7, 9, 4, 11, 21, 3, 9

mean $\Rightarrow \frac{7+9+4+11+21+3+9}{7} = \frac{64}{7} \simeq 9$ ✓

7, 9, 4, 11, 21, 3, 9, 1000

mean $\Rightarrow \frac{7+\dots+9+1000}{8} = \frac{1064}{8} \simeq 133$?

80% $\cup$ < 3m

20% > 10m

mean $\Rightarrow$ 6m

mod

median

7, 9, 4, 11, 21, 3, 9

mean $\Rightarrow \frac{7+9+4+11+21+3+9}{7} = \frac{64}{7} \simeq 9$ ✓

7, 9, 4, 11, 21, 3, 9, 1000

mean $\Rightarrow \frac{7+\dots+9+1000}{8} = \frac{1064}{8} \simeq 133$?

80% $\cup$ < 3m

20% > 10m

mean $\Rightarrow$ 6m

mod

median

جمع خطی :

۱- فشار خون
۲- درجه تب
۳- تعداد تپش قلب در دقیقه
۴- سن
۵- جنسیت

Batch

۶- بیماری A (y)

n = 6

m = 10,000

تعداد نمونه های آموزشی نفر ← 10,000

sample

جدول کشیدن X با

$Data \in \mathbb{R}^5$

ویژگی

feature

خصوصیت

$y \in \{Yes/NO\}$

	f_1 (فشار خون)	f_2 (درجه تب)	f_3 (WG)	. . .	f_n (بیماری y)
sample 1					Y/N
sample 2					
⋮	⋮				
sample m					

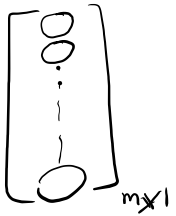
table

Matrix

2D Array

feature $\Longrightarrow$ vector

نمایش f_1 :



$f_1 \in \mathbb{R}^{10,000}$

$\vec{f_1} =$ 

$\vec{f_1}^T$

---

linear equation

$$\begin{cases} 4x_1 - 5x_2 = -13 \\ -2x_1 + 3x_2 = 9 \end{cases} \Longrightarrow Ax = b$$

$$A = \begin{bmatrix} 4 & -5 \\ -2 & 3 \end{bmatrix} \quad x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$b = \begin{bmatrix} -13 \\ 9 \end{bmatrix}$$

	آرد برنج	شکر	قیمت
شیرینی ۱	a	b	t_1
شیرینی ۲	c	d	t_2

$x_1 =$ شکر $= ?$

$x_2 =$ آرد $= ?$

$$\begin{cases} a x_1 + b x_2 = t_1 \\ c x_1 + d x_2 = t_2 \end{cases} \Rightarrow \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = ?$$

$$\Longrightarrow \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} t_1 \\ t_2 \end{pmatrix}$$

1- Basic Notation

a) $A \in \mathbb{R}^{m \times n} \Longrightarrow \begin{cases} m & \text{rows} \\ n & \text{columns} \end{cases}$

$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}_{m \times n}$ $a_{ij} \in \mathbb{R}$

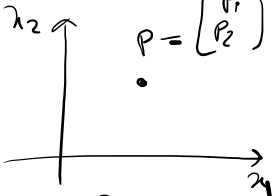
b) $x \in \mathbb{R}^n$

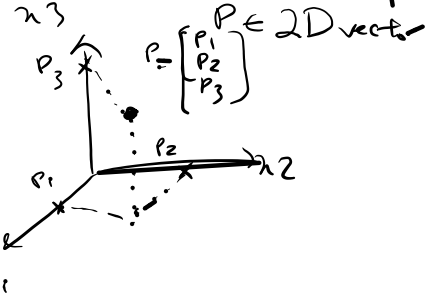
n-dimentional vector

→ Column vector $x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}_{n \times 1}$

→ row vector $x^T = \begin{bmatrix} x_1 & x_2 & \cdots & x_n \end{bmatrix}_{1 \times n}$

→ x_i x_i^T





c)

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ \vdots & & & \\ a_{m} & \cdots & \cdots & a_{mn} \end{bmatrix}$$

ستون j ام A

$A_{:j}$ ، $\vec{a}_j$

$$A = \begin{bmatrix} \uparrow & \uparrow & \uparrow & & \uparrow \\ \vec{a}_1 & a_2 & a_3 & \cdots & a_m \\ \downarrow & \downarrow & \downarrow & & \downarrow \end{bmatrix}_{C.W.M.}$$

→ Vector مرتبه 1
→ Matrix مرتبه 2
→ Tensor > 2

A ام i سطر $A_{i:}$ ، $\vec{a}_i^T$

$$A = \begin{bmatrix} \leftarrow & \vec{a}_1^T & \rightarrow \\ \leftarrow & \vec{a}_2^T & \rightarrow \\ & \vdots & \\ \leftarrow & \vec{a}_m^T & \rightarrow \end{bmatrix}_{R.W.M}$$

$\vec{a}_i^T$ $\vec{a}_j$

سطر i ام از ماتریس A

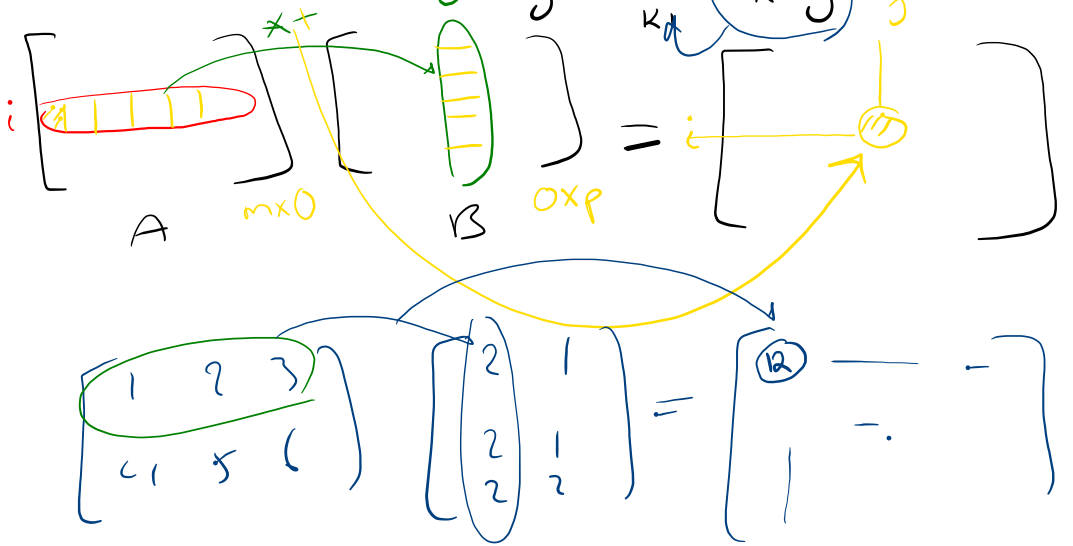
$\vec{a}_i^T \in \mathbb{R}^n$ بردار $a_i^T \in \mathbb{R}$ عضو

Matrix operation:

$A \in \mathbb{R}^{m \times n}$ $B \in \mathbb{R}^{n \times p}$

$C = AB \in \mathbb{R}^{m \times p}$

$c_{ij} = \sum_{k=1}^{n} A_{ik} B_{kj}$



→ $AB \neq BA$

vector – vector Products

$x, y \in \mathbb{R}^n$ $\quad x = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} \quad y = \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix}$ $\quad \vec{x} \cdot \vec{y} = \vec{y} \cdot \vec{x}$

$x^T y \in \mathbb{R}$ inner product (dot product)

$$x^T y = \sum_{i=1}^{n} x_i y_i \qquad \begin{bmatrix} x_1 & \cdots & x_n \end{bmatrix} \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix} = x_1 y_1 + x_2 y_2 + \cdots + x_n y_n$$

$$x^T y = \langle \vec{x}, \vec{y} \rangle = \langle \vec{x} \cdot \vec{y} \rangle = \vec{x} \cdot \vec{y} = \sum_{i=1}^{n} x_i y_i$$

$x^T y = y^T x \in \mathbb{R}$

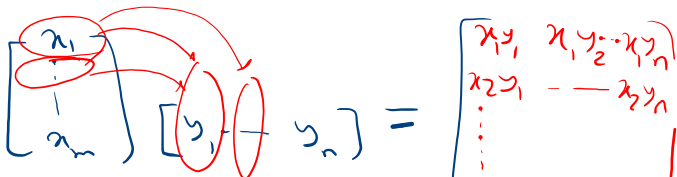
—

$$x^T y = \sum_i x_i y_i = \sum_i y_i x_i = y^T x$$

$x \in \mathbb{R}^m \quad y \in \mathbb{R}^n$ $\begin{cases} n \neq m \\ n = m \end{cases}$

Outer product

$xy^T \in \mathbb{R}^{m \times n}$



$$x = \begin{bmatrix} 1 \\ -5 \\ 4 \\ 3 \\ 2 \end{bmatrix} \quad \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \end{bmatrix}_{10} \quad A = \overbrace{\begin{bmatrix} 1 & 1 & \cdots & 1 \\ -5 & -5 & \cdots & -5 \\ 4 & 4 & \cdots & 4 \\ 3 & 3 & \cdots & 3 \\ 2 & 2 & \cdots & 2 \end{bmatrix}}^{10}$$

$$A = \begin{bmatrix} \uparrow & \uparrow & & \uparrow \\ x & x & \cdots & x \\ \downarrow & \downarrow & & \downarrow \end{bmatrix}$$

$x 1_{10}^T = A \qquad A^T$

$A = x 1^T$

5×10

$A^T = 1 x^T$

$x \in \mathbb{R}^m \qquad y \in \mathbb{R}$

$$x y^T = xy = \begin{bmatrix} y x_1 \\ y x_2 \\ \vdots \\ y x_m \end{bmatrix}$$

Matrix-vector product