In previous chapter, we discussed how a raw data set can be organized and summarized by tables and graphs.

Another method of summarizing data set precisely is to compute number (a single number).

Number that can be describe data sets are called “Descriptive measures”

There are four types of descriptive measures:

1. Measures of Central Tendency
2. Measures of Location
3. Measures of Dispersion
4. Shape of the distribution

Today we will just focus on “Measures of Central Tendency”. Now what is “Measures of central tendency”???? Three points must keep in mind…

- It is a summary measure
- Attempts to describe the whole data set with a single value
- Represents the center of the distribution/ center of the data set.

At a glance, “A measure of central tendency is a summary measure/statistic that attempts to describe the whole data set with a single value and that value represents the center of the distribution”.

There are three types of measures of central tendency:

1. Mean
2. Median
3. Mode

We will just focus on mean in this lecture.

Mean actually a simple average type. There is no specific definition for mean. But there are three there are three types of mean. What are they?

1. Arithmetic mean
2. Geometric mean
3. Harmonic mean

All of them are actually called "Average", but used for different situation.

In most of the case, we can use "Arithmetic mean". What is this?

It is simply sum of the observations divided by the total number of observations

For example, Consider the values 5, 3, 9, 2, 7, 5, 8.
Now the arithmetic mean for these seven values can be written as:

There are two types of formula of AM; one is for ungrouped data/ raw data (which are not organized), another is for grouped data (which is actually organized).

For ungrouped data: Let x_1, $x_2, \dots,$ x_n are some observations.

Arithmetic mean for ungrouped data can be written as,

$$\bar{X}_{AM} = \frac{x_1 + x_2 + \cdots + x_n}{n} = \frac{\sum x_i}{n}$$

For example: Consider the values **5, 3, 9, 2, 7, 5, 8**

$$\boldsymbol{Mean = \frac{(5+3+9+2+7+5+8)}{7} = 5.57\ Unit}$$

For grouped data: One additional term "f_i", frequency may add. Arithmetic mean for grouped data can be written as,

$$\bar{X}_{AM} = \frac{\sum(f_i \times x_i)}{\sum f_i}$$

Here,

$f_i = Frequency$;

$x_i = Class\ mid - value$

Example:

Class	f_i	Mid value	$f_i \times x_i$
5-9	4	7	28
9-13	3	11	33
13-17	3	15	45
17-21	3	19	57

Calculate mean or arithmetic mean or average from this table…

$$\bar{X}_{AM} = \frac{\sum(\boldsymbol{f_i \times x_i})}{\sum \boldsymbol{f_i}} = \frac{\mathbf{163}}{\mathbf{13}} = \mathbf{12.53}$$

This type of math/calculation is so easy. Right!!!! Now, if your get this,

Class	f_i	Mid value
5-9	4	7
9-13	3	11
13-17	3	15
17-21	3	19

Find the value of f_1, when the average in 12.53

[SELF PRACTICE MATH]

Geometric Mean: It is useful when dealing with data that exhibits exponential growth, or growth over the year, change over a period of times, or some geometric progression.

Like AM, there are two formulas for geometric mean. One is for ungrouped data; another is for grouped data.

For ungrouped data: n^{th} positive root of their product.

$$\bar{x}_{\mathrm{GM}} = (x_1 \times x_2 \times \ldots \times x_n)^{\frac{1}{n}}$$

For grouped data:

$$\bar{x}_{\mathrm{GM}} = \left(x_1^{f_1} \times x_2^{f_2} \times \ldots \times x_n^{f_n}\right)^{\frac{1}{\sum f_i}}$$

For example: 5, 3, 9, 2, 7, 5, 8

$$\bar{X}_{GM} = (5 \times 3 \times 9 \times 2 \times 7 \times 5 \times 8)^{\frac{1}{7}} = 4.98$$

Find the geometric mean for the following distribution:

Marks	**0-10**	**10-20**	**20-30**	**30-40**	**40-50**
f_i	5	7	15	25	8

If x_i are the percent value (rate) for a given time t,

$$Average\ growth\ rate = \{(1 + r_1) \times (1 + r_2) \times \ldots \times (1 + r_n)\}^{\frac{1}{n}} - 1$$

$$Average\ depreciation\ rate = \{(1 - r_1) \times (1 - r_2) \times \ldots \times (1 - r_n)\}^{\frac{1}{n}} - 1$$

Example: Suppose you have an investment that grew by 10% in the first year, 5% in the second year, 8% in the third year. What was the average growth rate for these three years.

Solution: Since, this is a geometric progression rate. So, geometric mean may use here.

Average growth rate, $\bar{X}_{GM} = \{(1 + 0.10) \times (1 + 0.05) \times (1 + 0.08)\}^{\frac{1}{3}} - 1$

$$\bar{X}_{GM} = 0.0748$$

Comment: Thus, the average growth rate is 7.48%.

Example: Let's consider an asset that depreciated by 15% in the first year, 8% in the second year, and 12% in the third year. What was the average depreciation rate over these three years.

[SELF PRACTICE]

Harmonic Mean: It is necessary to compute the average of some variables such as the average speed, average velocity, and so on.

HM for ungrouped data:

$$\bar{X} = \frac{\mathrm{n}}{\frac{1}{x_1} + \frac{1}{x_2} + \cdots + \frac{1}{x_n}}$$

HM for ungrouped data:

$$\bar{X} = \frac{\sum f_i}{\frac{f_1}{x_1} + \frac{f_2}{x_2} + \cdots + \frac{f_n}{x_n}}$$

For example: 5, 3, 9, 2, 7, 5, 8

$$\bar{X}_{HM} = \frac{7}{\frac{1}{5} + \frac{1}{3} + \cdots + \frac{1}{8}} = (???)$$

A car travels 50 miles at 40 mph, 60miles at 50mph and 40 miles at 60mph.What is the average speed of the trip?

Solution: Here, Distance: 50 miles, 60 miles, 40 miles

Speed: 40 mph, 50 mph, 60 mph

The mean (harmonic mean) can be written as,

$$HM = \frac{\sum f_i}{\sum \left(\frac{f_i}{x_i}\right)} = \frac{50 + 60 + 40}{\frac{50}{40} + \frac{60}{50} + \frac{40}{60}} = 48.08\, mph$$

Combined mean/pooled mean:

If $\bar{X}_1$ and $\bar{X}_2$ are the means with respective numbers of observations n_1 and n_2 of two data sets expressed in the same measuring units, then combined mean is given by,

$$\bar{X}_c = \frac{n_1\bar{X}_1 + n_2\bar{X}_2}{n_1 + n_2}$$

A consulting firm runs in two shifts. A random sample A of 13 employees has mean weekly salary 495$, and another random sample B of 10 employees has mean weekly salary 492$. Compute the arithmetic mean of the combined sample.

Solution:

$$\bar{X}_c = \frac{n_1\bar{X}_1 + n_2\bar{X}_2}{n_1 + n_2} = 493.7\ \$$$

$$AM \geq GM \geq HM$$

$$GM = \sqrt{AM \times HM}$$

Weighted mean: The weighted mean is a special case of the arithmetic mean. It occurs when there are several observations of the same value.

To explain- Suppose the Shumi's Hot Cake offers three different kinds of burger packages small, medium and large for Tk. 100, Tk. 125 and Tk. 150. Of the last 10 burgers sold 3 were small, 4 were medium and 3 were large. To find the mean price of the last 10 burger packages sold we can calculate using the usual formula of the arithmetic mean as follows,

$$\bar{X}_{AM} = \frac{(100 + 100 + 100 + 125 + 125 + 125 + 125 + 150 + 150 + 150)}{10} = 125$$

An easier way to find the mean selling price is to determine the weighted mean.

$$\bar{X}_{WM} = \frac{\sum(w_i \times x_i)}{\sum w_i}$$

$$\bar{X}_{WM} = \frac{(3 \times 100) + (4 \times 150) + (3 \times 150)}{10} = 125$$

Example: Madina Construction Company pays its part time employees hourly basis. For different level of employee, the hourly rate is Tk 50, Tk 75 and Tk 90. There are 260 hourly employees, 140 of which are paid at Tk 50 rate, 100 at Tk 75 and 20 at the Tk 90 rate What is the mean hourly rate paid to the employees?