1.

$\to T2\},g \quad [T1 \to T2]$,a :T1}

(a) {f:[T1

⊢(f(g a)) :T2

false, a:T1 , g:[T1→T2] ⟹ (g a):[T1→T2]

f:T1→T2 ⊢ (g a) :T2

T f T2

{ f:[T1 X T2 → T3]} ⊢ (lambda (x) (f x 100): T2→T3

false(f x:T2), f:[T1 X T2 → T3], 100: number

(x) f x [T1→T3] ≠ [T2 → T3] f

lambda →T2]} ⊢ (lambda (x) (f x)) number

100 T1

True, T1 (f x) : [T1→T2]

((lambda (x) (f x))) :[T1→T2]

$\{S : [T1$ $: [T1 \to T2]$

$X :$

b) $\{f: [T1 \times T2 \to T3], y: T2\} \vdash (\text{lambda}(x)$

$T_{new}, X: T1$, $(f\ x\ y): [T1 \times T2 \to T3]$

$(\text{lambda}(x)(f\ x\ y)): [T1 \to T3]$

$(f\ x\ y)): [T1 \to T3]$

c) $((\text{lambda}\ (f\ x1)\ (\text{if}\ x1\ (f\ 1\ x1)\ (f\ 3\ x1)))\ +\ \#t)\ T_0$

$(\text{lambda}\ (f\ x1)\ (\text{if}\ x1\ (f\ 1\ x1)\ (f\ 3\ x1)))$ $T1$

$(\text{if}\ x1\ (f\ 1\ x1)\ (f\ 3\ x1))$ $T2$

$x1$ Tx

$(f\ 1\ x1)$ $T3$

$(f\ 3\ x1)$ T_f

#1 T#1

essjon

Eq uation

((lambda (f x1) (if x1 (f 1 x1) (f 3 x1))) + #f)

T1 = [Tf * T#f] → T0

(lambda (f x1) (if x1 (f 1 x1) (f 3 x1)))

T1 = [Tf * Tx] → T2

(if x1 (f 1 x1) (f 3 x1))

T2 = T3

T3 = T4

Tx = Boolean

(f 1 x1)

Tf = [Tnum1 * Tx] → T3

Expr

(f 3 x1)

Tf = [Tnum3 *

Tx] → T4

+

T+ = [Number * Number] → Number

#f

T#f = Boolean

Equation	Substitution
$T1 = [Tf * T\#f] \rightarrow T0$	$\{\}$
$T1 = [Tf * TX] \rightarrow T2$	
$T2 = T3$	
$T3 = T4$	
$TX = Boolean$	
$Tf = [Tnum1 * TX] \rightarrow T3$	
$Tf = [Tnum3 * TX] \rightarrow T4$	
$T+ = [Number * Number] \rightarrow Number$	

Equation

$T1 = [Tf * TX] \rightarrow T2$

$T0\}$

$T2 = T3$

$T3 = T4$

$TX = Boolean$

$Tf = [Tnum1 * TX] \rightarrow T3$

$Tf = [Tnum3 * TX] \rightarrow T4$

$T+ = [Number * Number] \rightarrow Number$

Substitution

$\{T1 := [T+ * T\#f] \rightarrow$

Equation

$T2 : T3$

$T0\}$

$T3 = T4$

$TX = Boolean$

$Tf : [Tnum1 * TX] \to T3$

$Tf : [Tnum3 * TX] \to T4$

$T+ : [Number * Number] \to Number$

$Tf : T+$

$TX : T\#+$

$T0 = T2$

Substitution

$\{T1 := [T+ * T\#f] \to$

Equation

$T3 =$

$T4$

$T0\}$

$TX = Boolean$

$Tf : [Tnum1 * Tx] \rightarrow T3$

$Tf : [Tnum3 * Tx] \rightarrow T4$

$T+ : [Number * Number] \rightarrow Number$

$Tf : T+$

$Tx : T\#t$

$T0 = T2$

$T2 : T3$

Substitution

$\{T1 := [T+ * T\#t] \rightarrow$

Equation

$TX = Boolean$

$Tf = [Tnum1 * Tx] \rightarrow T3$

$Tf = [Tnum3 * Tx] \rightarrow T4$

$T+ = [Number * Number] \rightarrow Number$

$Tf = T+$

$Tx = T\$+$

$T0 = Tg$

$Tg = T3$

$T3 = T4$

Substitution

$\{T1 = [T+ * T\#f] \rightarrow T0\}$

Equation

$Tf = [Tnum1 * Tx] \to T3$

$Tf = [Tnum3 * Tx] \to T4$

$T+ = [Number * Number] \to Number$

$Tf = T+$

$Tx = T\#t$

$T0 = T2$

$T2 = T3$

$T3 = T4$

Substitution

$\{T1 := [T+ * T\#f] \to T0,$

$Tx := Bool\}$

Equation

$Tf = [Tnum3 * Tx] \to T4$

$T+ = [Number * Number] \to Number$

$Tf = T+$

$Tx = T\#f$

$T0 = T2$

$T2 = T3$

$T3 = T4$

Substitution

$\{T1 := [T+ * T\#f] \to T0,$

$Tf := [Tnum1 * Boolean] \to T3,$

$Tx := Boolean\}$

Equation

$T+ = [Number * Number] \to Number$

$Tf = T+$

$Tx = T\#f$

$T0 = T2$

$T2 = T3$

$T3 = T4$

$Tnum1 = Tnum3$

Substitution

$\{T1 := [T+ * T\#f] \to T0,$

$Tf := [Tnum1 * Boolean] \to T3,$

$Tx := Boolean\}$

Equation

$Tf = T+$

$Tx = T\$+$

$T0 = Tg$

$Tg = T3$

$T3 = T4$

$Tnum1 = Tnum3$

Substitution

$\{T1 := [[[number * number] \to number] * T\#+] \to T0,$

$Tf := [Tnum1 * Boolean] \to T3,$

$T+ := [Number * Number] \to Number,$

$Tx := Boolean\}$

, $Tf = T+$: כי נקבל

$Tf := [Tnum1 * Boolean] \to Number$

$T+ := [Number * Number] \to Number$ $\Rightarrow Tnum1 = Number$

$Boolean \neq Number$

כאשר נבדוק שוויון בין טיפוסים שונים נקבל שגיאה.

Boolean = Number Expression is not well typed.

((lambda (f1 x1 y1) (f1 x1 y1)) * 1 3) T0

(lambda (f1 x1 y1) (f1 x1 y1)) T1

(f1 x1 y1) T2

f1 Tf

x1 Tx

y1 Ty

* T*

1 Tnum1

3 Tnum3

Expression	Equation
((lambda (f1 x1 y1) (f1 x1 y1)) * 1 3)	$T1 = [T* * T_{num1} * T_{num3}] \to T0$
(lambda (f1 x1 y1) (f1 x1 y1))	$T1 = [Tf * Tx * Ty] \to T2$
(f1 x1 y1)	$Tf = [Tx * Ty] \to T2$
*	$T* = [Number * Number] \to Number$
1	$T_{num1} = Number$
3	$T_{num3} = Number$

Equation	Substitution
$T1 = [T* * T_{num1} * T_{num3}] \to T0$	$\{\}$

$T1 = [Tf * Tx * Ty] \to T2$

$Tf = [Tx * Ty] \to T2$

$T* = [Number * Number] \to Number$

$T_{num1} = Number$

$T_{num3} = Number$

Equation

$T1 = [Tf * Tx * Ty] \rightarrow T2$

$Tf = [Tx * Ty] \rightarrow T2$

$T* = [Number * Number] \rightarrow Number$

$Tnum1 = Number$

$Tnum3 = Number$

Substitution

$\{T1 := [T* * Tnum1 * Tnum3] \rightarrow T0\}$

Equation

$Tf = [Tx * Ty] \rightarrow T2$

$T* = [Number * Number] \rightarrow Number$

$Tnum1 = Number$

$Tnum3 = Number$

$Tf = T*$

$Tx = Tnum1$

$Ty = Tnum3$

$T0 = T2$

Substitution

$\{T1 := [T* * Tnum1 * Tnum3] \rightarrow T0\}$

Equation

$T* = [Number * Number] \to Number$

$T_{num1} = Number$

$T_{num3} = Number$

$T_f = T*$

$T_x = T_{num1}$

$T_y = T_{num3}$

$T0 = T2$

Substitution

$\{T1 := [T* * T_{num1} * T_{num3}] \to T0,$

$Tf := [Tx * Ty] \to T2\}$

Equation

$T_{num1} = Number$

$T_{num3} = Number$

$T_f = T*$

$T_x = T_{num1}$

$T_y = T_{num3}$

$T0 = T2$

Substitution

$\{T1 := [[[Number * Number] \to Number] * T_{num1} * T_{num3}] \to T0,$

$Tf := [Tx * Ty] \to T2$

$T* := [Number * Number] \to Number\}$

Equation

$T_{num3} = Number$

$T_f = T*$

$T_x = T_{num1}$

$T_y = T_{num3}$

$T0 = T2$

Substitution

$\{T1 := [[[Number * Number] \to Number] * Number * T_{num3}] \to T0,$

$Tf := [Tx * Ty] \to T2$

$T* := [Number * Number] \to Number,$

$T_{num1} := Number\}$

Equation

$T_f = T*$

$T_x = T_{num1}$

$T_y = T_{num3}$

$T0 = T2$

Substitution

$\{T1 := [[[Number * Number] \to Number] * Number * Number] \to T0,$

$Tf := [Tx * Ty] \to T2$

$T* := [Number * Number] \to Number,$

$T_{num1} := Number, T_{num3} := Number\}$

Equation

$T_x = T_{num1}$

$T_y = T_{num3}$

$T0 = T2$

$T_x = Number$

$T_y = Number$

$T2 = Number$

Substitution

$\{T1 := [[[Number * Number] \to Number] * Number * Number] \to T0,$

$Tf := [T_{num1} * Ty] \to T2$

$T* := [Number * Number] \to Number,$

$T_{num1} := Number, T_{num3} := Number\}$

Equation

$T_y = T_{num3}$

$T_0 = T_2$

$T_x = Number$

$T_y = Number$

$T_2 = Number$

Substitution

$\{T_1 := [[[Number * Number] \to Number] * Number * Number] \to T_0,$

$T_5 := [T_{num1} * T_y] \to T_2$

$T_* := [Number * Number] \to Number,$

$T_{num1} := Number, T_{num3} := Number,$

$T_x := T_{num1}\}$

Equation

$T_0 = T_2$

$T_x = Number$

$T_y = Number$

$T_2 = Number$

Substitution

$\{T_1 := [[[Number * Number] \to Number] * Number * Number] \to T_0,$

$T_5 := [T_{num1} * T_{num3}] \to T_2$

$T_* := [Number * Number] \to Number,$

$T_{num1} := Number, T_{num3} := Number,$

$T_x := T_{num1}, T_y := T_{num3}\}$

Equation

$T_x = Number$

$T_y = Number$

$T_2 = Number$

Substitution

$\{T_1 := [[[Number * Number] \to Number] * Number * Number] \to T_0,$

$T_5 := [T_{num1} * T_{num3}] \to T_2$

$T_* := [Number * Number] \to Number,$

$T_{num1} := Number, T_{num3} := Number,$

$T_x := T_{num1}, T_y := T_{num3}, T_0 := T_2\}$

Equation

$T_y = Number$

$T_2 = Number$

Substitution

$\{T_1 := [[[Number * Number] \to Number] * Number * Number] \to T_0,$

$T_5 := [Number * T_{num3}] \to T_2$

$T_* := [Number * Number] \to Number,$

$T_{num1} := Number, T_{num3} := Number,$

$T_x := Number, T_y := T_{num3}, T_0 := T_2\}$

Equation

T2 = Number

Substitution

{T1 := [[[Number * Number] -> Number] * Number * Number] -> T0

Tf := [Number * Number] -> T2,

T* := [Number * Number] -> Number,

Tnum1 := Number, Tnum3 := Number,

Tx := Number, Ty := Number, T0 := T2}

Equation

Substitution

{T1 := [[[Number * Number] -> Number] * Number * Number] -> Number,

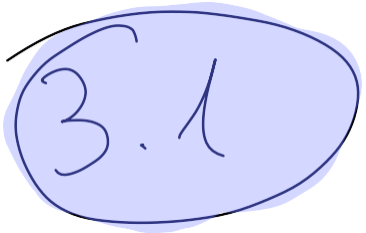
Tf := [Number * Number] -> Number,

T* := [Number * Number] -> Number,

Tnum1 := Number, Tnum3 := Number,

Tx := Number, Ty := Number, T2 := number, T0 := Number}

```
((lambda ([f1 : [Number * Number] -> Number] [x1 : Number] [y1 : Number])
   (f1 x1 y1) * 1 3)
```



Typing rule define:

For every: type environment _Tenv,
variable declaration var
expressions exp and
type expressions typ:
If _Tenv o {var : typ } |- exp : typ
Then _Tenv |- (define var exp) : void