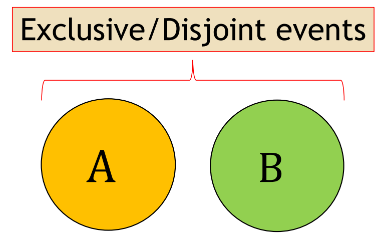
In previous class, we have learned about some basic terms and terminology about probability like, sample space, events and so on. In this lecture, we try to learn about some "probability laws" and try to solve some mathematical problems. First, probability laws. Two basic rules/laws of probability theory:

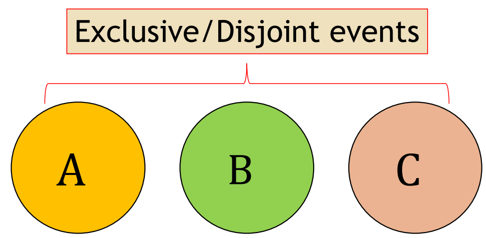
1. Additive rule: Always used for Joint, Disjoint, Exclusive events.
2. Multiplicative rule: Always used for Dependent, Independent events.

Additive rule

For disjoint events:
$P(A\ or\ B) = P(A \cup B) = P(A) + P(B)$

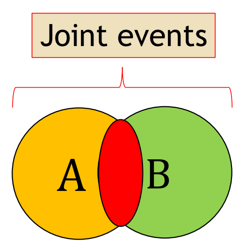


For disjoint events:
$P(A\ or\ B\ or\ C) = P(A \cup B \cup C \ldots)$
$= P(A) + P(B) + P(C)$

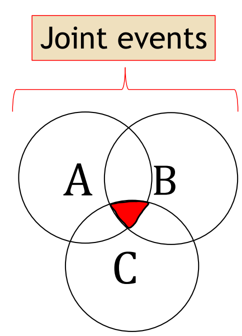


Additive rule

For joint events:
$P(A\ or\ B) = P(A \cup B)$
$= P(A) + P(B) - P(A \cap B)$



For joint events:
$P(A\ or\ B\ or\ C) = P(A \cup B \cup C)$
$= P(A) + P(B) + P(C) - P(A \cap B)$
$- P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$



Example: In a company, 60% of the employees have motorcycle, 40% has private car and 20% has both. What is the probability that the employee has either motorcycle or private car?

Solution: $P(M) = 0.6,\ P(C) = 0.4,\ P(M \cap C) = 0.2$

$\therefore P(M\ or\ C) = P(M \cup C) = P(M) + P(C) - P(M \cap C)$

$$= 0.6 + 0.4 - 0.2 = 0.8$$

Example: Mr. Ali feels that the probability that he will pass mathematics is 2/3 and statistics is 5/6. If the probability that he will pass both the course is 3/5, what is the probability that he will pass at least one of the course? (Ans: $\frac{9}{10}$)

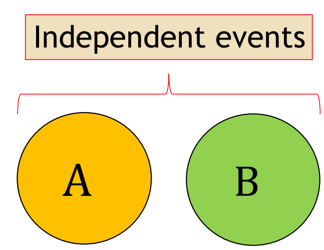
In a sample of 500 students, 320 said they had a stereo, 175 said they had a TV, and 100 said they had both. 5 said they had neither. If a student is selected at random, what is the probability that the student has only a stereo or TV? What is the probability that the student has both a stereo and TV? (Ans:$0.79,\ 0.20$)

Example: A sample survey was undertaken to investigate which papers A, B, and C people read. In survey of 100 people following results were obtained: 60 people read A, 40 people read B, 70 people read C, 32 people read A and B, 45 people read A and C, 38 people read B and C, 30 people read A, B, C. If a person is selected at random, find the probability,

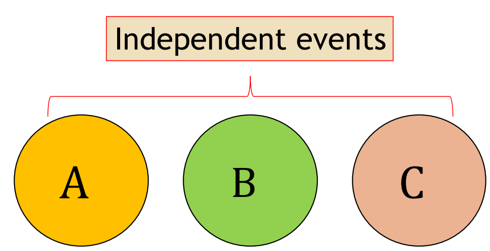
a) Read only newspaper A.
b) Read only one newspaper.
c) Read at least one newspaper.
d) Read at most one newspaper.

Multiplicative rule

For independent events:
$P(A \,\&\, B) = P(A \cap B) = P(A) \times P(B)$



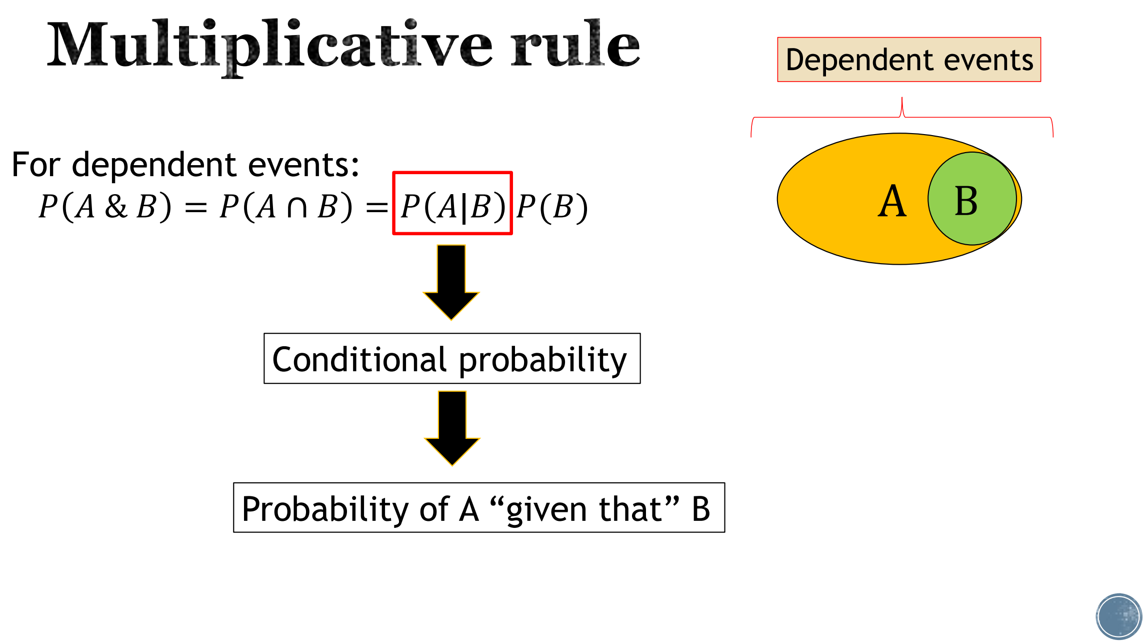
For independent events:
$P(A \,\&\, B \,\&\, C) = P(A \cap B \cap C \ldots)$
$= P(A) \times P(B) \times P(C)$



Example: If the probability that person A will be alive is 0.7 and the probability that person B will be alive is 0.5. What is the probability that they will both be alive in 20 years?

Solution: These are independent events

$$P(A \text{ and } B) = P(A \cap B) = P(A) \times P(B) = 0.35$$



Conditional probability refers to the chances that some outcome occurs given that another event has also/already occurred.

Let, $A = One\ event$, $B = Another\ event\ already\ occured$

It is often stated as the probability of A given B and is written as $P(A|B)$, where the probability of A depends on that of B happening.

$$\boxed{P(A|B) = \frac{P(A \text{ and } B)}{P(B)} = \frac{P(A \cap B)}{P(B)}}$$

Example: In a company, 60% of the employees have motorcycle, 40% has private car and 20% has both.

If it is known that the employee has a motorcycle, then what is the probability that the employee also has a car?

$$P(C|M) = \frac{P(M \cap C)}{P(M)} = \frac{0.2}{0.6} = \frac{1}{3} = 0.33$$

Suppose a balanced die is rolled once.

a) Find the probability that a number divisible by 3 is rolled given that the die comes up even.
b) Find the probability that the die comes up even given that a number divisible by 3 is rolled.
c) Find the probability that a number divisible by 3 is rolled given that die comes up at most 4.
d) Find the probability that the die comes up at most 4 given that a number divisible by 3 is rolled.

Conditional Probability

Sample space for rolling a die once, $S = \{1, 2, 3, 4, 5, 6\}$

a) Let,

$A = Number\ divisible\ by\ 3 = \{3, 6\}$

$B = Even\ number = \{2, 4, 6\}$

$P(A|B) = \frac{P(A \cap B)}{P(B)}$

$\therefore P(A|B) = \frac{\frac{1}{6}}{\frac{1}{2}} = 0.33$

Probability that a number divisible by 3 is rolled given that the die comes up even.

$A \cap B = \{3, 6\} \cap \{2, 4, 6\} = \{6\}$

$P(A \cap B) = \frac{1}{6}$

$B = \{2, 4, 6\}$

$P(B) = \frac{3}{6} = \frac{1}{2}$

Conditional Probability

1. In rainy season, it rains 70% of the days in Bangladesh. When it rains, 80% times it makes thunderstorms. What is the probability that, in a particular day of rainy season, it will rain and it will thunderstorm?

R be an event of rain

T be an event of Thunderstorms

$P(R) = 70\% = 0.7$

$P(R \text{ and } T) = P(R \cap T)$

$P(T|R) = 80\% = 0.8$

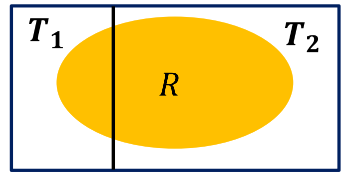
$$P(T|R) = \frac{P(T \cap R)}{P(R)}$$

$$P(T \cap R) = P(T|R) \times P(R)$$

$$P(T \cap R) = 0.8 \times 0.7 = 0.56$$

Example: Mr. Fahad and Mr. Khan has to tour abroad for their business frequently. Mr. Fahad tours 65% of the times in a year at abroad and Mr. Khan tours 50% of the times in a year at abroad. What is the probability that, on January 01, 2016, both Mr. Fahad and Mr. Khan will be at abroad?

Bayes' Theorem

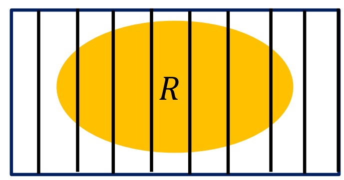


- Let, events T_1 and T_2 form partition of S. Let R be an event with $P(R) > 0$. Then,

$$P(T_1|R) = \frac{P(T_1 \cap R)}{P(R)} = \frac{P(T_1)P(R|T_1)}{P(T_1)P(R|T_1) + P(T_2)P(R|T_2)}$$

$$P(T_2|R) = \frac{P(T_2 \cap R)}{P(R)} = \frac{P(T_2)P(R|T_2)}{P(T_1)P(R|T_1) + P(T_2)P(R|T_2)}$$

Bayes' Theorem



- Let, events $T_1, T_2, \ldots$ form partition of S. Let R be an event with $P(R) > 0$. Then,

$$P(T_j|R) = \frac{P(T_j \cap R)}{P(R)} = \frac{P(T_j)P(R|T_j)}{P(T_1)P(R|T_1) + P(T_2)P(R|T_2) + \cdots}$$

60% of the students in a class are male. 5% of the males and 10% of the females are in the photography club. If a student is randomly selected from the class.

a) What is the probability that the student is in photography club?
b) If the randomly selected student is in the photography club, what is the chance that the student is male?

Solution

60% of the students in a class are male

40% of the students in a class are female

5% of the males are in the photography club

10% of the females are in the photography club

- $P(M) = 0.6$
- $P(F) = 0.4$
- $P(C|M) = 0.05$
- $P(C|F) = 0.10$

a) What is the probability that the student is in photography club?

$$P(C) = P(M)P(C|M) + P(F)P(C|F) = 0.07$$

b) If the randomly selected student is in the photography club, what is the chance that the student is male?

$$P(M|C) = \frac{P(M)P(C|M)}{P(C)} = 0.43$$

Contingency table

- A contingency table, also known as a cross-tabulation or crosstab, is a statistical table used to analyze and display the relationship between two categorical variables.

- For example, The question, "Do you like watching TV?" was asked of 100 people. Results are shown in the table.

	Yes	No	Total
Male	19	41	60
Female	12	28	40
Total	31	69	100

Joint Probability

- Both event occur together, $P(A\ or\ B) = P(A \cup B)$
- For example, The question, "Do you like watching TV?" was asked of 100 people. What is the probability of a randomly selected individual being a male who likes watching TV?

	Yes	No	Total
Male	19	41	60
Female	12	28	40
Total	31	69	100

Marginal Probability

- Single event occurring independently of any other events.
- For example, The question, "Do you like watching TV?" was asked of 100 people. What is the probability of a randomly selected individual like watching TV?

	Yes	No	Total
Male	19	41	60
Female	12	28	40
Total	31	69	100

Example

- Below given a contingency table for Smoking status and Cancer status.

	Cancer	Healthy	Total
Smoker	7860	1530	9390
Non smoker	5390	11580	16970
Total	13250	13110	26360

1. What is the probability that a randomly selected person is a smoker? (0.36)
2. What is the probability that a randomly selected person has cancer? (0.50)
3. What is the probability that a randomly selected person is both smoker and has cancer? (0.298)
4. If a person is smoker, what is the probability that he also has cancer? (0.84)