$$
\begin{cases}
a_{11}x_1 + a_{12}x_2 + \cdots + a_{1m}x_m = b_1 \\
\vdots \qquad \ddots \\
a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nm}x_m = b_n
\end{cases}
$$

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1m} \\ \vdots & & \ddots & \\ a_{n1} & a_{n2} & \cdots & a_{nm} \end{bmatrix}$$

$$\vec{x} = \begin{bmatrix} x_1 \\ \vdots \\ x_m \end{bmatrix} \qquad \vec{b} = \begin{bmatrix} b_1 \\ \vdots \\ b_n \end{bmatrix}$$

$$\Downarrow$$

$$\vec{b} = A\vec{x}$$

$$\Downarrow$$

$$\boxed{A\vec{x} = \vec{b}} \Longrightarrow \text{معادله ماتریسی}$$

$$A \in \mathbb{R}^{n \times m} \qquad \vec{x} \in \mathbb{R}^{m} \qquad \vec{b} \in \mathbb{R}^{n}$$

$$A\vec{x}=\vec{b} \xrightarrow{\vec{x}=?} A^{-1}A\vec{x}=A^{-1}b \Longrightarrow I\vec{x}=A^{-1}b \Rightarrow \boxed{\vec{x}=A^{-1}b}$$ ✓

وجود داشته باشد

0- A is square

1- A Full rank

$A\in\mathbb{R}^{n\times m}$ $\quad n\neq m$

1

$$A=\begin{bmatrix}1&2\\3&4\\5&6\end{bmatrix}_{3\times2} \longrightarrow \begin{bmatrix}\ \end{bmatrix}_{\substack{3\times3\\2\times2}}$$

$$AA^T=\begin{bmatrix}\ \end{bmatrix}_{3\times2}\times\begin{bmatrix}\ \end{bmatrix}_{2\times3}=\begin{bmatrix}\ \end{bmatrix}_{3\times3}$$

$$\boxed{A^TA}=\begin{bmatrix}\ \end{bmatrix}_{2\times3}\times\begin{bmatrix}\ \end{bmatrix}_{3\times2}=\begin{bmatrix}\ \end{bmatrix}_{2\times2}$$

$$A\vec{x}=\vec{b} \qquad A\in\mathbb{R}^{n\times m} \qquad \vec{x}\in\mathbb{R}^{m} \qquad \vec{b}=\mathbb{R}^{n}$$

$$(A^TA)\vec{x}=A^T\vec{b} \Rightarrow A'\vec{x}=A^Tb \Rightarrow \vec{x}=A'^{-1}A^Tb$$

$$\underbrace{(A^TA)^{-1}(A^TA)}_{I}\vec{x}=(A^TA)^{-1}A^Tb \Rightarrow \boxed{\vec{x}=(A^TA)^{-1}A^Tb}$$

$$A^{\tau}=(A^TA)^{-1}A^T$$

شبه معکوس A' $\Longleftarrow$ مربع نیست A

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ \boxed{5 \quad 6} \end{bmatrix} \qquad x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \qquad b = \begin{bmatrix} -3 \\ -5 \\ -7 \end{bmatrix}$$

$$\text{rank}(A) = 2$$

$x_1 = ?$ $\quad 1$

$x_2 = ?$ $\quad -2$

$$\vec{x} = \underline{(A^T A)}^{-1} A^T \vec{b}$$

$$\downarrow$$

$$A^t A$$

$$\begin{cases} x_1 + 2x_2 = -3 \\ 3x_1 + 4x_2 = -5 \\ 5x_1 + 6x_2 = -7 \end{cases}$$

$$A=\begin{bmatrix}1 & 1 & 1\\ 1 & -1 & 2\\ 0 & 1 & 3\\ 3 & 1 & 5\end{bmatrix} \qquad x=\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} \qquad \vec{b}=\begin{bmatrix}3\\ 2\\ 2\\ 8\end{bmatrix}$$

$\vec{x}=?$

$$\boxed{\vec{x}=(A^T A)^{-1}A^T\vec{b}}$$

$r_4 = 2r_1 + r_2$

```
import numpy as np
A = np.array(____)
b = np.array(____).reshape(-1,1)
x = np.linalg.inv(A.T@A)@A.T@b
```

$\boxed{\text{numpy}}$

1) pip install numpy
easy_install numpy

? A^{-1} ?

$(A^T A)^{-1}$?

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 2 \\ \cancel{2} & \cancel{0} & 3 \end{bmatrix}_{3\times 3} \qquad \vec{b} = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$$

$\vec{x} = A^{-1}\vec{b}$ X

$r_3 = r_1 + r_2$

$A^{-1} \Rightarrow \boxed{\text{rank}(A)} = \cancel{3}$ X 2

X

$A^{-1} = \frac{1}{|A|} \text{adj}(A)$ X

روشهای [illegible]

① $\det(I) = 1$

$$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 1 \times 1 - 0 \times 0 = 1$$

$$I_3 = \begin{bmatrix} \boxed{1 \quad 0 \quad 0} \\ 0 \quad 1 \quad 0 \\ 0 \quad 0 \quad 1 \end{bmatrix} = 1 \times \underbrace{\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}}_{1} \underbrace{- 0 \times \begin{vmatrix} 0 & 0 \\ 0 & 1 \end{vmatrix} + 0 \times \begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix}}_{0} = 1$$

$I_n \Rightarrow \det(I_n) = 1$

② $A = \begin{bmatrix} — a_1^T — \\ — a_2^T — \\ \vdots \\ — a_m^T — \end{bmatrix}$

$$\left| \begin{bmatrix} — t a_1^T — \\ — a_2^T — \\ \vdots \\ — a_m^T — \end{bmatrix} \right| = t|A|$$

2

③ $$\left| \begin{bmatrix} — a_2^T — \\ — a_1^T — \\ \vdots \\ — a_m^T — \end{bmatrix} \right| = -|A|$$

4) $A \in \mathbb{R}^{n \times n} \implies |A| = |A^T|$

5) $A, B \in \mathbb{R}^{n \times n}$

$\implies |AB| = |A||B|$

6) if $|A| = 0 \implies A \implies$ singular [illegible]

7) $A \in \mathbb{R}^{n \times n}$ & $|A| \neq 0 \implies |A^{-1}| = \dfrac{1}{|A|}$

Quadratic Forms:

معادله خطی ماتریسی

Ax

$A \in \mathbb{R}^{n \times n}$, $x \in \mathbb{R}^{n}$

$\boxed{x^T A x} \in \mathbb{R}$

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \quad x = \begin{bmatrix} -5 \\ 6 \end{bmatrix}$$

$$\underbrace{\underset{1\times 2}{\begin{bmatrix} -5 & 6 \end{bmatrix}} \underset{2\times 2}{\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}}} \begin{bmatrix} -5 \\ 6 \end{bmatrix}$$

$$\underset{1\times 2}{\begin{bmatrix} 13 & 14 \end{bmatrix}} \underset{2\times 1}{\begin{bmatrix} -5 \\ 6 \end{bmatrix}} = -65 + 84 = \boxed{19}$$

1- Positive definit (PD)

$A \in \mathbb{R}^{n \times n}$ if $\vec{x} \in \mathbb{R}^n$ & $\vec{x} \neq \vec{0}$ $\quad \vec{x}^T A \vec{x} > 0$

2- negative definit (ND)

$\vec{x}^T A \vec{x} < 0$

$\vec{x}^T A \vec{x}$ مشتق ماتریس

$A \to PD \Rightarrow A^{-1}$