Measures of Dispersion

Md. Ismail Hossain Riday

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- It gives only partial information of the data set.

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- Student B: 100, 100, 0, 0

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On average both students are equal in getting final exam score

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On average both students are equal in getting final exam score

So it is also important to describe "How much the observation vary from one to another

Dispersion/ Variation

Measures of Dispersion

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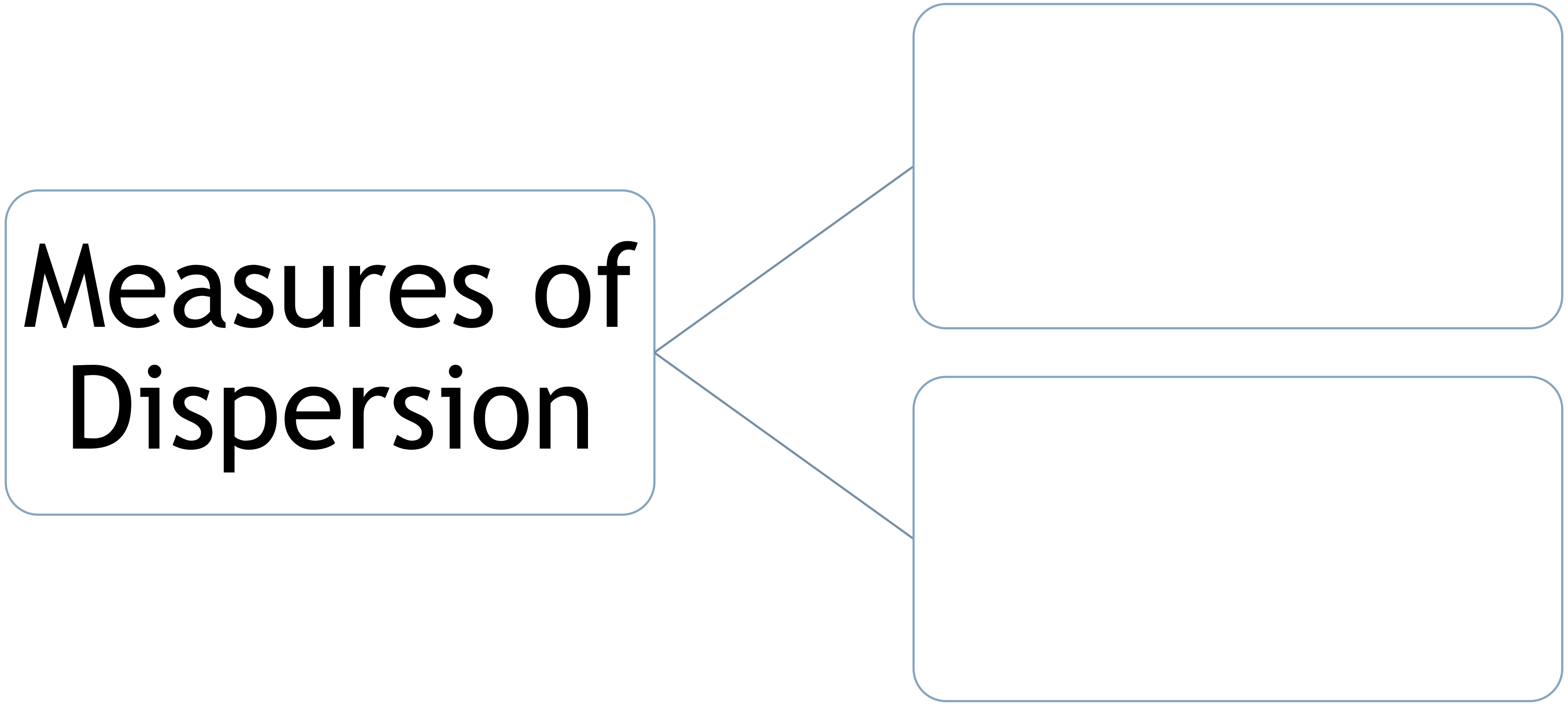
- Descriptive measures that indicate the amount of variation in a data set are called measures of dispersion.

Why do we need?

- To determine the reliability of an average

- To compare the variability of two or more data sets.

Types of Measures of Dispersion



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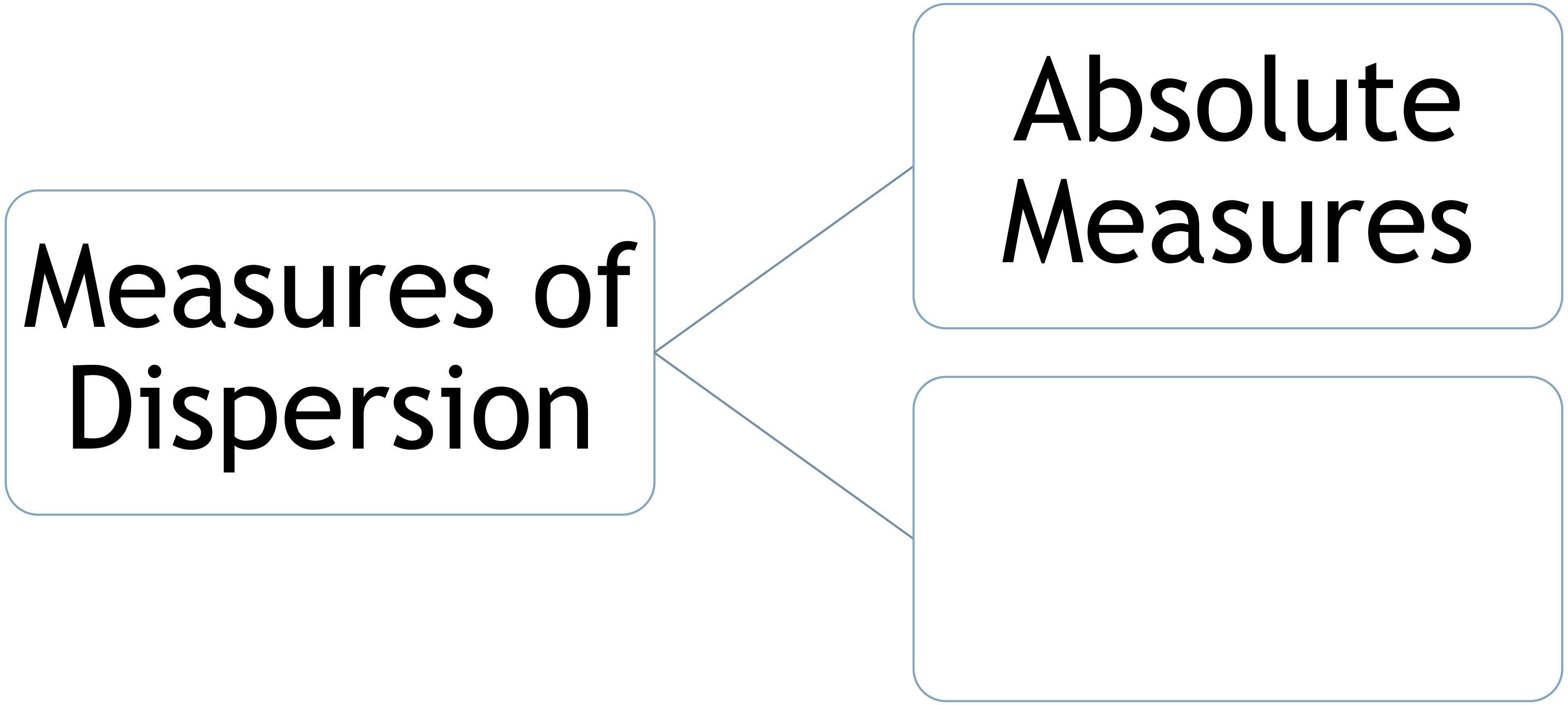

Types of Measures of Dispersion

- Measures of Dispersion
  - Absolute Measures
  - Relative Measures

Absolute measures

- Only measure the inherent variation of a data set

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Range

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Mean Deviation

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Variance

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$$7.5, 4.5, 10.1, 9.6, and\ 5.5$$

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$$(Range = ????)$$

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$$Range = 10.1 - 4.5$$

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- For example, weights of 5 newly born babies (in pounds)

$$7.5, 4.5, 10.1, 9.6, and\ 5.5$$

$$Range = 10.1 - 4.5 = 5.6\ pounds$$

Mean Deviation

- Let $x_1, x_2, \ldots, x_n$ be n observations, where $\bar{x}$ is average/ or mean

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- $MD = \frac{\sum f_i \, |x_i - \bar{x}|}{\sum f_i}$ [for grouped data]

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x_i	$\bar{x}$
1	$\frac{\sum x_i}{n} = 3$
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4	
5	

Mean Deviation

- For example, $x = 1,2,4,5$. Calculate the mean deviation.

x_i	$\bar{x}$	$\lvert x_i - \bar{x} \rvert$
1	$\frac{\sum x_i}{n} = 3$	
2		
4		
5		

Mean Deviation

- For example, $x = 1,2,4,5$. Calculate the mean deviation.

x_i	$\bar{x}$	$\lvert x_i - \bar{x} \rvert$
1	$\frac{\sum x_i}{n} = 3$	2
2		1
4		1
5		2

$$MD = \frac{\sum \lvert x_i - \bar{x} \rvert}{n} = \frac{6}{4} = 1.5$$

Variance

- Let $x_1, x_2, \dots, x_n$ be n observations

Variance

- Let $x_1, x_2, \ldots, x_n$ be n observations
- $\sigma^2 = \frac{\sum(x_i - \mu)^2}{N}$ [Population variance]
- $s^2 = \frac{\sum(x_i - \bar{x})^2}{n-1}$ [Sample variance]

Standard Deviation

- Square root of variance

Standard Deviation

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- $\sigma = \sqrt{\frac{\sum(x_i-\mu)^2}{N}}$ [Population standard deviation]
- $s = \sqrt{\frac{\sum(x_i-\bar{x})^2}{n-1}}$[Sample variance]

Standard Deviation

- For example: weights of 10 newly born babies (in pounds)

$$7.5, 4.5, 10.1, 9.6, 5.5, 6.6, 7.8, 5.9, 6.0, 5.5$$

- a) Calculate the standard deviation

- b) Interpretation

Standard Deviation

x_i	$\bar{x}$	$(x_i - \bar{x})^2$

Standard Deviation

x_i	$\bar{x}$	$(x_i - \bar{x})^2$
7.5		0.36
4.5		0.57
10.1		10.24
9.6		7.29
5.5	$\frac{\sum x_i}{n} = 6.9$	1.96
6.6		0.09
7.8		0.81
5.9		1
6.0		0.81
5.5		1.96

Standard Deviation

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- $$7.5, 4.5, 10.1, 9.6, 5.5, 6.6, 7.8, 5.9, 6.0, 5.5$$
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Standard Deviation

- For example: weights of 10 newly born babies (in pounds)
- $$7.5, 4.5, 10.1, 9.6, 5.5, 6.6, 7.8, 5.9, 6.0, 5.5$$
- a) Calculate the standard deviation
- $s = \sqrt{\frac{\sum(x_i - \bar{x})^2}{n-1}} = \sqrt{\frac{30.28}{10-1}} = \sqrt{3.36} = 1.83\ pounds$
- b) Interpretation

The average variation of the weights of the newly born babies from the mean weights is 1.83 pounds.

Relative measures

- To compare the variation of two or more data sets having different or same units

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Coefficient of MD

Coefficient of variance

Coefficient of QD

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Coefficient of variance (CV)

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- Between standard deviation (SD) and mean

Coefficient of variance (CV)

- It is a ratio
- Between standard deviation (SD) and mean
- Expressed as percent

$$\text{General formula of CV} = \frac{SD}{mean} \times 100$$

Coefficient of variance (CV)

- $CV = \frac{\sigma}{\mu} \times 100\,;[For\ population]$
- $CV = \frac{s}{\bar{x}} \times 100\,;[For\ sample]$

Higher CV =
High variation

Lower CV =
Low Variation

More consistent

More uniform

More homogeneous

More stable

Coefficient of variance (CV)

- Section A: Mean = 30 and SD = 4

$$\boxed{CV_A = \frac{4}{30} \times 100 = 13.3\%}$$

- Section B: Mean = 25 and SD = 6

$$CV_B = \frac{6}{25} \times 100 = 24\%$$

- Which section is more consistent in getting final exam mark?

Since, the CV of A is lower than the CV of B.
Thus, section A is more consistent comparatively section B

Thank You