1 Inertial Reference Frame

In an inertial frame

$$\ddot{\mathbf{x}} = 0 \text{ when } \mathbf{F} = 0$$

If S is an inertial frame the rotated frame S' is also inertial. In other words, $d^2\mathbf{x}/dt^2 = 0$, after rotation, $\mathbf{x}' = R\mathbf{x}$.

Let $\mathbf{x} = (x_1, x_2, x_3)^T$, $\mathbf{x}' = (x_1', x_2', x_3')^T$. A rotation angles about x, y, z axes are α, β, γ, the matrix representation is

$$\begin{aligned}
&R = R_z(\gamma)R_y(\beta)R_x(\alpha) \\
&= \begin{pmatrix} \cos\gamma & -\sin\gamma & 0 \\ \sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha \\ 0 & \sin\alpha & \cos\alpha \end{pmatrix} \\
&\equiv \begin{pmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{pmatrix}
\end{aligned}$$

Then,

$$\begin{pmatrix} x_1' \\ x_2' \\ x_3' \end{pmatrix} = \begin{pmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} R_{11}x_1 + R_{12}x_2 + R_{13}x_3 \\ R_{21}x_1 + R_{22}x_2 + R_{23}x_3 \\ R_{31}x_1 + R_{32}x_2 + R_{33}x_3 \end{pmatrix}$$

Thus, $\ddot{x}_1' = R_{11}\ddot{x}_1 + R_{12}\ddot{x}_2 + R_{13}\ddot{x}_3 = 0$, which indicates $\ddot{\mathbf{x}}' = 0$.