This chapter established the set theory of hoshino version.

Definition -2.1 (Minimum). *Let S a set and $n \in S$, n is minimum if for any $m \in S$, $n \leq m$.*

Theorem -2.1. *Let S be a non-empty set which consists of natural number, show that there is $n \in S$ such that n is minimum.*

Proof. Suppose there no such $n \in S$ where n is minimum, then for any $n \in S$, n is not minimum, then for any $n \in S$, there is $m \in S$ such that $n > m$. Therefore, for any $n \in S$, we can obtain a smaller element m.

Let $n \in S$, then we can get a smaller element m, and do the same thing on m. We will finally reach $0 \in S$, nut here is no any natural number that smaller than 0, but we can still obtain a m such that $m < 0$, this is unacceptible.

So S has a smallest element. □

Theorem -2.2. *Let S a set and S_i a collection of set, and $C_i = S \setminus S_i$ a collection of complements. Then*

$$\bigcup C_i = \bigcup (S \setminus S_i) = S \setminus (\bigcap S_i)$$

Proof. ($\supseteq$) For any element $x \in \bigcup C_i$, we know x must belongs to some C_α, therefore $x \notin S \setminus C_\alpha = S_\alpha$, so $x \notin \bigcap S_i$, therefore $x \in S \setminus (\bigcap S_i)$.

($\subseteq$) For any element $x \in S \setminus (\bigcap S_i)$, we know $x \in S$ but $s \notin \bigcap S_i$, therefore, x must not belongs to some S_α, therefore $x \in S \setminus S_\alpha = C_\alpha$, so $x \in \bigcup C_i$. □

Theorem -2.3. *Let S a set and S_i a collection of set, and $C_i = S \setminus S_i$ a collection of complements. Then*

$$\bigcap C_i = \bigcap (S \setminus S_i) = S \setminus (\bigcup S_i)$$

Proof. ($\supseteq$) For any element $x \in \bigcap C_i$, we know x belongs to all C_i, for any $S_i = S \setminus C_i$, $x \notin S_i$, so $x \notin \bigcup S_i$, therefore $s \in S \setminus (\bigcup S_i)$. ($\subseteq$) For any element $x \in S \setminus (\bigcup S_i)$, we know $x \in S$ but $x \notin \bigcup S_i$, so none of them contains x, therefore every $S \setminus S_i$ contains x, that is, $x \in \bigcap (S \setminus S_i) = \bigcap C_i$. □

Theorem -2.4. *Suppose $f : A \to B$, then for any $V, W \subseteq A$ we have:*

- $f(V \cap W) \subseteq f(V) \cap f(W)$
- $f(V \cup W) = f(V) \cup f(W)$

Proof.

- For any $x \in f(V \cap W)$, we there is $y \in V \cap W$ such that $f(y) = x$, then $x \in f(V)$ since $y \in V$ and $x \in f(W)$ since $y \in W$.

- ($\supseteq$) For any $x \in f(V) \cup f(W)$, we know $x \in f(V)$ or $x \in f(W)$, we may suppose $x \in f(V)$. Then there is $y \in V$ such that $f(y) = x$, therefore $x \in f(V \cup W)$ since $y \in V \cup W$.

  ($\subseteq$) For any $x \in f(V) \cup f(W)$, we may suppose $x \in f(V)$. Then there is $a \in V$ such that $f(a) = x$, then $a \in V \cup W$ and $x \in f(V \cup W)$.

$\square$