

# NOTE ON HYPERGRAPHS

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[?]

**Notation 0.1.** We employ the following notations.

- For each category  $\mathbf{C}$ , we write  $\mathbf{C}^\cong$  for the core groupoid of  $\mathbf{C}$ .
- For each groupoid  $\mathbf{G}$ , we write  $\pi_0(\mathbf{G})$  for the set of connected components of  $\mathbf{G}$ .
- For each category  $\mathbf{C}$  and an object  $c \in \mathbf{C}$ , we write  $\mathbf{C}_{c/}$  and  $\mathbf{C}/_c$  for the under category and the over category.
- For each category  $\mathbf{C}$  and an object  $c \in \mathbf{C}$ , the domain part defines a functor  $\mathbf{C}/_c \xrightarrow{\Sigma_c} \mathbf{C}$ , and we write  $c^*$  and  $\Pi_c$  for the right adjoints of  $\Sigma_c$  and  $c^*$  respectively, if exists.  
The dual of  $\Sigma_c$ ,  $c^*$ , and  $\Pi_c$  are denoted by  $\mathcal{Z}_c: \mathbf{C}_{c/} \rightarrow \mathbf{C}$ ,  $c_*: \mathbf{C} \rightarrow \mathbf{C}_{c/}$ , and  $\Pi_c: \mathbf{C} \rightarrow \mathbf{C}_{c/}$ .
- We write  $\mathbf{Set}$  for the category of sets.
- For each nonnegative integer  $n$ , we write  $\bar{n}$  for the set  $\{1, \dots, n\}$ .
- We write  $\mathbf{FinSet}$  for the category defined as follows.
  - $\text{Obj}(\mathbf{FinSet}) = \mathbb{N}$ .
  - $\mathbf{FinSet}(n, m) = \mathbf{Set}(\bar{n}, \bar{m})$ .
  - $n \mapsto \bar{n}$  extends to a fully faithful functor  $\mathbf{FinSet} \hookrightarrow \mathbf{Set}$ .
- For each category  $\mathbf{C}$  with finite coproducts, we write  $- + -$  for the binary coproduct and  $\emptyset$  for the initial object.  
We write  $\nabla_c: c + c \rightarrow c$  for the codiagonal.

**Definition 0.2.** Suppose we are given a set  $\Lambda \in \mathbf{Set}$ . We define a category  $\mathbf{FinSet}_\Lambda$  by the following pullback.

$$\begin{array}{ccc} \mathbf{FinSet}_\Lambda & \longrightarrow & \mathbf{Set}_{/\Lambda} \\ \downarrow & \lrcorner & \downarrow \Sigma_\Lambda \\ \mathbf{FinSet} & \hookrightarrow & \mathbf{Set} \end{array}$$

An element of  $\mathbf{FinSet}_\Lambda$  is called a  $\lambda$ -labelled finite set.

**Proposition 0.3.** For each  $\Lambda \in \mathbf{Set}$ ,  $\mathbf{FinSet}_\Lambda$  has finite colimits.

*Proof.* Both  $\mathbf{FinSet} \hookrightarrow \mathbf{Set}$  and  $\mathbf{Set}_{/\Lambda} \xrightarrow{\Sigma_\Lambda} \mathbf{Set}$  create finite colimits.  $\square$

**Definition 0.4.** Let  $\mathbf{C}$  be a category with finite colimits. We write  $\mathbf{Cosp}[\mathbf{C}]$  for the category defined as follows.

- $\text{Obj}(\mathbf{Cosp}[\mathbf{C}]) = \text{Obj } \mathbf{C}$ .
- For each  $c, c' \in \mathbf{C}$ , the homset is defined as

$$\mathbf{Cosp}[\mathbf{C}](c, c') = \pi_0 \left( (\mathbf{C}_{c+c'/})^\cong \right).$$

- The identity on  $c$  is represented by the codiagonal  $\nabla: c + c \rightarrow c$ .
- The composition is obtained by applying  $\pi_0(-)^\cong$  to the following functor.

$$\mathbf{C}_{c+c'/} \times \mathbf{C}_{c'+c''/} \xrightarrow{+} \mathbf{C}_{c+c'+c'+c''/} \xrightarrow{(c + \nabla_{c'} + c'')^*} \mathbf{C}_{c+c'+c''/} \xrightarrow{\mathcal{Z}_{\text{inc}_{1,3}}} \mathbf{C}_{c+c''/}$$

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