Our new chapter “Basic concepts of probability”.

Firstly, we need to learn “What is probability”. In simple way,

Probability is the likeliness/chance of occurring any event(s).

For example, you flip coin. Now, there are two possible questions may arise

1. What is the chance of it landing heads up? The answer of this question is “Probability”
2. What is the chance of it landing tails up? The answer of this question is “Probability”

Before going to the in depth of probability concept, we need to know some basic terms. First one is experiment. Sir, what is “Experiment”???

An experiment refers to a specific action, process, or phenomenon that leads to observable outcomes.

For example,

1. Measuring distance from Dhaka to Chattogram
2. Tossing a fair coin

For first one, “Measuring distance from Dhaka to Chattogram”. In this experiment the outcome (observable outcome) can be predict in advanced/ the outcome is fixed. If you conduct this experiment more than one time, you will get similar findings.

But for second one, Flipping a fair coin. In this experiment the outcome (observable outcome) cannot be predict in advanced/ the outcome is not fixed. If you conduct this experiment more than one time, you will not get similar findings.

Based on this, we can say that, there are two types of experiment.

1. Deterministic: Whose outcome is predictable in advance
2. Probabilistic/Random: Whose outcome is not predictable

Some examples:

Is tossing a coin a random experiment or deterministic experiment?

Ans: Tossing a coin is an experiment. There are two possible outcomes (head or tail). These outcomes are unpredictable before the coin is tossed. Therefore, this is a random experiment.

Is drawing a card from well shuffled deck of cards a random experiment or deterministic experiment?

Ans: Drawing a card from a well-shuffled deck of cards is a random experiment. The randomness arises from the uncertainty of which card will be drawn, even though the deck is well-shuffled. The outcome (the specific card drawn) is not predictable beforehand, making it a random event.

Is multiplying 4 and 8 on a calculator a random experiment or deterministic experiment?

Ans: Multiplying 4 and 8 on a calculator is a deterministic experiment. In a deterministic experiment, the outcome is certain and predictable based on the given inputs and the rules of the operation. In this case, multiplying 4 and 8 will always result in the same answer: 32. There is no randomness or uncertainty involved in this calculation, making it a deterministic process.

Another basic terms, “Sample space”.

First, let's look at an example. Someone is flipping a coin. So, what possible results can he get? Either heads or tails. Right!

In other words, heads could be one outcome, or tails could be one outcome. What is outcome? Result of an experiment

If we consider all possible outcomes of this experiment, that means, if we combine heads and tails into one set, create a set, then we can call this set a sample space.

So, what does the book's definition say? Sample space is a set in which all possible outcomes of my experiment will be present.

We denote it with a capital S.

For the example we show in previous slide, the sample space would be: $S = \{H, T\}$; $from\ previous\ example$

Another example:

- For example, consider the experiment of tossing two coins. Write the sample space of this experiment.
- $S = \{(H,H),(H,T),(T,H),(T,T)\}$

Next term "Event"

Here, let's also look at an example.

When we take a part of the sample space, create a subset, we can call that subset an event. In other words, any subset of a sample space is an event.

Events can take various forms...

Union of events $(A \cup B)$

Intersection of events $(\mathrm{A} \cap B)$

Complement of an event $(\bar{A}\ or\ A')$

Exclusive event

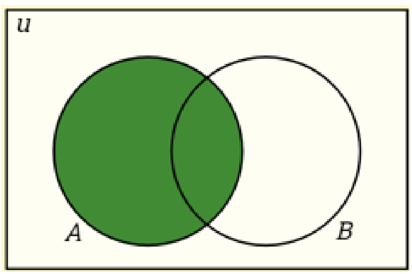
Exhaustive event

Equally likely event

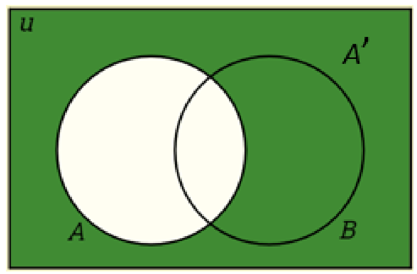
<u>Exclusive events:</u> **Two events are called mutually exclusive if both the events cannot occur simultaneously in a single trial. In other words, if one of those events occurs, the other event will not occur.**

<u>Exhaustive events:</u> **Exhaustive events are those, which includes all possible outcomes.**

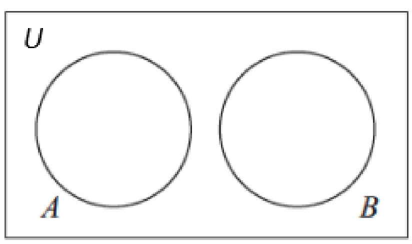
<u>Equally likely events:</u> **The events of a random experiment are called equally likely if the chance of occurring those events is all equal.**



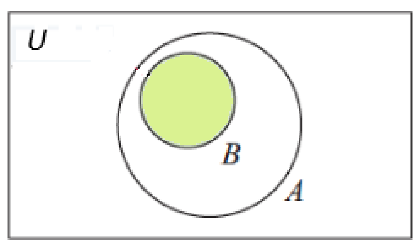
Set A



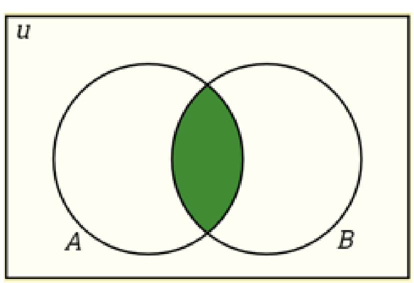
A' the complement of A



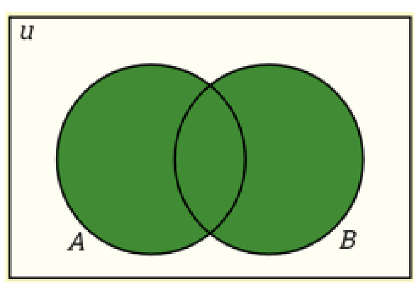
A and B are disjoint sets



B is proper subset of A $B \subset A$



Both A and B $A \cap B$
A intersect B



Either A or B $A \cup B$
A union B

Now, sir, how do we calculate probability of some event?

We then define our favorable event and assign probability to the event using one of the following 3 basic approaches-

1. Classical Approach
2. Frequency Approach
3. Subjective Approach

Just formula.

$$\boldsymbol{P(Event) = \frac{Number\ of\ occurance/outcome\ in\ the\ event}{Total\ number\ of\ outcomes\ in\ the\ sample\ space}}$$

Simple example,

$$S = \{1,\ 2,\ 3,\ 4,\ 5,\ 6\}$$

$$A = \{2,\ 4,\ 6\}$$

$$B = \{1,\ 3,\ 5\}$$

$$P(A) = \frac{3}{6}$$

$$P(B) = \frac{3}{6}$$

Example:

Example: A committee consists of five executives of which three women (W_1, W_2, W_3) and two men (M_1, M_2). A random sample of two executives needs to be selected at random without replacement from which chairman and a secretary would be selected. Set up the sample space and find the probability that

a) W_1 and W_2 will be selected
b) M_1 will be selected
c) M_1 will not be selected
d) W_1 or M_1 will be selected

Classical approach

a) W_1 and W_2 will be selected

- Solution: Sample space of this experiment is
- $S = \{(W_1W_2), (W_1W_3), (W_1M_1), (W_1M_2), (W_2W_3), (W_2M_1), (W_2M_2), (W_3M_1), (W_3M_2), (M_1M_2)\}$
- Total number of outcomes in the experiment = 10

a) Let, $A = Event\ of\ selecting\ W_1\ and\ W_2 = \{W_1W_2\}$

$$\therefore P(A) = \frac{1}{10} = 0.1$$

Classical approach

b) M_1 will be selected

- Solution: Sample space of this experiment is
- $S = \{(W_1W_2), (W_1W_3), (W_1M_1), (W_1M_2), (W_2W_3), (W_2M_1), (W_2M_2), (W_3M_1), (W_3M_2), (M_1M_2)\}$
- Total number of outcomes in the experiment = 10

b) Let, $B = Event\ of\ selecting\ M_1 = \{W_1M_1, W_2M_1, W_3M_1, M_1M_2\}$

$$\therefore P(B) = \frac{4}{10} = 0.4$$

c) M_1 will not be selected

Classical approach

- Solution: Sample space of this experiment is
- $S = \{(W_1W_2),(W_1W_3),(W_1M_1),(W_1M_2),(W_2W_3),(W_2M_1),(W_2M_2),(W_3M_1),(W_3M_2),(M_1M_2)\}$
- Total number of outcomes in the experiment = 10

c) Let, $B = Event\ of\ selecting\ M_1 = \{W_1M_1, W_2M_1, W_3M_1, M_1M_2\}$

$$\therefore P(\bar{B}) = 1 - P(B) = 1 - \frac{4}{10} = 0.6$$

d) $W_1\ or\ M_1$ will be selected

Classical approach

- Solution: Sample space of this experiment is
- $S = \{(W_1W_2),(W_1W_3),(W_1M_1),(W_1M_2),(W_2W_3),(W_2M_1),(W_2M_2),(W_3M_1),(W_3M_2),(M_1M_2)\}$
- Total number of outcomes in the experiment = 10

d) Let,

$$C = Event\ of\ selecting\ W_1 = \{(W_1W_2),(W_1W_3),(W_1M_1),(W_1M_2)\}$$

$$B = Event\ of\ selecting\ M_1 = \{W_1M_1, W_2M_1, W_3M_1, M_1M_2\}$$

$C\ or\ B = C \cup B = \{(W_1W_2),(W_1W_3),(W_1M_1),(W_1M_2), W_2M_1, W_3M_1, M_1M_2\}$

$P(C \cup B) = \frac{7}{10} = 0.7$

Frequency approach:

A random experiment is repeated n times under same condition

An event "A" occurs m times

According to frequency approach

Probability of A, $P(A) = \lim_{n\to\infty} \frac{m}{n}$

For example;

In a dice throwing experiment, S= {1, 2, 3, 4, 5, 6}

And our favorable event is E= {2}

Let, 2 occurred a total of 998 times out of total 6000 trials. Therefore $P(E) = \lim_{n\to\infty} \frac{998}{6000} = \frac{1}{6}$

Subjective approach:

Based on the judgement (personal experience, prior information and belief etc.), one can assign probability to an event E of a random experiment.

For example; on a day of summer someone made a statement on probability that rain will occur on that day is .70, based on his previous experience.

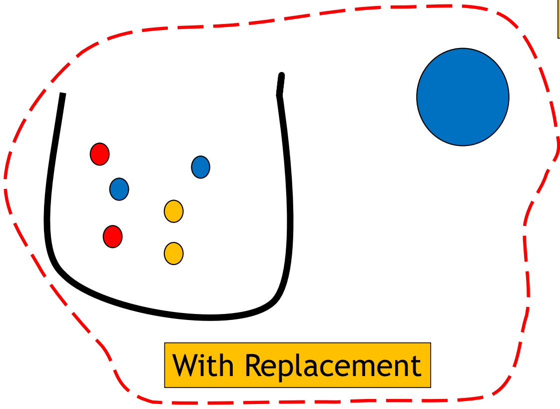
Axioms of Probability:

Probability of an event follows three axioms:

1. $P(A) \geq 0$ (Axiom of positivizes)
2. $P(S) = 1$ (Axiom of certainty) $S = \{1,2,3\}$
3. $P(A_1 \cup A_2 \cup \dots) = P(A_1) + P(A_2) + \cdots = \sum P(A_i)$

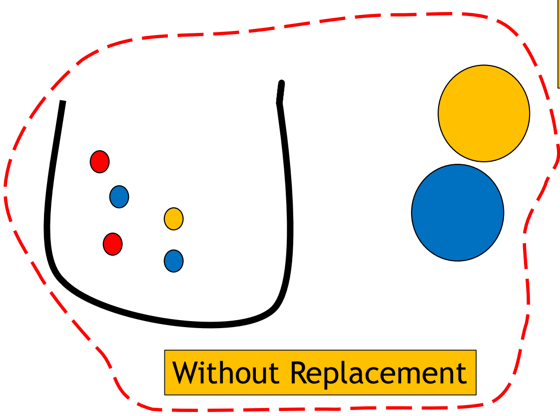
With and Without Replacement

If the elements of a sample are drawn randomly one by one and after each draw the element is returned to the population = With Replacement



With and Without Replacement

If the elements of a sample are drawn randomly one by one and after each draw the element is not returned to the population = With Out Replacement



With and Without Replacement

- A box contains 20 bulbs, of which 5 are defective. If 3 of the bulbs are selected at random without replacement, what is the probability that all three bulbs are defective?

Solution:

Probability of bulb is defective, $P(D) = \frac{5}{20} \times \frac{4}{19} \times \frac{3}{18} = 0.0088$

With and Without Replacement

- A box contains 20 bulbs, of which 5 are defective. If 3 of the bulbs are selected at random without replacement, what is the probability that all three bulbs are defective?

Another Solution:

$\therefore$ 3 bulbs out of 20 bulbs can be draw in ${}^{20}C_3 = 1140$ ways

$\therefore$ 3 defective bulbs out of 5 defective bulbs can e draw in ${}^{5}C_3 = 10$ ways

$\therefore$ Probability of defective bulbs, $P(D) = \frac{10}{1140} = 0.0088$