Q_2 - s_2 - q_2)dt \tag{8}$$

$$r_3 = r_{3,0} + f\int_{t_0}^{t}(Q_3 + q_1 + s_1 + q_2 - s_3 - q_3)dt = r_{2,0} + f\int_{t_0}^{t}(Q_2 - s_2 - q_2)dt \tag{9}$$

Using the time intervals given the formulas change to

$$r_{1,t} = r_{1,t-1} + f \cdot l_t(Q_{1,t} - s_{1,t} - q_{1,t}) \tag{10}$$

$$r_{2,t} = r_{2,t-1} + f \cdot l_t(Q_{2,t} - s_{2,t} - q_{2,t}) \tag{11}$$

$$r_{3,t} = r_{3,t-1} + f \cdot l_t(Q_{3,t} + q_{1,t} + s_{1,t} + q_{2,t} - s_{3,t} - q_{3,t}) \tag{12}$$

In the first time period the value of $q_2$ is not known. So the initial value for $q_{2,0} = 0$.

# 4 constraints

$$P_t = \sum_{t=1}^{3} P_{t,i} \forall t \in [1,4] and \forall i \in [1,3] \tag{13}$$

- $\beta_{A,1} = 2.058$
- $\beta_{A,2} = 0.9$

$$Q_{min,A,i} \leq q_{A,i,t} \leq Q_{max,A,i} \tag{14}$$

$$R_{min,t} \leq r_{i,t} \leq R_{max,t} \tag{15}$$

# 5 table analysis

## 5.1 final reservoir level

- $r_{1,4} = 306.92 Mm^3$
- $r_{2,4} = 235.14 Mm^3$
- $r_{3,4} = 20 Mm^3$

## 5.2 generation per period

### Generation unit 1

- $p_{1,1} = 0.00$
- $p_{1,2} = 106.77$
- $p_{1,3} = 49.99$
- $p_{1,4} = 0.00$

### Generation unit 2

- $p_{2,1} = 0.01$
- $p_{2,2} = 116.62$
- $p_{2,3} = 100.02$
- $p_{2,4} = 100.01$

### Generation unit 3

- $p_{3,1} = 99.99$
- $p_{3,2} = 116.6$
- $p_{3,3} = 99.99$
- $p_{3,4} = 99.99$

## 5.3 spillage

Spillage occur in reservoir 3 in the two last timesteps.

## 5.4 production dispatch

# 6 shadow prices

## 6.1 marginal costs

shadow production prices

- period 1,3,4: 3673.5 NOK/MWh
- period 2: 7375.6 NOK/MWh

marginal costs

- period 1,3,4: 612.1 NOK/MWh
- period 2: 1229.67 NOK/MWh

### 6.2 discharge constraints

### 6.3 reservoir constraints zero - why?