NOTE ON POLYNOMIALS

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Notation 0.1. We employ the following notations.

- Sets are regarded as discrete categories, and categories are regarded as locally discrete 2-categories.
- **Set** is the large category of small sets.
- We write $\mathfrak{CAT}$ for the huge 2-category of large categories.
- For each set S and a functor $X\colon S \longrightarrow \mathbf{Set}$, we write X_s for the image of $s \in S$ under X. Moreover, we write $(X_s)_{s:S}$ for X.
- For each set S and functors $X, Y\colon S \longrightarrow \mathbf{Set}$, a natural transformation $f\colon X \Longrightarrow Y$ is denoted as a family of functions $(f_s)_{s:S}$.

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1. POLYNOMIALS

Definition 1.1. We define a category **Poly** as the Grothendieck construction of the following 2-functor.

$$\mathbf{Set}^{\mathrm{op}} \xrightarrow{[-,\mathbf{Set}]^{\mathrm{op}}} \mathfrak{CAT}$$

A ***polynomial*** is an object in **Poly**. We write

$$(-)_\circ\colon \mathbf{Poly} \longrightarrow \mathbf{Set}$$

for the fibration corresponding to the 2-functor. For each polynomial A, we write $A = \sum_{a:A_\circ} y^{A_a}$. ■

Definition 1.2. Define a functor $- \triangleleft -\colon \mathbf{Poly} \times \mathbf{Poly} \longrightarrow \mathbf{Poly}$ as follows.

- Let A, B be polynomials.
$$(A \triangleleft B)_\circ = \sum_{a:A_\circ} [A_a, B_\circ]$$
$$(A \triangleleft B)_{a,t} = \sum_{u:A_a} B_{t(u)}$$
for any $a : A_\circ$ and $t\colon A_a \longrightarrow B_\circ$.
- Let $F\colon A \longrightarrow X$, $G\colon B \longrightarrow Y$ be morphisms in **Poly**.
$$(F \triangleleft G)_\circ\colon \sum_{a:A_\circ} [A_a, B_\circ] \longrightarrow \sum_{x:X_\circ} [X_x, Y_\circ] : (a, t) \mapsto (F_\circ(a), F_a \mathbin{⨾} t \mathbin{⨾} G_\circ)$$
$$(F \triangleleft G)_{a,t}\colon \sum_{v:X_{F_\circ(a)}} Y_{G_\circ(t(F_a(v)))} \longrightarrow \sum_{u:A_a} B_{t(u)} : (v, r) \mapsto (F_a(v), G_{t(F_a(v))}(r))$$

■

Definition 1.3. ■

References

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