

This chapter established the set theory of hoshino version.

Definition -2.1 (Minimum). *Let S a set and $n \in S$, n is minimum if for any $m \in S$, $n \leq m$.*

Theorem -2.1. *Let S be a non-empty set which consists of natural number, show that there is $n \in S$ such that n is minimum.*

Proof. Suppose there no such $n \in S$ where n is minimum, then for any $n \in S$, n is not minimum, then for any $n \in S$, there is $m \in S$ such that $n > m$. Therefore, for any $n \in S$, we can obtain a smaller element m .

Let $n \in S$, then we can get a smaller element m , and do the same thing on m . We will finally reach $0 \in S$, but here is no any natural number that smaller than 0, but we can still obtain a m such that $m < 0$, this is unacceptable.

So S has a smallest element. \square

Theorem -2.2. *Let S a set and S_i a collection of set, and $C_i = S \setminus S_i$ a collection of complements. Then*

$$\bigcup C_i = \bigcup (S \setminus S_i) = S \setminus \left(\bigcap S_i \right)$$

Proof. (\supseteq) For any element $x \in \bigcup C_i$, we know x must belongs to some C_α , therefore $x \notin S \setminus C_\alpha = S_\alpha$, so $x \notin \bigcap S_i$, therefore $x \in S \setminus \left(\bigcap S_i \right)$.

(\subseteq) For any element $x \in S \setminus \left(\bigcap S_i \right)$, we know $x \in S$ but $x \notin \bigcap S_i$, therefore, x must not belongs to some S_α , therefore $x \in S \setminus S_\alpha = C_\alpha$, so $x \in \bigcup C_i$. \square

Theorem -2.3. *Let S a set and S_i a collection of set, and $C_i = S \setminus S_i$ a collection of complements. Then*

$$\bigcap C_i = \bigcap (S \setminus S_i) = S \setminus \left(\bigcup S_i \right)$$

Proof. (\supseteq) For any element $x \in \bigcap C_i$, we know x belongs to all C_i , for any $S_i = S \setminus C_i$, $x \notin S_i$, so $x \notin \bigcup S_i$, therefore $x \in S \setminus \left(\bigcup S_i \right)$. (\subseteq) For any element $x \in S \setminus \left(\bigcup S_i \right)$, we know $x \in S$ but $x \notin \bigcup S_i$, so none of them contains x , therefore every $S \setminus S_i$ contains x , that is, $x \in \bigcap (S \setminus S_i) = \bigcap C_i$. \square