Regression

In the previous class, we covered the concept of "correlation," which helps us measure the relationship between two variables, denoted as "x" and "y." A positive correlation implies that as "x" increases, "y" also increases, or vice versa. However, determining which variable is influencing the other is challenging

To answer this, we'll explore another concept called "Regression."

Sir, what is “Regression”?

Regression means relationship between two or more variables (similar to the correlation).

Which types of relationship? “Cause and Effect relationship”. That means, in regression you can figure out the actual variable which is influencer and which one is influenced by other.

That means, there are two types of variables in regression

1) Causal variable (mainly termed as independent variable)
2) Affected variable (mainly termed as dependent variable)

At a glance, Regression refers to the cause-and-effect relationship between two or more variables, where affected variables are dependent variables and causal variables are independent variables.

This relationship is mathematically expressed, highlighting the impact of independent variables on dependent variables.

$$\boxed{\boldsymbol{Dependent\ variable\ = Y}}$$

$$\boxed{\boldsymbol{Independent\ variable\ = X}}$$

There is a positive relationship between income and expenditure, i.e. an increase in income increases expenditures.

As increase in income causes an increase in expenditures

"Income" as independent variable (*X*) and "Expenditures" as dependent variable (*Y*).

Correlation vs Regression:

Correlation	Regression
1. Correlation is a statistical measure that determines the association between two variables.	1. Regression describe how to numerically relate an independent variable to the dependent variable.
2. There is no dependent variable and independent variable.	2. Must be one dependent variable and one independent variable.
3. To represent the linear relationship between variables.	3. To represent the cause-and-effect relationship between variables.

Types of Regression: Two types of regression on the basis of variables

1) Simple regression (One dependent vs one independent)
2) Multiple regression (One dependent vs more than one independent)

Simple Regression:

- One dependent variable vs One independent variable
- Expressed this relationship as a mathematical form
- The mathematical form is,

$$Y_i = \alpha + \beta X_i + \epsilon_i \quad ; i = 1,2,3, \dots, n$$

Here,

$Y_i = Dependent\ variable$

$X_i = Independent\ variable$

$\alpha = Intercept\ coefficient$

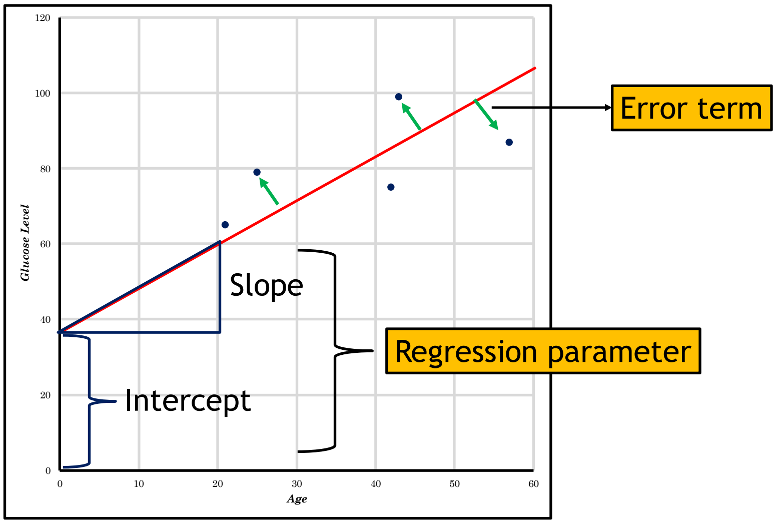
$\beta = Slope\ coefficient$

$\epsilon_i = Error\ term$

Multiple regression: $Y = \alpha + \beta_1 X_1 + \cdots + \beta_k X_k + \epsilon$

Simple regression

$Y_i = \alpha + \beta X_i + \epsilon_i$



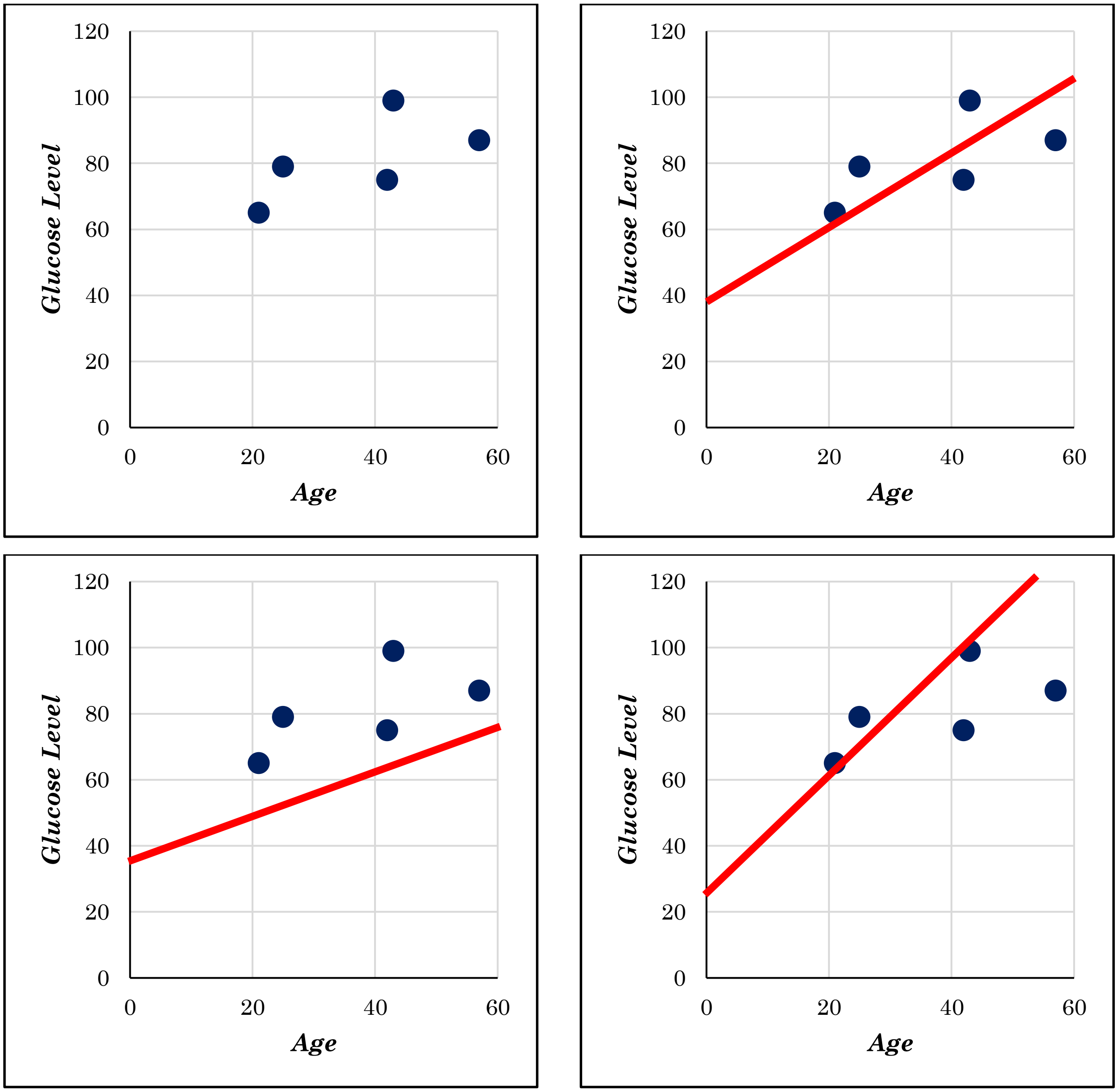
Here $\beta = Slope$ and $\alpha = Intercept$ are unknown regression parameter, which we want to estimate. Now the question is “How to estimate/calculate the unknown regression parameters?”

We can estimate regression parameters in two ways:

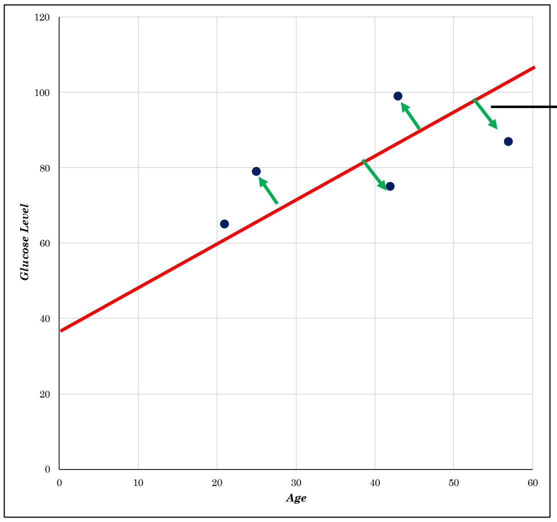
1) Least Square Method
2) Graphical Method

What is “Least Square Method”? “Least” means “Minimum”.

Now look at this graph,



Estimating parameters



Error term

Let the estimator of α and β is $\hat{\alpha}$ and $\hat{\beta}$.

$$\hat{\beta} = \frac{\sum(x_i - \bar{x})(y_i - \bar{y})}{\sum(x_i - \bar{x})^2} = \frac{n\sum x_i y_i - \sum x_i \sum y_i}{n\sum x_i^2 - (\sum x_i)^2}$$

$\widehat{Y_i} = \hat{\alpha} + \hat{\beta} X_i$

Fitted model

$$\hat{\alpha} = \bar{y} - \hat{\beta}\bar{x}$$

Advertising cost (x_i)	**Sales revenue (y_i)**	x_i^2	$x_i \times y_i$
2	7	4	14
1	3	1	3
3	8	9	24
4	10	16	40
$\sum x_i = 10$	$\sum y_i = 28$	$\sum x_i^2 = 30$	$\sum x_i y_i = 81$

$$\hat{\beta} = \frac{n\sum x_i y_i - \sum x_i \sum y_i}{n\sum x_i^2 - (\sum x_i)^2} = \frac{(4 \times 81) - (10 \times 28)}{(4 \times 30) - (10)^2} = 2.2$$

$\hat{\beta} = 2.2$ means that an increase of \$1 million in advertising cost, the average sales revenue will increase \$2.2 million.

$$\hat{\alpha} = \bar{Y} - \hat{\beta}\bar{X} = 7 - (2.2 \times 2.5) = 1.5$$

$\hat{\alpha} = 1.5$ means that, if there is no advertising cost then average sales revenue would be \$1.5 million

$Fitted\ model\colon\ \widehat{Y_i} = 1.5 + 2.2X_i$

$If\ X = 9\colon\ \widehat{Y_i} = 1.5 + 2.2 \times 9 = 21.3$

Example 2:

Investment (x)	**Profit (y)**	x^2	y^2
5	3	25	9
10	4	100	16
15	8	225	64
20	12	400	144
25	18	625	324
75	45	$\sum x_i^2 = 1375$	$\sum y_i^2 = 557$

$$\beta = 0.76$$

$$\alpha = -2.4$$

$$\hat{y} = -2.4 + 0.76x$$

Goodness of fit:

- How precise such predictions are?
- Is this regression equation being useful for prediction?
- To answer these questions, we need “Coefficient of Determination”- Denoted by $\boldsymbol{R^2}$

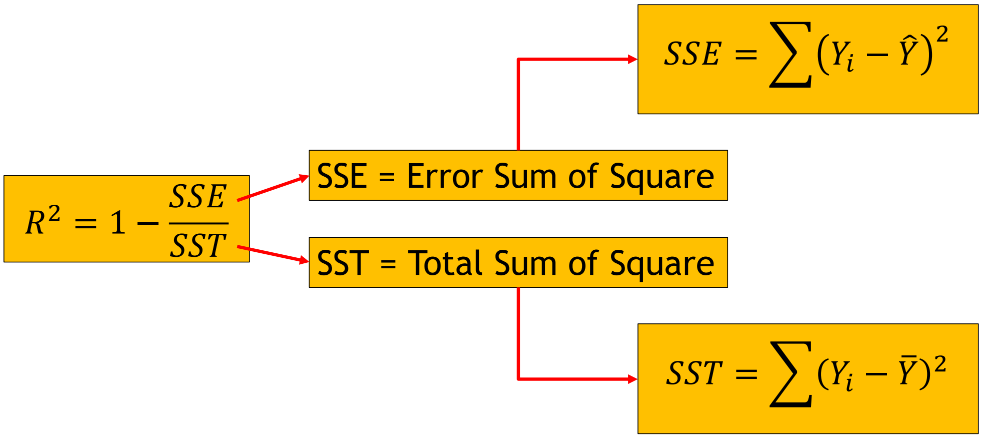
Coefficient of determination: The coefficient of determination tells the percent of the variation in the dependent variable that is explained (determined) by the model and the explanatory variable. It is denoted by R^2.

Range of $\boldsymbol{R^2}$: $[\boldsymbol{0}\ \boldsymbol{to}\ \boldsymbol{1}]$

$\boldsymbol{R^2 = 0: Equation\ is\ not\ useful\ for\ predictions}$

$\boldsymbol{R^2 = 1: Equation\ is\ useful\ for\ predictions}$

Coefficient of determination



$$R^2 = 1 - \frac{SSE}{SST}$$

SSE = Error Sum of Square

$$SSE = \sum (Y_i - \hat{Y})^2$$

SST = Total Sum of Square

$$SST = \sum (Y_i - \bar{Y})^2$$

Coefficient of determination

$Fitted\ model: \widehat{Y_i} = 1.5 + 2.2X_i$

Advertising cost (x_i)	Sales revenue (y_i)	$\hat{y} = 1.5 + 2.2x_i$	$(y_i - \hat{y}_i)^2$	$(y_i - \bar{y})^2$
2	7	5.9	1.21	0
1	3	3.7	0.49	16
3	8	8.1	0.01	1
4	10	10.3	0.09	9
$\sum x_i = 10$	$\sum y_i = 28$		$SSE = 1.8$	$SST = 26$

$$SSE = \sum (Y_i - \hat{Y})^2$$

$$SST = \sum (Y_i - \bar{Y})^2$$

Coefficient of determination

$$R^2 = 1 - \frac{SSE}{SST}$$

→

$$R^2 = 1 - \frac{1.8}{26}$$

→

$$R^2 = 0.93$$

Interpretation of R^2: For example, $R^2 = 0.93$ or 93%. It indicates that, almost 93% of the variability of the dependent variables explained by the independent variables.

Error Calculation

$Fitted\ model\colon\ \widehat{Y}_i = 1.5 + 2.2X_i$

Advertising cost (x_i)	Sales revenue (y_i)	$\hat{y} = 1.5 + 2.2x_i$	$\epsilon_i = (y_i - \hat{y}_i)$
2	7	5.9	1.1
1	3	3.7	-0.7
3	8	8.1	-0.1
4	10	10.3	-0.3

Properties of Regression coefficient:

1. Regression coefficient has unit.
2. Regression coefficients are not symmetric function, $i.e.,\ \beta_{yx} \neq \beta_{xy}$