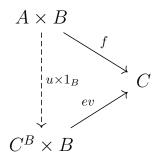
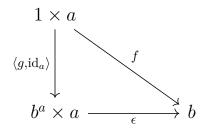
1 Exponential

Definition 1.1. Let C a category. For any $B, C \in C$, (C^B, ev) forms an exponential where $C^B \in C$ and $ev : C^B \times B \to C$, if for any object $A \in C$ and $f : A \times B \to C$, there is a unique $u : A \to C^B$ such that $f = ev \circ (u \times 1_B)$. In other words, the follow diagram commutes.

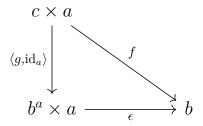


Suppose we are in **Set**, we know for any function $f:A\to B$, there is an element in some set that represents this function. In programming perspect, we know it is an element of type $A\to B$. What if we generialize it to other category? In order to represent an element of some set/type/object, we can use global element, that is, a morphism from terminal object. We denote the object that represents the morphisms from a to b by b^a , then a morphism $f:a\to b$ should be represented by $g:1\to b^a$. Furthermore, we can apply the function to some argument, that is, $\epsilon:b^a\times a\to b$, we should require such g respects this behaviour, and we get this diagram commutes:



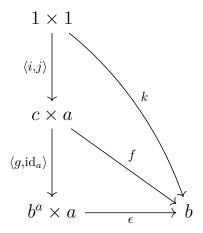
It says, if we take an element g that represents f, then we can apply g to for any "element" of a, it should behave like we apply f to that "element" of a.

We may loose the requirement that 1 exists, we can use any object c:



The morphism g corresponds to the currying in programming.

Then we can choose some elements of a and c:



The diagram says: For a function object $g \circ i$, apply it to element j by ϵ , should equivalent to we apply f to them, and the result is the element k:

$$\begin{array}{ll} \epsilon \circ \langle g \circ i, j \rangle & \text{(applying through function object)} \\ = f \circ \langle i, j \rangle & \text{(applying directly)} \\ = k & \text{(the result)} \end{array}$$

Furthermore, we can observe that the morphism f and the "curring" morphism g is one-one corresponding. It notices us that there is a underlying adjunction:

$$\mathcal{C}(c \times a, b) \cong \mathcal{C}(c, b^a)$$

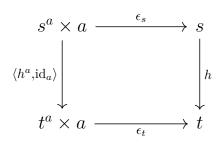
It would be more clear if we rewrite them:

$$L_a c = c \times a$$

$$R_a b = b^a$$

$$C(L_a c, b) \cong C(c, R_a b)$$

But we need to show that they are functors. L_a is a functor cause $-\times -$ is a functor (see chapter product). For any morphism $h: s \to t$, we need to provide a morphism $h^a: s^a \to t^a$, one reasonable choice is:



It is easy to check the functoriality by the uniqueness of h^a .