Unabigious mapping between hypergraph and grid representations of architectural objects.

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Abstract

Hypergraphs offer very powerful tool in terms of consistent description of architectural objecst with arbitrary complexity. For practical application one is sometimes forced to operate on simpler structures such as grids. In such cases an unambiguous mapping is required to connect hypergraph and grid descriptions. In this chapter a description of such formal mapping is presented as well as a practical realisation with regard to grid of convex cells.

1 A hypergraph-grid representation mapping

For a given hypergraph H and a grid G we would like to find a mapping between them. Namely for each grid cell g_i of a grid G find its corresponding hypervertex v_j of hypergrah H. We require for this procedure to be:

- Unambiguous - if the hypergraph and the grid does not change, the mapping should not change as well,
- Spatially local - relation between grid cells and hypervertices should not depend on elements distant in terms of represented space.

1.1 A background vertex

Often a grid will represent larger physical space then corresponding hypergraph. In such situations it is hard to ensure that each grid cell will have suitable hypervertex. In order to mitigate this we introduce **a background**

hypervertex b. Its role is to affiliate grid cells that have no corresponding hypervertices. We therefore introduce an extended set of hypervertices V':

$$V' = V \cup \{b\}.$$

Depending on a situation hypervertex b will have different interpretation. When H represents an indoor object - for example a room in a building, grid cells outside the room will be affiliated with b, thus b will represent all outside cells. On the other side - if H represents some outdoor installation such as playground in a park, b will be affiliated with all cells that are part of the park, but not part of the playground itself.

2 An affiliation relation

We are now ready to define explicitly **an affiliation relation** $\sim_A$. Given an extended set of hypervertices V' and a set of grid cells G an affiliation relation $\sim_A$ forms a subset of their Cartesian product:

$$\sim_A \subseteq V' \times G.$$

We demand following conditions to be satisfied by an affiliation relation $\sim_A$:

- $\forall g_j \; b \sim_A g_j$,
- If there is a real connection between space represented by hypervertex v_i and grid cell g_j, then $g_j \sim_A v_i$.

It should be noted that with a single grid cell more then one hypervertex can be affiliated. The first condition ensures that all grid cells are affiliated with at least one hypervertex. Example how such affiliation relation can be constructed for real life examples is shown in the next section.

2.1 An affiliated set and an ordering relation

For each grid cell g_i and an affiliation relation $\sim_A$ we can construct **an affiliated set** A_{g_i} - a set of all hypervertices affiliated with a given grid cell:

$$A_{g_i} = \{v_j \ : \ v_j \sim_A g_i\}.$$

As we want to have an unambiguous mapping between grid and hypergraph structures - for each grid cell g_i and its corresponding affiliated set A_{g_i} we introduce a relation $>_{g_i}$ with following properties:

- Antisymmetry - $\forall_{v_j,v_k \in A_{g_i}}$ if $v_j >_{g_i} v_k$ then $\neg\ v_k >_{g_i} v_j$,
- Transitivity - $\forall_{v_j,v_k,v_l \in A_{g_i}}$ if $v_j >_{g_i} v_k$ and $v_k >_{g_i} v_l$ then $v_j >_{g_i} v_l$,
- Totality - $\forall_{v_j,v_k \in A_{g_i}}$ if $v_j \neq v_k$ then $v_j >_{g_i} v_k$ or $v_k >_{g_i} v_j$,
- Distinguishability - $\forall_{v_j \in A_{g_i}}$ if $v_j \neq b$ then $v_j >_{g_i} b$.

A relation having such properties impose a strict total order on an affiliated set A_{g_i}, thus there is always the bottom element namely hypervertex b and the top element which we will call the top affiliated hypervertex - v_{g_i}. As for each grid cell g_i there is only one top affiliated hypervertex v_{g_i} it is the mapping we were searching for.

Success of this procedure depends on the construction of $\sim_A$ and $>_{g_i}$ relations. It should be also noted that spatial locality is not guaranteed and should be taken into consideration during construction of these relations. In the following section we present a possible construction of such relations for 3D hypergraph to 2D grid mapping illustrated on a real life case.

3 A mapping between simple room and corresponding grid

In this section we will show how an affiliation and ordering relations can be explicitly constructed and applied to a hypergraph representing a simple transient room. As can be seen in Fig. 1 the room is L-shaped with two exits and consists of a small cupboard and one table. A synthesized hypergraph of a room as a whole can be seen in Fig. **??**. The hypergrah of a room solid consists of six wall hypervertices $w_1, ..., w_6$, two door hypervertices d_1, d_2, floor and ceiling hypervertices f_1, f_2. The hypergraph of the cupboard represented as a cuboid consists of four vertical faces $c_2, ..., c_5$ as well as top and bottom surfaces c_1, c_6. The table is modeled as a cylinder and consists of two horizontal surfaces t_1, t_3 and circular horizontal surface hypervertex t_2. Schematic drawings of the objects and their corresponding hypergraphs are shown in Fig. **??** and **??**. For clarity only simple and external edges are presented for all hypergraphs.

For a given room a square grid has been selected as can bee seen on Fig. 1. The grid G here is a set of oriented square surfaces of the same size, lying in the same plane corresponding with floor plane. It consists of eight columns and seven rows, labeled with letters and numbers respectively. The aim of such grid mapping can be to provide a basis for a pedestrian dynamics simulation.

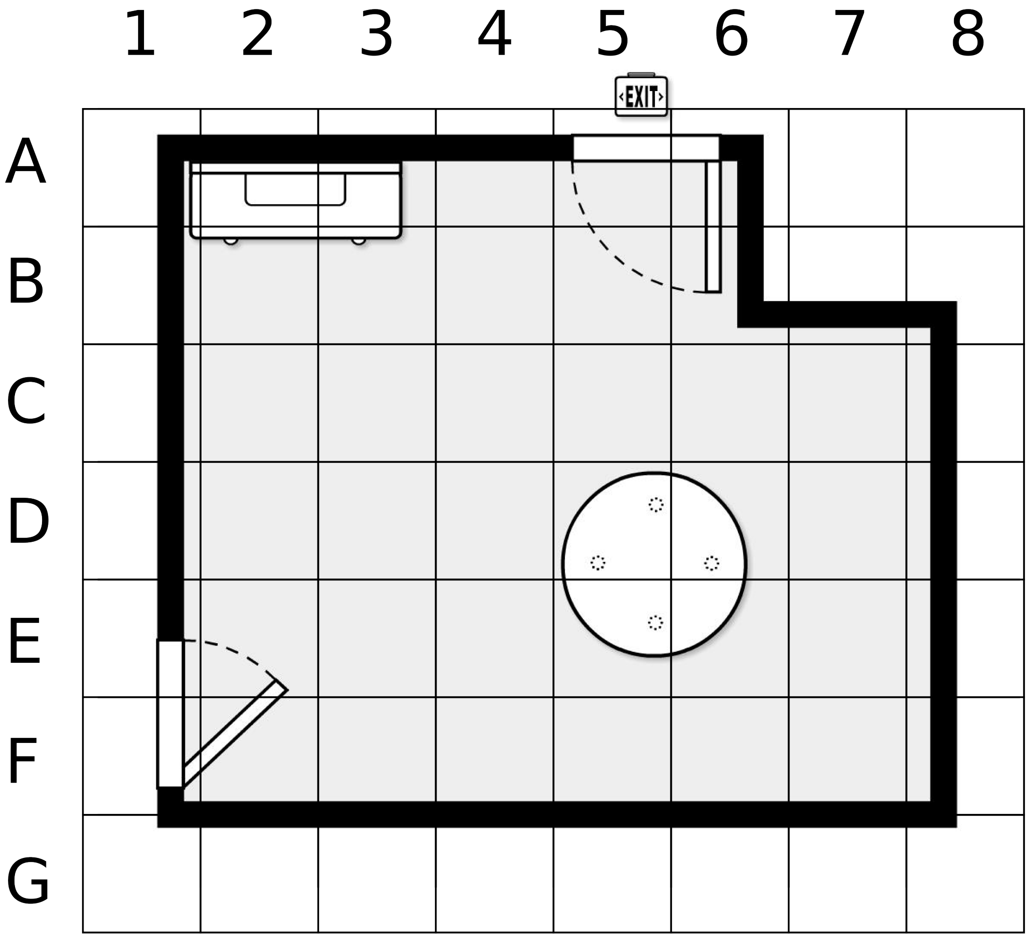

Figure 1: A simple transient room with two exits, one table and one cupboard. A regular rectangular grid to which mapping will be done is shown as well. It consists of 8 columns and 7 rows labeled with numbers and letters correspondingly.

3.1 An affiliation relation and affiliated sets

We demand of an affiliation relation to affiliate surfaces that correspond to each other. Let us denote considered grid cells by g_i, their normal vectors by $\hat{g}_i$ and the grid plane by P_G. We can now construct an affiliation relation in the following way:

- For each hypervertex v_j and its corresponding surface s_j we calculate projection of the surface s_j onto the grid plane P_G and denote it by s_j^p.
- For each surface s_j and each grid cell g_i we calculate minimal Euclidean distance between them and denote it by $d_{s_j \leftrightarrow g_j}$.
- $v_j \sim_A g_i$ if and only if $s_j^p \cap g_i \neq \varnothing$ and $d_{s_j \leftrightarrow g_j} \leq h_{max}$.
- h_{max} is the cut-off value taken as a typical maximum height of the human (approximately 2 m, less then height of the room).

Affiliated sets for each grid cell will be composed of the hypervertices corresponding to the surfaces that have non-empty projection onto the grid cell and are close to it.

For the considered room let us focus on a few chosen grid cells namely **A1** ,**A6**, **C4** and **A8**. The affiliated sets will have following composition:

- $A_{A1} = \{f_1, c_1, c_2, c_3, c_5, w_6, w_1, b\}$. Surface corresponding to f_2 is farther from the grid then cut-off value h_{max}.
- $A_{A6} = \{d_1, w_2, f_1, b\}$.
- $A_{C4} = \{f_1, b\}$.
- $A_{A8} = \{b\}$.

3.2 An ordering relation

Natural intuition is to order the affiliated set simply by the area of the projection of the surface s_j onto the grid cell g_i. However, in practical applications information about unusual affiliations very often is more important. For example, if there is an obstacle in the room it will heavily influence pedestrian movement and is crucial to be taken into account even if its size is small respective to the grid cell size.

With this reasoning in mind we introduce **a priority** p_{v_i} of hypervertex v_i as its attribute. Priority of the hypervertex depends on its relative influence. Continuing our example with a grid for pedestrian dynamics simulation we introduce three priority classes:

- High - doors and other connection between rooms.
- Medium - wall, furniture and other surfaces that are obstacles for the pedestrians.
- Low - surfaces that are easily accessible for pedestrian.

Now we can define ordering relation $>_{g_i}$ for a grid cell g_i and its affiliated set A_{g_i}:

- For two different hypervertices $v_j, v_k \in A_{g_i}$ and their corresponding surfaces s_j, s_k, if v_j has higher priority class then v_k then $v_j >_{g_i} v_k$ and vice versa.
- If two different hypervertices have the same priority then $v_j >_{g_i} v_k$ if $s_j^p > s_k^p$ and vice versa, where the last relation we understand as s_j^p having larger area then s_k^p.
- If two different hypervertices have the same priority and same projected area then $v_j >_{g_i} v_k$ if $d_{s_j \leftrightarrow g_i} > d_{s_k \leftrightarrow g_i}$ and vice versa.

In the following example the hypervertices corresponding to wall surfaces will be assumed to have a low priority. This is motivated by the fact that considered grid size is quite large and background hypervertex b corresponds to inaccessible exterior. Otherwise the mapping would not preserve the topology of accessibility.

Continuing our example for the cells **A1** ,**A6**, **C4** and **A8** let us determine top affiliated hypervertices:

- $A_{A1} = c_6$ - four hypervertices have medium priority here. Both c_1 and c_6 have the same projection area, but c_6 is above c_1.
- $A_{A6} = d_1$ - only one hypervertex in this affiliated set has high priority so it will be the top affiliated hypervertex
- $A_{C4} = f_1$ - hypervertex corresponding to the floor is always superior to background b with regard to ordering relation.
- $A_{A8} = b$ - in this situation there are no other affiliated hypervertices so the top affiliated hypervertex will be the background one.

In the Fig. 2 we can see with what priority hypervertices were assigned to each cell. Purple, red and green color correspond to high, medium and low priority correspondingly. For cells colored gray the top affiliated hypervertex was the background hypervertex b.

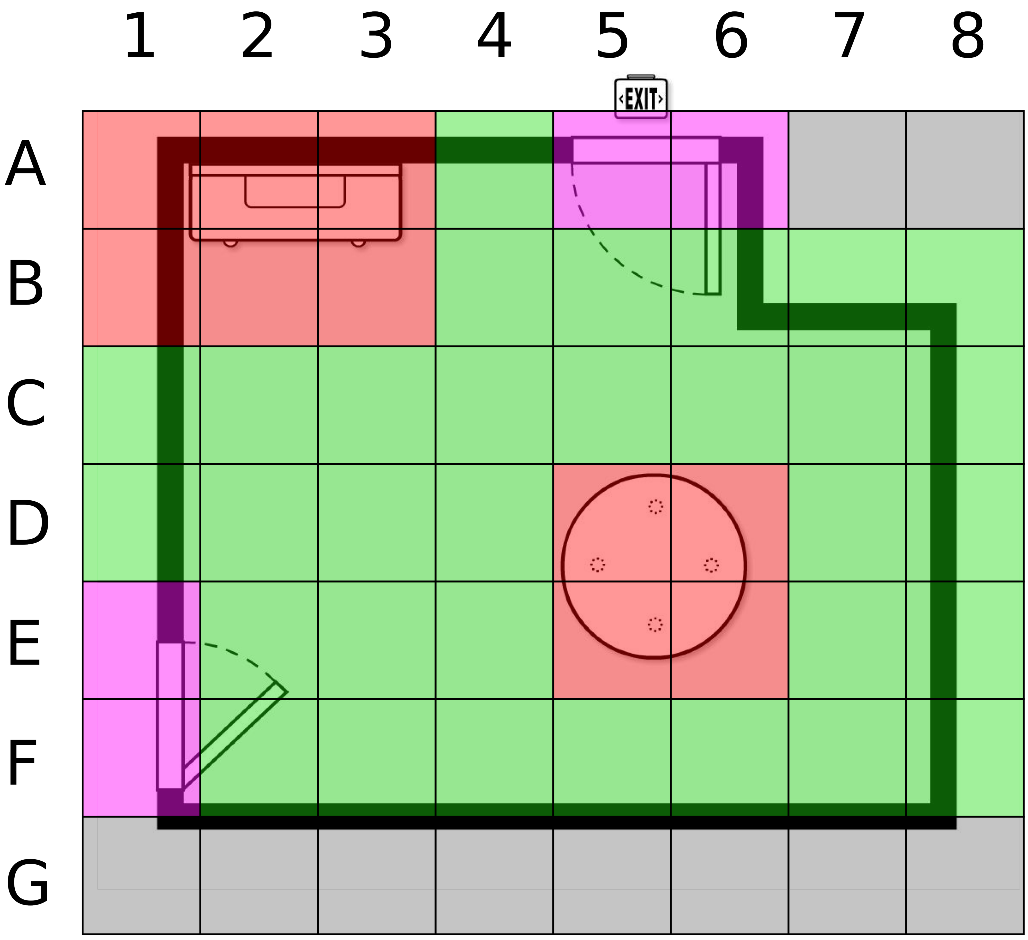

Figure 2: A simple transient room and a regular square grid to which a mapping was done. Different colors correspond to different priority classes of mapped hypervertices. High - purple, medium - red, low - green. Grey cells are affiliated with background hypervertex b.