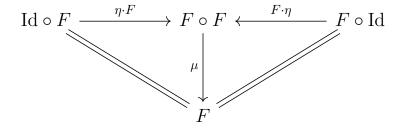
## 1 Monad

A monad is an endofunctor F equipped with:

- identity  $\eta: \mathrm{Id} \to F$ , which unsually called "pure" or "return".
- multiplication  $\mu: F \circ F \to F$ , which usually called "join". and the identity law:



the associative law:

$$(F \circ F) \circ F = F \circ (F \circ F)$$

$$\downarrow^{F \cdot \mu}$$

$$F \circ F = \mu \qquad F \circ F$$

We can easily see that it is a monoid on the category of endofunctor. If you confuse with "what the element of an endofunctor is", try global element, just like  $\eta$ .

Suppose  $\alpha, \beta : \operatorname{Id} \to F$  are two "elements" of F, then the multiplication of them will be  $\mu \circ (\alpha \cdot \beta) : \operatorname{Id} \to F$ , the vertical composition acts the role of function application.

## 1.1 Free Monad

Just like a list is a free monoid (on set), we also have free monad (free monoid on endofunctor):

List (Free Monoid)	Free Monad
I: List  A	$pure : Id \to FreeMonad F$
$(::):A\otimes(\mathrm{List}\ A)\to\mathrm{List}\ A$	free: $F \circ (FreeMonad F) \rightarrow FreeMonad F$