Supplementary Material

(Not intended for publication)

Understanding High-Wage and Low-Wage Firms

Horng Chern Wong

CREST - École Polytechnique

Email: horngchernwong@gmail.com

Shapley Decomposition Method

Recall the firm wage premium equation, written in natural logs:

$$\phi = wm + arph - pm + leo$$

This structural wage equation can now be seen as a linear regression. To assess the relative importance of each dimension of firm heterogeneity on the cross-sectional variation in firm wage premia, I implement the Shapley Decomposition of R^2 (Shorrocks, 1982, 2013). With four dimensions of heterogeneity, an analysis of variance approach generates ten variance and covariance terms, with potential negative contributions of certain variables, depending on the joint distribution of the explanatory variables. The Shapley approach offers simplicity in terms of the interpretation of the contribution of each dimension of heterogeneity, as it partitions the total R^2 into the marginal contributions of each variable. This gives four partial R^2's, one for each dimension of firm heterogeneity. Moreover, the partial R^2's never take negative values.

In cooperative game theory, the Shapley value is the unique solution to distributing the total surplus generated by a coalition of players. The idea is to view each variable (dimension of firm heterogeneity) as a player in a coalition, and the total R^2 as the total surplus. The Shapley decomposition then applies the Shapley value to partition the total R^2, based on each variable's marginal contribution. It is based on the following axioms, under which the Shapley value is derived:

- Efficiency: the entire surplus is distributed.
- Symmetry: any two players (variables) with same marginal contribution to the total surplus obtains the same share.
- Monotonicity: the total surplus is non-decreasing in the number of players.
- Null player: the null player does not obtain a share of the surplus.

The partial R^2 of a variable $X_j = \{wm, arph, pm, leo\}$ can then be written as:

$$R^2(x_j) = \sum_{T \subseteq V \setminus \{X_j\}} \frac{k! \cdot (p-k-1)!}{p!} \left(R^2(T \cup \{X_j\}) - R^2(T) \right)$$

where p denotes the number of variables, which is equal to four in this case; T is a regression with k number of variables, and V is the set of all combinations of regressor variables excluding X_j.

Wage-Posting and Wage-Bargaining Frameworks

Random Search Wage-Bargaining Framework

The structural framework presented in Section 2 does not take a stance on the specific frictions generating upward-sloping labor supply curves. I present here a model in which labor markets are characterized by search frictions and wages are set via bargaining over the match surplus. I derive the firm wage premium equation from this model and discuss the interpretation of the wage markdown in this model. I draw from the multiworker-firm random search models of Mortensen (2010), Elsby, Michaels, and Ratner (2018), and Schaal (2017) in which workers are allowed to search on-the-job. I assume that there are no aggregate shocks.

Matching in the labor market is governed by a matching function $\Lambda_t = \Lambda(\bar{U}_t + \xi(\bar{H}_t - \bar{U}_t), \bar{V}_t)$, where $\bar{H}$ and $\bar{U}$ denote total skill-adjusted population of workers and unemployed workers, and $\bar{V}$ denotes aggregate vacancies. The search intensity of employed workers is ξ. Labor market tightness is the ratio of vacancies to jobseekers $\theta_t \equiv \frac{\bar{V}_t}{\bar{U}_t + \xi(\bar{H}_t - \bar{U}_t)}$. The vacancy contact rate is then $q(\theta_t) = \Lambda(\theta_t^{-1})$, and the unemployed and employed worker job finding rates are $f(\theta_t)$ and $\xi f(\theta_t)$.

On the firm side, the hiring rate for a firm providing a value V_{jt}^e to its workers is:

$$\lambda(V_{jt}^e) = q(\theta_t) \left[\frac{\bar{U}_t}{\bar{U}_t + \xi(\bar{H}_t - \bar{U}_t)} + \frac{\xi(\bar{H}_t - \bar{U}_t)}{\bar{U}_t + \xi(\bar{H}_t - \bar{U}_t)} G_E(V_{jt}^e) \right]$$

where $G_E(.)$ denotes the cumulative distribution function of the realized value of employment to workers across employed workers. Similarly, the separation rate of this firm is:

$$s(V_{jt}^e) = \delta_s + (1-\delta_s)\xi f(\theta_t)\left(1 - F_V(V_{jt}^e)\right)$$

where δ_s is an exogenous separation rate, and $F_V(.)$ is the cumulative distribution function of the offered value of employment to workers among vacancies.

The unemployed worker's value function is:

$$U_t = b + \beta[(1-f(\theta_{t+1}))U_{t+1} + f(\theta_{t+1})E_t(V_{t+1}^e)]$$

which is a function of the flow value of unemployment b and the expected utility next period. Since there are no aggregate shocks, $U_t = U_{t+1}$. The employed worker's value function is:

$$\begin{aligned} V_{jt}^e = & \, u(\Phi_{jt}, A_{jt}) \\ & + \beta\{\delta_s U_{t+1} + (1-\delta_s)E_t[(1-\xi f(\theta_{t+1}))V_{jt+1}^e \\ & + \xi f(\theta_{t+1})F(V_{jt+1}^e)V_{jt+1}^e \\ & + \xi f(\theta_{t+1})(1-F(V_{jt+1}^e))E_t(V_{t+1}^e | V_{t+1}^e \geq V_{jt+1}^e)]\} \end{aligned}$$

which depends on the wage Φ_{jt} and non-wage amenities A_{jt} this period through a constant returns to scale utility function $u(.)$, the expected utility next period if the worker is exogenously separated from the firm, and the expected utility if the worker is not exogenously separated. The last component depends on the expected utility of being employed at the same firm, and the expected utility of moving to a new employer conditional on the new employer offering a higher utility. I assume that: (i) the flow utility function $u(.,.)$ is homogenous of degree one in its inputs, (ii) there is no savings mechanism, (iii) the value of non-wage amenities is proportional to worker efficiency, $A_{ijt} = E_{it}A_{jt}$, and (iv) worker efficiency is allowed vary over time due to random shocks: $E_{it+1} = E_{it} + \zeta_{it+1}$, where ζ_{it+1} is a mean-zero random shock.[1] Therefore, the value of unemployment and employment

[1]Alternatively, for a more realistic human capital accumulation process, one can also envision a model in which each worker i's efficiency grows at a deterministic rate, and

is proportional to worker efficiency. A worker with efficiency E_{it} obtains a value of $E_{it}U_t$ while unemployed and $E_{it}V^e_{jt}$ while employed.

The firm's profit maximization problem can be written as:

$$\Pi_{jt} = \max_{K_{jt}, M_{jt}, V_{jt}} P_{jt}Y_{jt} - R^K_t K_{jt} - P^m_t M_{jt} - \Phi(H_{jt})H_{jt} - c_t(V_{jt})V_{jt} + \beta E_t[\Pi_{jt+1}]$$

subject to the law of motion for employment:

$$H_{jt} = (1 - s(V^e_{jt}))H_{jt-1} + \lambda(V^e_{jt})V_{jt} \tag{1}$$

The average skill of workers at firm j is denoted as $\bar{E}_j$. The vacancy posting cost function $c_t(V_{jt})$ is assumed to be twice differentiable, monotonically increasing in vacancies $c'_t(V_{jt}) > 0$, and the marginal cost of vacancies is increasing $c''_t(V_{jt}) > 0$.

Wages are determined via Stole and Zwiebel (1996) bargaining between the firm and the marginal worker over the marginal match surplus. This generalizes the Nash bargaining protocol in models with constant marginal returns to labor to the case of diminishing marginal returns to labor. Employers do not make counteroffers. The bargained wage $\Phi(H_{jt})$ is a function of the firm's size, since diminishing marginal returns to labor implies that, all else equal, the marginal revenue product of labor, and hence total match surplus, is decreasing in firm size. The marginal surplus to be bargained over is:

$$\kappa_{jt}J_{jt} = (1 - \kappa_{jt})(V^e_{jt} - U_t)$$

where κ_{jt} is the worker's relative bargaining weight, which is allowed to differ across firms, and $J_{jt} \equiv \frac{\partial \Pi_{jt}}{\partial H_{jt}}$ is the firm's marginal surplus from an additional skill-adjusted worker. I obtain the following familiar equation for the firm's wage

the worker may receive an exogenous death shock, in which case the worker is replaced by a newly-born worker in the model. See, for example, Bagger, Fontaine, Postel-Vinay, and Robin (2014).

(premium):

$$\Phi_{jt} = \kappa_{jt}(MRPH_{jt} - \frac{\partial \Phi_{jt}}{\partial H_{jt}} H_{jt} + \beta E_t[(1 - s(\Phi_{jt+1}, A_{jt+1}))J_{jt+1}]) + (1 - \kappa_{jt})W^r_{jt}$$

This equation shows that the firm's wage is a weighted average of the value of the worker to the firm and the worker's reservation wage.

Combining the wage bargaining protocol with the first-order condition with respect to vacancies, I rearrange the above firm wage equation to obtain the firm wage premium equation, in which the firm's wage markdown component can be written as:

$$WM_{jt} = \frac{\left(\frac{\kappa_{jt}}{1-\kappa_{jt}}\right)\left(1 + \frac{W^r_{jt}}{\Phi_{jt} - W^r_{jt}}\right)}{1 + (1 - |\frac{\partial \Phi_{jt}}{\partial H_{jt}} \frac{H_{jt}}{\Phi_{jt}}|)\left(\frac{\kappa_{jt}}{1-\kappa_{jt}}\right)\left(1 + \frac{W^r_{jt}}{\Phi_{jt} - W^r_{jt}}\right) - \beta E_t\left(\frac{(1-s(\Phi_{jt+1}, A_{jt+1}))J_{jt+1}}{c_{V,jt}V_{jt} + c(V_{jt})}\right)\lambda(\Phi_{jt}, A_{jt})} \tag{2}$$

Note that $\frac{\partial \Phi_{jt}}{\partial H_{jt}} \frac{H_{jt}}{W_{jt}} < 0$ is no longer the inverse labor supply elasticity. It takes a negative value. This is because, with multilateral bargaining, the firm bargains with all of its worker over the marginal surplus of a match. With diminishing marginal returns to labor, if the firm and worker do not agree on a wage, the match is not formed, and the marginal revenue product of labor is higher for the remaining workers. This is an additional channel on top of workers' bargaining weight from which workers extract rents from the match.

The numerator of the wage markdown shows that the firm's wage markdown depends on its workers' relative bargaining power (κ_{jt}) and the reservation wage (W^r_{jt}). The higher the workers' bargaining power or reservation wage, the higher the fraction of marginal revenue product of labor workers obtain (higher wage markdown). The denominator shows that the wage markdown is also increasing in the expected future value of the worker to the firm.

Random Search Wage-Posting Framework

I now replace the wage-setting protocol of the random search framework above with wage-posting and discuss the determinants of the wage markdown. This model provides one microfoundation for the wage markdown derived from the structural framework presented in Section 2.

The firm's profit maximization problem is:

$$\Pi_{jt} = \max_{K_{jt}, M_{jt}, V_{jt}, \Phi_{jt}} P_{jt}Y_{jt} - R_t^K K_{jt} - P_t^m M_{jt} - \Phi_{jt}H_{jt} - c_t(V_{jt})V_{jt} + \beta E_t[\Pi_{jt+1}]$$

subject to the law of motion for employment:

$$H_{jt} = (1 - s(\Phi_{jt}, A_{jt}))H_{jt-1} + \lambda(\Phi_{jt}, a_{jt})V_{jt} \tag{3}$$

The wage markdown in this model is as follows:

$$WM_{jt} = \frac{\epsilon_{jt}^H}{1 + \epsilon_{jt}^H - \beta E_t \left(\frac{(1 - s(\Phi_{jt+1}, A_{jt+1}))J_{jt+1}}{c_{V,jt}V_{jt} + c(V_{jt})} \right) \lambda(\Phi_{jt}, A_{jt})}$$

where the firm-specific labor supply elasticity (ϵ_{jt}^H) can be written as:

$$\epsilon_{jt}^H = \frac{\lambda(\Phi_{jt}, A_{jt})V_{jt}}{H_{jt}} \epsilon_{\Phi,jt}^\lambda - \frac{s(\Phi_{jt}, A_{jt})H_{jt-1}}{H_{jt}} \epsilon_{\Phi,jt}^s > 0$$

which depends on the elasticity of the firm's hiring rate with respect to the firm's wage ($\epsilon_{\Phi,jt}^\lambda > 0$) weighted by the share of new hires among its workforce, minus the elasticity of the firm's separation rate with respect to the firm's wage ($\epsilon_{\Phi,jt}^s < 0$) weighted by the share of workers who separate from the firm among its workforce.

Directed Search Wage-Posting Framework

The random search model assumes that workers have no information about wages when they search for a job. An alternative assumption is that workers observe the full menu of wages in the economy when searching for jobs – directed or competitive search (Moen, 1997) and Kaas and Kircher (2015). I now replace random search with directed search in the otherwise identical wage-posting model. I show in this environment that the firm wage premium equation can be obtained and the wage markdown is identical as the model with random search.[2] The following timing assumption applies. First, idiosyncratic firm productivity and worker efficiency shocks are realized. Next, firms post wages and workers decide on where to search. Then, matching and separations take place. Finally, production begins.

In this model, workers can choose the firm or market at which she searches for a job by trading off offered utility and job-finding probability. The worker observes the offered utility V_{jt}^e at each firm j, before matching takes place. The pre-matching value to the unemployed worker is:

$$U_t^{bm} = \max_{V_{jt}^e}(1 - f(\theta(V_{jt}^e)))U_t + f(\theta(V_{jt}^e))V_{jt}^e$$

and the pre-matching value to a worker employed at firm j is:

$$V_{jt}^{e,bm} = \max_{V_{kt}^e} \delta_s U_t + (1 - \delta_s)\left[(1 - sf(\theta(V_{kt}^e)))\, V_{jt}^e + sf(\theta(V_{kt}^e))V_{kt}^e\right]$$

where the offered utility $V(W_{jt}, A_{jt})$ at any firm j depends on both the offered wages and non-wage amenities. As I show in the next subsection, no two workers with different utility V^e will search for employment at the same firm. Relative to a worker with lower utility, the worker with a higher utility will search for employment at a firm that offers an even higher utility, at the cost of a lower probability of this employment relationship materializing.

Firms post wages taking into account its effect on both recruitment and retention. Each firm recruits from other firms who offer a lower utility to their employees. From the employed worker's value function above, given the value of employment at a firm that offers $\underline{V}_t^e$, this worker optimally searches for employment at firm j, where $V_{jt}^e > \underline{V}_t^e$. Denote this unique solution as $V_{jt}^e = v(\underline{V}_t^e)$.

[2]For a comprehensive discussion of the theory and applications of directed search, see Wright, Kircher, Julien, and Guerrieri (2018).

Therefore, firm j recruits workers from this market. Similarly, firm j loses workers due to quits to a higher utility firm who pays $\overline{V}^e_t$. The optimal search strategy of a worker employed at firm j is then $\overline{V}^e_t = v(V^e_{jt})$. Next, note that the firm-specific separation rate is now $s_{jt} = \delta_s + (1-\delta_s)sf(\theta(\overline{V}^e_t))$. Using the law of motion for employment, the firm-specific "labor supply" curve is then:

$$
\begin{aligned}
H_{jt} &= (1 - s(\overline{V}^e_t))H_{jt-1} + q(\underline{V}^e_t)V_{jt} \\
&= (1 - s(V^e_{jt}))H_{jt-1} + q(V^e_{jt})V_{jt}
\end{aligned}
$$

The second line obtains by inverting the employed worker's optimal search function $v(.)$, which is monotonically increasing in its argument. Solving for the firm wage premium equation gives the same wage markdown expression as the random search wage-posting model above.

Workers' Search Behavior in a Directed Search Model

I now show that in the directed search model above, relative to workers employed at lower offered utility firms, workers employed at a higher offered utility firm will choose to search for employment at a firm that offers even higher utility, at the cost of a lower probability of finding employment there (see Wright et al. (2018)). Consider worker 1 employed at firm 1, searching optimally for employment at firm j; and worker 2 employed at firm 2, searching optimally for employment at firm k. Suppose that firm 2 offers a strictly higher utility than firm 1, $V^e_{2t} > V^e_{1t}$. The utility of either workers can be written as:

$$V^e_{2t} = U_t + \left(\frac{1-\delta_s}{\delta_s}\right) sf(V^e_{kt})[V^e_{kt} - V^e_{2t}]$$

$$V^e_{1t} = U_t + \left(\frac{1-\delta_s}{\delta_s}\right) sf(V^e_{jt})[V^e_{jt} - V^e_{1t}]$$

Under utility maximization:

$$f(V^e_{kt})[V^e_{kt} - V^e_{2t}] \geq f(V^e_{jt})[V^e_{jt} - V^e_{2t}]$$

$$f(V^e_{kt})[V^e_{kt} - V^e_{1t}] \leq f(V^e_{jt})[V^e_{jt} - V^e_{1t}]$$

which implies that:

$$f(V^e_{kt})[V^e_{1t} - V^e_{2t}] > f(V^e_{jt})[V^e_{1t} - V^e_{2t}]$$

Since the utility of worker 2 is strictly larger than that of worker 1

$$f(V^e_{kt}) < f(V^e_{jt})$$

Therefore, relative to worker 1, worker 2 who is employed at a higher utility firm searches for a job at a firm which has an even higher offered utility, at the cost of a lower probability of matching.

Workplace-Differentiation Monopsonistic Wage-Posting Framework

This section presents a static monopsonistic model based on the imperfect substitutability of firm-specific non-wage amenities (Card, Cardoso, Heining, and Kline, 2018). Worker i's indirect utility when employed at firm j is:

$$u_{ijt} = \gamma \ln(W_{ijt}) + a_{jt} + \eta_{ijt}$$

where $W_{ijt} = E_{it}\Phi_{jt}$ is the wage obtained by worker i with efficiency E_{it} earning a wage premium Φ_{jt}. The common value of the firm-specific non-wage amenity a_{jt}. Worker's preferences over non-wage amenities are subject to idiosyncratic shocks η_{ijt}, which is identically and independently drawn from a type I extreme value distribution.

Each worker i maximizes utility by choosing where to work:

$$j = \arg\max_j u_{ijt}$$

The firm-specific labor supply curve is then:

$$\frac{H_{jt}}{\bar{H}_t} = \frac{exp(\gamma \ln(\Phi_{jt}) + a_{jt})}{\sum_{k=1}^{J} exp(\gamma \ln(\Phi_{kt}) + a_{kt})}$$

Firm j's profit-maximization problem is:

$$\Pi_{jt} = \max_{K_{jt}, M_{jt}, \Phi_{jt}} P_{jt} Y_{jt} - R_t^K K_{jt} - P_t^m M_{jt} - \Phi(H_{jt}) H_{jt} + \beta E_t[\Pi_{jt+1}]$$

subject to the firm-specific labor supply curve. This model gives the firm wage premium equation and the following expression for the wage markdown:

$$WM_{jt} = \frac{\epsilon_{jt}^h}{1 + \epsilon_{jt}^h}$$

where the labor supply elasticity $\epsilon_{jt}^h = \gamma(1 - \frac{H_{jt}}{\bar{H}_t})$ depends on the labor market share of firm j. This equation shows that the wage markdown is decreasing in the firm's labor market share, as firm's with a high market share face a low labor supply elasticity. This expression provides a mapping between labor market shares, labor market concentration, and wages (Azar, Marinescu, and Steinbaum, 2020; Benmelech, Bergman, and Kim, 2020).

Distributions of Estimated Firm Characteristics

	Mean	Median	Variance	25th Pct	75th Pct
Wage markdown	0.85	0.82	0.14	0.69	0.96
Price-cost markup	1.27	1.26	0.12	1.14	1.39
Labor elasticity of output	0.41	0.41	0.02	0.57	0.24
Material elasticity of output	0.53	0.53	0.02	0.68	0.37
Service elasticity of output	0.53	0.53	0.02	0.68	0.37
Capital elasticity of output	0.06	0.06	0.00	0.09	0.03
log Average revenue product of labor	4.52	4.49	0.25	4.20	4.81
log Marginal revenue product of labor	2.83	2.82	0.09	2.67	2.98
Number of firms	243,453				

Table 1: Summary statistics of firm characteristics within 2-digit sectors in 2014. The overall mean of each dimension is kept constant.

Wage markdowns	Mean	Median	90th	75th	25th	10th	Variance
Aggregate	0.85	0.80	1.17	0.97	0.67	0.55	0.15
Construction	0.87	0.84	1.11	0.97	0.72	0.62	0.11
Manufacturing	0.82	0.78	1.04	0.90	0.67	0.58	0.13
Non-Financial	0.87	0.81	1.17	0.98	0.69	0.58	0.15
Transportation	0.77	0.73	0.91	0.81	0.66	0.58	0.14
Wholesale-Retail	0.88	0.81	1.32	1.04	0.62	0.48	0.20

Table 2: Distribution of estimated wage markdowns by sector in 2014.

Price-cost markups	Mean	Median	90th	75th	25th	10th	Variance
Aggregate	1.28	1.24	1.62	1.41	1.11	1.00	0.13
Construction	1.26	1.26	1.52	1.39	1.12	1.00	0.06
Manufacturing	1.19	1.17	1.39	1.27	1.07	0.98	0.08
Non-Financial	1.26	1.21	1.53	1.33	1.12	1.03	0.08
Transportation	1.14	1.12	1.33	1.22	1.02	0.95	0.04
Wholesale-Retail	1.38	1.27	1.74	1.58	1.17	1.00	0.24

Table 3: Distribution of estimated price-cost markups by sector in 2014.

Labor elasticity of output	Mean	Median	90th	75th	25th	10th	Variance
Aggregate	0.26	0.24	0.43	0.33	0.17	0.11	0.02
Construction	0.30	0.30	0.42	0.37	0.24	0.17	0.01
Manufacturing	0.29	0.29	0.41	0.35	0.23	0.16	0.01
Non-Financial	0.32	0.27	0.56	0.42	0.21	0.17	0.02
Transportation	0.32	0.32	0.45	0.39	0.25	0.18	0.01
Wholesale-Retail	0.16	0.15	0.25	0.20	0.12	0.09	0.01

Table 4: Distribution of estimated labor elasticities of output by sector in 2014.

Material elasticity of output	Mean	Median	90th	75th	25th	10th	Variance
Aggregate	0.41	0.41	0.72	0.55	0.26	0.13	0.05
Construction	0.31	0.32	0.39	0.36	0.28	0.22	0.01
Manufacturing	0.37	0.40	0.52	0.46	0.30	0.21	0.02
Non-Financial	0.28	0.25	0.53	0.48	0.12	0.03	0.04
Transportation	0.12	0.16	0.22	0.20	0.06	0.00	0.01
Wholesale-Retail	0.63	0.64	0.81	0.75	0.54	0.45	0.03

Table 5: Distribution of estimated material input elasticities of output by sector in 2014.

Service elasticity of output	Mean	Median	90th	75th	25th	10th	Variance
Aggregate	0.28	0.26	0.48	0.36	0.19	0.13	0.02
Construction	0.33	0.31	0.48	0.39	0.25	0.20	0.01
Manufacturing	0.28	0.27	0.42	0.34	0.21	0.17	0.01
Non-Financial	0.35	0.32	0.55	0.44	0.24	0.19	0.02
Transportation	0.49	0.47	0.70	0.59	0.38	0.30	0.02
Wholesale-Retail	0.19	0.18	0.28	0.24	0.12	0.08	0.01

Table 6: Distribution of estimated service input elasticities of output by sector in 2014.

Capital elasticity of output	Mean	Median	90th	75th	25th	10th	Variance
Aggregate	0.05	0.05	0.09	0.07	0.02	0.01	0.00
Construction	0.07	0.07	0.09	0.08	0.05	0.03	0.00
Manufacturing	0.05	0.05	0.08	0.07	0.04	0.02	0.00
Non-Financial	0.05	0.03	0.12	0.08	0.01	0.01	0.00
Transportation	0.07	0.08	0.12	0.10	0.05	0.02	0.00
Wholesale-Retail	0.02	0.02	0.04	0.03	0.01	0.00	0.00

Table 7: Distribution of estimated capital elasticities of output by sector in 2014.

Average revenue product of labor	Mean	Median	90th	75th	25th	10th	Variance
Aggregate	4.52	4.44	5.39	4.90	4.08	3.79	0.37
Construction	4.32	4.28	4.90	4.58	4.02	3.79	0.20
Manufacturing	4.33	4.27	4.99	4.62	3.99	3.77	0.25
Non-Financial	4.25	4.24	4.89	4.57	3.89	3.59	0.27
Transportation	4.24	4.20	4.78	4.47	3.97	3.78	0.18
Wholesale-Retail	5.03	5.02	5.79	5.43	4.61	4.26	0.37

Table 8: Distribution of log average revenue product of labor in efficiency units by sector in 2014.

Marginal revenue product of labor	Mean	Median	90th	75th	25th	10th	Variance
Aggregate	2.83	2.82	3.20	2.99	2.64	2.46	0.10
Construction	2.82	2.81	3.11	2.96	2.69	2.56	0.06
Manufacturing	2.86	2.86	3.17	3.00	2.71	2.56	0.09
Non-Financial	2.79	2.79	3.15	2.96	2.61	2.43	0.11
Transportation	2.90	2.89	3.17	2.99	2.80	2.69	0.06
Wholesale-Retail	2.82	2.81	3.33	3.07	2.57	2.33	0.18

Table 9: Distribution of log marginal revenue product of labor in efficiency units by sector in 2014.

Shapley Decomposition Results by Sector

	Aggregate	Construction	Manufacturing
wm	0.25	0.26	0.24
arph	0.35	0.47	0.43
pm	0.11	0.09	0.08
leo	0.29	0.17	0.24
R^2	1	1	1
Number of firms	4,907,010	1,005,476	1,126,657

	Non-Financial	Transportation	Wholesale & Retail
wm	0.20	0.28	0.32
arph	0.39	0.41	0.36
pm	0.14	0.09	0.09
leo	0.29	0.22	0.23
R^2	1	1	1
Number of firms	1,126,785	178,487	1,469,287

Table 11: Shapley decomposition of the firm wage premium distribution by sectors, 1995-2014.

Ensemble Decomposition Results

$$V(\phi) = CV(\ln WM, \phi) + CV(-\ln PM, \phi) + CV(\ln LEO, \phi) + CV(\ln ARPH, \phi)$$

	Aggregate	Construction	Manufacturing	Non-Financial	Transportation	Wholesale & Retail
$CV(\ln WM, \phi)$	0.006	0.009	0.003	0.003	0.004	0.007
$CV(-\ln PM, \phi)$	-0.001	-0.002	-0.001	-0.002	-0.001	-0.002
$CV(\ln LEO, \phi)$	-0.002	-0.007	-0.005	-0.003	-0.004	-0.003
$CV(\ln ARPH, \phi)$	0.006	0.011	0.009	0.008	0.009	0.008
Number of firms	243,453	49,175	45,558	52,818	7,039	68,661

Table 12: Ensemble decomposition of the firm wage premium distribution by sectors (2014).

Standard Variance Decomposition Results

$$
\begin{aligned}
V(\phi) = \; & V(wm) + V(pm) + V(leo) + V(arph) \\
& + 2CV(wm, -pm) + 2CV(wm, leo) + 2CV(wm, arph) \\
& + 2CV(-pm, leo) + 2CV(-pm, arph) + 2CV(leo, arph) \\
= \; & V(wm) + V(mrph) + 2CV(wm, mrph)
\end{aligned}
$$

	wm	$arph$	pm	leo	$mrph$
wm	0.102				
$arph$	-0.051	0.367			
pm	-0.036	0.004	0.038		
leo	-0.010	-0.289	-0.017	0.279	
$mrph$	-0.096	0.057	-0.034	0.008	0.100

Table 13: Firm heterogeneity variance-covariance matrix in 2014 (aggregate).

	wm	$arph$	$-pm$	leo	$mrph$
wm	0.062				
$arph$	0.021	0.190			
$-pm$	-0.018	0.003	0.028		
leo	-0.014	-0.160	-0.014	0.181	
$mrph$	-0.053	0.032	0.016	0.007	0.055

Table 14: Firm heterogeneity variance-covariance matrix in 2014 (construction).

	wm	$arph$	$-pm$	leo	$mrph$
wm	0.078				
$arph$	0.034	0.232			
$-pm$	-0.021	-0.019	0.024		
leo	-0.021	-0.170	0.015	0.171	
$mrph$	-0.076	0.043	0.020	0.016	0.079

Table 15: Firm heterogeneity variance-covariance matrix in 2014 (manufacturing).

	wm	$arph$	$-pm$	leo	$mrph$
wm	0.095				
$arph$	-0.016	0.249			
$-pm$	-0.025	-0.019	0.033		
leo	0.050	-0.207	-0.010	0.244	
$mrph$	-0.092	0.024	0.023	0.047	0.095

Table 16: Firm heterogeneity variance-covariance matrix in 2014 (non-financial services).

	wm	$arph$	$-pm$	leo	$mrph$
wm	0.050				
$arph$	-0.011	0.156			
$-pm$	-0.009	-0.011	0.023		
leo	-0.048	-0.148	-0.004	0.195	
$mrph$	-0.046	-0.002	0.008	0.043	0.049

Table 17: Firm heterogeneity variance-covariance matrix in 2014 (transportation).

	wm	$arph$	$-pm$	leo	$mrph$
wm	0.169				
$arph$	-0.106	0.347			
$-pm$	-0.071	0.007	0.052		
leo	0.015	-0.241	-0.009	0.213	
$mrph$	-0.161	0.114	0.068	-0.019	0.163

Table 18: Firm heterogeneity variance-covariance matrix in 2014 (wholesale and retail).

Cross-sectional Correlations by Sector

	wm	$arph$	$-pm$	leo	$mrph$
wm	1				
$arph$	0.194	1			
$-pm$	-0.432	0.034	1		
leo	-0.132	-0.864	0.199	1	
$mrph$	-0.904	-0.313	0.416	0.070	1

Table 19: Firm heterogeneity correlation matrix in 2014 (construction).

	wm	$arph$	$-pm$	leo	$mrph$
wm	1				
$arph$	0.251	1			
$-pm$	-0.479	-0.256	1		
leo	-0.182	-0.854	-0.239	1	
$mrph$	-0.960	0.315	0.464	0.141	1

Table 20: Firm heterogeneity correlation matrix in 2014 (manufacturing).

	wm	$arph$	$-pm$	leo	$mrph$
wm	1				
$arph$	-0.103	1			
$-pm$	-0.459	-0.209	1		
leo	0.331	-0.837	-0.107	1	
$mrph$	-0.969	0.156	0.420	-0.311	1

Table 21: Firm heterogeneity correlation matrix in 2014 (non-financial services).

	wm	$arph$	$-pm$	leo	$mrph$
wm	1				
$arph$	-0.123	1			
$-pm$	-0.260	-0.180	1		
leo	-0.484	-0.845	-0.063	1	
$mrph$	-0.923	-0.024	0.241	0.441	1

Table 22: Firm heterogeneity correlation matrix in 2014 (transportation).

	wm	$arph$	$-pm$	leo	$mrph$
wm	1				
$arph$	-0.437	1			
$-pm$	-0.751	0.053	1		
leo	0.080	-0.884	0.084	1	
$mrph$	-0.971	0.477	0.739	-0.099	1

Table 23: Firm heterogeneity correlation matrix in 2014 (wholesale and retail).

Bibliography

Azar, J., I. Marinescu, and M. I. Steinbaum (2020). Labor market concentration. *Journal of Human Resources*, forthcoming.

Bagger, J., F. Fontaine, F. Postel-Vinay, and J. Robin (2014). Tenure, Experience, Human Capital, and Wages: A Tractable Equilibrium Search Model of Wage Dynamics. *American Economic Review 104*(6), 1551–1596.

Benmelech, E., N. Bergman, and H. Kim (2020). Strong Employers and Weak Employees: How Does Employer Concentration Affect Wages? *Journal of Human Resources (forthcoming)*.

Card, D., A. Cardoso, J. Heining, and P. Kline (2018). Firms and Labor Market Inequality: Evidence and Some Theory. *Journal of Labor Economics 36*(1), 13–70.

Elsby, M., R. Michaels, and M. Ratner (2018). Vacancy Chains. *Working paper*.

Kaas, L. and P. Kircher (2015). Efficient Firm Dynamics in a Frictional Labor Market. *American Economic Review 105*, 3030–3060.

Moen, E. (1997). Competitive Search Equilibrium. *Journal of Political Economy 105*(2), 385–341.

Mortensen, D. (2010). Wage Dispersion in the Search and Matching Model. *American Economic Review: Papers and Proceedings 100*(2), 338–342.

Schaal, E. (2017). Uncertainty and Unemployment. *Econometrica 85*(6), 1675–1721.

Shorrocks, A. (1982). Inequality Decomposition by Factor Components. *Econometrica 50*(1), 193–211.

Shorrocks, A. (2013). Decomposition Procedures for Distributional Analysis: A Unified Framework Based on the Shapley Value. *The Journal of Economic Inequality 11*(1), 99–126.

Stole, L. and J. Zwiebel (1996). Intra-Firm Bargaining under Non-Binding Contracts. *Review of Economic Studies 63*(3), 375–410.

Wright, R., P. Kircher, B. Julien, and V. Guerrieri (2018). Directed Search: A Guided Tour. *Working paper*.