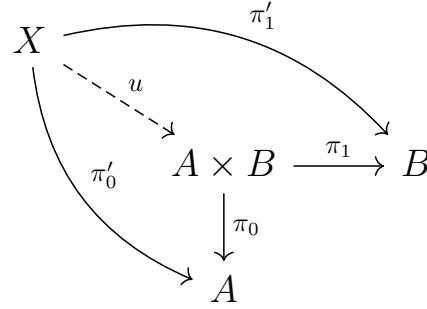


1 Product

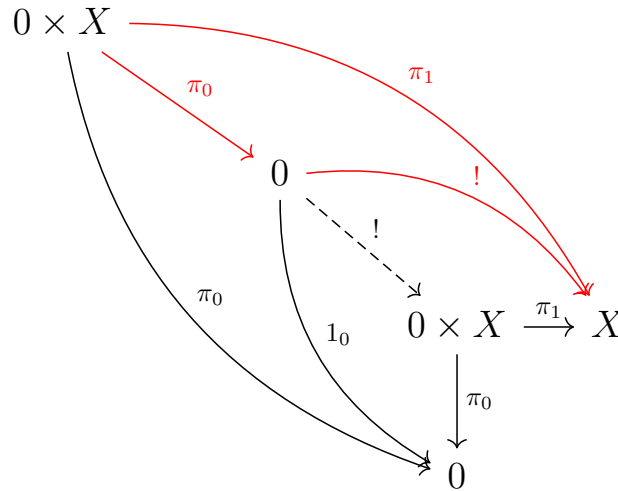
Definition 1.1 (Product). Let \mathcal{C} a category and $A, B \in \mathcal{C}$, $(A \times B, \pi_0, \pi_1)$ forms a product of A and B where $A \times B \in \mathcal{C}$, $\pi_0 : A \times B \rightarrow A$ and $\pi_1 : A \times B \rightarrow B$, if for any $X \in \mathcal{C}$ with $\pi'_0 : X \rightarrow A$ and $\pi'_1 : X \rightarrow B$, there is a unique arrow $u : X \rightarrow A \times B$ such that the following diagram commutes:



Furthermore, a product of A and B is a limit of diagram:

$$(A \quad B)$$

One may trying to show that $0 \times X \simeq 0$ by:



However, the red triangle needs not to commutes, that is, the arrow π_0 from $(0 \times X, \pi_0, \pi_1)$ to $(0, 1_0, !)$ may not exist.

Definition 1.2 (Product of Arrow). Suppose $(A \times B, \pi_0, \pi_1)$ and $(C \times D, \pi_2, \pi_3)$ are two product, and $f : A \rightarrow C$, $g : B \rightarrow D$. The product of arrow $f \times g$ is a

unique arrow from $A \times B$ to $C \times D$ such that the following diagram commutes:

$$\begin{array}{ccccc}
 A & \xleftarrow{\pi_0} & A \times B & \xrightarrow{\pi_1} & B \\
 \downarrow f & & \downarrow f \times g & & \downarrow g \\
 C & \xleftarrow{\pi_2} & C \times D & \xrightarrow{\pi_3} & D
 \end{array}$$

We may consider \times a functor from $\mathcal{C} \times \mathcal{C} \rightarrow \mathcal{C}$ if \mathcal{C} has any product. Then the \times acts on any morphism $\langle f, g \rangle : (a, b) \rightarrow (c, d)$ is the product of arrows we showed above. The functoriality can be proved by the uniqueness of the factor morphism.