Statistics - Formulas and Tables

The following definitions are given for univariate or bivariate data sets with n observations ($x_1, x_2, .., x_n$), resp ($x_1/y_1, x_2/y_2, .., x_n/y_n$),

Descriptive Statistics

Univariate Data – Raw Data

- *(Empirical) Quantile (**p-Quantile,** $\tilde{x}_p$):*

  For a set of ordinal data $x_1, x_2, \dots, x_n$ *with* $x_{(1)} \leq x_{(2)} \leq \dots \leq x_n$ *and* $p \in (\mathbf{0}, \mathbf{1})$*:*

  $$\tilde{x}_p = x_{(k)} \qquad if\ np \notin \mathbb{N} \Longrightarrow np < k < np + 1$$
  $$\tilde{x}_p \in \{ x_{(k)},\ x_{(k+1)} \} \qquad if\ np \in \mathbb{N} \Longrightarrow k = np$$

  For a set of metric data $x_1, x_2, \dots, x_n$ *with* $x_{(1)} \leq x_{(2)} \leq \dots \leq x_n$ *and* $p \in (\mathbf{0}, \mathbf{1})$*:*

  $$\tilde{x}_p = x_{(k)} \qquad if\ np \notin \mathbb{N} \Longrightarrow np < k < np + 1$$
  $$\tilde{x}_p = \frac{1}{2}\left(x_{(k)} + x_{(k+1)} \right) \qquad if\ np \in \mathbb{N} \Longrightarrow k = np$$

- *Interquartile Range (**IQR**):*

  $$IQR = \tilde{x}_{0.75} - \tilde{x}_{0.25}$$

- *Lower Fence:*

  $$Lower\ Fence = \tilde{x}_{0.25} - 1.5 * IQR$$

- *Upper Fence:*

  $$Upper\ Fence = \tilde{x}_{0.75} + 1.5 * IQR$$

- *Arithmetic Mean:*

Population Mean: $\bar{\mu} = \frac{\sum x_i}{n}$ *Sample Mean:* $\bar{\chi} = \frac{\sum x_i}{n}$

- *Geometric Mean:* $\bar{x}_{geo} = \sqrt[n]{\prod x_i}$ *with* $x_1, \ldots, x_n > 0$.

- *Harmonic Mean:* $\bar{x}_{harm} = \frac{n}{\sum \frac{1}{x_i}}$

- *Weighted Arithmetic Mean:* $\bar{x} = \sum g_i \bullet x_i$ *with weights* $g_1, \ldots, g_n \geq 0$ *and* $\sum g_i = 1$

- *Weighted Geometric Mean:* $\bar{x}_{geo.w} = \prod x_i^{g_i}$ *with weights* $g_1, \ldots, g_n \geq 0$ *and* $\sum g_i = 1$

- *Weighted Harmonic Mean:* $\bar{x}_{harm.w} = \frac{1}{\sum_{i=1}^{n} \frac{g_i}{x_i}}$ *with weights* $g_1, \ldots, g_n \geq 0$ *and*

$\sum g_i = 1$

- *Population Variance:* $\sigma^2 = \frac{1}{n} \sum_{i=1}^{n} (x_i - \bar{\mu})^2$ *with* $\bar{\mu}$ *= arithmetic mean*

- *Sample Variance:* $s^2 = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{\chi})^2$ *with* $\bar{\chi}$ *= arithmetic mean*

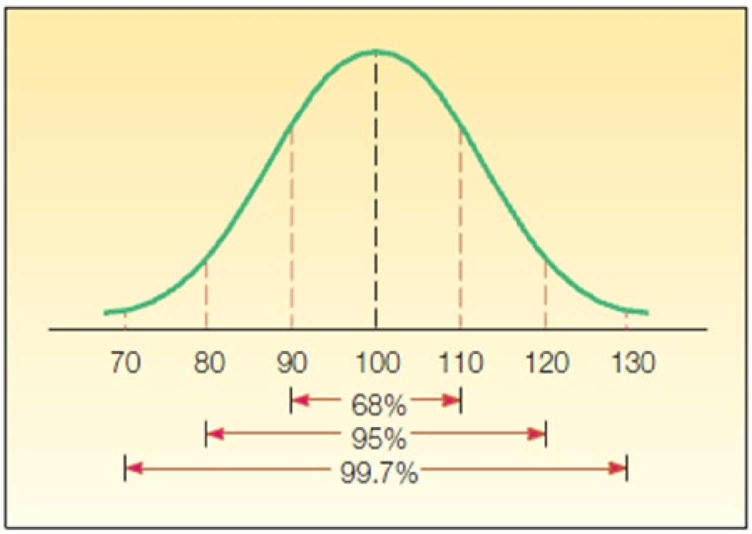
- *Standard Deviation:* $\sigma_n = \sqrt{\sigma^2}$ *resp.* $s_n = \sqrt{s^2}$

- *Median Absolute Deviation:* $d = \frac{1}{n} \sum_{i=1}^{n} | x_i - \tilde{x}_n |$ *with* $\tilde{x}$ *= median*

- *Coefficient of Variation:* $V = \frac{S_n}{\bar{x}}$

- *Standard Score:* $z_i = \frac{x_i - \bar{x}}{s}$

- *Empirical Rule:* *Illustrated by the following figure* *<u>for a bell shaped distriubution with</u> a <u>mean of 100</u> and a <u>standard deviation of 10</u>*



Univariate Data – Grouped Data

- *Absolute Class frequencies:*

  n(K_j) *= number of observed values that belong to class K_j*

  With M = number of classes:

$$\sum_{j=1}^{M} \boldsymbol{n}(\boldsymbol{K_j}) = \boldsymbol{n}$$

- *Relative Class frequencies:*

  f(K_j) = n(K_j) / n

  With M = number of classes

$$\sum_{j=1}^{M} \boldsymbol{f}(\boldsymbol{K_j}) = \boldsymbol{1}$$

- *Class midpoint:*

  class midpoint of class j,

  $\overline{\boldsymbol{v}}_{\boldsymbol{j}}$***= (lower class limit + next lower class limit)/2***

- *Arithmetic Mean for Grouped Data:*

  Population Mean: $\mu_{grouped} = \sum_{j=1}^{M} \boldsymbol{f}(\boldsymbol{Kj})\overline{\boldsymbol{v}}_{\boldsymbol{j}}$

  Sample Mean: $\overline{\boldsymbol{x}}_{grouped} = \sum_{j=1}^{M} \boldsymbol{f}(\boldsymbol{Kj})\overline{\boldsymbol{v}}_{\boldsymbol{j}}$

- *Geometric Mean for Grouped Data:* $\overline{x}_{geo,\,grouped} = \prod_{J=1}^{M} \overline{v}_j{}^{f(Kj)}$

- *Harmonic Mean for Grouped Data:* $\overline{x}_{harm,\,grouped} = \frac{1}{\sum_{j=1}^{M} \frac{f(Kj)}{\overline{v}_j}}$

- *Population Variance for Grouped Data:*

$$\sigma^2{}_{grouped} = \frac{\sum_{j=1}^{M} n(Kj)\left(\overline{v}_j - \mu_{grouped}\right)2}{\sum_{j=1}^{M} n(Kj)}$$

- *Sample Variance for Grouped Data:*

$$s^2{}_{grouped} = \frac{\sum_{j=1}^{M} n(Kj)\left(\overline{v}_j - \overline{x}_{grouped}\right)2}{\left(\sum_{j=1}^{M} n(Kj)\right) - 1}$$

- *Standard Deviation:* $\sigma_n = \sqrt{\sigma^2}$ *resp.* $s_n = \sqrt{s^2}$

Bivariate Data

- *Correlation Coefficient:*

$$r_{xy} = \frac{s_{xy}}{s_x \cdot s_y} = \frac{\sum(x_i - \overline{x})(y_i - \overline{y})}{\sqrt{\sum(x_i - \overline{x})^2}\sqrt{\sum(y_i - \overline{y})^2}}$$

- *Least-Squares Regression*

  For a line described by $\boxed{\hat{y} = b_0 + b_1 x}$

  find the constants b_0 and b_1, such that $\boxed{\sum (y - \hat{y})^2}$

  is as small as possible,

  with $y = observed\_value$ *and* $\hat{y} = predicted\_value$ *)*

$$b_1 = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2} = r_{xy} * \frac{S_y}{S_x} \qquad b_0 = \frac{\sum y - (b_1 \cdot \sum x)}{n}$$

- *Coefficient of Determination (for the least-squares regression model):*

$$R^2 = r^2$$

Probability

- *Empirical Probability:*

$$P(E) \approx \frac{frequency\ of\ E}{number\ of\ trials\ of\ experiment}$$

- *Classical Probability:*

$$P(E) \approx \frac{number\ of\ ways\ that\ E\ can\ occur}{number\ of\ possible\ outcomes} = \frac{N(E)}{N(S)}$$

- *Addition Rule for Disjoint Events*

$$P(E\ or\ F) = P(E) + P(F)$$

- *Addition Rule for n Disjoint Events*

$$P(E\ or\ F\ or\ G\ or\ \ldots) = P(E) + P(F) + P(G) + \ldots$$

- *General Addition Rule*

$$P(E\ or\ F) = P(E) + P(F) - P(E\ and\ F)$$

- *Complement Rule:*

$$P(E^c) = 1 - P(E)$$

- *Multiplication Rule for Independent Events*

$$P(E \text{ and } F) = P(E) \cdot P(F)$$

- *Multiplication Rule for n Independent Events*

$$P(E \text{ and } F \text{ and } G\ldots) = P(E) \cdot P(F) \cdot P(G)\ldots$$

- *Conditional Probability Rule*

$$P(F|E) = \frac{P(E \text{ and } F)}{P(E)} = \frac{N(E \text{ and } F)}{N(E)}$$

- *General Multiplication Rule:*

$$P(E \text{ and } F) = P(E) \cdot P(F|E)$$

$$\text{and}$$

$$P(E \text{ and } F) = P(F) \cdot P(E|F)$$

- *Bayes Theorem:*

$$P(E \text{ and } F) =$$

$$P(E) \; x \; P(F|E) = P(F) \; x \; P(E|F)$$

$$\Rightarrow$$

$$P(F|E) = \frac{(P(F) \; x \; P(E|F))}{P(E)}$$

$$\text{and}$$

$$P(E|F) = \frac{(P(E) \; x \; P(F|E))}{P(F)}$$

Counting

- *Factorial*

$$n! = n \cdot (n-1) \cdot (n-2) \ldots \cdot 3 \cdot 2 \cdot 1$$

- *Permutation of n objects taken r at a time*

$$n\,Pr = \frac{n!}{(n-r)!}$$

- *Combination of n objects taken r at a time*

$$n\,Cr = \frac{n!}{r!\,(n-r)!}$$

- *Permutations with Repetition*

$$\frac{n!}{n_1!\; n_2! \ldots\ldots n_k\;!}$$

Discrete Probability Distributions

- *Mean (Expected Value) of a **Discrete** Random Variable*

$$\mu_x = \sum x \cdot P(x)$$

- ***Variance** of a **Discrete** Random Variable*

$$\sigma_x^2 = \sum (x-\mu)^2 \cdot P(x) = \sum x^2 \cdot P(x) - \mu_x^2$$

- *Criteria for a **Binomial** Probability Experiment*

    1. *The experiment is performed a fixed number of times (trials).*
    2. *The trials are independent.*
    3. *For each trial, there are two mutually exclusive outcomes, success or failure.*
    4. *The probability of success is the same for each trial of the experiment.*

- ***Binomial** Probability Distribution Function*

$$P(x) = {}_nC_x\, p^x (1-p)^{n-x}$$

where x is the number of successes, p is the probability of success and n is the number of independent trials

- *Mean and Standard Deviation of a **Binomial** Random Variable*

$$\mu_x = np \qquad \text{and} \qquad \sigma_x = \sqrt{np(1-p)}$$

- *Rule of thumb for the normal approximation of the **binomial** probability distribution*

$$np(1-p) \geq 10$$

- *Criteria for a **Hypergeometric** Probability Experiment*

    1. *A sample of size **n** is randomly selected without replacement from a population of **N** items.*
    2. *In the population, **k** items can be classified as successes, and **N - k** items can be classified as failures.*

    → *The result of each draw in the selection process (the elements of the population being sampled) can be classified into one of two mutually exclusive categories: Success or Failure (The random variable **X** represents the number of successes in the sample).*

    → *The probability of a success changes on each draw, as each draw decreases the population (sampling without replacement from a finite population).*

- ***Hypergeometric*** *Probability Distribution Function*

$$P(X=x) = P(x) = \frac{{}_kC_x * {}_{N-k}C_{n-x}}{{}_NC_n} = \frac{\binom{k}{x} * \binom{N-k}{n-x}}{\binom{N}{n}}$$

*where **N** is population size, **n** is sample size, **x** is the number of successes in the sample, **k** is the number of elements of type x in N.*

- *Mean and Standard Deviation of a **Hypergeometric** Random Variable*

$$\mu_x = \frac{nk}{N} \quad \text{and} \quad \sigma_x = \sqrt{\frac{nk(N-k)(N-n)}{N^2(N-1)}}$$

The Normal Distribution

- ***Standardizing a Normal Random Variable***

$$z = \frac{x-\mu}{\sigma}$$

- *Finding the Score*

$$x = \mu + z\sigma$$

- ***Drawing a Normal Probability Plot*** *- The expected proportion of observations less than or equal to the ith data value in an ordered list of values:*

$$f_i = \frac{i-0.375}{n+0.25}$$

i: index,the position of the data value in an ordered list

n: number of observed values

Sampling Distributions

- *Mean and Standard Deviation of the Sampling Distribution of* $\bar{x}$ *:*

$$\mu_{\bar{x}} = \mu \quad \text{and} \quad \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

- *Sample Proportion:*

$$\hat{p} = \frac{x}{n}$$

- *Mean and Standard Deviation of the Sampling Distribution of* $\hat{p}$:

$$\mu_{\hat{p}} = p \qquad \text{and} \qquad \sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$$

Confidence Intervals

- *A* ***(1-α). 100%*** *confidence interval about* ***μ*** *with* ***σ*** ***known*** *is*

$$\bar{x} \pm z_{\frac{\alpha}{2}} \cdot \frac{\sigma}{\sqrt{n}}$$

- *A* ***(1-α). 100%*** *confidence interval about* ***μ*** *with* ***σ*** ***unknown*** *is*

$$\bar{x} \pm t_{\frac{\alpha}{2}} \cdot \frac{s}{\sqrt{n}}.$$

Note: $t_{\frac{\alpha}{2}}$ *is computed using* ***n-1*** *degrees of freedom.*

- *A **(1-α).100%** confidence interval about p is*

$$\hat{p} \pm z_{\frac{\alpha}{2}} \cdot \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

Sample Size

- *To estimate the population mean with a margin of error **E** at a **(1-α)100%** level of confidence:*

$$n = \left(\frac{z_{\frac{\alpha}{2}} \cdot \sigma}{E}\right)^2$$

rounded up to the next integer.

- *To estimate the population proportion with a margin of error **E** at a **(1-α).100%** level of confidence:*

$$n = \hat{p}\,(1-\hat{p})\,\left({}^{z_{\frac{\alpha}{2}}}\!/_{E}\right)^2$$

rounded up to the next integer, where $\hat{p}$ *is a prior estimate of the population proportion, or*

$$n = 0.25\left({}^{z_{\frac{\alpha}{2}}}\!/_{E}\right)^2$$

rounded up to the next integer when no prior estimate of p *is available.*

Hypothesis Tests

Location Tests (Testing Claims Regarding a Population Mean)

Test	Test regarding..	Reqirements*	Test Statistic
For one sample/population			
Single sample t-test	**Mean**	▪ Normally distributed population or sample of size n > 29 ▪ **Variance of the population is not known**	$t_0 = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$
Single sample z-test	**Mean**	▪ Normally distributed population or sample of size n > 29 ▪ **Variance of the population is known**	$z_0 = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$
For two independent samples/populations			
Welch-Test	**Means**	▪ Normally distributed populations or samples of size n > 29 ▪ Variances of the populations are **not** known and not equal	$t_0 = \frac{(\bar{x}_1 - \bar{x}_2) - (y_1 - y_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$ Note: The P-Value is for t_0 is found using **the smaller of n_1-1 or n_2-1 degrees of freedom**
For two dependent samples/populations			
Two-sample t-test	**Means**	▪ The **difference** between paired observations is normally distributed or the samples are of size n > 29 ▪ **The population variance of the difference between paired observations is not known**	$t_0 = \frac{\bar{d} - \mu_d}{s_d/\sqrt{n}}$ *Where $\bar{d}$ is the mean and s_d the standard deviation of the differenced data.* Note: The P-Value for t_0 is found using **n-1 degrees of freedom**

Two-sample z-test	**Means**	▪ The **difference** between paired observations is normally distributed or the samples are of size n > 29 ▪ **The population variance of the difference between paired observations is known**	$$z_0 = \frac{\bar{d} - \mu_d}{\sigma_d / \sqrt{n}}$$ *Where* $\bar{d}$ *is the mean of the differenced data and* σ_d *the population standard deviation of the differences between paired observations.*

Table1: Location tests.
* All tests require simple random sampling

Testing Claims Regarding a Population Proportion

Test	Test regarding..	Reqirements*	Test Statistic
For one sample/population			
Single sample z-test (normal approximation of the binomial test statistic, Binomialtest)	**Proportion**	▪ **$np_0(1- p_0) \geq 10$** ▪ The sample values are independent of each other	$$z_0 = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$$

Table2: Tests regarding a population proportion
* All tests require simple random sampling

Testing Claims Regarding a Single Population Parameter (Test Statistics)

- *single mean, σ known*

$$z_0 = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}$$

- *single mean, σ unknown*

$$t_0 = \frac{\bar{x} - \mu_0}{s / \sqrt{n}}$$

- *single proportion*

$$z_0 = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1 - p_0)}{n}}}$$

Testing Claims Regarding the Difference of Two Population Parameters

- *Difference of* ***two population means****, Test Statistic for* ***Matched-Pairs data***

$$t_0 = \frac{\bar{d} - \mu_d}{s_d / \sqrt{n}}$$

where $\bar{d}$ is the mean and s_d the standard deviation of the differenced data.

Note: The P-value for t_0 is found using ***n-1 degrees of freedom****.*

- *Difference of two population means,* ***Confidence Interval for Matched-Pairs data***

$$\bar{d} \pm t_{\alpha/2} \cdot \frac{s_d}{\sqrt{n}}$$

Note: $t_{\alpha/2}$ *is found using* ***n-1 degrees of freedom****.*

- ***Comparing Two Means****, Test Statistic for* ***Independent Sampling***

  *(**Welch's t**)*

$$t_0 = \frac{(\bar{x}_1 - \bar{x}_2) - (y_1 - y_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

Note: The P-value for t_0 *is found using* ***the smaller of*** n_1***-1 or*** n_2***-1 degrees of freedom****.*

- *Difference of two population means,* ***Confidence Interval for Independent Samples***

$$(\bar{x}_1 - \bar{x}_2) \pm t_{\alpha/2} \cdot \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

Note: $t_{\alpha/2}$ *is found using the smaller of* n_1*-1 or* n_2*-1 degrees of freedom.*

- *Comparing Two Population Proportions, Test Statistic for Independent Sampling*

$$z_0 = \frac{\hat{p}_1 - \hat{p}_2 - (p_1 - p_2)}{\sqrt{\hat{p}(1-\hat{p})}\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \quad where\ \hat{p} = \frac{x_1 + x_2}{n_1 + n_2}$$

- *Compairing Two Population Proportions, Confidence Interval for Independent Samples*

$$(\hat{p}_1 - \hat{p}_2) \pm z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$$

- *Comparing Two Population Proportions, Test Statistic for Dependent Sampling*

$$z_0 = \frac{|f_{11} - f_{21}| - 1}{\sqrt{f_{11} + f_{21}}}$$

The frequencies have to be chosen according to table 1.

		Treatment B	
		Success	Failure
Treatment A	Success	f_{11}	f_{12}
	Failure	f_{21}	f_{22}

Table3: Frequencies obtained from dependent samples.

Testing Hypotheses Regarding Distributions

- *Expected Counts (when testing for goodness of fit, i.e. testing hypotheses regarding the distribution of a categorical variable based on a single population)*

$$E_i = \mu_1 = np_i \quad for \quad i = 1, 2, 3, \dots, k$$

- *Degrees of Freedom (when testing for goodness of fit, i.e. testing hypotheses regarding the distribution of a categorical variable based on a single population)*

$$With\ \ k\ different\ categories: k - 1$$

- *Expected Frequencies (for independence or homogeneity of proportions , i.e. testing hypotheses regarding the distribution of two categorical variables based on a single population or the distribution of a categorical variable based on two or more populations) based on the cross tabulation of the data:*

$$Expected\ frequency \quad = \frac{(row\ total)(column\ total)}{table\ total}$$

- *Degrees of Freedom (for independence or homogeneity of proportions , i.e. testing hypotheses regarding the distribution of two categorical variables based on a single population or the distribution of a categorical variable based on two or more populations) based on the cross tabulation of the data*

$$With\ \ r\ different\ rows\ r\ and\ c\ different\ columns:$$

$$(r - 1)(c - 1)$$

- *Chi-Square Test Statistic*

$$X_0{}^2 = \frac{(observed - expected)^2}{expected} = \sum \frac{(O_i - E_i)^2}{E_i}$$

$$i = 1, 2, 3, \dots, k$$

All $E_i \geq 1$ *and no more than 20% less than 5.*

Testing the Significance of the Least-Squares Regression Model

- *Standard Error of the Estimate*

$$s_e = \sqrt{\frac{\sum(yi - \hat{y}i)^2}{n-2}} = \sqrt{\frac{\sum residuals^2}{n-2}}$$

- *Standard Error of b_1*

$$s_{b_1} = \frac{s_e}{\sqrt{\sum(x_1 - \bar{x})^2}}$$

- *Test statistic for the Slope of the Least-Squares Regression Line*

$$t_0 = \frac{b_1 - \beta_1}{s_e \Big/ \sqrt{\sum(x_i - \bar{x})^2}} = \frac{b_1 - \beta_1}{s_{b_1}}$$

- *Confidence Interval about the Slope of the Least-Squares Regression Line*

$$b_1 \pm t_{\alpha/2} \cdot \frac{s_e}{\sqrt{\sum(x_i - \bar{x})^2}}$$

- *Confidence Interval about the Mean Response of y, $\hat{y}$*

$$\hat{y} \pm t_{\alpha/2} \cdot s_e \sqrt{\frac{1}{n} + \frac{(x^* - \bar{x})}{\sqrt{\sum(x_i - \bar{x})^2}}}$$

where x^* is the given value oft her explanatory variable and $t_{\alpha/2}$ is the critical value with n-2 degrees of freedom.

- *Prediction Interval about an Individual Response, $\hat{y}$*

$$\hat{y} \pm t_{\alpha/2} \cdot s_e \sqrt{1 + \frac{1}{n} + \frac{(x^* - \bar{x})}{\sqrt{\Sigma (x_i - \bar{x})}^2}}$$

where x^* is the given value oft her explanatory variable and $t_{\alpha/2}$ is the critical value with n-2 degrees of freedom.

Standard Normal Distribution

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
−3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
−3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
−3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
−3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
−2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
−2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
−2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
−2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
−2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
−2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
−2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
−2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
−2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
−2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
−1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
−1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
−1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
−1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
−1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
−1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
−1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
−1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
−1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
−1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
−0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
−0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
−0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
−0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
−0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
−0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
−0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
−0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
−0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
−0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

Standard Normal Distribution

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
−3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
−3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
−3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
−3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
−2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
−2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
−2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
−2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
−2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
−2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
−2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
−2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
−2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
−2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
−1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
−1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
−1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
−1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
−1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
−1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
−1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
−1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
−1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
−1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
−0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
−0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
−0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
−0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
−0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
−0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
−0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
−0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
−0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
−0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974

t-Distribution
Area in Right Tail

df	0.25	0.20	0.15	0.10	0.05	0.025	0.02	0.01	0.005	0.0025	0.001	0.0005
1	1.000	1.376	1.963	3.078	6.314	12.706	15.894	31.821	63.657	127.321	318.309	636.619
2	0.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.089	22.327	31.599
3	0.765	0.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.215	12.924
4	0.741	0.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610
5	0.727	0.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773	5.893	6.869
6	0.718	0.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317	5.208	5.959
7	0.711	0.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029	4.785	5.408
8	0.706	0.889	1.108	1.397	1.860	2.306	2.449	2.896	3.355	3.833	4.501	5.041
9	0.703	0.883	1.100	1.383	1.833	2.262	2.398	2.821	3.250	3.690	4.297	4.781
10	0.700	0.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	3.581	4.144	4.587
11	0.697	0.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497	4.025	4.437
12	0.695	0.873	1.083	1.356	1.782	2.179	2.303	2.681	3.055	3.428	3.930	4.318
13	0.694	0.870	1.079	1.350	1.771	2.160	2.282	2.650	3.012	3.372	3.852	4.221
14	0.692	0.868	1.076	1.345	1.761	2.145	2.264	2.624	2.977	3.326	3.787	4.140
15	0.691	0.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.286	3.733	4.073
16	0.690	0.865	1.071	1.337	1.746	2.120	2.235	2.583	2.921	3.252	3.686	4.015
17	0.689	0.863	1.069	1.333	1.740	2.110	2.224	2.567	2.898	3.222	3.646	3.965
18	0.688	0.862	1.067	1.330	1.734	2.101	2.214	2.552	2.878	3.197	3.610	3.922
19	0.688	0.861	1.066	1.328	1.729	2.093	2.205	2.539	2.861	3.174	3.579	3.883
20	0.687	0.860	1.064	1.325	1.725	2.086	2.197	2.528	2.845	3.153	3.552	3.850
21	0.686	0.859	1.063	1.323	1.721	2.080	2.189	2.518	2.831	3.135	3.527	3.819
22	0.686	0.858	1.061	1.321	1.717	2.074	2.183	2.508	2.819	3.119	3.505	3.792
23	0.685	0.858	1.060	1.319	1.714	2.069	2.177	2.500	2.807	3.104	3.485	3.768
24	0.685	0.857	1.059	1.318	1.711	2.064	2.172	2.492	2.797	3.091	3.467	3.745
25	0.684	0.856	1.058	1.316	1.708	2.060	2.167	2.485	2.787	3.078	3.450	3.725
26	0.684	0.856	1.058	1.315	1.706	2.056	2.162	2.479	2.779	3.067	3.435	3.707
27	0.684	0.855	1.057	1.314	1.703	2.052	2.158	2.473	2.771	3.057	3.421	3.690
28	0.683	0.855	1.056	1.313	1.701	2.048	2.154	2.467	2.763	3.047	3.408	3.674
29	0.683	0.854	1.055	1.311	1.699	2.045	2.150	2.462	2.756	3.038	3.396	3.659
30	0.683	0.854	1.055	1.310	1.697	2.042	2.147	2.457	2.750	3.030	3.385	3.646
31	0.682	0.853	1.054	1.309	1.696	2.040	2.144	2.453	2.744	3.022	3.375	3.633
32	0.682	0.853	1.054	1.309	1.694	2.037	2.141	2.449	2.738	3.015	3.365	3.622
33	0.682	0.853	1.053	1.308	1.692	2.035	2.138	2.445	2.733	3.008	3.356	3.611
34	0.682	0.852	1.052	1.307	1.691	2.032	2.136	2.441	2.728	3.002	3.348	3.601
35	0.682	0.852	1.052	1.306	1.690	2.030	2.133	2.438	2.724	2.996	3.340	3.591
36	0.681	0.852	1.052	1.306	1.688	2.028	2.131	2.434	2.719	2.990	3.333	3.582
37	0.681	0.851	1.051	1.305	1.687	2.026	2.129	2.431	2.715	2.985	3.326	3.574
38	0.681	0.851	1.051	1.304	1.686	2.024	2.127	2.429	2.712	2.980	3.319	3.566
39	0.681	0.851	1.050	1.304	1.685	2.023	2.125	2.426	2.708	2.976	3.313	3.558
40	0.681	0.851	1.050	1.303	1.684	2.021	2.123	2.423	2.704	2.971	3.307	3.551
50	0.679	0.849	1.047	1.299	1.676	2.009	2.109	2.403	2.678	2.937	3.261	3.496
60	0.679	0.848	1.045	1.296	1.671	2.000	2.099	2.390	2.660	2.915	3.232	3.460
70	0.678	0.847	1.044	1.294	1.667	1.994	2.093	2.381	2.648	2.899	3.211	3.435
80	0.678	0.846	1.043	1.292	1.664	1.990	2.088	2.374	2.639	2.887	3.195	3.416
90	0.677	0.846	1.042	1.291	1.662	1.987	2.084	2.368	2.632	2.878	3.183	3.402
100	0.677	0.845	1.042	1.290	1.660	1.984	2.081	2.364	2.626	2.871	3.174	3.390
1000	0.675	0.842	1.037	1.282	1.646	1.962	2.056	2.330	2.581	2.813	3.098	3.300
z	0.674	0.842	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807	3.090	3.291

Chi-Square (χ^2) Distribution

Degrees of Freedom	Area to the Right of Critical Value									
	0.995	0.99	0.975	0.95	0.90	0.10	0.05	0.025	0.01	0.005
1	—	—	0.001	0.004	0.016	2.706	3.841	5.024	6.635	7.879
2	0.010	0.020	0.051	0.103	0.211	4.605	5.991	7.378	9.210	10.597
3	0.072	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.345	12.838
4	0.207	0.297	0.484	0.711	1.064	7.779	9.488	11.143	13.277	14.860
5	0.412	0.554	0.831	1.145	1.610	9.236	11.070	12.833	15.086	16.750
6	0.676	0.872	1.237	1.635	2.204	10.645	12.592	14.449	16.812	18.548
7	0.989	1.239	1.690	2.167	2.833	12.017	14.067	16.013	18.475	20.278
8	1.344	1.646	2.180	2.733	3.490	13.362	15.507	17.535	20.090	21.955
9	1.735	2.088	2.700	3.325	4.168	14.684	16.919	19.023	21.666	23.589
10	2.156	2.558	3.247	3.940	4.865	15.987	18.307	20.483	23.209	25.188
11	2.603	3.053	3.816	4.575	5.578	17.275	19.675	21.920	24.725	26.757
12	3.074	3.571	4.404	5.226	6.304	18.549	21.026	23.337	26.217	28.300
13	3.565	4.107	5.009	5.892	7.042	19.812	22.362	24.736	27.688	29.819
14	4.075	4.660	5.629	6.571	7.790	21.064	23.685	26.119	29.141	31.319
15	4.601	5.229	6.262	7.261	8.547	22.307	24.996	27.488	30.578	32.801
16	5.142	5.812	6.908	7.962	9.312	23.542	26.296	28.845	32.000	34.267
17	5.697	6.408	7.564	8.672	10.085	24.769	27.587	30.191	33.409	35.718
18	6.265	7.015	8.231	9.390	10.865	25.989	28.869	31.526	34.805	37.156
19	6.844	7.633	8.907	10.117	11.651	27.204	30.144	32.852	36.191	38.582
20	7.434	8.260	9.591	10.851	12.443	28.412	31.410	34.170	37.566	39.997
21	8.034	8.897	10.283	11.591	13.240	29.615	32.671	35.479	38.932	41.401
22	8.643	9.542	10.982	12.338	14.041	30.813	33.924	36.781	40.289	42.796
23	9.260	10.196	11.689	13.091	14.848	32.007	35.172	38.076	41.638	44.181
24	9.886	10.856	12.401	13.848	15.659	33.196	36.415	39.364	42.980	45.559
25	10.520	11.524	13.120	14.611	16.473	34.382	37.652	40.646	44.314	46.928
26	11.160	12.198	13.844	15.379	17.292	35.563	38.885	41.923	45.642	48.290
27	11.808	12.879	14.573	16.151	18.114	36.741	40.113	43.195	46.963	49.645
28	12.461	13.565	15.308	16.928	18.939	37.916	41.337	44.461	48.278	50.993
29	13.121	14.256	16.047	17.708	19.768	39.087	42.557	45.722	49.588	52.336
30	13.787	14.953	16.791	18.493	20.599	40.256	43.773	46.979	50.892	53.672
40	20.707	22.164	24.433	26.509	29.051	51.805	55.758	59.342	63.691	66.766
50	27.991	29.707	32.357	34.764	37.689	63.167	67.505	71.420	76.154	79.490
60	35.534	37.485	40.482	43.188	46.459	74.397	79.082	83.298	88.379	91.952
70	43.275	45.442	48.758	51.739	55.329	85.527	90.531	95.023	100.425	104.215
80	51.172	53.540	57.153	60.391	64.278	96.578	101.879	106.629	112.329	116.321
90	59.196	61.754	65.647	69.126	73.291	107.565	113.145	118.136	124.116	128.299
100	67.328	70.065	74.222	77.929	82.358	118.498	124.342	129.561	135.807	140.169