

1 Compactness

Exercise 1.1. *Let $\{V_\alpha\}$ be an open cover of a topological space \mathcal{X} . Show that $W \subseteq \mathcal{X}$ is open iff $W \cap V_\alpha$ is open for any $V_\alpha \in \{V_\alpha\}$.*

Proof. (\Rightarrow) is trivial.

(\Leftarrow) For any point $x \in W$, we have $x \in V_x$ since $\{V_\alpha\}$ is open cover. Consider $\bigcup_{x \in W} (W \cap V_x)$, it is a union of open sets, and it is a subset of W , and it contains all points of W , so W is open. \square