General

power station 1

Costs of starting and stoping are assumed negligible (= 0).

- $K_1 = 500 + 45 \cdot P_1 + 0.075 \cdot P_1^2 [EUR/h]$
- $P_{1_{min}} = 20MW$
- $P_{1_{max}} = 100MW$

power station 2

- $K_2 = 750 + 52 \cdot P_2 + 0.025 \cdot P_2^2 [EUR/h]$
- $P_{2_{min}} = 40MW$
- $P_{2_{max}} = 200MW$

1 optimal operation of both stations

minimum costs, where K1 and K2 intersect ($K_1 = K_2$). This is at a production value of $P = 169.5MW$

2 Production alternatives

2.1 operation possibilities

In table 1 the possibilities for plant combination at different production intervals is listed.

Interval (MW)	K_1	K_2	$K_1 + K_2$
20..40	✓	x	x
40..60	✓	✓	x
60..100	✓	✓	✓
100..200	x	✓	✓
200..300	x	x	✓

Tabell 1: Operation possibilities for different production

2.2 cost function

$$K_1 = 500 + 45 \cdot P_1 + 0.075 P_1^2 \quad (1)$$

$$K_2 = 750 + 52 \cdot P_2 + 0.025 P_2^2 \quad (2)$$

The equations 1 and 2 can be derived and the equal marginal costs calculated.

$$\frac{dK_1}{dP_1} = 45 + 0.15 \cdot P_1 \quad (3)$$

$$\frac{dK_2}{dP_2} = 52 + 0.05 \cdot P_2 \quad (4)$$

The total costs are minimal where equations 3 and 4 are the same. For production of less than 40MW the operation of plant 1 only is possible. So the cost function is not listed seperately.

2.2.1 interval 40MW-60MW

The Lagrange function can be written as

$$L = -(K_1 + K_2) + \lambda(P_1 + P_2 - P_{max}) \tag{5}$$

In this formulary 5 the cost functions for the two generation units can be filled in and the Lagrange function be derived afterwards.

$$L = -(500 + 45 \cdot P_1 + 0.075 \cdot P_1^2 + 750 + 52 \cdot P_2 + 0.025 \cdot P_2^2 + \lambda(P_1 + P_2 - 60)$$

$$L = -(1250 + 45 \cdot P_1 + 52 \cdot P_2 + 0.075 \cdot P_1^2 + 0.025 \cdot P_2^2) + \lambda(P_1 + P_2 - 60)$$

$$\frac{dL}{dP_1} = -(45 + 0.15P_1) + \lambda = 0$$

$$\frac{dL}{dP_2} = -(52 + 0.05P_2) + \lambda = 0$$

With this derivates the optimal production amount for P_1 and P_2 can be calculated.

$$45 + 0.15P_1 = 52 + 0.05P_2$$

$$P_1 = \frac{1}{3}(P_2 + 140)$$

Now the constraint $P_1 + P_2 = 60$ can be filled in.

$$\frac{1}{3}(P_2 + 140) + P_2 = 60$$

$$P_2 = 10$$

This value is the optimum, but it is not ok due to missmatching the constraint. So the production level has to be set to the smallest possible value P_2 can procuce for 2-station operation. The production therefore is

- $P_2 = P_{2_{min}} = 40MW$
- $P_1 = 60MW - 40MW = 20MW$

Both values are meeting with the constraints, so this is the optimal solution for two station operation. The costs are calculated as follows:

- K_1 only: $K_1(60) = 500 + 45 \cdot 60 + 0.075 \cdot 60^2 = 3470EUR$
- K_2 only: $K_2(60) = 750 + 52 \cdot 60 + 0.025 \cdot 60^2 = 3960EUR$
- $K_1 + K_2$: $K = 1250 + 45 \cdot 20 + 0.075 \cdot 20^2 + 52 \cdot 40 + 0.025 \cdot 40^2 = 4300EUR$

2.2.2 interval 60MW-100MW

This interval is calculated using tables and increasing the production amount with regard to the actual costs. The calculation steps are listed in table 2. The costs for the production of 100MW are therefore

- K_1 only: $K_1(100) = 500 + 45 \cdot 100 + 0.075 \cdot 100^2 = 5750EUR$
- K_2 only: $K_2(100) = 750 + 52 \cdot 100 + 0.025 \cdot 100^2 = 6200EUR$
- $K_1 + K_2$: $K = 1250 + 45 \cdot 60 + 0.075 \cdot 60^2 + 52 \cdot 40 + 0.025 \cdot 40^2 = 6340EUR$

(P_1,P_2) [MW]	$\frac{dK_1}{dP_1}$	$\frac{dK_2}{dP_2}$	difference	comment
(0,0)	45	52	$\Delta P_1 = +20$ $\Delta P_2 = +40$	costs without production
(20,40)	48	54	$\Delta P_1 = +20$	lower production limit, increasing P_1 due to lower costs
(40,40)	51	54	$\Delta P_1 = +20$	increase P_1, costs are still lower
(60,40)	54	54		costs are equal, sum of production is 100MW

Tabell 2: Calculation of production amount in interval 60MW-100MW

(P_1,P_2) [MW]	$\frac{dK_1}{dP_1}$	$\frac{dK_2}{dP_2}$	difference	comment
(60,40)	54	54	$\Delta P = +10$	end point of last calculation, starting point for this one. The costs are equal, so both stations get the same amount of production increase.
(70,50)	55.5	54.5	$\Delta P_2 = +10$	due to cheaper production of P_2 increase production of this generation unit
(70,60)	55.5	55	$\Delta P_2 = +10$	P_2 still cheaper
...	...	...	...	...
(80,100)	57	57	$\Delta P_1 = +3$ $\Delta P_2 = +10$	the amount of power production (200MW) is nearly reached, so decrease the step size
(83,110)	57.45	57.5	$\Delta P_1 = +1$ $\Delta P_2 = +6$	P_1 is not much cheaper, so increase P_2 little bit more
(84,116)	57.6	57.8	$\Delta P_1 = +1$ $\Delta P_2 = -1$	the production amount is reached, but the costs are not equal, so customizing
(85,115)	57.75	57.75		equal costs

Tabell 3: Calculation of production amount in interval 100MW-200MW

2.2.3 interval 100MW-200MW

For the interval from 100MW to 200MW the calculation is also done in a table. The steps are listed in table 3. Because of the fact, that the calculation principle is always the same, some calculation steps in the middle are not listed. As in the table can be seen, the optimal combination for 200MW of produced power is $P_1 = 85MW$ and $P_2 = 115MW$. The costs for the two possible generation options are

- K_2 only: $K_2(200) = 750 + 52 \cdot 200 + 0.025 \cdot 200^2 = 12750EUR$
- $K_1 + K_2$: $K = 1250 + 45 \cdot 85 + 0.075 \cdot 85^2 + 52 \cdot 115 + 0.025 \cdot 115^2 = 11927.5EUR$

2.2.4 interval 200MW-300MW

In this calculation step the wanted power is in a range, that it is only possible to generate with both generation plants. For 300MW of power both stations have to be utilized full. Thus the costs are

- $K_1 + K_2$: $K = 1250 + 45 \cdot 100 + 0.075 \cdot 100^2 + 52 \cdot 200 + 0.025 \cdot 200^2 = 17900EUR$

2.3 best combination, operation for generation unit 1

To get the best generation option the costs are important. Best alternative is that where the costs are lowest.

- 20-40MW:K_1 (only possibility)
- 40-60MW: K_1
- 60-100MW: K_1
- 100-200MW: $K_1 + K_2$
- 200-300MW: $K_1 + K_2$

3 calculation with emissions

The maximal allowed emissions are $E_{max} = 120t$, the wanted power level 250MW. For calculating the optimal combination of the generation units with respect to the emissions, the Lagrange function has to be modified.

- K_1: $e_1 = 0.55t/MWh$
- K_2: $e_2 = 0.45t/MWh$

$$L = -(1250+45\cdot P_1+52\cdot P_2+0.075\cdot P_1^2+0.025\cdot P_2^2)+\lambda(P_1+P_2-250)+\mu(120-0.55\cdot P_1-0.45\cdot P_2) \quad (6)$$

This modified Lagrange function in equation 6 can now be derived.

$$\frac{dL}{dP_1} = -45 + \lambda - 0.55 \cdot \mu$$

$$\frac{dL}{dP_2} = -52 + \lambda - 0.45 \cdot \mu$$

With respect to the constraints the new values for the production values of P_1 and P_2 can be calculated.

$$0.55 \cdot P_1 + 0.45 \cdot P_2 \leq 120$$

$$0.55(250 - P_2) + 0.45 \cdot P_2 \leq 120$$

$$-0.1 \cdot P_2 \leq -17.5$$

$$P_2 \geq 175$$

Thus the production values are

- $P_1 = 75MW$
- $P_2 = 175MW$

The production amounts are modified in that way, that the production plant is penalized by the emission constraint. Production amounts are moving to K_2 so that K_2 is now producing more power than without the emission term.