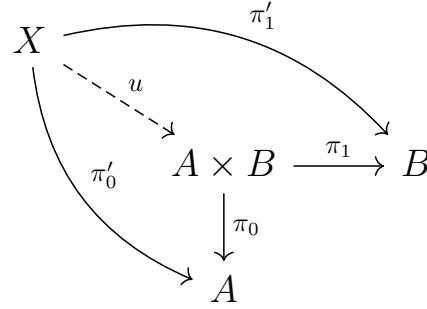


# 1 Product

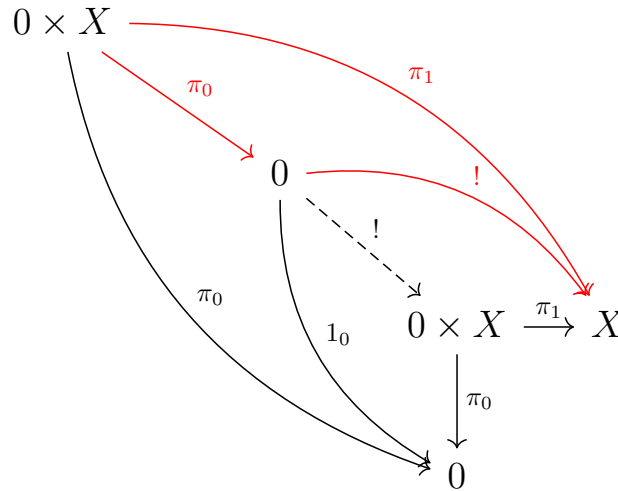
**Definition 1.1** (Product). *Let  $\mathcal{C}$  a category and  $A, B \in \mathcal{C}$ ,  $(A \times B, \pi_0, \pi_1)$  forms a product of  $A$  and  $B$  where  $A \times B \in \mathcal{C}$ ,  $\pi_0 : A \times B \rightarrow A$  and  $\pi_1 : A \times B \rightarrow B$ , if for any  $X \in \mathcal{C}$  with  $\pi'_0 : X \rightarrow A$  and  $\pi'_1 : X \rightarrow B$ , there is a unique arrow  $u : X \rightarrow A \times B$  such that the following diagram commutes:*



*Furthermore, a product of  $A$  and  $B$  is a limit of diagram:*

$$(A \quad B)$$

One may trying to show that  $0 \times X \simeq 0$  by:



However, the red triangle needs not to commutes, that is, the arrow  $\pi_0$  from  $(0 \times X, \pi_0, \pi_1)$  to  $(0, 1_0, !)$  may not exist.

**Definition 1.2** (Product of Arrow). *Suppose  $(A \times B, \pi_0, \pi_1)$  and  $(C \times D, \pi_2, \pi_3)$  are two product, and  $f : A \rightarrow C$ ,  $g : B \rightarrow D$ . The product of arrow  $f \times g$  is a*

unique arrow from  $A \times B$  to  $C \times D$  such that the following diagram commutes:

$$\begin{array}{ccccc}
 A & \xleftarrow{\pi_0} & A \times B & \xrightarrow{\pi_1} & B \\
 \downarrow f & & \downarrow f \times g & & \downarrow g \\
 C & \xleftarrow{\pi_2} & C \times D & \xrightarrow{\pi_3} & D
 \end{array}$$