(2)L 守恒, 有

$$Rv\cos\alpha=(R+h)v_0=rv_\theta=r\frac{\mathrm{d}\theta}{\mathrm{d}t}\longrightarrow\begin{cases}\dfrac{\mathrm{d}\theta}{\mathrm{d}t}=\dfrac{(R+h)v_0}{r}\\ v=\dfrac{R+h}{R\cos\alpha}v_0\end{cases}$$

E 守恒, 有

$$\frac{1}{2}mv_0^2-\frac{GmM}{R+h}=\frac{1}{2}m(v(r))^2-\frac{GmM}{r}=\frac{1}{2}mv^2-\frac{GmM}{R}$$

解得 $\boxed{v=\dfrac{\sqrt{2GhM+hRv_0^2+R^2v_0^2}}{\sqrt{hR+R^2}}}$ 与 $\boxed{\cos\alpha=\dfrac{v_0(h+R)^2}{\sqrt{R(h+R)}\sqrt{2GhM+Rv_0^2(h+R)}}}$, 同时由于 $v(r)=v_\theta^2+v_r^2=\left(\dfrac{\mathrm{d}\theta}{\mathrm{d}t}\right)^2+\left(\dfrac{\mathrm{d}r}{\mathrm{d}t}\right)^2$

$$\frac{1}{2}mv_0^2-\frac{GmM}{R+h}=\frac{1}{2}m\left(\left(\frac{\mathrm{d}\theta}{\mathrm{d}t}\right)^2+\left(\frac{\mathrm{d}r}{\mathrm{d}t}\right)^2\right)-\frac{GmM}{r}$$

解得

$$\frac{\mathrm{d}r}{\mathrm{d}t}=-\sqrt{-\frac{2GM}{h+R}-v_0^2+\frac{2GM}{r}-\frac{v_0^2(h+R)^2}{r^2}}$$

$$\mathrm{d}r=\mathrm{d}t\left(-\sqrt{-\frac{2GM}{h+R}-v_0^2+\frac{2GM}{r}-\frac{v_0^2(h+R)^2}{r^2}}\right)$$

$$r\mathrm{d}r=\mathrm{d}t\left(-\sqrt{-\frac{2GMr^2}{h+R}-r^2v_0^2+2GMr-v_0^2(h+R)^2}\right)$$

$$\frac{r\mathrm{d}r}{-\sqrt{\left(-\dfrac{2GM}{h+R}-v_0^2\right)r^2+2GMr-v_0^2(h+R)^2}}=\mathrm{d}t$$

同时积分, 初状态 $t=0,r=R+h$, 末状态 $t=t,r=R$。

$$\int_0^t\mathrm{d}t=\int_{R+h}^{R}\frac{r\mathrm{d}r}{-\sqrt{\left(-\dfrac{2GM}{h+R}-v_0^2\right)r^2+2GMr-v_0^2(h+R)^2}}$$

$$\int_0^t\mathrm{d}t=\int_{R}^{R+h}\frac{r\mathrm{d}r}{\sqrt{\left(-\dfrac{2GM}{h+R}-v_0^2\right)r^2+2GMr-v_0^2(h+R)^2}}$$

利用

$$\int\frac{x\mathrm{d}x}{\sqrt{a+bx+cx^2}}=\frac{\sqrt{a+bx+cx^2}}{c}-\frac{b}{2(-c)^{3/2}}\arcsin\frac{2cx+b}{\sqrt{\Delta}}+C\;(c<0,C\text{ 为积分常数})$$

代入数据 $c=-\dfrac{2GM}{h+R}-v_0^2<0,b=2GM,a=-v_0^2(h+R)^2$

$$\boxed{\begin{aligned}t=&\frac{R+h}{2GM-v_0^2(R+h)}\sqrt{\frac{2GMRh}{R+h}-v_0^2(2R+h)h}\\&+GM\left(\frac{R+h}{2GM-v_0^2(R+h)}\right)^{3/2}\left(\frac{\pi}{2}+\arcsin\frac{v_0^2R(R+h)-GM(R-h)}{GM(R+h)-v_0^2(R+h)^2}\right)\end{aligned}}$$