



DeepLearning.AI

Math for Machine Learning

Probability and Statistics - Week 4

W4 Lesson 1



DeepLearning.AI

Confidence Interval

**Confidence Interval
(Known Standard Deviation)**

Confidence Interval - Intuition

Confidence Interval - Intuition

Statistopia

10,000 people

Confidence Interval - Intuition

Statistopia

10,000 people

Estimate μ

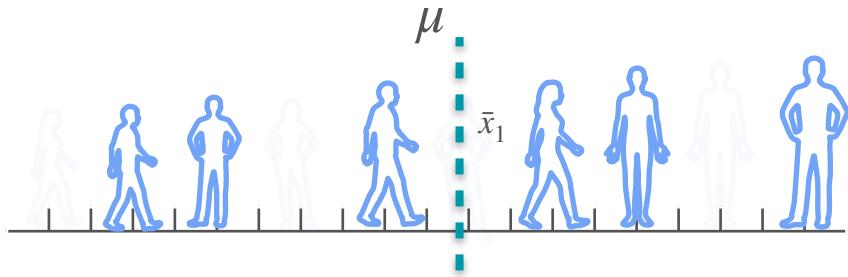
Confidence Interval - Intuition

Statistopia

10,000 people

Estimate μ
(mean height of the population)

Confidence Interval - Intuition

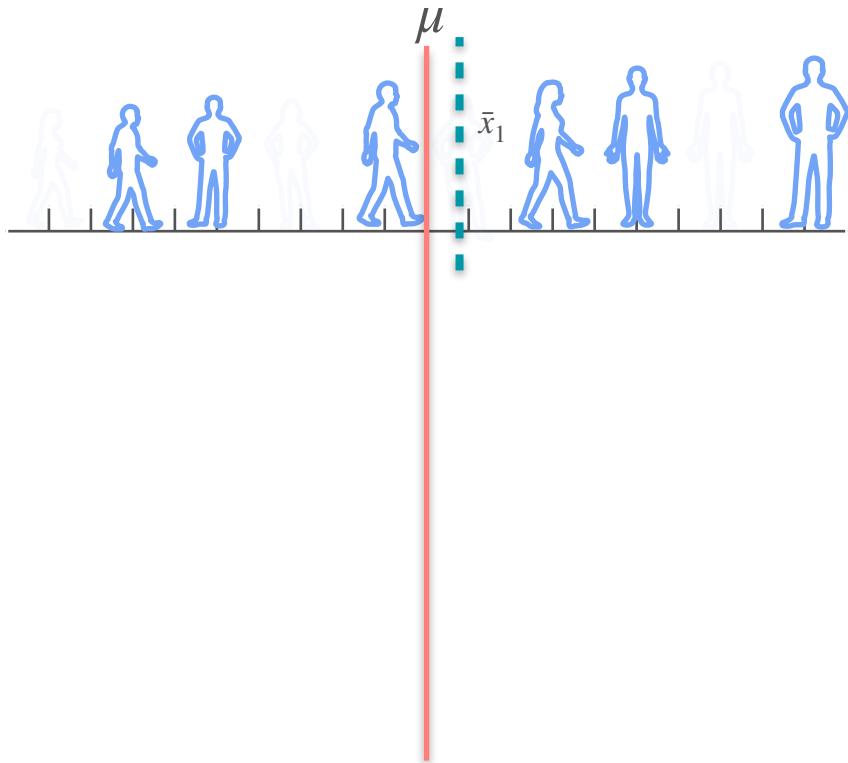


Statistopia

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Estimate μ
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Confidence Interval - Intuition

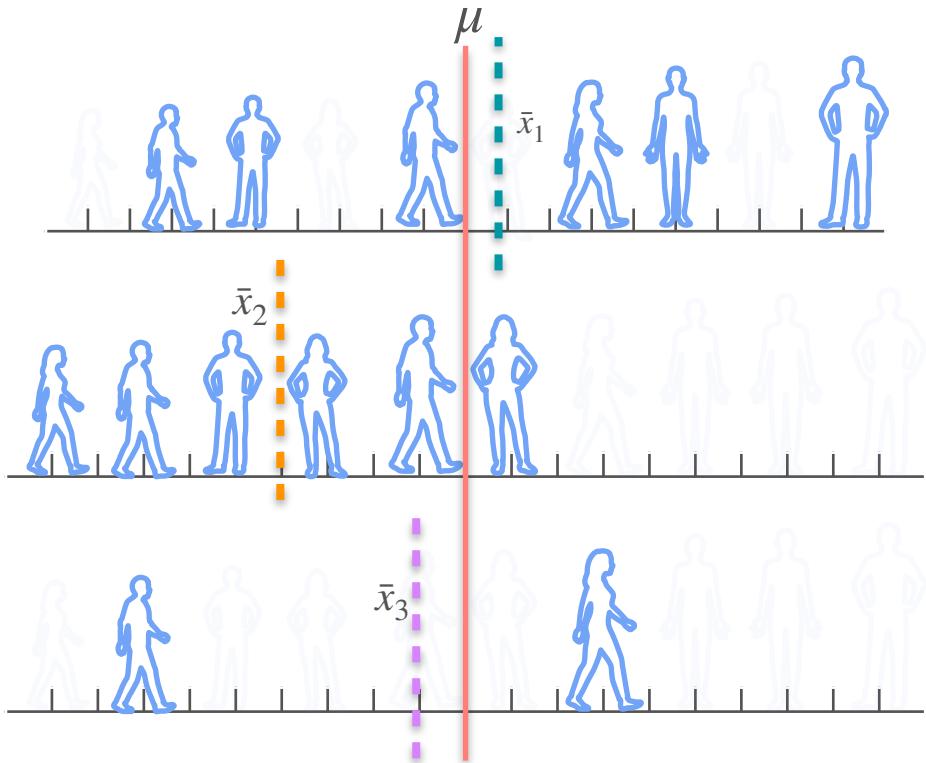


Statistopia

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Confidence Interval - Intuition

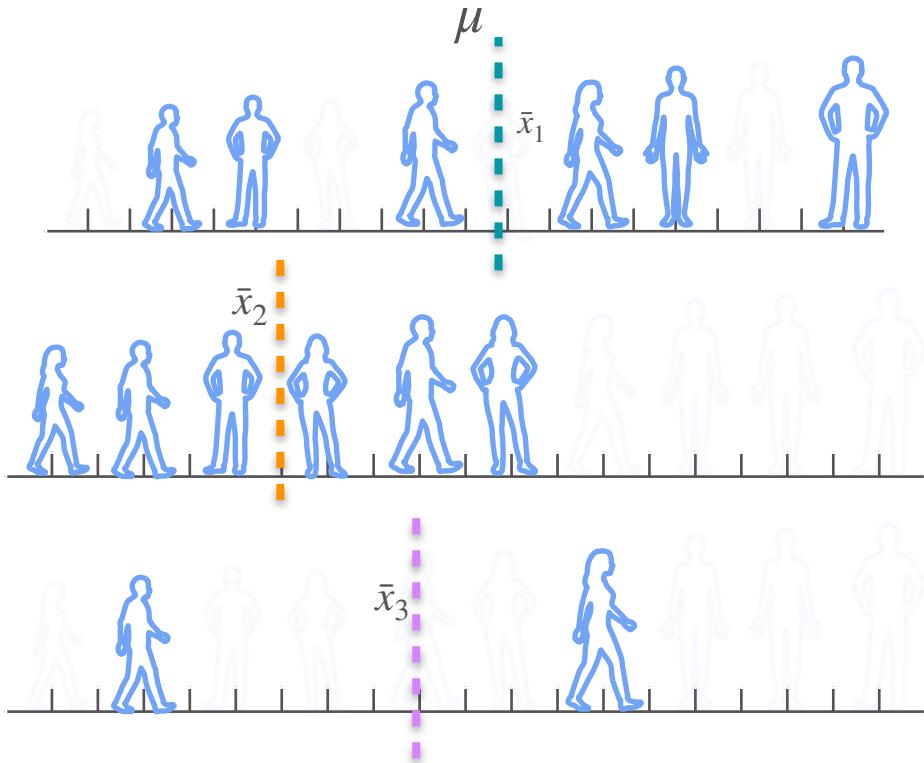


Statistopia

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Estimate μ
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Confidence Interval - Intuition

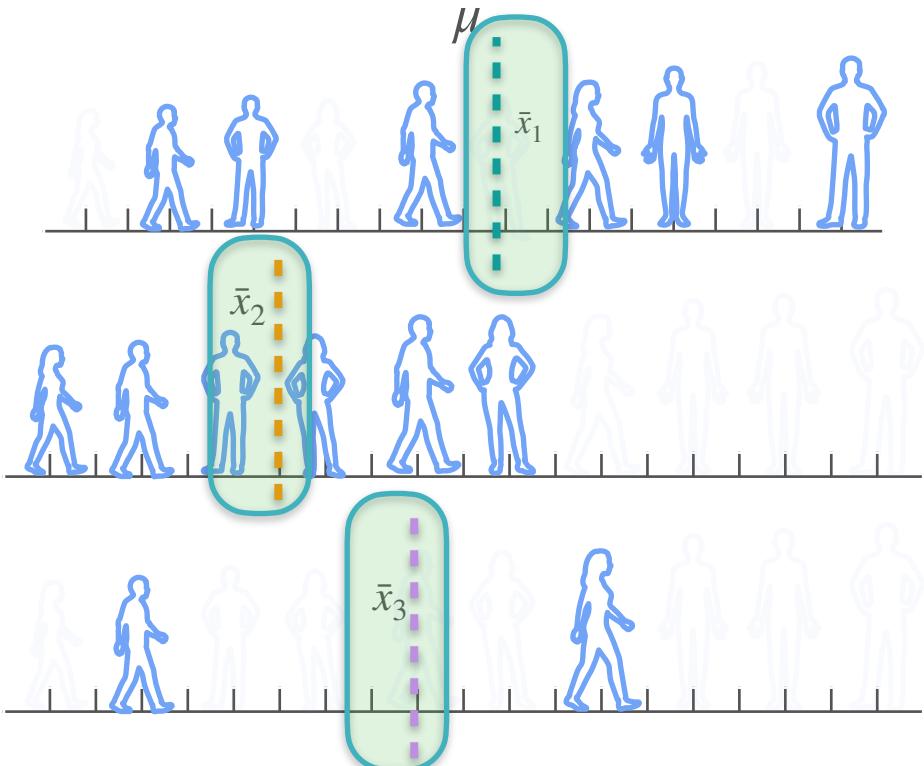


Statistopia

10,000 people

Estimate μ
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Confidence Interval - Intuition



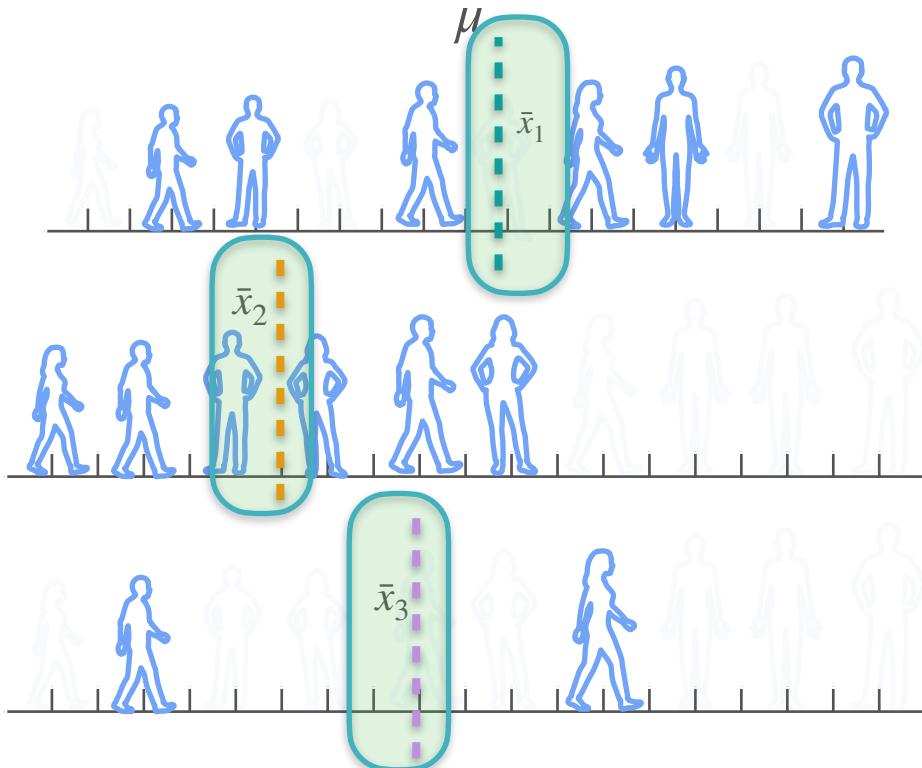
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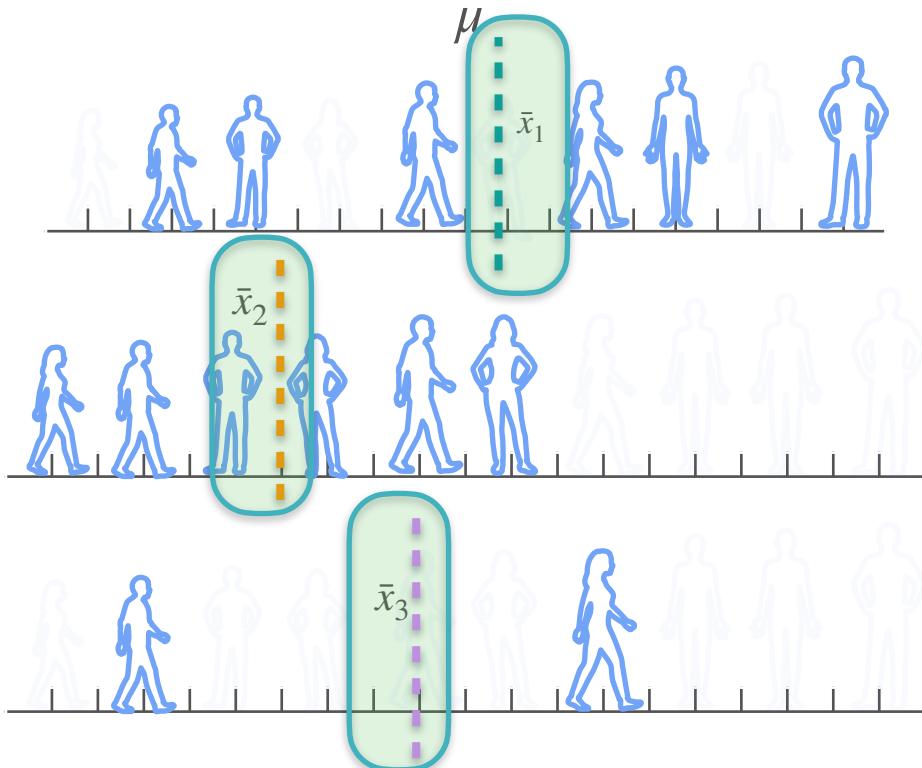
Can you use these sample means with
some degree of certainty?

Confidence Interval - Intuition



Can you use these sample means with some degree of certainty?

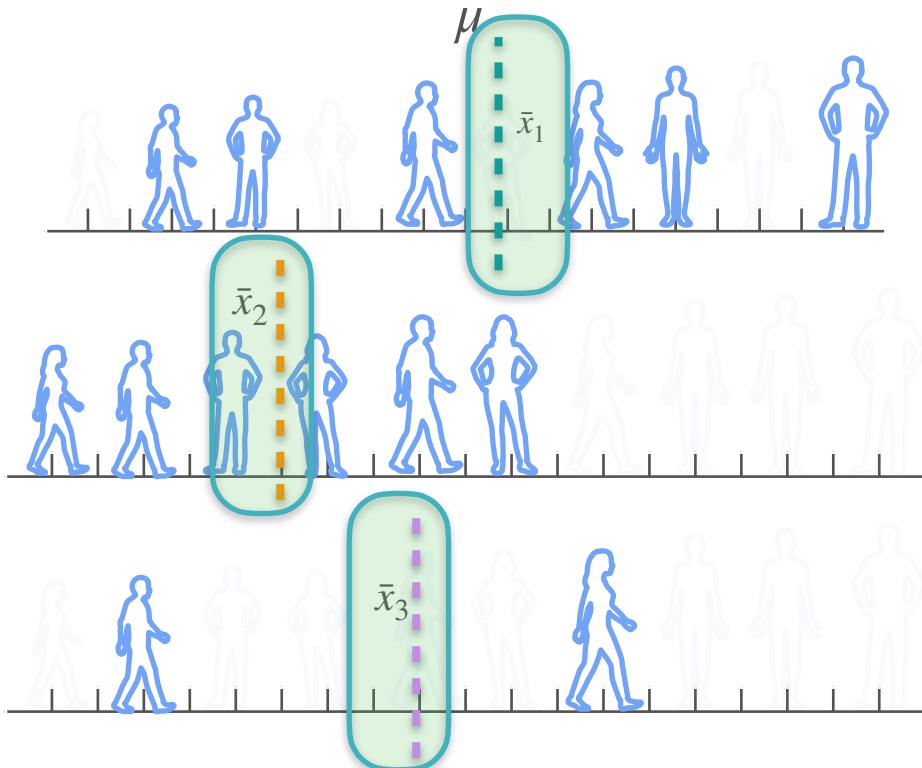
Confidence Interval - Intuition



Can you use these sample means with some degree of certainty?

**Confidence
Interval**

Confidence Interval - Intuition



Can you use these sample means with some degree of certainty?

Confidence Interval

$$\text{lower limit} < \bar{x} < \text{upper limit}$$

Confidence Interval - Intuition

Confidence Interval - Intuition

$n = 1$



Confidence Interval - Intuition

$$n = 1$$



$$\bar{x}$$

Confidence Interval - Intuition

$n = 1$



\bar{x}

Central Limit Theorem

Confidence Interval - Intuition

$$n = 1$$

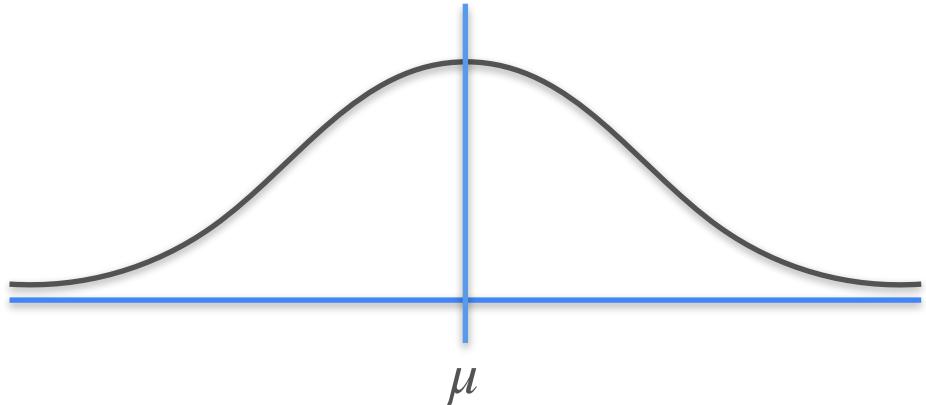


$$\bar{x}$$

Central Limit Theorem

population standard deviation (σ)

$$\bar{X} \sim \mathcal{N}(\mu, \sigma^2)$$



Confidence Interval - Intuition

$$n = 1$$

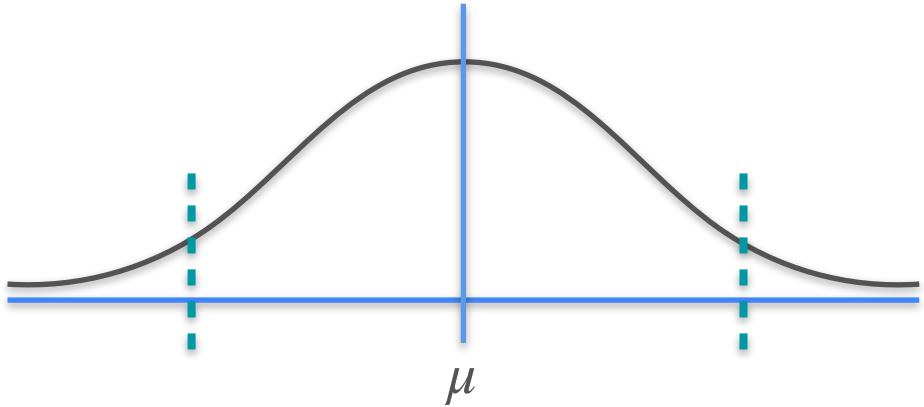


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Confidence Interval - Intuition

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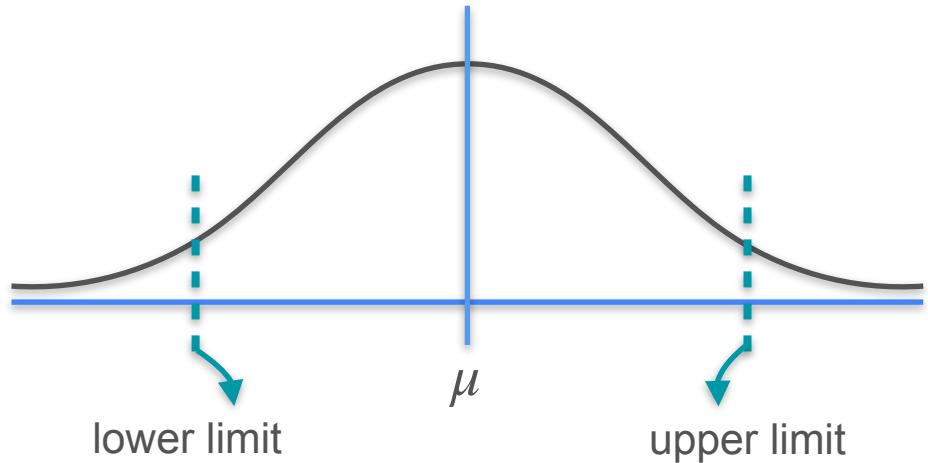


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Confidence Interval - Intuition

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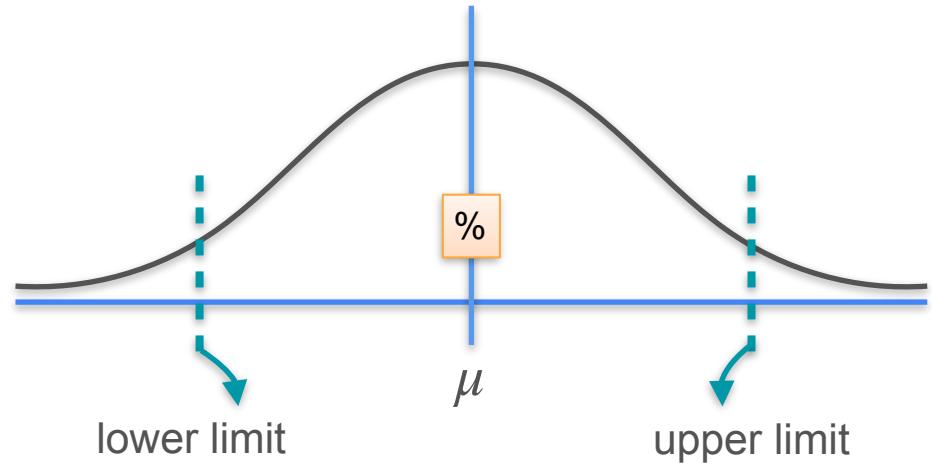


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Confidence Interval - Intuition

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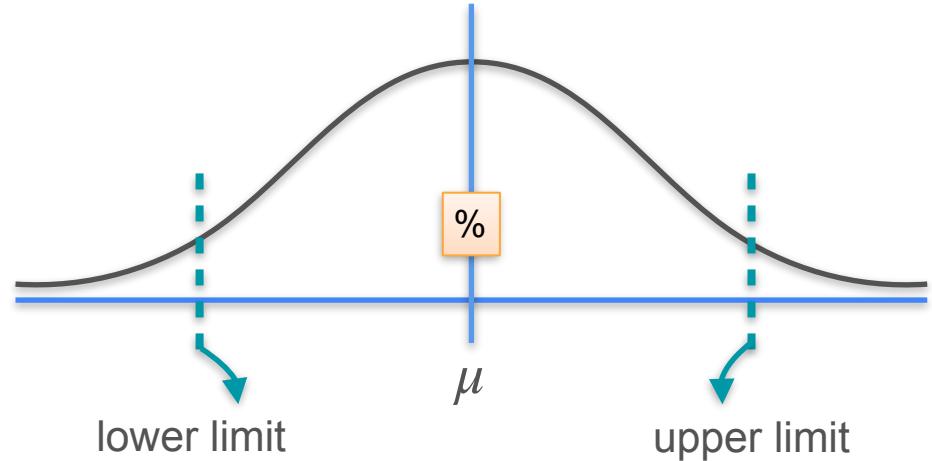
$$\bar{x}$$

α
significance level

Central Limit Theorem

population standard deviation (σ)

$$\bar{X} \sim \mathcal{N}(\mu, \sigma^2)$$



Confidence Interval - Intuition

$n = 1$



\bar{x}

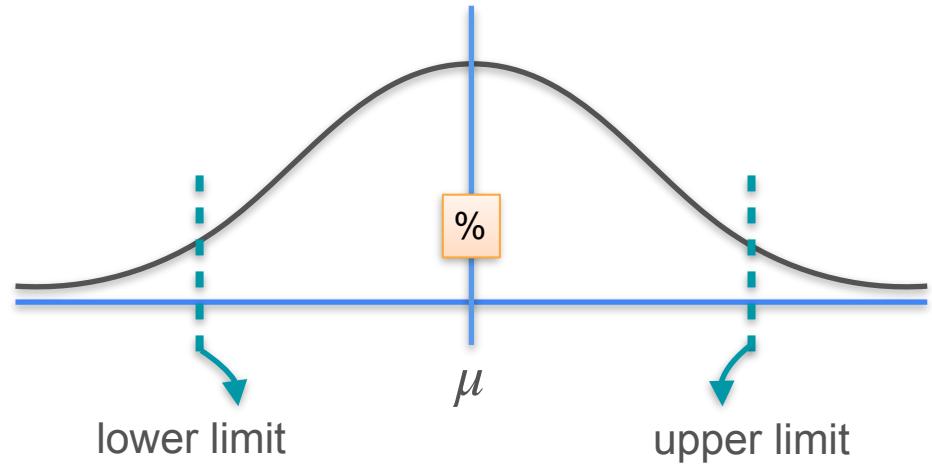
$1 - \alpha$

α
significance level

Central Limit Theorem

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Confidence Interval - Intuition

$n = 1$



\bar{x}

α
significance level

Confidence level

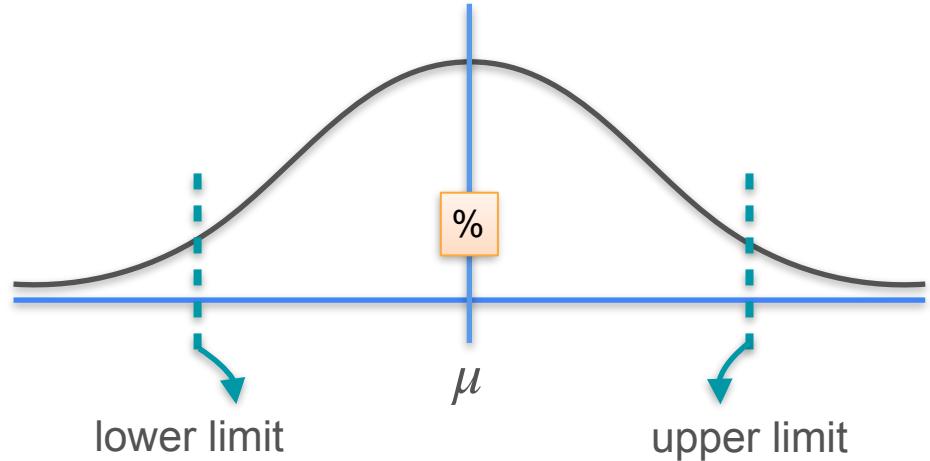


$1 - \alpha$

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Confidence Interval - Intuition

$n = 1$



\bar{x}

α
significance level

Confidence level

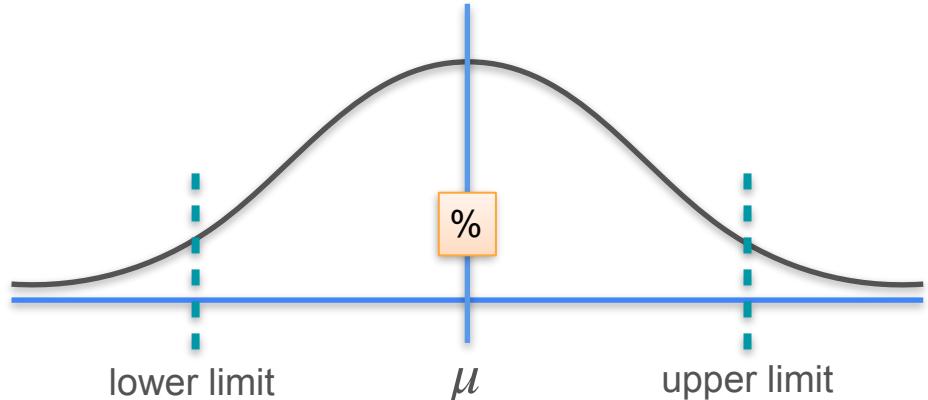


$1 - \alpha$

Central Limit Theorem

population standard deviation (σ)

$$\bar{X} \sim \mathcal{N}(\mu, \sigma^2)$$



Confidence Interval - Intuition

$n = 1$



\bar{x}

$\alpha = 0.05$
significance level

Confidence level

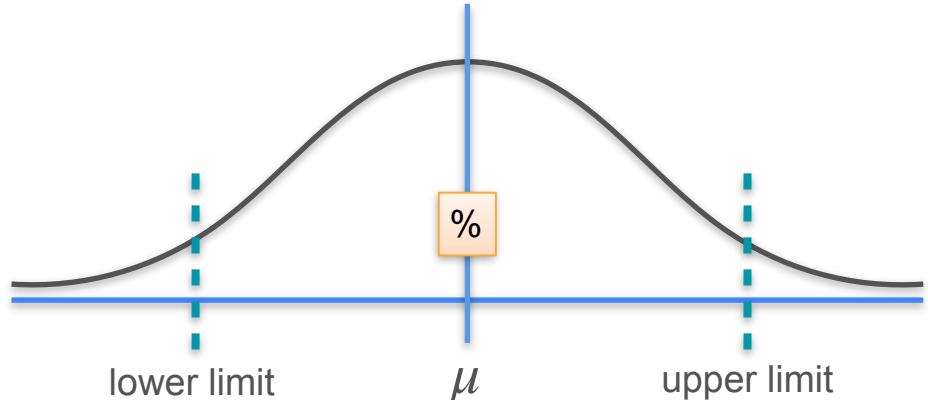


$1 - \alpha$

Central Limit Theorem

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Confidence Interval - Intuition

$$n = 1$$



Confidence level



$$\bar{x}$$

$$\alpha = 0.05$$

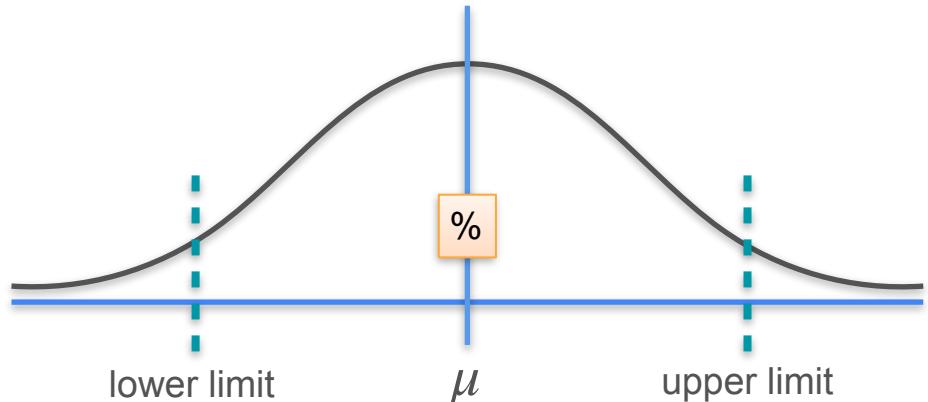
significance level

$$1 - \alpha = 0.95$$

Central Limit Theorem

population standard deviation (σ)

$$\bar{X} \sim \mathcal{N}(\mu, \sigma^2)$$



Confidence Interval - Intuition

$$n = 1$$



Confidence level

95%

$$\bar{x}$$

$$1 - \alpha$$

$$\alpha = 0.05$$

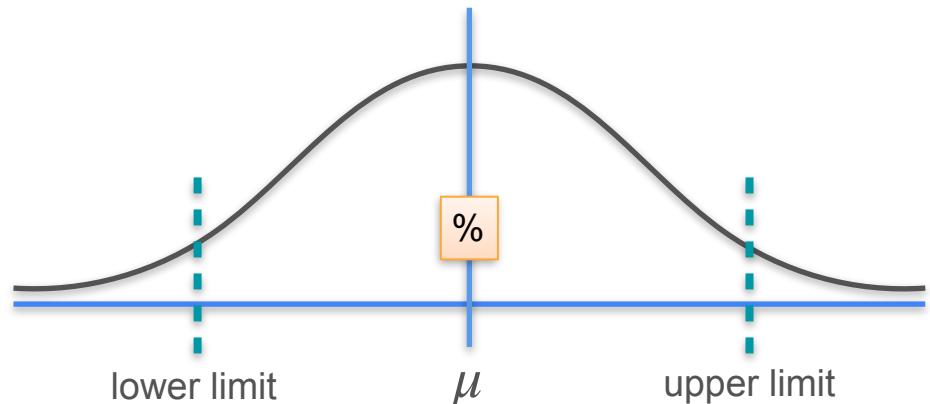
significance level

$$1 - 0.05 = 0.95$$

Central Limit Theorem

population standard deviation (σ)

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Confidence Interval - Intuition

$$n = 1$$



$$\bar{x}$$

$$\alpha = 0.05$$

significance level

Confidence level

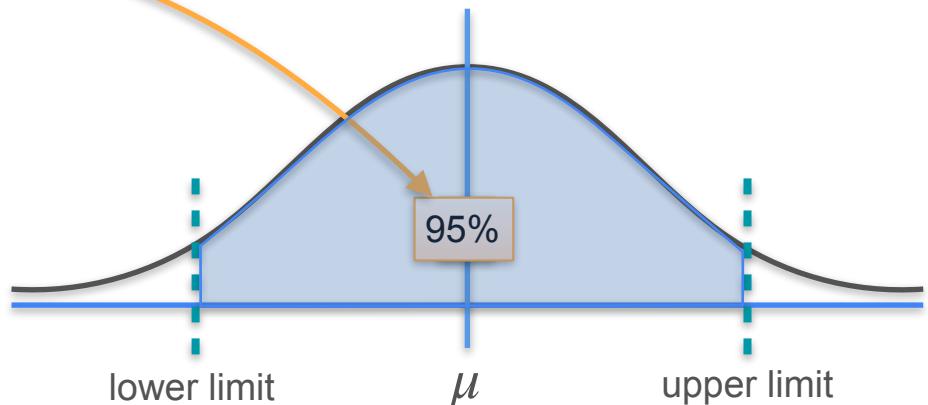
95%

$$1 - \alpha = 0.95$$

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Confidence Interval - Intuition

$$n = 1$$



$$\bar{x}$$

$$\alpha = 0.05$$

significance level

Confidence level

95%

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Central Limit Theorem

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$$1 - \alpha$$

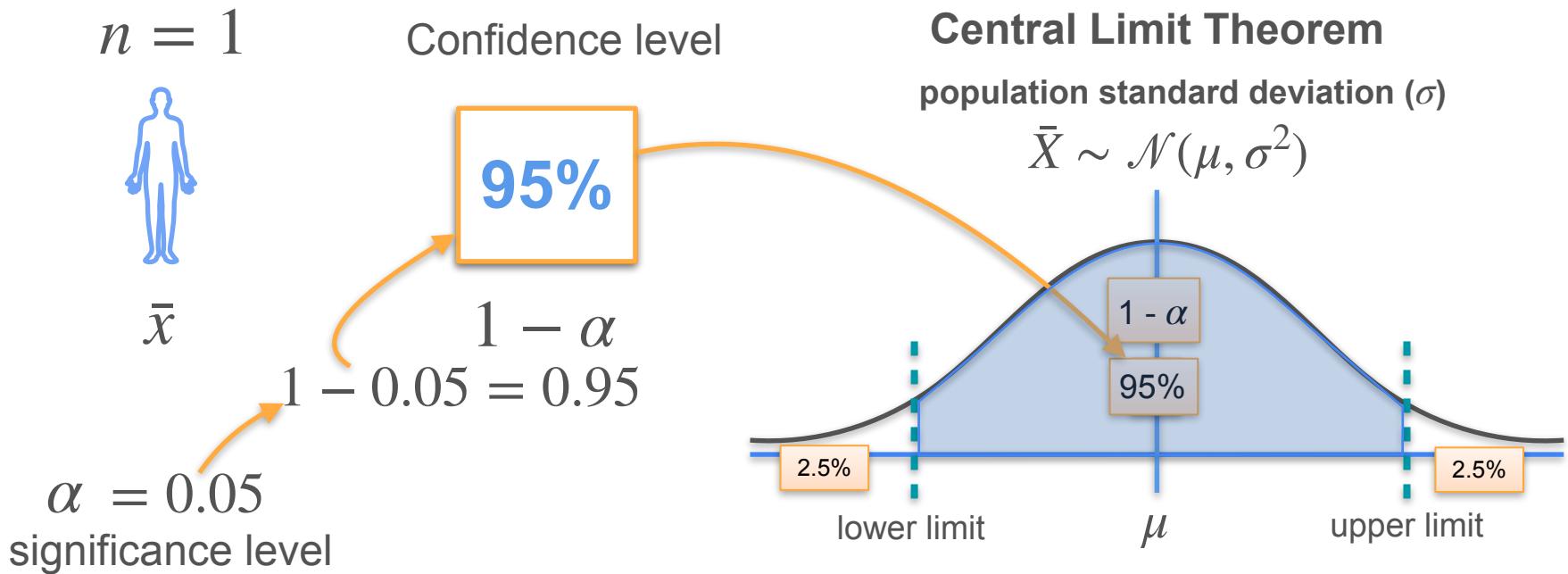
95%

lower limit

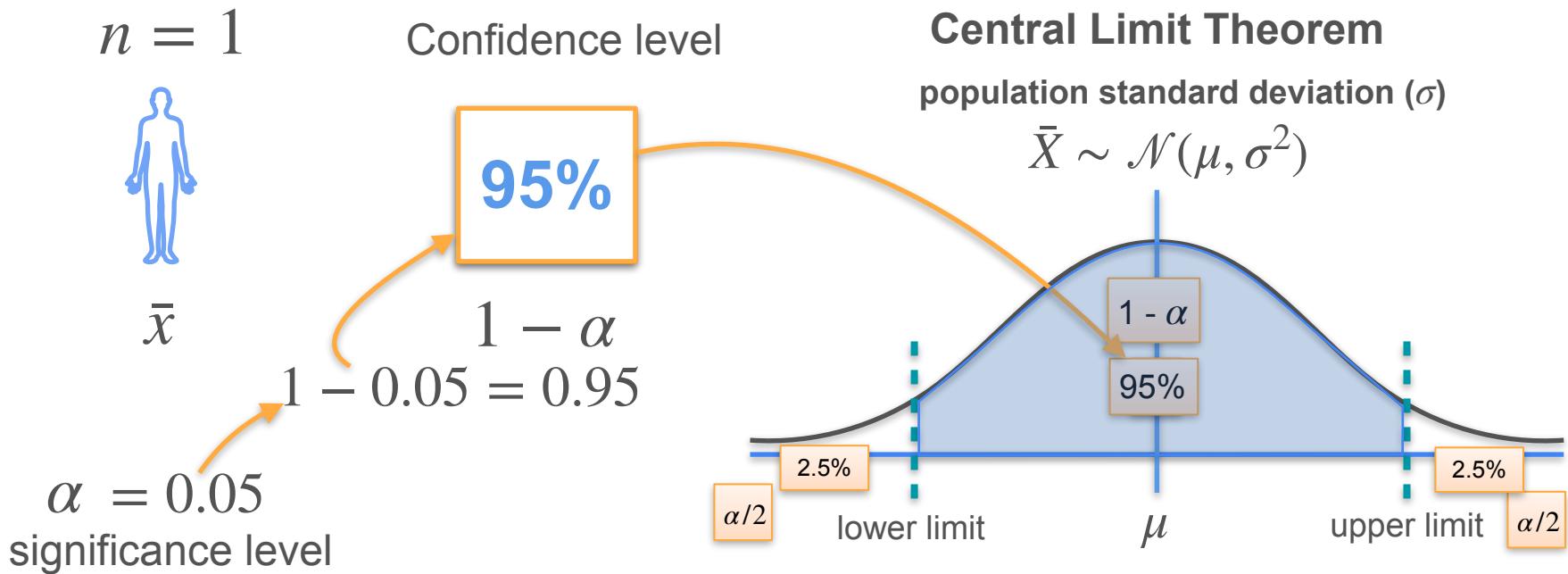
$$\mu$$

upper limit

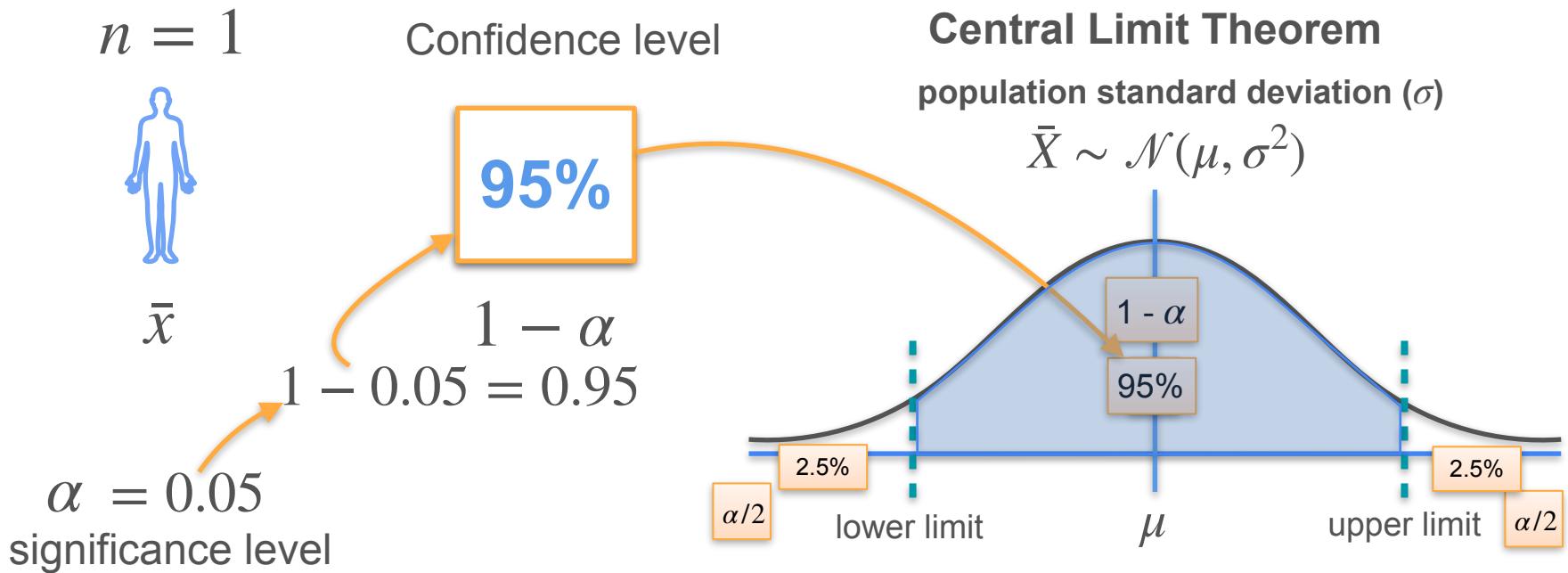
Confidence Interval - Intuition



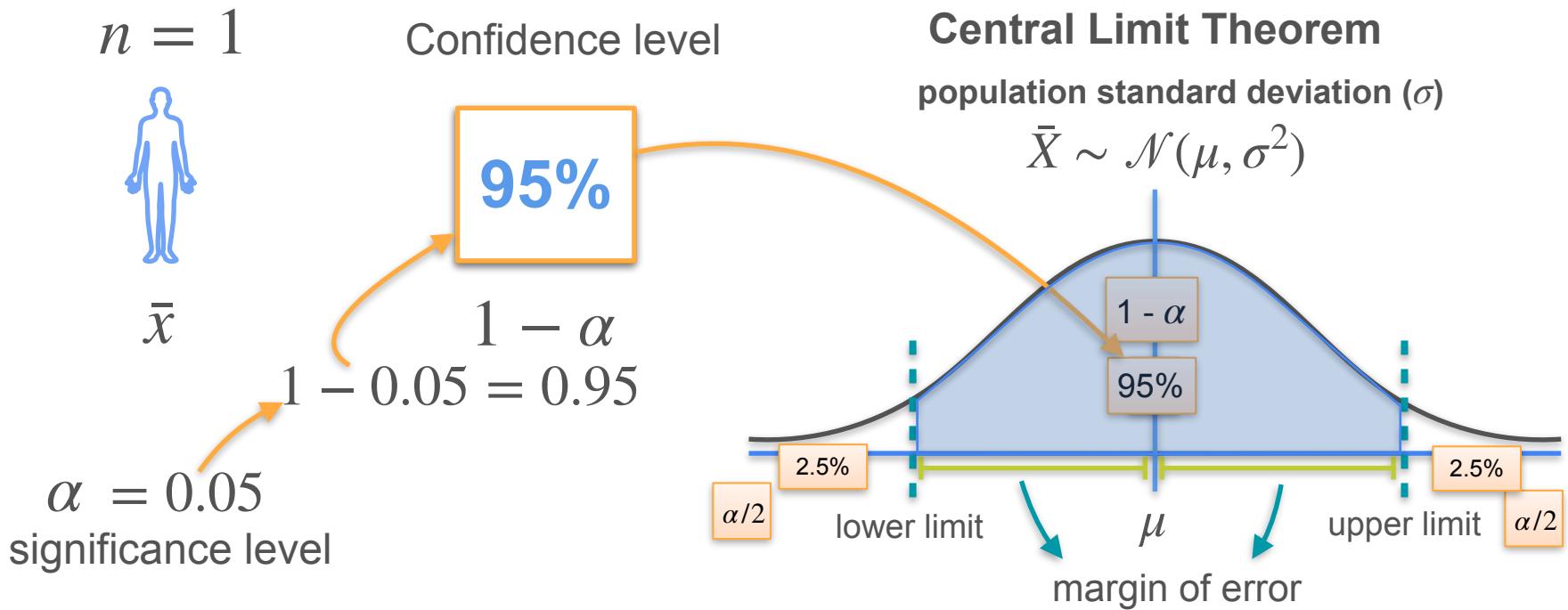
Confidence Interval - Intuition



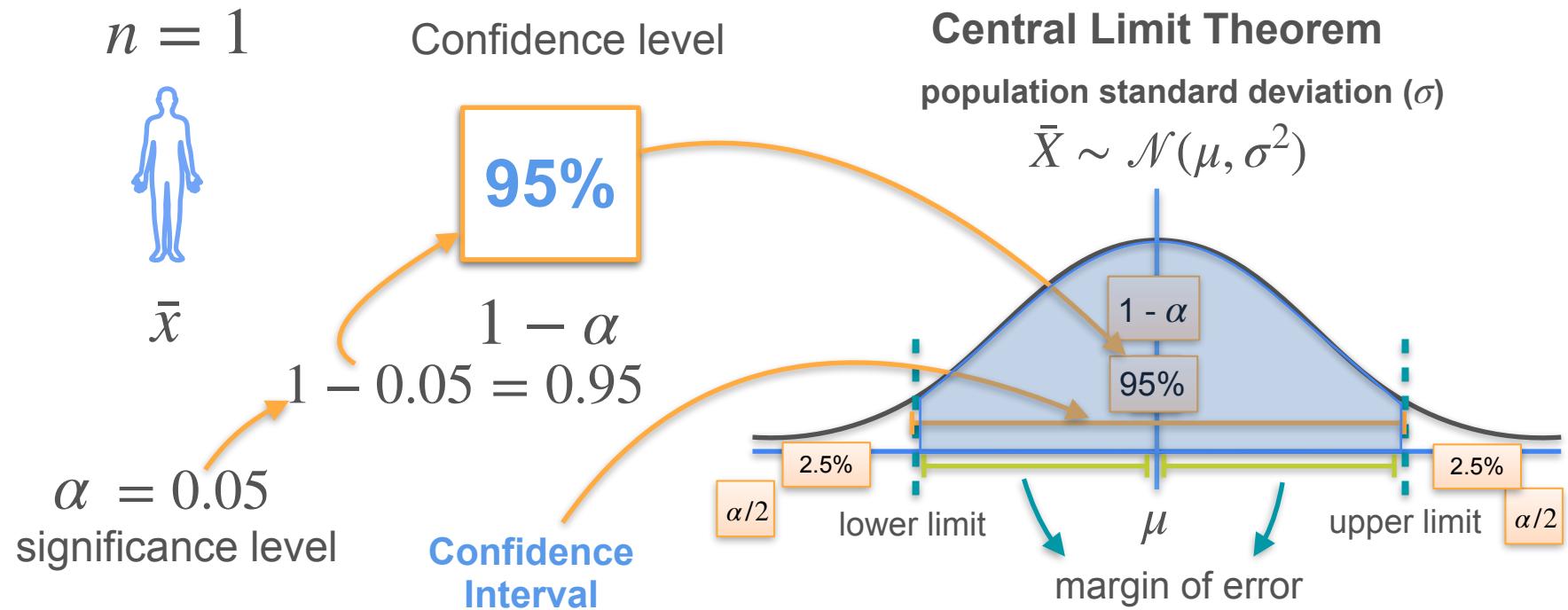
Confidence Interval - Intuition



Confidence Interval - Intuition



Confidence Interval - Intuition



Confidence Interval - Intuition

$$n = 1$$

Known σ

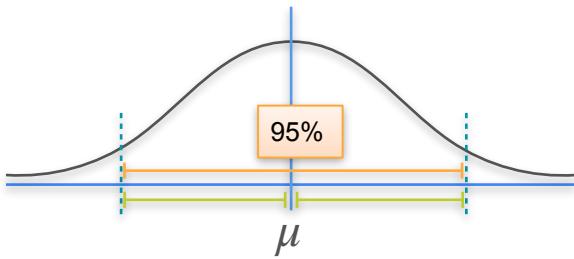
95%

Confidence Interval - Intuition

$$n = 1$$

Known σ

95%



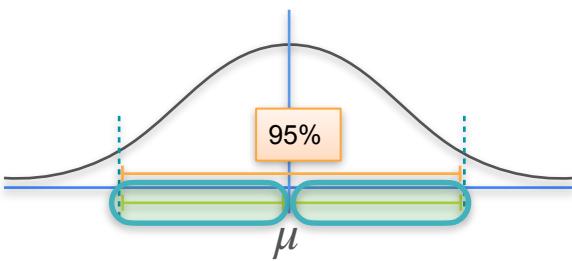
Confidence Interval - Intuition

$$n = 1$$

Known σ

95%

Margin of error



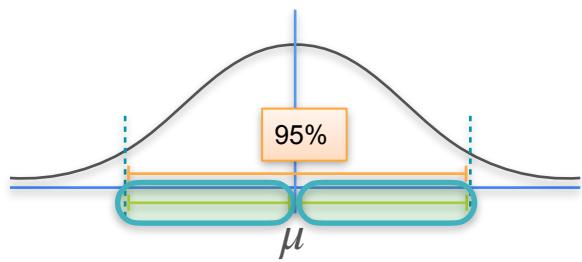
Confidence Interval - Intuition

$n = 1$

Known σ

95%

Margin of error



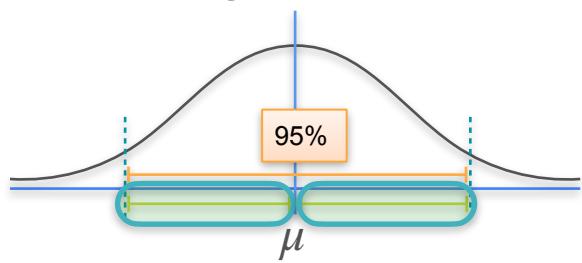
Confidence Interval - Intuition

$n = 1$

Known σ

95%

Margin of error



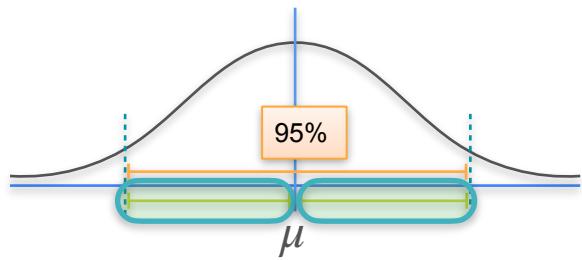
Confidence Interval - Intuition

$n = 1$

Known σ

95%

Margin of error



\bar{x}_1



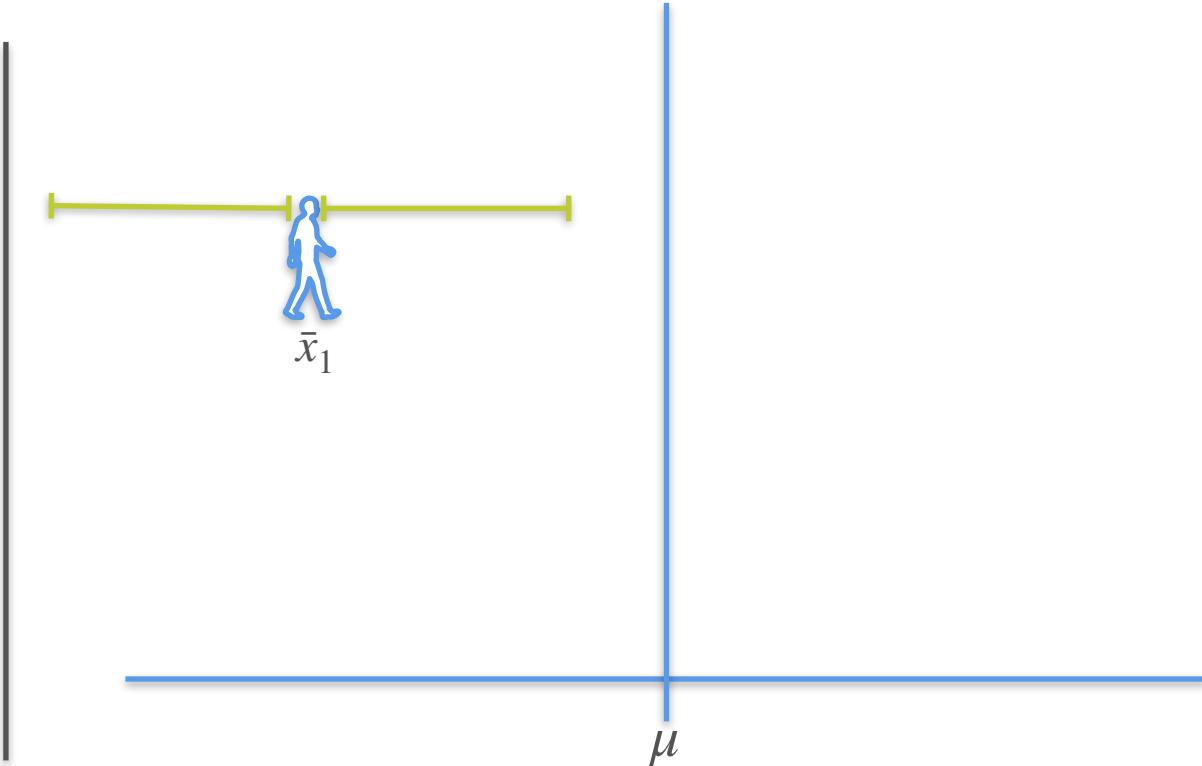
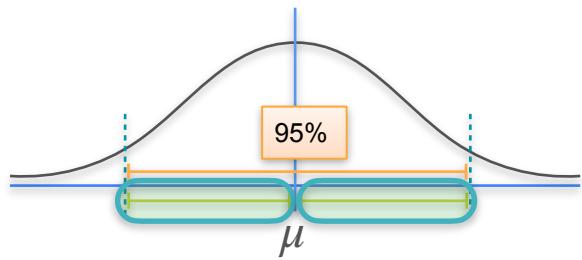
Confidence Interval - Intuition

$n = 1$

Known σ

95%

Margin of error



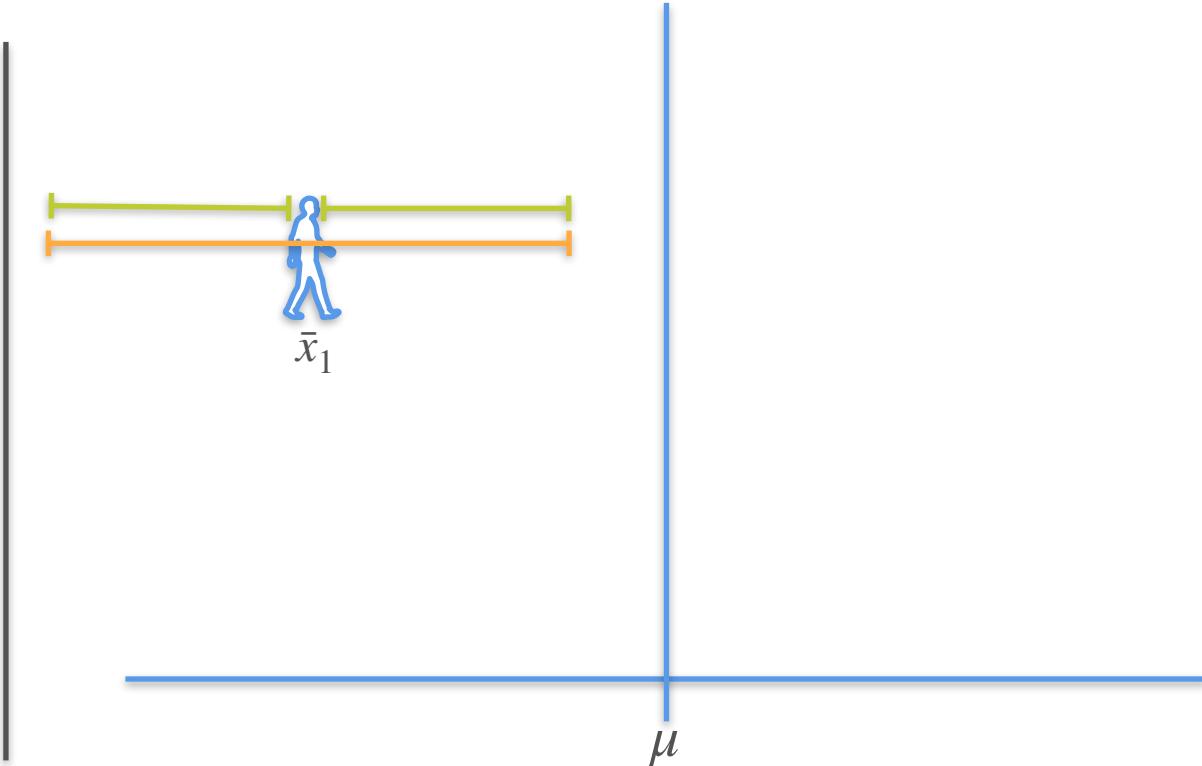
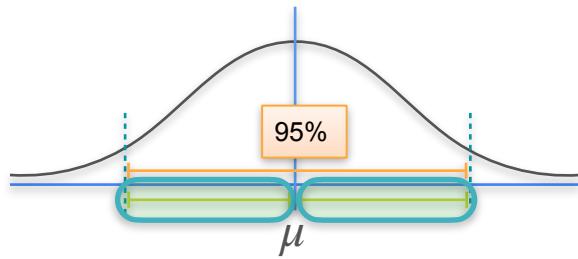
Confidence Interval - Intuition

$n = 1$

Known σ

95%

Margin of error



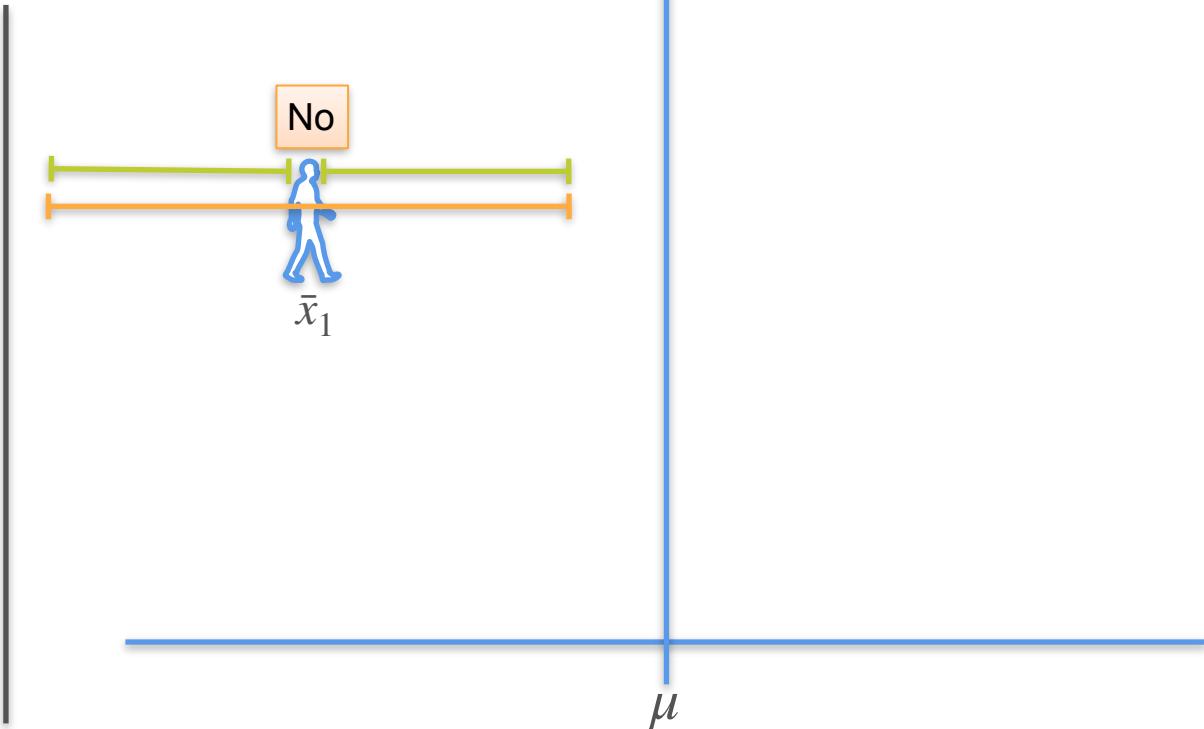
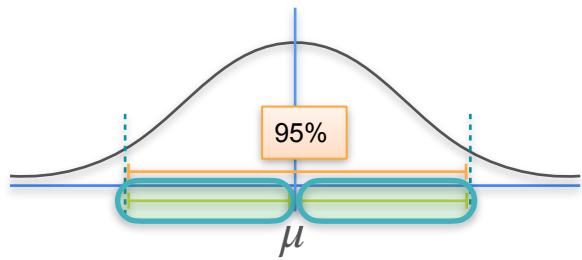
Confidence Interval - Intuition

$n = 1$

Known σ

95%

Margin of error



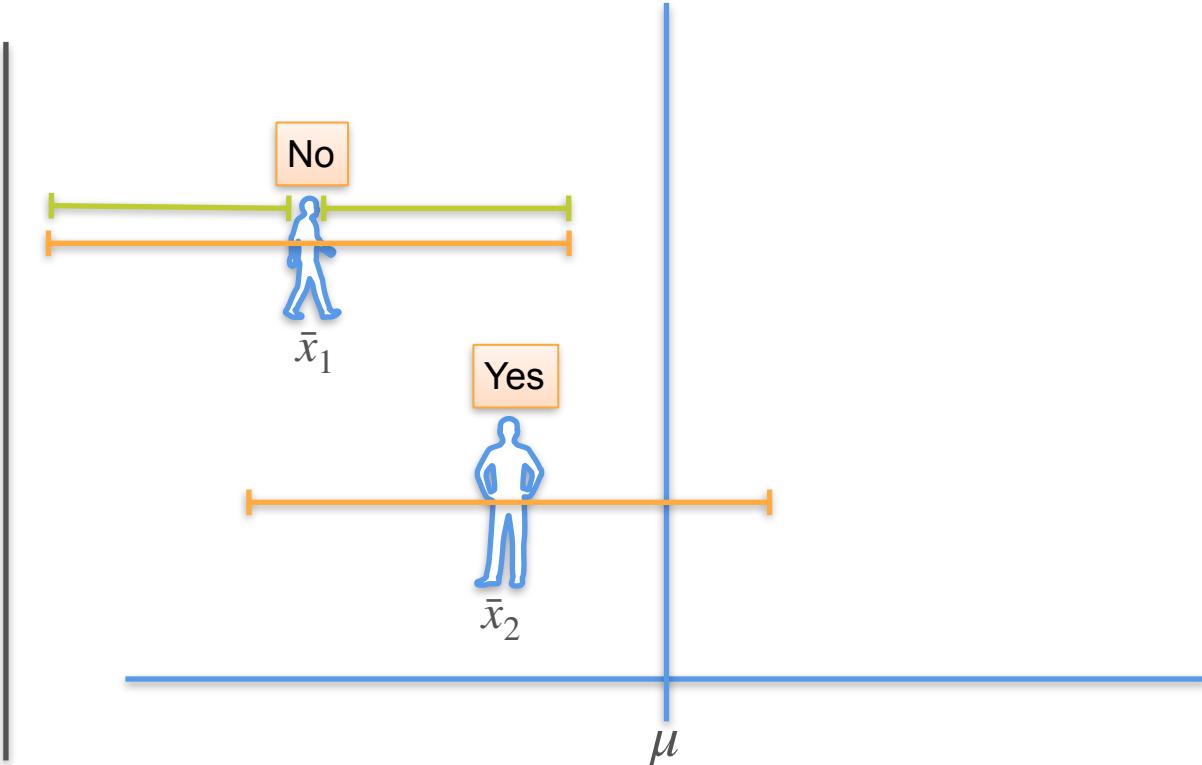
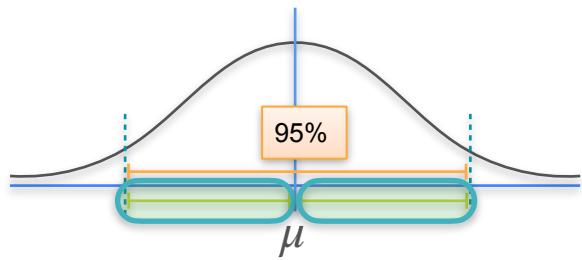
Confidence Interval - Intuition

$n = 1$

Known σ

95%

Margin of error



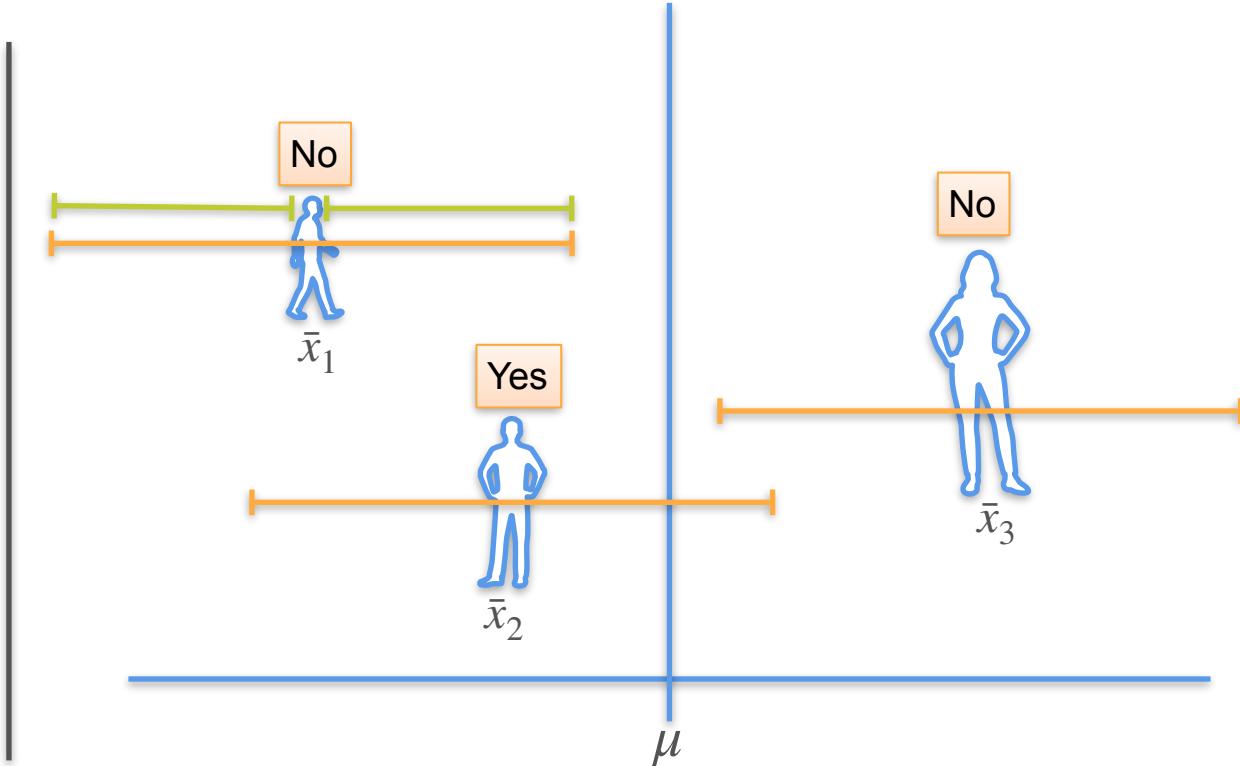
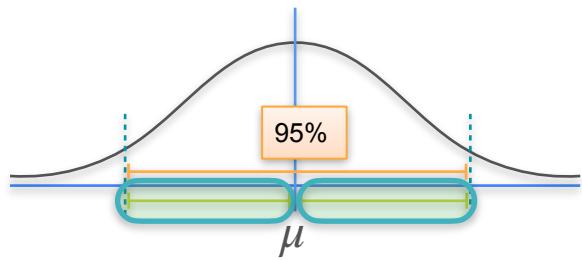
Confidence Interval - Intuition

$n = 1$

Known σ

95%

Margin of error



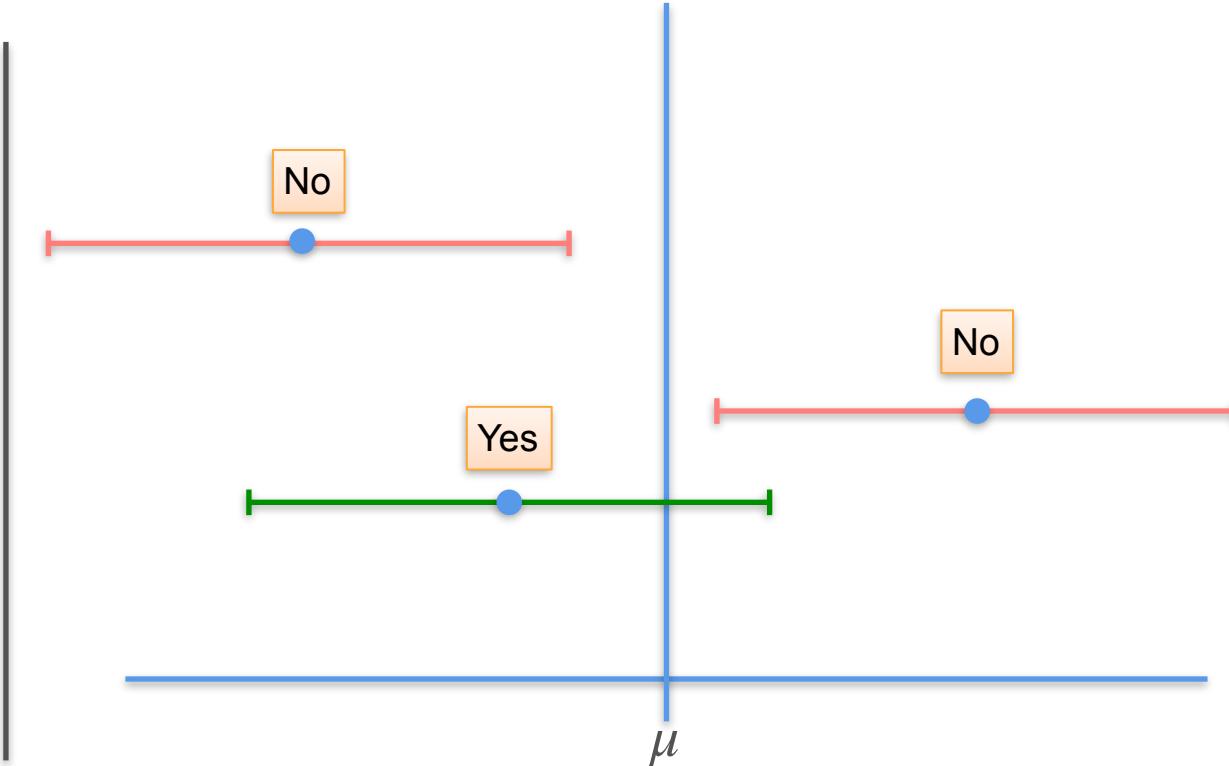
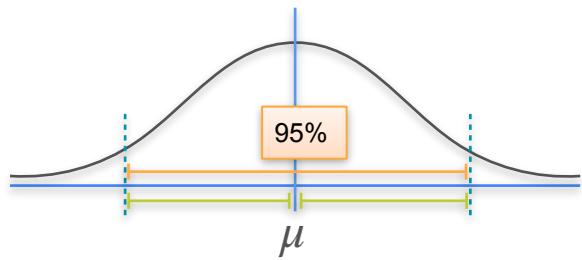
Confidence Interval - Intuition

$n = 1$

Known σ

95%

Margin of error



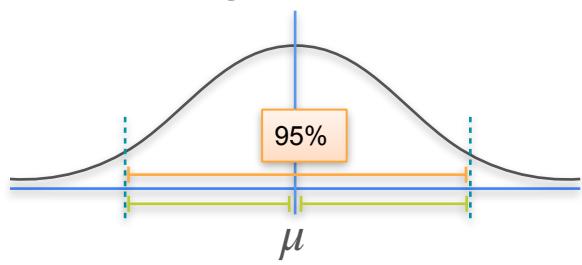
Confidence Interval - Intuition

$n = 1$

Known σ

95%

Margin of error



μ

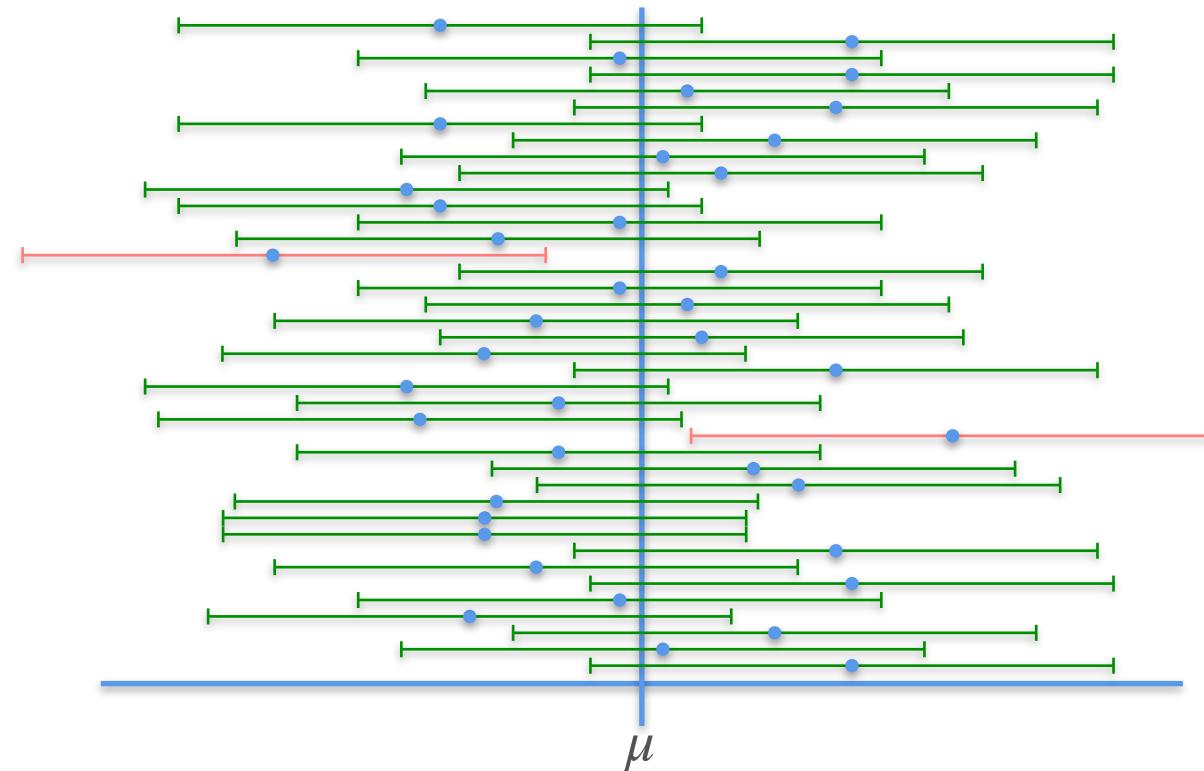
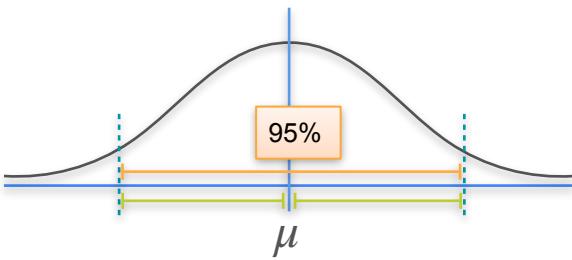
Confidence Interval - Intuition

$n = 1$

Known σ

95%

Margin of error



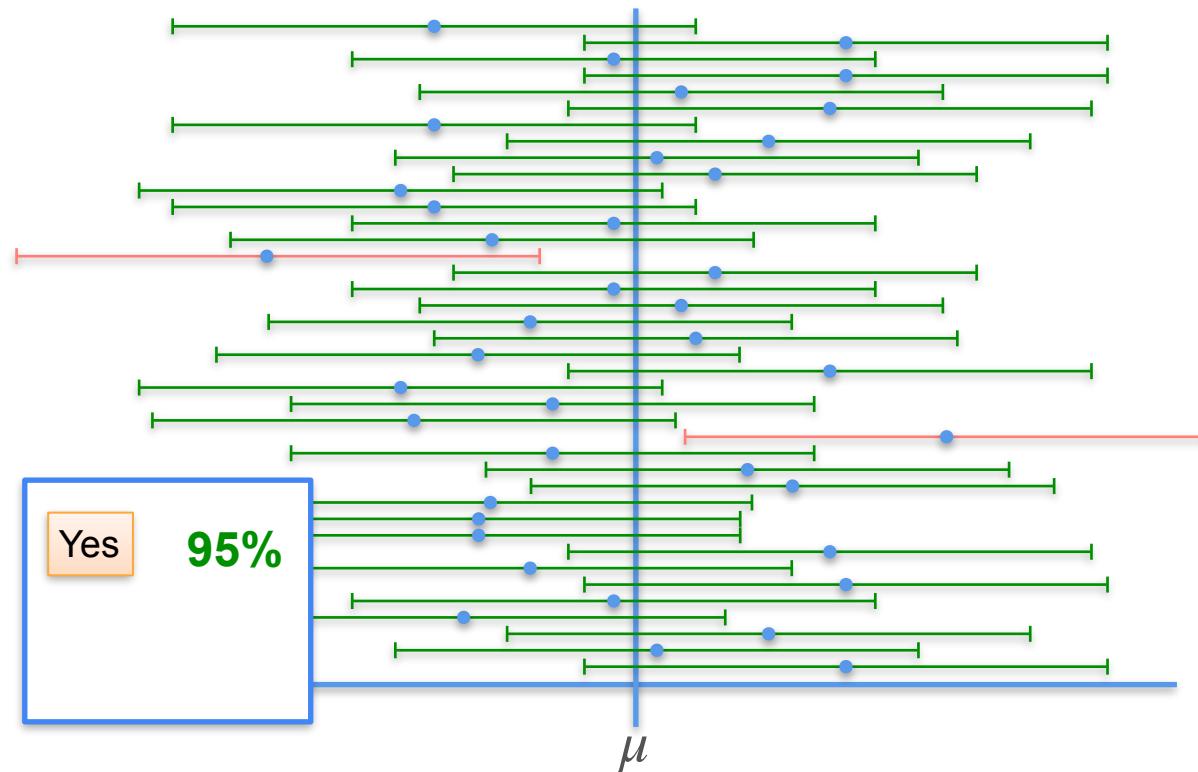
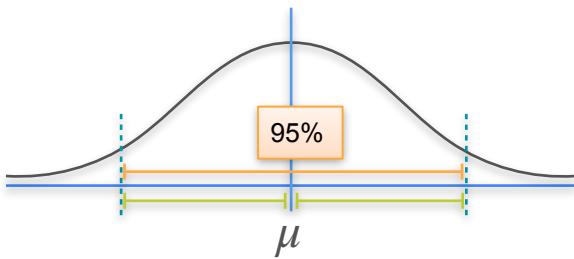
Confidence Interval - Intuition

$n = 1$

Known σ

95%

Margin of error



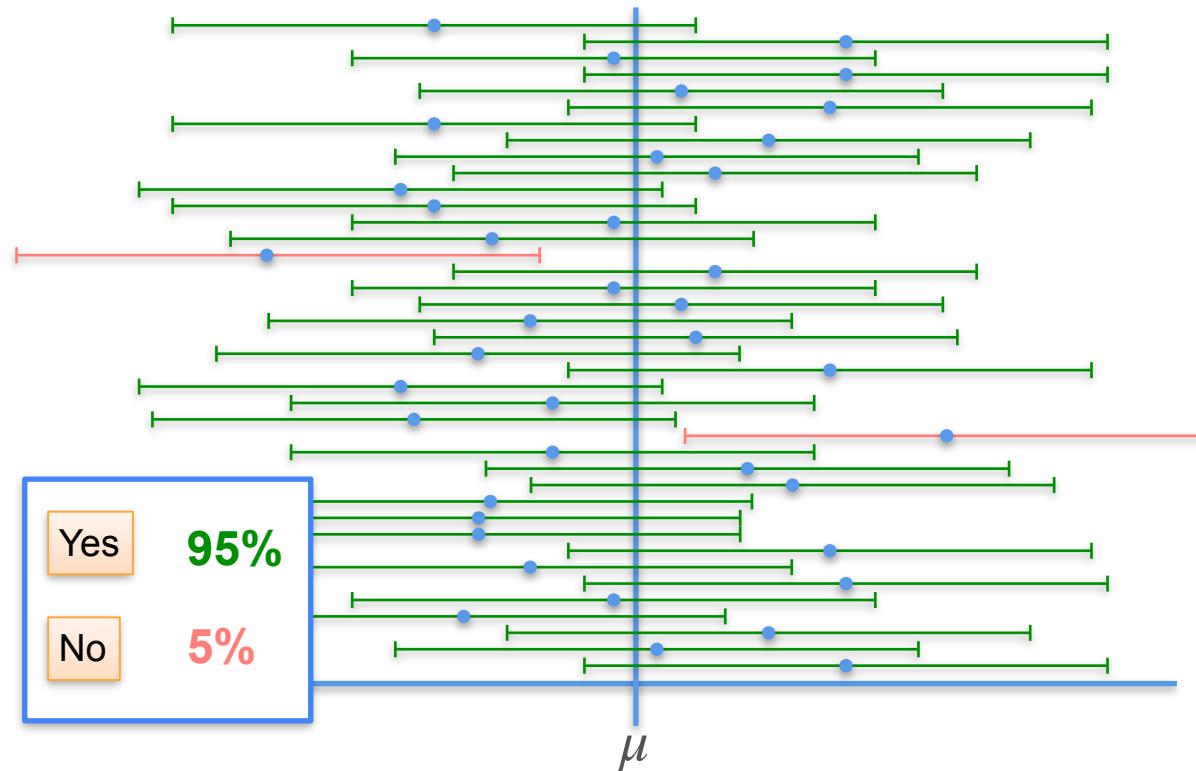
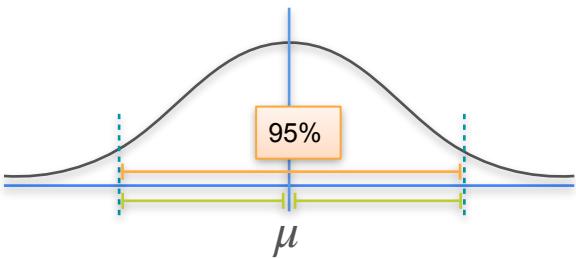
Confidence Interval - Intuition

$n = 1$

Known σ

95%

Margin of error



Confidence Interval - Intuition

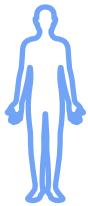
$$n = 1$$



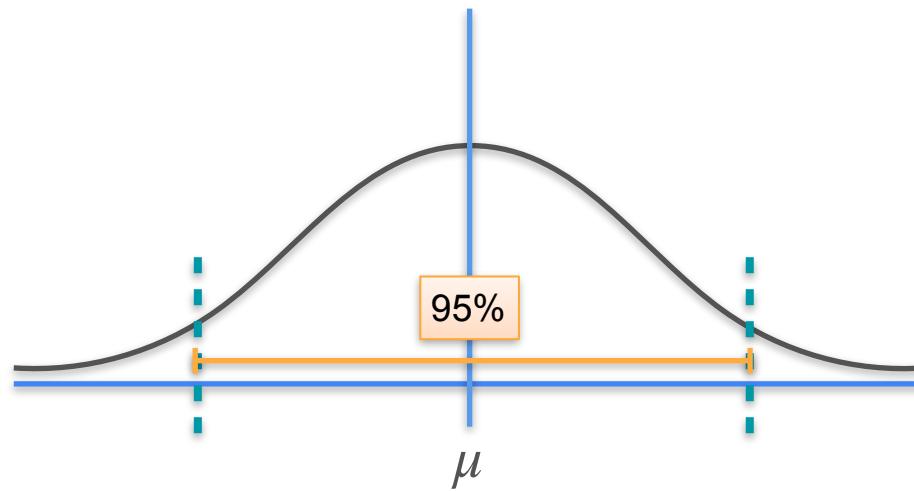
$$\bar{x}$$

Confidence Interval - Intuition

$$n = 1$$

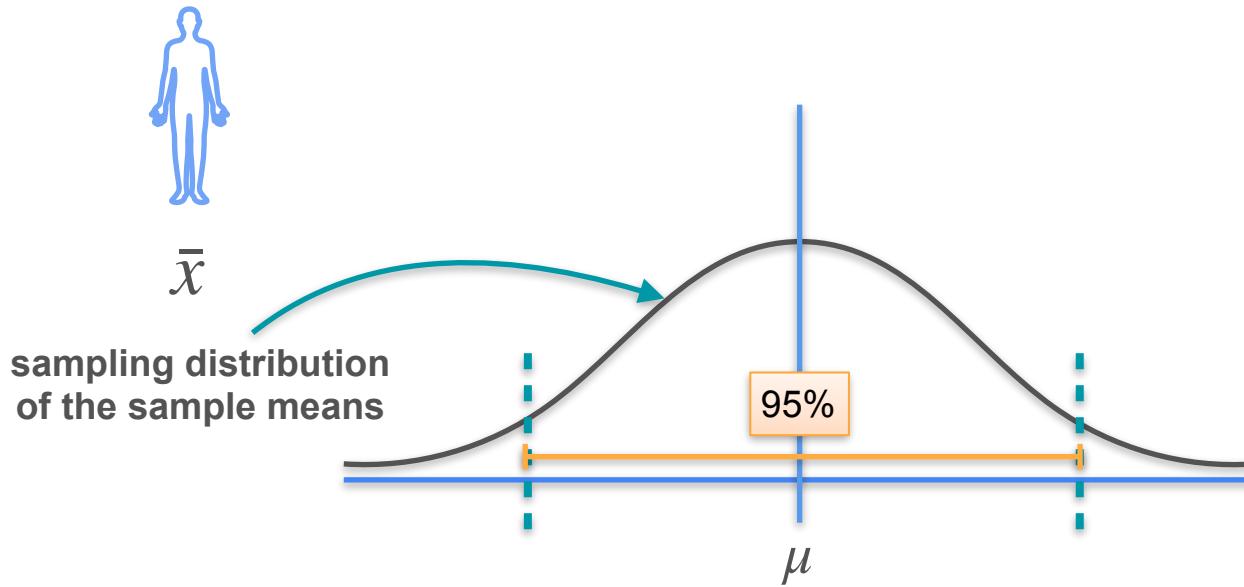


$$\bar{x}$$



Confidence Interval - Intuition

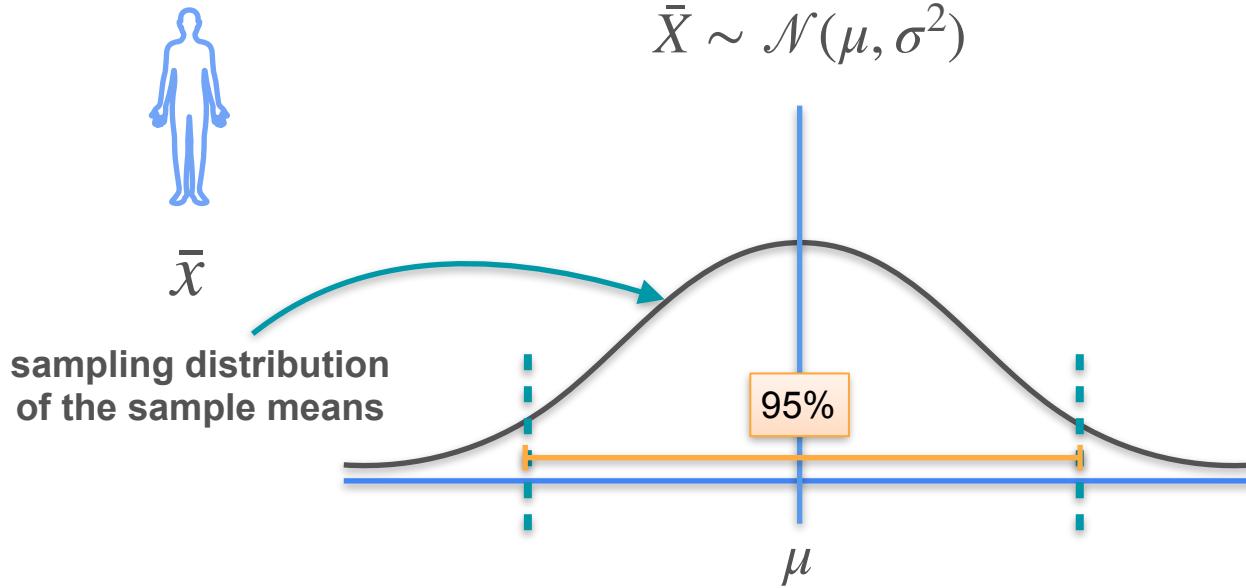
$$n = 1$$



Confidence Interval - Intuition

$n = 1$

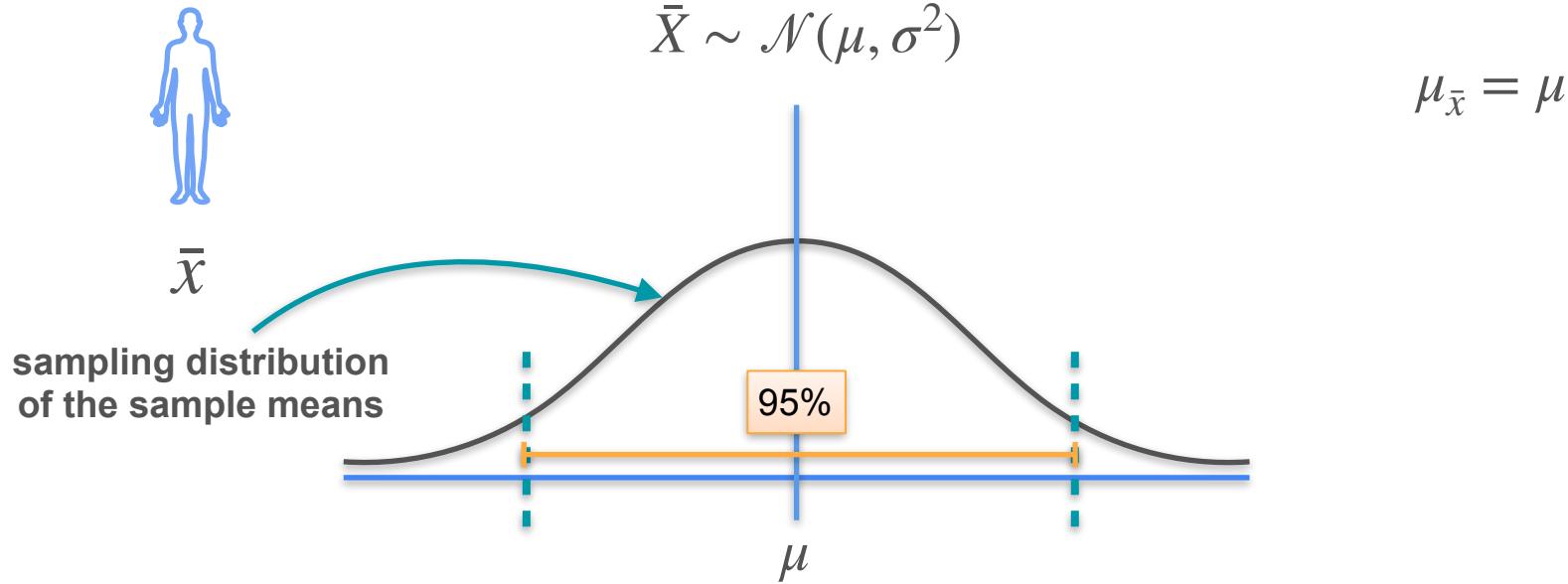
Central Limit Theorem



Confidence Interval - Intuition

$$n = 1$$

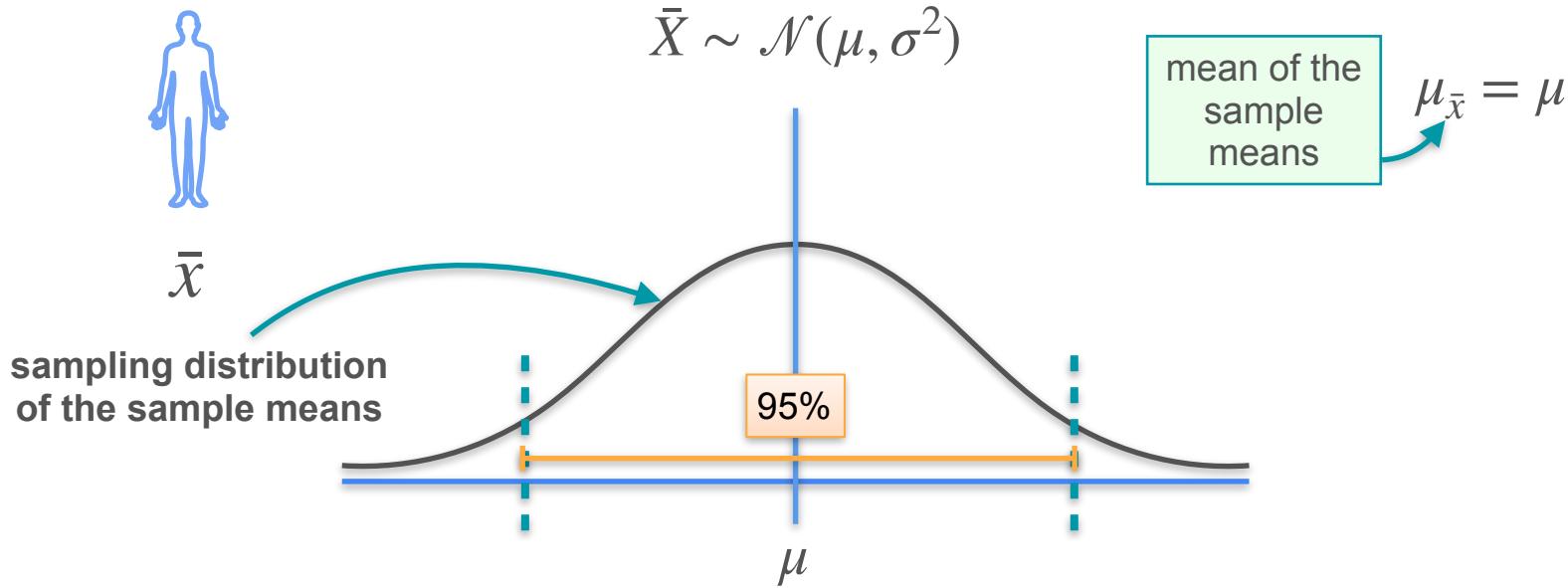
Central Limit Theorem



Confidence Interval - Intuition

$$n = 1$$

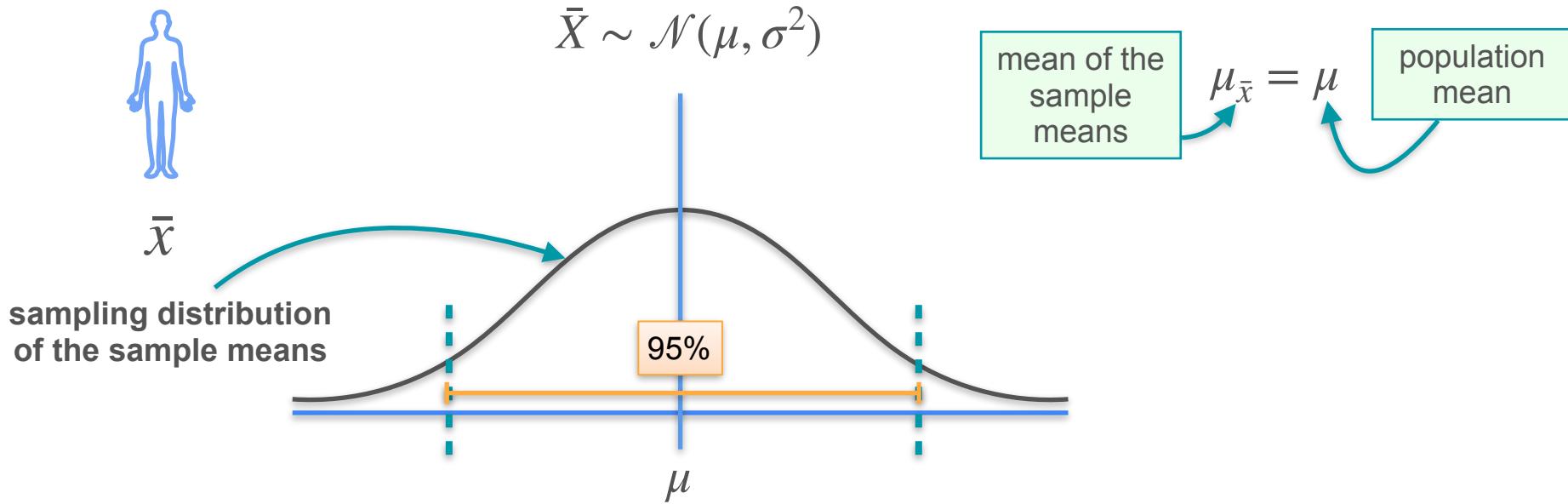
Central Limit Theorem



Confidence Interval - Intuition

$$n = 1$$

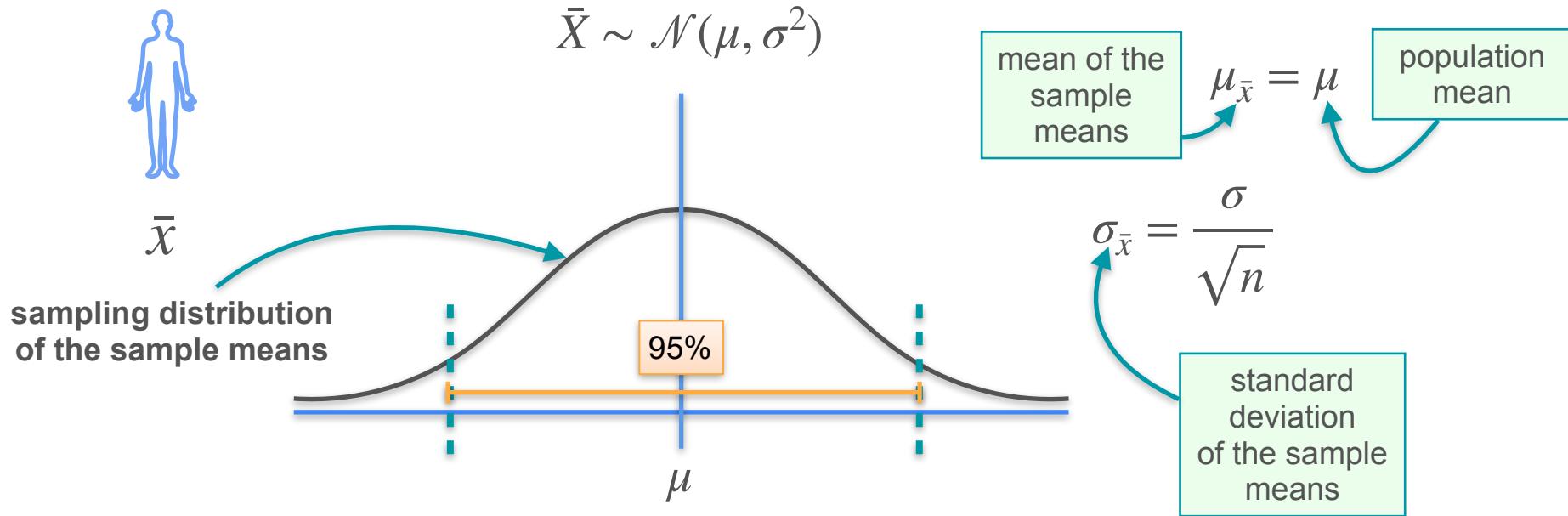
Central Limit Theorem



Confidence Interval - Intuition

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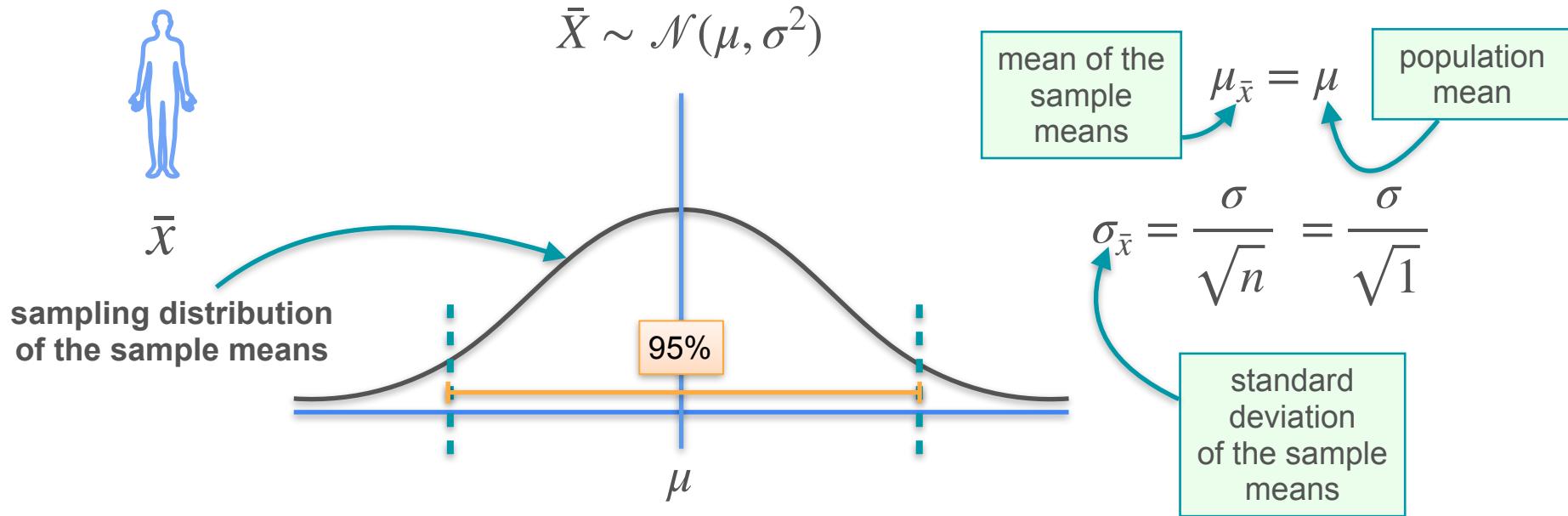
Central Limit Theorem



Confidence Interval - Intuition

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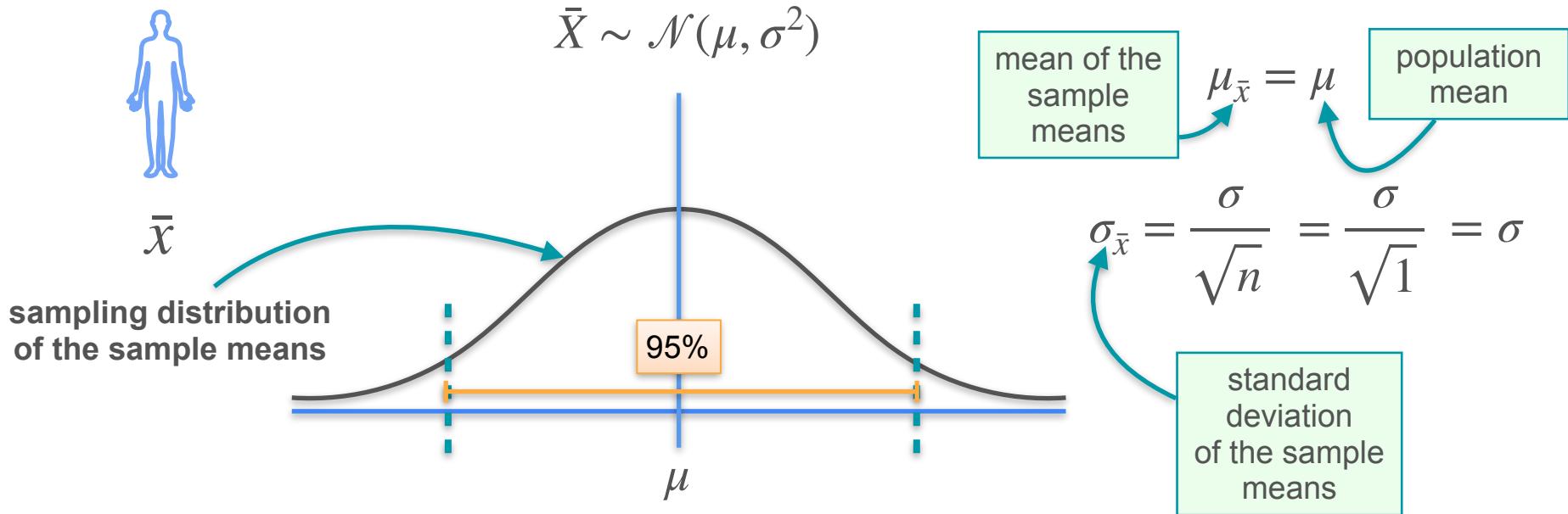
Central Limit Theorem



Confidence Interval - Intuition

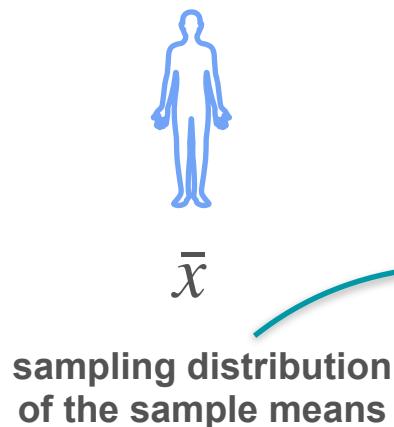
$n = 1$

Central Limit Theorem



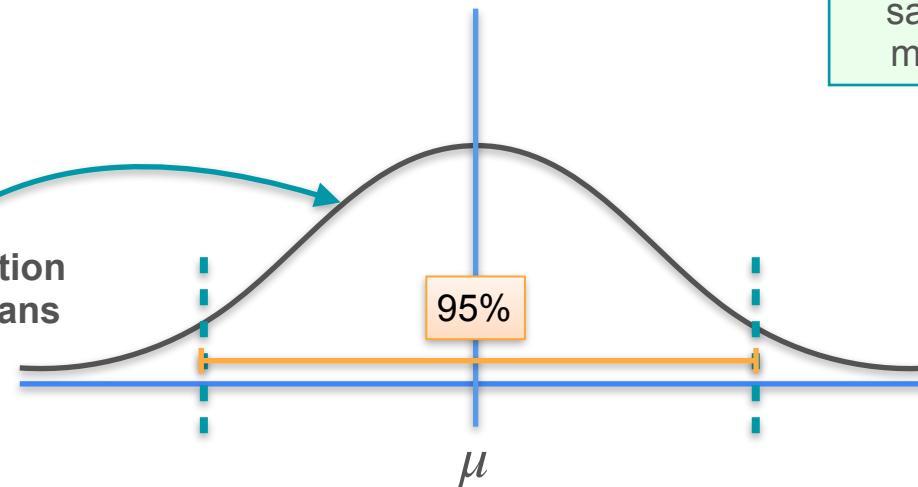
Confidence Interval - Intuition

$n = 1$



Central Limit Theorem

$$\bar{X} \sim \mathcal{N}(\mu, \sigma^2)$$



Population standard deviation (σ)

mean of the
sample
means

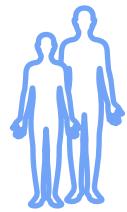
$$\mu_{\bar{x}}$$

population
mean

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{\sigma}{\sqrt{1}} = \sigma$$

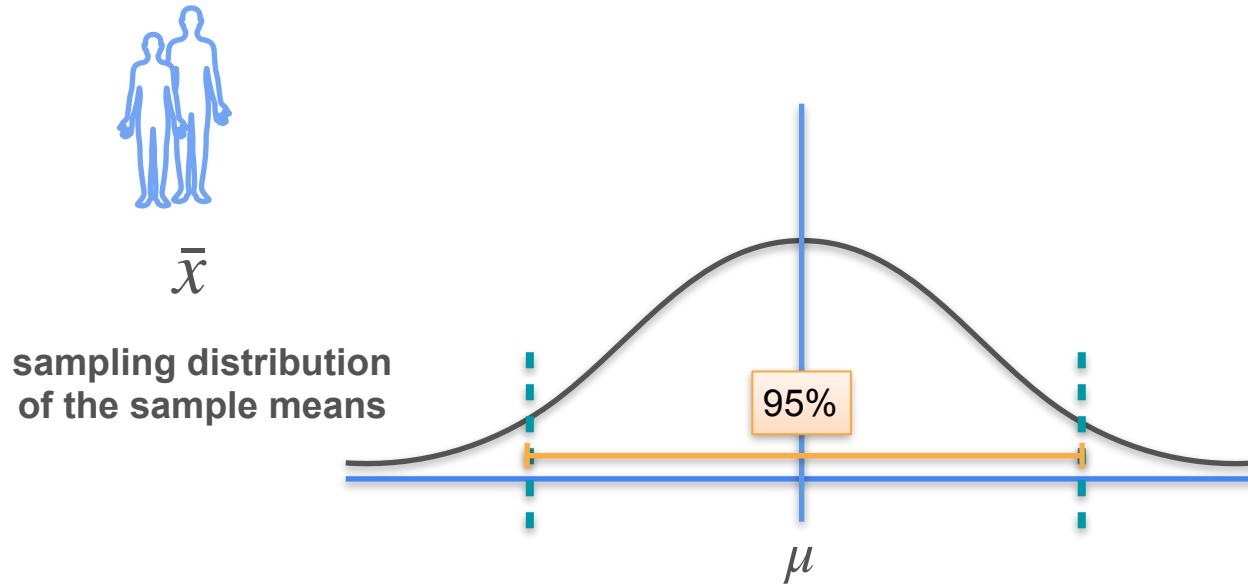
standard
deviation
of the sample
means

Confidence Interval - Intuition



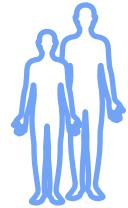
Central Limit Theorem

Population standard deviation (σ)



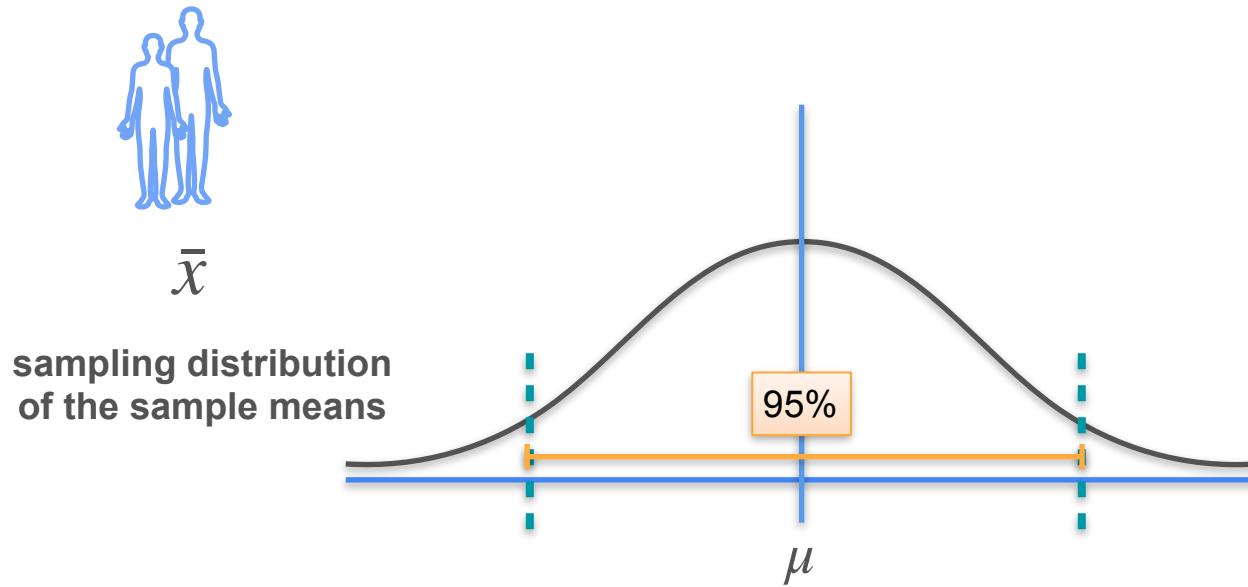
Confidence Interval - Intuition

$n = 2$



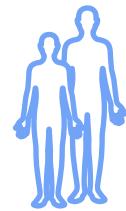
Central Limit Theorem

Population standard deviation (σ)



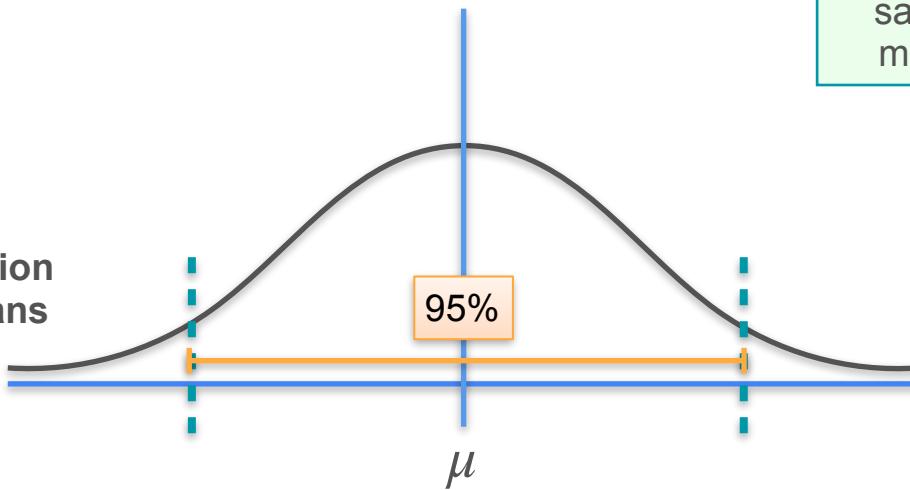
Confidence Interval - Intuition

$n = 2$



\bar{x}
sampling distribution
of the sample means

Central Limit Theorem



Population standard deviation (σ)

mean of the
sample
means

$$\mu_{\bar{x}} = \mu$$

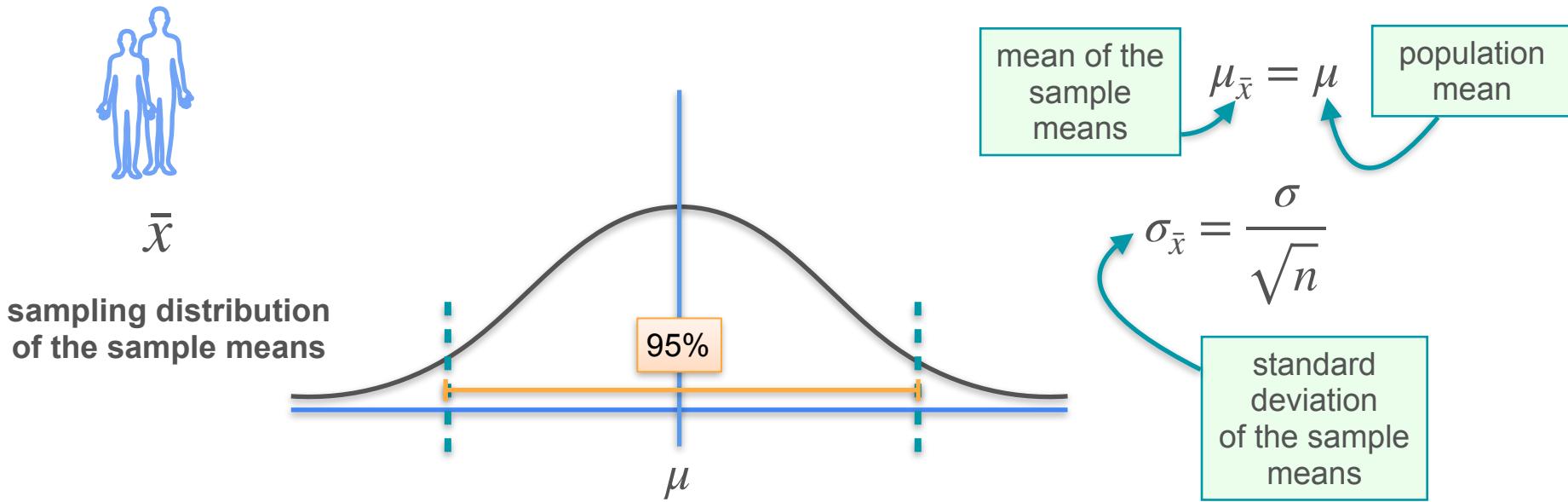
population
mean

Confidence Interval - Intuition

$n = 2$

Central Limit Theorem

Population standard deviation (σ)

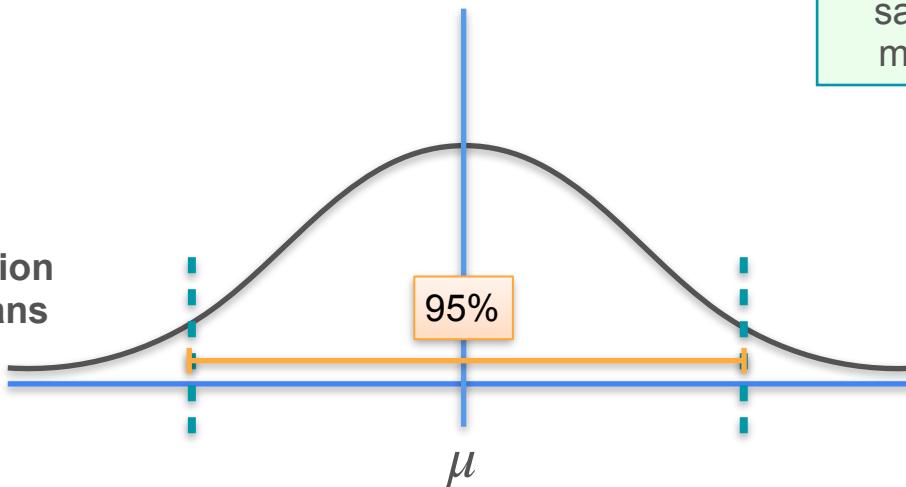


Confidence Interval - Intuition

$n = 2$


 \bar{x}
sampling distribution
of the sample means

Central Limit Theorem



Population standard deviation (σ)

mean of the
sample
means

$$\mu_{\bar{x}} = \mu$$

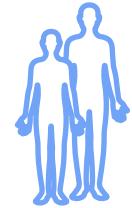
population
mean

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{\sigma}{\sqrt{2}}$$

standard
deviation
of the sample
means

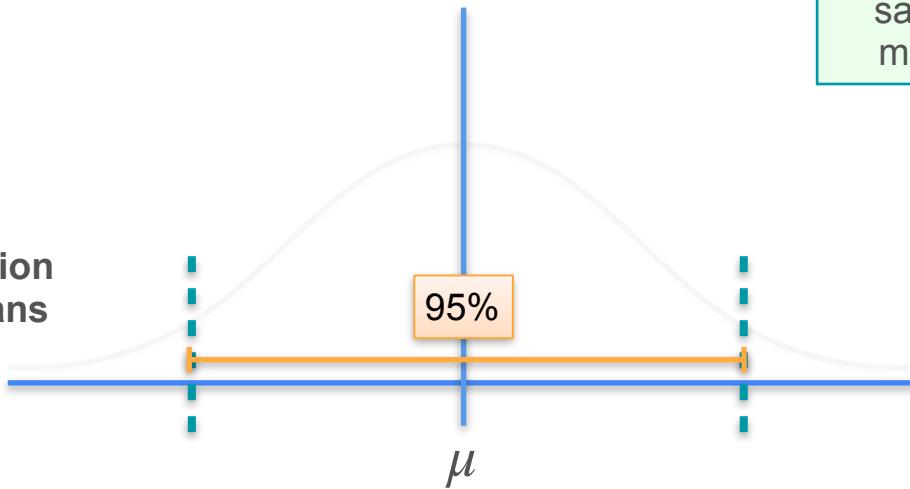
Confidence Interval - Intuition

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\bar{x}
sampling distribution
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Central Limit Theorem



Population standard deviation (σ)

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standard
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of the sample
means

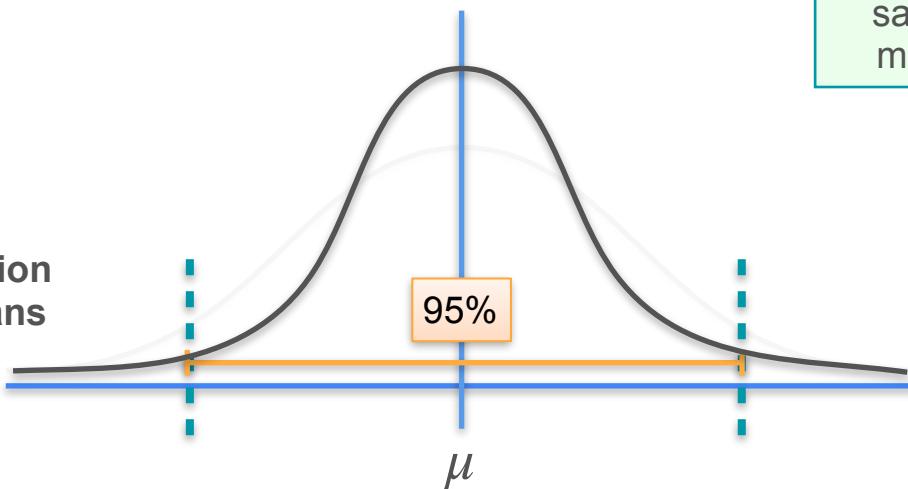
Confidence Interval - Intuition

$n = 2$

Central Limit Theorem

Population standard deviation (σ)


 \bar{x}
sampling distribution
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mean of the
sample
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$$\mu_{\bar{x}} = \mu$$

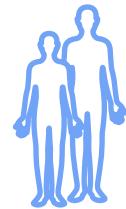
population
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$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{\sigma}{\sqrt{2}}$$

standard
deviation
of the sample
means

Confidence Interval - Intuition

$n = 2$

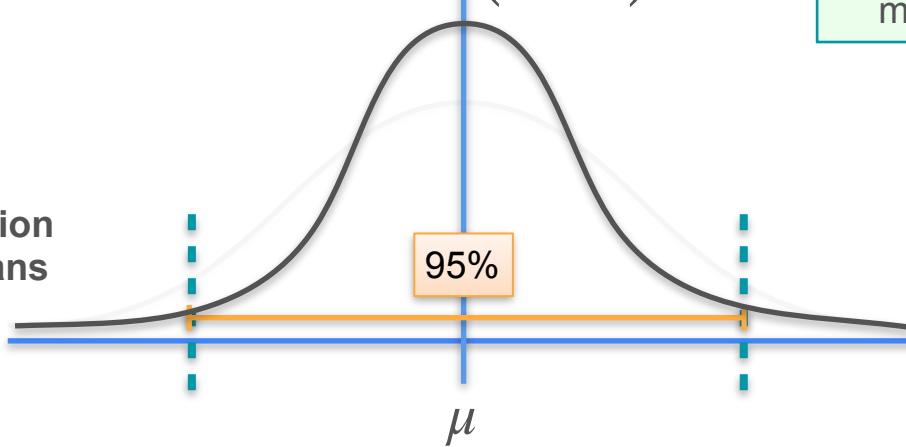


\bar{x}

sampling distribution
of the sample means

Central Limit Theorem

$$\bar{X} \sim \mathcal{N}\left(\mu, \frac{\sigma^2}{n}\right)$$



Population standard deviation (σ)

mean of the
sample
means

$$\mu_{\bar{x}} = \mu$$

population
mean

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{\sigma}{\sqrt{2}}$$

standard
deviation
of the sample
means

Confidence Interval - Intuition

$n = 2$

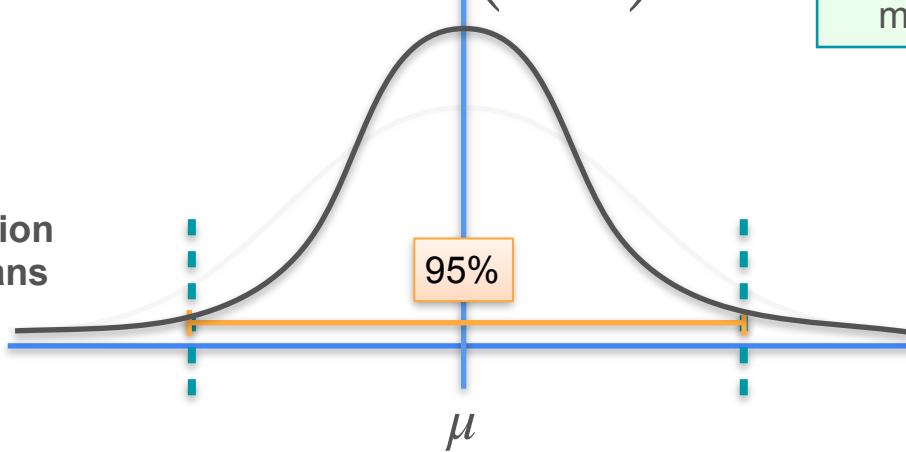


\bar{x}

sampling distribution
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Central Limit Theorem

$$\bar{X} \sim \mathcal{N}\left(\mu, \frac{\sigma^2}{2}\right)$$



Population standard deviation (σ)

mean of the
sample
means

$$\mu_{\bar{x}} = \mu$$

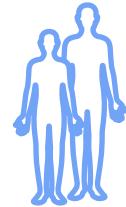
population
mean

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{\sigma}{\sqrt{2}}$$

standard
deviation
of the sample
means

Confidence Interval - Intuition

$n = 2$

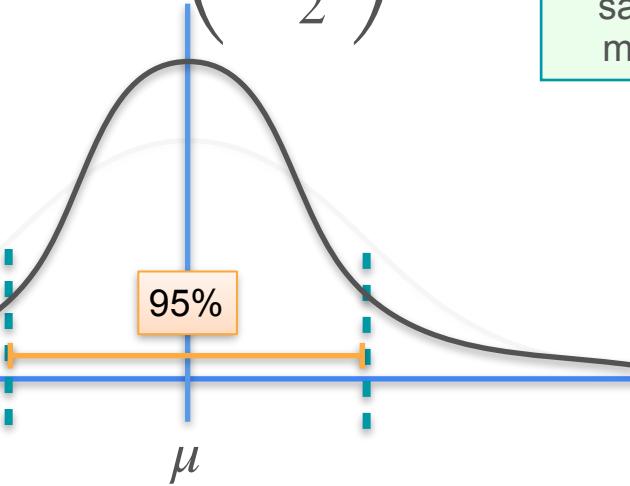


\bar{x}

sampling distribution
of the sample means

Central Limit Theorem

$$\bar{X} \sim \mathcal{N}\left(\mu, \frac{\sigma^2}{2}\right)$$



Population standard deviation (σ)

mean of the
sample
means

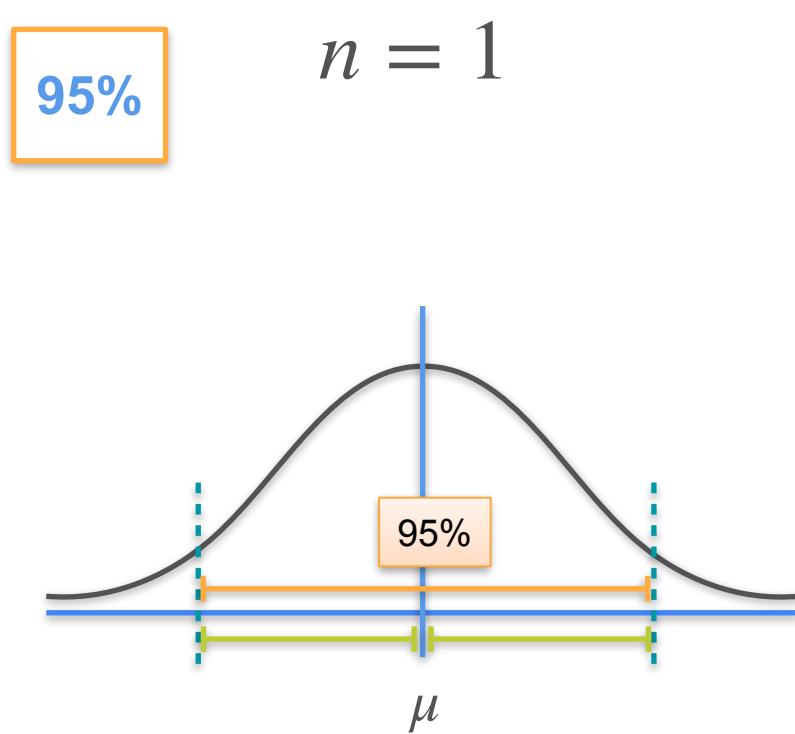
$$\mu_{\bar{x}} = \mu$$

population
mean

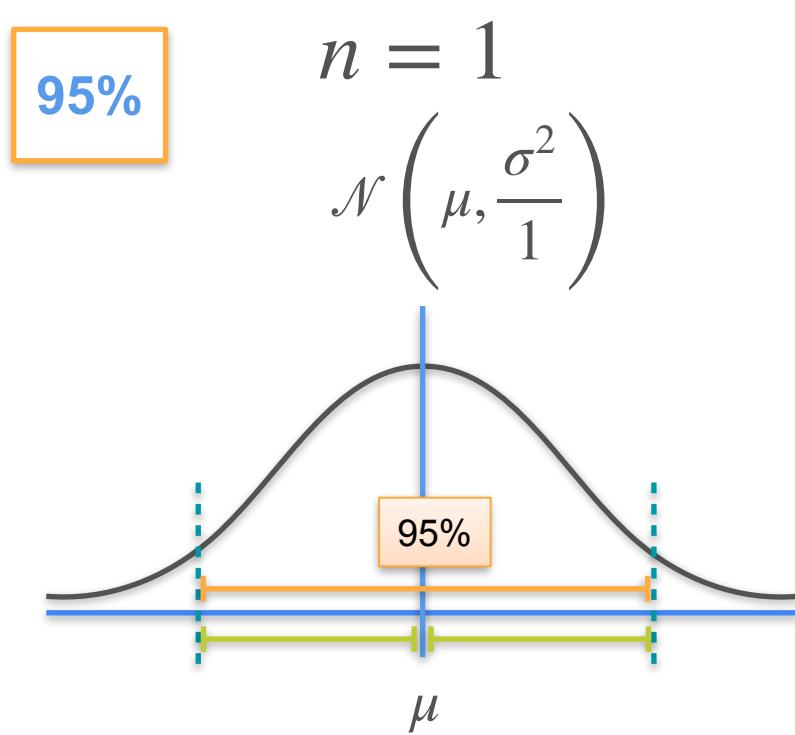
$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{\sigma}{\sqrt{2}}$$

standard
deviation
of the sample
means

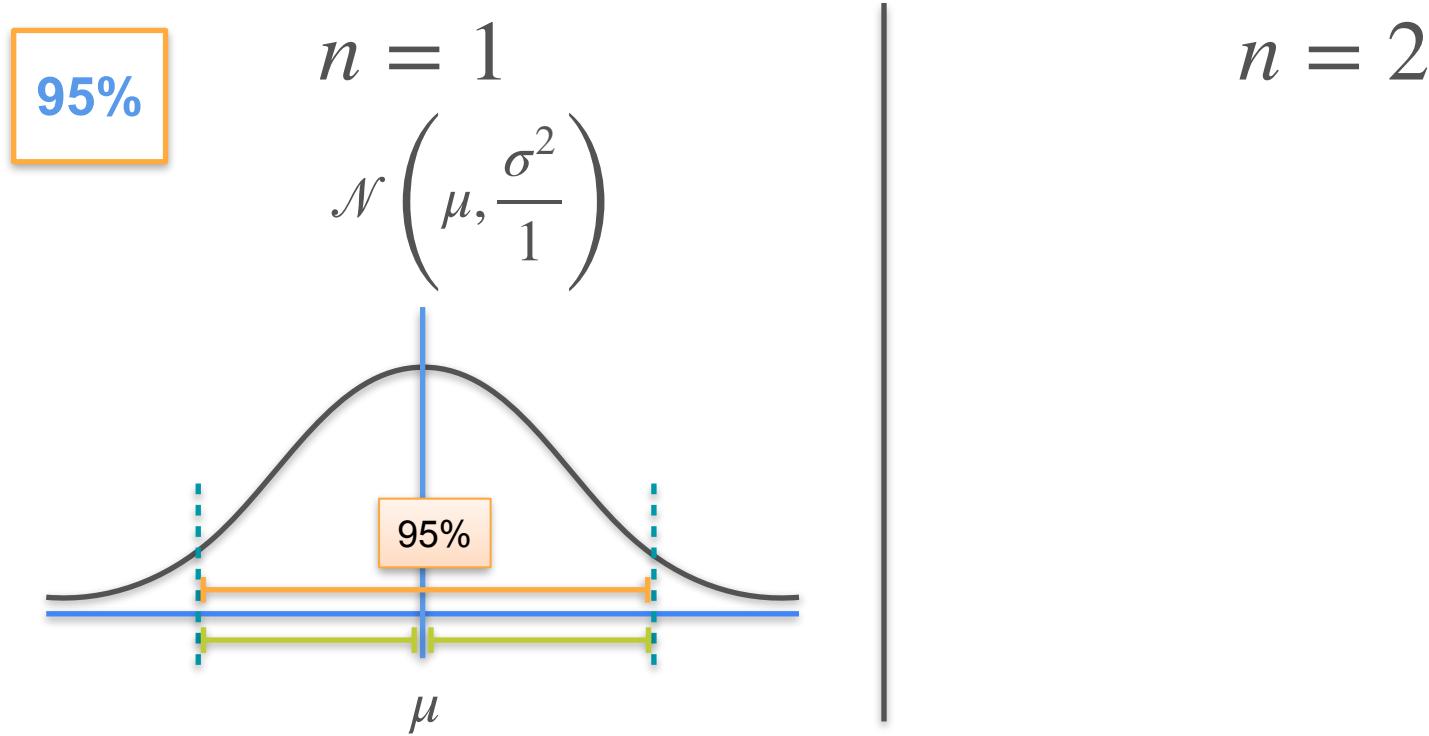
Confidence Interval - Intuition



Confidence Interval - Intuition



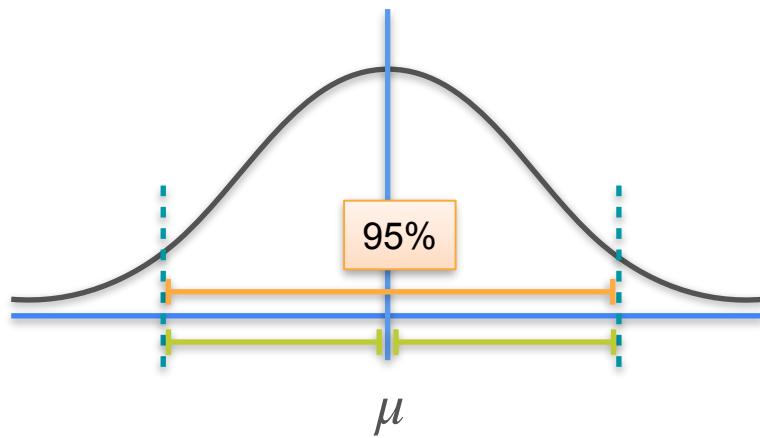
Confidence Interval - Intuition



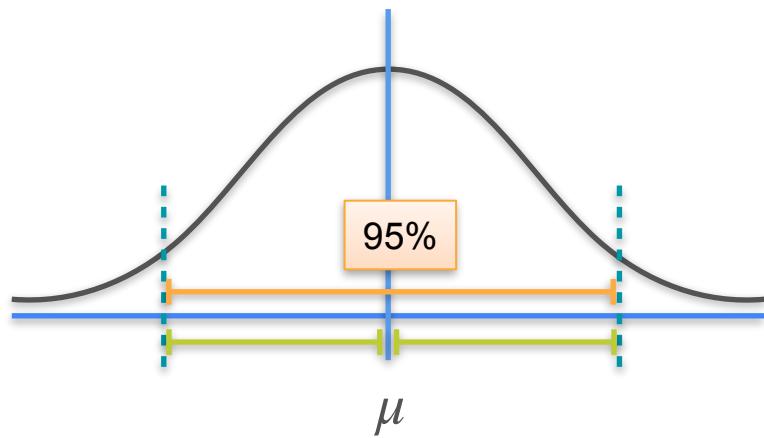
Confidence Interval - Intuition

95%

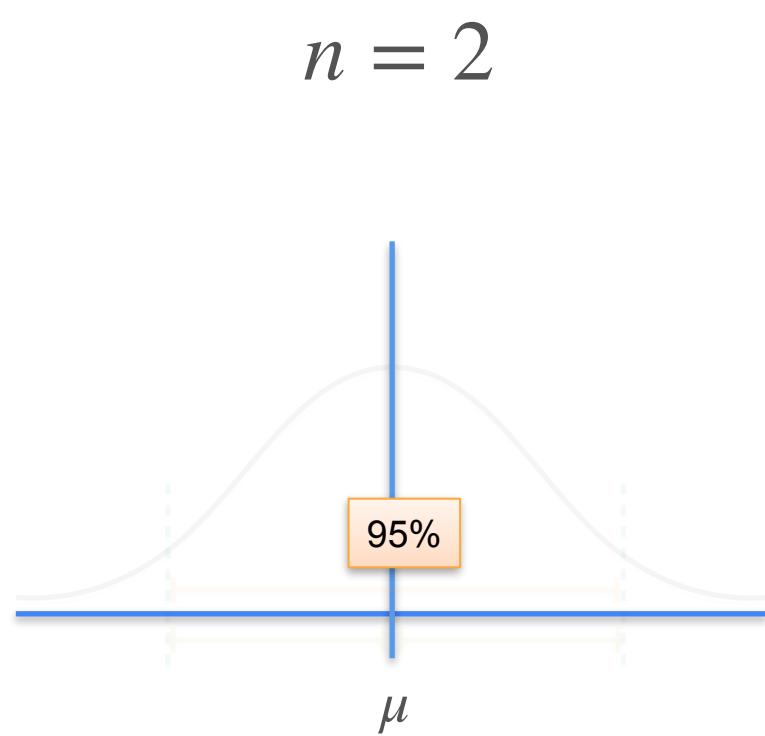
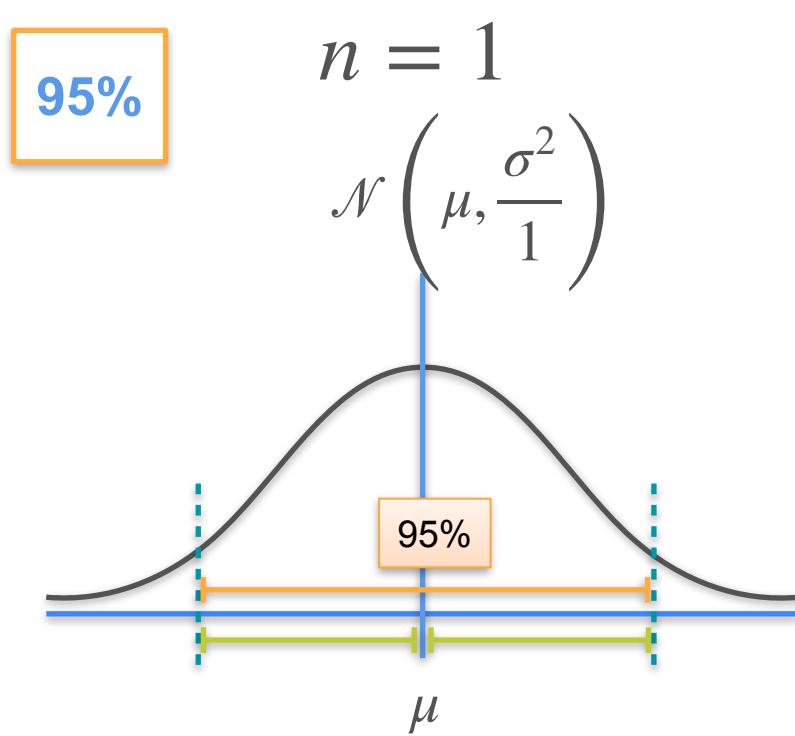
$$n = 1$$
$$\mathcal{N}\left(\mu, \frac{\sigma^2}{1}\right)$$



$n = 2$



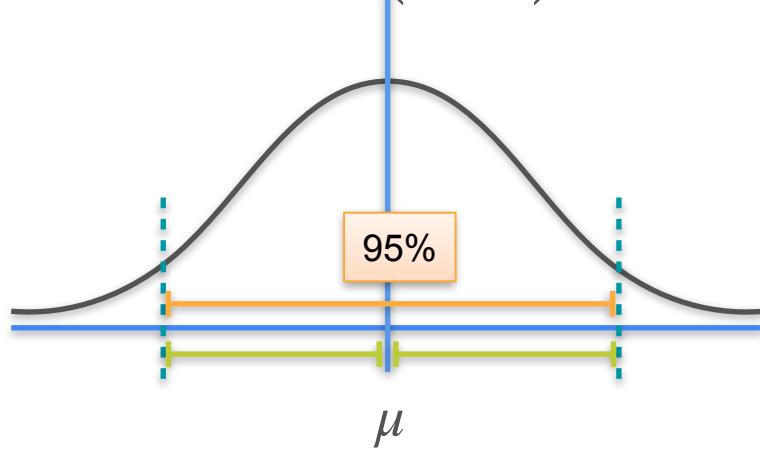
Confidence Interval - Intuition



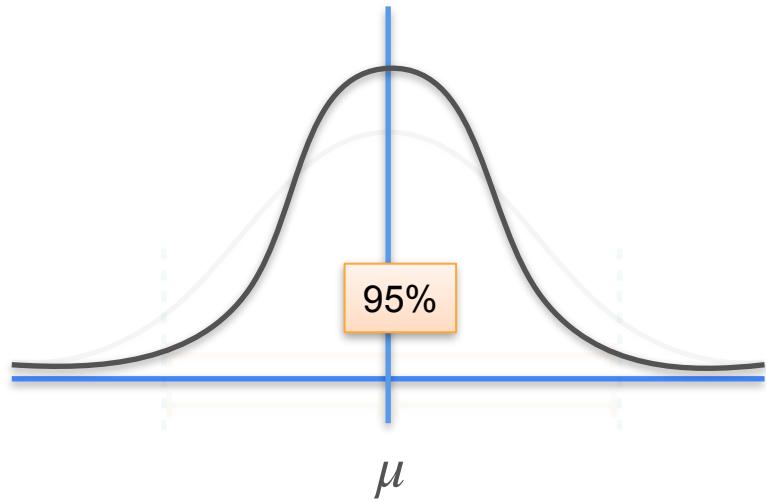
Confidence Interval - Intuition

95%

$$n = 1$$
$$\mathcal{N}\left(\mu, \frac{\sigma^2}{1}\right)$$



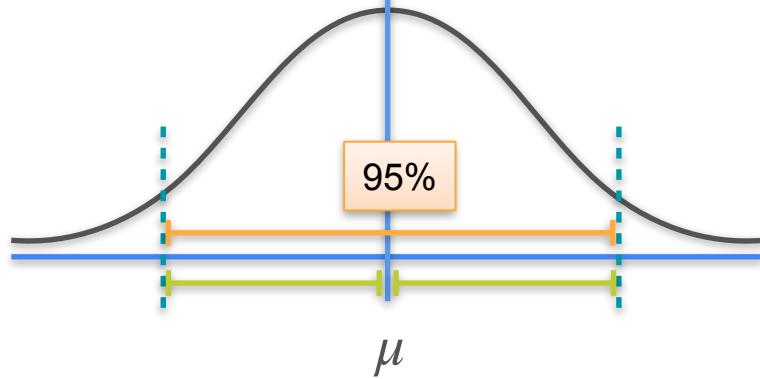
$n = 2$



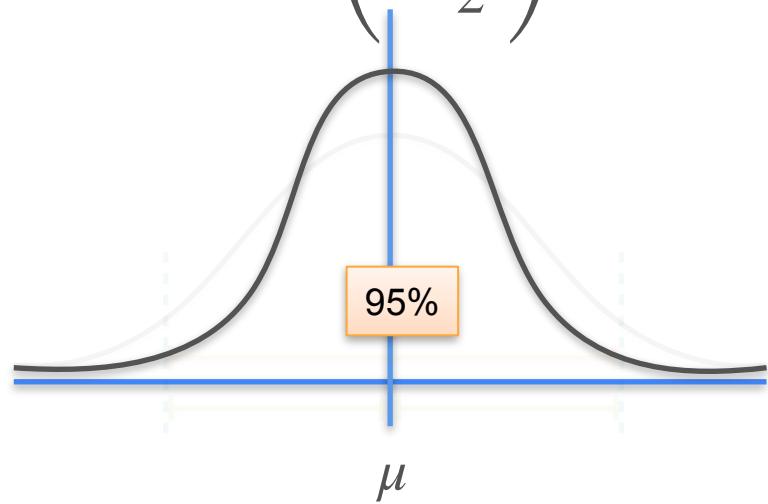
Confidence Interval - Intuition

95%

$$n = 1$$
$$\mathcal{N}\left(\mu, \frac{\sigma^2}{1}\right)$$



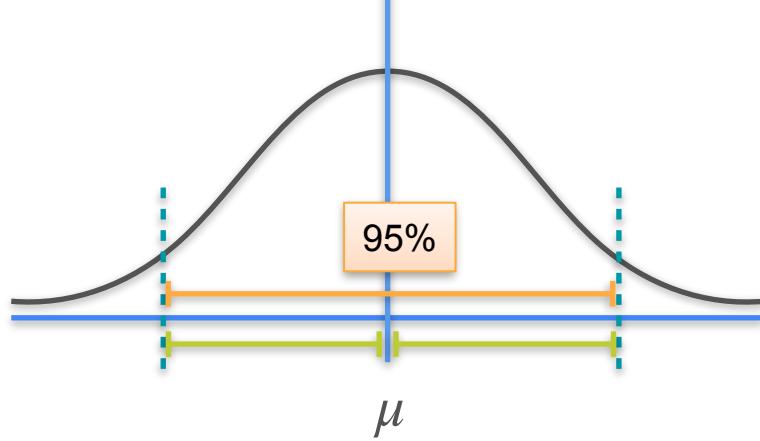
$$n = 2$$
$$\mathcal{N}\left(\mu, \frac{\sigma^2}{2}\right)$$



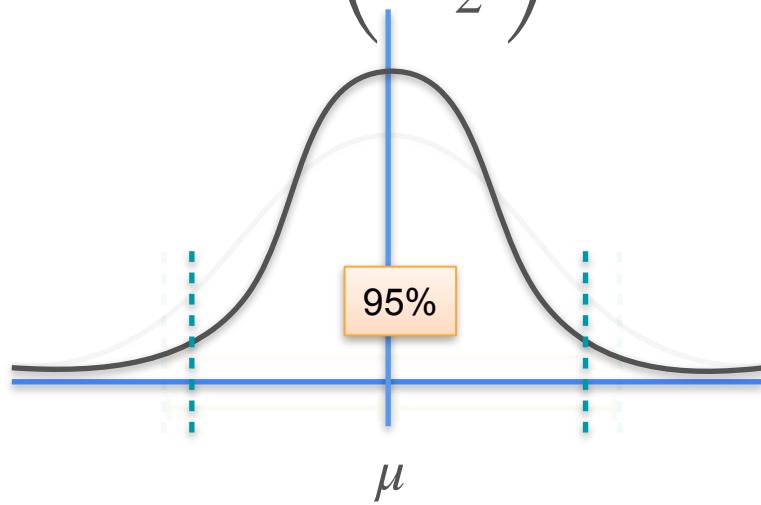
Confidence Interval - Intuition

95%

$$n = 1$$
$$\mathcal{N}\left(\mu, \frac{\sigma^2}{1}\right)$$



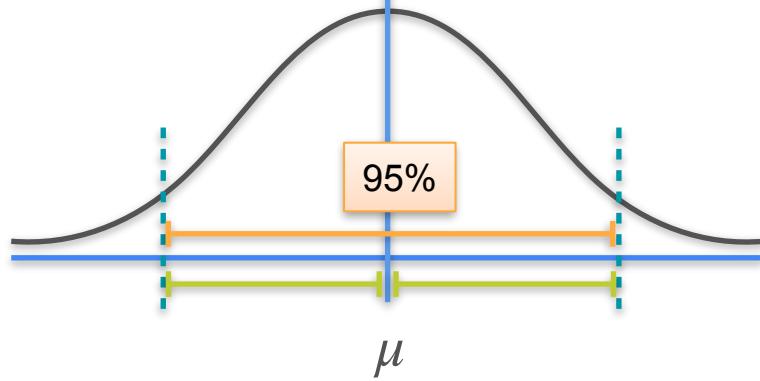
$$n = 2$$
$$\mathcal{N}\left(\mu, \frac{\sigma^2}{2}\right)$$



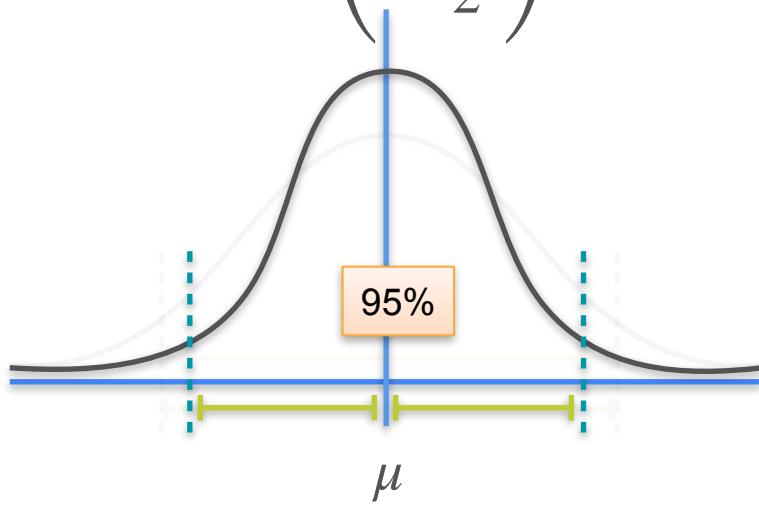
Confidence Interval - Intuition

95%

$$n = 1$$
$$\mathcal{N}\left(\mu, \frac{\sigma^2}{1}\right)$$



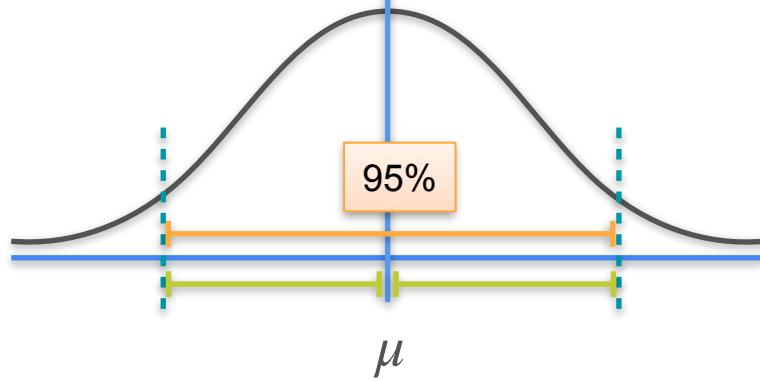
$$n = 2$$
$$\mathcal{N}\left(\mu, \frac{\sigma^2}{2}\right)$$



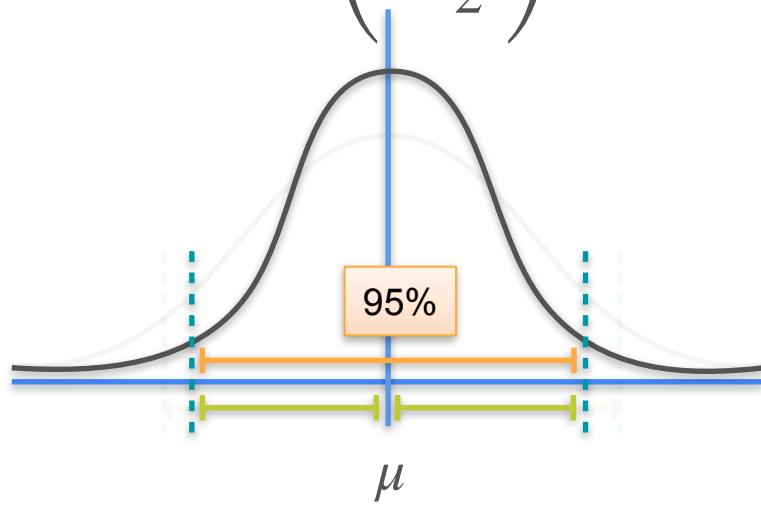
Confidence Interval - Intuition

95%

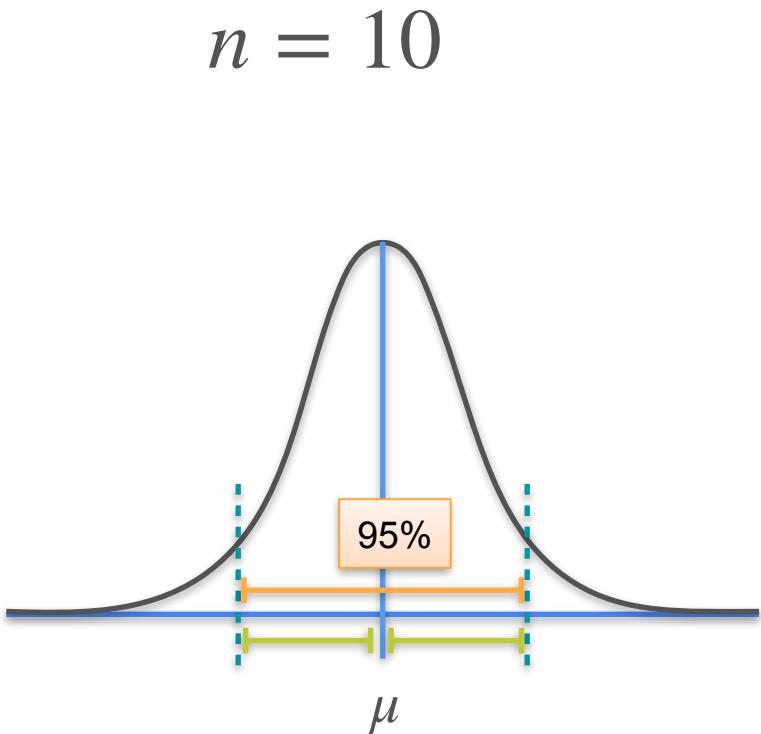
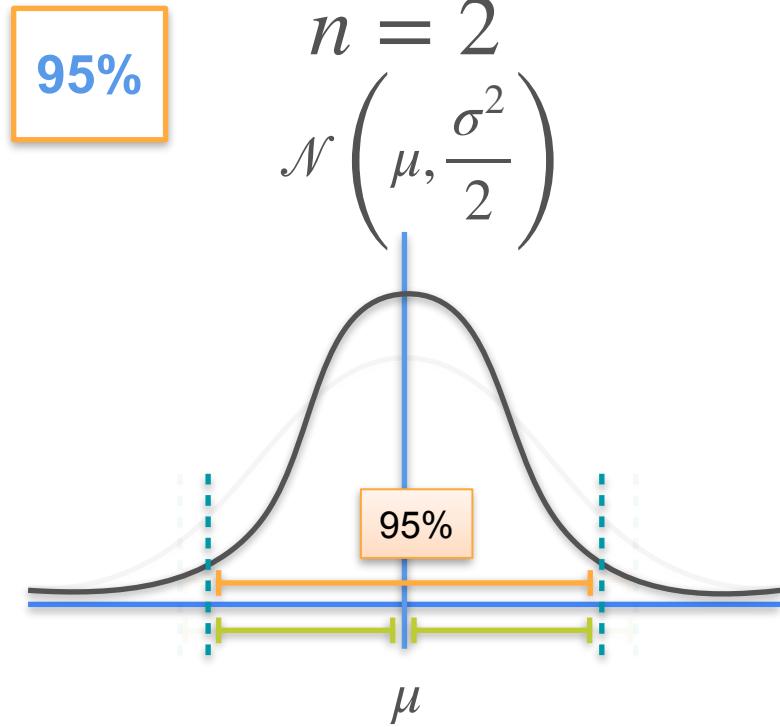
$$n = 1$$
$$\mathcal{N}\left(\mu, \frac{\sigma^2}{1}\right)$$



$$n = 2$$
$$\mathcal{N}\left(\mu, \frac{\sigma^2}{2}\right)$$



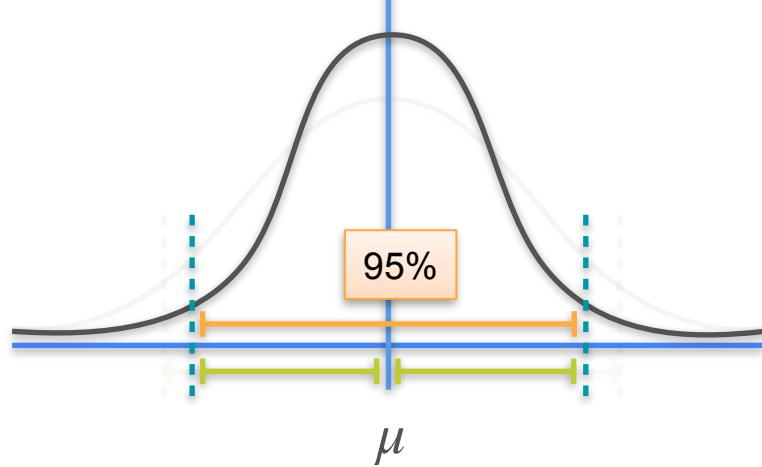
Confidence Interval - Intuition



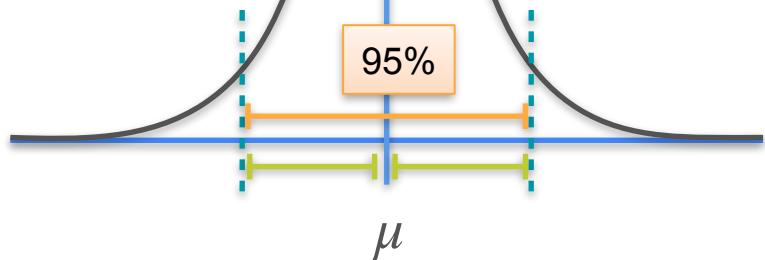
Confidence Interval - Intuition

95%

$$n = 2$$
$$\mathcal{N}\left(\mu, \frac{\sigma^2}{2}\right)$$



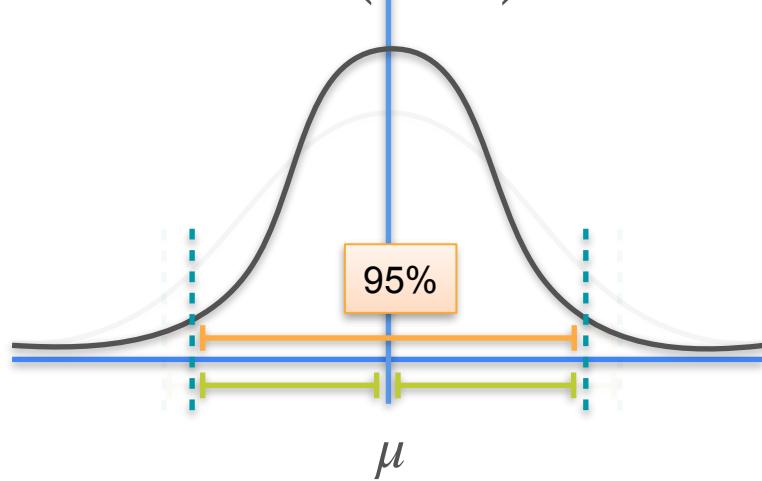
$$n = 10$$
$$\mathcal{N}\left(\mu, \frac{\sigma^2}{10}\right)$$



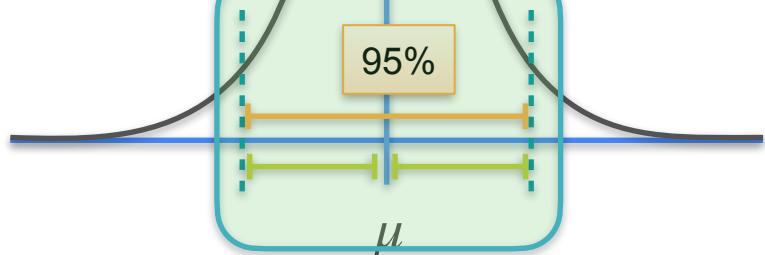
Confidence Interval - Intuition

95%

$$n = 2$$
$$\mathcal{N}\left(\mu, \frac{\sigma^2}{2}\right)$$



$$n = 10$$
$$\mathcal{N}\left(\mu, \frac{\sigma^2}{10}\right)$$



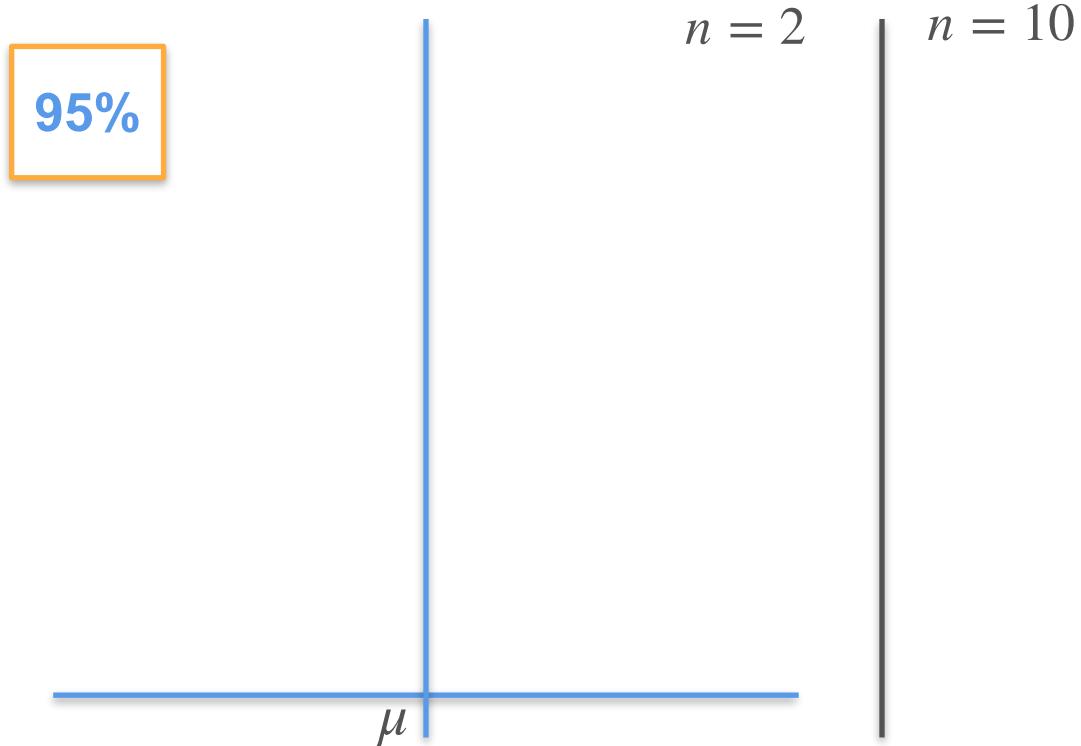
Confidence Interval - Intuition

95%

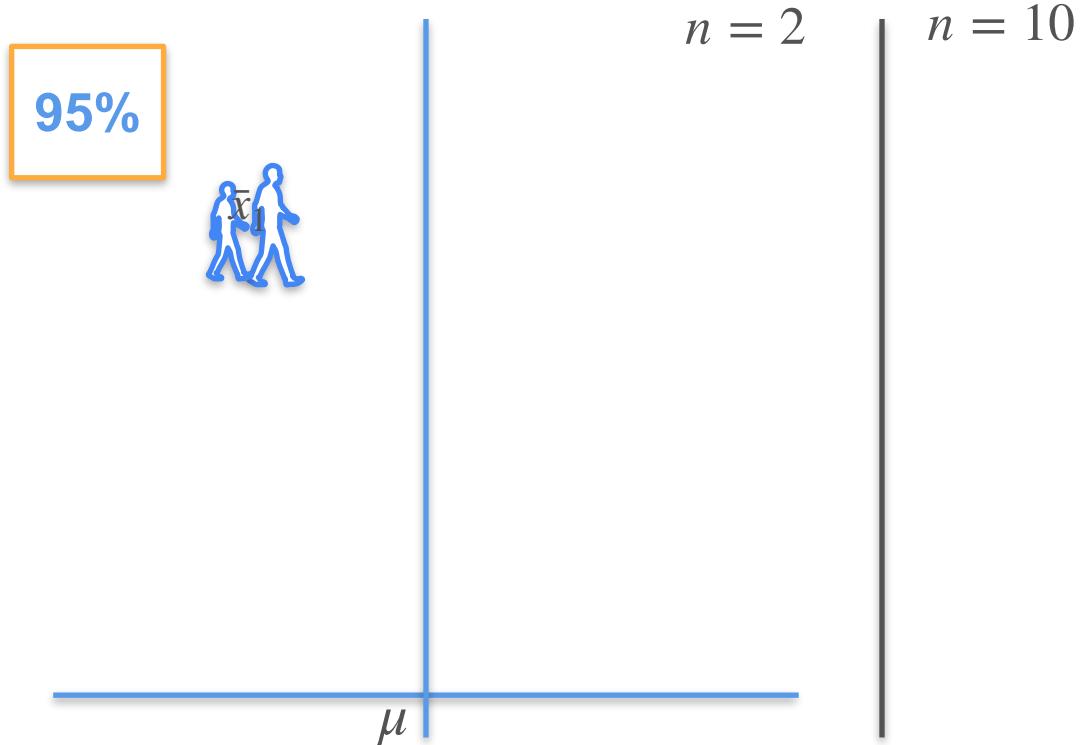
$n = 2$

$n = 10$

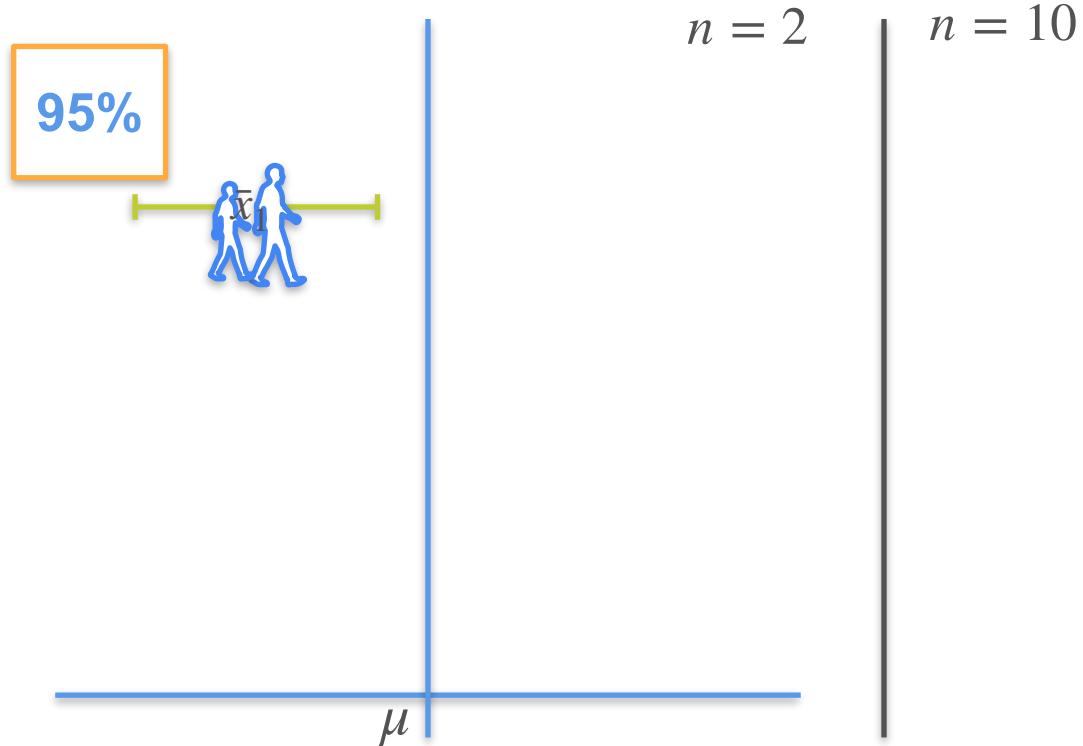
Confidence Interval - Intuition



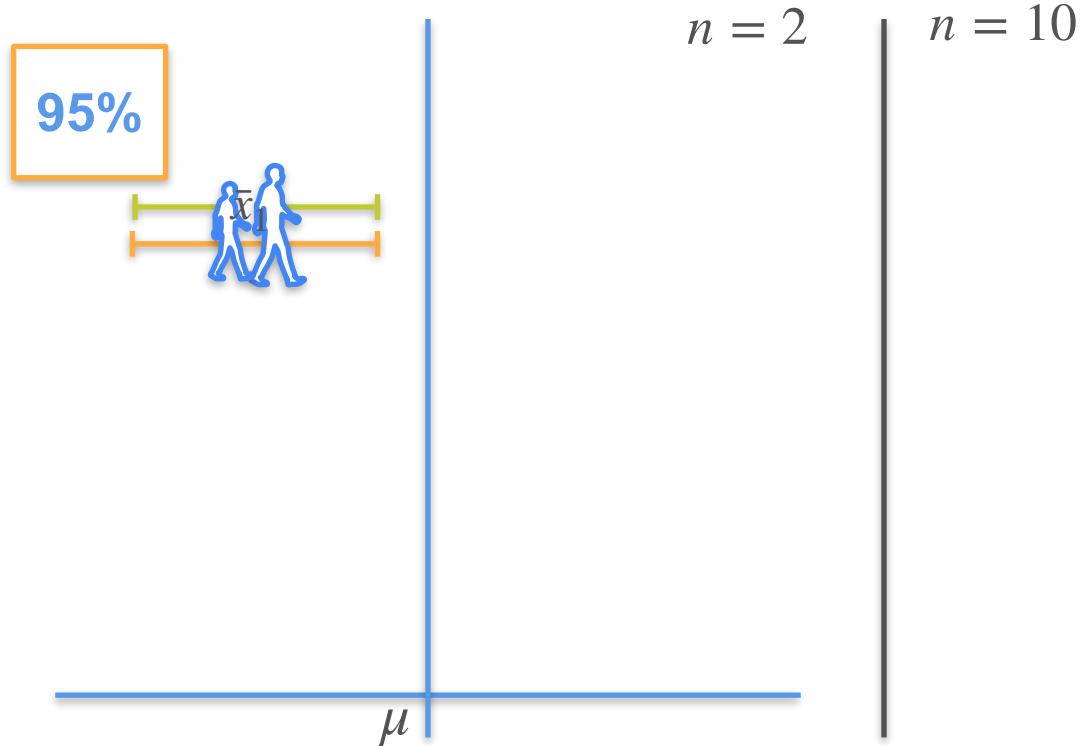
Confidence Interval - Intuition



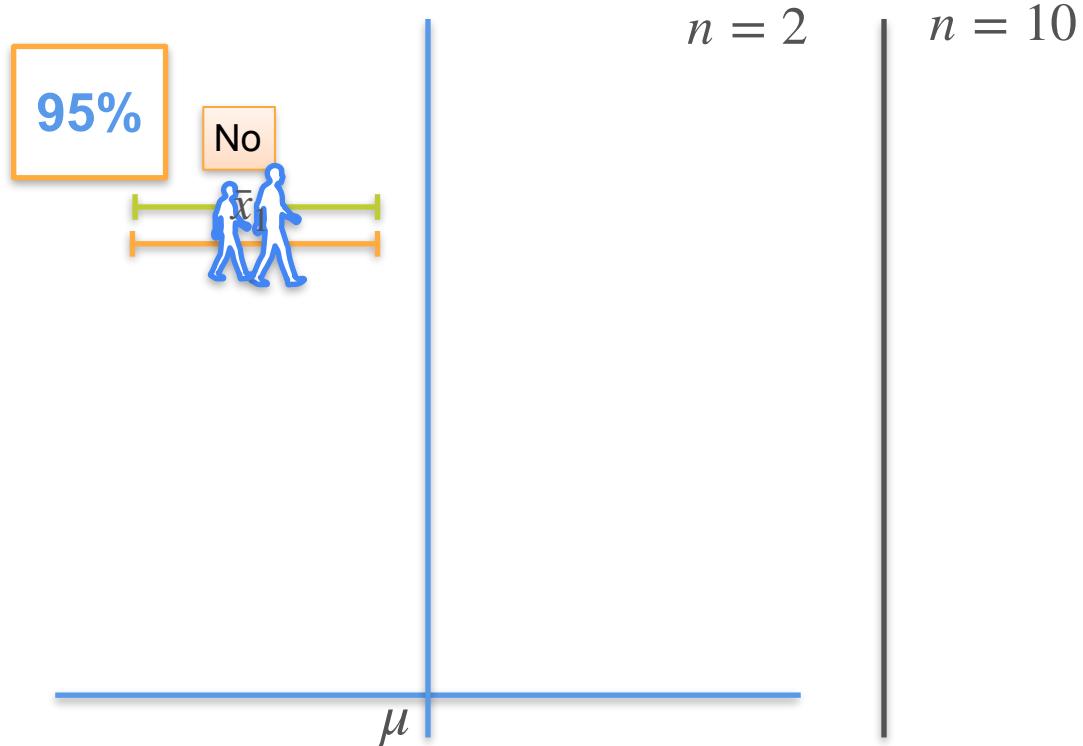
Confidence Interval - Intuition



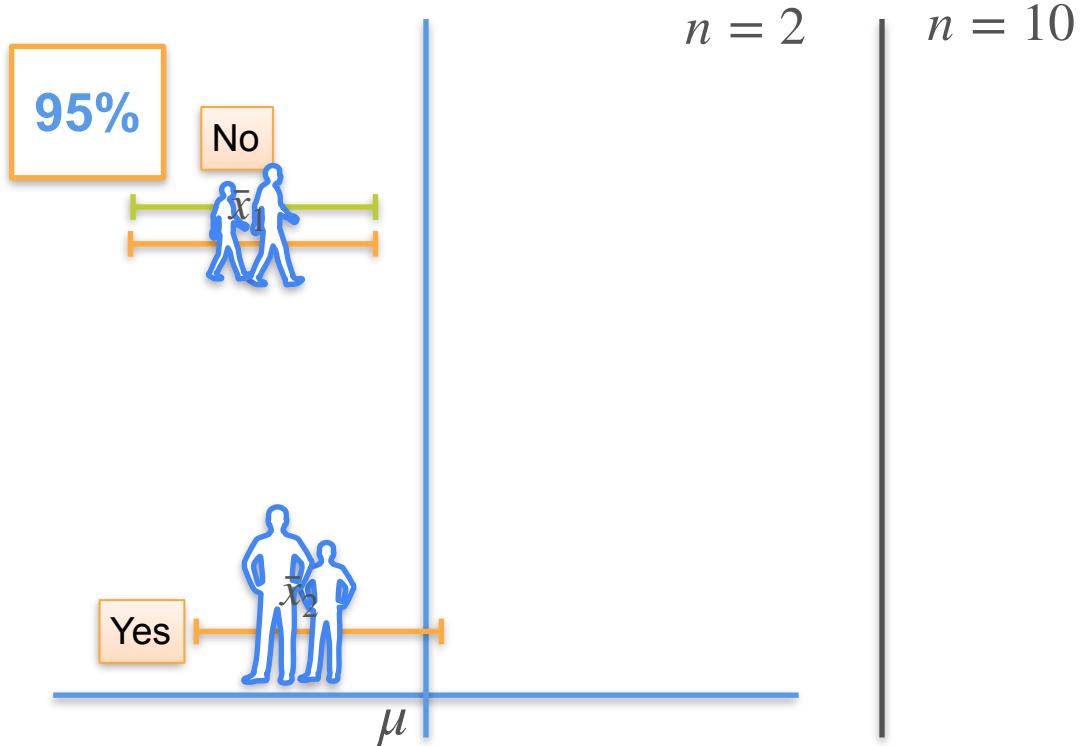
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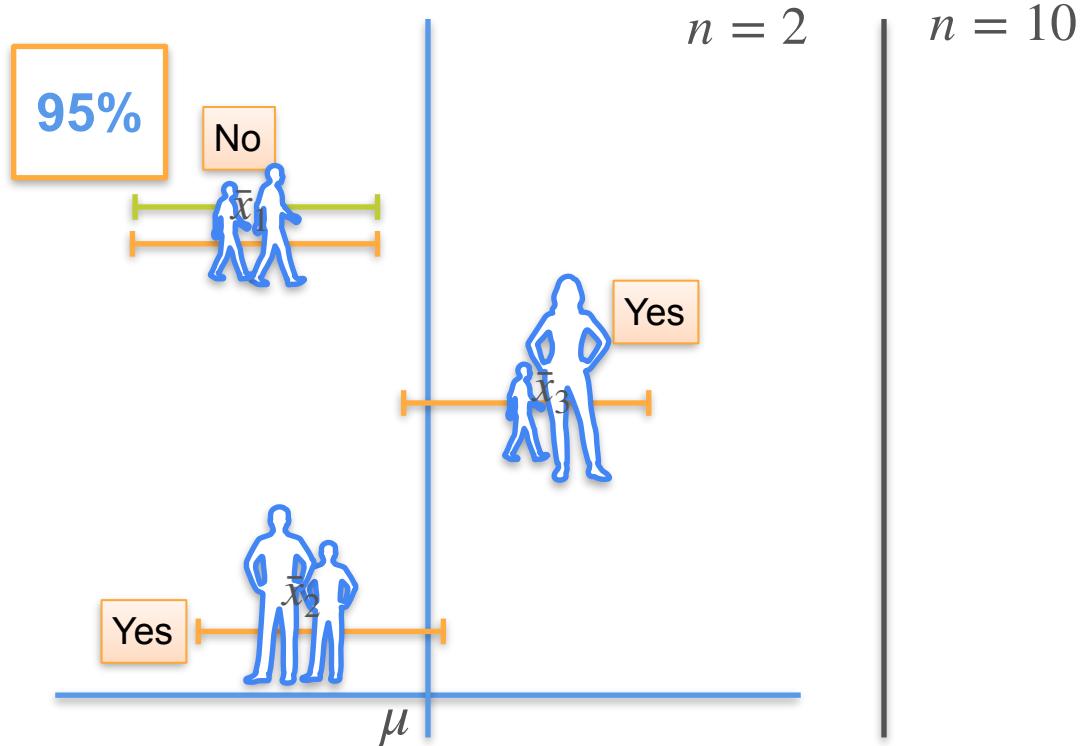
Confidence Interval - Intuition



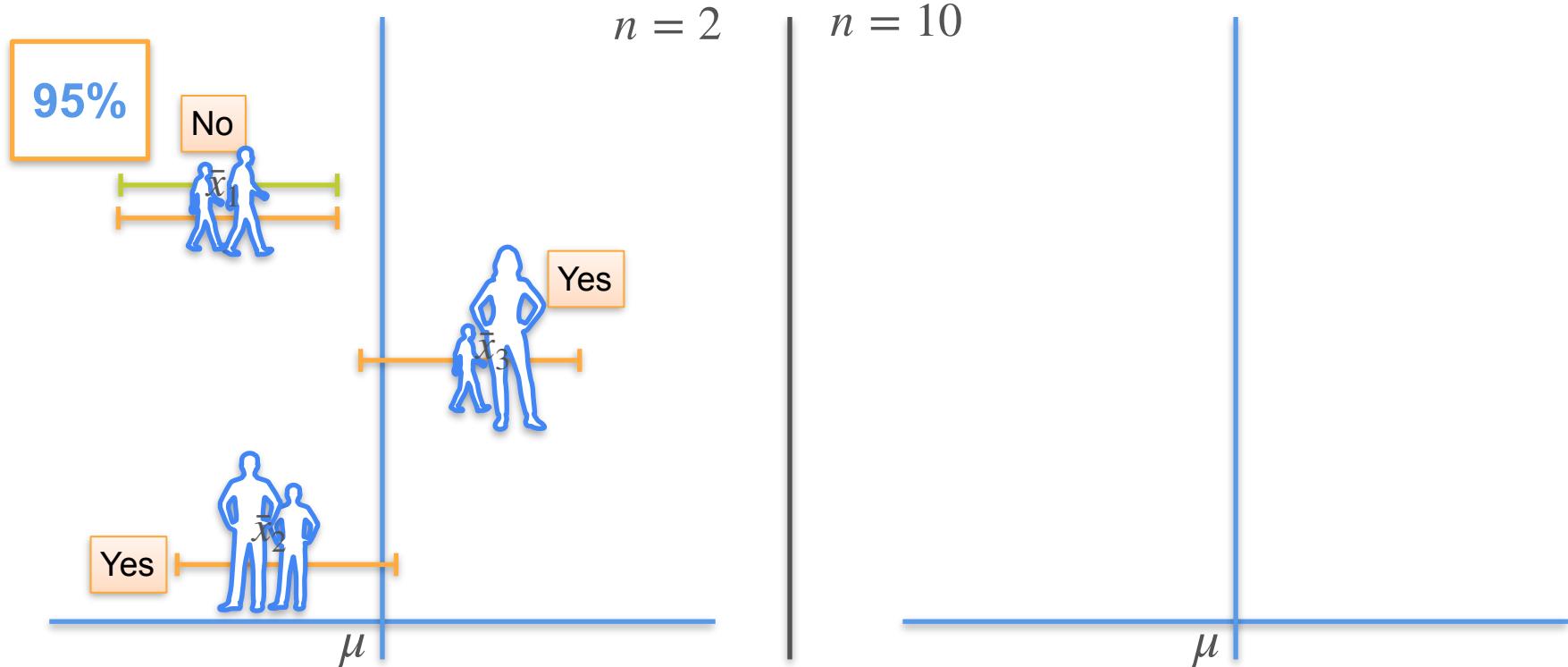
Confidence Interval - Intuition



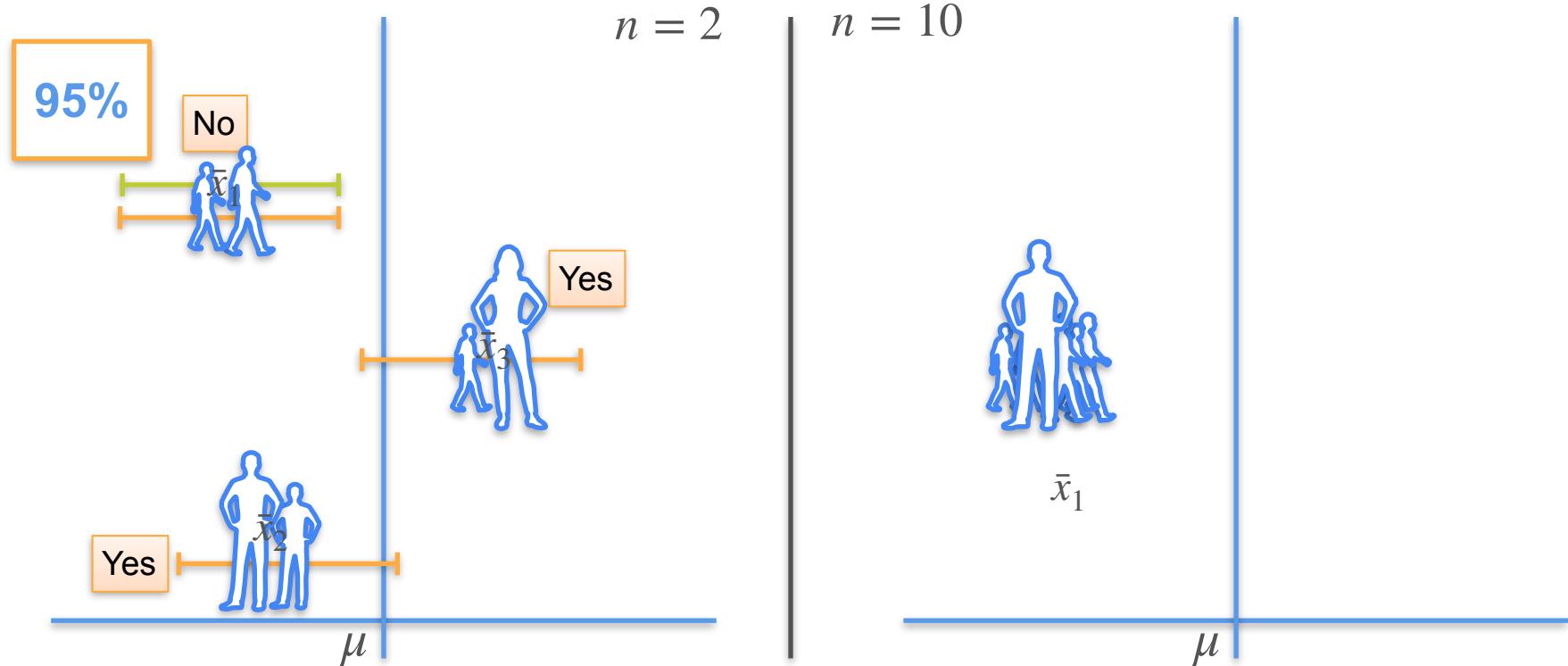
Confidence Interval - Intuition



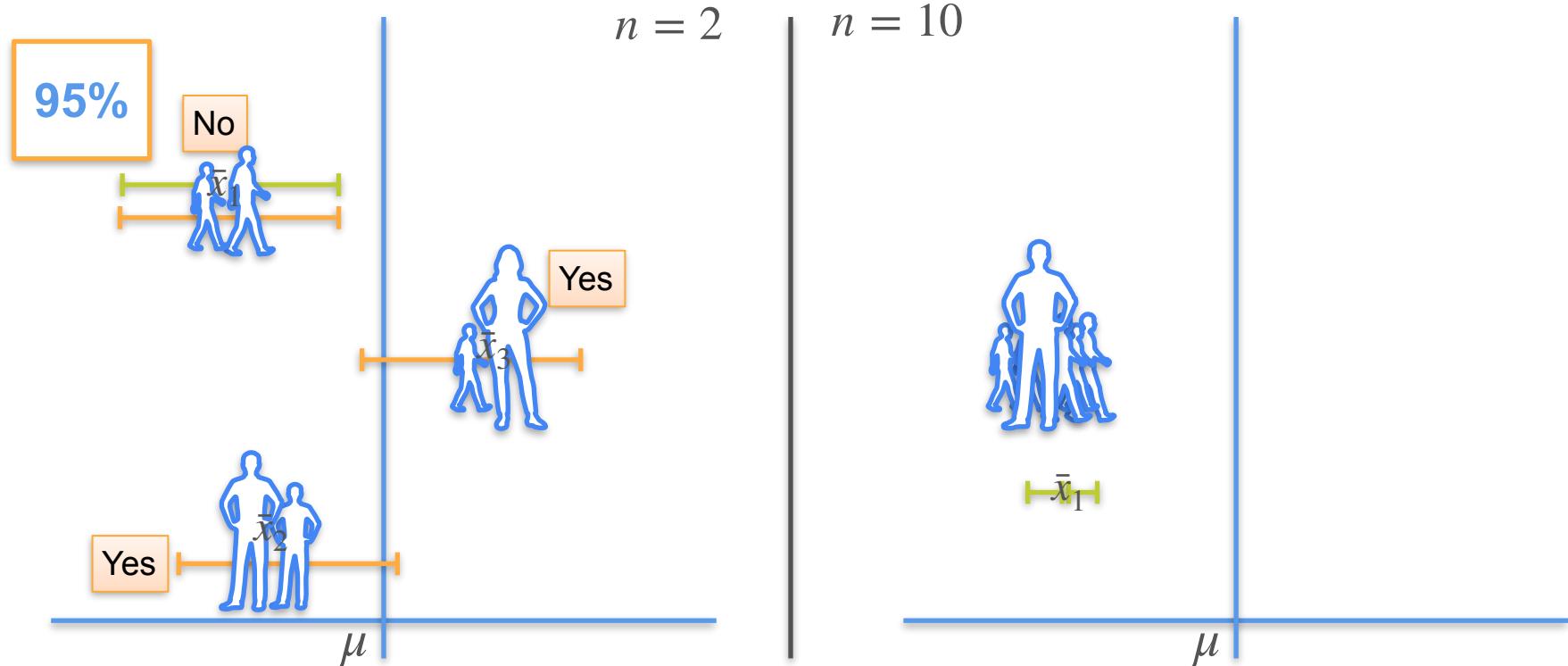
Confidence Interval - Intuition



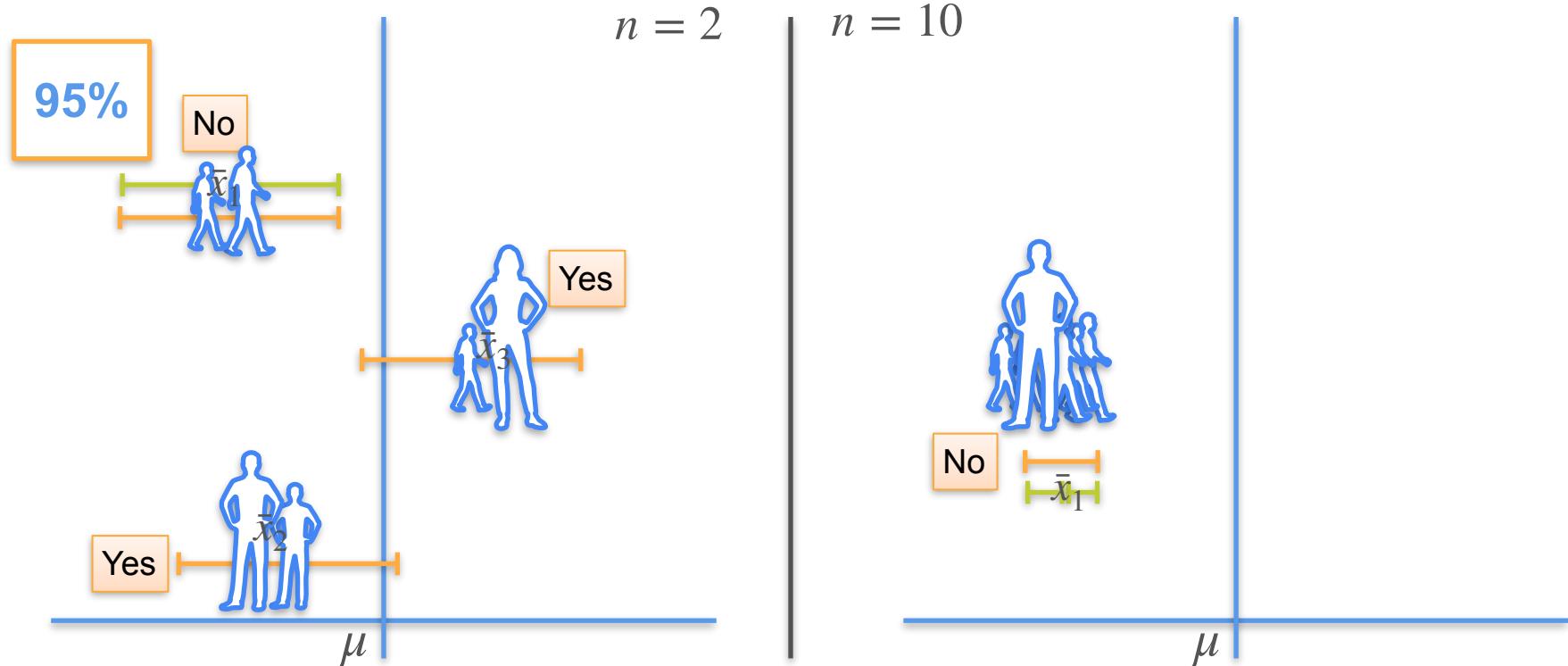
Confidence Interval - Intuition



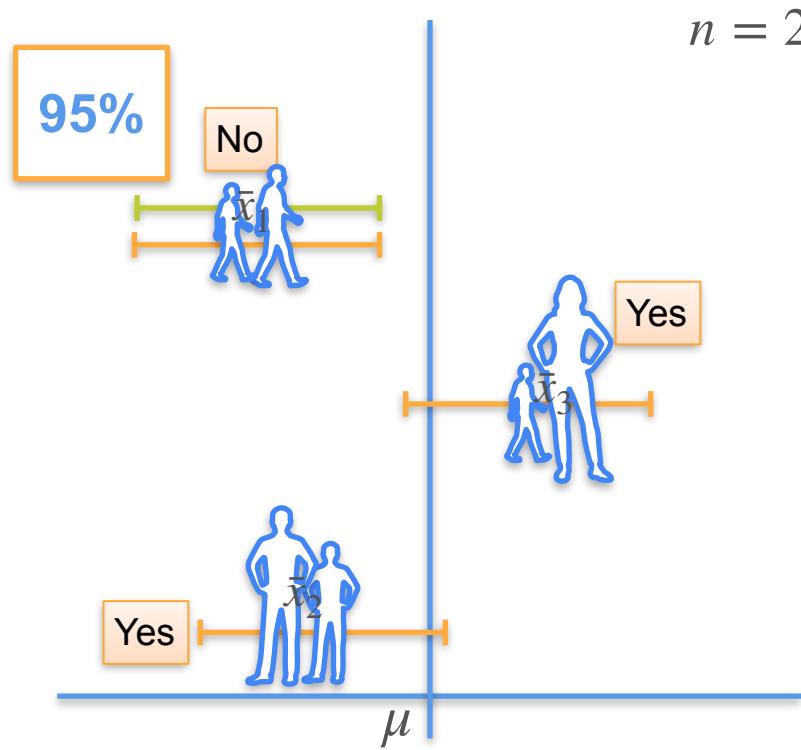
Confidence Interval - Intuition



Confidence Interval - Intuition

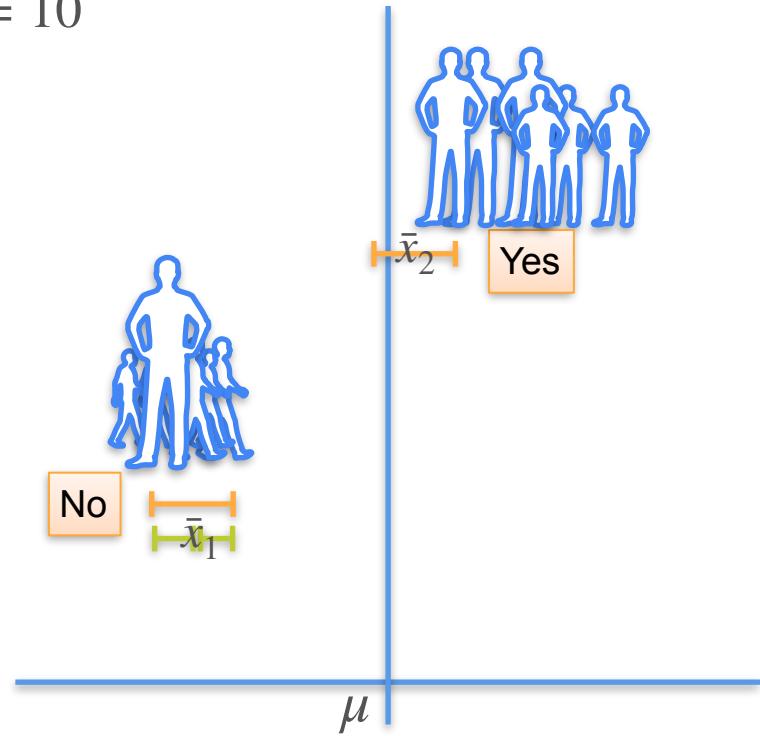


Confidence Interval - Intuition

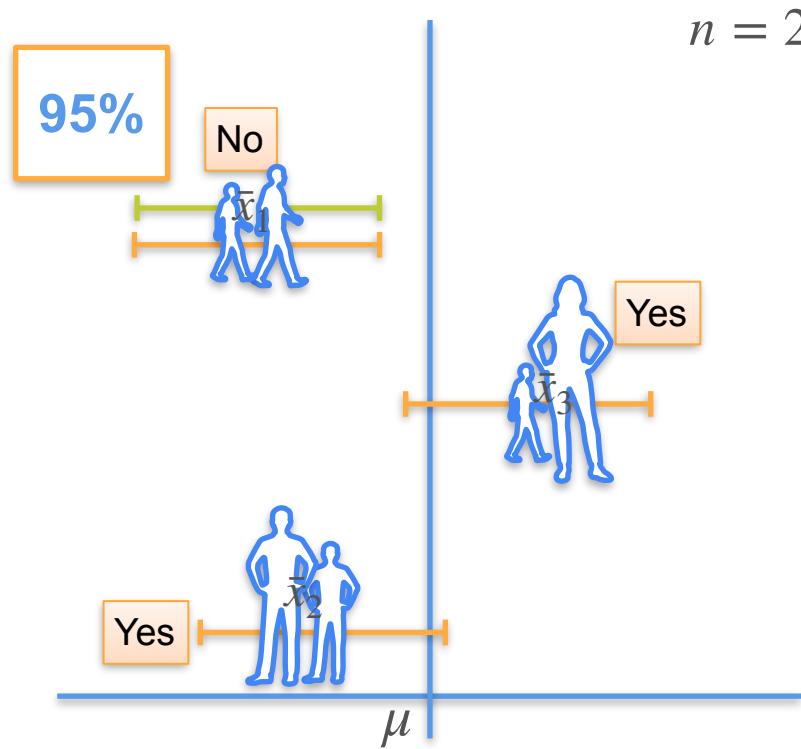


$n = 2$

$n = 10$

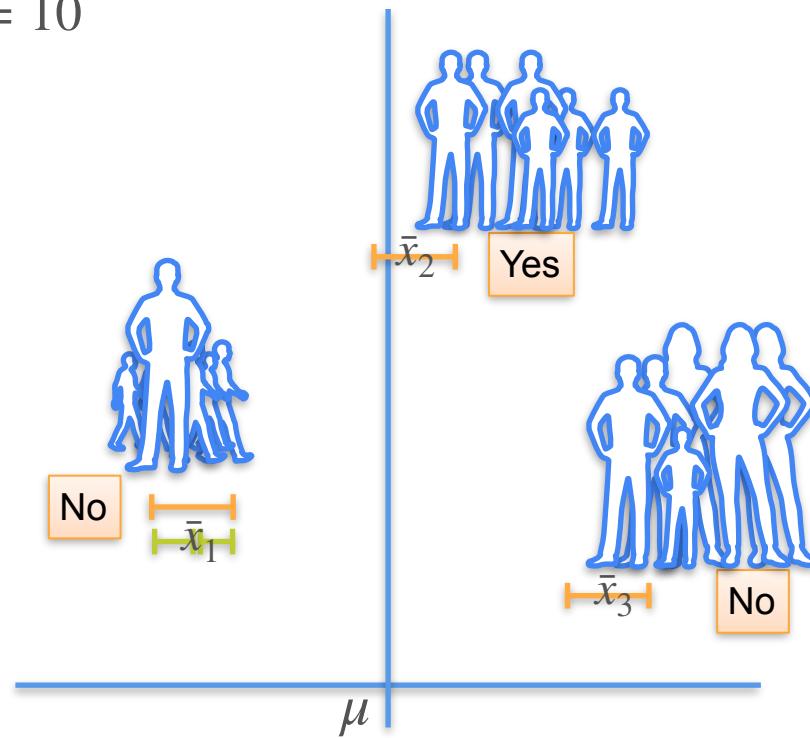


Confidence Interval - Intuition

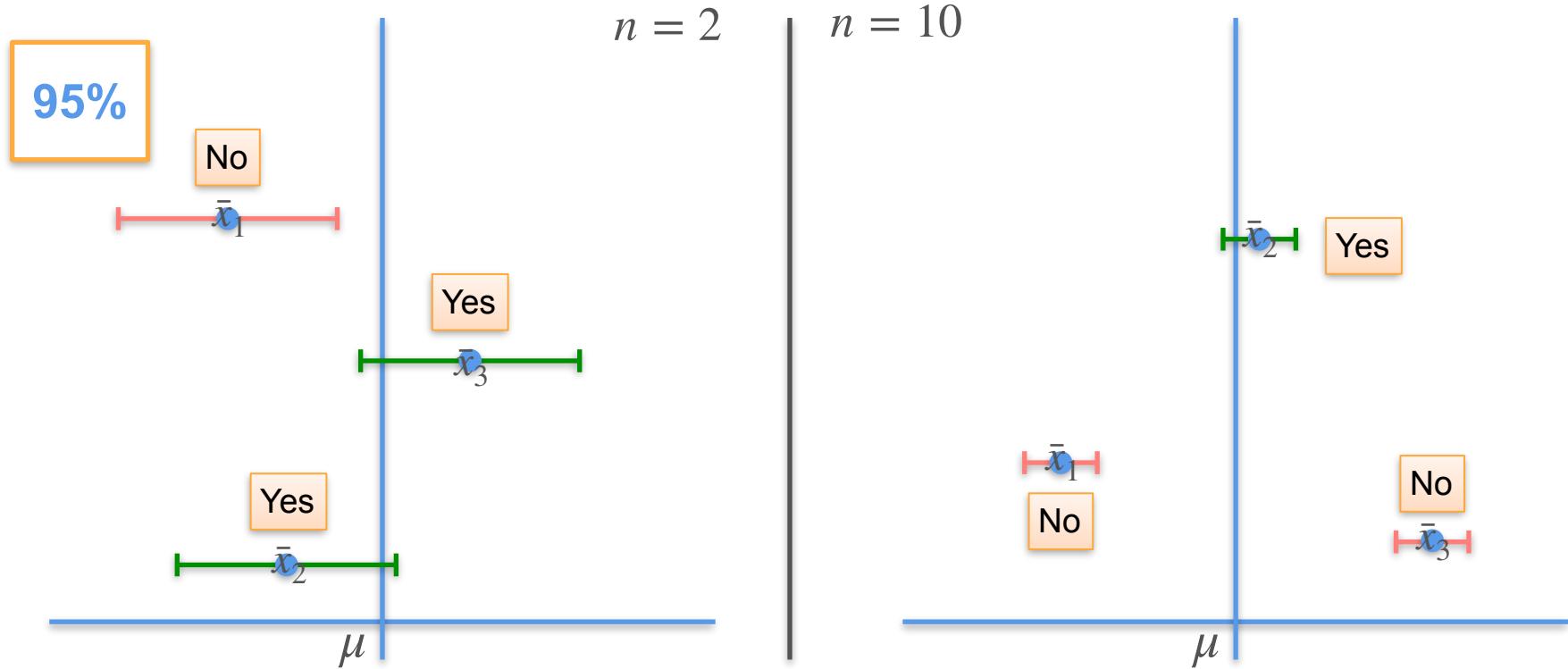


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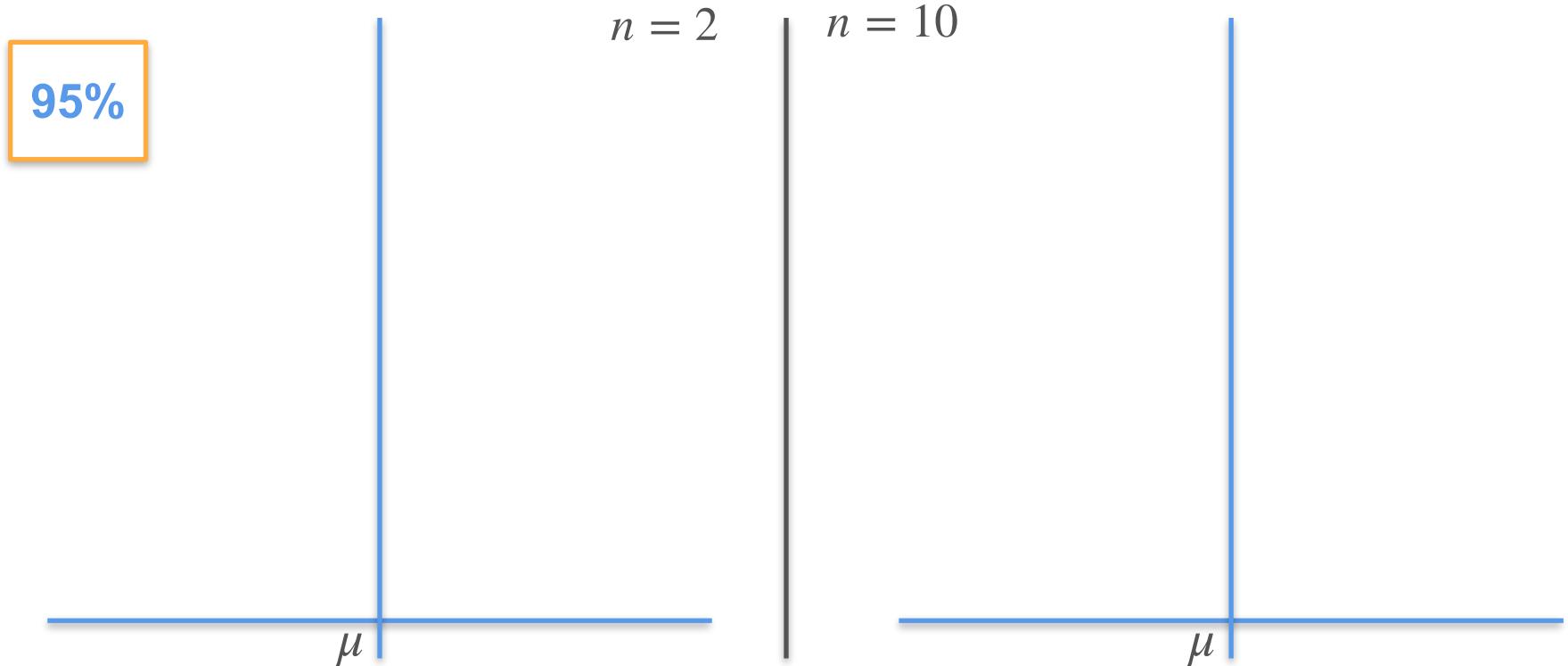
$n = 10$



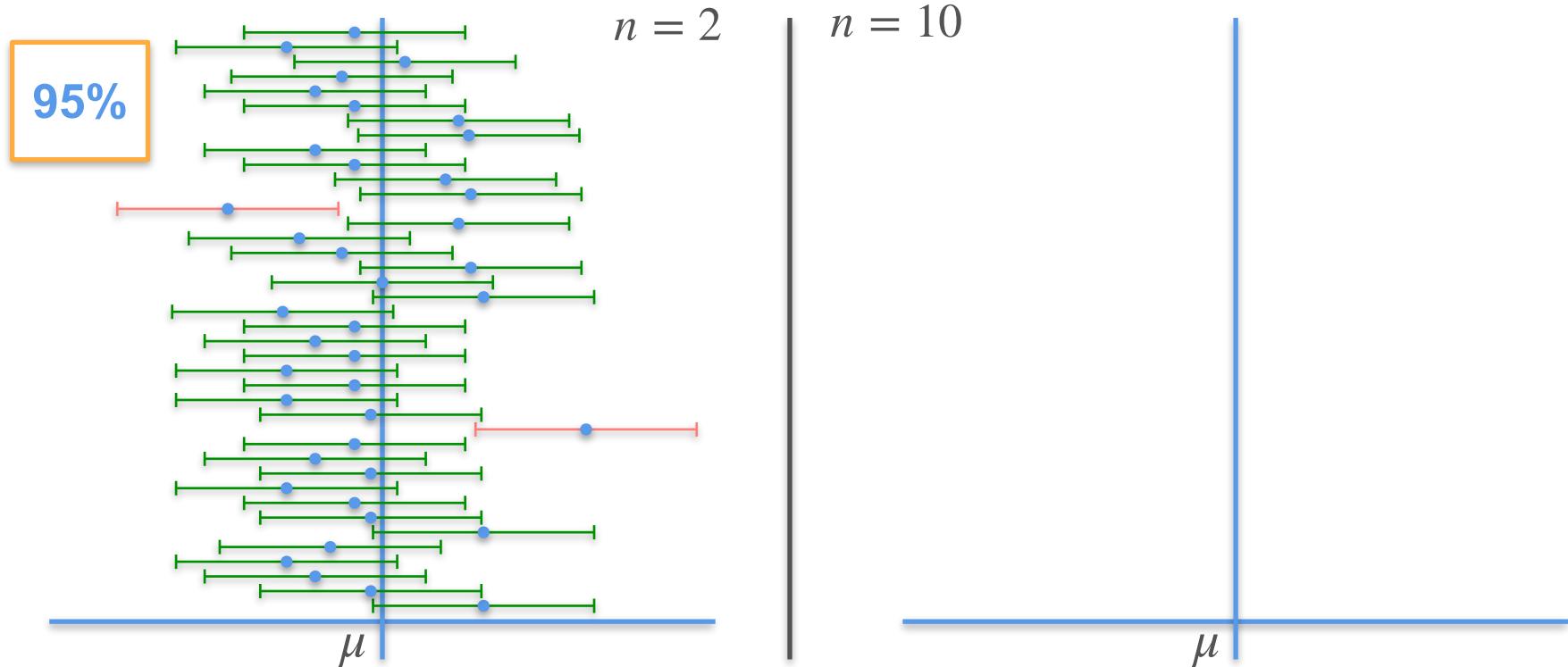
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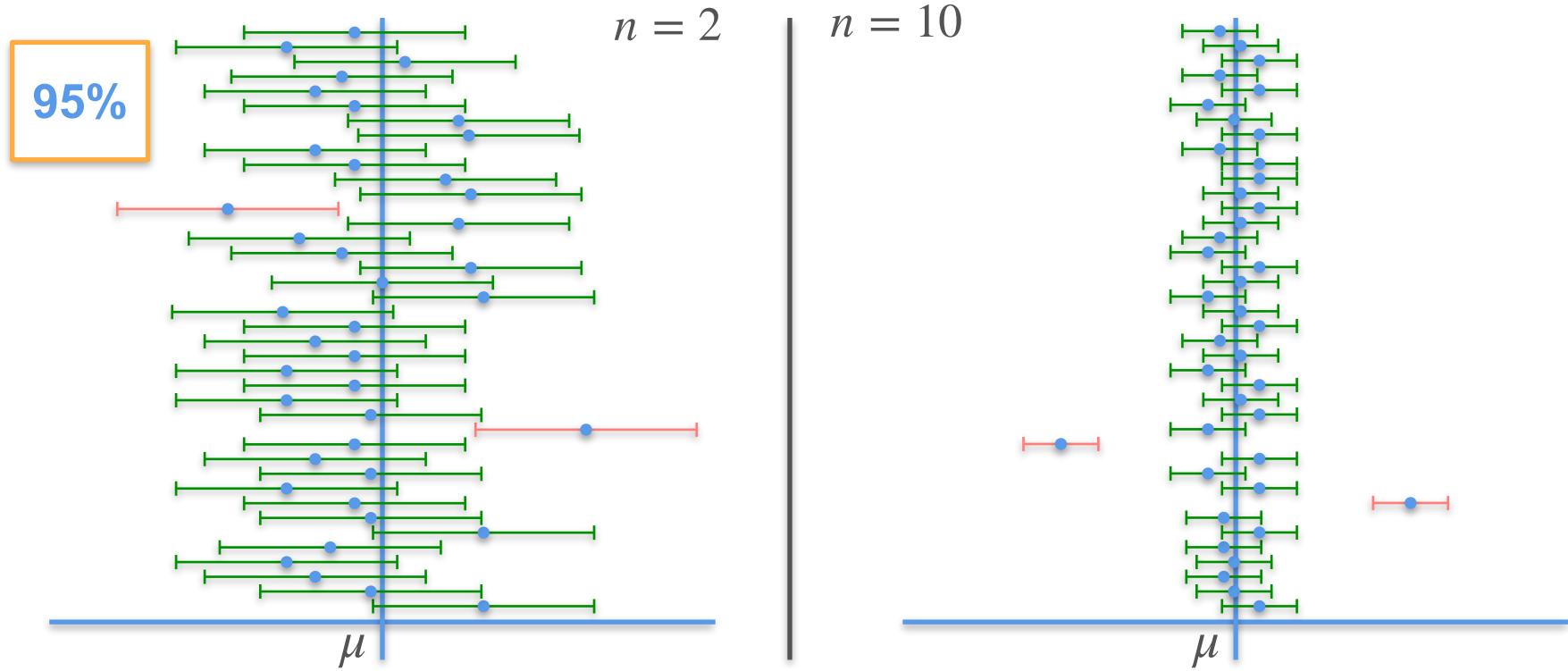
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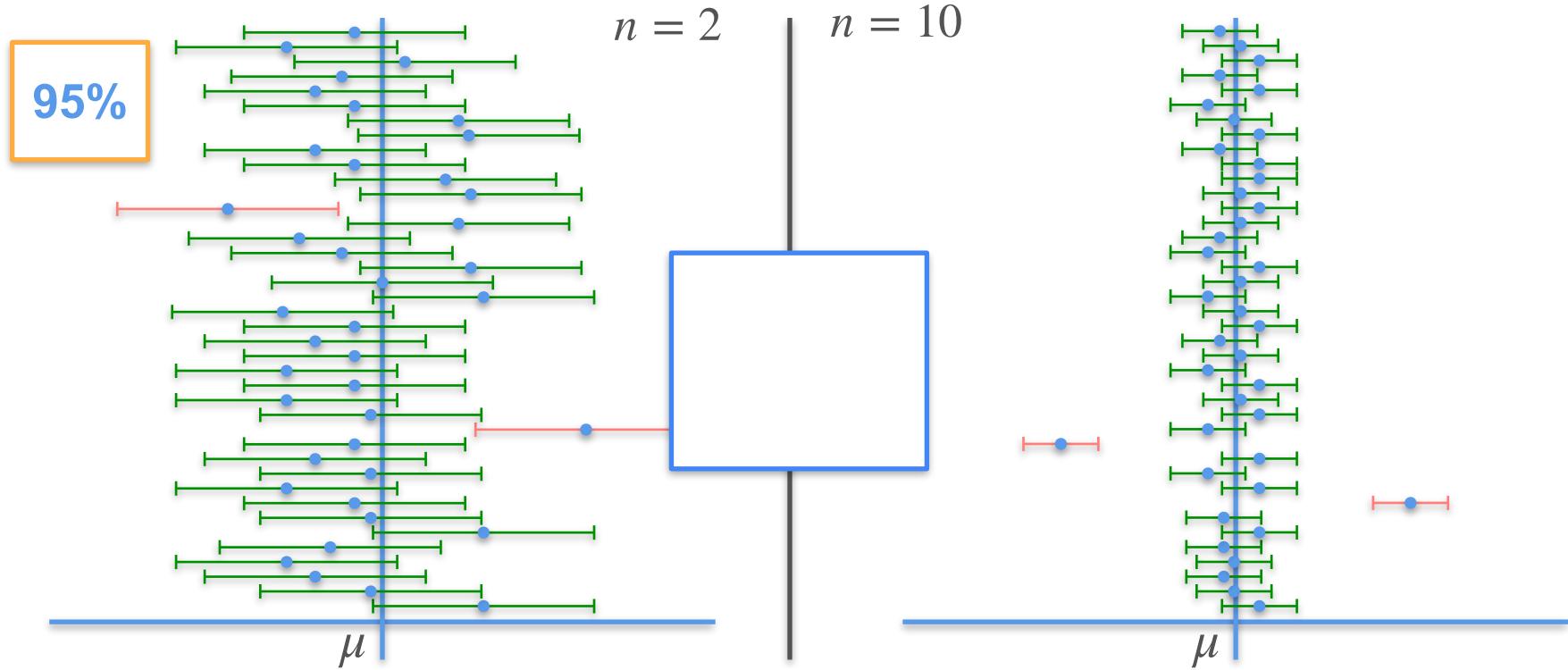
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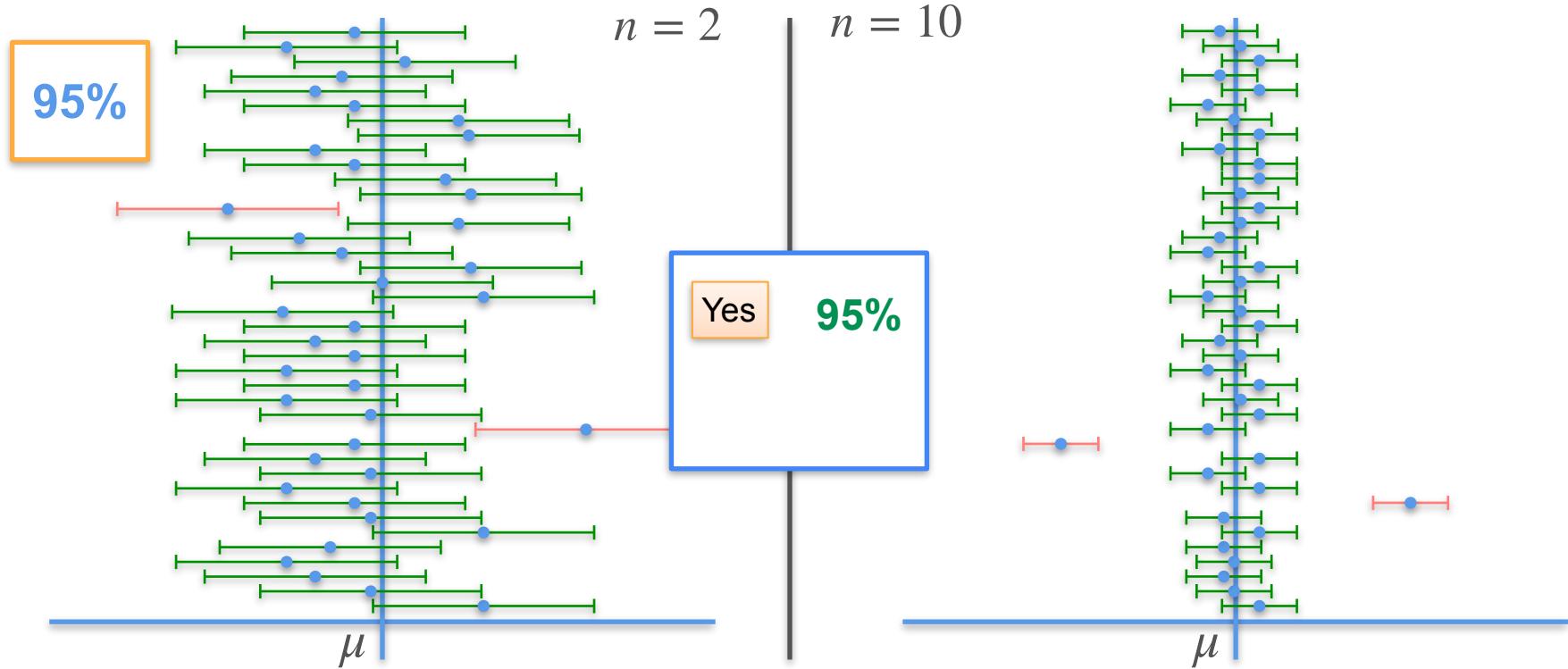
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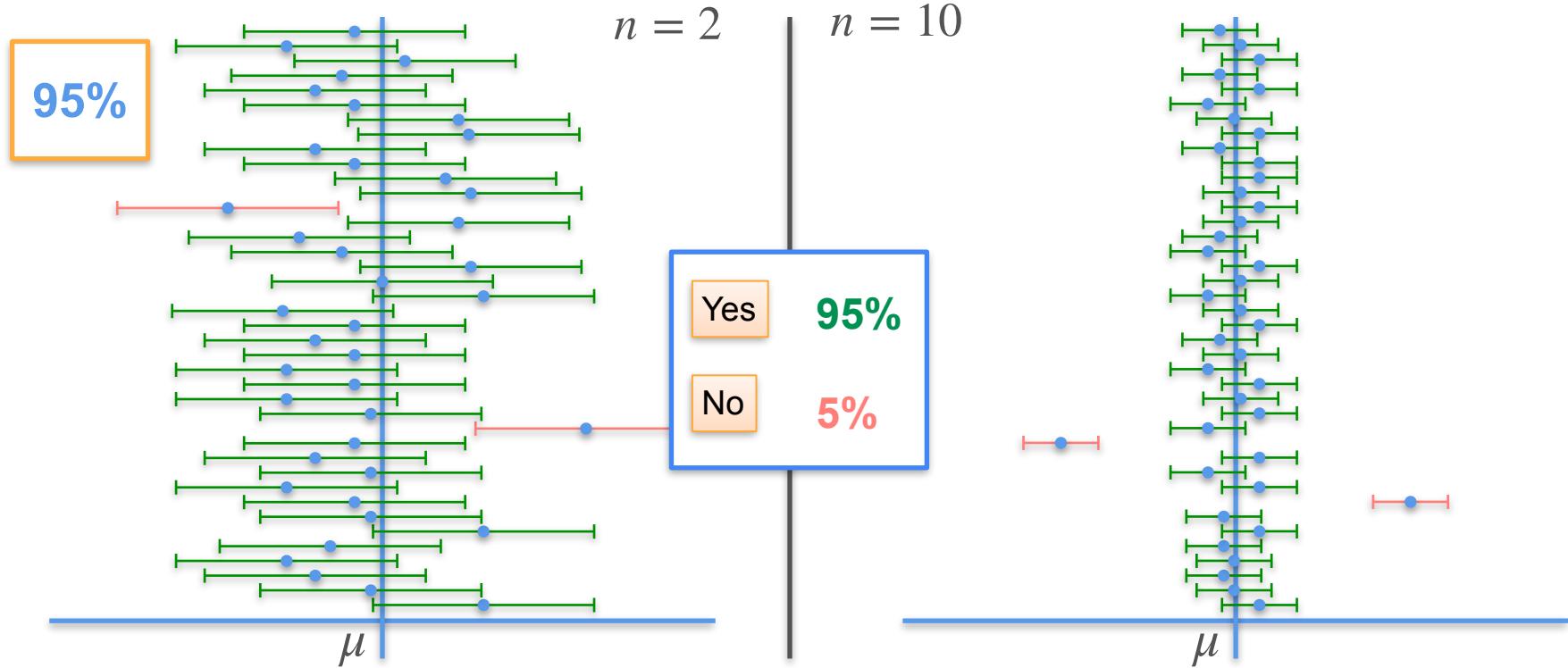
Confidence Interval - Intuition



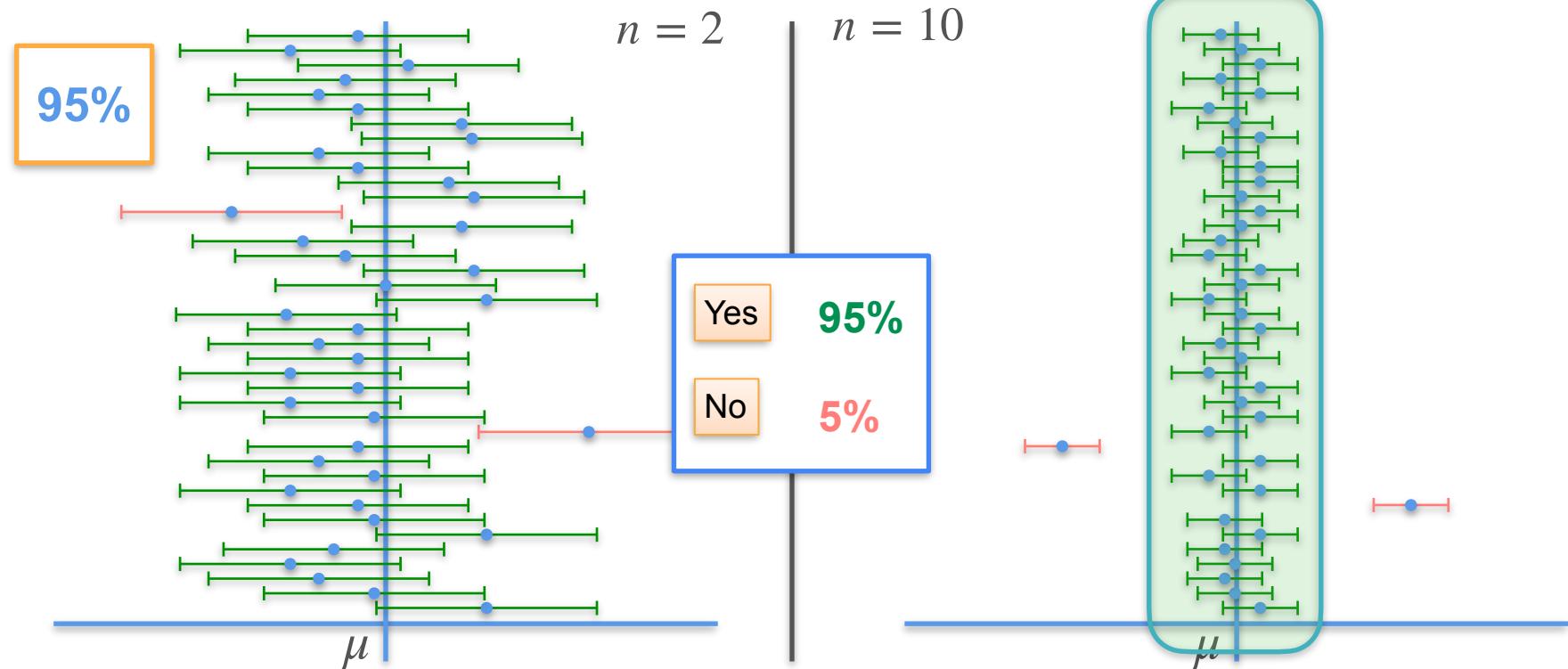
Confidence Interval - Intuition



Confidence Interval - Intuition



Confidence Interval - Intuition



Effect of the Sample Size

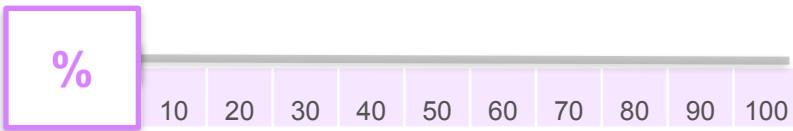


Effect of the Sample Size

sample size

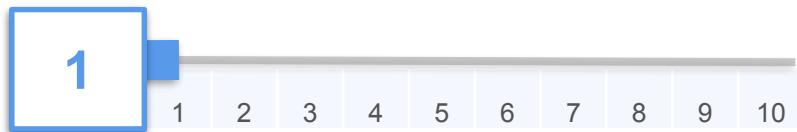


Confidence level

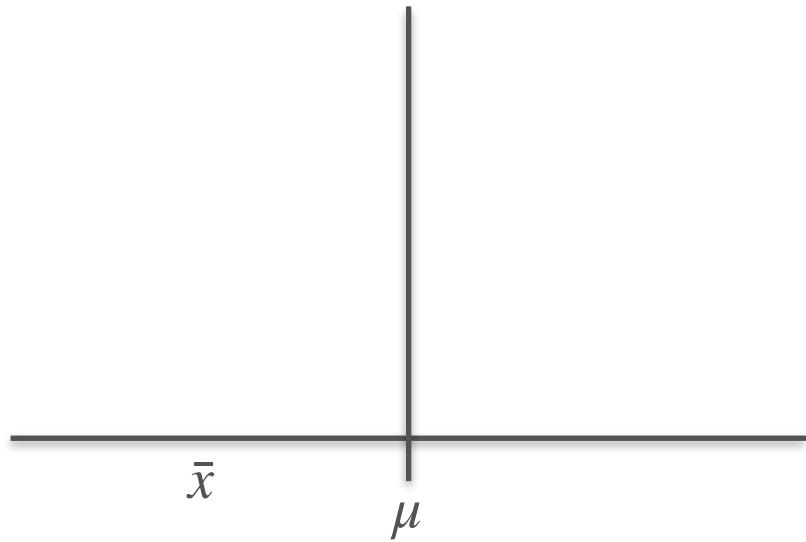
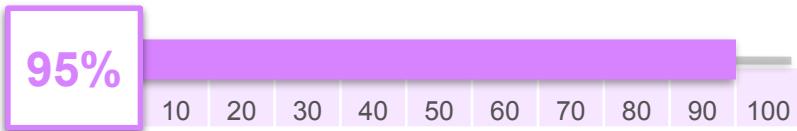


Effect of the Sample Size

sample size

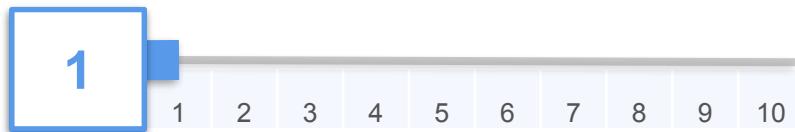


Confidence level

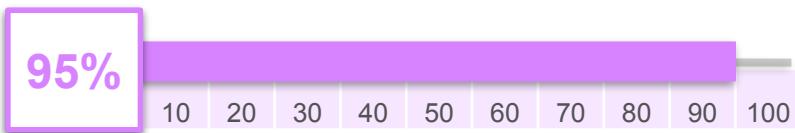


Effect of the Sample Size

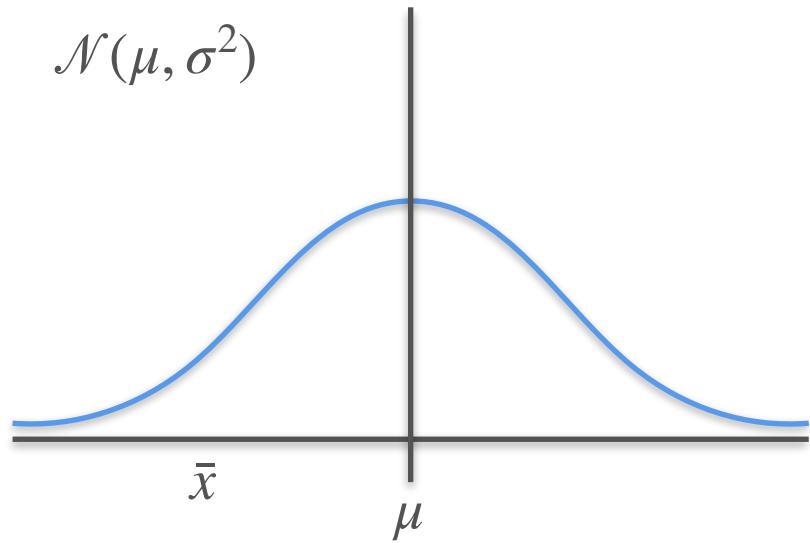
sample size



Confidence level

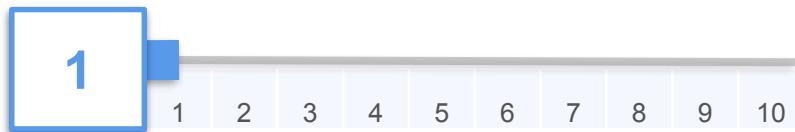


$$\mathcal{N}(\mu, \sigma^2)$$

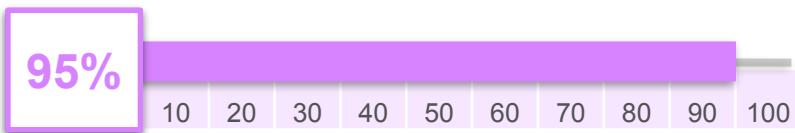


Effect of the Sample Size

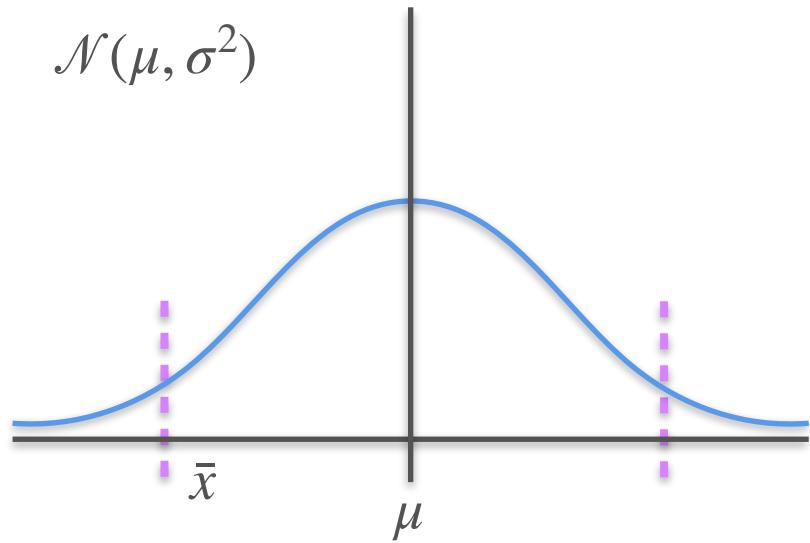
sample size



Confidence level

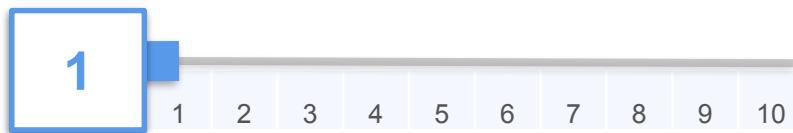


$$\mathcal{N}(\mu, \sigma^2)$$

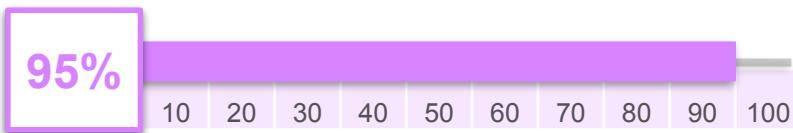


Effect of the Sample Size

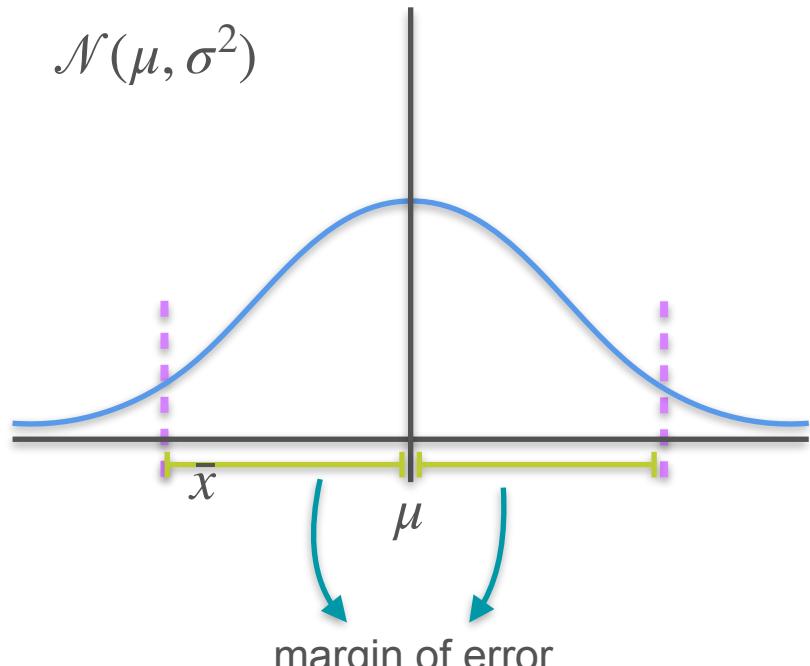
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Confidence level

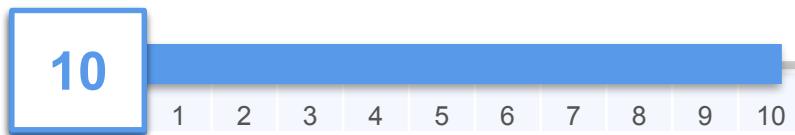


$$\mathcal{N}(\mu, \sigma^2)$$

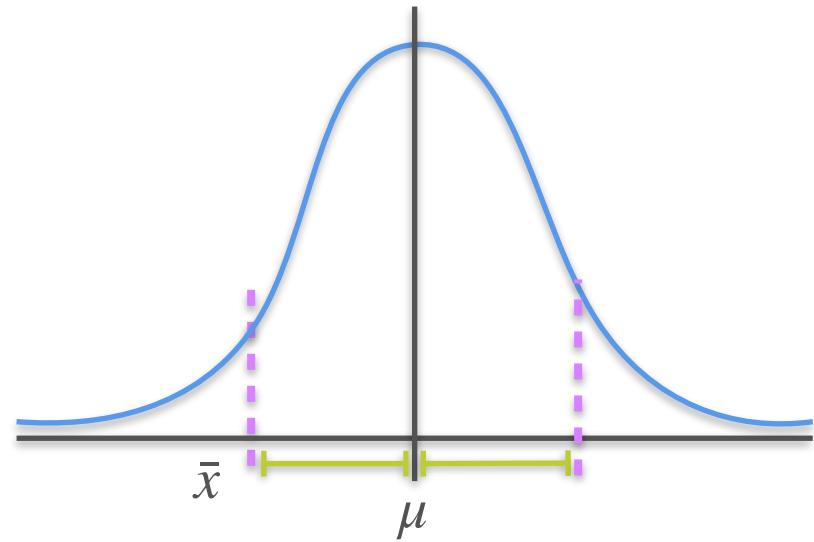
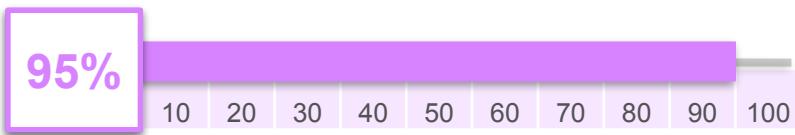


Effect of the Sample Size

sample size

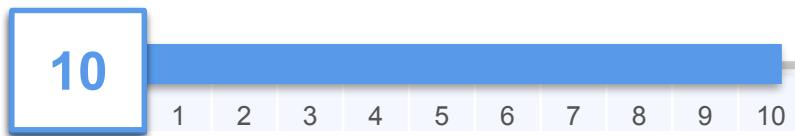


Confidence level

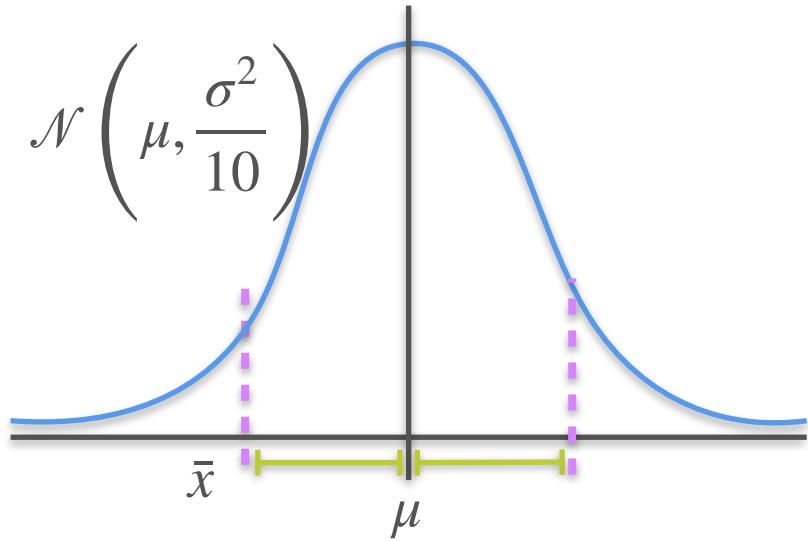
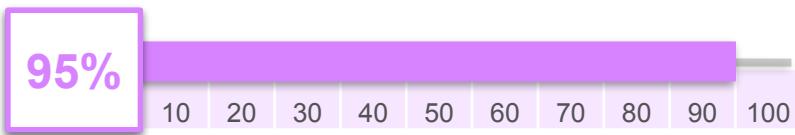


Effect of the Sample Size

sample size

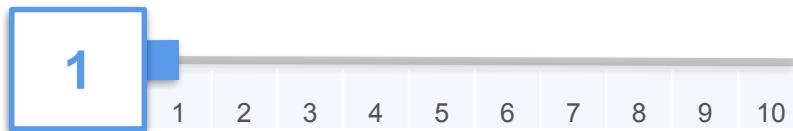


Confidence level

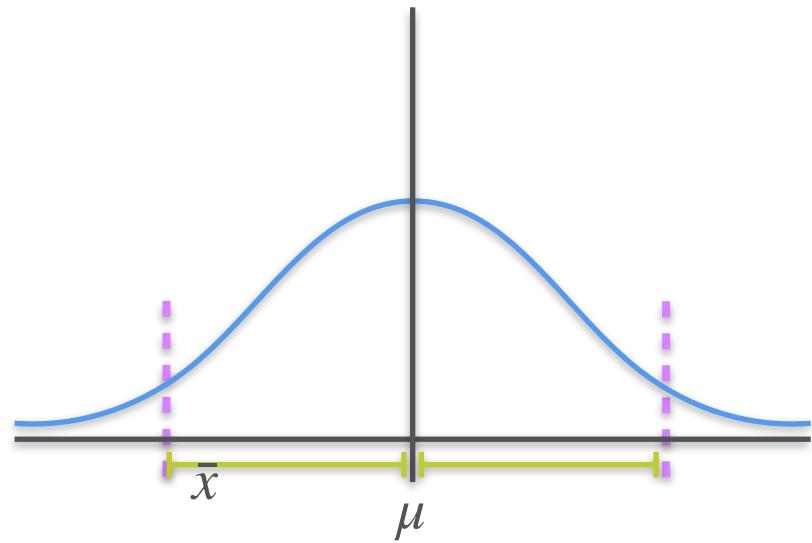
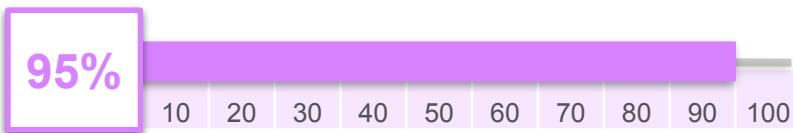


Effect of the Sample Size

sample size

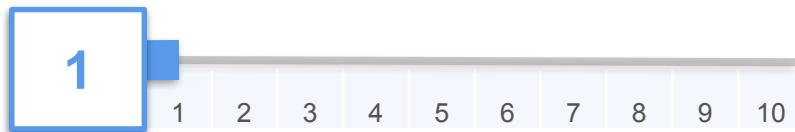


Confidence level

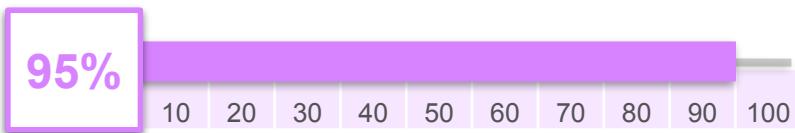


Effect of the Sample Size

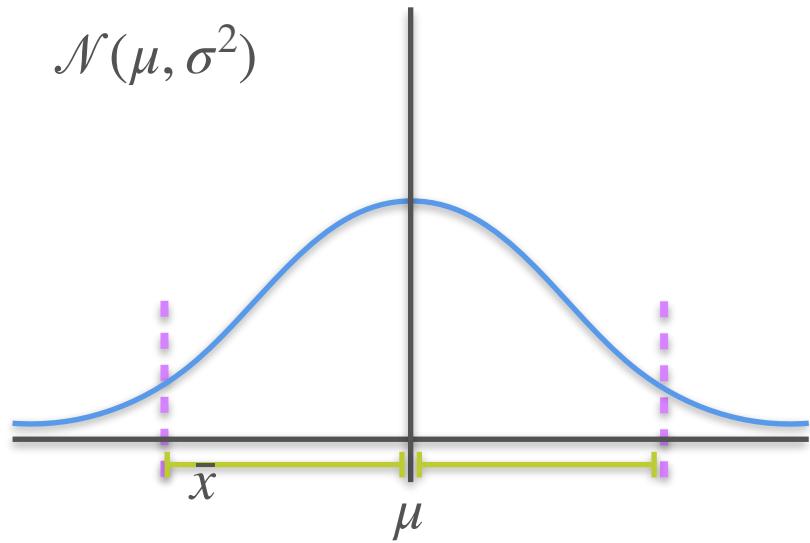
sample size



Confidence level

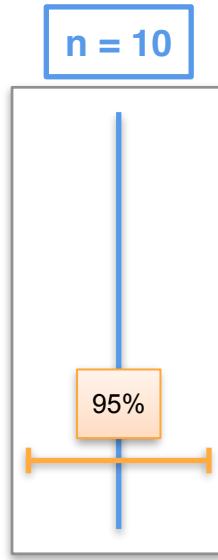
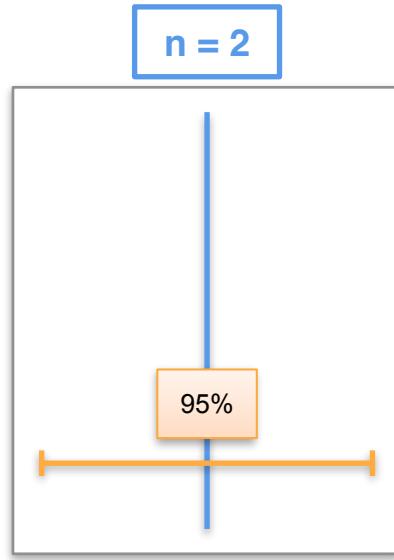
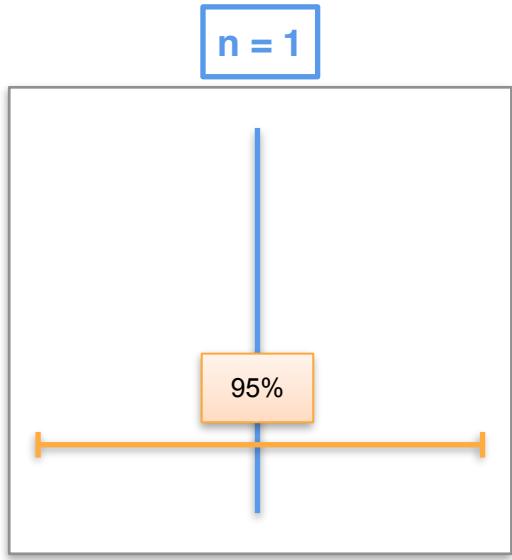


$$\mathcal{N}(\mu, \sigma^2)$$

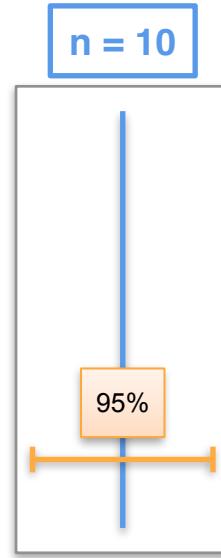
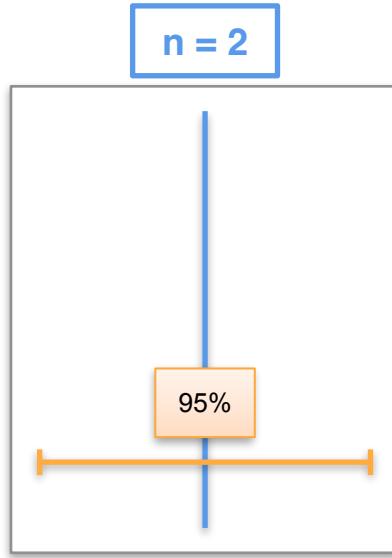
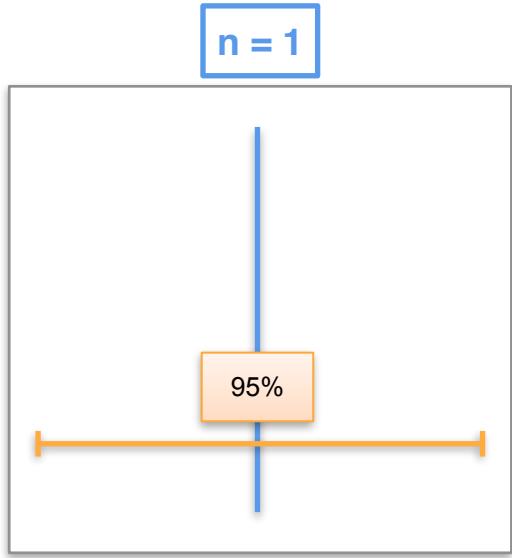


Effect of the Sample Size

Effect of the Sample Size



Effect of the Sample Size



As n increases, the confidence interval shrinks

Effect of the Confidence Level



Effect of the Confidence Level

$n = 1$



Effect of the Confidence Level

$n = 1$

$\mathcal{N}(\mu, \sigma^2)$



Effect of the Confidence Level

$n = 1$

95%

$\mathcal{N}(\mu, \sigma^2)$

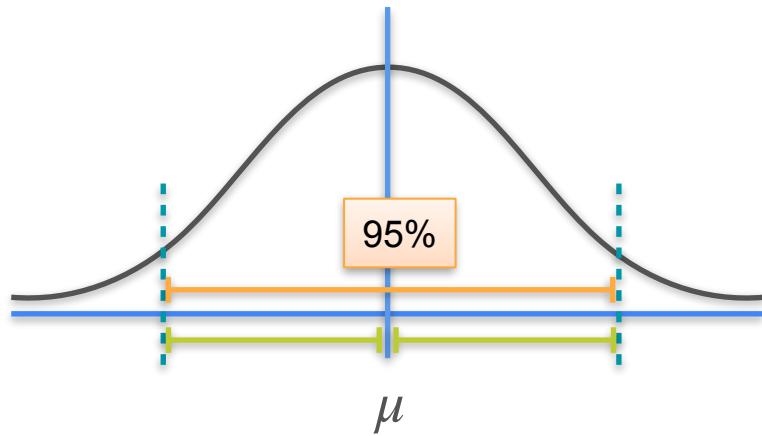


Effect of the Confidence Level

$n = 1$

95%

$\mathcal{N}(\mu, \sigma^2)$

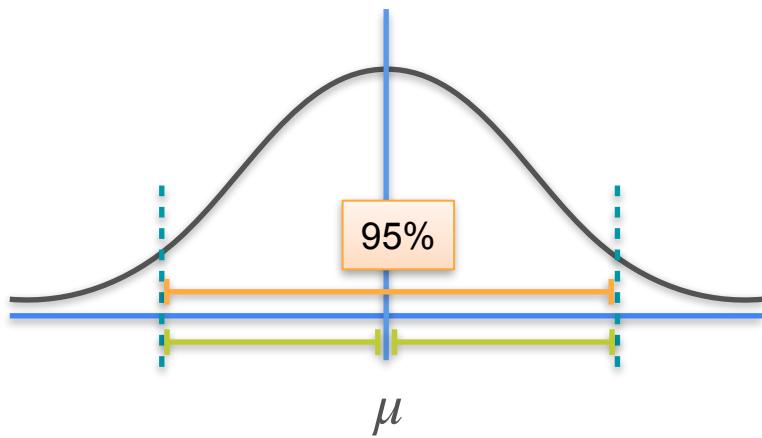
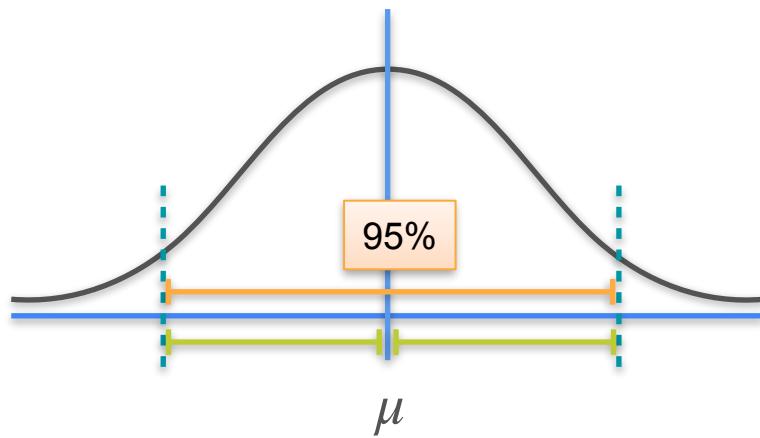


Effect of the Confidence Level

$$n = 1$$

$$\mathcal{N}(\mu, \sigma^2)$$

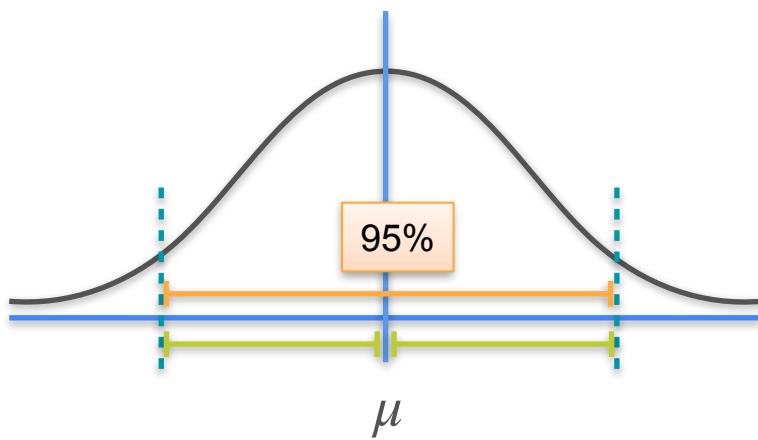
95%



Effect of the Confidence Level

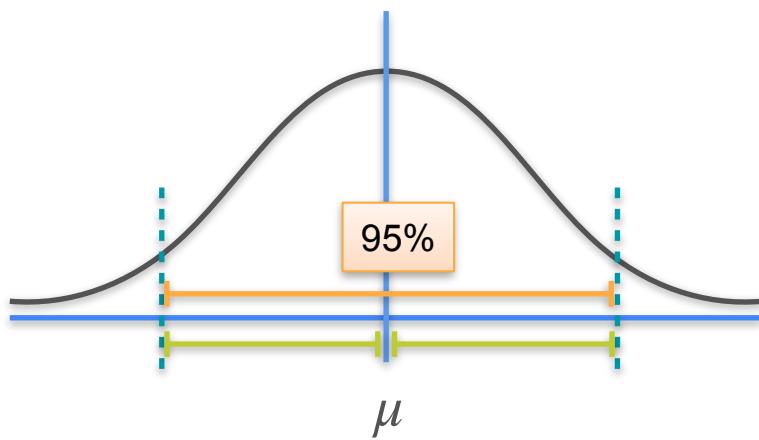
$$n = 1$$

$$\mathcal{N}(\mu, \sigma^2)$$



95%

70%

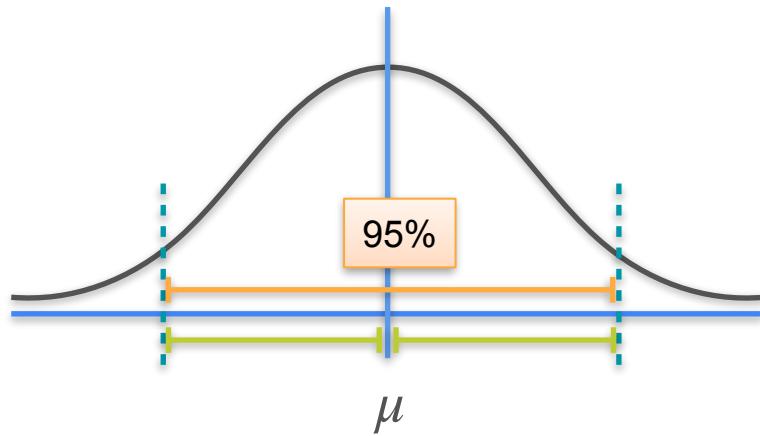


Effect of the Confidence Level

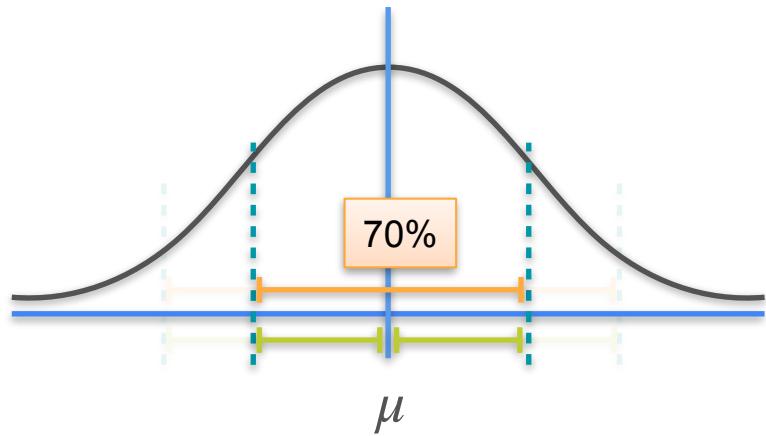
$$n = 1$$

$$\mathcal{N}(\mu, \sigma^2)$$

95%



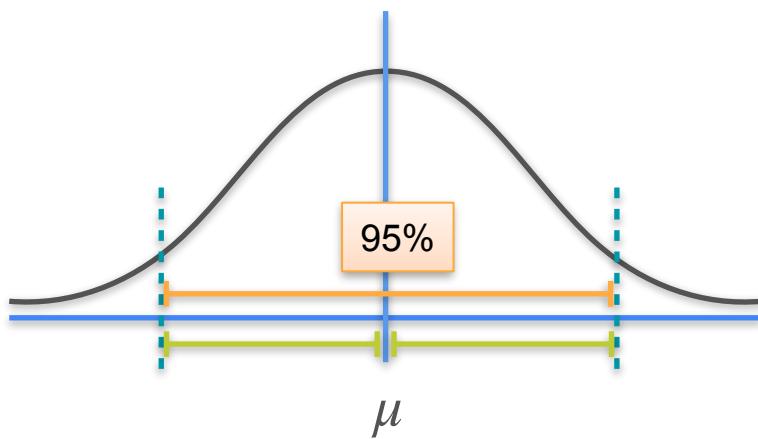
70%



Effect of the Confidence Level

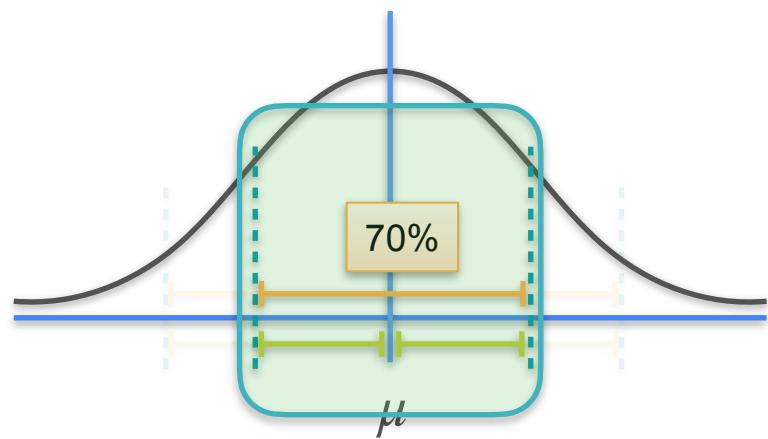
$$n = 1$$

$$\mathcal{N}(\mu, \sigma^2)$$



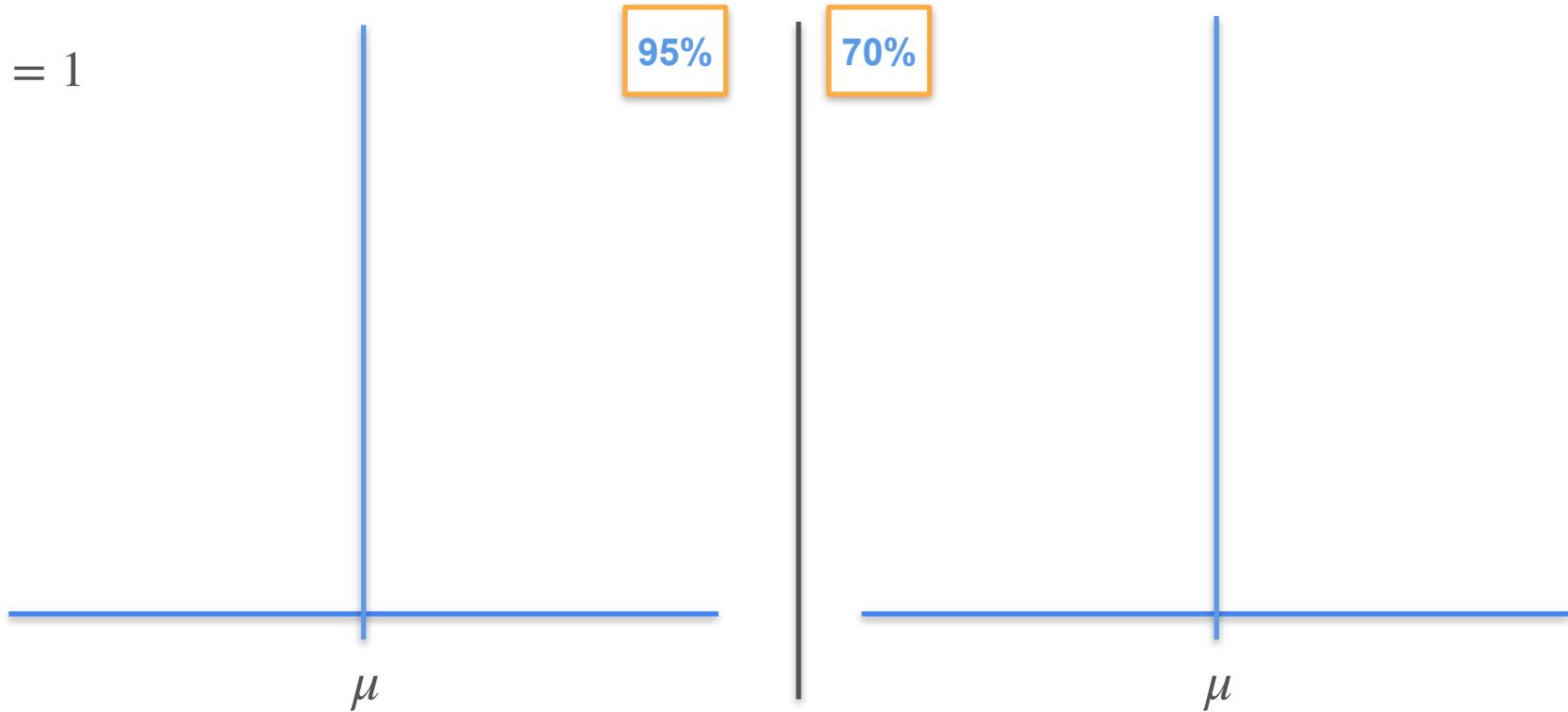
95%

70%

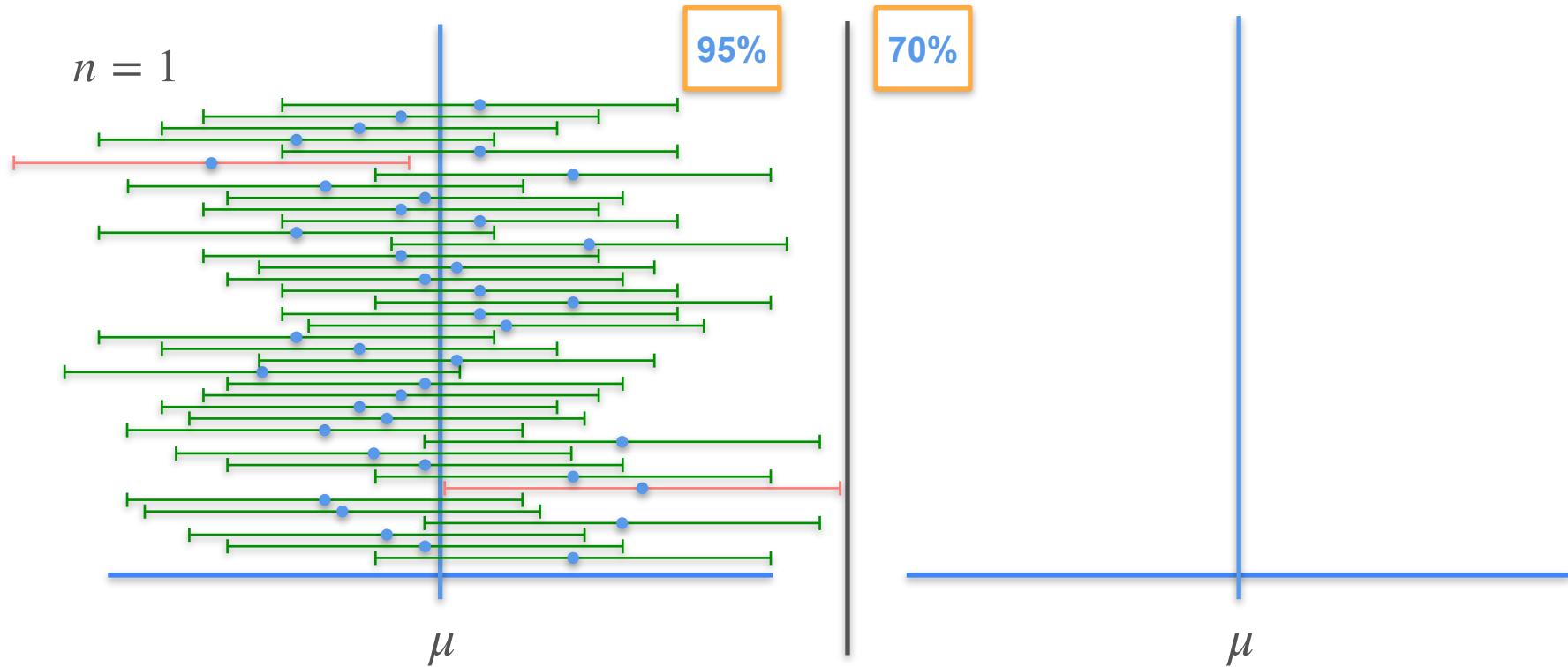


Effect of the Confidence Level

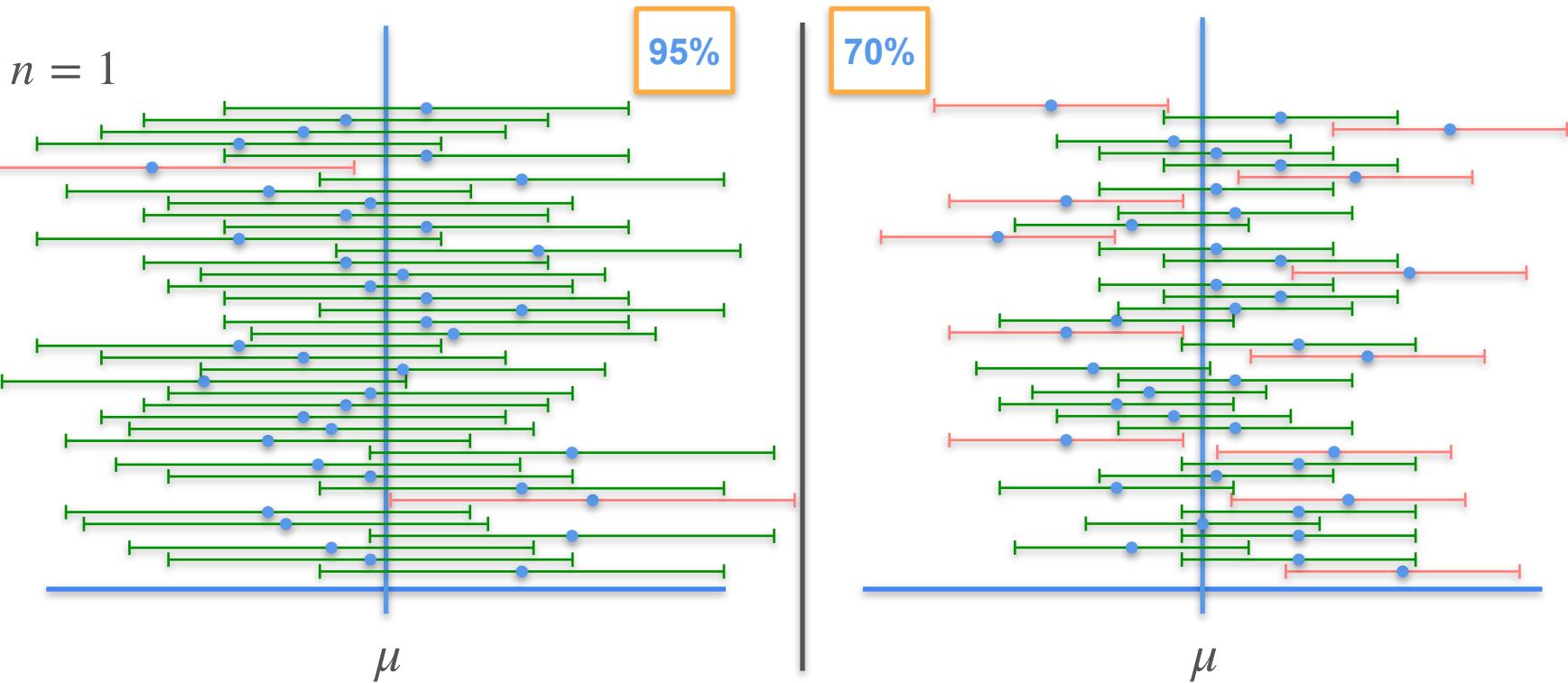
$n = 1$



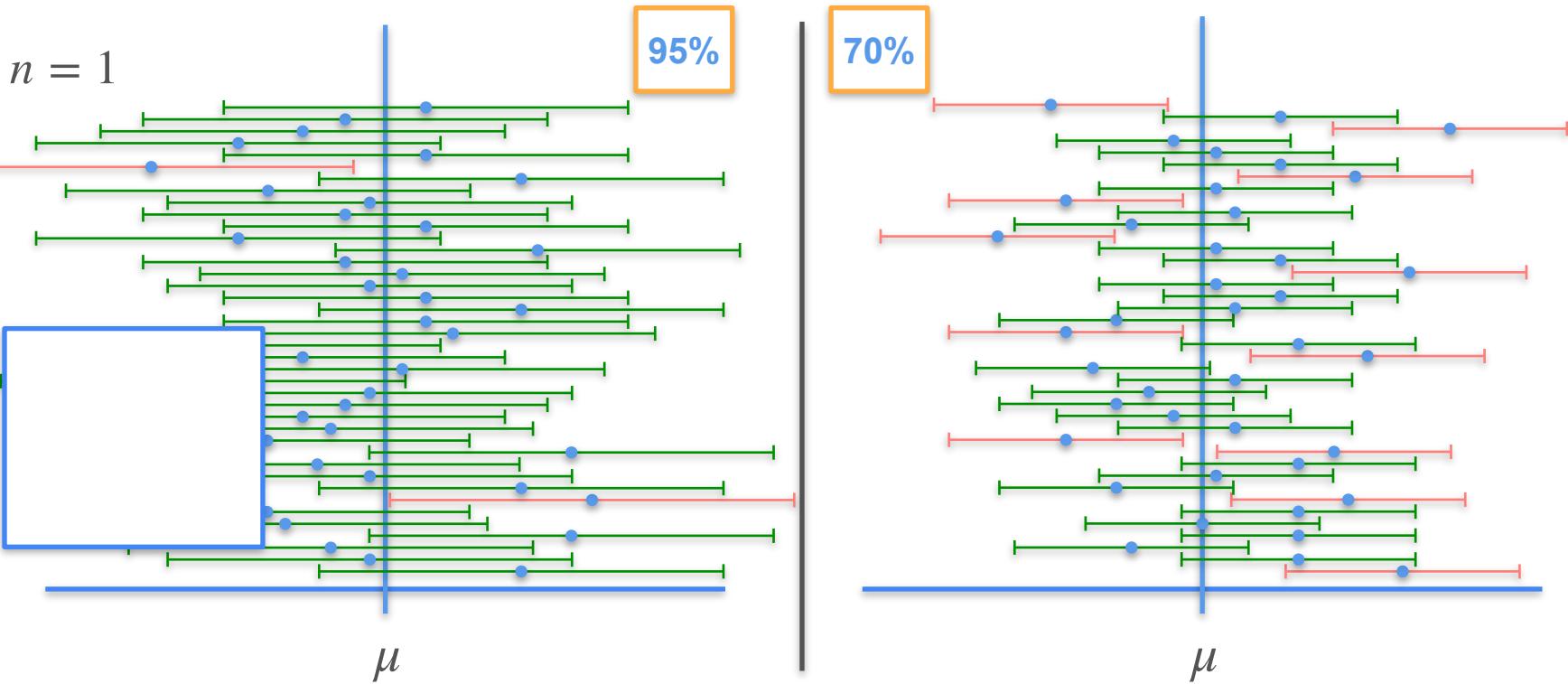
Effect of the Confidence Level



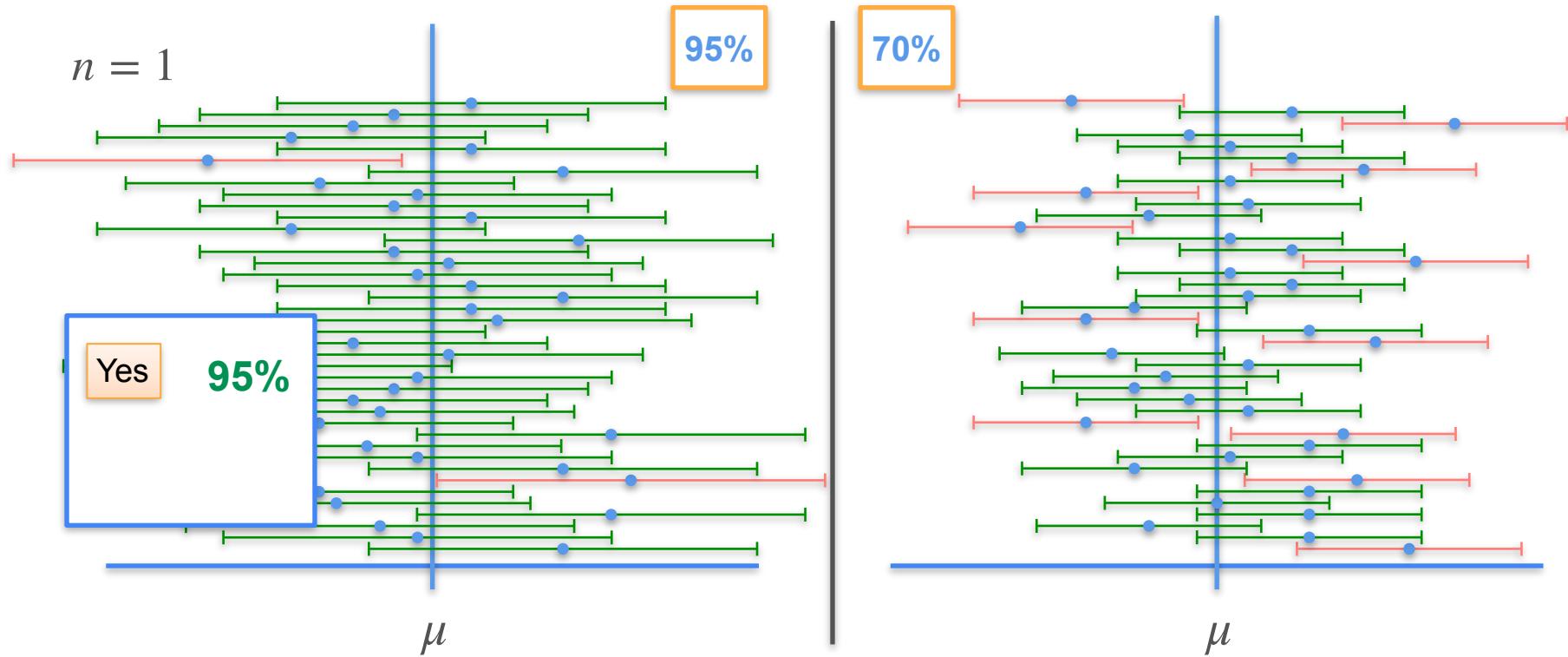
Effect of the Confidence Level



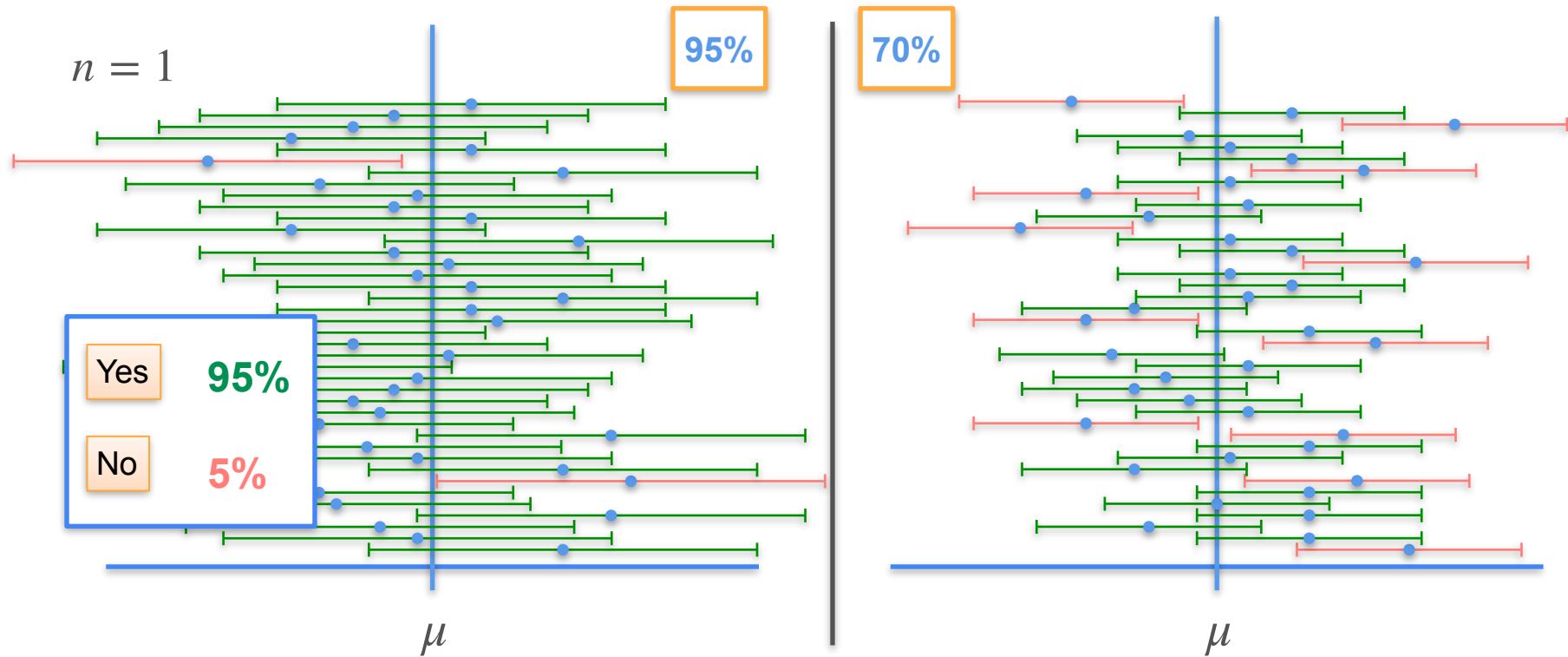
Effect of the Confidence Level



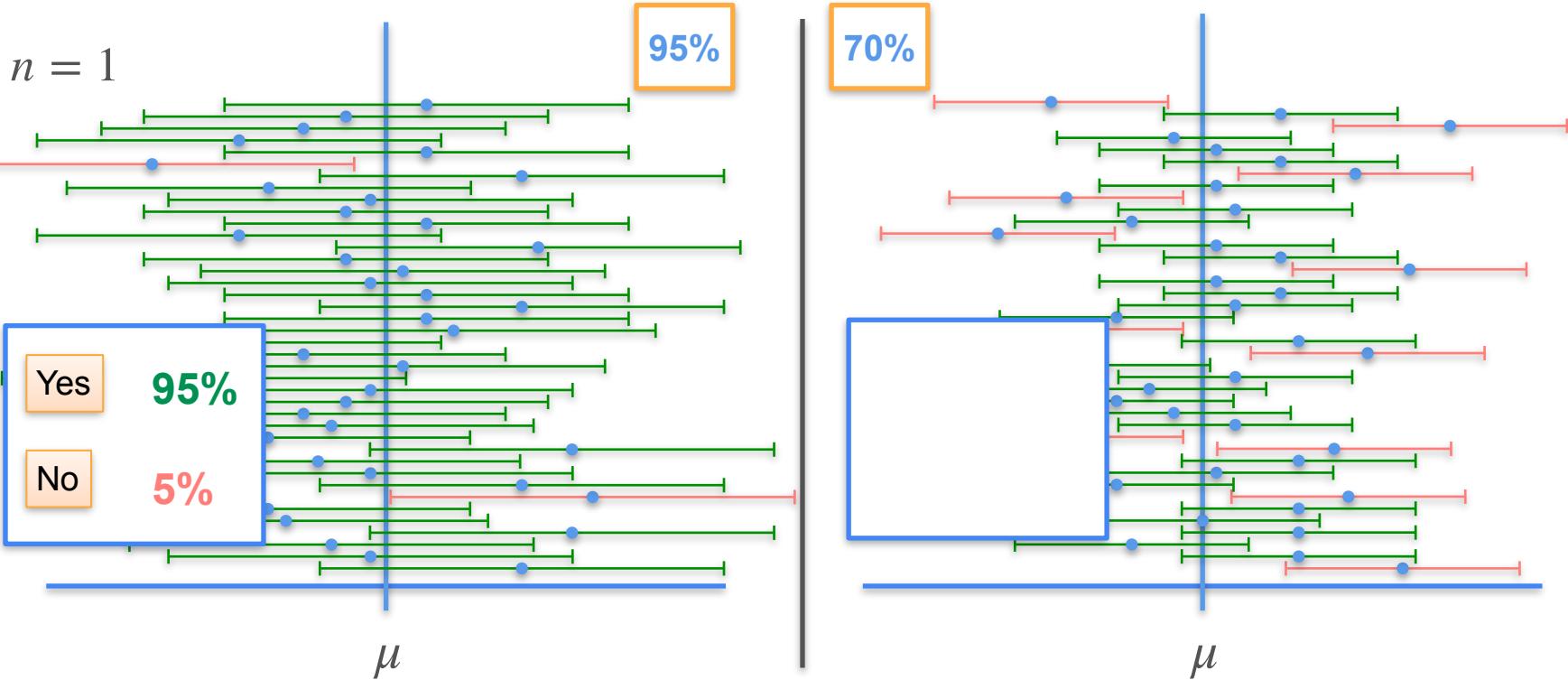
Effect of the Confidence Level



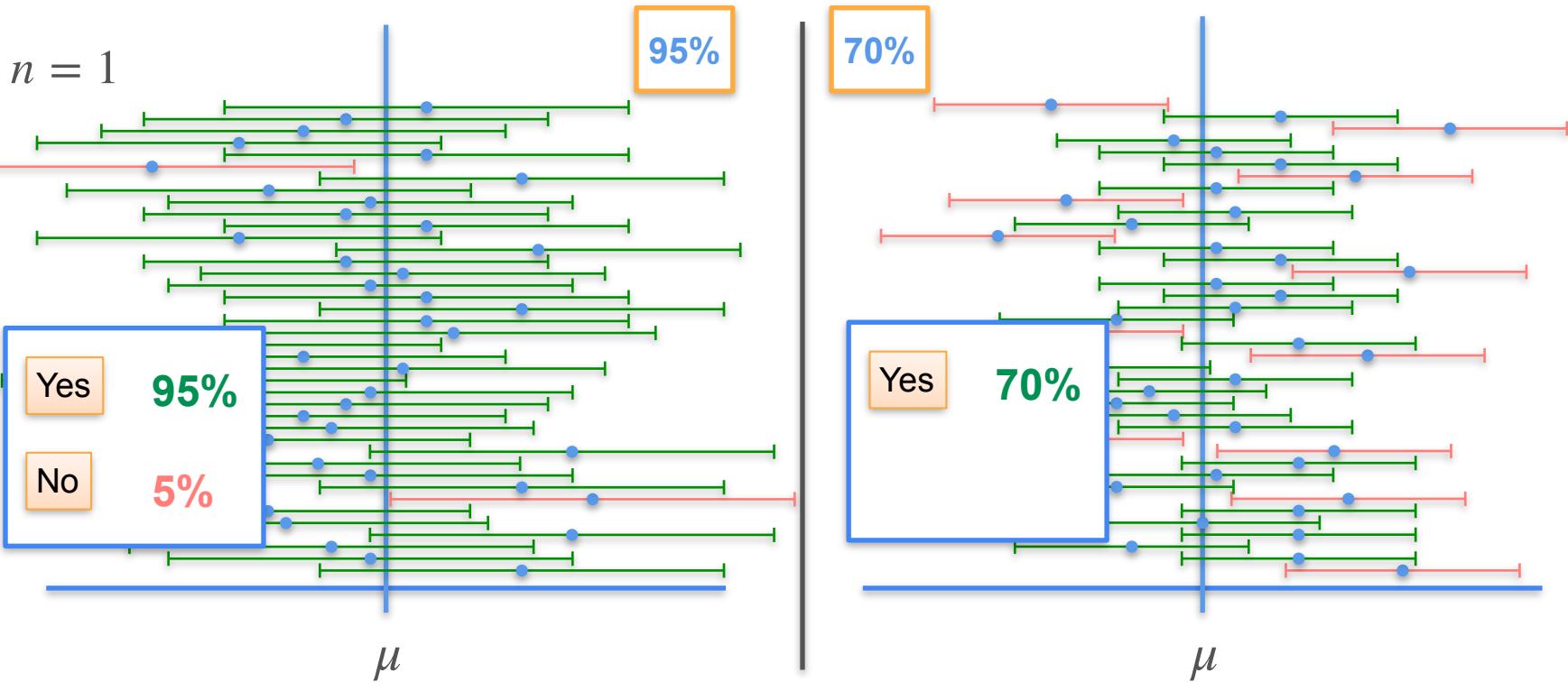
Effect of the Confidence Level



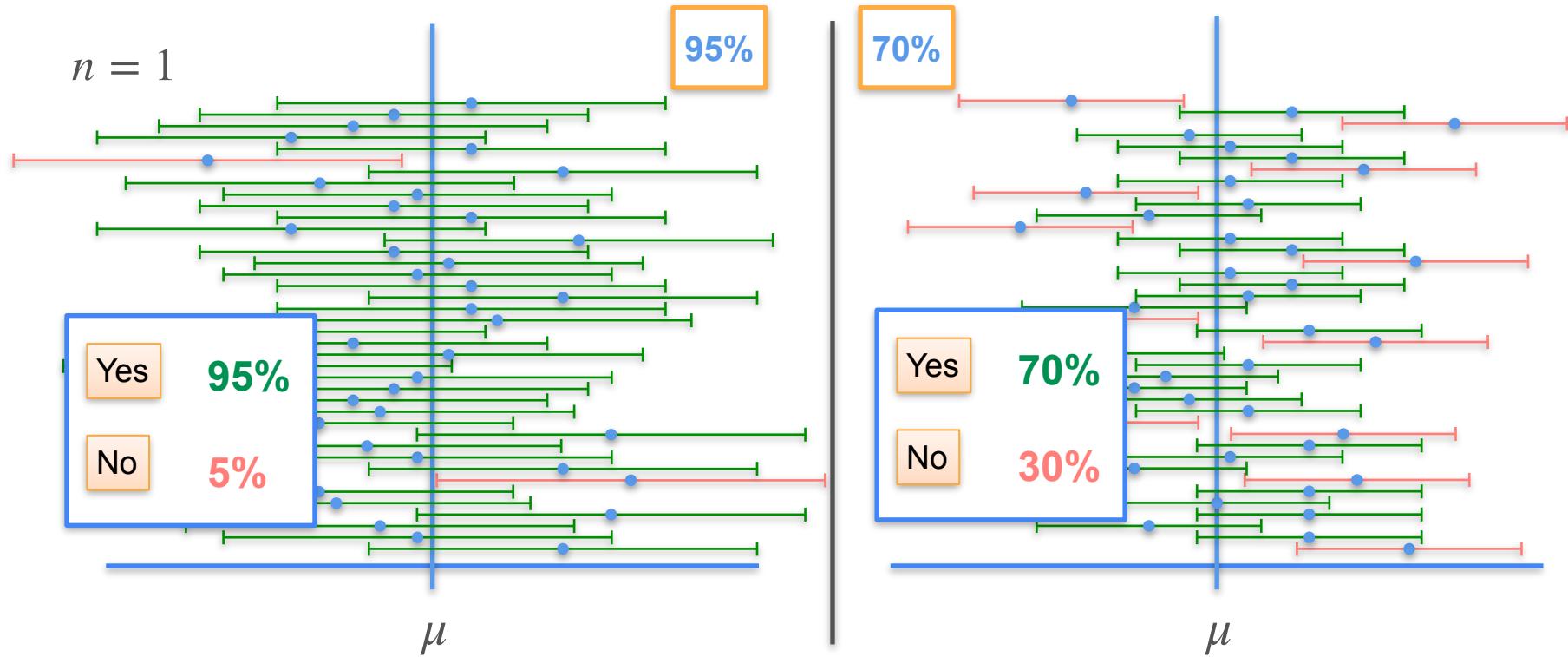
Effect of the Confidence Level



Effect of the Confidence Level

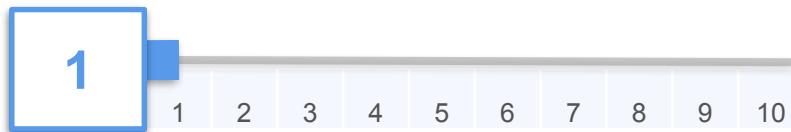


Effect of the Confidence Level

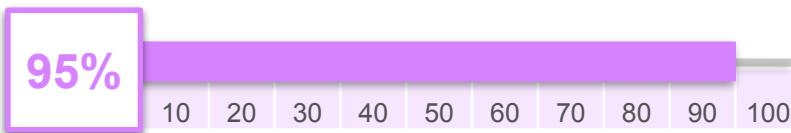


Effect of the Confidence Level

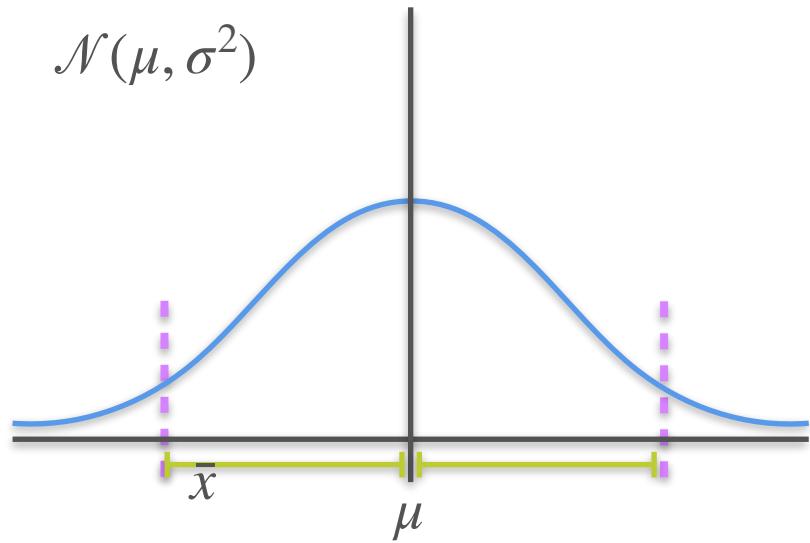
sample size



Confidence level



$$\mathcal{N}(\mu, \sigma^2)$$

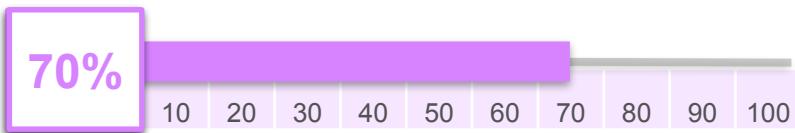


Effect of the Confidence Level

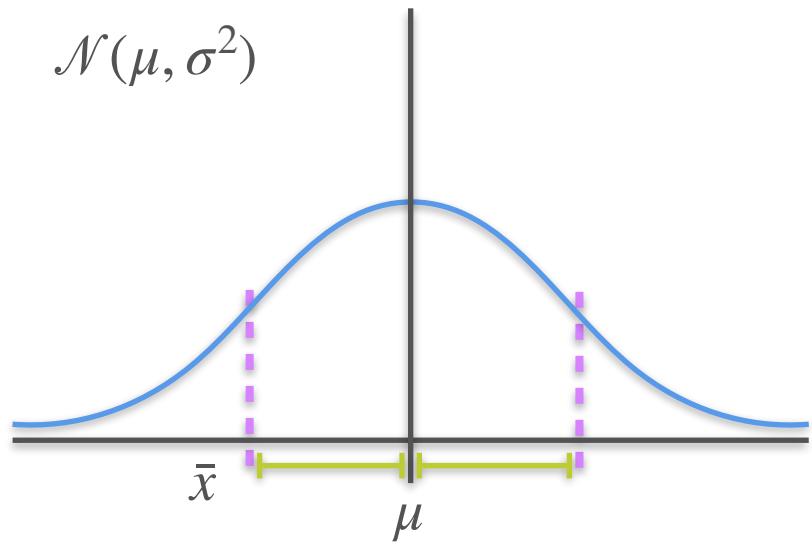
sample size



Confidence level

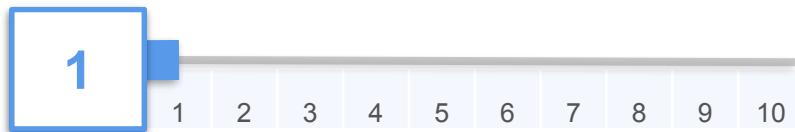


$$\mathcal{N}(\mu, \sigma^2)$$

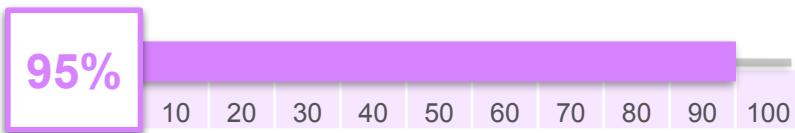


Effect of the Confidence Level

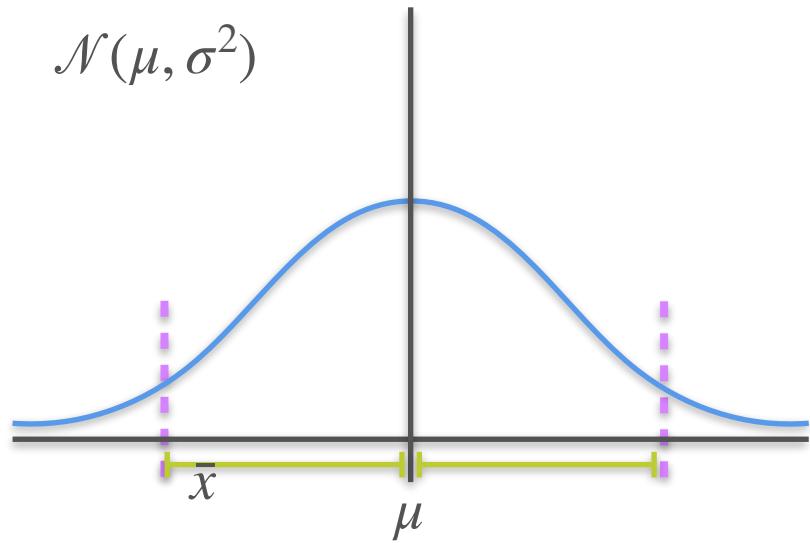
sample size



Confidence level



$$\mathcal{N}(\mu, \sigma^2)$$

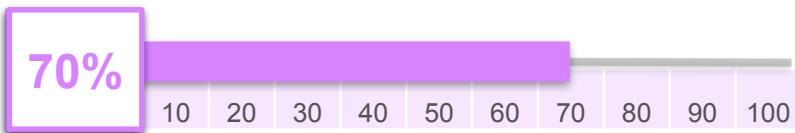


Effect of the Confidence Level

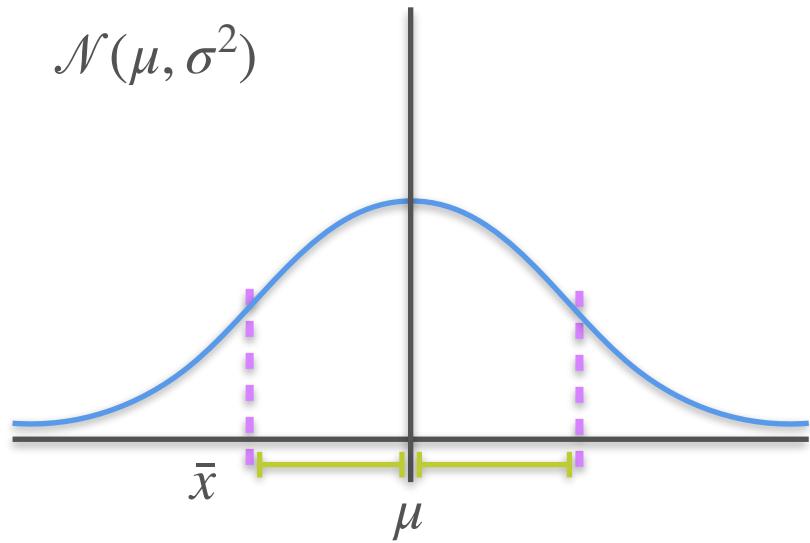
sample size



Confidence level



$$\mathcal{N}(\mu, \sigma^2)$$



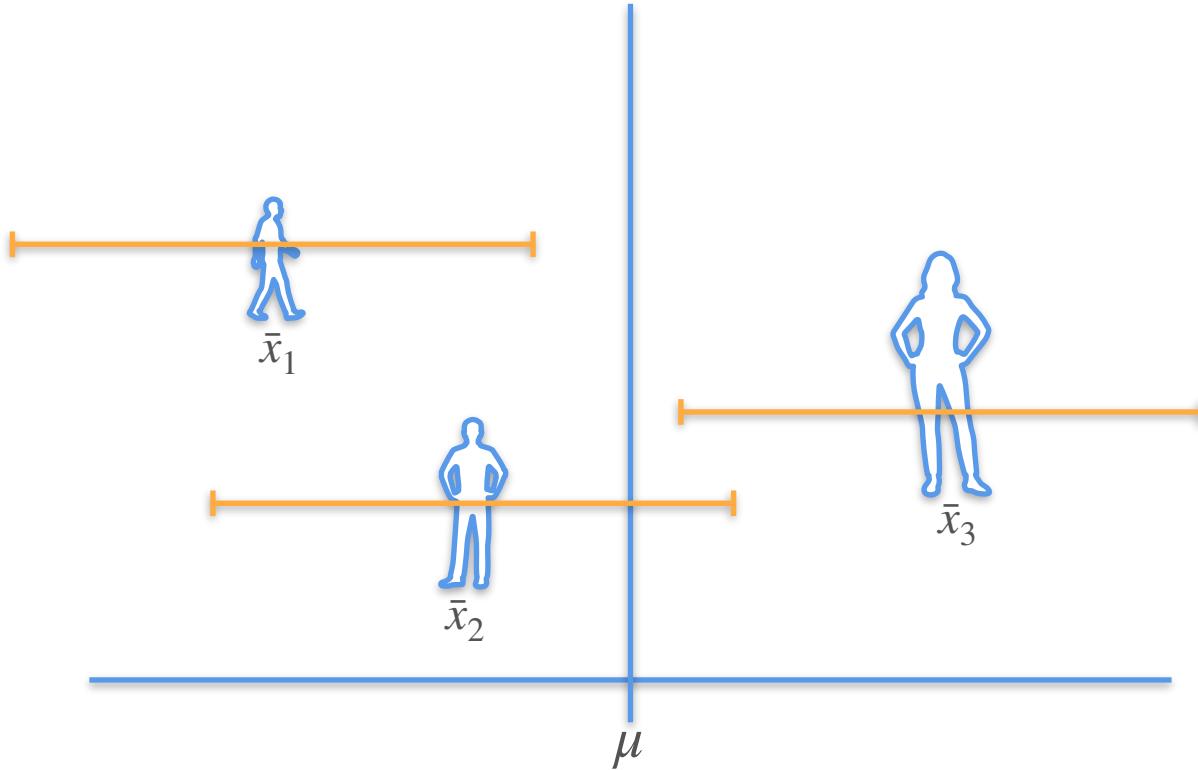


DeepLearning.AI

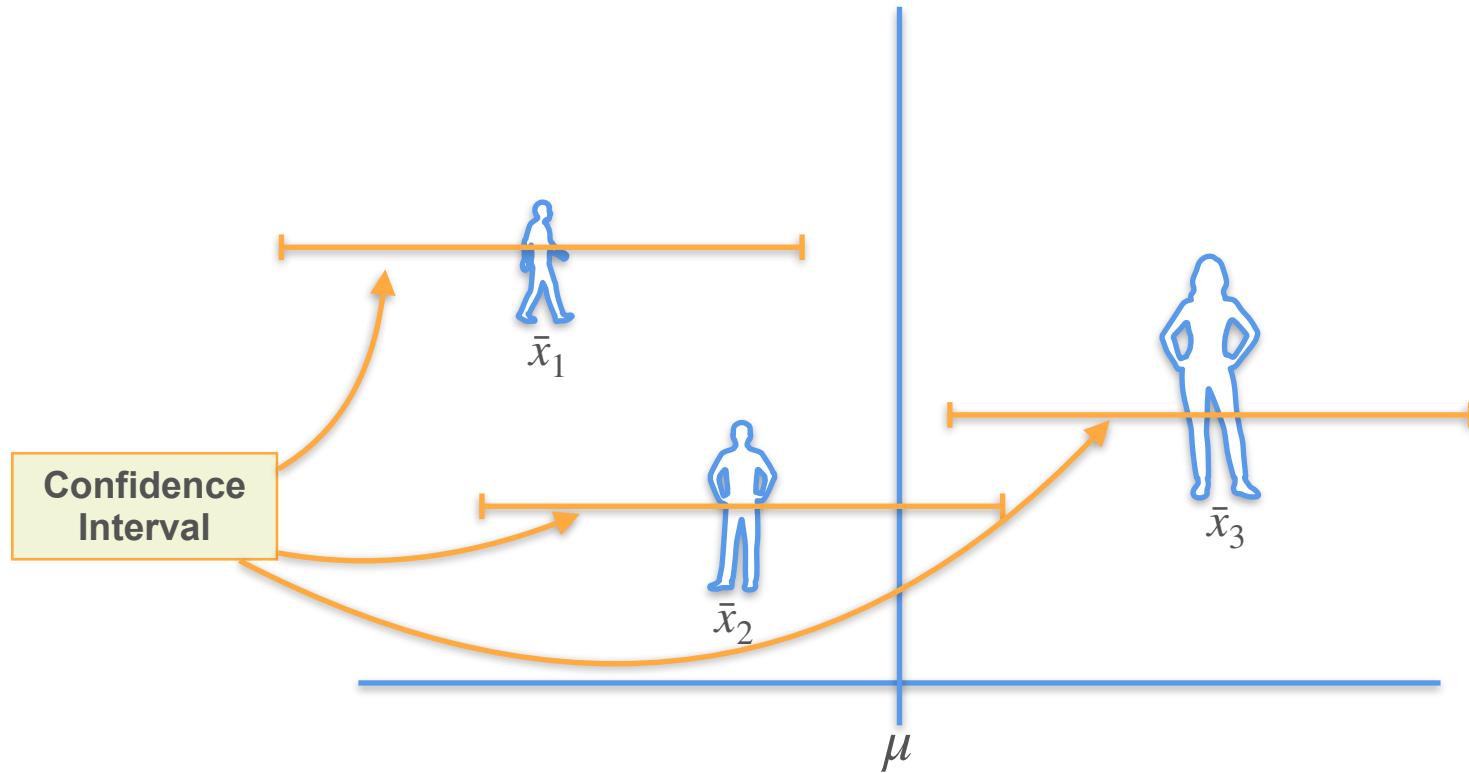
Confidence Interval

Margin of Error

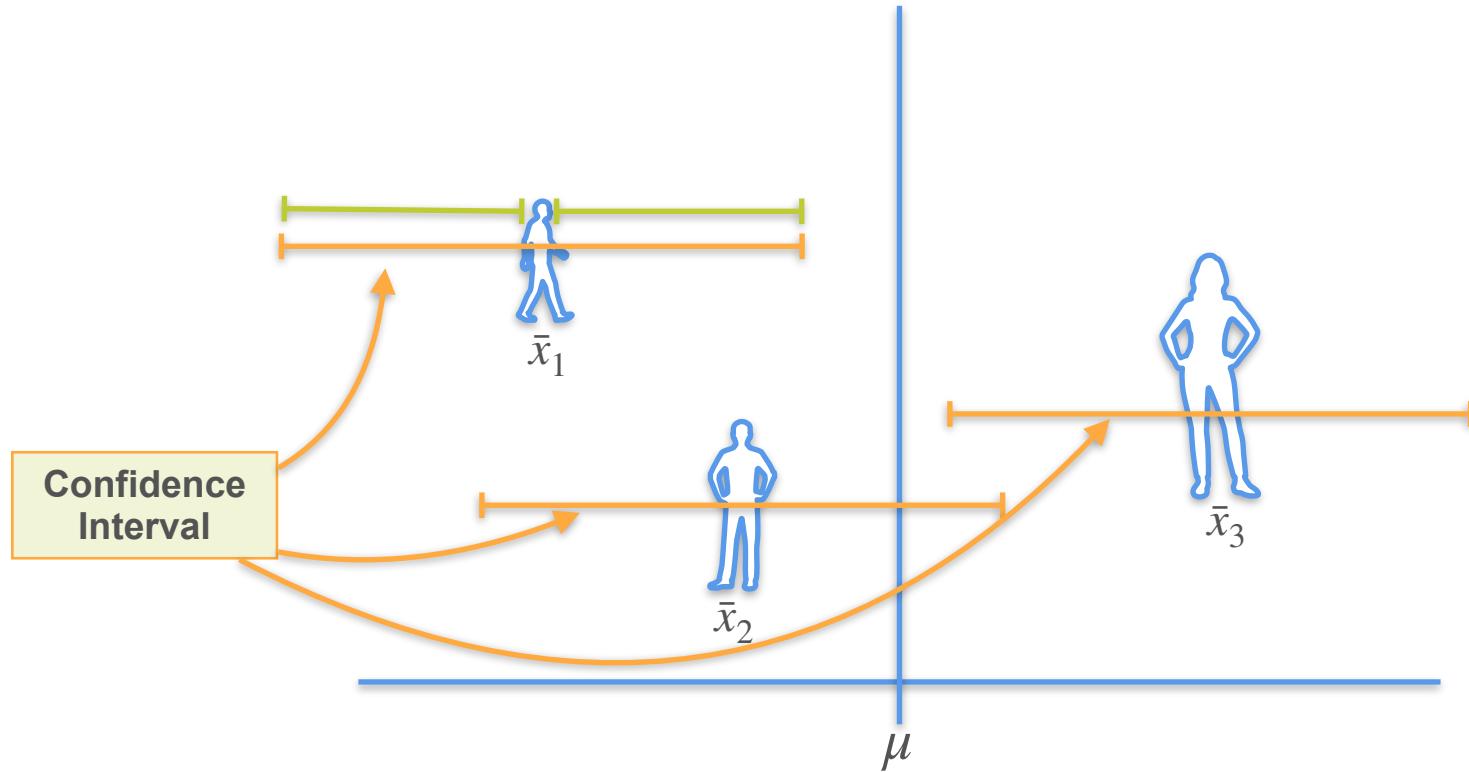
Margin of Error - Introduction



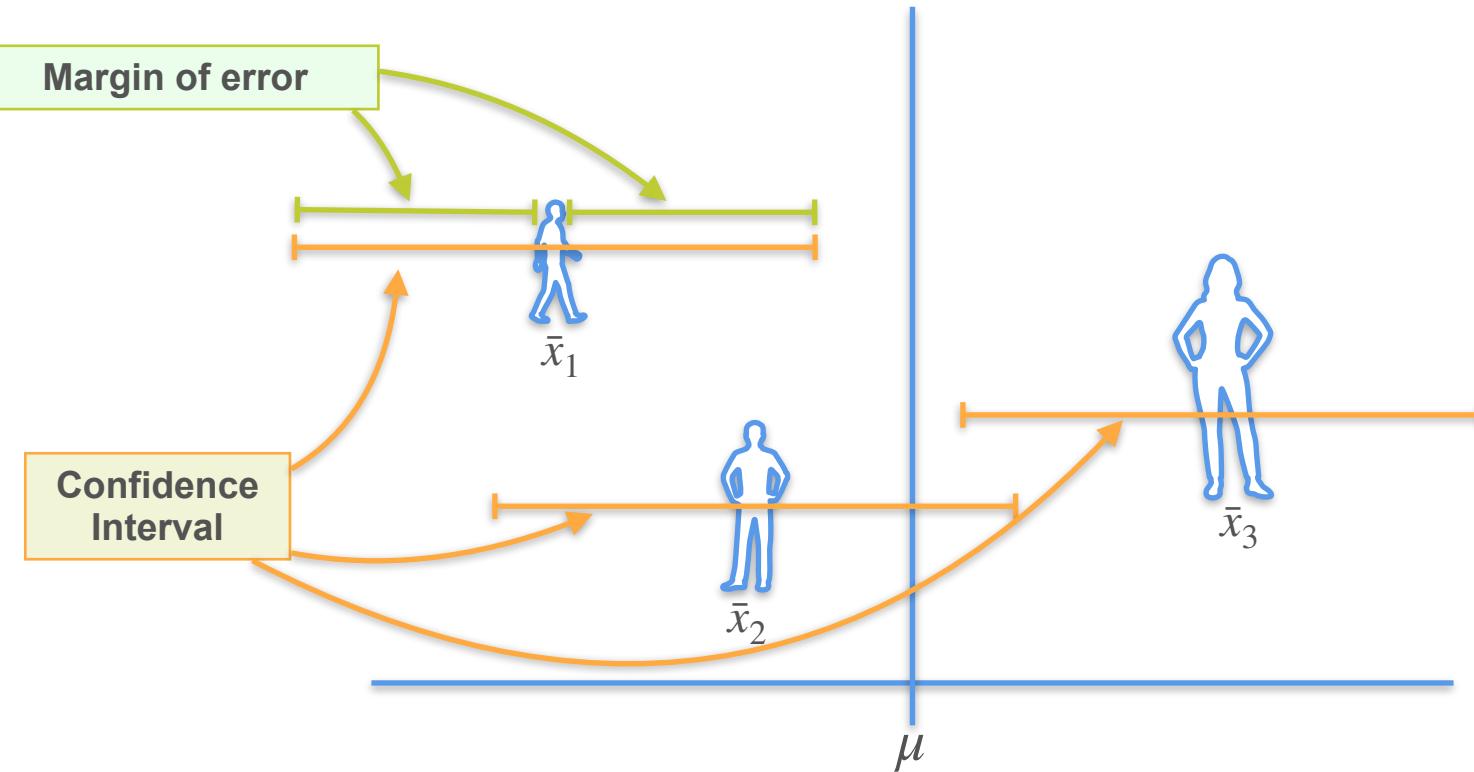
Margin of Error - Introduction



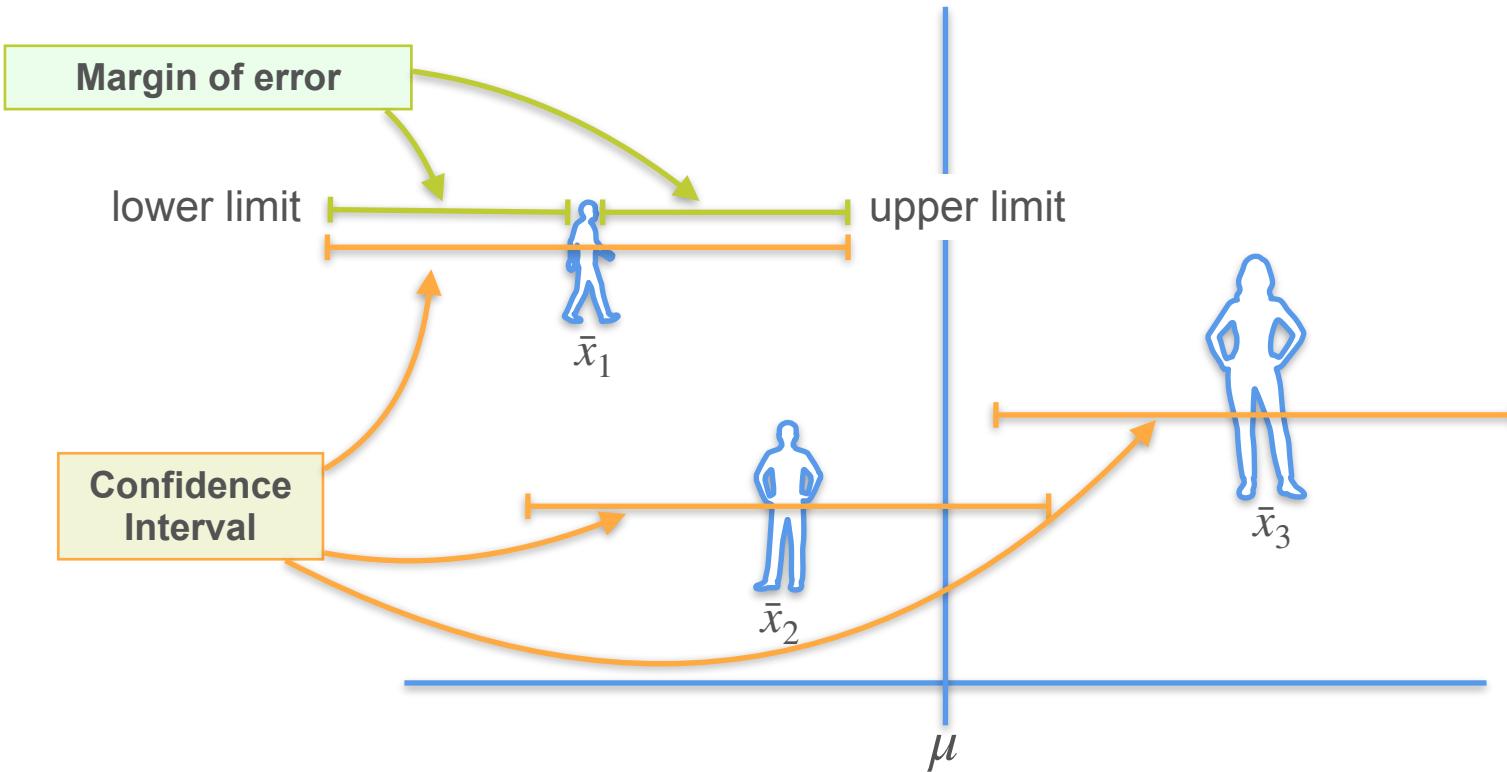
Margin of Error - Introduction



Margin of Error - Introduction



Margin of Error - Introduction



Margin of Error

Margin of Error

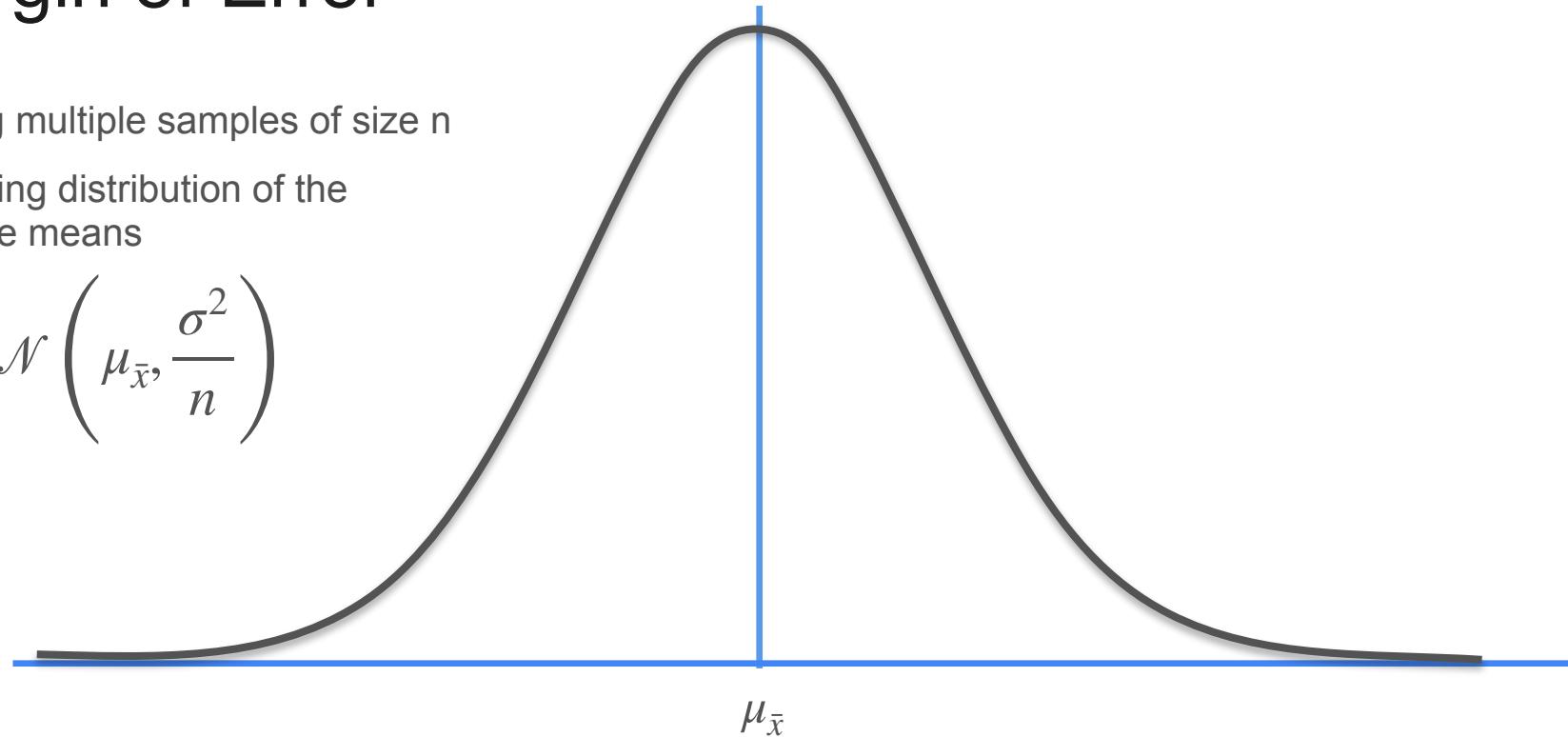
Taking multiple samples of size n
sampling distribution of the
sample means

$$\bar{X} \sim \mathcal{N}\left(\mu_{\bar{x}}, \frac{\sigma^2}{n}\right)$$

Margin of Error

Taking multiple samples of size n
sampling distribution of the
sample means

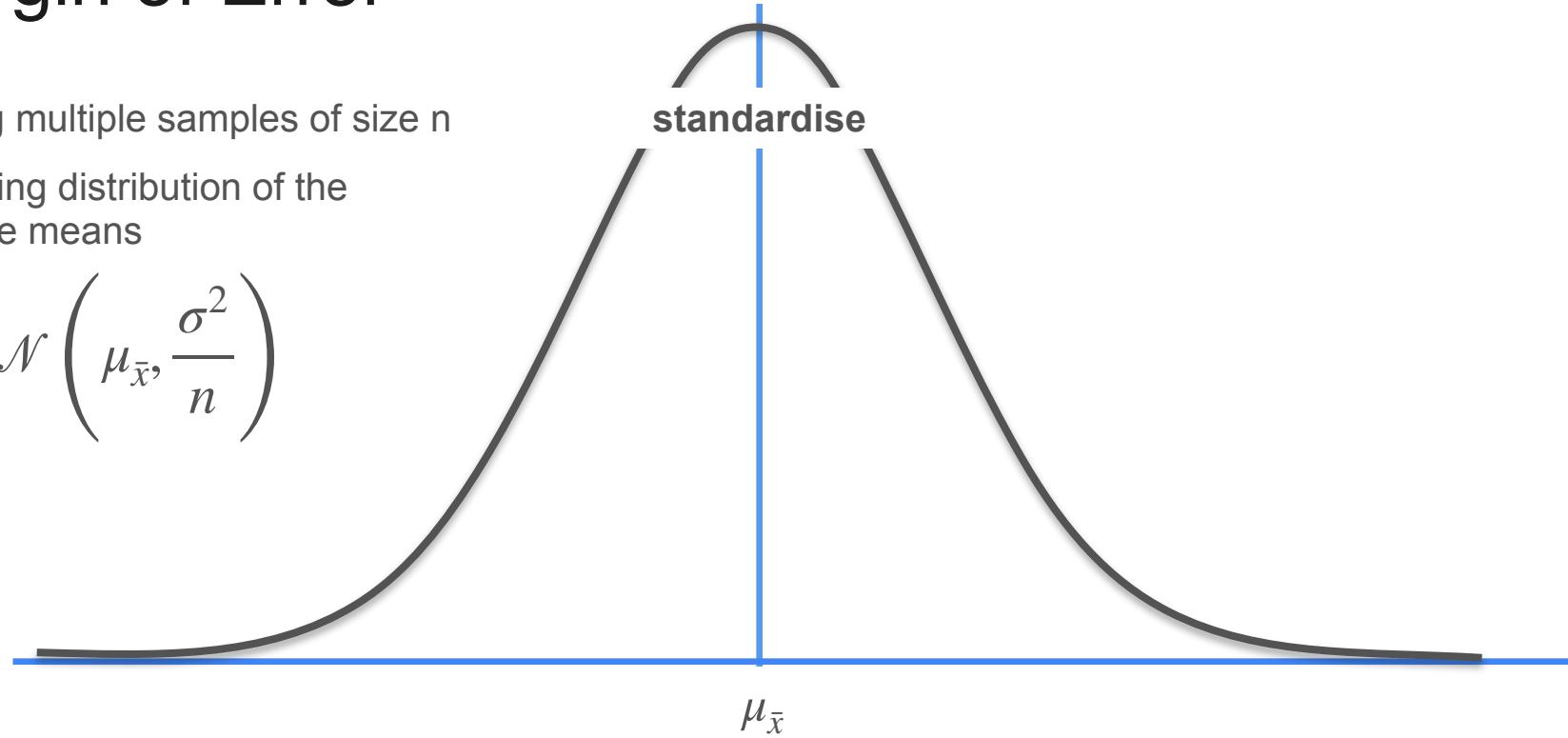
$$\bar{X} \sim \mathcal{N}\left(\mu_{\bar{x}}, \frac{\sigma^2}{n}\right)$$



Margin of Error

Taking multiple samples of size n
sampling distribution of the
sample means

$$\bar{X} \sim \mathcal{N}\left(\mu_{\bar{x}}, \frac{\sigma^2}{n}\right)$$



Margin of Error

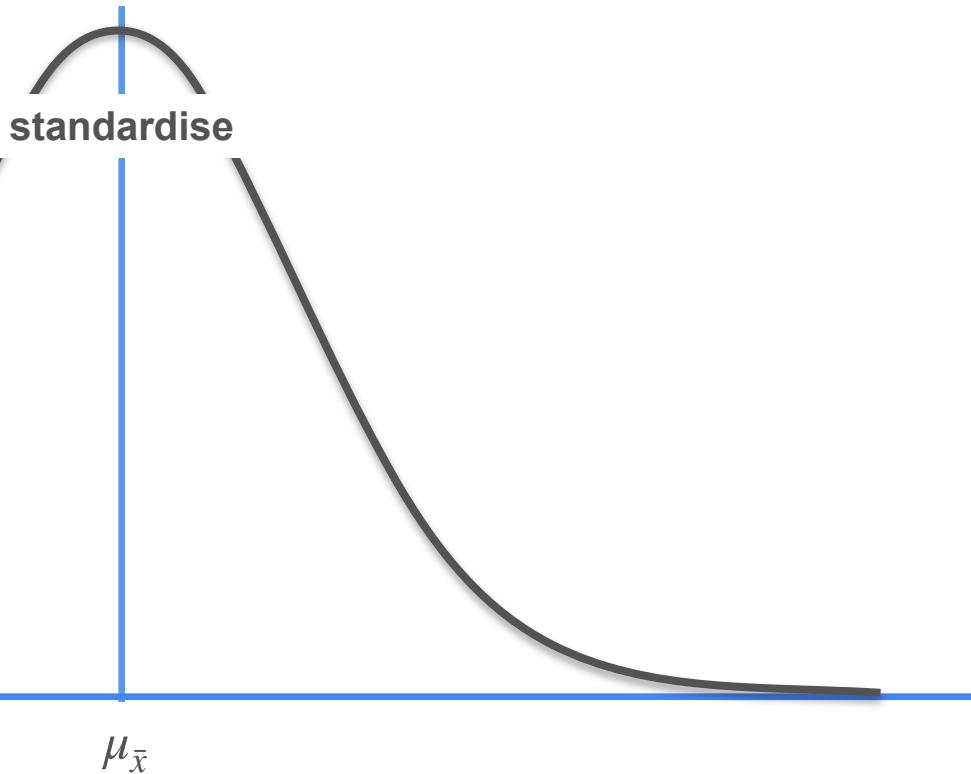
Taking multiple samples of size n
sampling distribution of the
sample means

$$\bar{X} \sim \mathcal{N}\left(\mu_{\bar{x}}, \frac{\sigma^2}{n}\right)$$

$$\mu = 0$$

$$\sigma = 1$$

$$\bar{X} \sim \mathcal{N}(0,1)$$



Margin of Error

Taking multiple samples of size n

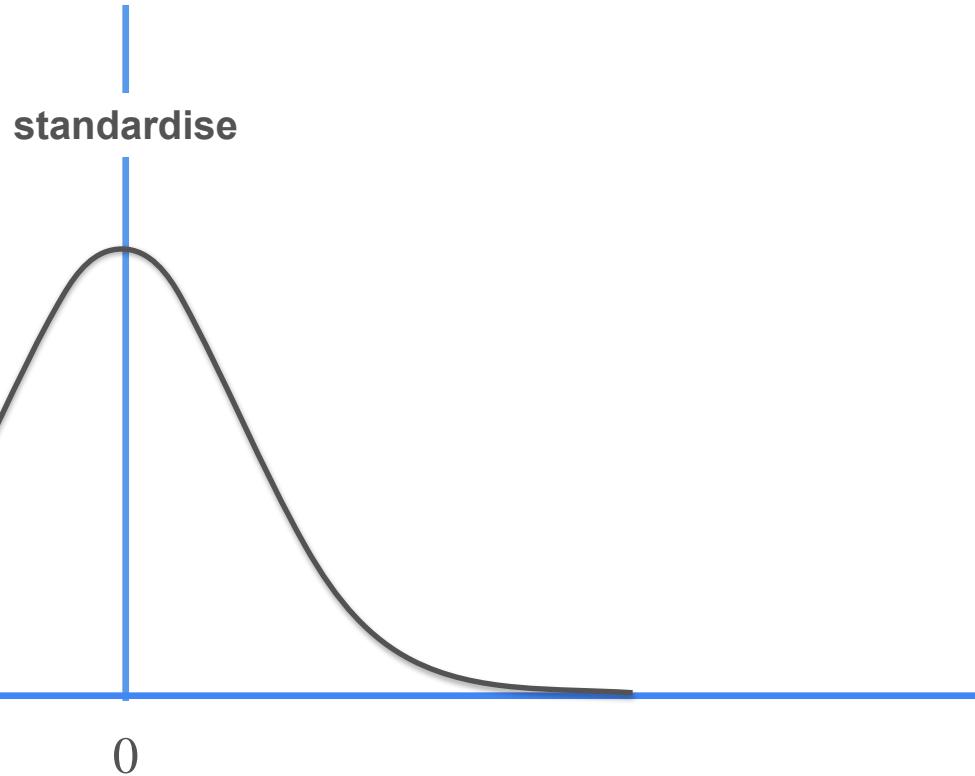
sampling distribution of the sample means

$$\bar{X} \sim \mathcal{N}\left(\mu_{\bar{x}}, \frac{\sigma^2}{n}\right)$$

$$\mu = 0$$

$$\sigma = 1$$

$$\bar{X} \sim \mathcal{N}(0,1)$$



Margin of Error

Taking multiple samples of size n

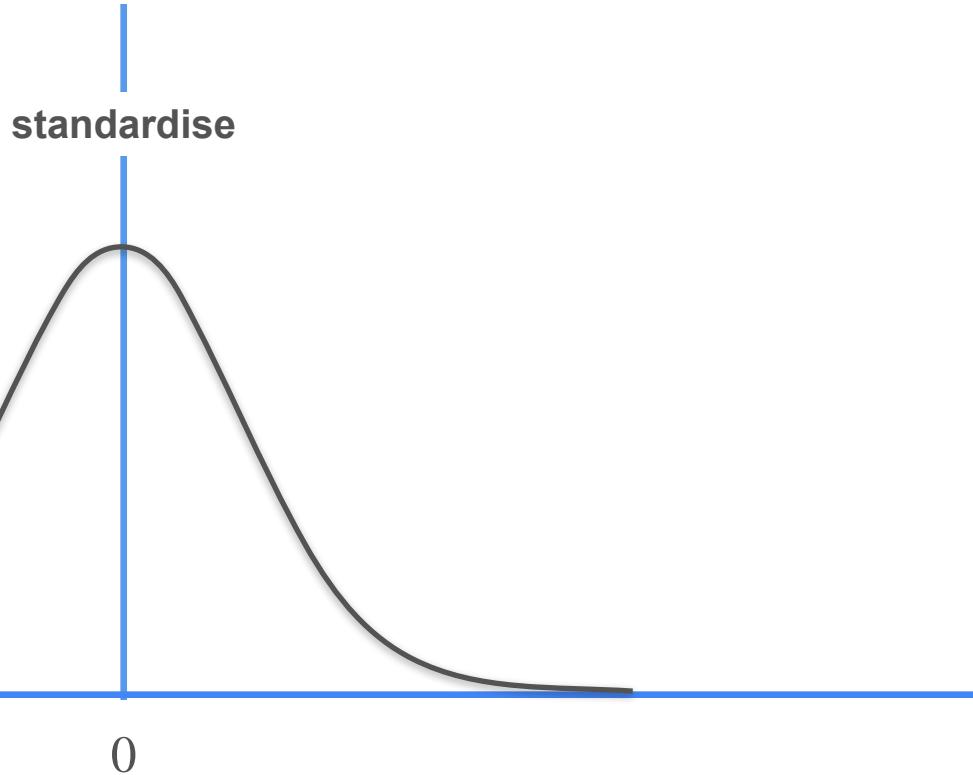
sampling distribution of the sample means

$$\bar{X} \sim \mathcal{N}\left(\mu_{\bar{x}}, \frac{\sigma^2}{n}\right)$$

$$\mu = 0$$

$$\sigma = 1$$

$$\bar{X} \sim \mathcal{N}(0, 1^2)$$

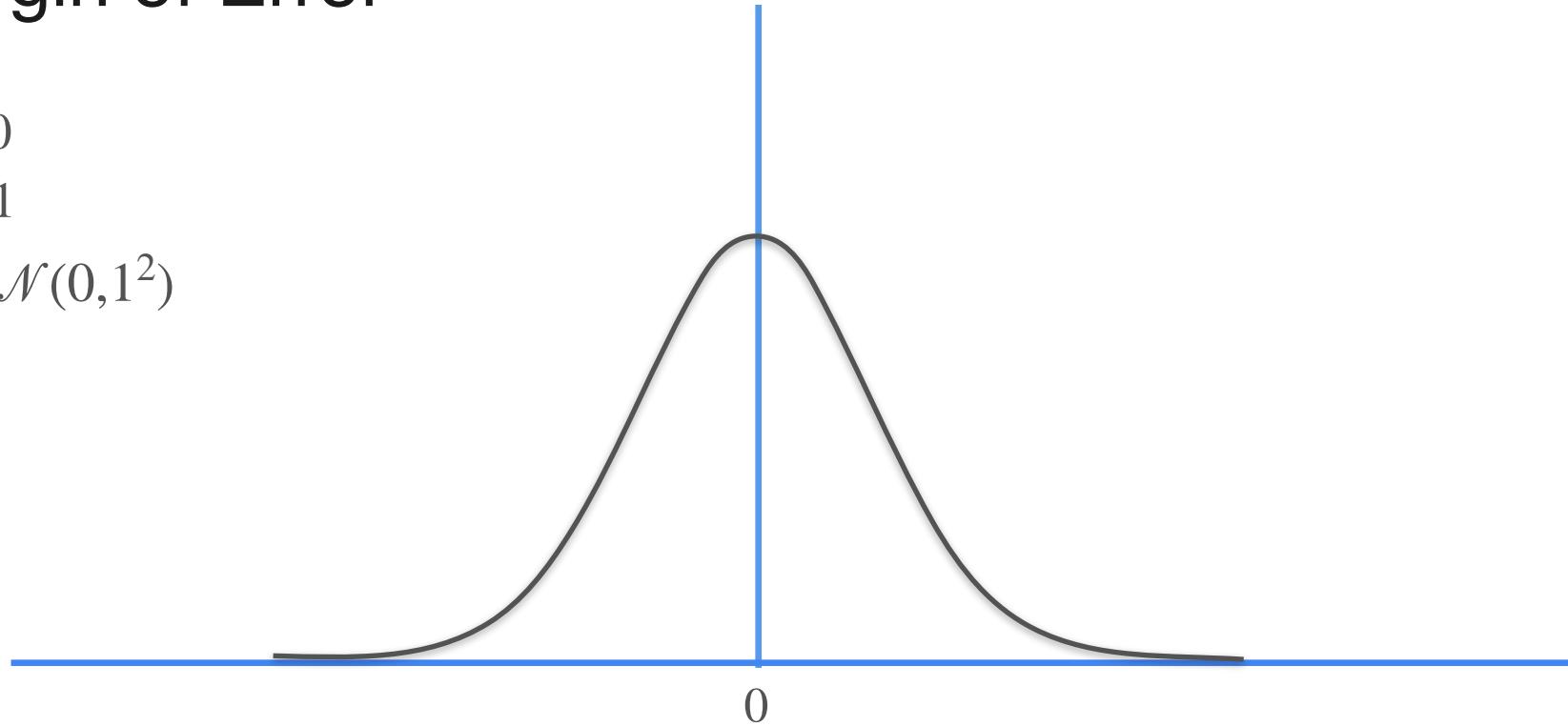


Margin of Error

$$\mu = 0$$

$$\sigma = 1$$

$$\bar{X} \sim \mathcal{N}(0, 1^2)$$

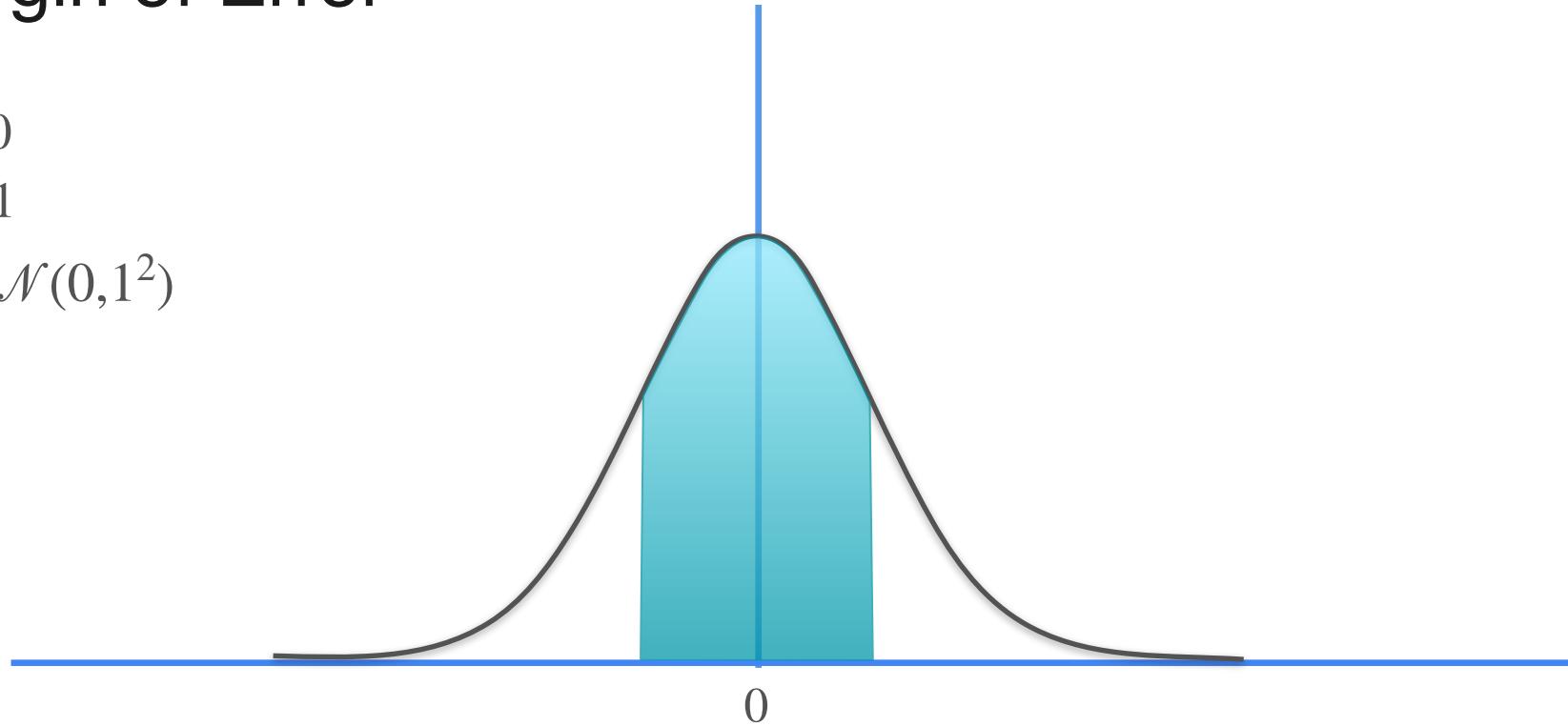


Margin of Error

$$\mu = 0$$

$$\sigma = 1$$

$$\bar{X} \sim \mathcal{N}(0, 1^2)$$

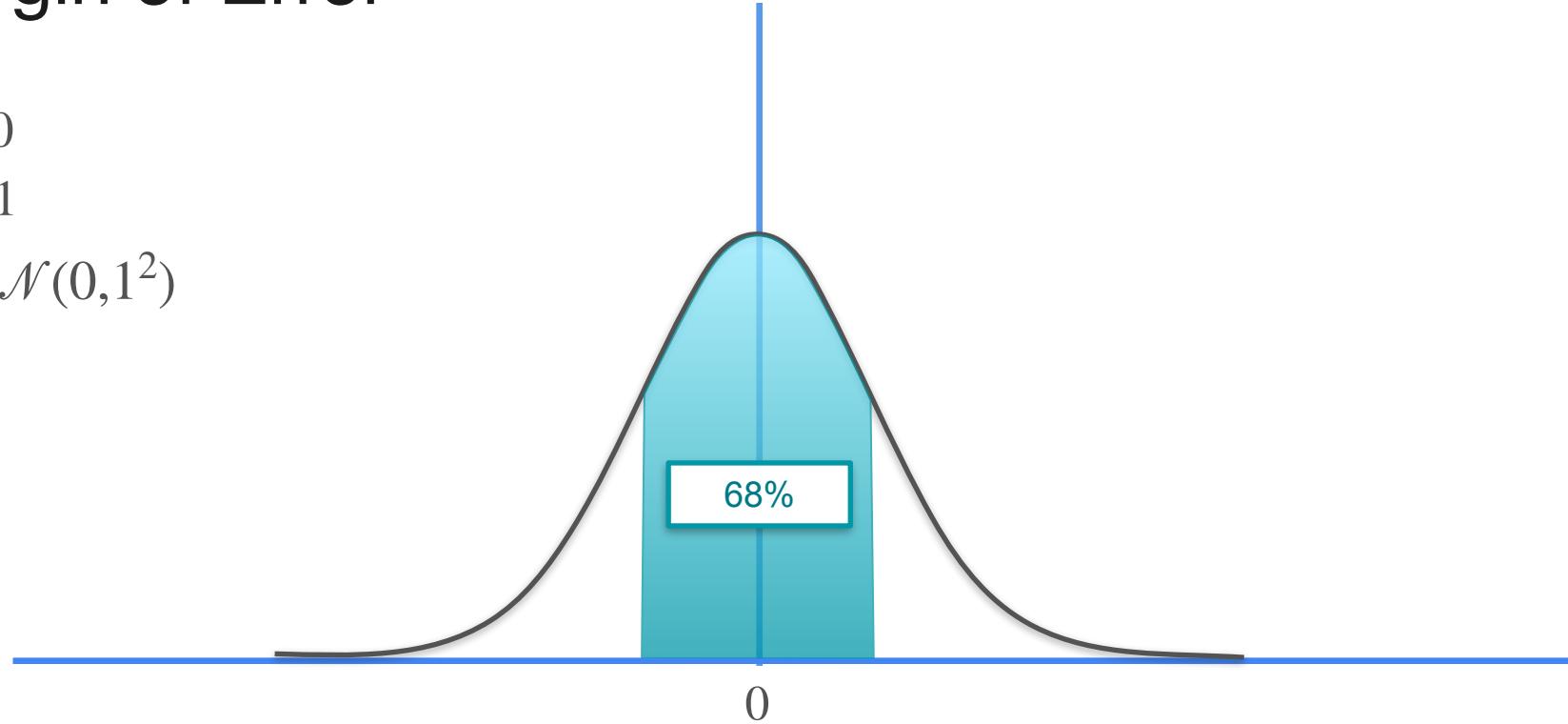


Margin of Error

$$\mu = 0$$

$$\sigma = 1$$

$$\bar{X} \sim \mathcal{N}(0, 1^2)$$

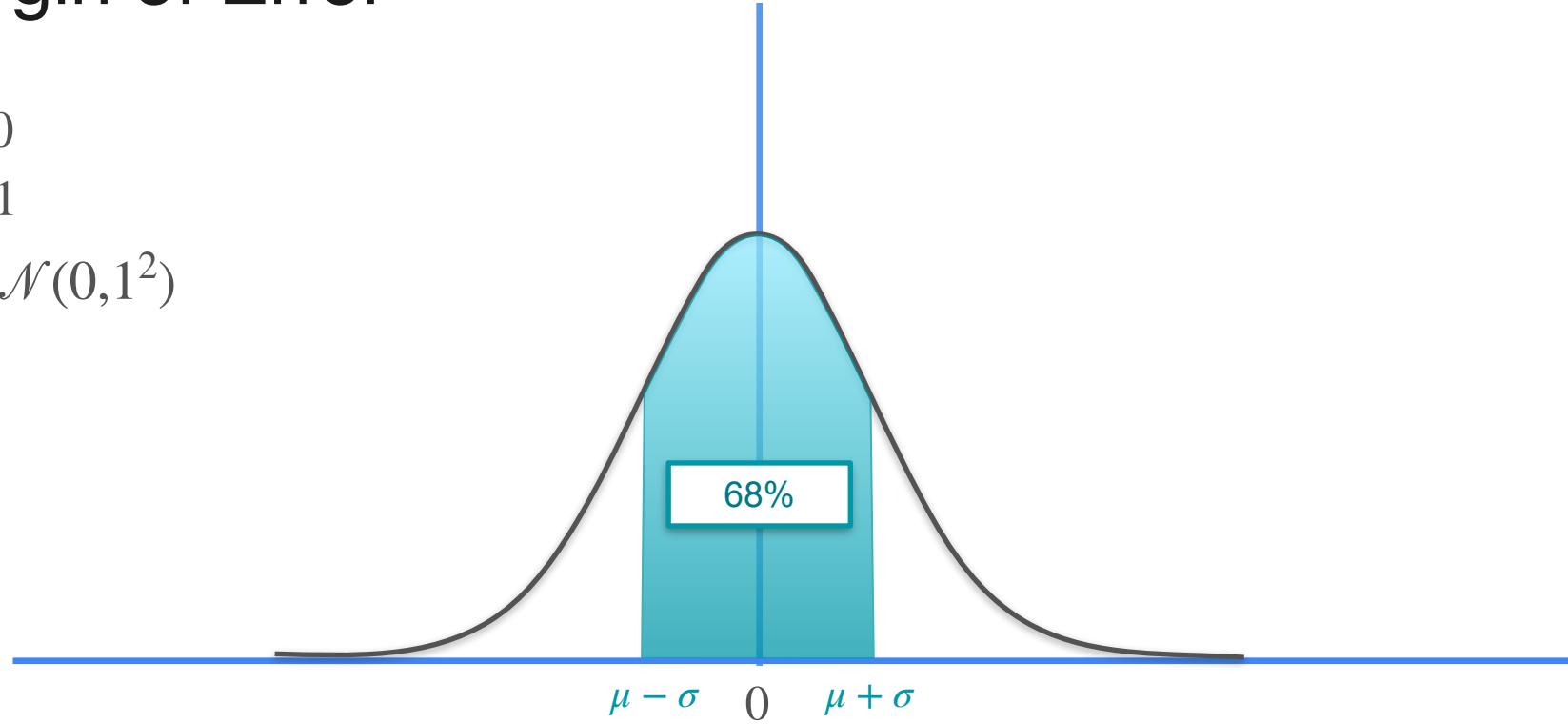


Margin of Error

$$\mu = 0$$

$$\sigma = 1$$

$$\bar{X} \sim \mathcal{N}(0, 1^2)$$

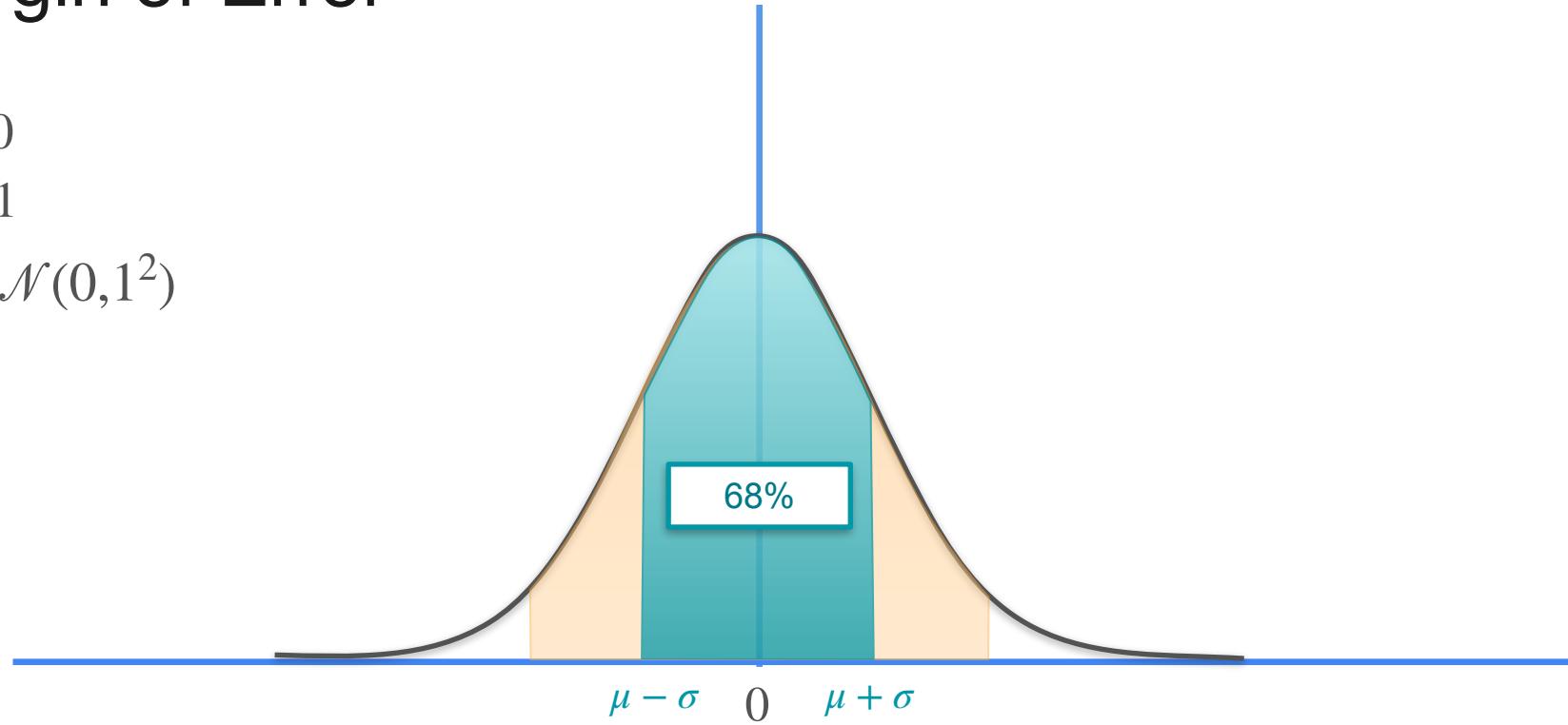


Margin of Error

$$\mu = 0$$

$$\sigma = 1$$

$$\bar{X} \sim \mathcal{N}(0, 1^2)$$

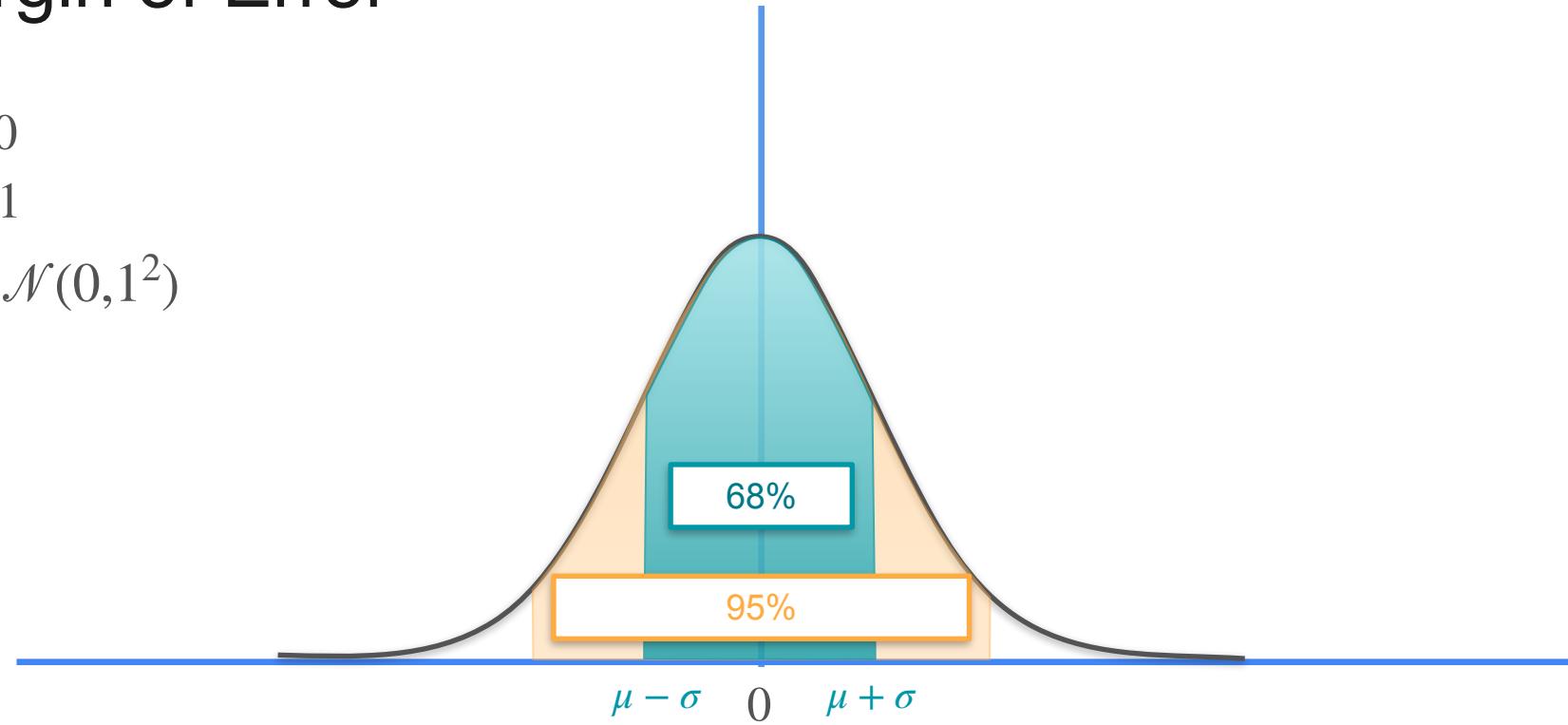


Margin of Error

$$\mu = 0$$

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$$\bar{X} \sim \mathcal{N}(0, 1^2)$$

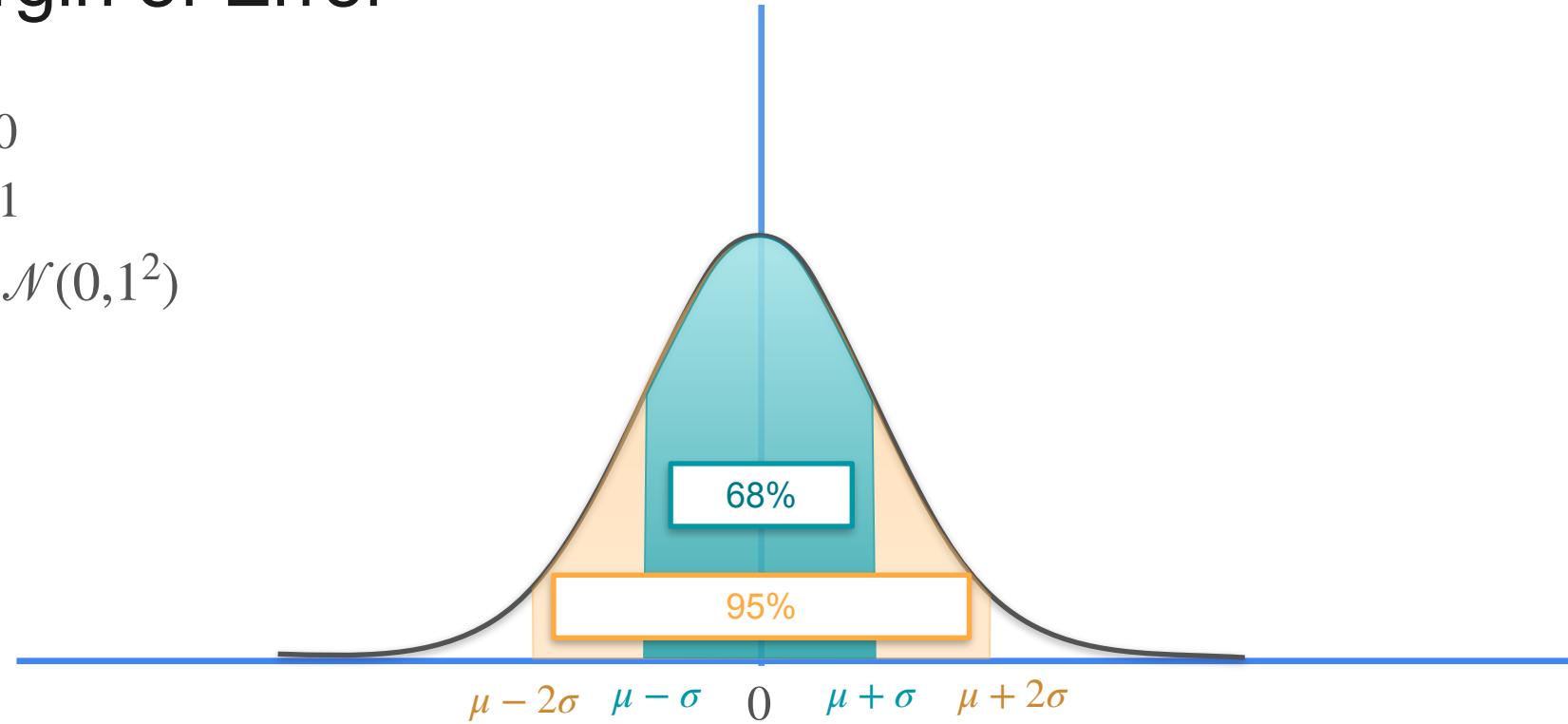


Margin of Error

$$\mu = 0$$

$$\sigma = 1$$

$$\bar{X} \sim \mathcal{N}(0, 1^2)$$

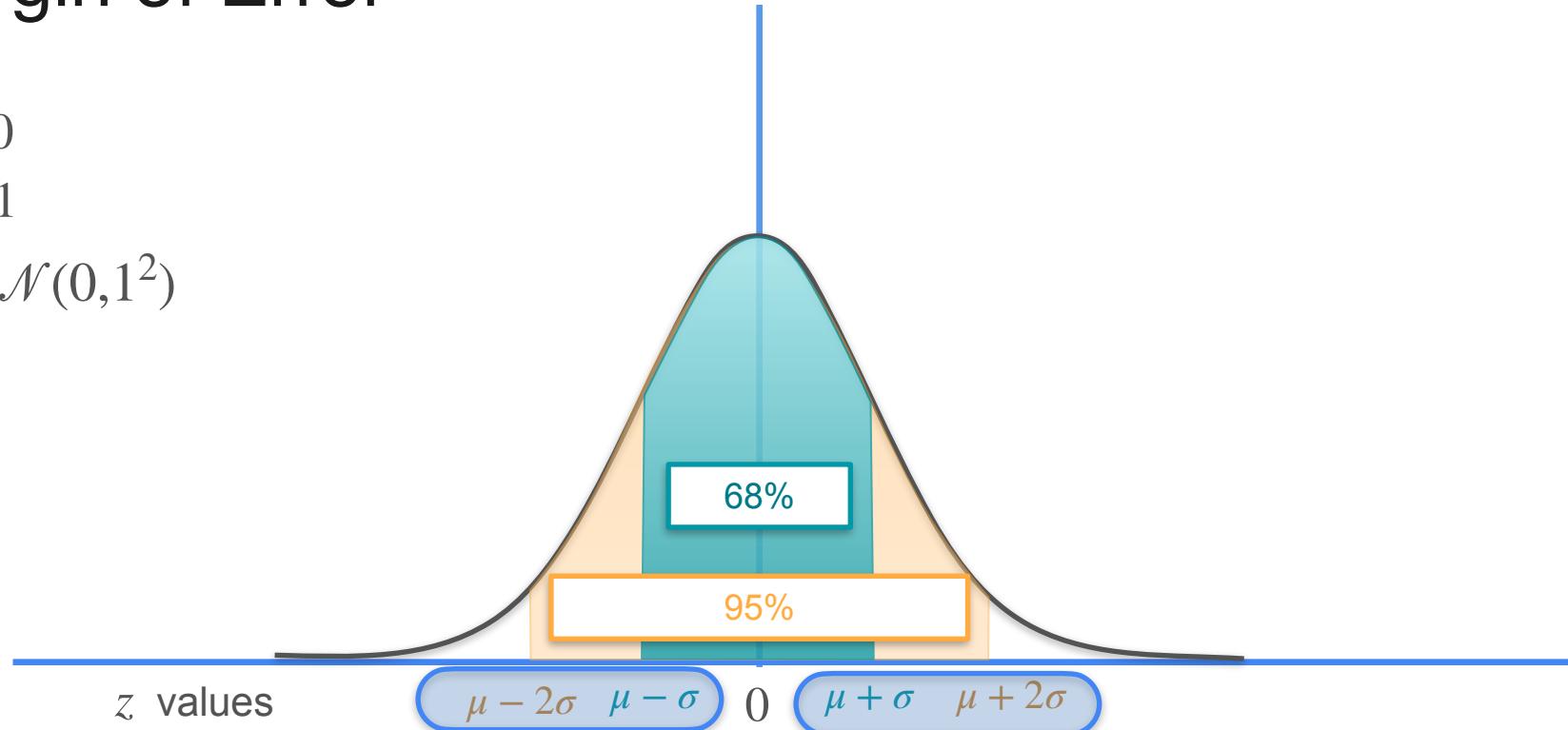


Margin of Error

$$\mu = 0$$

$$\sigma = 1$$

$$\bar{X} \sim \mathcal{N}(0, 1^2)$$

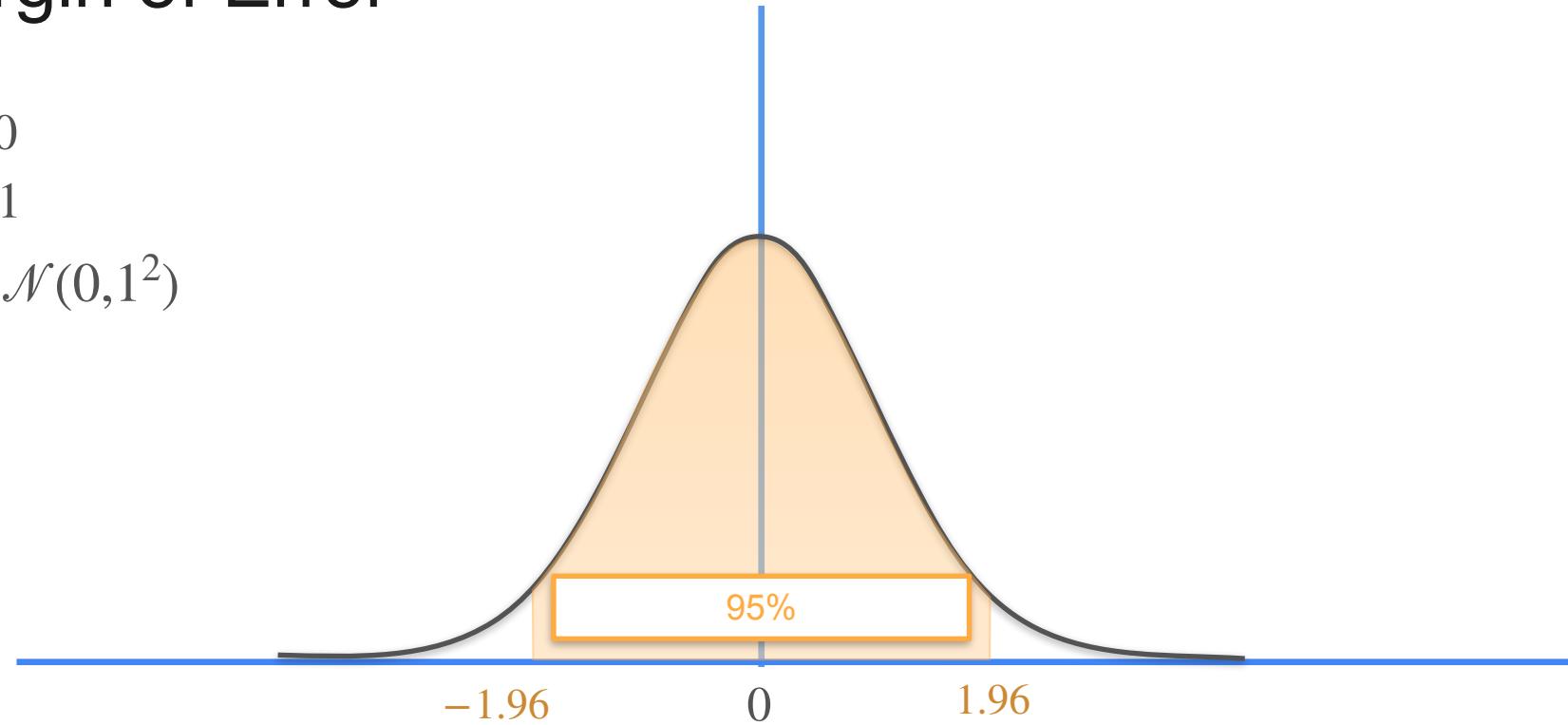


Margin of Error

$$\mu = 0$$

$$\sigma = 1$$

$$\bar{X} \sim \mathcal{N}(0, 1^2)$$

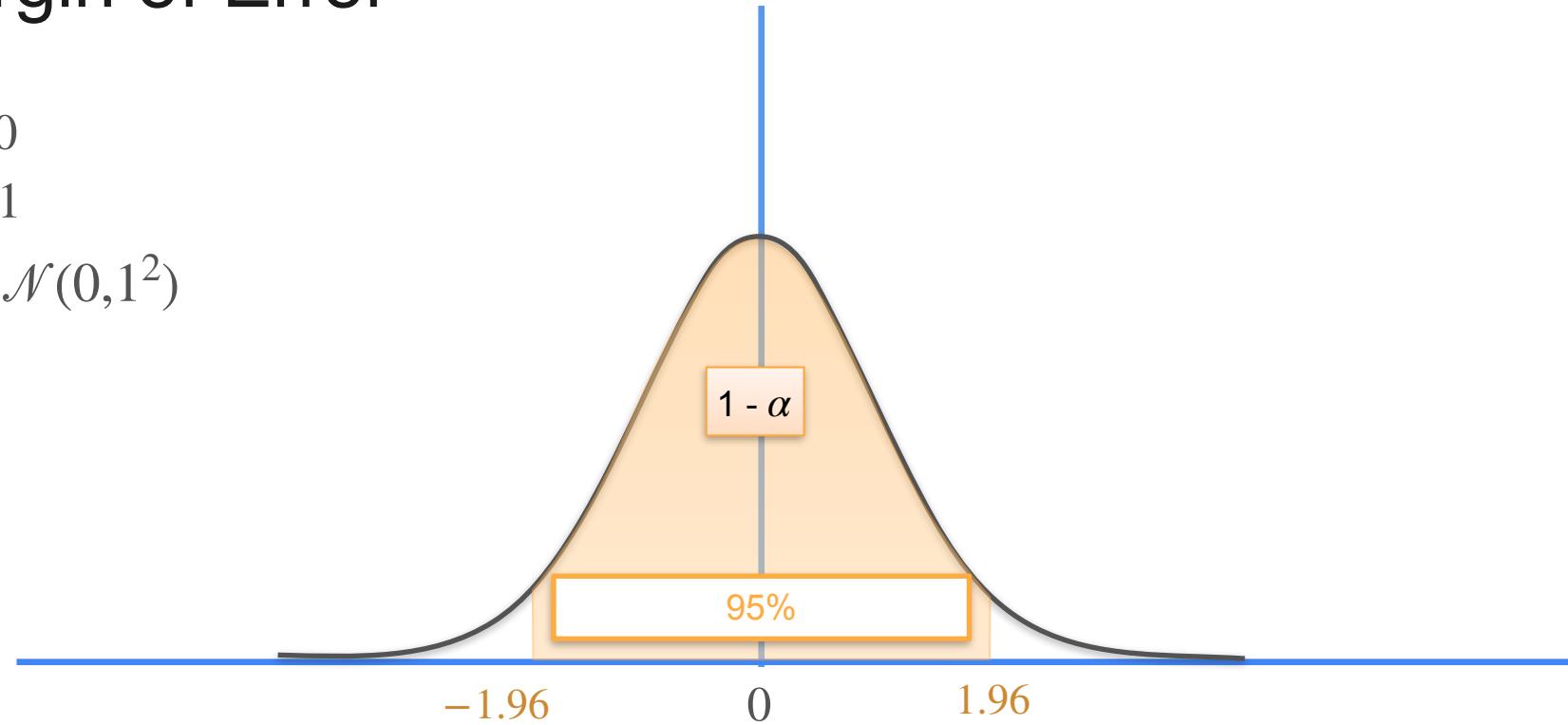


Margin of Error

$$\mu = 0$$

$$\sigma = 1$$

$$\bar{X} \sim \mathcal{N}(0, 1^2)$$

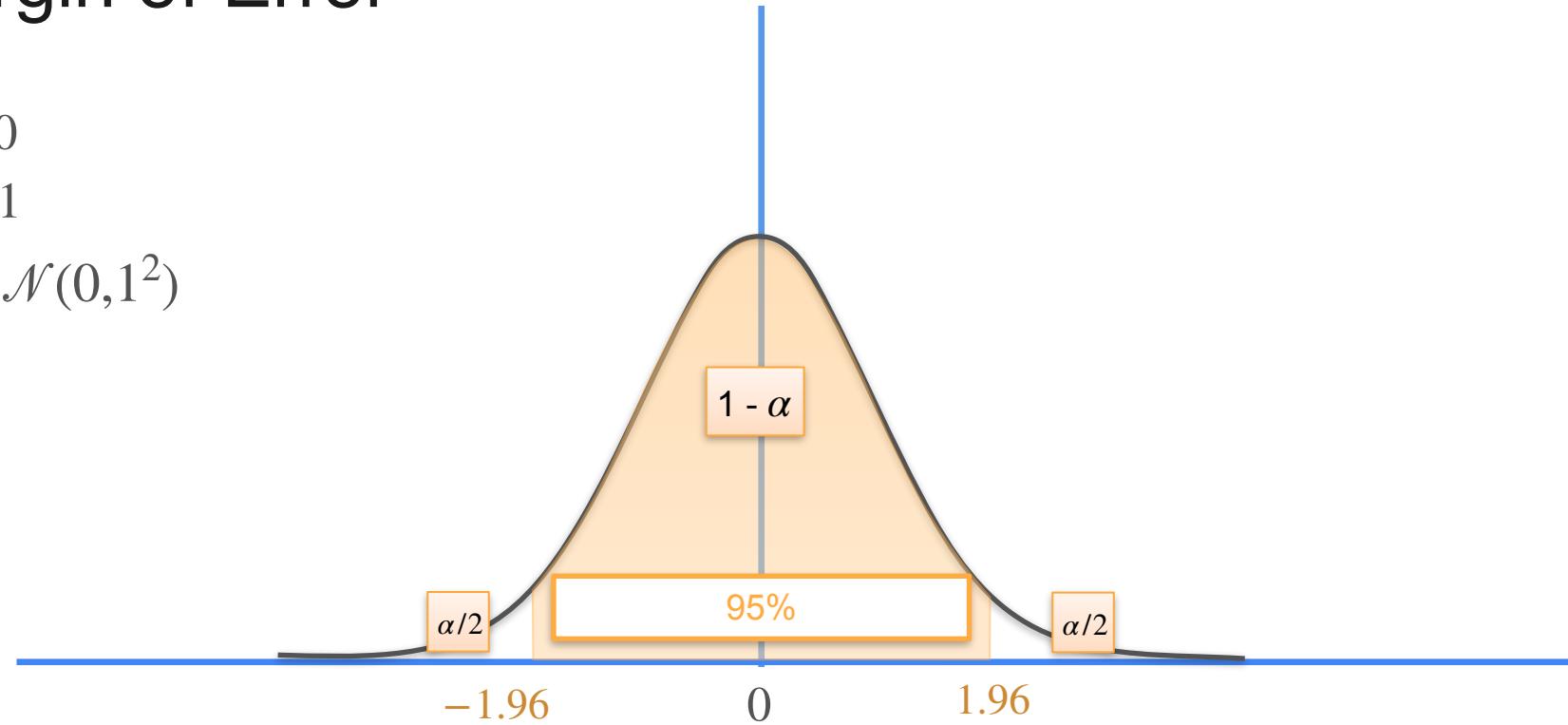


Margin of Error

$$\mu = 0$$

$$\sigma = 1$$

$$\bar{X} \sim \mathcal{N}(0, 1^2)$$



Margin of Error

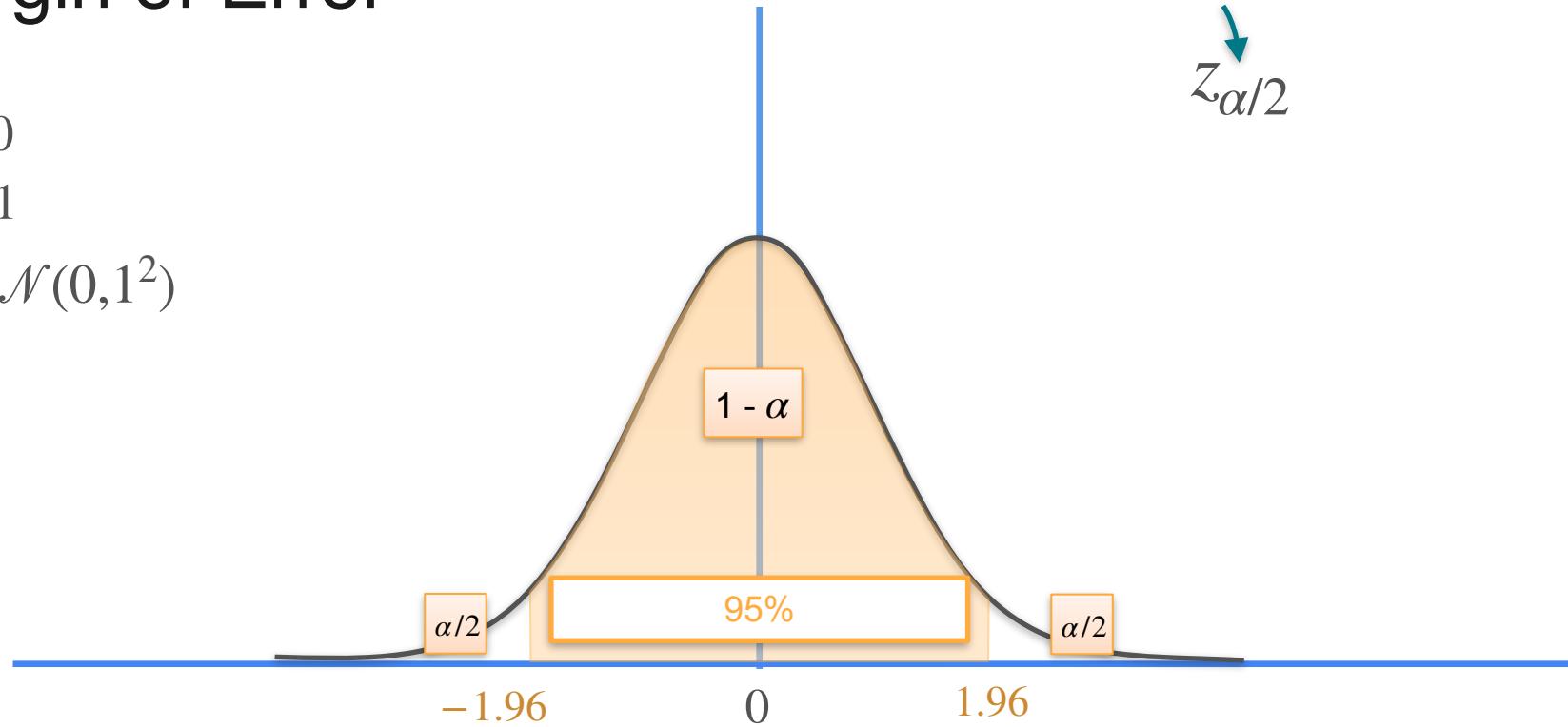
$$\mu = 0$$

$$\sigma = 1$$

$$\bar{X} \sim \mathcal{N}(0, 1^2)$$

Critical Value

$$z_{\alpha/2}$$

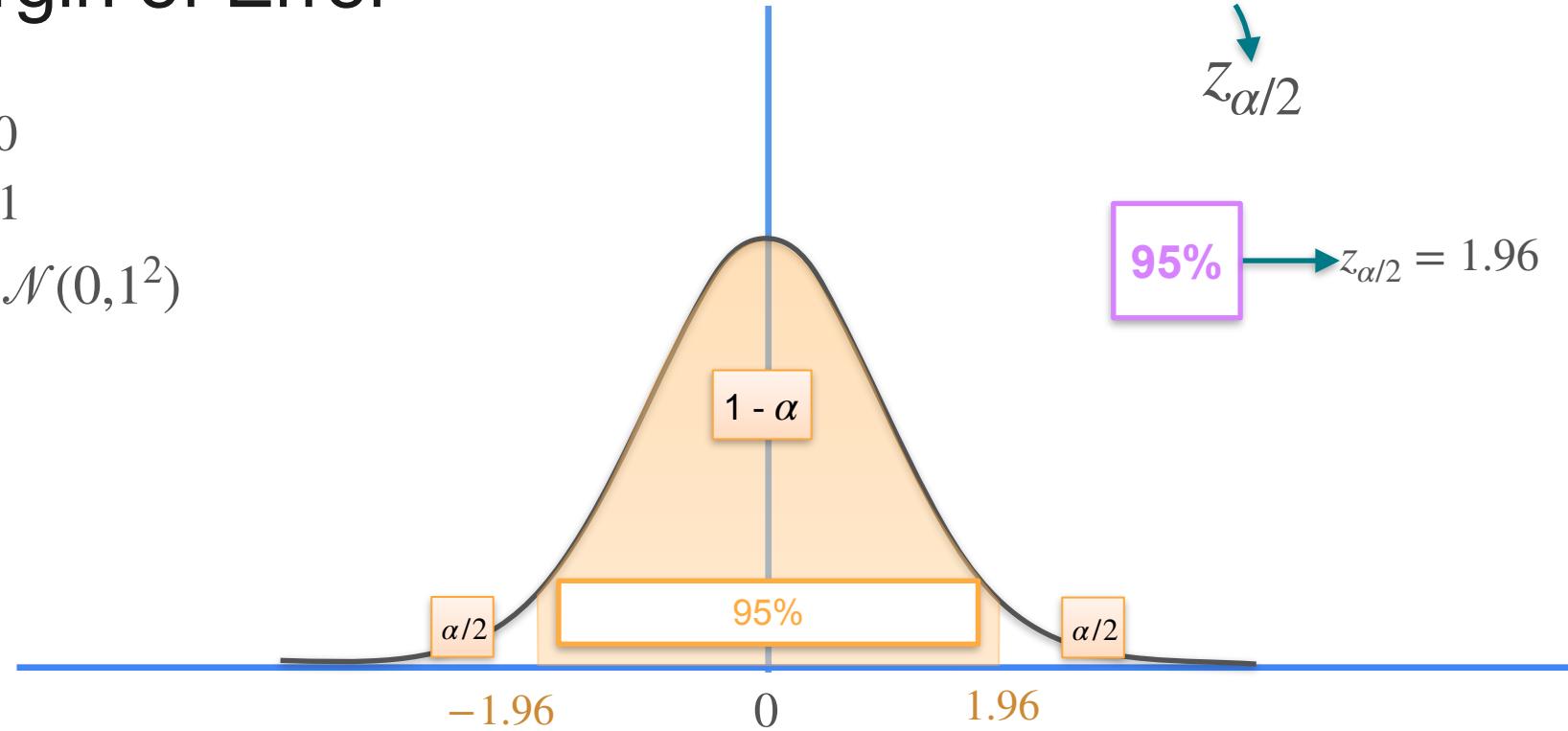


Margin of Error

$$\mu = 0$$

$$\sigma = 1$$

$$\bar{X} \sim \mathcal{N}(0, 1^2)$$



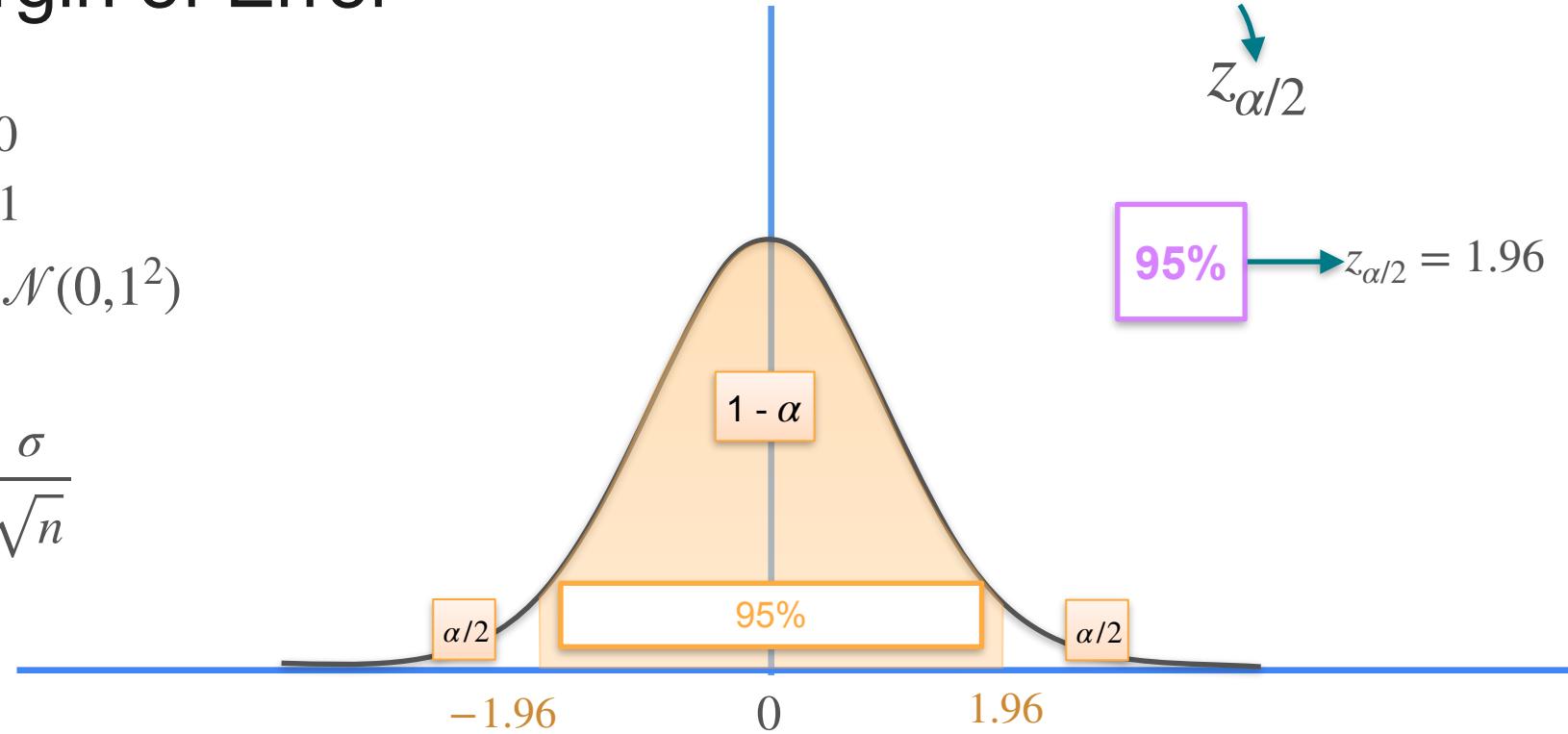
Margin of Error

$$\mu = 0$$

$$\sigma = 1$$

$$\bar{X} \sim \mathcal{N}(0, 1^2)$$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$



Margin of Error

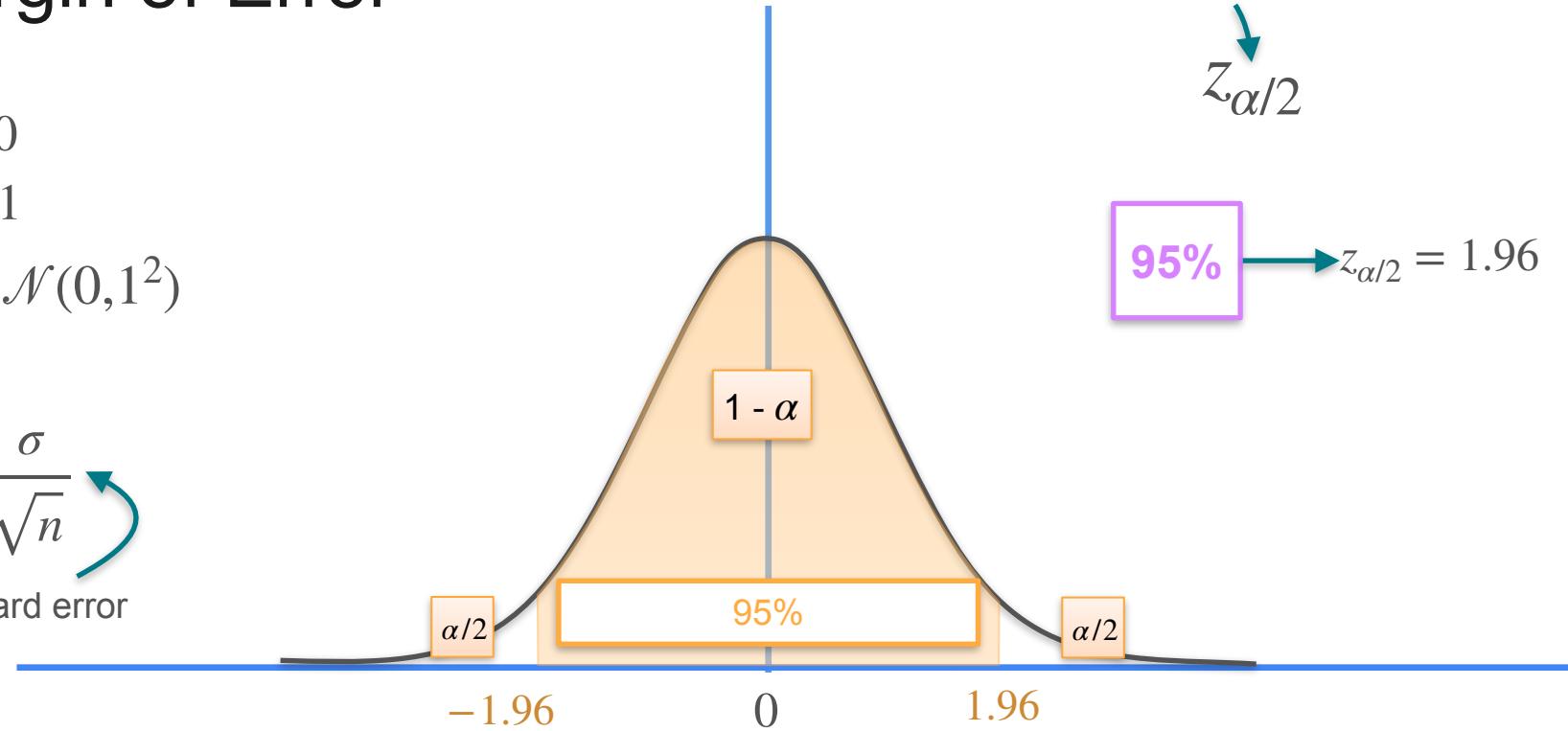
$$\mu = 0$$

$$\sigma = 1$$

$$\bar{X} \sim \mathcal{N}(0, 1^2)$$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

Standard error



Margin of Error

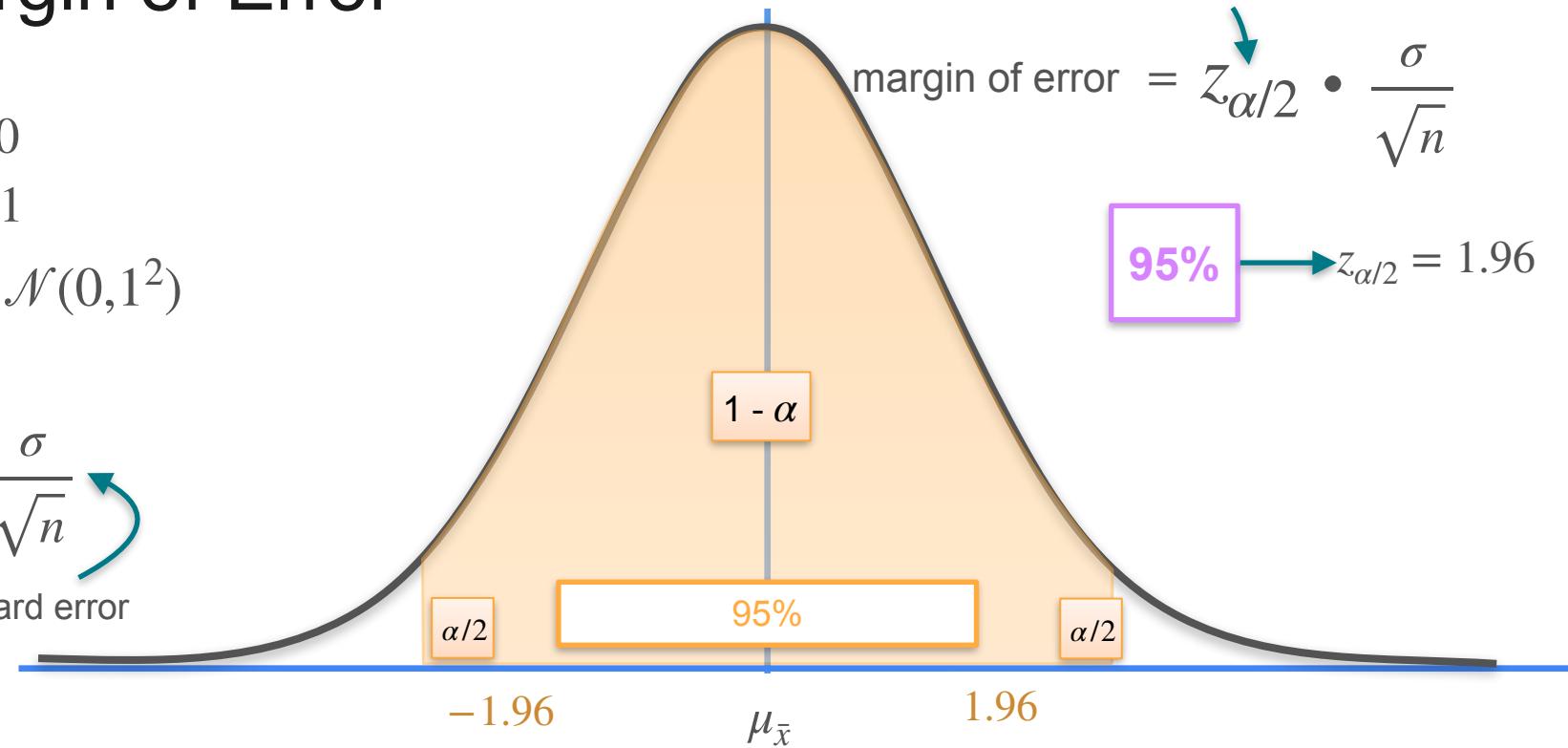
$$\mu = 0$$

$$\sigma = 1$$

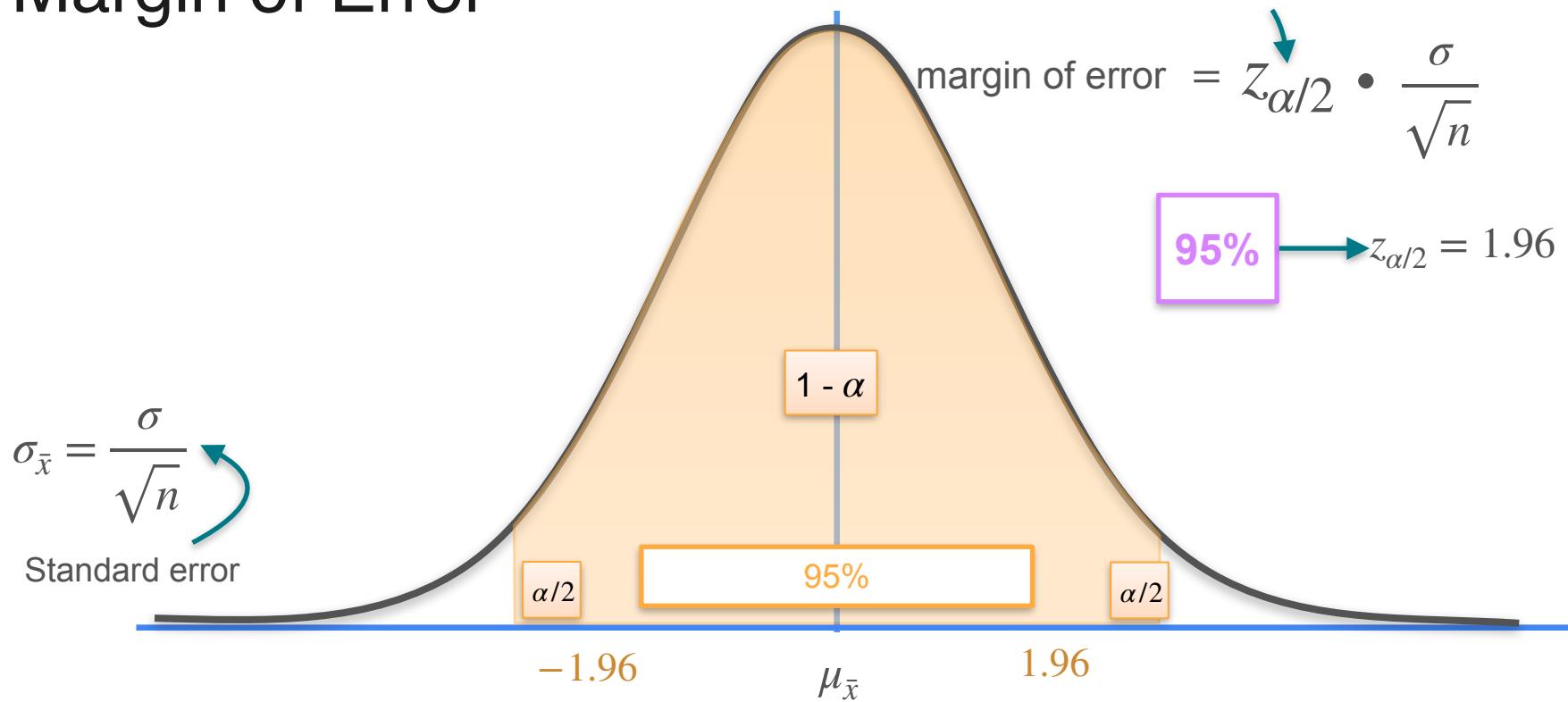
$$\bar{X} \sim \mathcal{N}(0, 1^2)$$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

Standard error



Margin of Error



Margin of Error

$$\mu_{\bar{x}} = \mu$$

$$\bar{X} \sim \mathcal{N}\left(\mu_{\bar{x}}, \frac{\sigma^2}{n}\right)$$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

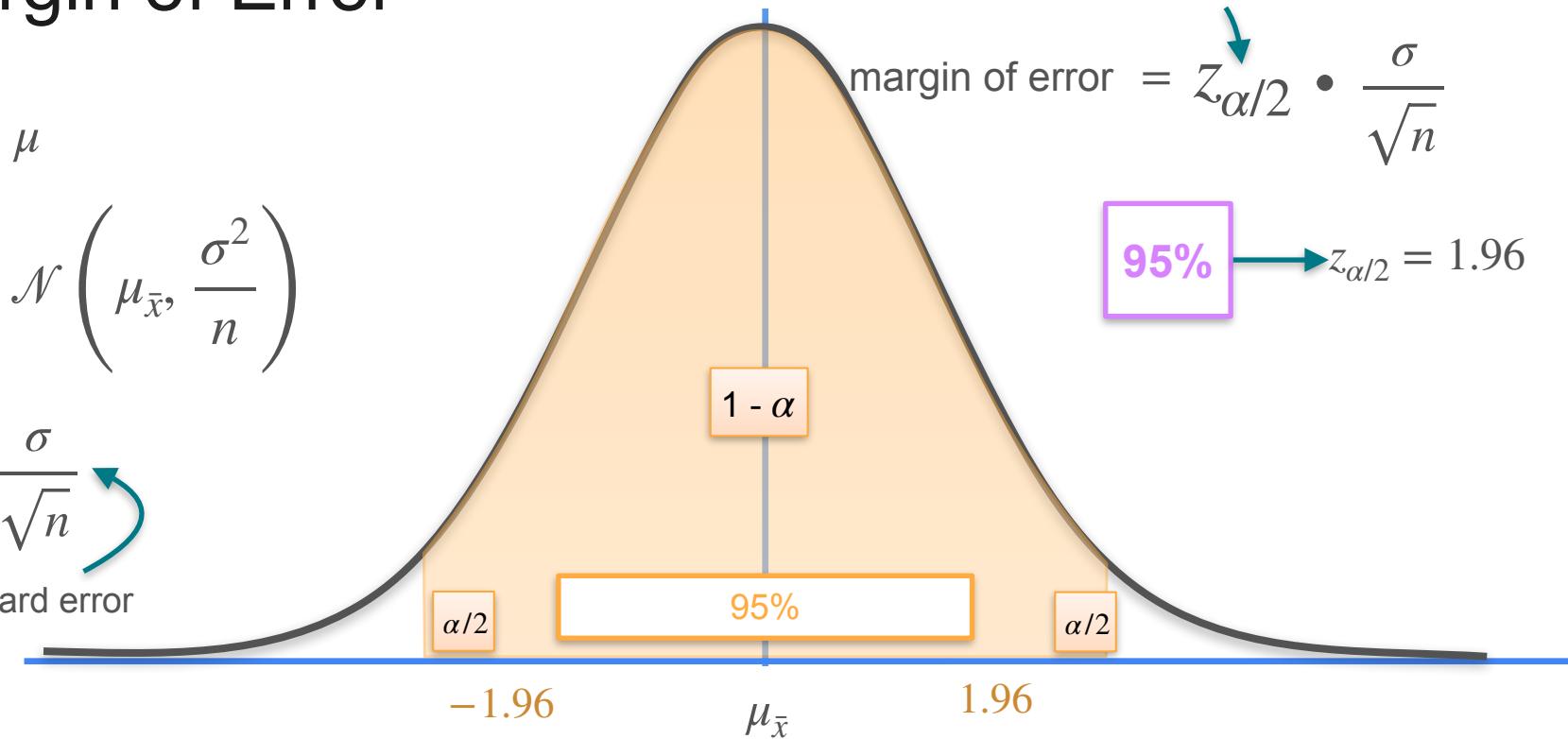
Standard error

Critical Value

$$\text{margin of error} = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

95%

$$z_{\alpha/2} = 1.96$$



Margin of Error

$$\mu_{\bar{x}} = \mu$$

$$\bar{X} \sim \mathcal{N}\left(\mu_{\bar{x}}, \frac{\sigma^2}{n}\right)$$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

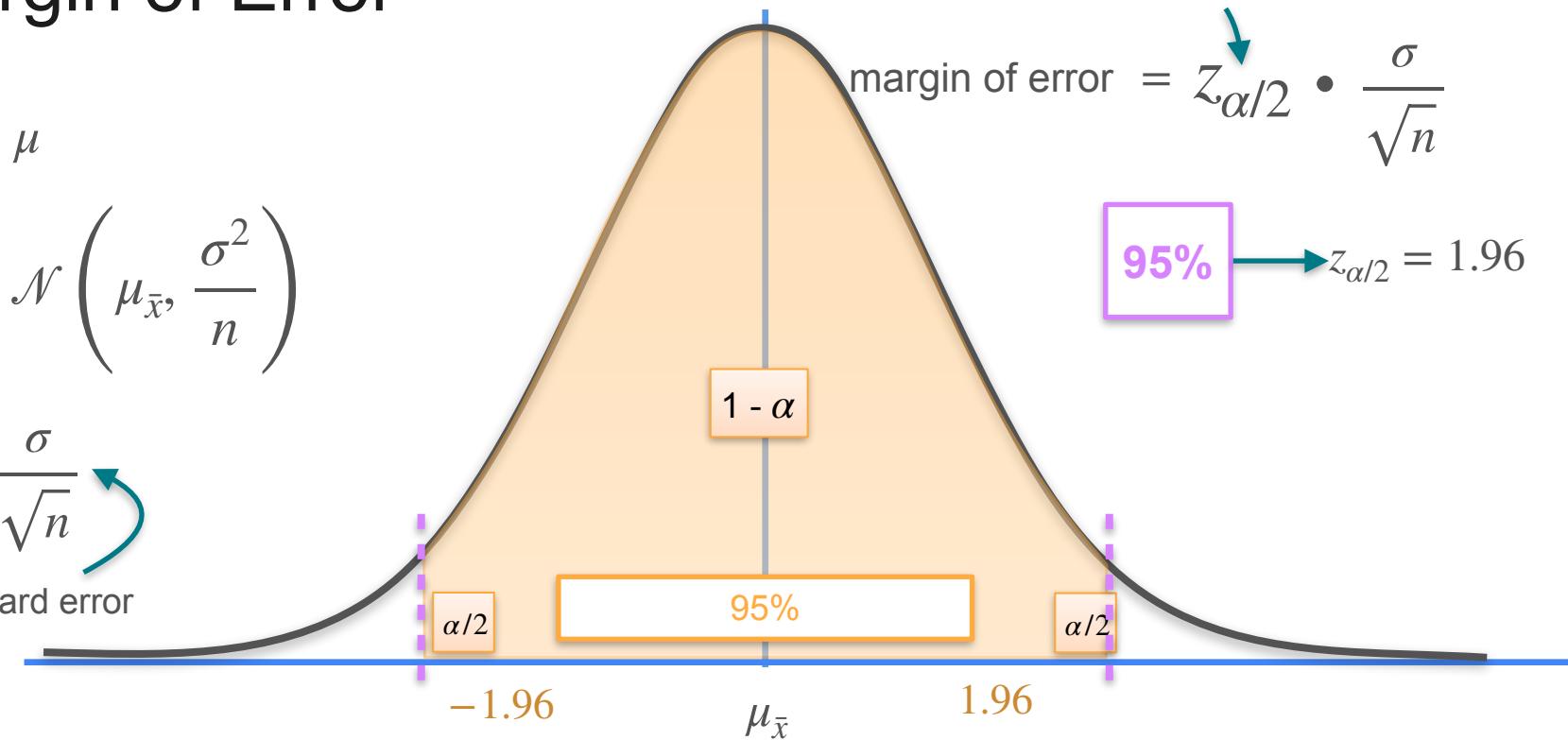
Standard error

Critical Value

$$\text{margin of error} = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

95%

$$z_{\alpha/2} = 1.96$$



Margin of Error

$$\mu_{\bar{x}} = \mu$$

$$\bar{X} \sim \mathcal{N}\left(\mu_{\bar{x}}, \frac{\sigma^2}{n}\right)$$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

Standard error

Critical Value

$$\text{margin of error} = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

95%

$$z_{\alpha/2} = 1.96$$

$1 - \alpha$

95%

$$z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

$\alpha/2$

-1.96

$\mu_{\bar{x}}$

1.96

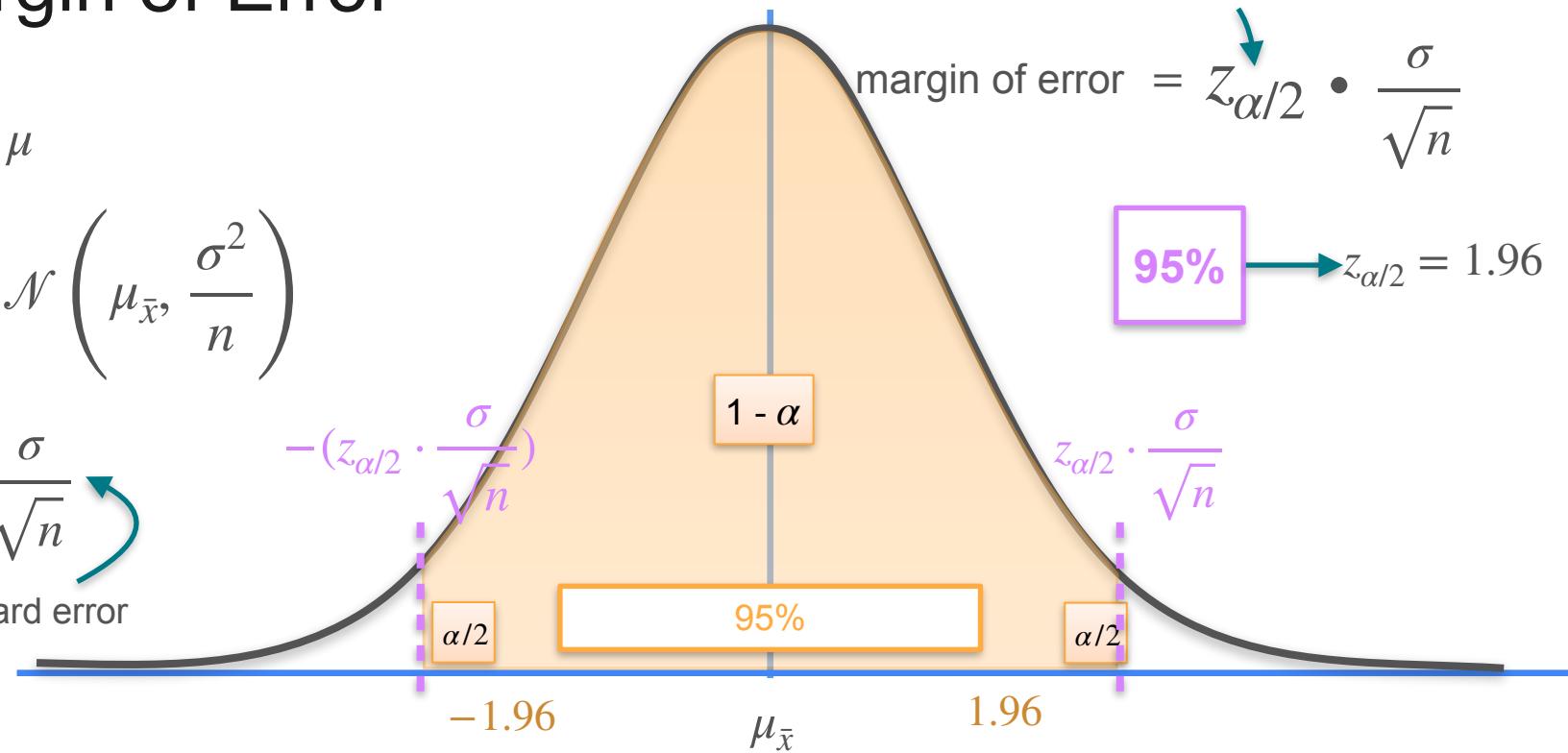
Margin of Error

$$\mu_{\bar{x}} = \mu$$

$$\bar{X} \sim \mathcal{N}\left(\mu_{\bar{x}}, \frac{\sigma^2}{n}\right)$$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

Standard error



Margin of Error

$$\mu_{\bar{x}} = \mu$$

$$\bar{X} \sim \mathcal{N}\left(\mu_{\bar{x}}, \frac{\sigma^2}{n}\right)$$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

Standard error

Critical Value

$$\text{margin of error} = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

95%

$$z_{\alpha/2} = 1.96$$

$$-(z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}})$$

$1 - \alpha$

$$z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

-1.96

$\mu_{\bar{x}}$

1.96



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Confidence Interval

**Confidence Interval -
Calculation Steps**

Confidence Interval - Calculation Steps

STEPS:

Confidence Interval - Calculation Steps

STEPS:

- Find the sample mean

Confidence Interval - Calculation Steps

STEPS:

- Find the sample mean

$$\bar{x}$$

Confidence Interval - Calculation Steps

STEPS:

- Find the sample mean
- Define a desired confidence level ($1 - \alpha$)

$$\bar{x}$$

Confidence Interval - Calculation Steps

STEPS:

- Find the sample mean
- Define a desired confidence level ($1 - \alpha$)

\bar{x}

95%

Confidence Interval - Calculation Steps

STEPS:

- Find the sample mean
- Define a desired confidence level ($1 - \alpha$)
- Get the critical value ($z_{\alpha/2}$)

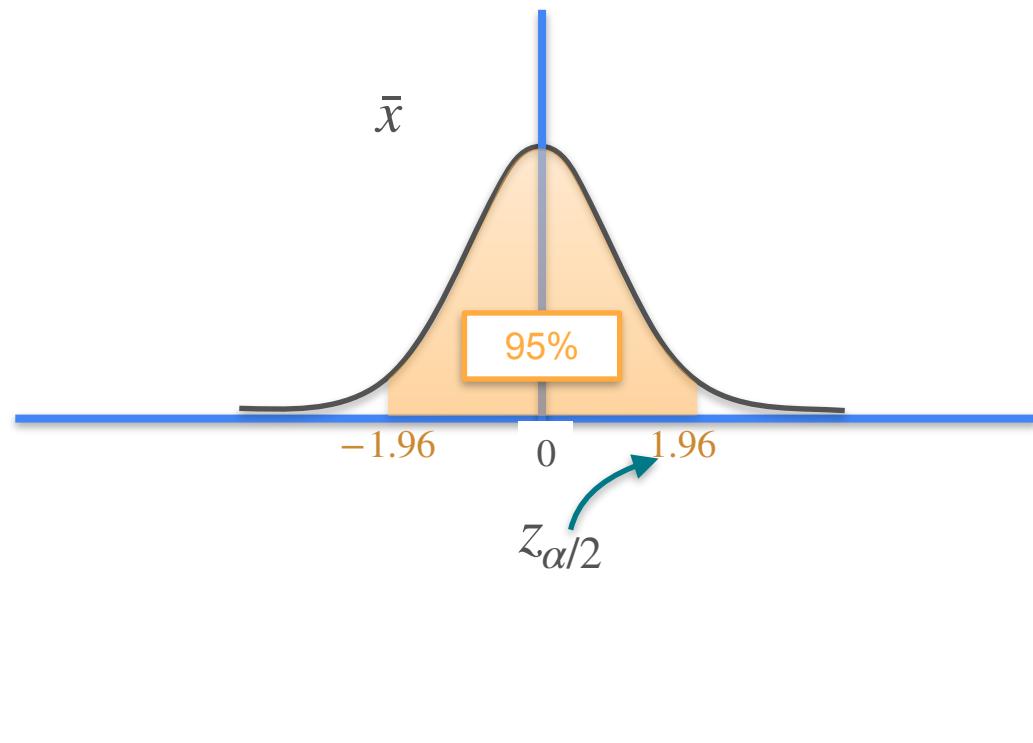
\bar{x}

95%

Confidence Interval - Calculation Steps

STEPS:

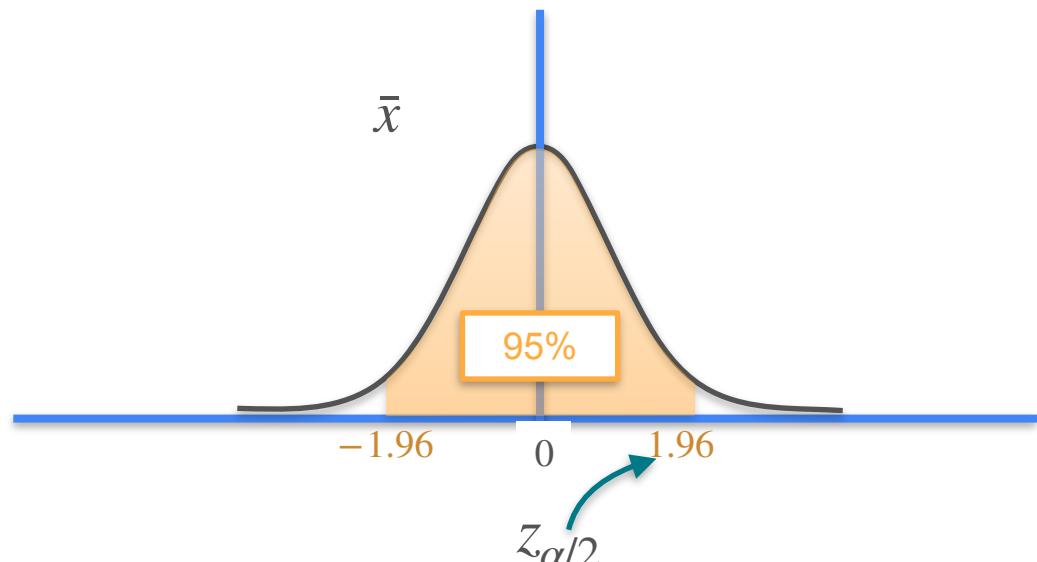
- Find the sample mean
- Define a desired confidence level ($1 - \alpha$)
- Get the critical value ($z_{\alpha/2}$)



Confidence Interval - Calculation Steps

STEPS:

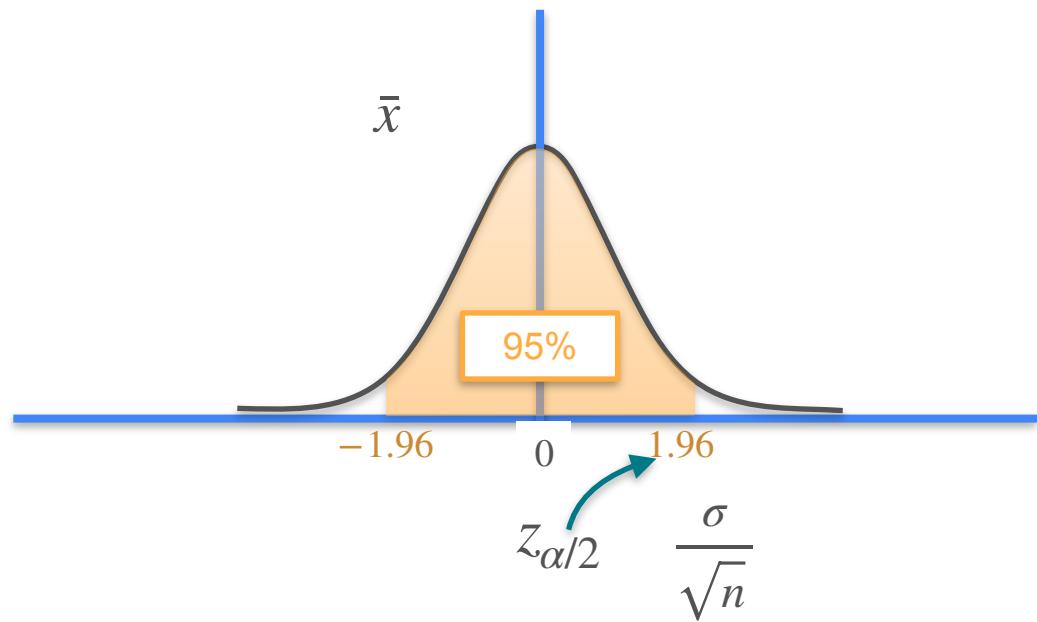
- Find the sample mean
- Define a desired confidence level ($1 - \alpha$)
- Get the critical value ($z_{\alpha/2}$)
- Find the standard error ($\frac{\sigma}{\sqrt{n}}$)



Confidence Interval - Calculation Steps

STEPS:

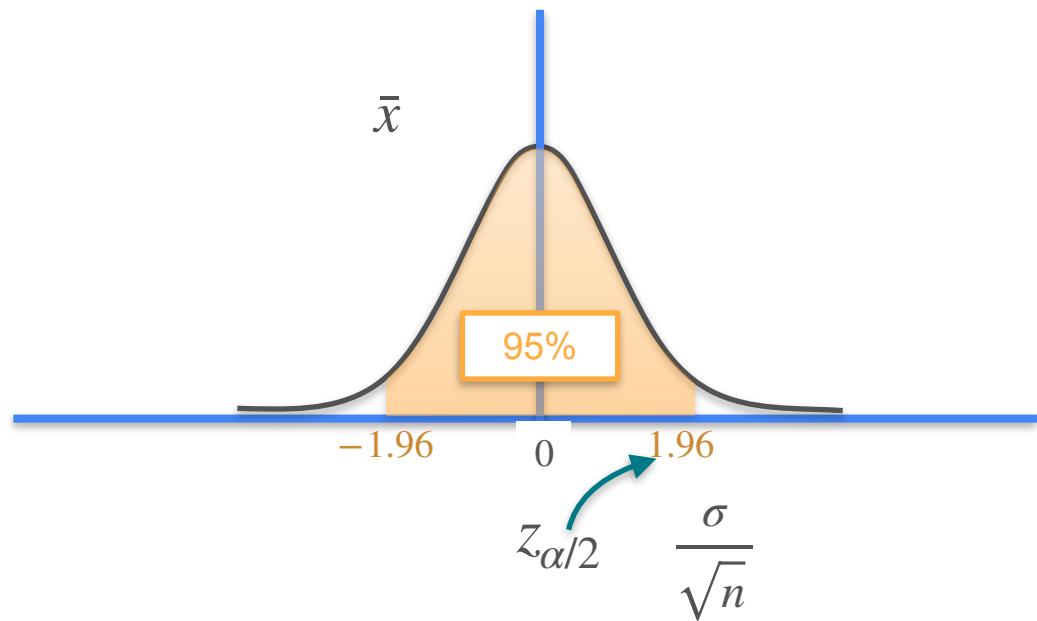
- Find the sample mean
- Define a desired confidence level ($1 - \alpha$)
- Get the critical value ($z_{\alpha/2}$)
- Find the standard error ($\frac{\sigma}{\sqrt{n}}$)



Confidence Interval - Calculation Steps

STEPS:

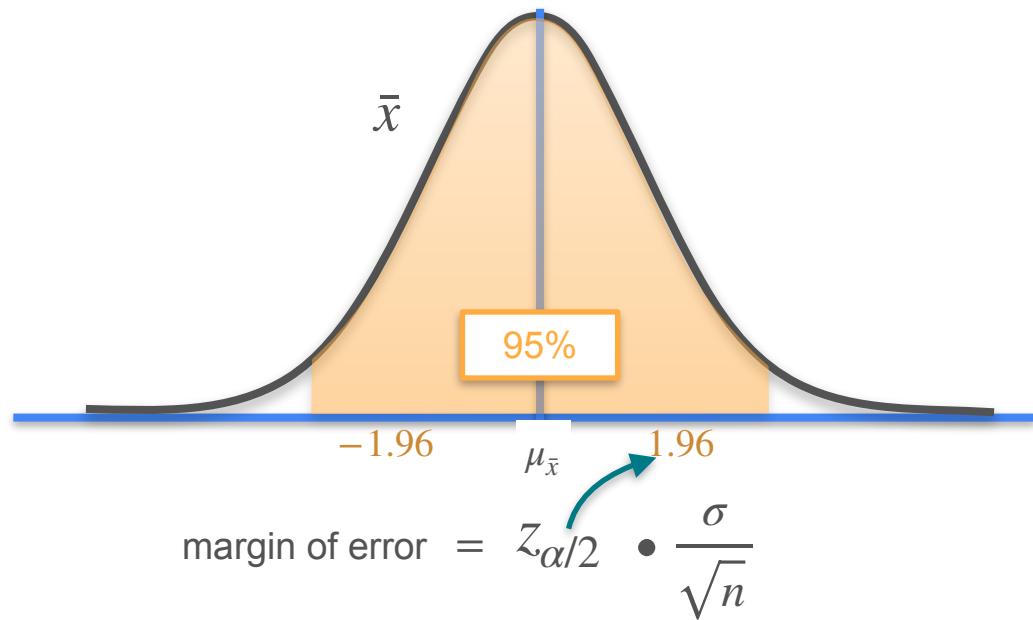
- Find the sample mean
- Define a desired confidence level ($1 - \alpha$)
- Get the critical value ($z_{\alpha/2}$)
- Find the standard error ($\frac{\sigma}{\sqrt{n}}$)
- Find the margin of error



Confidence Interval - Calculation Steps

STEPS:

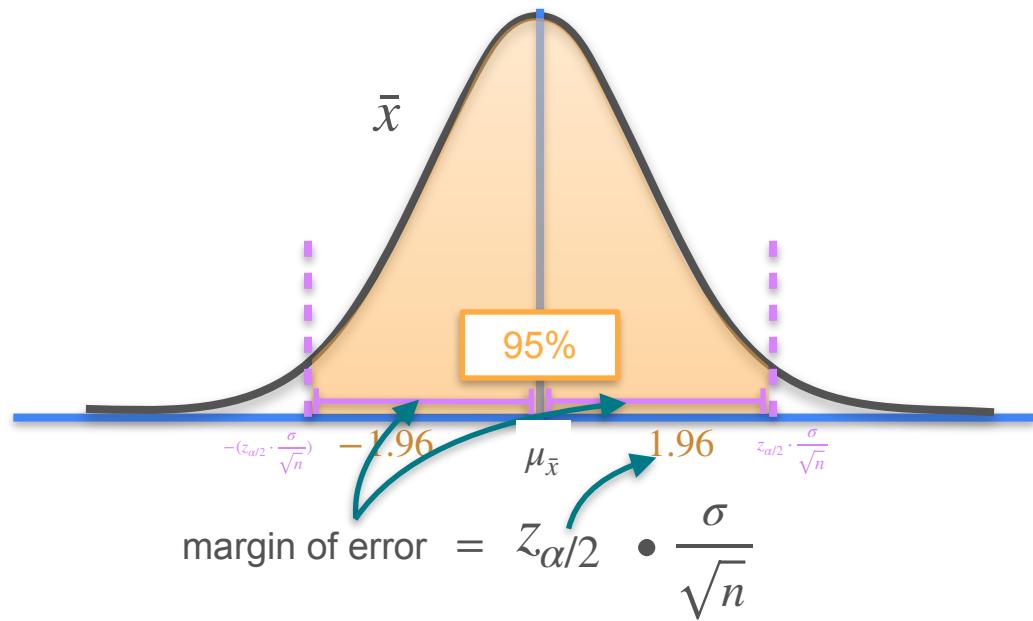
- Find the sample mean
- Define a desired confidence level ($1 - \alpha$)
- Get the critical value ($z_{\alpha/2}$)
- Find the standard error ($\frac{\sigma}{\sqrt{n}}$)
- Find the margin of error



Confidence Interval - Calculation Steps

STEPS:

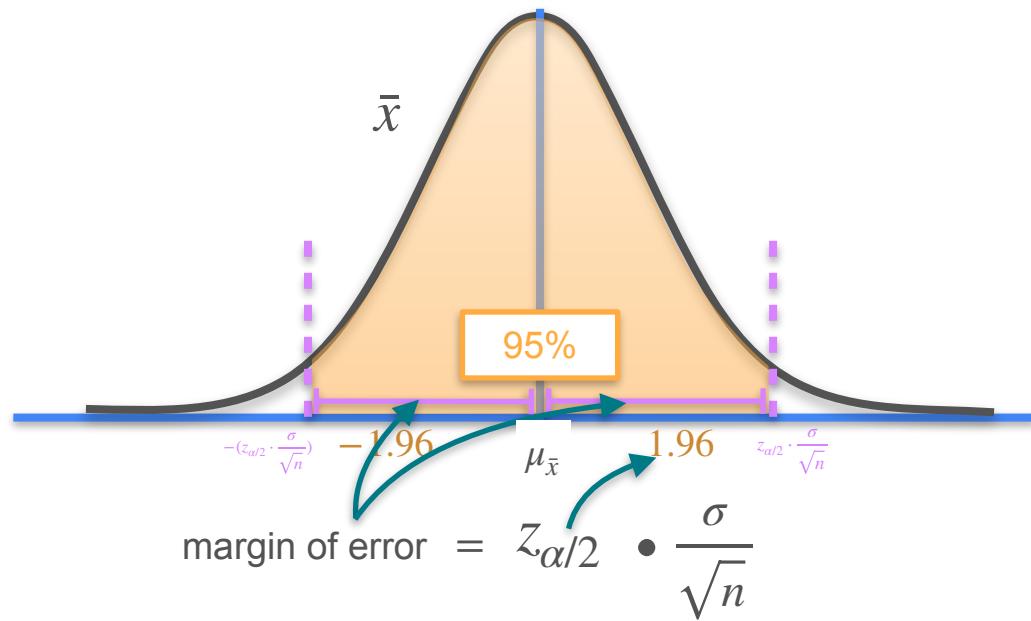
- Find the sample mean
- Define a desired confidence level ($1 - \alpha$)
- Get the critical value ($z_{\alpha/2}$)
- Find the standard error ($\frac{\sigma}{\sqrt{n}}$)
- Find the margin of error



Confidence Interval - Calculation Steps

STEPS:

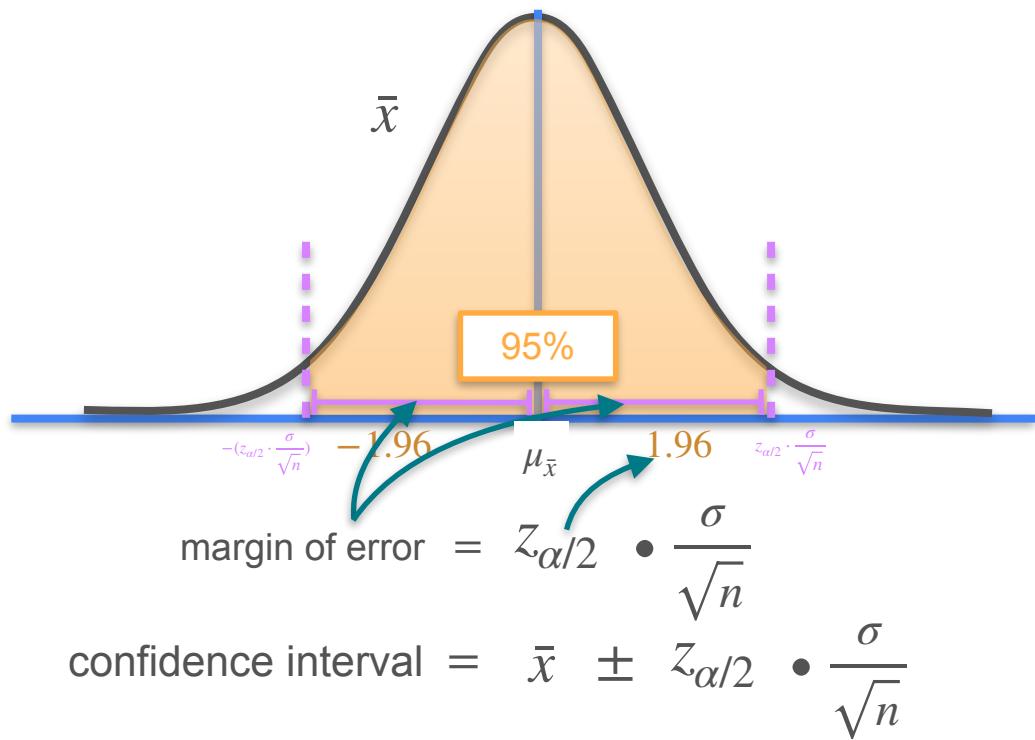
- Find the sample mean
- Define a desired confidence level ($1 - \alpha$)
- Get the critical value ($z_{\alpha/2}$)
- Find the standard error ($\frac{\sigma}{\sqrt{n}}$)
- Find the margin of error
- Add/subtract the margin of error to the sample mean



Confidence Interval - Calculation Steps

STEPS:

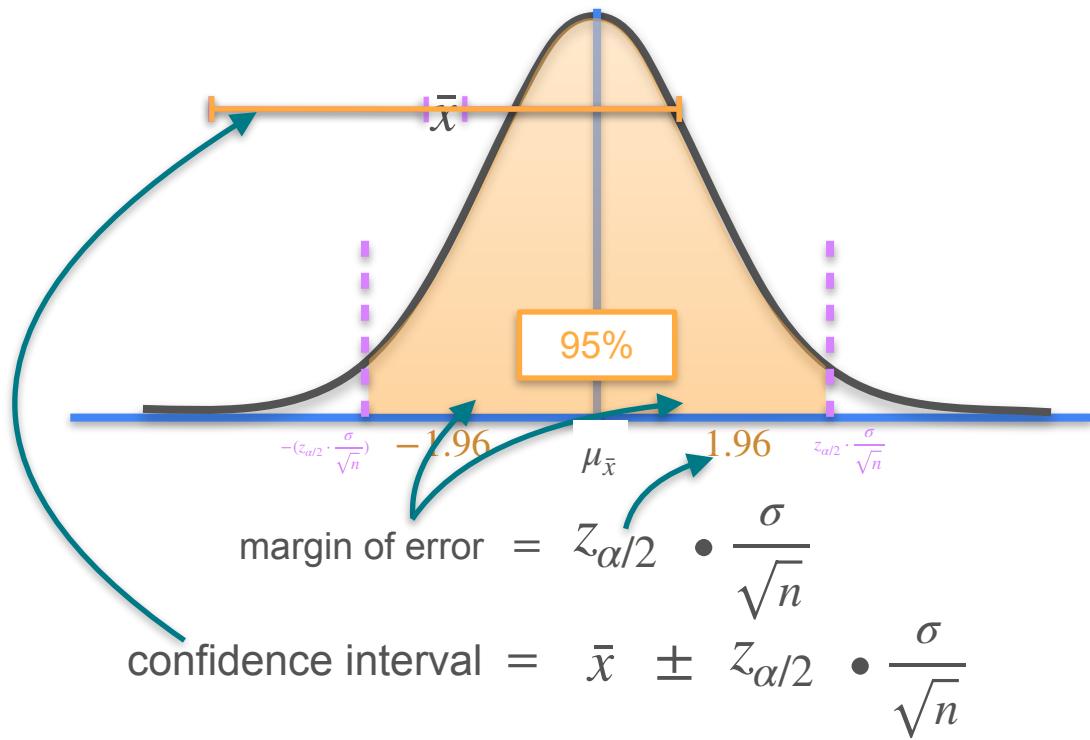
- Find the sample mean
- Define a desired confidence level ($1 - \alpha$)
- Get the critical value ($z_{\alpha/2}$)
- Find the standard error ($\frac{\sigma}{\sqrt{n}}$)
- Find the margin of error
- Add/subtract the margin of error to the sample mean



Confidence Interval - Calculation Steps

STEPS:

- Find the sample mean
- Define a desired confidence level ($1 - \alpha$)
- Get the critical value ($z_{\alpha/2}$)
- Find the standard error ($\frac{\sigma}{\sqrt{n}}$)
- Find the margin of error
- Add/subtract the margin of error to the sample mean



Confidence Interval - Calculation Steps

STEPS:

- Find the sample mean
- Define a desired confidence level ($1 - \alpha$)
- Get the critical value ($z_{\alpha/2}$)
- Find the standard error ($\frac{\sigma}{\sqrt{n}}$)
- Find the margin of error
- Add/subtract the margin of error to the sample mean

$$\text{confidence interval} = \bar{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

Confidence Interval - Calculation Steps

STEPS:

- Find the sample mean
- Define a desired confidence level ($1 - \alpha$)
- Get the critical value ($z_{\alpha/2}$)
- Find the standard error ($\frac{\sigma}{\sqrt{n}}$)
- Find the margin of error
- Add/subtract the margin of error to the sample mean

$$\text{confidence interval} = \bar{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

Assumptions

Confidence Interval - Calculation Steps

STEPS:

- Find the sample mean
- Define a desired confidence level ($1 - \alpha$)
- Get the critical value ($z_{\alpha/2}$)
- Find the standard error ($\frac{\sigma}{\sqrt{n}}$)
- Find the margin of error
- Add/subtract the margin of error to the sample mean

$$\text{confidence interval} = \bar{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

Assumptions

- Simple random sample

Confidence Interval - Calculation Steps

STEPS:

- Find the sample mean
- Define a desired confidence level ($1 - \alpha$)
- Get the critical value ($z_{\alpha/2}$)
- Find the standard error ($\frac{\sigma}{\sqrt{n}}$)
- Find the margin of error
- Add/subtract the margin of error to the sample mean

$$\text{confidence interval} = \bar{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

Assumptions

- Simple random sample
- Sample size > 30 or population is approximately normal



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Confidence Interval

**Confidence Interval -
Example**

Confidence Interval - Example

Confidence Interval - Example

Statistopia

6,000 adults

Confidence Interval - Example

Statistopia

6,000 adults

Random Selection

49



Confidence Interval - Example

Statistopia

6,000 adults

Random Selection



$$\bar{x} = 170\text{cm}$$

Confidence Interval - Example

Statistopia

6,000 adults

Random Selection



$$\bar{x} = 170\text{cm}$$

$$\sigma = 25\text{cm}$$

Confidence Interval - Example

Statistopia

6,000 adults

Random Selection



$$\bar{x} = 170\text{cm}$$

$$\sigma = 25\text{cm}$$

Calculate a 95% confidence interval for the average height of adults on Statistopia.

Confidence Interval - Example

Statistopia

6,000 adults

Random Selection



$$\bar{x} = 170\text{cm}$$

$$\sigma = 25\text{cm}$$

95%

$$\rightarrow z_{\alpha/2} = 1.96$$

Calculate a 95% confidence interval for the average height of adults on Statistopia.

Confidence Interval - Example

Random Selection

49



$$\sigma = 25\text{cm}$$

95% → $z_{\alpha/2} = 1.96$

Confidence Interval - Example

Random Selection

49



margin of error =

$$\sigma = 25\text{cm}$$

95%

$$\rightarrow z_{\alpha/2} = 1.96$$

Confidence Interval - Example

Random Selection

49



$$\text{margin of error} = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

$$\sigma = 25\text{cm}$$

95%

$$z_{\alpha/2} = 1.96$$

Confidence Interval - Example

Random Selection

49



$$\sigma = 25\text{cm}$$

$$\begin{aligned}\text{margin of error} &= z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} \\ &= 1.96 \cdot \frac{25}{\sqrt{49}}\end{aligned}$$

95%

$$z_{\alpha/2} = 1.96$$

Confidence Interval - Example

Random Selection

49



$$\sigma = 25\text{cm}$$

95%

$$z_{\alpha/2} = 1.96$$

$$\begin{aligned}\text{margin of error} &= z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} \\ &= 1.96 \cdot \frac{25}{\sqrt{49}} \\ &= 1.96 \cdot \frac{25}{7}\end{aligned}$$

Confidence Interval - Example

Random Selection

49



$$\sigma = 25\text{cm}$$

95%

$$z_{\alpha/2} = 1.96$$

$$\begin{aligned}\text{margin of error} &= z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} \\ &= 1.96 \cdot \frac{25}{\sqrt{49}} \\ &= 1.96 \cdot \frac{25}{7} \\ &= 7\end{aligned}$$

Confidence Interval - Example

Random Selection



$$\sigma = 25\text{cm}$$

95%

$$z_{\alpha/2} = 1.96$$

Confidence Interval

$$\begin{aligned}\text{margin of error} &= z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} \\ &= 1.96 \cdot \frac{25}{\sqrt{49}} \\ &= 1.96 \cdot \frac{25}{7} \\ &= 7\end{aligned}$$

Confidence Interval - Example

Random Selection



49

$\sigma = 25cm$

95%

$$z_{\alpha/2} = 1.96$$

Confidence Interval

$170cm \pm \text{margin of error}$

$$\text{margin of error} = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

$$= 1.96 \cdot \frac{25}{\sqrt{49}}$$

$$= 1.96 \cdot \frac{25}{7}$$

$$= 7$$

Confidence Interval - Example

Random Selection

2500



$$\sigma = 10\text{cm}$$

Confidence Interval

$$170\text{cm} \pm \text{margin of error}$$

$$\text{margin of error} = 7$$

95%

$$\rightarrow z_{\alpha/2} = 1.96$$

Confidence Interval - Example

Random Selection

2500



$$\sigma = 10cm$$

Confidence Interval

$170cm \pm \text{margin of error}$

margin of error = 7

Confidence Interval

$$95\% \rightarrow z_{\alpha/2} = 1.96$$

Confidence Interval - Example

Random Selection

2500



$$\sigma = 10\text{cm}$$

Confidence Interval

$$170\text{cm} \pm \text{margin of error}$$

$$\text{margin of error} = 7$$

Confidence Interval

$$170\text{cm} - 7 = 163\text{cm}$$

95%

$$\rightarrow z_{\alpha/2} = 1.96$$

Confidence Interval - Example

Random Selection

2500



$$\sigma = 10\text{cm}$$

95%

$$z_{\alpha/2} = 1.96$$

Confidence Interval

$$170\text{cm} \pm \text{margin of error}$$

$$\text{margin of error} = 7$$

Confidence Interval

$$170\text{cm} - 7 = 163\text{cm}$$

$$170\text{cm} + 7 = 177\text{cm}$$

Confidence Interval - Example

Random Selection

2500



$$\sigma = 10\text{cm}$$

95% → $z_{\alpha/2} = 1.96$

Confidence Interval

$$170\text{cm} \pm \text{margin of error}$$

$$\text{margin of error} = 7$$

Confidence Interval

$$170\text{cm} - 7 = 163\text{cm}$$

$$170\text{cm} + 7 = 177\text{cm}$$

$$163\text{cm} < \mu < 177\text{cm}$$



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Confidence Interval

Calculating Sample Size

Calculating Sample Size

6,000 adults

95%

$$\rightarrow z_{\alpha/2} = 1.96$$

$$\bar{x} = 170\text{cm} \quad \sigma = 25\text{cm}$$

Calculating Sample Size

6,000 adults

Random Selection

49



95%

$$\rightarrow z_{\alpha/2} = 1.96$$

$$\bar{x} = 170\text{cm} \quad \sigma = 25\text{cm}$$

Calculating Sample Size

6,000 adults

Random Selection

Margin of error: 7cm

49



$$95\% \rightarrow z_{\alpha/2} = 1.96$$

$$\bar{x} = 170\text{cm} \quad \sigma = 25\text{cm}$$

Calculating Sample Size

6,000 adults

Random Selection

49

Margin of error: 7cm

$$\bar{x} \pm 7\text{cm}$$



95% → $z_{\alpha/2} = 1.96$

$$\bar{x} = 170\text{cm} \quad \sigma = 25\text{cm}$$

Calculating Sample Size

6,000 adults

95%

$$\rightarrow z_{\alpha/2} = 1.96$$

$$\bar{x} = 170cm \quad \sigma = 25cm$$

Random Selection

49



Margin of error: 7cm

$$\bar{x} \pm 7cm$$

$$163cm < \mu < 177cm$$

Calculating Sample Size

6,000 adults

95%

$$\rightarrow z_{\alpha/2} = 1.96$$

$$\bar{x} = 170cm \quad \sigma = 25cm$$

Random Selection

49



Margin of error: 7cm

$$\bar{x} \pm 7cm$$

$$163cm < \mu < 177cm$$

Calculating Sample Size

6,000 adults

95%

$$\rightarrow z_{\alpha/2} = 1.96$$

$$\bar{x} = 170\text{cm} \quad \sigma = 25\text{cm}$$

Random Selection

49



Margin of error: 7cm

$$\bar{x} \pm 7\text{cm}$$

$$163\text{cm} < \mu < 177\text{cm}$$

Margin of error: 3 cm

$$\bar{x} \pm 3\text{cm}$$

Calculating Sample Size

6,000 adults

95%

$$\rightarrow z_{\alpha/2} = 1.96$$

$$\bar{x} = 170\text{cm} \quad \sigma = 25\text{cm}$$

Random Selection



Margin of error: 7cm

$$\bar{x} \pm 7\text{cm}$$

$$163\text{cm} < \mu < 177\text{cm}$$

Margin of error: 3 cm

$$\bar{x} \pm 3\text{cm}$$

Calculating Sample Size

6,000 adults

95%

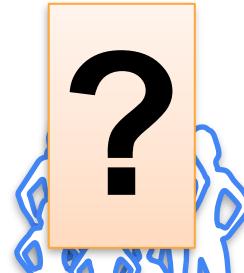
$$\rightarrow z_{\alpha/2} = 1.96$$

$$\bar{x} = 170\text{cm}$$

$$\sigma$$

What is the smallest sample size to obtain the
desired margin of error?

Random Selection



Margin of error: 7cm

$$\bar{x} \pm 7\text{cm}$$

$$163\text{cm} < \mu < 177\text{cm}$$

Margin of error: 3 cm

$$\bar{x} \pm 3\text{cm}$$

Calculating Sample Size

6,000 adults

95%

$$z_{\alpha/2} = 1.96$$

$$\bar{x} = 170\text{cm} \quad \sigma = 25\text{cm}$$

Margin of error: 3 cm

Calculating Sample Size

6,000 adults

$$\text{margin of error} = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

95%

$$z_{\alpha/2} = 1.96$$

$$\bar{x} = 170\text{cm} \quad \sigma = 25\text{cm}$$

Margin of error: 3 cm

Calculating Sample Size

6,000 adults

$$\text{margin of error} = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

95%

$$z_{\alpha/2} = 1.96$$

$$\bar{x} = 170\text{cm} \quad \sigma = 25\text{cm}$$

Margin of error: 3 cm

Calculating Sample Size

6,000 adults

$$\text{margin of error} = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

95%

$$z_{\alpha/2} = 1.96$$

$$\bar{x} = 170\text{cm} \quad \sigma = 25\text{cm}$$

3

Margin of error: 3 cm

Calculating Sample Size

6,000 adults

$$\text{margin of error} = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

95%

$$z_{\alpha/2} = 1.96$$

$$3 =$$

$$\bar{x} = 170\text{cm} \quad \sigma = 25\text{cm}$$

Margin of error: 3 cm

Calculating Sample Size

6,000 adults

95%

$$z_{\alpha/2} = 1.96$$

$$\bar{x} = 170\text{cm} \quad \sigma = 25\text{cm}$$

Margin of error: 3 cm

$$\text{margin of error} = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

$$3 = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

Calculating Sample Size

6,000 adults

$$\text{margin of error} = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

95%  $z_{\alpha/2} = 1.96$

$$\bar{x} = 170\text{cm} \quad \sigma = 25\text{cm}$$

$$3 = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

Margin of error: 3 cm

Calculating Sample Size

6,000 adults

$$\text{margin of error} = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

95%  $z_{\alpha/2} = 1.96$

$$\bar{x} = 170\text{cm} \quad \sigma = 25\text{cm}$$

$$3 = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

$$3 \geq$$

Margin of error: 3 cm

Calculating Sample Size

6,000 adults

95%

$$z_{\alpha/2} = 1.96$$

$$\bar{x} = 170\text{cm} \quad \sigma = 25\text{cm}$$

Margin of error: 3 cm

$$\text{margin of error} = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

$$3 = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

$$3 \geq z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

Calculating Sample Size

6,000 adults

95%

$$z_{\alpha/2} = 1.96$$

$$\bar{x} = 170\text{cm} \quad \sigma = 25\text{cm}$$

Margin of error: 3 cm

$$\text{margin of error} = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

$$3 = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

$$3 \geq z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

Calculating Sample Size

6,000 adults

$$\text{margin of error} = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

95%

$$z_{\alpha/2} = 1.96$$

$$3 \geq z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

$$\bar{x} = 170\text{cm} \quad \sigma = 25\text{cm}$$

Margin of error: 3 cm

Calculating Sample Size

6,000 adults

$$\text{margin of error} = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

95%

$$z_{\alpha/2} = 1.96$$

$$\bar{x} = 170\text{cm} \quad \sigma = 25\text{cm}$$

Margin of error: 3 cm

$$3 \geq z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

$$3 \geq 1.96 \times \frac{25}{\sqrt{n}}$$

Calculating Sample Size

6,000 adults

$$\text{margin of error} = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

95%

$$z_{\alpha/2} = 1.96$$

$$\bar{x} = 170\text{cm} \quad \sigma = 25\text{cm}$$

Margin of error: 3 cm

$$3 \geq z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

$$\frac{3}{1.96} \geq \frac{25}{\sqrt{n}}$$

Calculating Sample Size

6,000 adults

$$\text{margin of error} = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

95%

$$z_{\alpha/2} = 1.96$$

$$\bar{x} = 170\text{cm} \quad \sigma = 25\text{cm}$$

Margin of error: 3 cm

$$3 \geq z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

$$\frac{3}{1.96 \times 25} \geq \frac{1}{\sqrt{n}}$$

Calculating Sample Size

6,000 adults

95% $\rightarrow z_{\alpha/2} = 1.96$

$$\bar{x} = 170\text{cm} \quad \sigma = 25\text{cm}$$

Margin of error: 3 cm

$$\text{margin of error} = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

$$3 \geq z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

$$\frac{1.96 \times 25}{3} \leq \frac{\sqrt{n}}{1}$$

Calculating Sample Size

6,000 adults

$$\text{margin of error} = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

95%  $z_{\alpha/2} = 1.96$

$$\frac{1.96 \times 25}{3} \leq \frac{\sqrt{n}}{1}$$

$$\bar{x} = 170\text{cm} \quad \sigma = 25\text{cm}$$

Margin of error: 3 cm

Calculating Sample Size

6,000 adults

$$\text{margin of error} = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

95%

$$z_{\alpha/2} = 1.96$$

$$\bar{x} = 170\text{cm} \quad \sigma = 25\text{cm}$$

Margin of error: 3 cm

$$\frac{1.96 \times 25}{3} \leq \frac{\sqrt{n}}{1}$$

$$\left(\frac{1.96 \times 25}{3} \right)^2$$

Calculating Sample Size

6,000 adults

$$\text{margin of error} = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

95%  $z_{\alpha/2} = 1.96$

$$\bar{x} = 170\text{cm} \quad \sigma = 25\text{cm}$$

Margin of error: 3 cm

$$\frac{1.96 \times 25}{3} \leq \frac{\sqrt{n}}{1}$$

$$\left(\frac{1.96 \times 25}{3} \right)^2 \leq n$$

Calculating Sample Size

6,000 adults

95%

$$z_{\alpha/2} = 1.96$$

$$\bar{x} = 170\text{cm} \quad \sigma = 25\text{cm}$$

Margin of error: 3 cm

$$\text{margin of error} = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

$$\frac{1.96 \times 25}{3} \leq \frac{\sqrt{n}}{1}$$

$$n \geq \left(\frac{1.96 \times 25}{3} \right)^2$$

Calculating Sample Size

6,000 adults

$$\text{margin of error} = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

95%  $z_{\alpha/2} = 1.96$

$$n \geq \left(\frac{1.96 \times 25}{3} \right)^2$$

$$\bar{x} = 170\text{cm} \quad \sigma = 25\text{cm}$$

Margin of error: 3 cm

Calculating Sample Size

6,000 adults

$$\text{margin of error} = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

95%  $z_{\alpha/2} = 1.96$

$$\bar{x} = 170\text{cm} \quad \sigma = 25\text{cm}$$

$$n \geq \left(\frac{1.96 \times 25}{3} \right)^2$$

$$n \geq 266.78 \approx 267$$

Margin of error: 3 cm

Calculating Sample Size

$$\text{margin of error} = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

$$n \geq \left(\frac{1.96 \times 25}{3} \right)^2$$

Calculating Sample Size

$$\text{margin of error} = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

$$n \geq \left(\frac{1.96 \times 25}{3} \right)^2$$

$$n \geq \left(\frac{z_{\alpha/2} \cdot \sigma}{MOE} \right)^2$$



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Confidence Interval

**Difference Between
Confidence and Probability**

Difference Between Confidence and Probability

Difference Between Confidence and Probability

\bar{x}

Difference Between Confidence and Probability

95%
Confidence
Level



Difference Between Confidence and Probability

95%
Confidence
Level



The confidence interval contains the true population parameter approximately 95% of the time.

Difference Between Confidence and Probability

95%
Confidence
Level



The confidence interval contains the true population parameter approximately 95% of the time.



Difference Between Confidence and Probability

95%
Confidence
Level



The confidence interval contains the true population parameter approximately 95% of the time.



There's a 95% probability that the population parameter falls within the confidence interval.



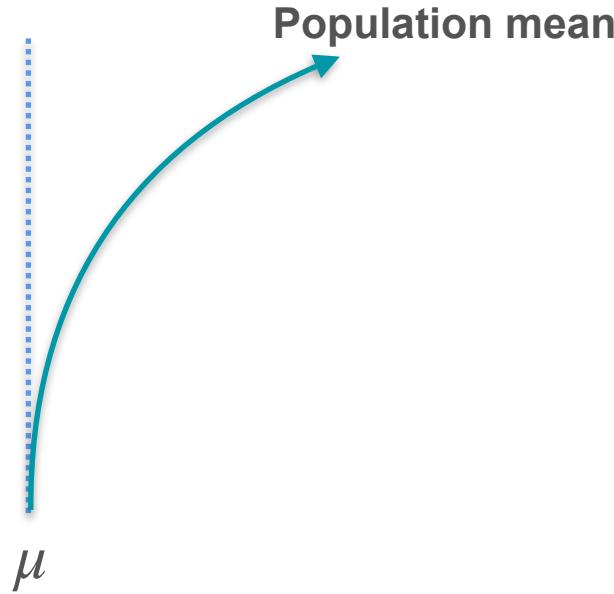
Difference Between Confidence and Probability

95%
Confidence
Level



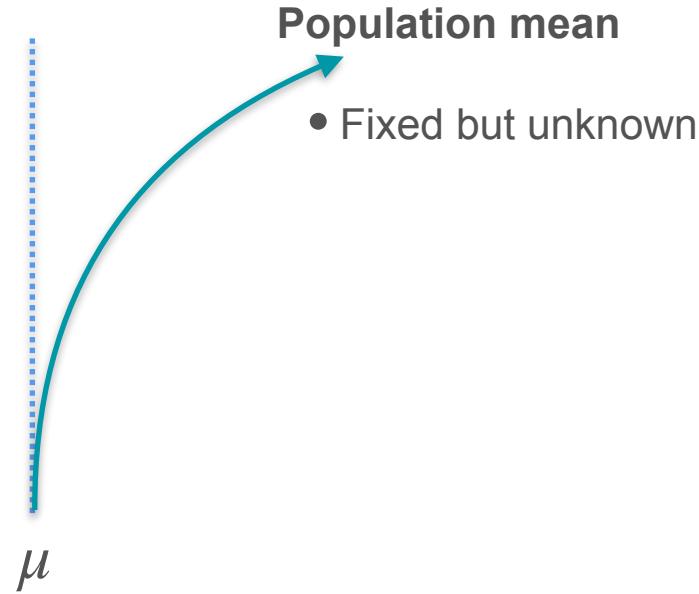
Difference Between Confidence and Probability

95%
Confidence
Level



Difference Between Confidence and Probability

95%
Confidence
Level



Difference Between Confidence and Probability

95%
Confidence
Level

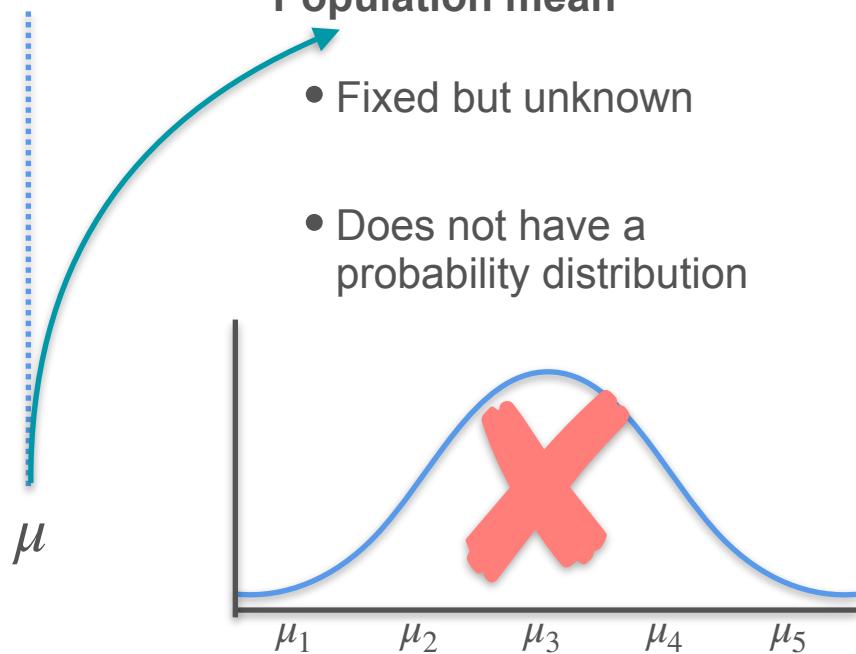


Population mean

- Fixed but unknown
- Does not have a probability distribution

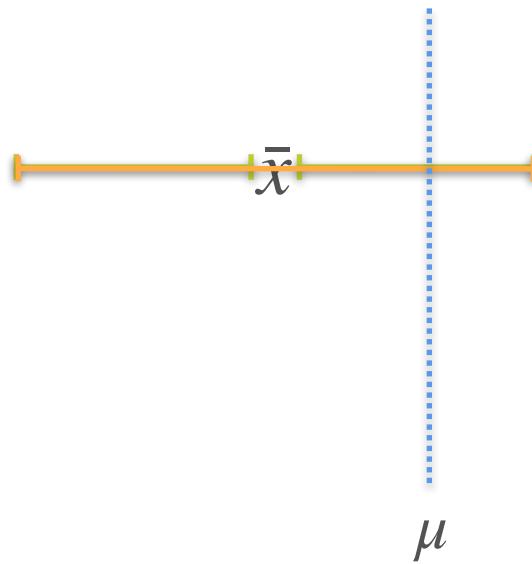
Difference Between Confidence and Probability

95%
Confidence
Level



Difference Between Confidence and Probability

95%
Confidence
Level

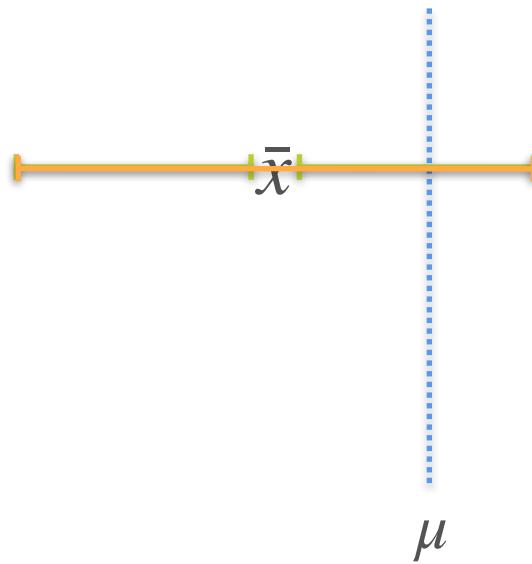


Population mean

- Fixed but unknown
- Does not have a probability distribution
- In the interval

Difference Between Confidence and Probability

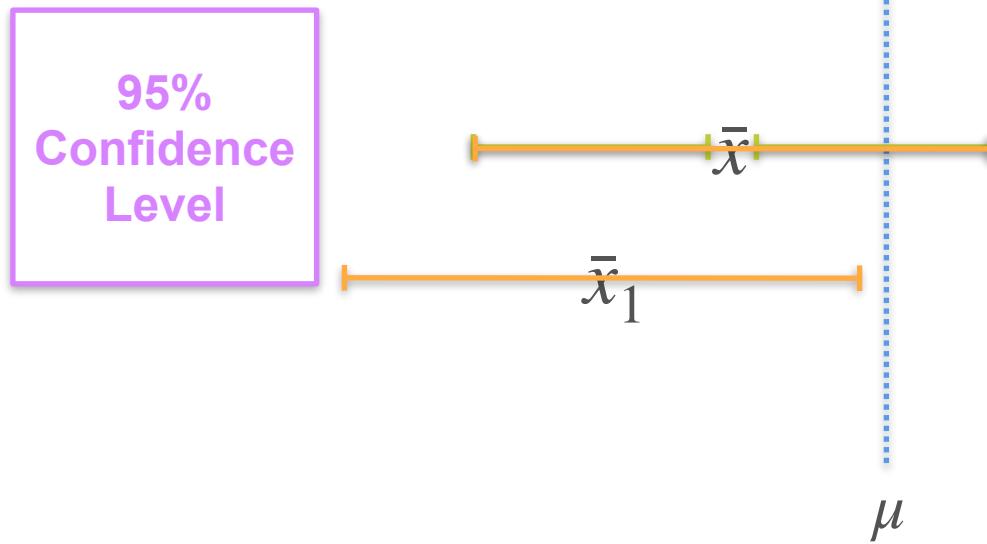
95%
Confidence
Level



Population mean

- Fixed but unknown
- Does not have a probability distribution
- In the interval or not

Difference Between Confidence and Probability

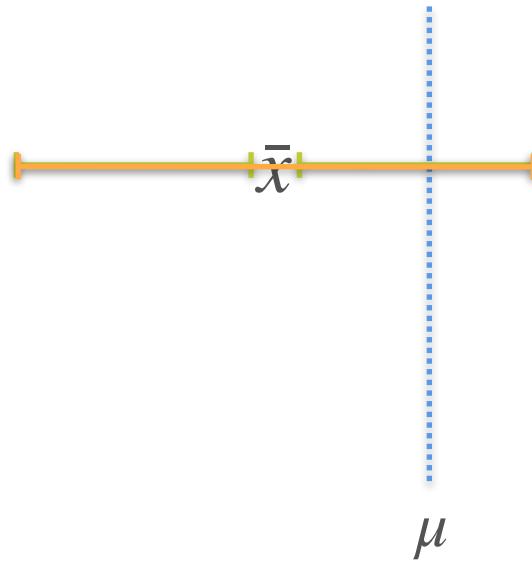


Population mean

- Fixed but unknown
- Does not have a probability distribution
- In the interval or not

Difference Between Confidence and Probability

95%
Confidence
Level

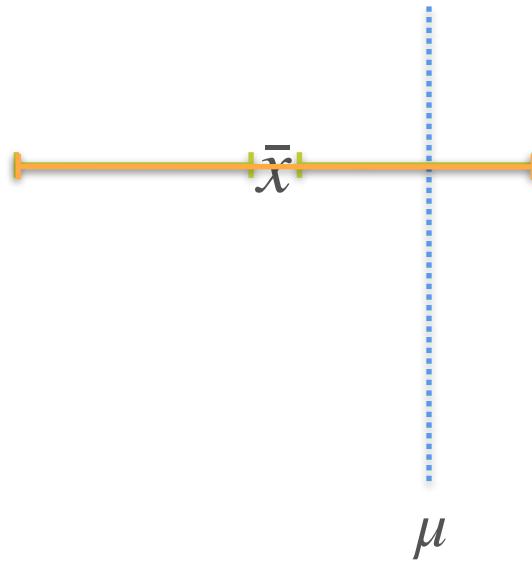


Population mean

- Fixed but unknown
- Does not have a probability distribution
- In the interval or not
- Does not fall within a specific interval 95% of the time

Difference Between Confidence and Probability

95%
Confidence
Level



Population mean

- Fixed but unknown
- Does not have a probability distribution
- In the interval or not
- Does not fall within a specific interval 95% of the time

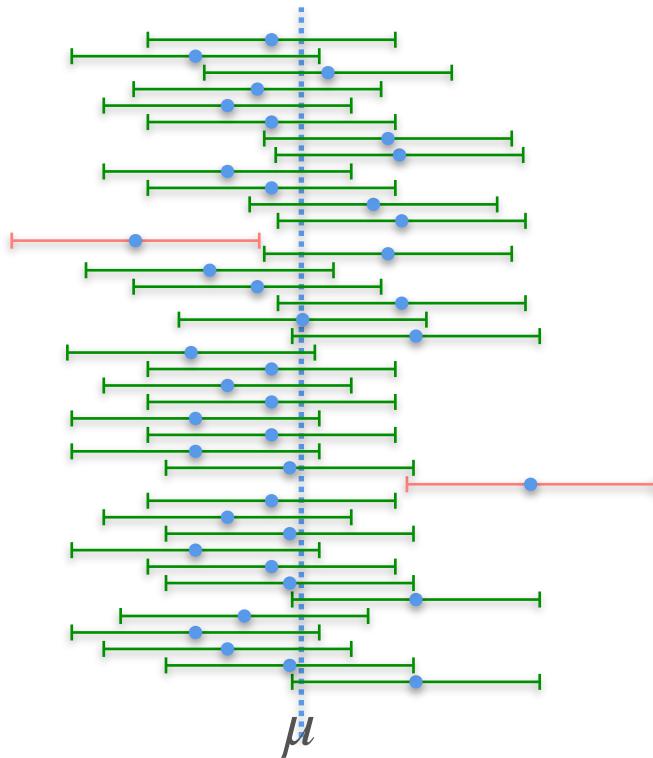
Difference Between Confidence and Probability

95%
Confidence
Level



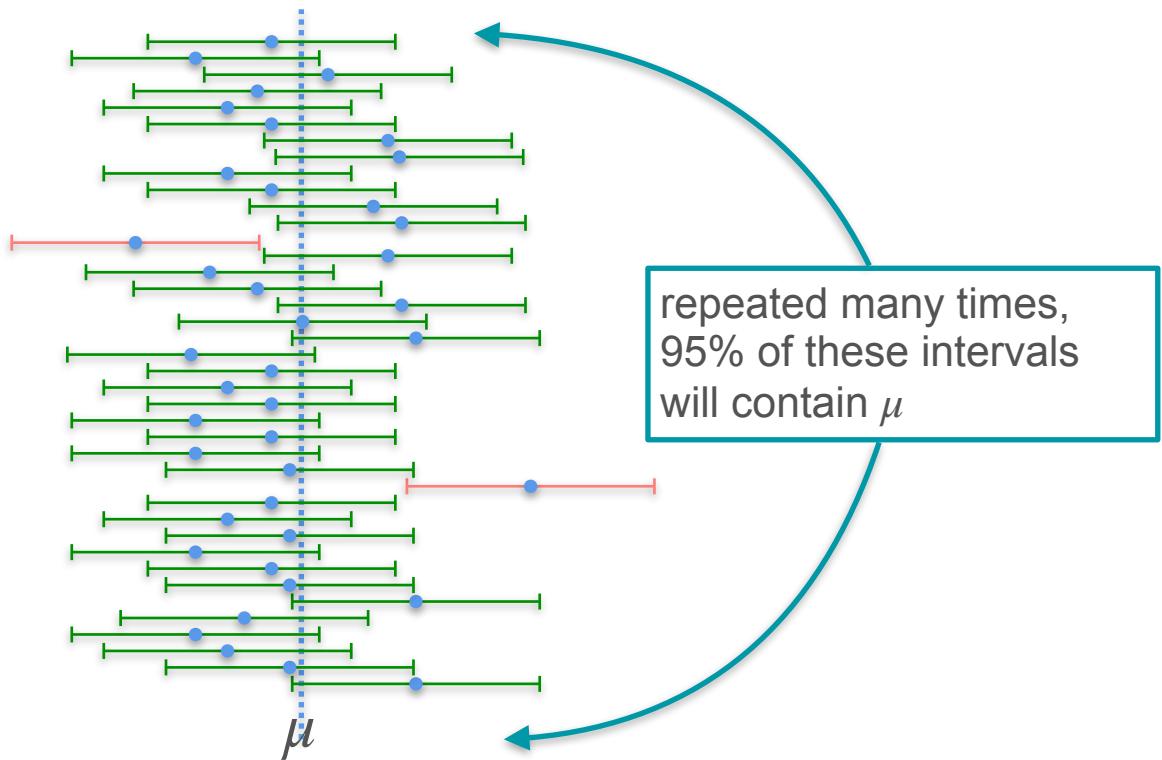
Difference Between Confidence and Probability

95%
Confidence
Level



Difference Between Confidence and Probability

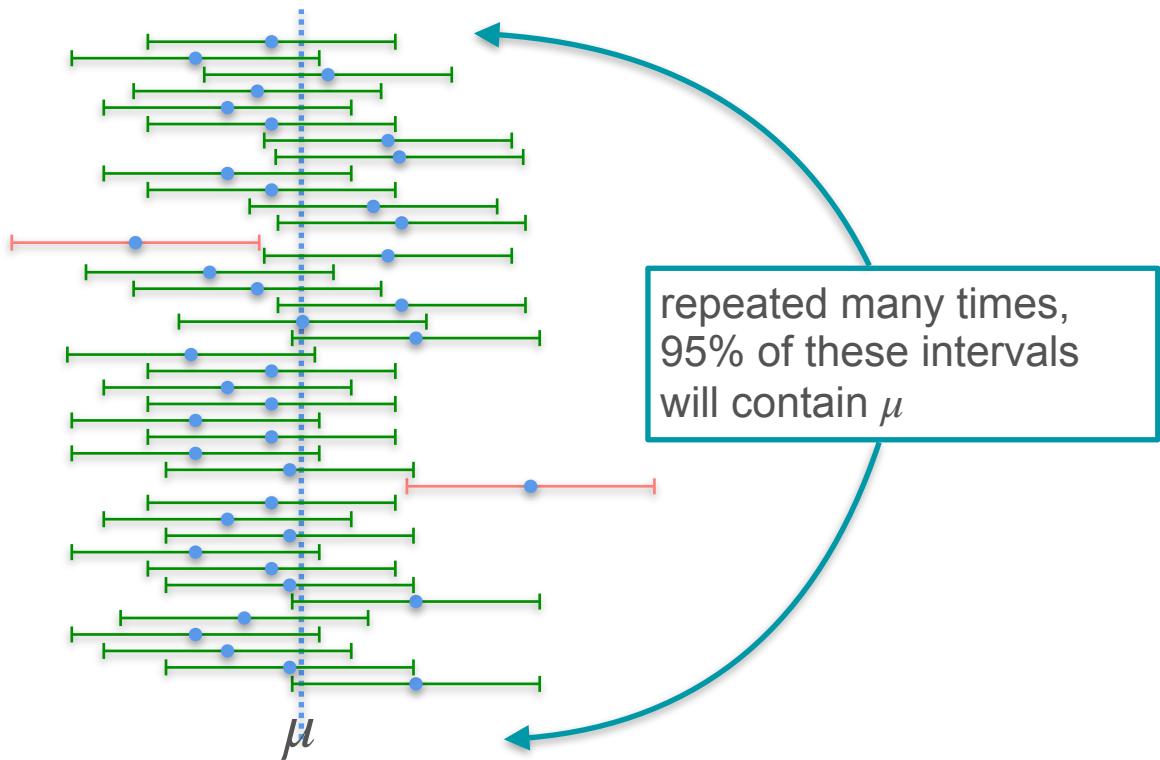
95%
Confidence
Level



Difference Between Confidence and Probability

95%
Confidence
Level

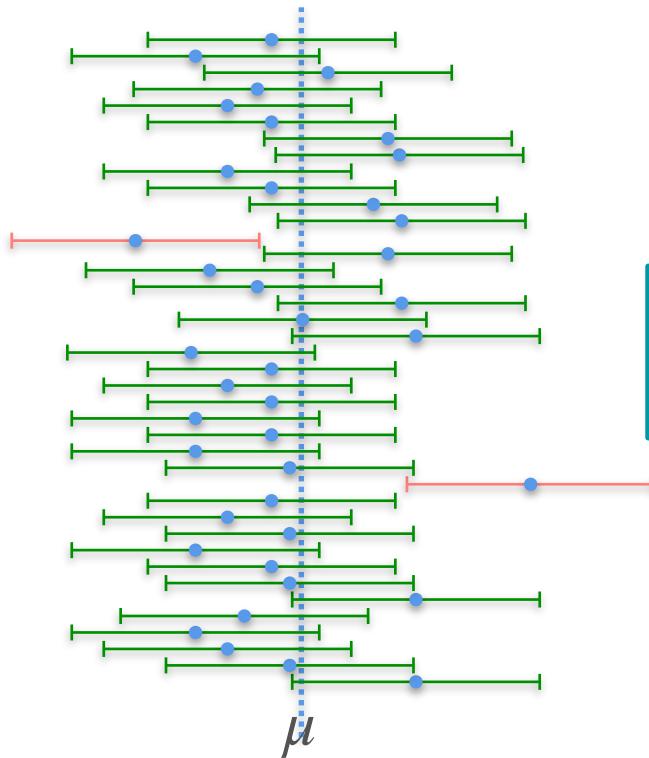
success rate for constructing
the confidence interval



Difference Between Confidence and Probability

95%
Confidence
Level

success rate for constructing
the confidence interval

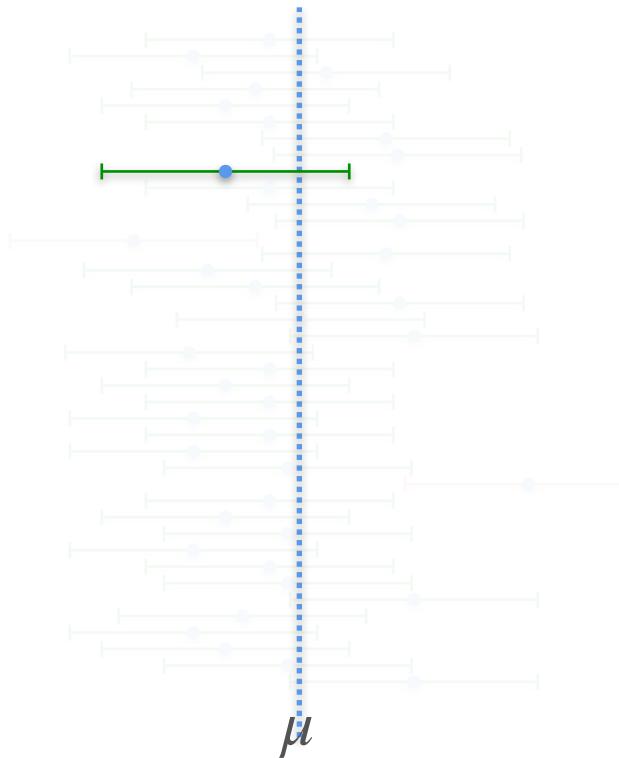


repeated many times,
95% of these intervals
will contain μ

Difference Between Confidence and Probability

95%
Confidence
Level

success rate for constructing
the confidence interval

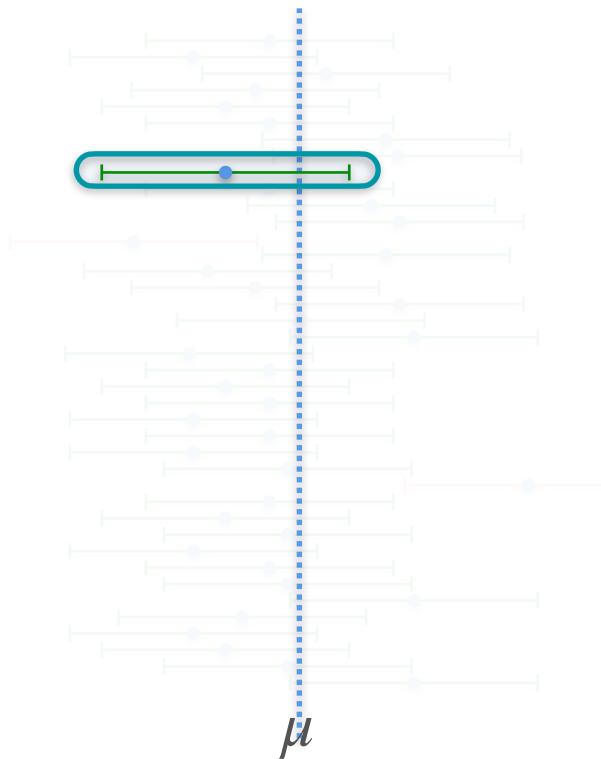


repeated many times,
95% of these intervals
will contain μ

Difference Between Confidence and Probability

95%
Confidence
Level

success rate for constructing
the confidence interval



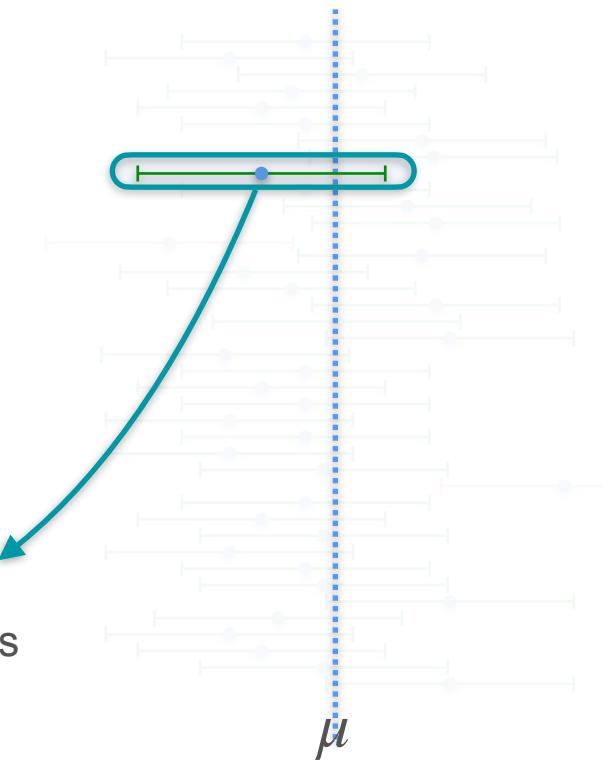
repeated many times,
95% of these intervals
will contain μ

Difference Between Confidence and Probability

95%
Confidence
Level

success rate for constructing
the confidence interval

not the probability that
one specific intervals contains
the population mean



repeated many times,
95% of these intervals
will contain μ



DeepLearning.AI

Confidence Interval

**Confidence Interval
(Unknown Standard Deviation)**

Confidence Interval - t Distribution



Confidence Interval - t Distribution

$$\bar{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$



Confidence Interval - t Distribution

known σ

$$\bar{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$



Confidence Interval - t Distribution

known σ

$$\bar{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

$$\frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$



Confidence Interval - t Distribution

known σ

$$\bar{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

$$\frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

normal
distribution



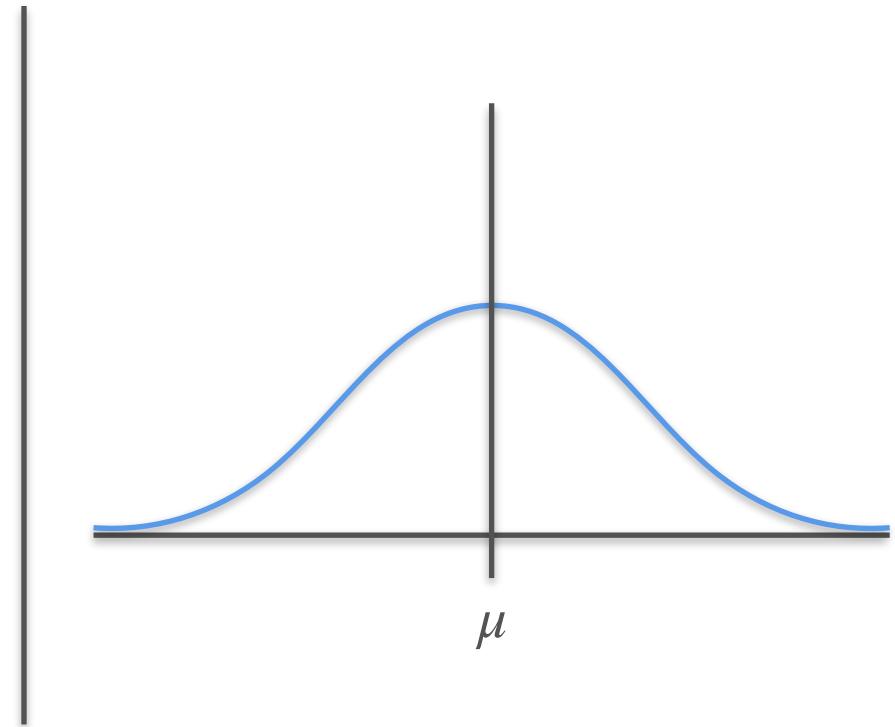
Confidence Interval - t Distribution

known σ

$$\bar{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

$$\frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

normal
distribution



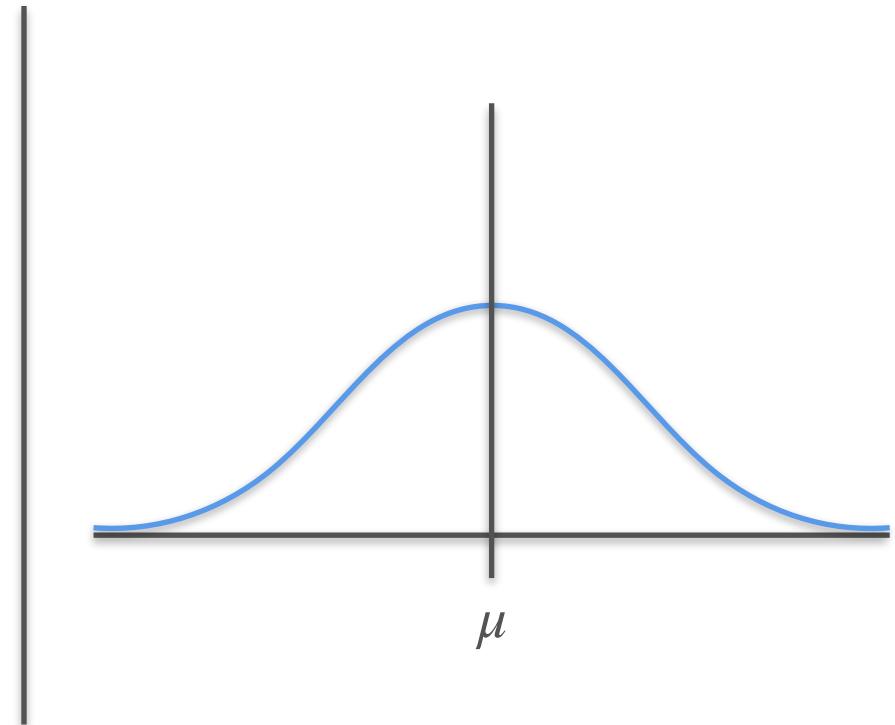
Confidence Interval - t Distribution

known σ

$$\bar{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

$$\frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

normal
distribution



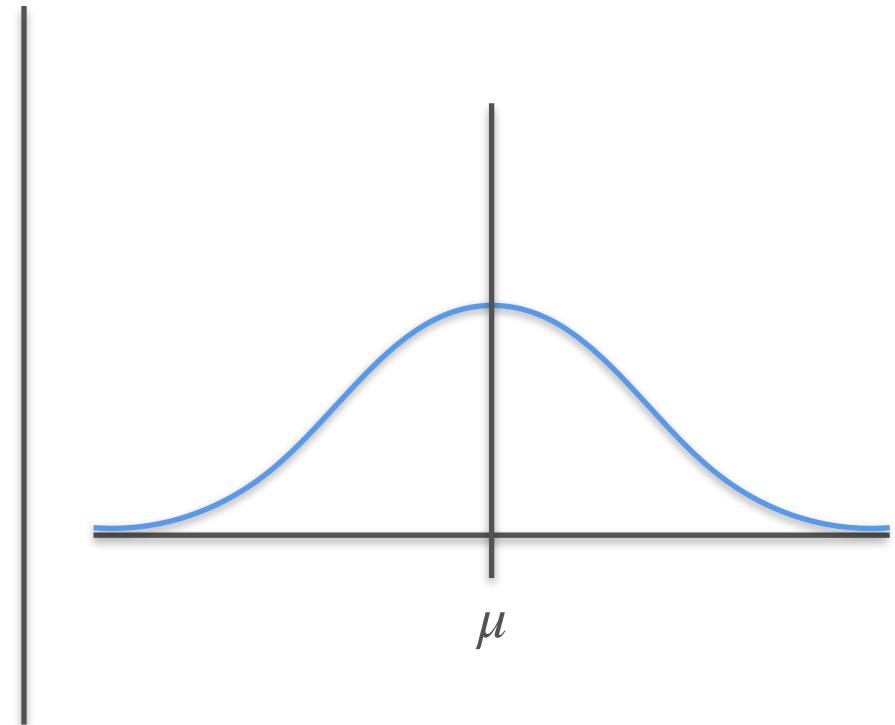
Confidence Interval - t Distribution

known σ

$$\bar{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

$$\frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

normal
distribution



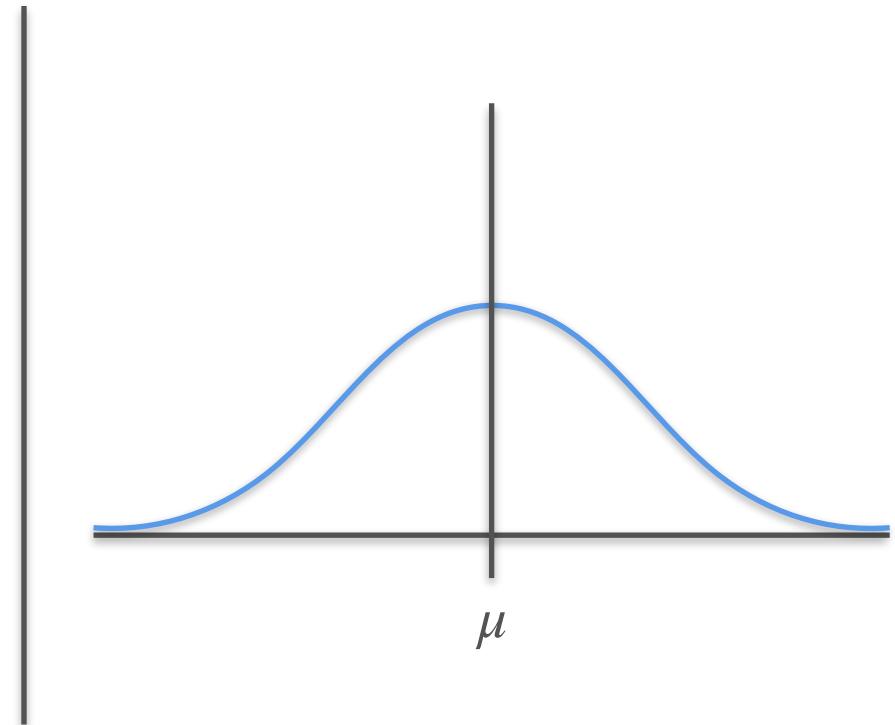
Confidence Interval - t Distribution

known σ ?

$$\bar{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

$$\frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

normal
distribution



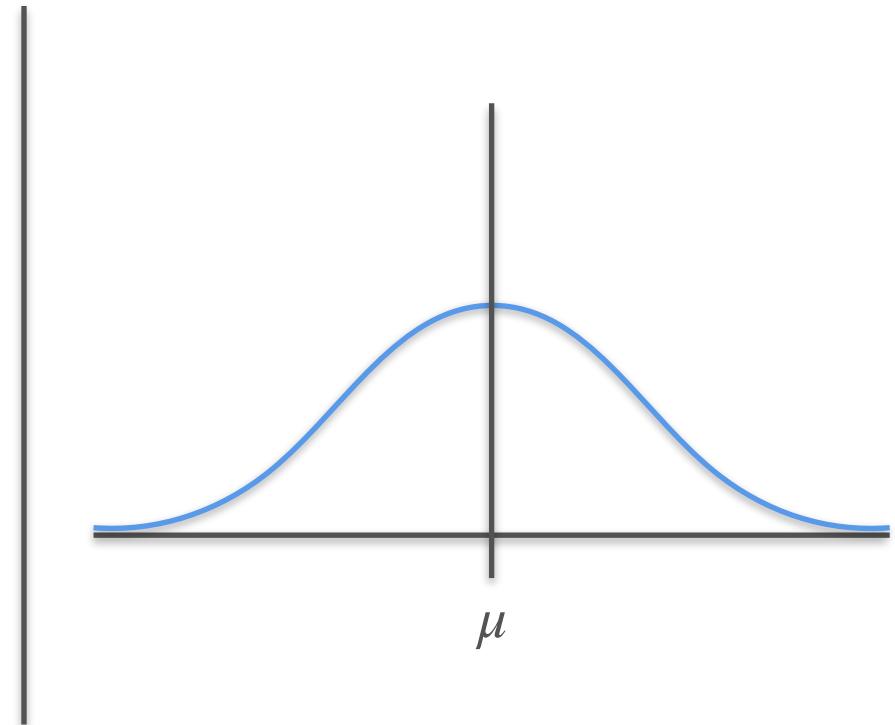
Confidence Interval - t Distribution

known σ ? $\rightarrow s$

$$\bar{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

$$\frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

normal
distribution



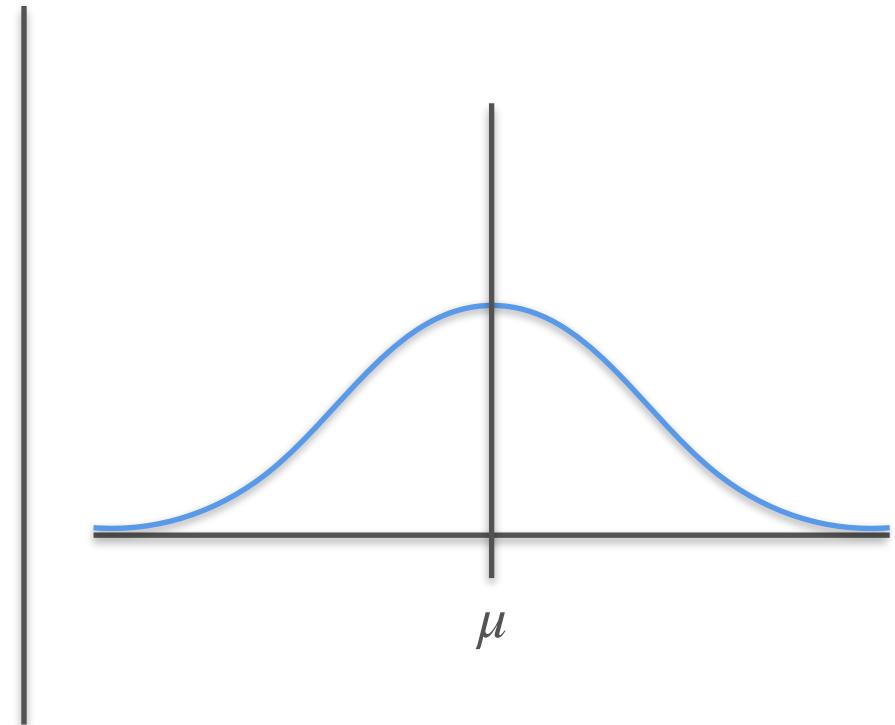
Confidence Interval - t Distribution

known σ ? $\rightarrow s$

$$\bar{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

$$\frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} \xrightarrow{\text{dotted arrow}} \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$$

normal
distribution



Confidence Interval - t Distribution

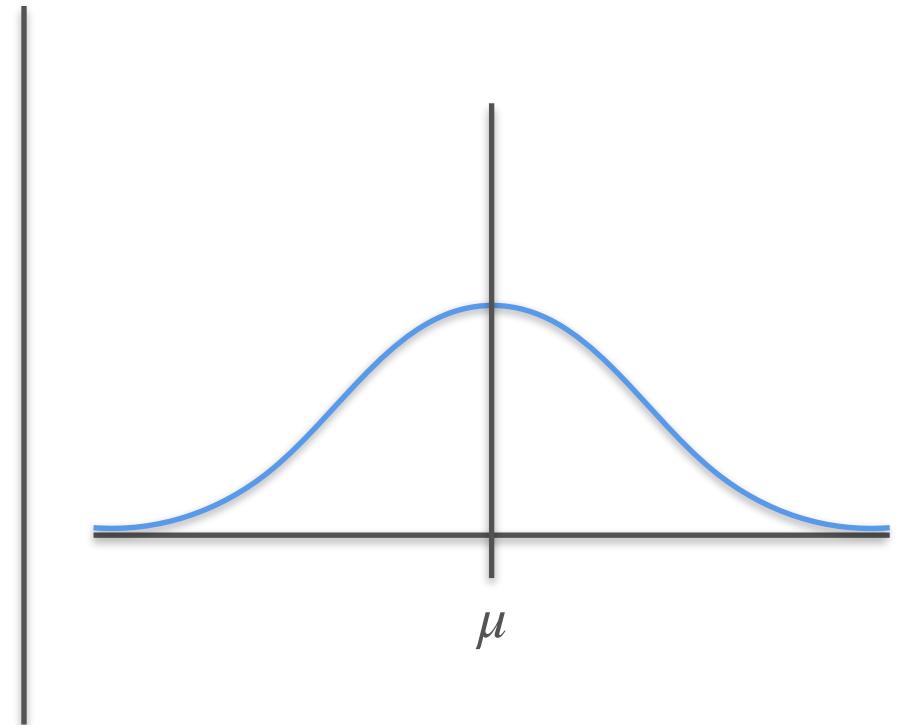
known σ ? $\rightarrow s$

$$\bar{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

$$\frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} \xrightarrow{\text{dotted arrow}} \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$$

normal
distribution

not a normal
distribution



Confidence Interval - t Distribution

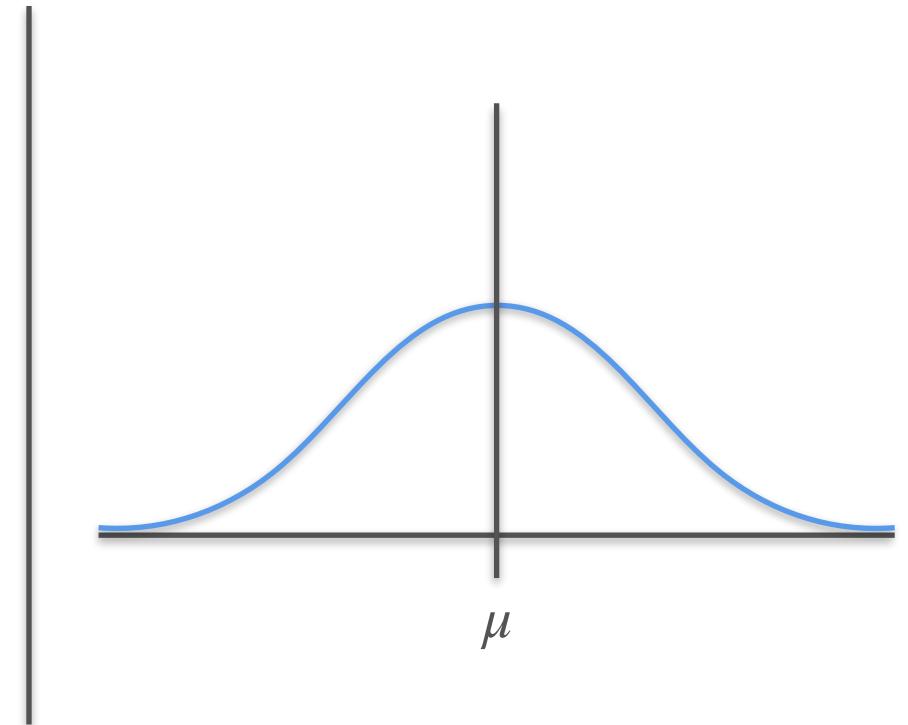
known σ ? $\rightarrow s$

$$\bar{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

$$\frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} \xrightarrow{\text{dotted arrow}} \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$$

normal
distribution

not a normal
distribution
**student's t
distribution**



Confidence Interval - t Distribution

known σ ? $\rightarrow s$

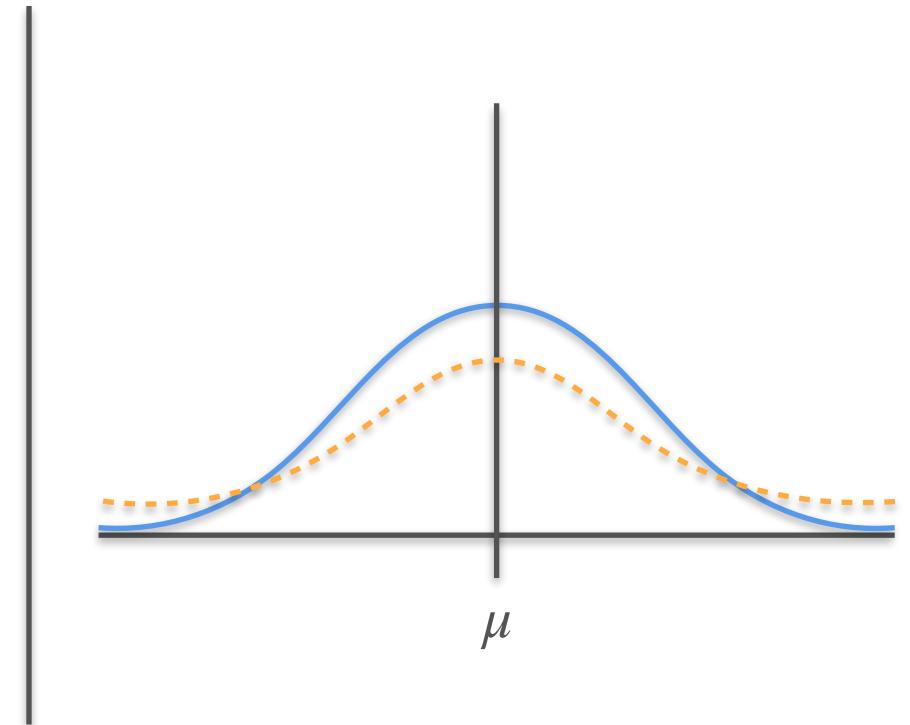
$$\bar{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

$$\frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} \xrightarrow{\text{dotted arrow}} \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$$

normal distribution

not a normal distribution

student's t distribution



Confidence Interval - t Distribution

known σ ? $\rightarrow s$

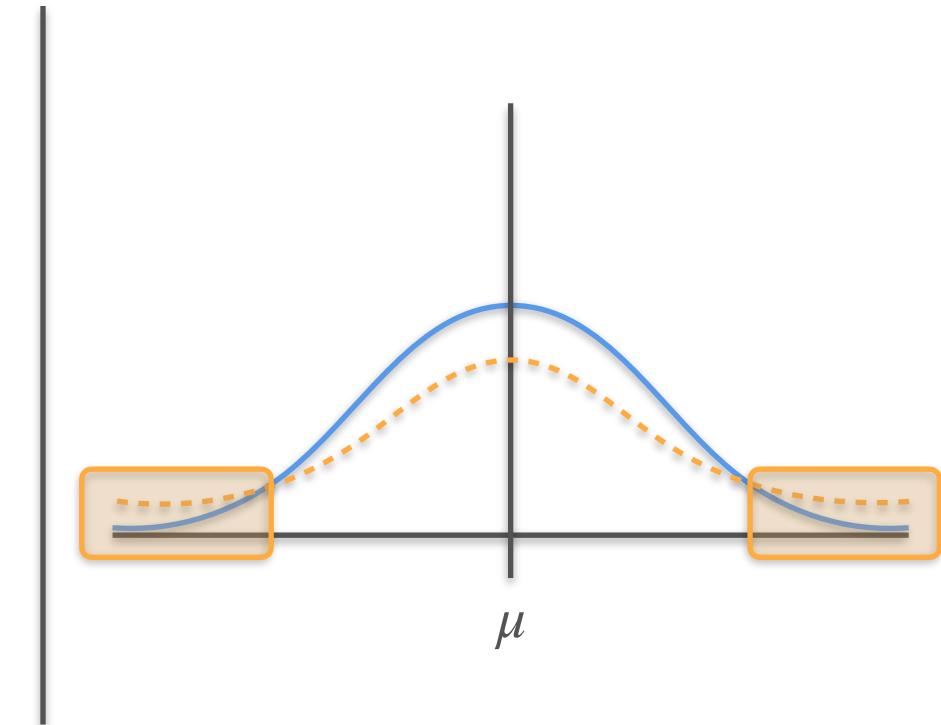
$$\bar{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

$$\frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} \xrightarrow{\text{dotted arrow}} \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$$

normal distribution

not a normal distribution

student's t distribution



Confidence Interval - t Distribution

known σ

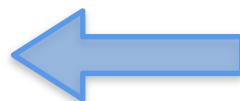
$$\frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

normal
distribution

$$\bar{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

Confidence Interval - t Distribution

known σ



$$\frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

normal
distribution

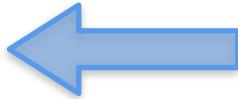
$$\bar{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$



Confidence Interval - t Distribution

known σ

$$\frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$



normal
distribution

$$\bar{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

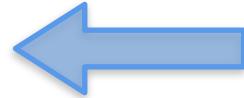


Confidence Interval - t Distribution

known σ

$$\frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

normal
distribution



$$\bar{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

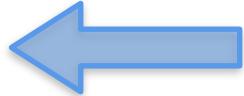
Confidence Interval - t Distribution

known σ

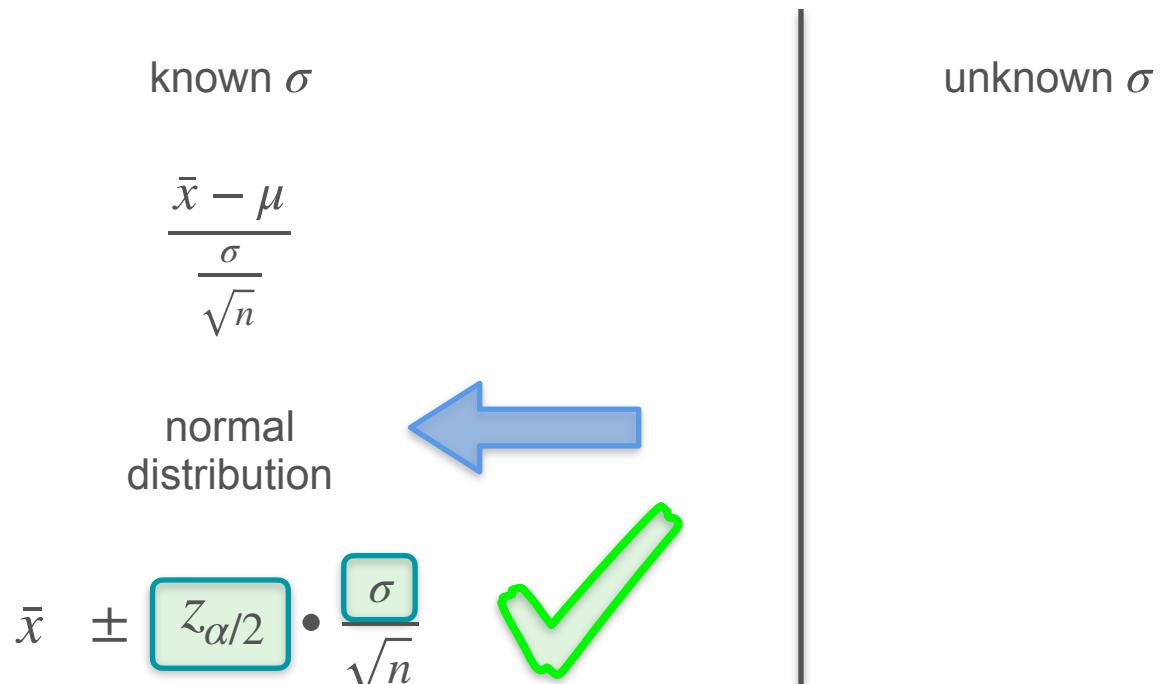
$$\frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

normal
distribution

$$\bar{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$



Confidence Interval - t Distribution



Confidence Interval - t Distribution

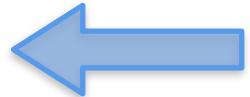
known σ

$$\frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

normal distribution

$$\bar{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

unknown σ  replace with s



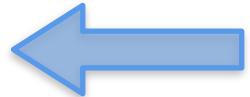
Confidence Interval - t Distribution

known σ

$$\frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

normal distribution

$$\bar{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$



unknown σ  replace with s

$$\frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$



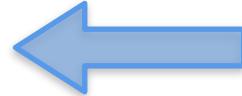
Confidence Interval - t Distribution

known σ

$$\frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

normal distribution

$$\bar{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$



unknown σ replace with s

$$\frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$$

Confidence Interval - t Distribution

known σ

$$\frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

normal distribution

$$\bar{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$



unknown σ  replace with s

$$\frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} \quad \xrightarrow{\text{dotted arrow}} \quad \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$$

student's t distribution



Confidence Interval - t Distribution

known σ

$$\frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

normal distribution

$$\bar{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$



unknown σ replace with s

$$\frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} \xrightarrow{\text{dotted arrow}} \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$$

student's t distribution

$$\bar{x} \pm z_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$$

Confidence Interval - t Distribution

known σ

$$\frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

normal distribution

$$\bar{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$



unknown σ replace with s

$$\frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$$

student's t distribution

$$\bar{x} \pm z_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$$

Confidence Interval - t Distribution

known σ

$$\frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

normal distribution

$$\bar{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$



unknown σ replace with s

$$\frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$$

student's t distribution

$$\bar{x} \pm z_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$$



Confidence Interval - t Distribution

known σ

$$\bar{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$



unknown σ

$$\bar{x} \pm z_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$$



Confidence Interval - t Distribution

known σ

$$\bar{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$



unknown σ

$$\bar{x} \pm z_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$$



$$\bar{x} \pm t_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$$

Confidence Interval - t Distribution

known σ

$$\bar{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$



unknown σ

$$\bar{x} \pm z_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$$



$$\bar{x} \pm t_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$$



Confidence Interval - t Distribution

known σ

$$\bar{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

unknown σ

$$\bar{x} \pm t_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$$

Confidence Interval - t Distribution

known σ

$$\bar{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

unknown σ

$$\bar{x} \pm t_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$$

Confidence Interval - t Distribution

known σ

$$\bar{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

unknown σ

$$\bar{x} \pm t_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$$

Confidence Interval - t Distribution

known σ

$$\bar{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

unknown σ

$$\bar{x} \pm t_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$$

Confidence Interval - t Distribution

known σ

$$\bar{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

unknown σ

$$\bar{x} \pm t_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$$

Confidence Interval - t Distribution

unknown σ

$$\bar{x} \pm t_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$$



Confidence Interval - t Distribution

unknown σ

$$\bar{x} \pm t_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$$

Confidence Interval - t Distribution

unknown σ

$$\bar{x} \pm t_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$$

degrees of freedom

Confidence Interval - t Distribution

unknown σ

$$\bar{x} \pm t_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$$

degrees of freedom

$$n - 1$$

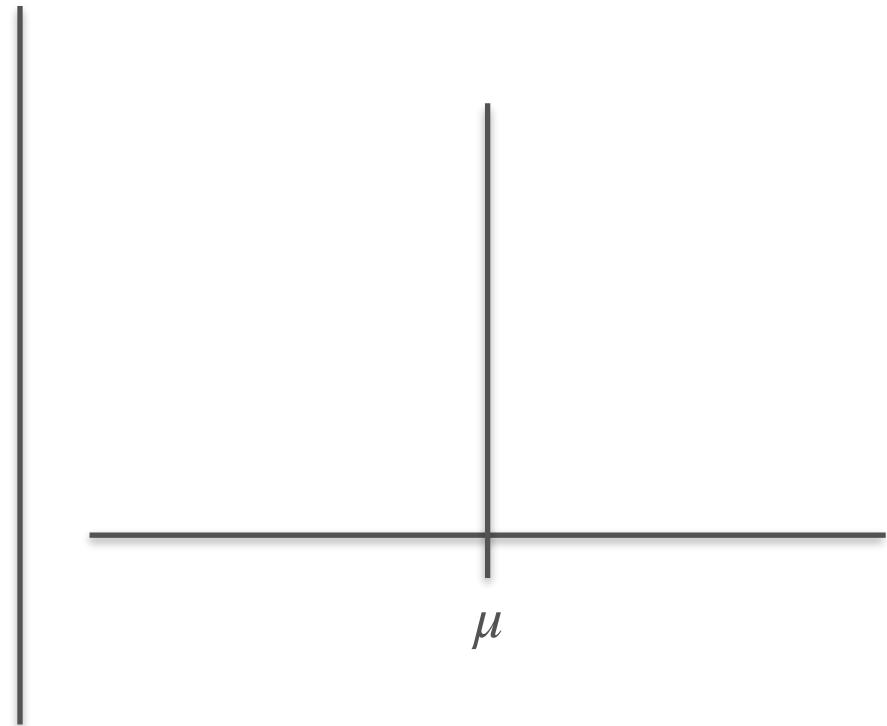
Confidence Interval - t Distribution

unknown σ

$$\bar{x} \pm t_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$$

degrees of freedom

$$n - 1$$



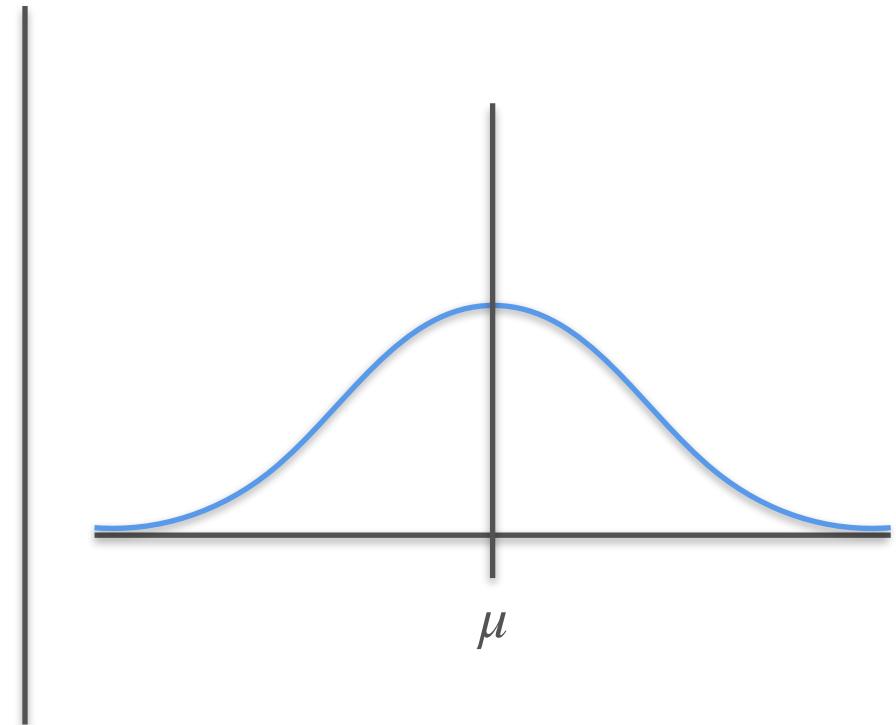
Confidence Interval - t Distribution

unknown σ

$$\bar{x} \pm t_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$$

degrees of freedom

$$n - 1$$



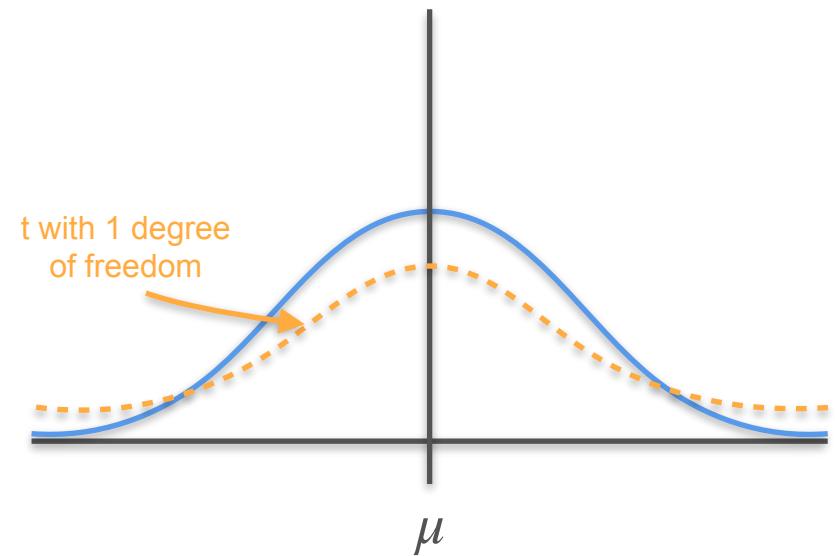
Confidence Interval - t Distribution

unknown σ

$$\bar{x} \pm t_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$$

degrees of freedom

$$n - 1$$



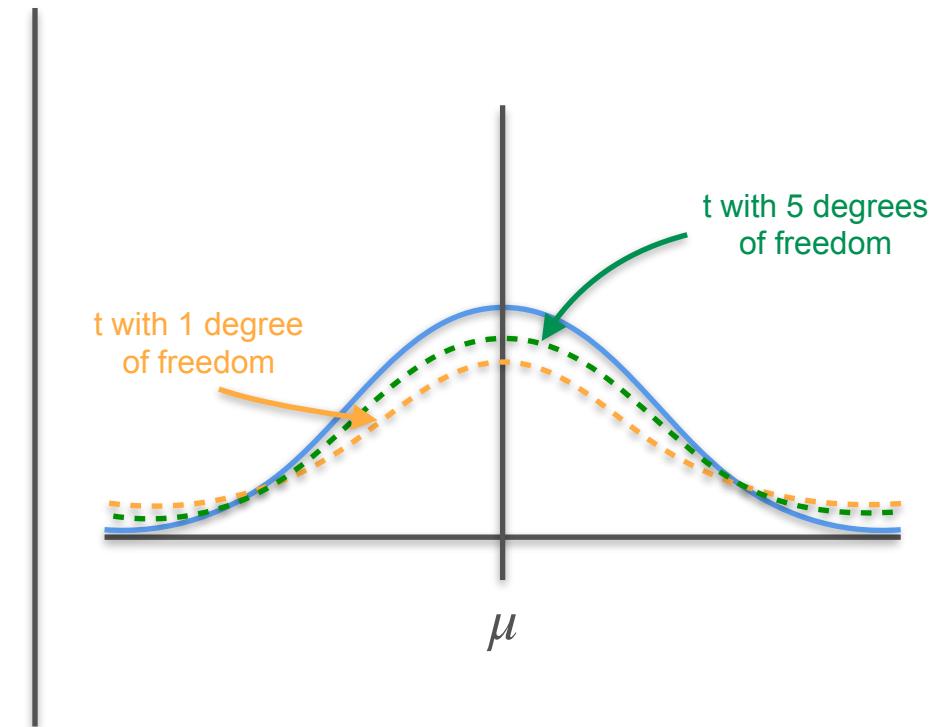
Confidence Interval - t Distribution

unknown σ

$$\bar{x} \pm t_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$$

degrees of freedom

$$n - 1$$



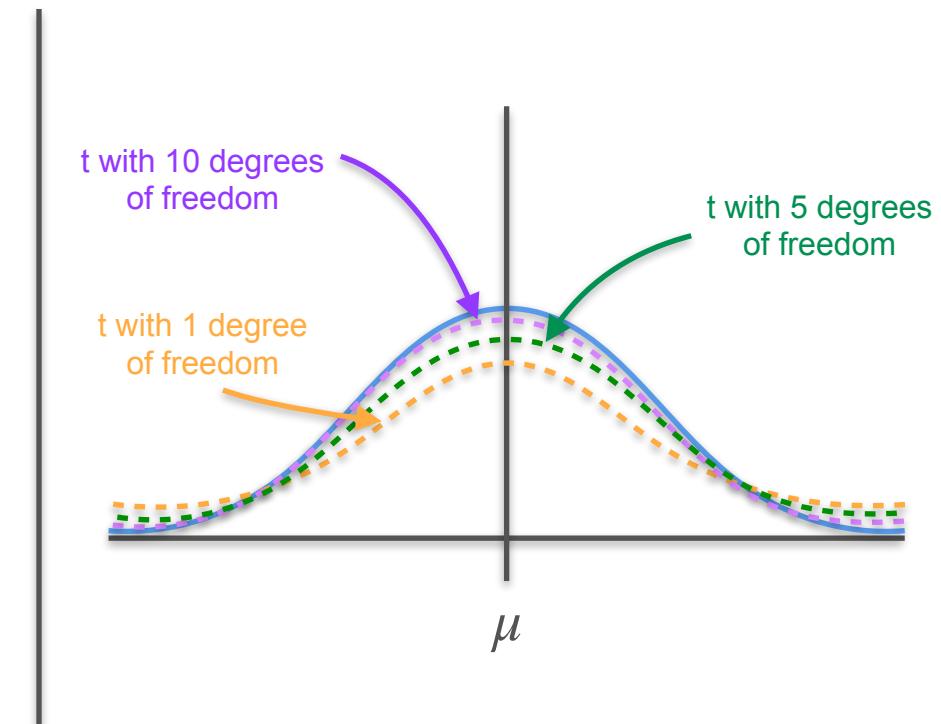
Confidence Interval - t Distribution

unknown σ

$$\bar{x} \pm t_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$$

degrees of freedom

$$n - 1$$





DeepLearning.AI

Confidence Interval

Confidence Intervals for Proportion

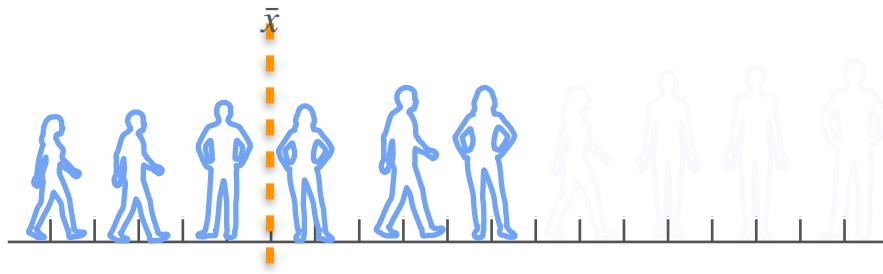
Confidence Interval for Proportions

Confidence Interval for Proportions

Confidence Interval for Means

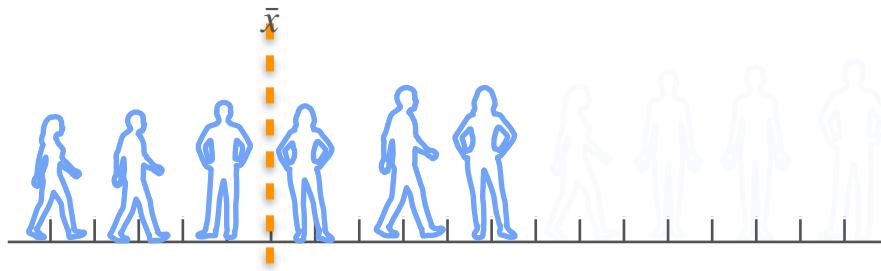
Confidence Interval for Proportions

Confidence Interval for Means



Confidence Interval for Proportions

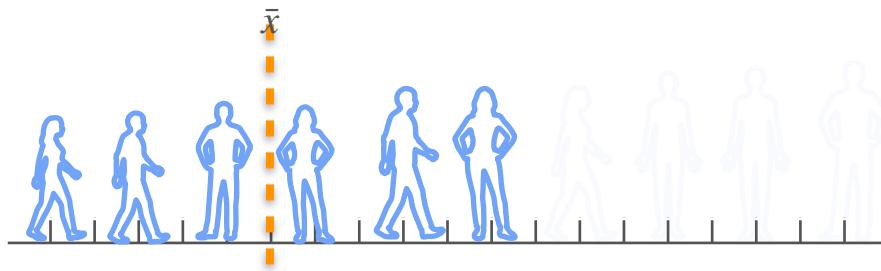
Confidence Interval for Means



$$\text{confidence interval} = \bar{x} \pm \text{margin of error}$$

Confidence Interval for Proportions

Confidence Interval for Means

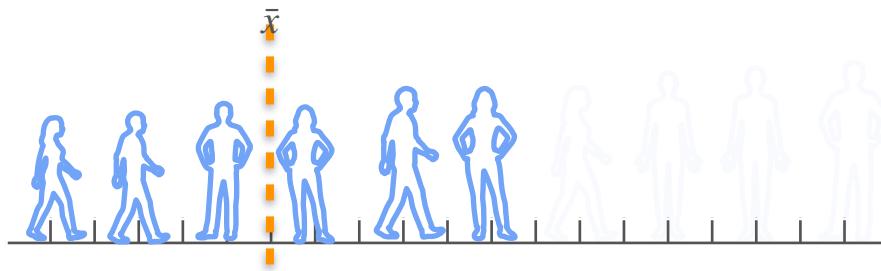


confidence interval = $\bar{x} \pm$ margin of error

margin of error =

Confidence Interval for Proportions

Confidence Interval for Means

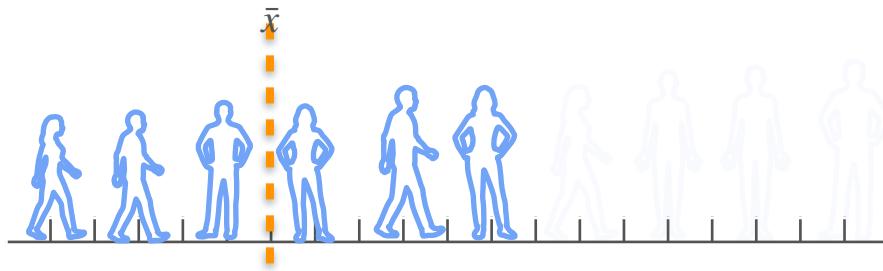


confidence interval = $\bar{x} \pm$ margin of error

$$\text{margin of error} = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

Confidence Interval for Proportions

Confidence Interval for Means

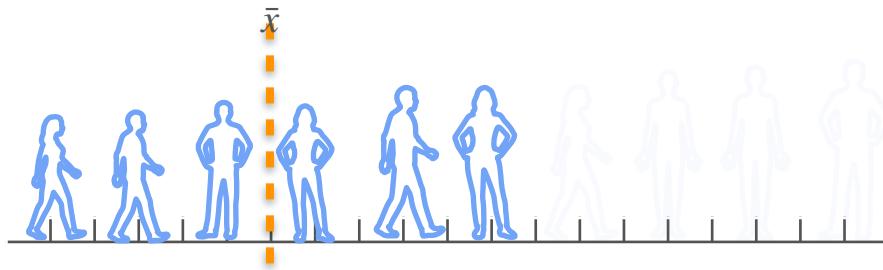


$$\text{confidence interval} = \bar{x} \pm \text{margin of error}$$

$$\text{margin of error} = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

Confidence Interval for Proportions

Confidence Interval for Means



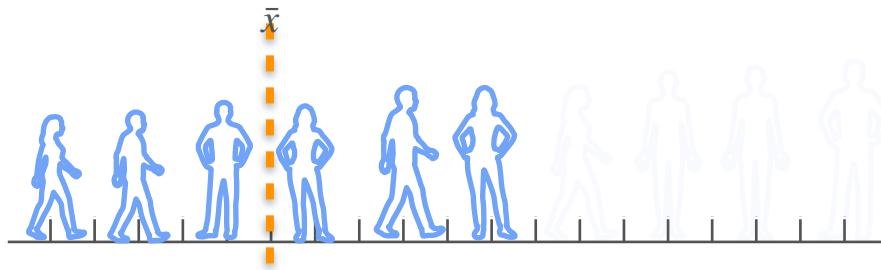
$$\text{confidence interval} = \bar{x} \pm \text{margin of error}$$

Confidence Interval for Proportions

$$\text{margin of error} = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

Confidence Interval for Proportions

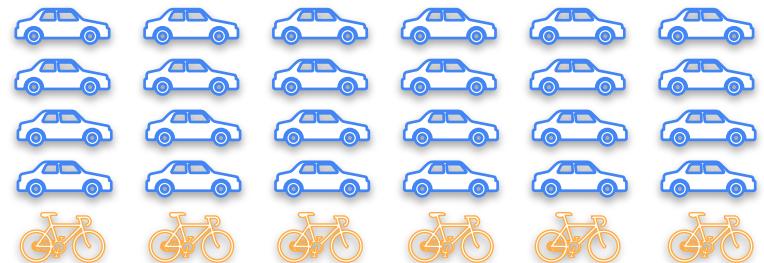
Confidence Interval for Means



$$\text{confidence interval} = \bar{x} \pm \text{margin of error}$$

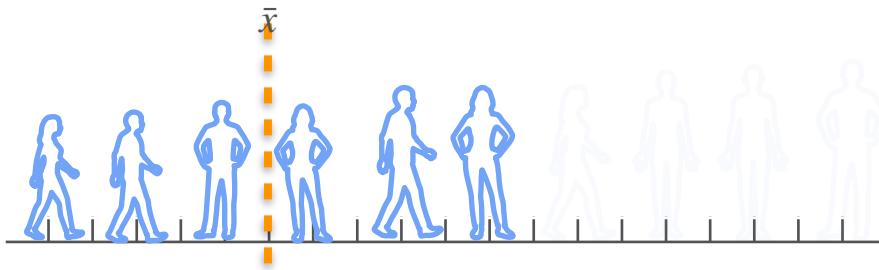
$$\text{margin of error} = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

Confidence Interval for Proportions



Confidence Interval for Proportions

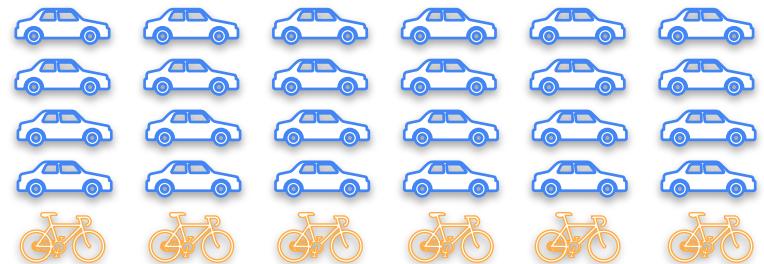
Confidence Interval for Means



$$\text{confidence interval} = \bar{x} \pm \text{margin of error}$$

$$\text{margin of error} = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

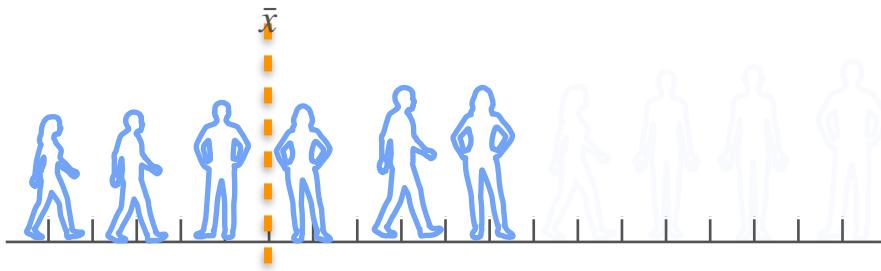
Confidence Interval for Proportions



$$n = 30$$

Confidence Interval for Proportions

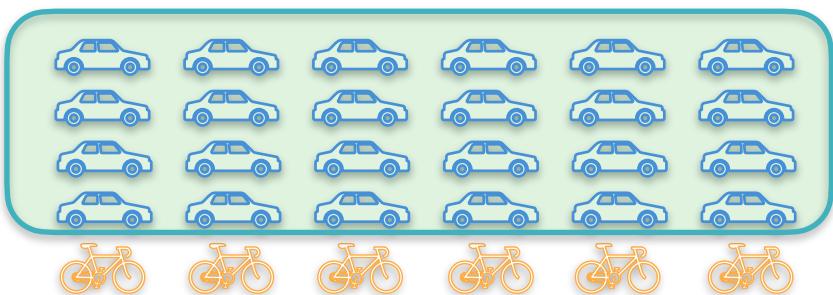
Confidence Interval for Means



$$\text{confidence interval} = \bar{x} \pm \text{margin of error}$$

$$\text{margin of error} = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

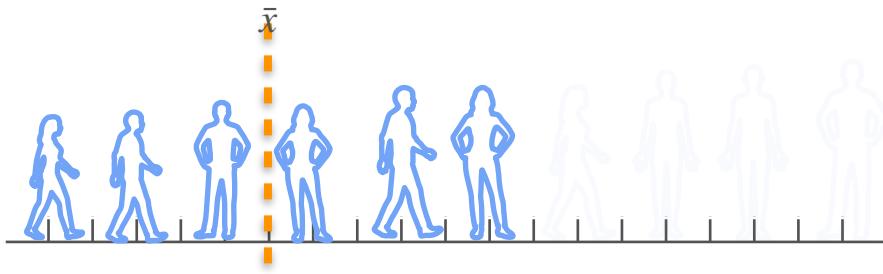
Confidence Interval for Proportions



$$n = 30$$

Confidence Interval for Proportions

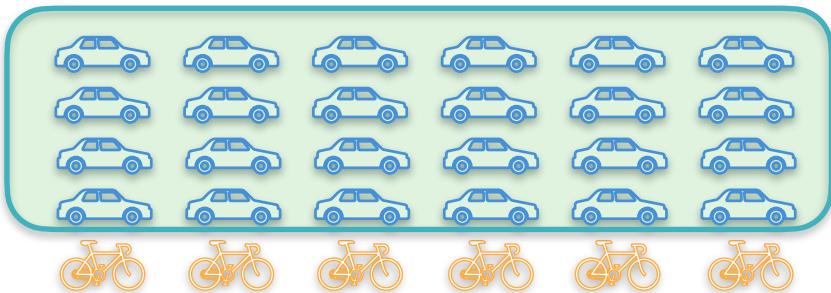
Confidence Interval for Means



$$\text{confidence interval} = \bar{x} \pm \text{margin of error}$$

$$\text{margin of error} = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

Confidence Interval for Proportions

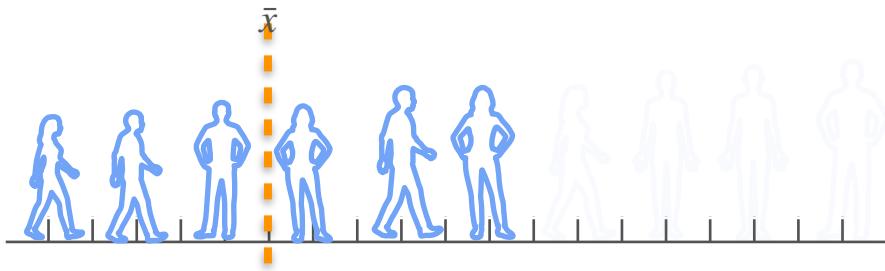


$$x = 24$$

$$n = 30$$

Confidence Interval for Proportions

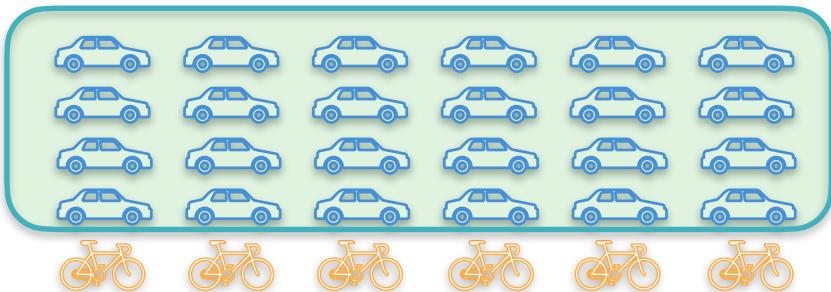
Confidence Interval for Means



$$\text{confidence interval} = \bar{x} \pm \text{margin of error}$$

$$\text{margin of error} = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

Confidence Interval for Proportions



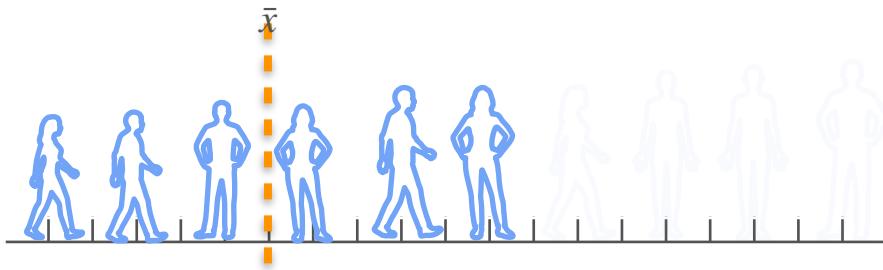
$$x = 24$$

$$n = 30$$

$$\hat{p} = \frac{x}{n} = \frac{24}{30} = 80\%$$

Confidence Interval for Proportions

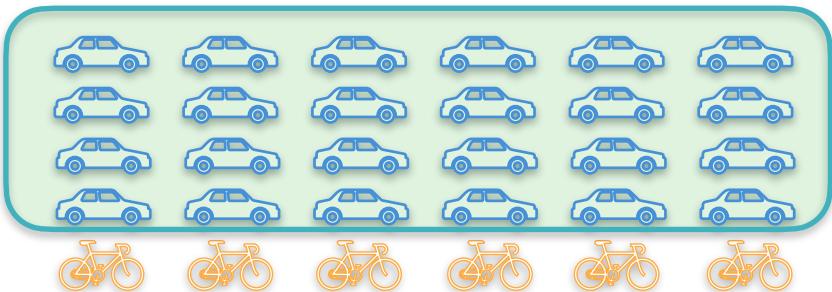
Confidence Interval for Means



$$\text{confidence interval} = \bar{x} \pm \text{margin of error}$$

$$\text{margin of error} = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

Confidence Interval for Proportions



$$x = 24$$

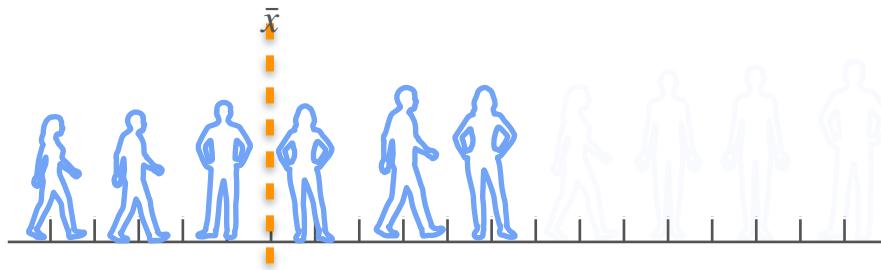
$$n = 30$$

$$\hat{p} = \frac{x}{n} = \frac{24}{30} = 80\%$$

How do you calculate a 95% confidence interval for this sample proportion?

Confidence Interval for Proportions

Confidence Interval for Means



confidence interval = $\bar{x} \pm$ margin of error

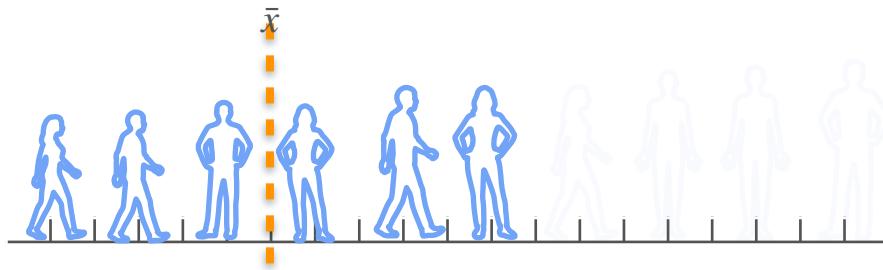
$$\text{margin of error} = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

Confidence Interval for Proportions

$$\hat{p} = \frac{x}{n} = \frac{24}{30} = 80\%$$

Confidence Interval for Proportions

Confidence Interval for Means



confidence interval = $\bar{x} \pm$ margin of error

$$\text{margin of error} = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

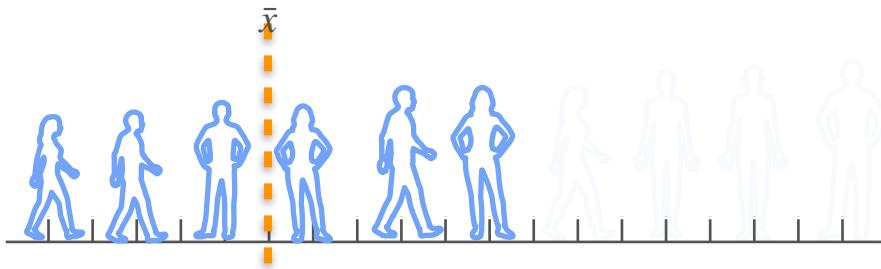
Confidence Interval for Proportions

$$\hat{p} = \frac{x}{n} = \frac{24}{30} = 80\%$$

confidence interval =

Confidence Interval for Proportions

Confidence Interval for Means



$$\text{confidence interval} = \bar{x} \pm \text{margin of error}$$

$$\text{margin of error} = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

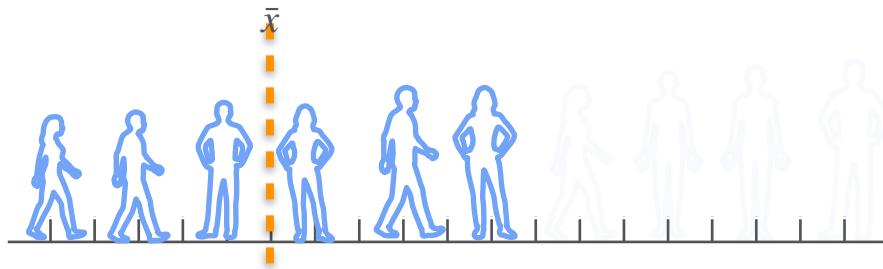
Confidence Interval for Proportions

$$\hat{p} = \frac{x}{n} = \frac{24}{30} = 80\%$$

$$\text{confidence interval} = \hat{p}$$

Confidence Interval for Proportions

Confidence Interval for Means



$$\text{confidence interval} = \bar{x} \pm \text{margin of error}$$

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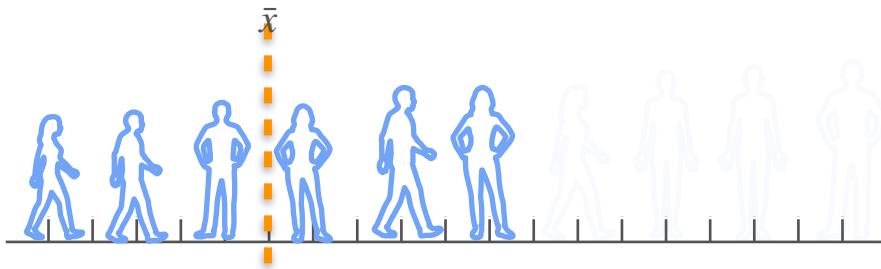
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Confidence Interval for Proportions

Confidence Interval for Means



$$\text{confidence interval} = \bar{x} \pm \text{margin of error}$$

$$\text{margin of error} = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

Confidence Interval for Proportions

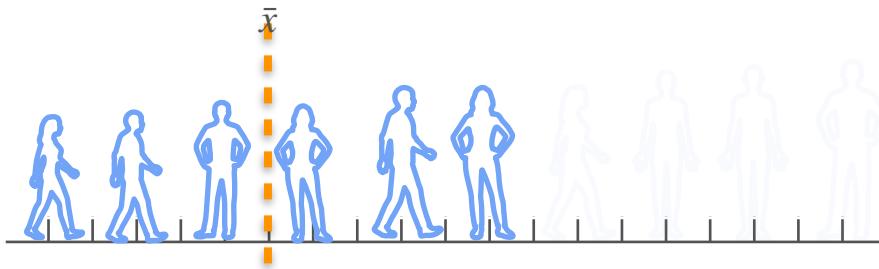
$$\hat{p} = \frac{x}{n} = \frac{24}{30} = 80\%$$

$$\text{confidence interval} = \hat{p} \pm \text{margin of error}$$

$$\text{margin of error} =$$

Confidence Interval for Proportions

Confidence Interval for Means



$$\text{confidence interval} = \bar{x} \pm \text{margin of error}$$

$$\text{margin of error} = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

Confidence Interval for Proportions

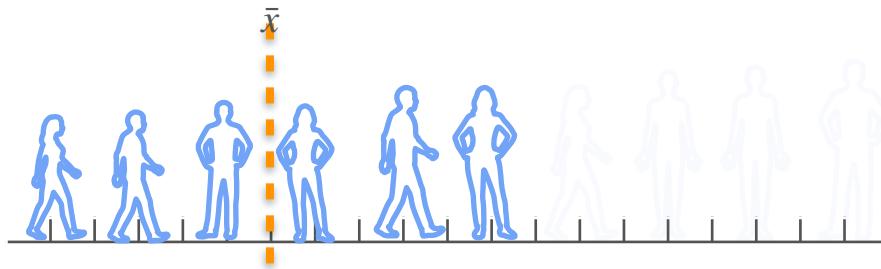
$$\hat{p} = \frac{x}{n} = \frac{24}{30} = 80\%$$

$$\text{confidence interval} = \hat{p} \pm \text{margin of error}$$

$$\text{margin of error} = z_{\alpha/2} \cdot$$

Confidence Interval for Proportions

Confidence Interval for Means



$$\text{confidence interval} = \bar{x} \pm \text{margin of error}$$

$$\text{margin of error} = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

Confidence Interval for Proportions

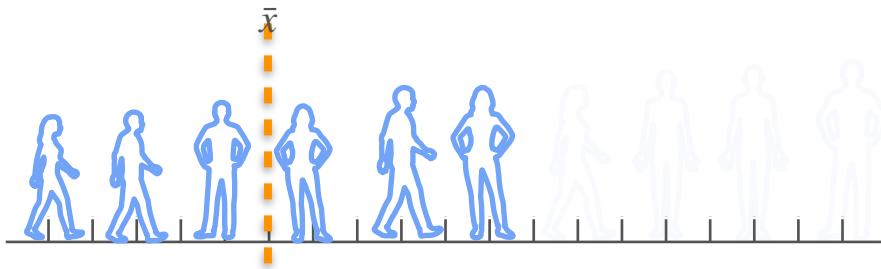
$$\hat{p} = \frac{x}{n} = \frac{24}{30} = 80\%$$

$$\text{confidence interval} = \hat{p} \pm \text{margin of error}$$

$$\text{margin of error} = z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}}$$

Confidence Interval for Proportions

Confidence Interval for Means



confidence interval = $\bar{x} \pm$ margin of error

$$\text{margin of error} = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

Confidence Interval for Proportions

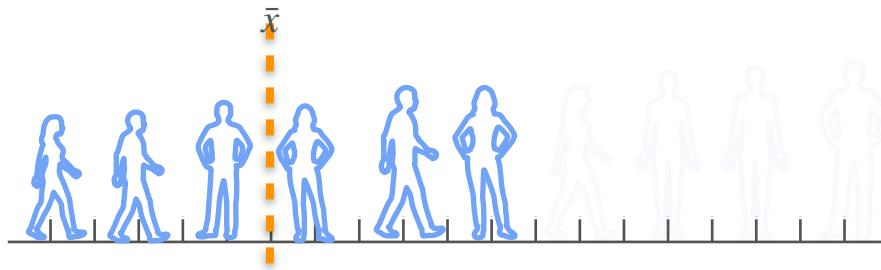
$$\hat{p} = \frac{x}{n} = \frac{24}{30} = 80\%$$

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$$\text{margin of error} = z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}}$$

Confidence Interval for Proportions

Confidence Interval for Means



$$\text{confidence interval} = \bar{x} \pm \text{margin of error}$$

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Confidence Interval for Proportions

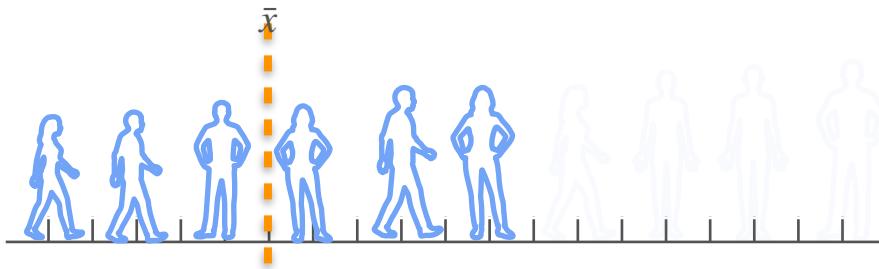
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Confidence Interval for Proportions

Confidence Interval for Means



$$\text{confidence interval} = \bar{x} \pm \text{margin of error}$$

$$\text{margin of error} = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

Confidence Interval for Proportions

$$\hat{p} = \frac{x}{n} = \frac{24}{30} = 80\%$$

$$\text{confidence interval} = \hat{p} \pm \text{margin of error}$$

$$\text{margin of error} = z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}}$$

standard error

Confidence Interval for Proportions

Confidence Interval for Proportions

Confidence Interval for Proportions

Confidence Interval for Proportions

confidence interval =

Confidence Interval for Proportions

Confidence Interval for Proportions

confidence interval = \hat{p}

Confidence Interval for Proportions

Confidence Interval for Proportions

confidence interval = $\hat{p} \pm$ margin of error

Confidence Interval for Proportions

Confidence Interval for Proportions

confidence interval = $\hat{p} \pm$ margin of error

margin of error =

Confidence Interval for Proportions

Confidence Interval for Proportions

confidence interval = $\hat{p} \pm$ margin of error

margin of error = $z_{\alpha/2} \cdot$

Confidence Interval for Proportions

Confidence Interval for Proportions

confidence interval = $\hat{p} \pm$ margin of error

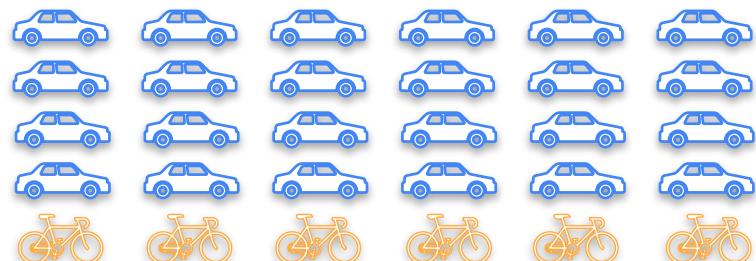
margin of error = $z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}}$

Confidence Interval for Proportions

Confidence Interval for Proportions

confidence interval = $\hat{p} \pm$ margin of error

$$\text{margin of error} = z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}}$$

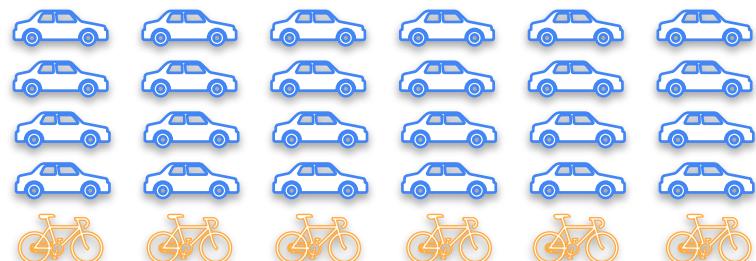


Confidence Interval for Proportions

Confidence Interval for Proportions

confidence interval = $\hat{p} \pm$ margin of error

$$\text{margin of error} = z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}}$$



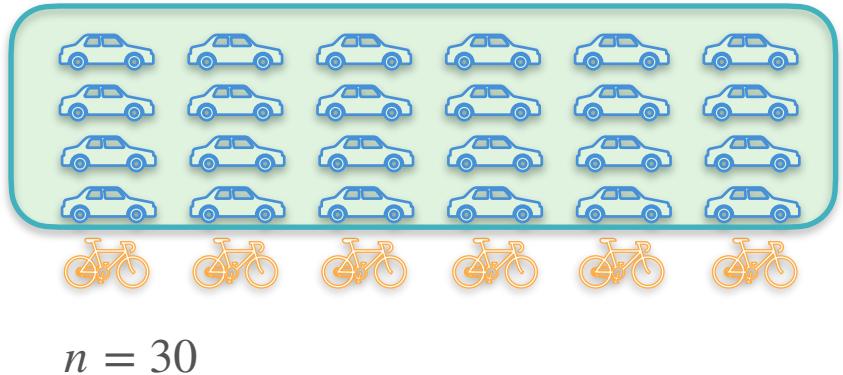
$$n = 30$$

Confidence Interval for Proportions

Confidence Interval for Proportions

$$\text{confidence interval} = \hat{p} \pm \text{margin of error}$$

$$\text{margin of error} = z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}}$$

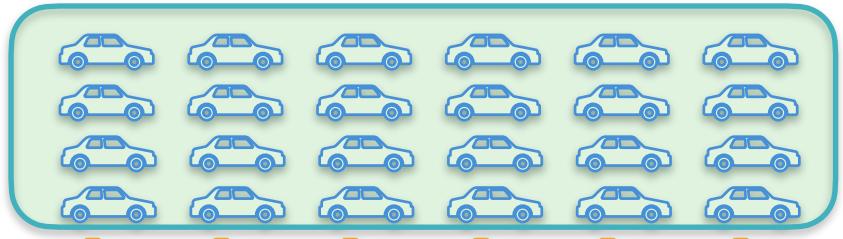


Confidence Interval for Proportions

Confidence Interval for Proportions

$$\text{confidence interval} = \hat{p} \pm \text{margin of error}$$

$$\text{margin of error} = z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}}$$



$$n = 30$$

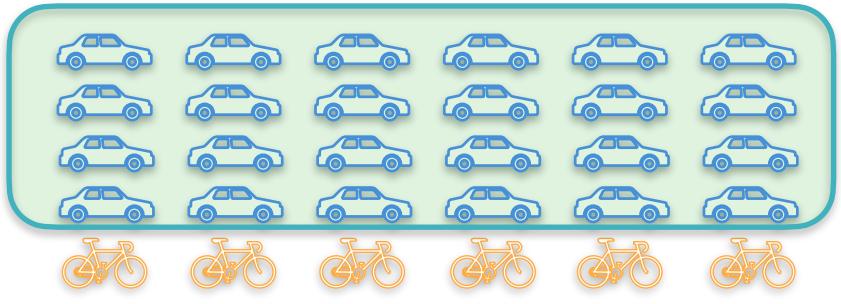
$$\hat{p} = 80\% = 0.8$$

Confidence Interval for Proportions

Confidence Interval for Proportions

confidence interval = $\hat{p} \pm$ margin of error

$$\text{margin of error} = z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}}$$



$$n = 30 \quad \hat{p} = 80\% = 0.8$$

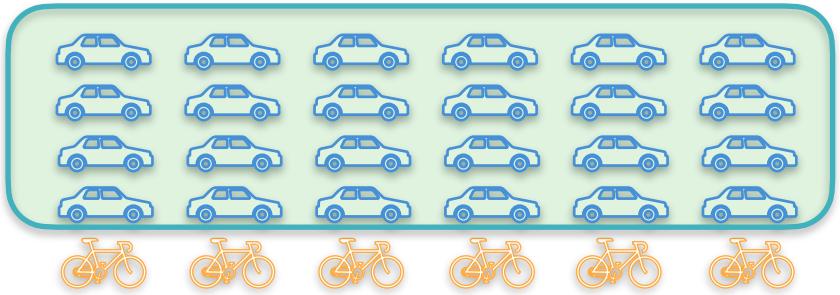
Calculate a 95% confidence interval for this sample proportion

Confidence Interval for Proportions

Confidence Interval for Proportions

confidence interval = $\hat{p} \pm$ margin of error

$$\text{margin of error} = z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}}$$



$$n = 30 \quad \hat{p} = 80 \% = 0.8$$

Calculate a 95% confidence interval for this sample proportion

95%

$$\rightarrow z_{\alpha/2} = 1.96$$

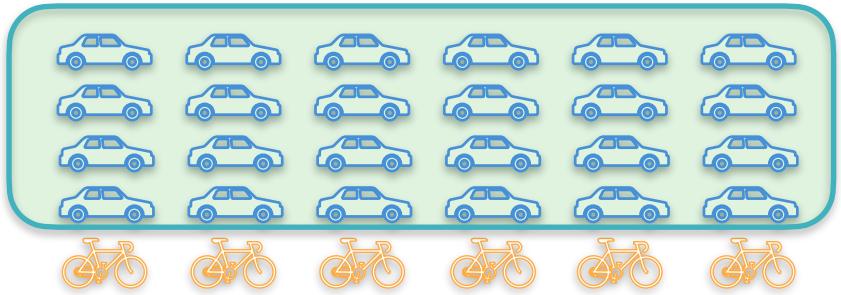
Confidence Interval for Proportions

Confidence Interval for Proportions

confidence interval = $\hat{p} \pm$ margin of error

$$\text{margin of error} = z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}}$$

margin of error =



$$n = 30 \quad \hat{p} = 80\% = 0.8$$

Calculate a 95% confidence interval for this sample proportion

95%

$$\rightarrow z_{\alpha/2} = 1.96$$

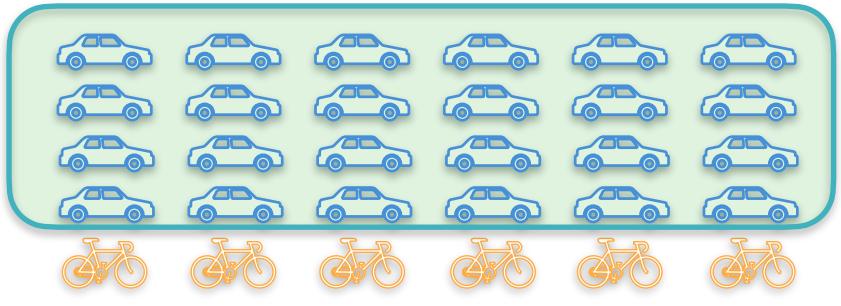
Confidence Interval for Proportions

Confidence Interval for Proportions

confidence interval = $\hat{p} \pm$ margin of error

$$\text{margin of error} = z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}}$$

margin of error = 1.96



$$n = 30 \quad \hat{p} = 80 \% = 0.8$$

Calculate a 95% confidence interval for this sample proportion

$$95\% \rightarrow z_{\alpha/2} = 1.96$$

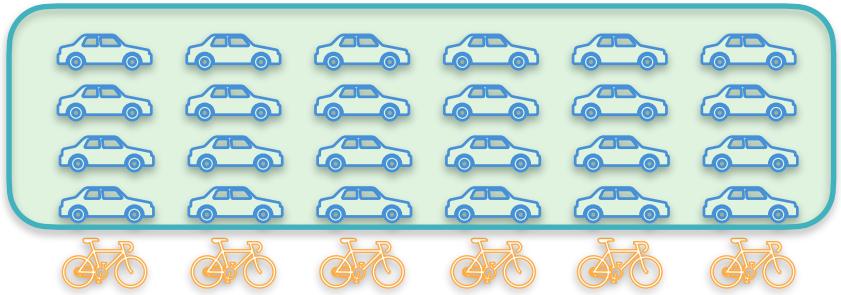
Confidence Interval for Proportions

Confidence Interval for Proportions

$$\text{confidence interval} = \hat{p} \pm \text{margin of error}$$

$$\text{margin of error} = z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}}$$

$$\text{margin of error} = 1.96 \cdot \sqrt{\frac{0.8(1 - 0.8)}{30}}$$



$$n = 30 \quad \hat{p} = 80\% = 0.8$$

Calculate a 95% confidence interval for this sample proportion

95%

$$z_{\alpha/2} = 1.96$$

Confidence Interval for Proportions

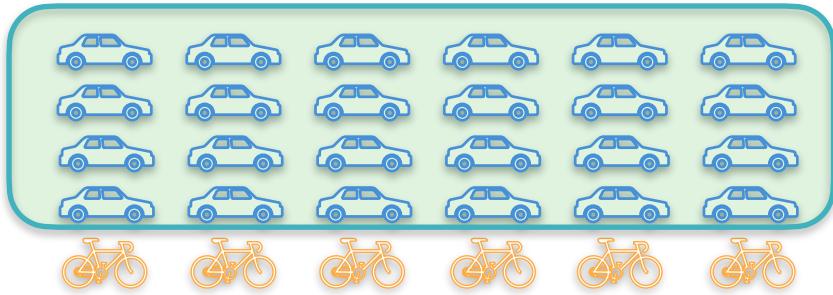
Confidence Interval for Proportions

confidence interval = $\hat{p} \pm$ margin of error

$$\text{margin of error} = z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}}$$

$$\text{margin of error} = 1.96 \cdot \sqrt{\frac{0.8(1 - 0.8)}{30}}$$

$$\text{margin of error} = 0.14$$



$$n = 30 \quad \hat{p} = 80\% = 0.8$$

Calculate a 95% confidence interval for this sample proportion

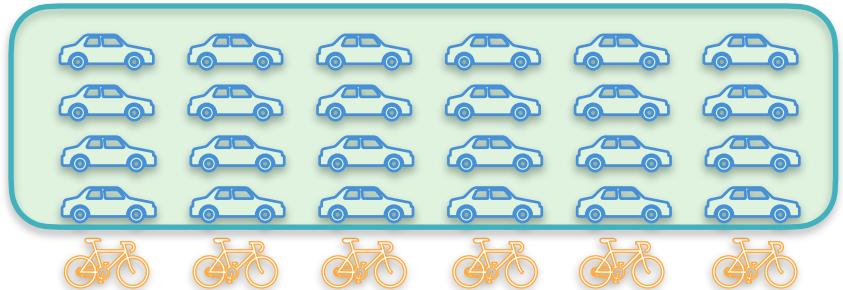
95% → $z_{\alpha/2} = 1.96$

Confidence Interval for Proportions

Confidence Interval for Proportions

$$\text{confidence interval} = \hat{p} \pm \text{margin of error}$$

$$\text{margin of error} = 0.14$$



$$n = 30 \quad \hat{p} = 80\% = 0.8$$

Calculate a 95% confidence interval for this sample proportion

95%

$$\rightarrow z_{\alpha/2} = 1.96$$

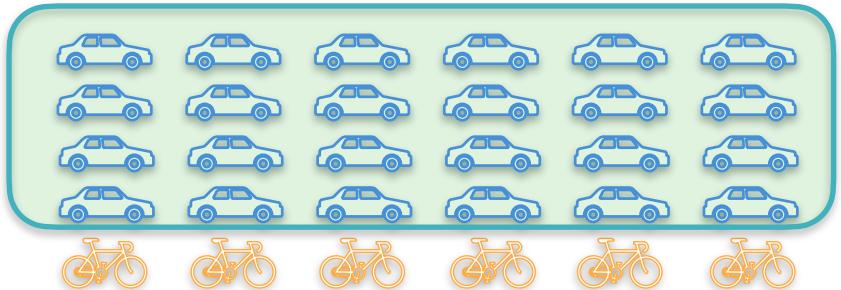
Confidence Interval for Proportions

Confidence Interval for Proportions

$$\text{confidence interval} = \hat{p} \pm \text{margin of error}$$

$$\text{margin of error} = 0.14$$

$$\text{confidence interval} =$$



$$n = 30 \quad \hat{p} = 80\% = 0.8$$

Calculate a 95% confidence interval for this sample proportion

$$95\% \rightarrow z_{\alpha/2} = 1.96$$

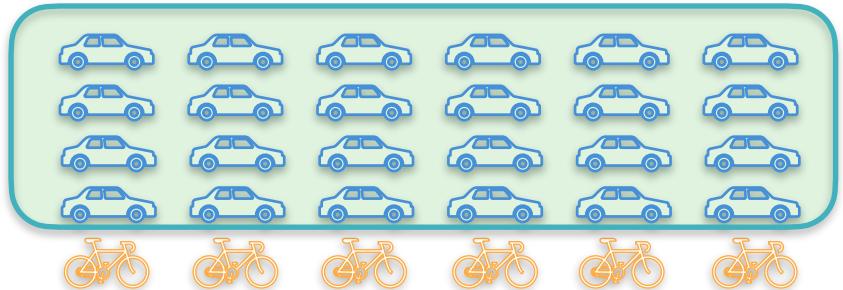
Confidence Interval for Proportions

Confidence Interval for Proportions

$$\text{confidence interval} = \hat{p} \pm \text{margin of error}$$

$$\text{margin of error} = 0.14$$

$$\text{confidence interval} = 0.8 \pm 0.14$$



$$n = 30 \quad \hat{p} = 80\% = 0.8$$

Calculate a 95% confidence interval for this sample proportion

95%

$$\rightarrow z_{\alpha/2} = 1.96$$

Confidence Interval for Proportions

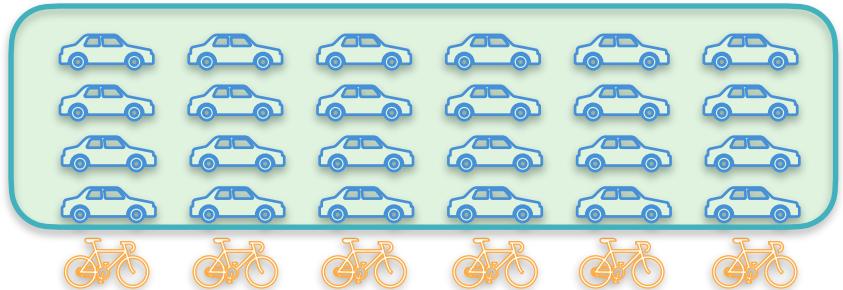
Confidence Interval for Proportions

$$\text{confidence interval} = \hat{p} \pm \text{margin of error}$$

$$\text{margin of error} = 0.14$$

$$\text{confidence interval} = 0.8 \pm 0.14$$

$$0.66 < p < 0.94$$



$$n = 30 \quad \hat{p} = 80\% = 0.8$$

Calculate a 95% confidence interval for this sample proportion

$$95\% \rightarrow z_{\alpha/2} = 1.96$$

Confidence Interval for Proportions

Confidence Interval for Proportions

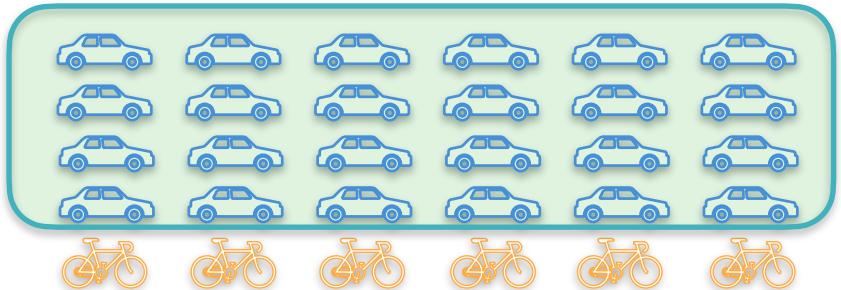
$$\text{confidence interval} = \hat{p} \pm \text{margin of error}$$

$$\text{margin of error} = 0.14$$

$$\text{confidence interval} = 0.8 \pm 0.14$$

$$0.66 < p < 0.94$$

$$66\% < p < 94\%$$



$$n = 30 \quad \hat{p} = 80\% = 0.8$$

Calculate a 95% confidence interval for this sample proportion

$$95\% \rightarrow z_{\alpha/2} = 1.96$$

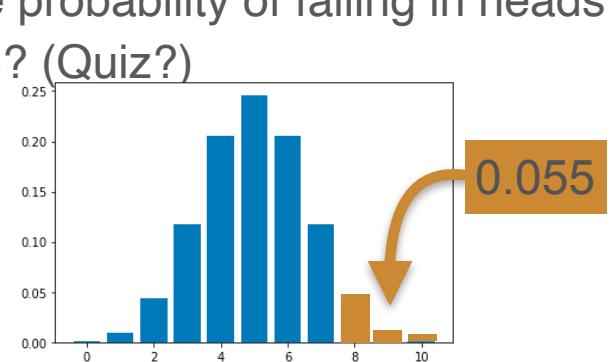
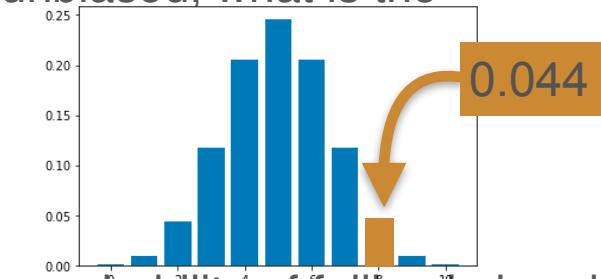
W4 Lesson 2

Hypothesis Testing

- Data Science interview
 - I throw a coin 10 times. I get heads 8 times.
 - Quiz 1: Do you think the coin is biased, or not?
 - Keep both answers correct. Specify that we don't have enough information.
 - More information: We agree to say that something is *unlikely* if the probability of it happening is less than 5%.

Hypothesis Testing

- Quiz: If we were to assume that the coin is unbiased, what is the probability that it falls in heads 8 times?
 - Answer: $(10 \text{ choose } 8) * 0.5^{10}$
this is for unbiased
 - However, should we instead look at the probability of falling in heads 8 or more times? How much is this one? (Quiz?)
 - Answer:
 $((10 \text{ choose } 8) +$
 $(10 \text{ choose } 9) +$
 $(10 \text{ choose } 10)) * 0.5^{10}$

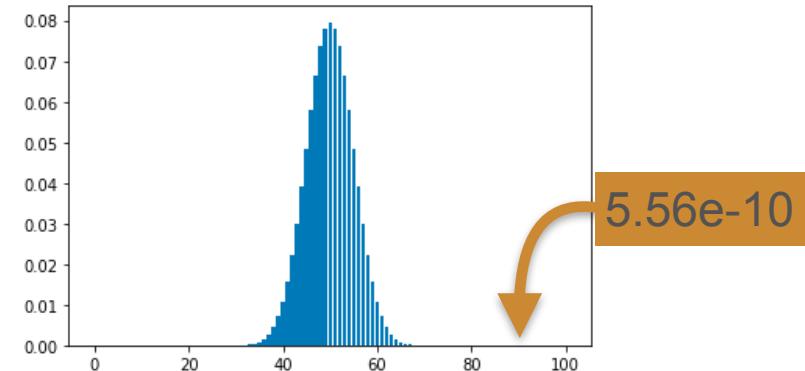


Hypothesis Testing

- If we assume that the coin is fair (**null hypothesis**)
- Then the probability that it lands in heads 8 times (**outcome**) is 0.055
- That probability is higher than 5%
 - Therefore the coin *could* potentially be fair (i.e., **we can't reject the null hypothesis**)

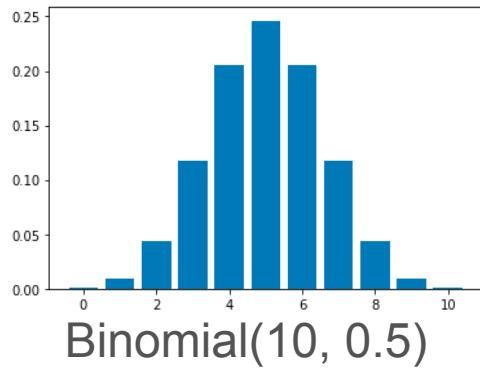
Hypothesis Testing

- Now, same problem except that we toss the coin 100 times and get heads 80 times.
- Probability of 80 or more heads:
 - Sum from $i=80$ to 100 of $(100 \text{ choose } i) * 0.5^{100}$
 - The probability is TINY
- Therefore, we reject the null hypothesis
 - We conclude the coin is not biased
- Problem: A huge sum of tiny numbers to calculate
- Solution: Approximate with a Gaussian!

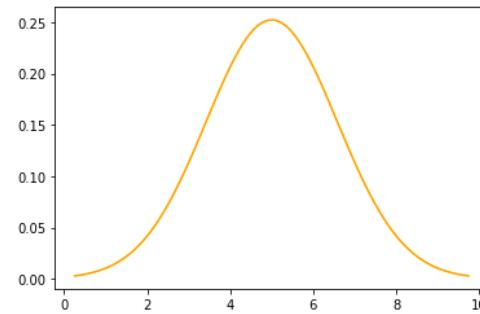


Central Limits Theorem (-Ish)

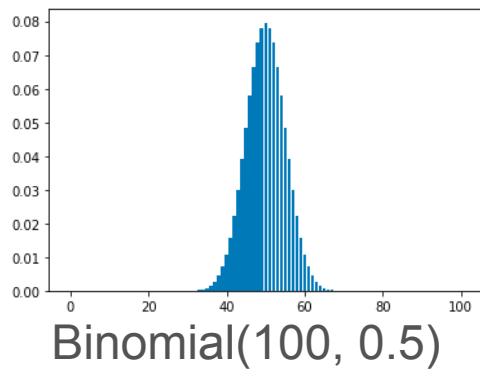
This is called on W₃



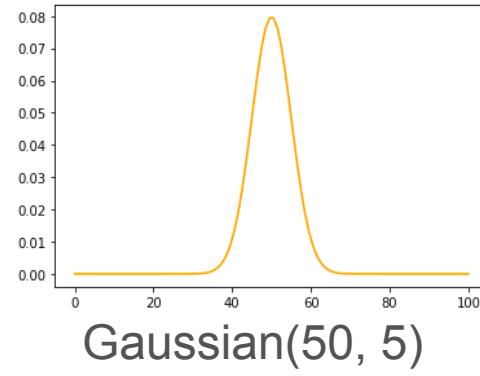
Binomial(10, 0.5)



Gaussian($5, \sqrt{2.5}$)

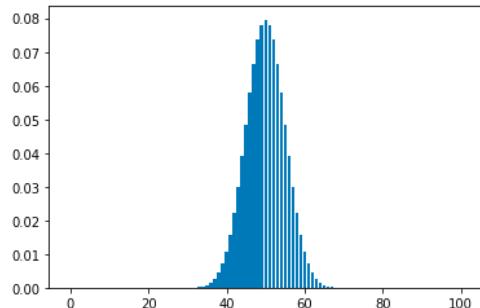


Binomial(100, 0.5)

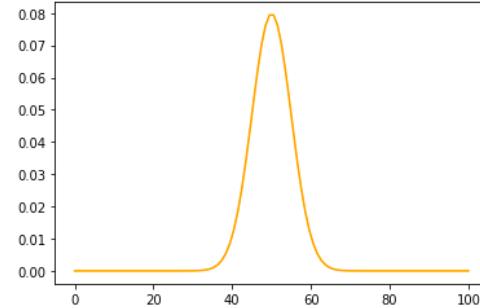


Gaussian($50, 5$)

Central Limits Theorem (-Ish)



Binomial(n, p)



Gaussian(?, ?)

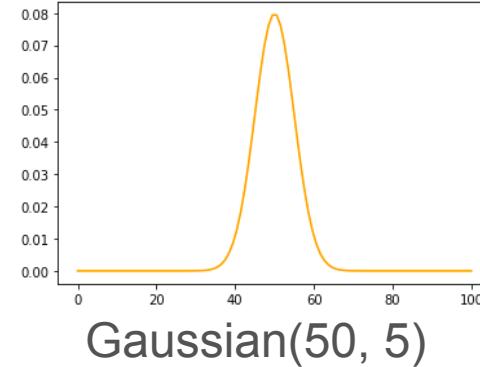
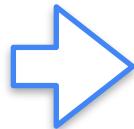
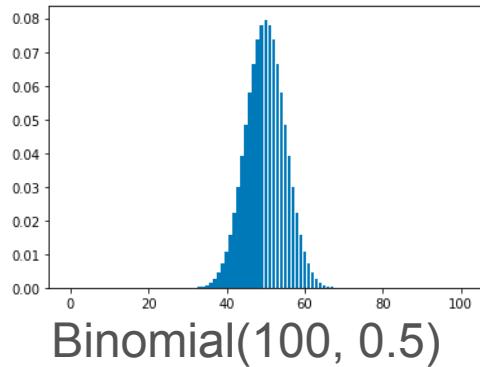
$\text{Gaussian}(np, \sqrt{np(1 - p)})$



Mean = np
Variance = $np(1-p)$

Gaussian with:
mean= np
Variance = $np(1-p)$

Approximating the Binomial With a Gaussian

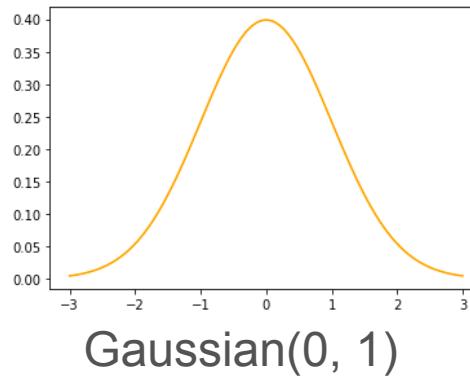
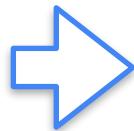
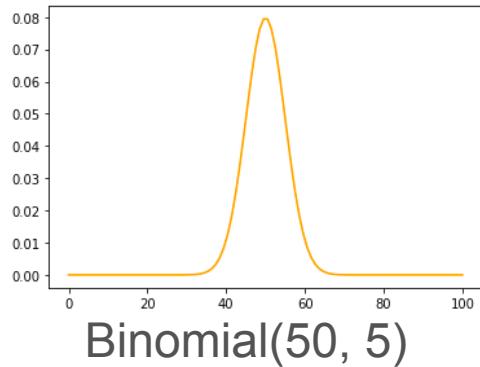


Sum of binomials from 80 to 100

Area under the curve from 80 to 100

Can be calculated using CDF!

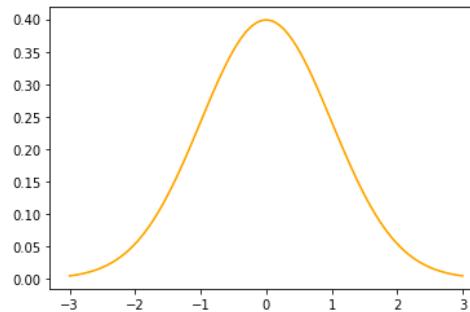
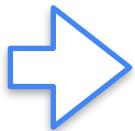
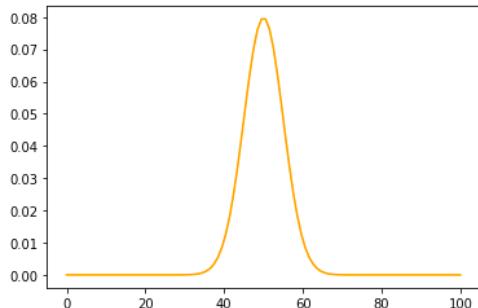
How To Do It? Normalize



X

$$\frac{X - \text{mean}}{\text{std}}$$
$$\frac{X - 50}{5}$$

How To Do It? Normalize



X

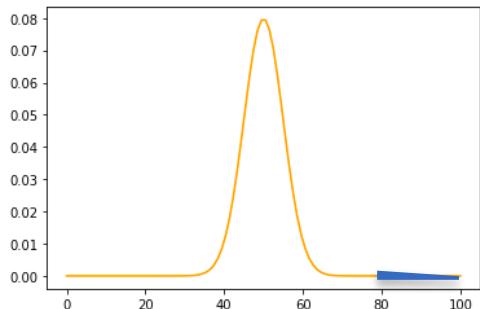
$$\mu = np$$

$$\sigma = np(1 - p)$$

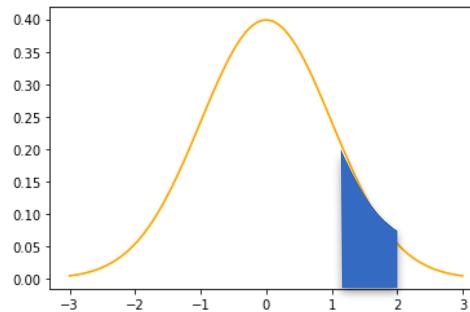
$$\frac{X - \text{mean}}{\text{std}}$$

$$\frac{X - \mu}{\sigma}$$

How To Do It? Normalize



$\text{Binomial}(n, p)$



$\text{Gaussian}(0, 1)$

$P(80 < X < 100)$ 

$$P\left(\frac{80 - \mu}{\sigma} < \frac{X - \mu}{\sigma} < \frac{100 - \mu}{\sigma}\right)$$

$$P\left(\frac{80 - 50}{5} < \frac{X - 50}{5} < \frac{100 - 50}{5}\right)$$

$$= P(12 < Z < 20)$$

How To Do It? Normalize

$$\begin{aligned} P\left(\frac{80 - 50}{25} < \frac{X - 50}{25} < \frac{100 - 50}{25}\right) &= P(12 < Z < 20) \\ &= \Phi(20) - \Phi(12) \\ &= \text{almost } 0 \end{aligned}$$

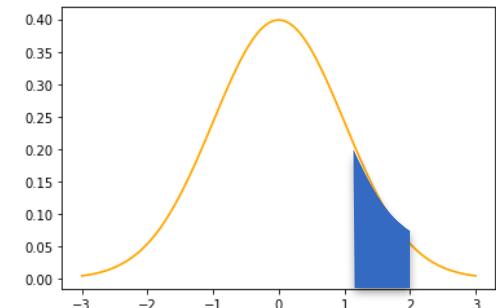
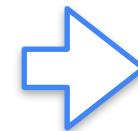
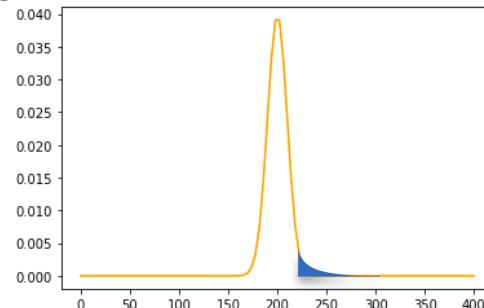
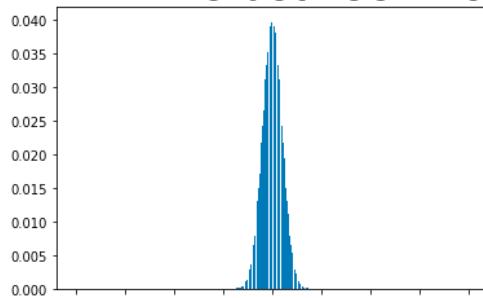
Conclusion: The coin must be biased

Why?

Because we reject the **null hypothesis** which says the coin is not biased.

Another Example

I toss a fair coin 400 times. What is the probability that the number of heads is between 250 and 350?



We can use this
on top of this

$$0.9332 - 0.6915$$

$$P\left(\frac{250-200}{100} < \frac{X-200}{100} < \frac{350-200}{100}\right)$$

$$P\left(0.5 < \frac{X-200}{100} < 1.5\right)$$

$$= P(0.5 < Z < 1.5)$$

$$= 0.2417$$



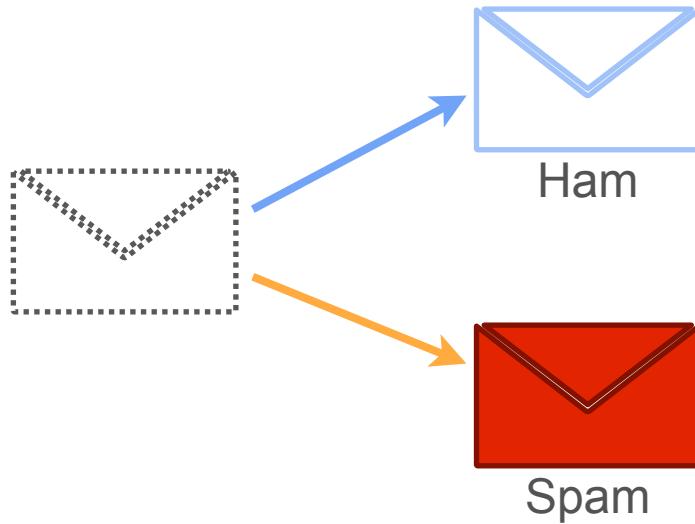
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Hypothesis Testing

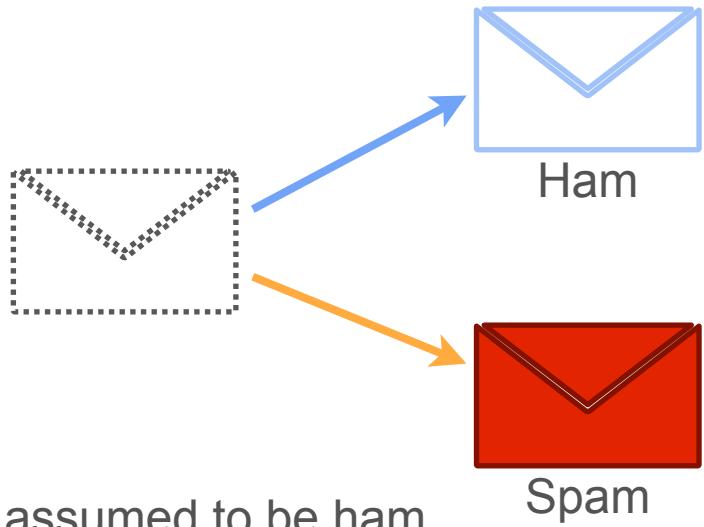
Defining hypothesis

Motivation

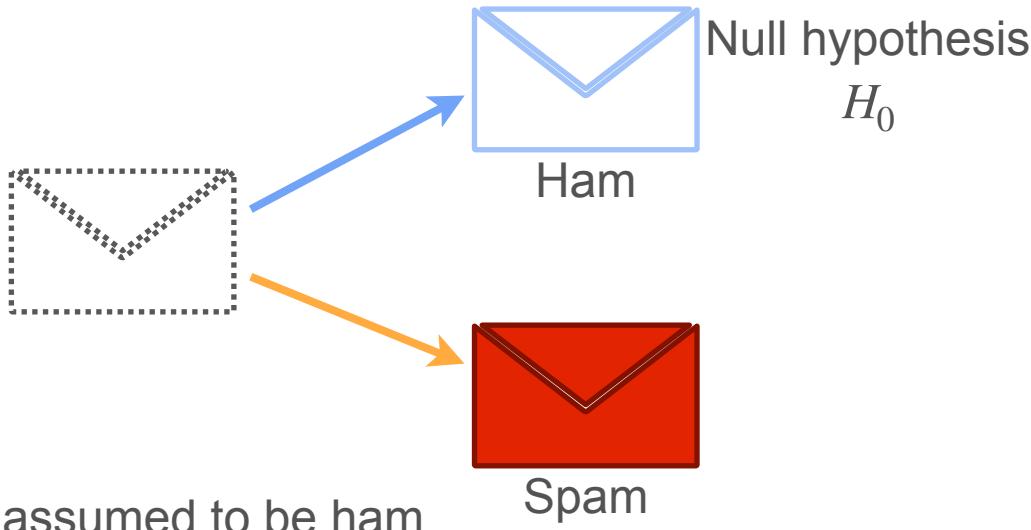
Motivation



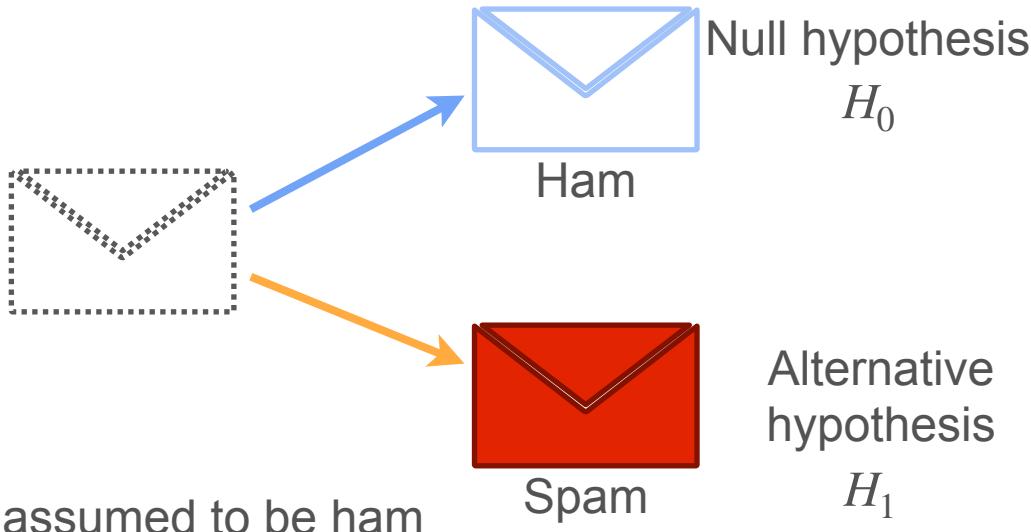
Motivation



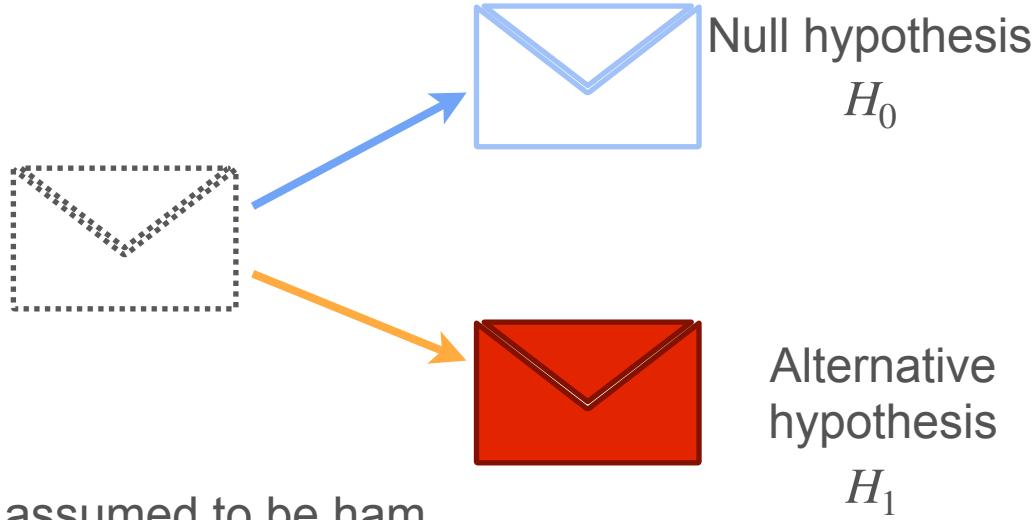
Motivation



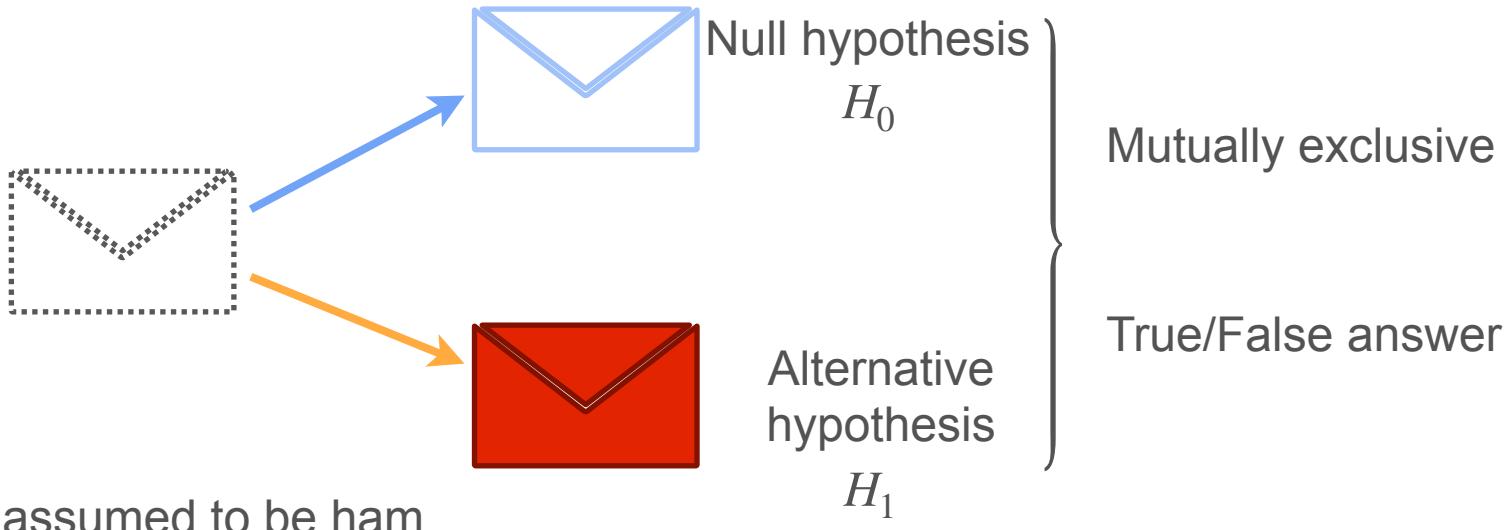
Motivation



Motivation

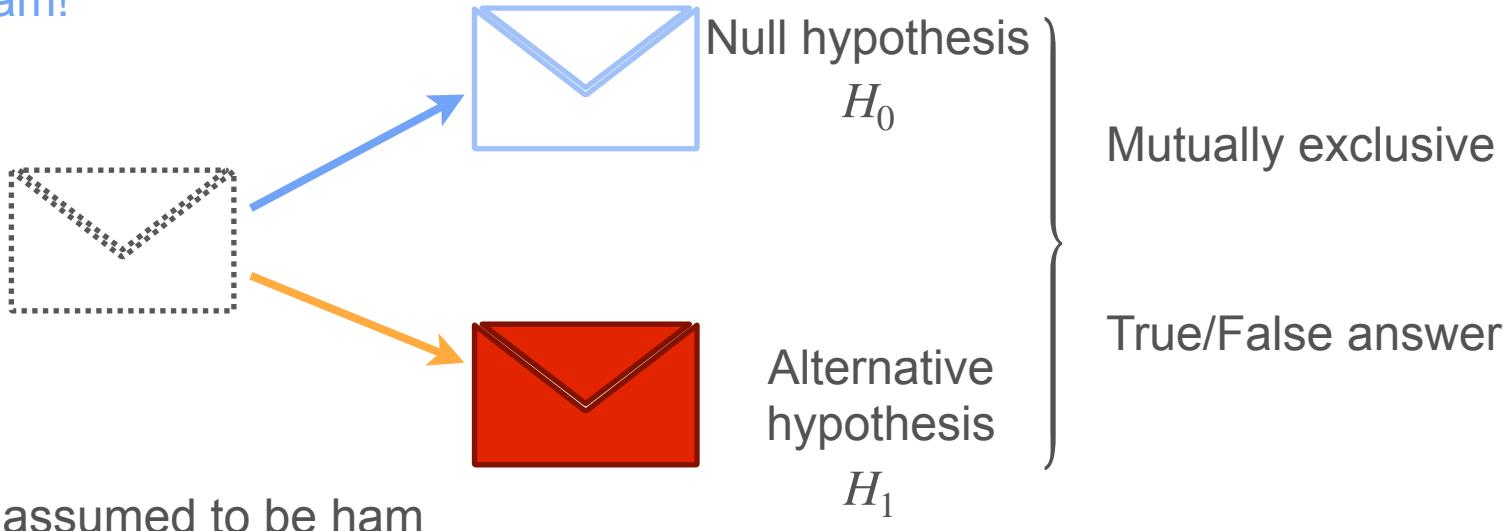


Motivation



Motivation

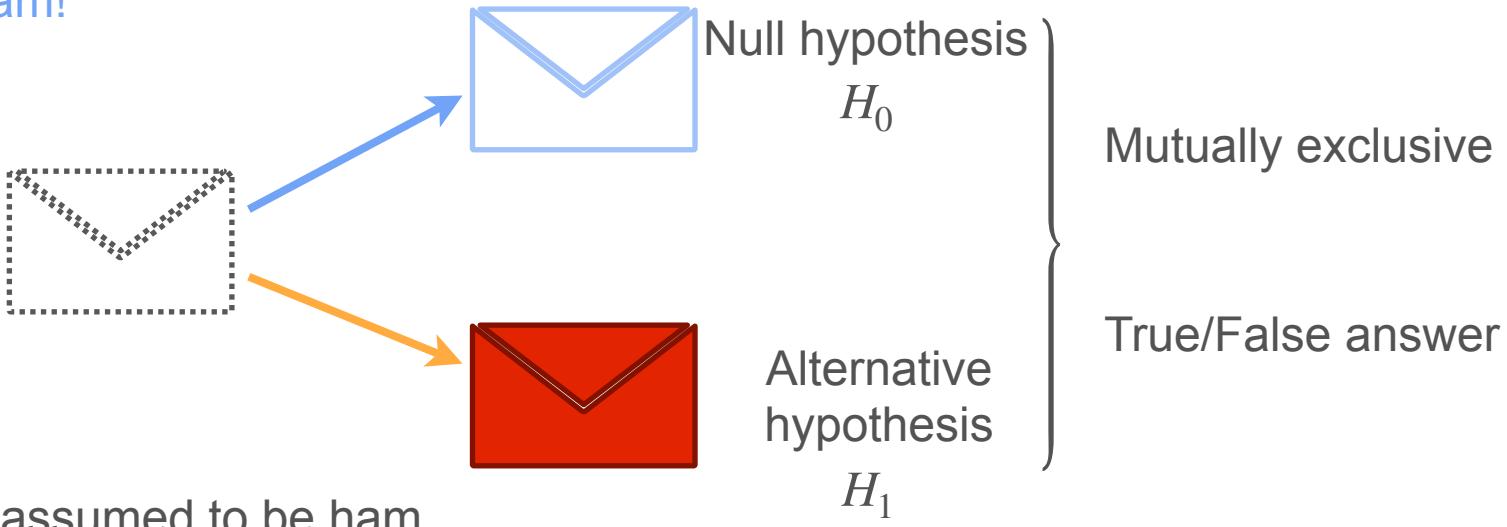
When rejecting that the email is not spam, you are accepting that the email is spam!



Motivation

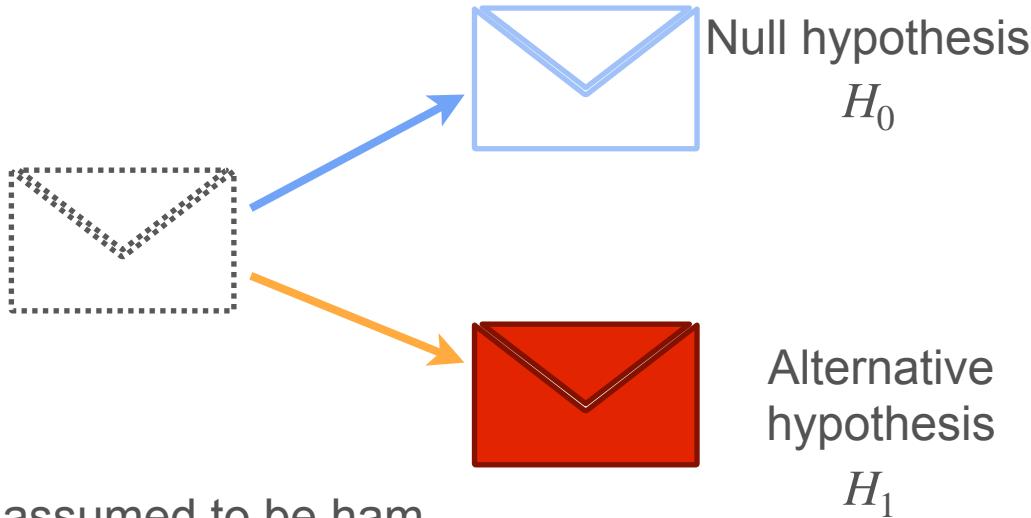
When rejecting that the email is not spam, you are accepting that the email is spam!

By failing to reject that the email IS spam, you are **not** accepting that it's ham



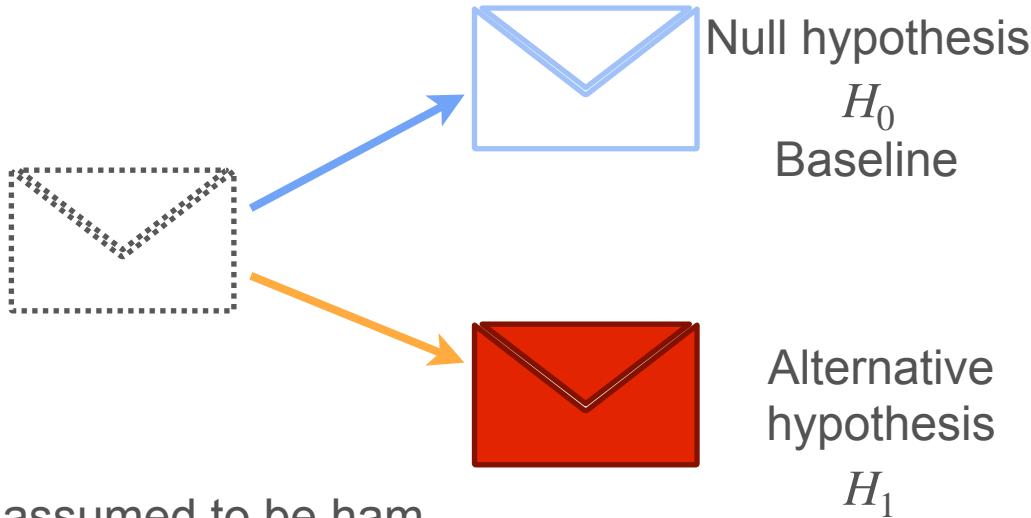
Motivation

Not labeling the email spam, doesn't mean the email is ham!



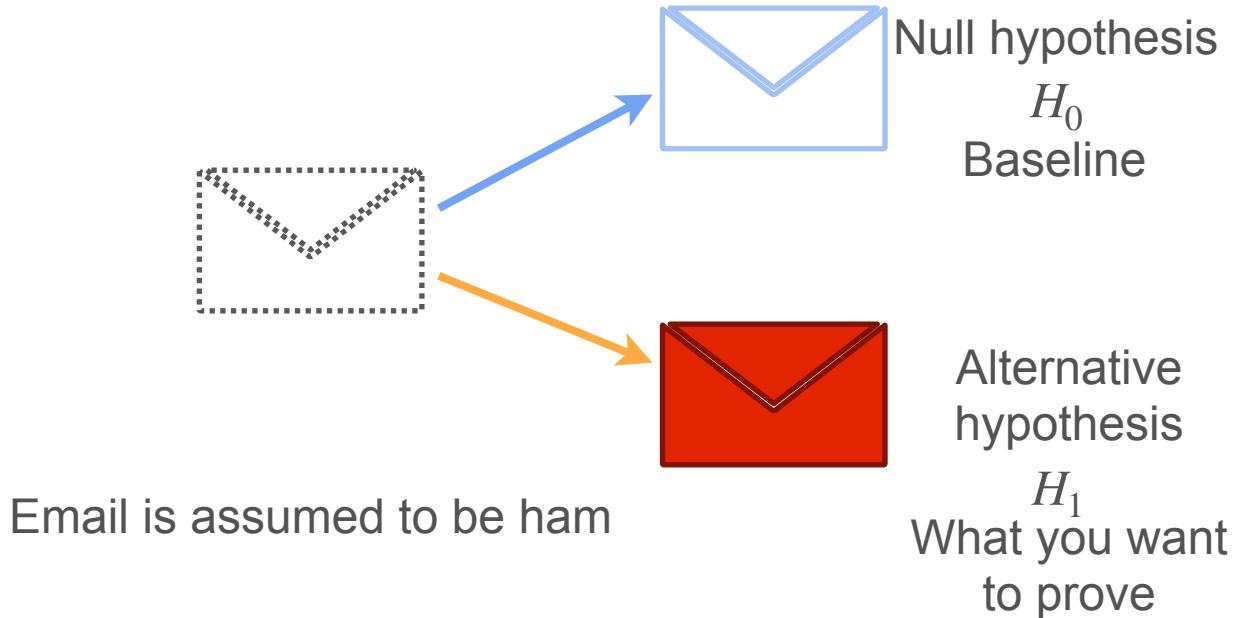
Motivation

Not labeling the email spam, doesn't mean the email is ham!



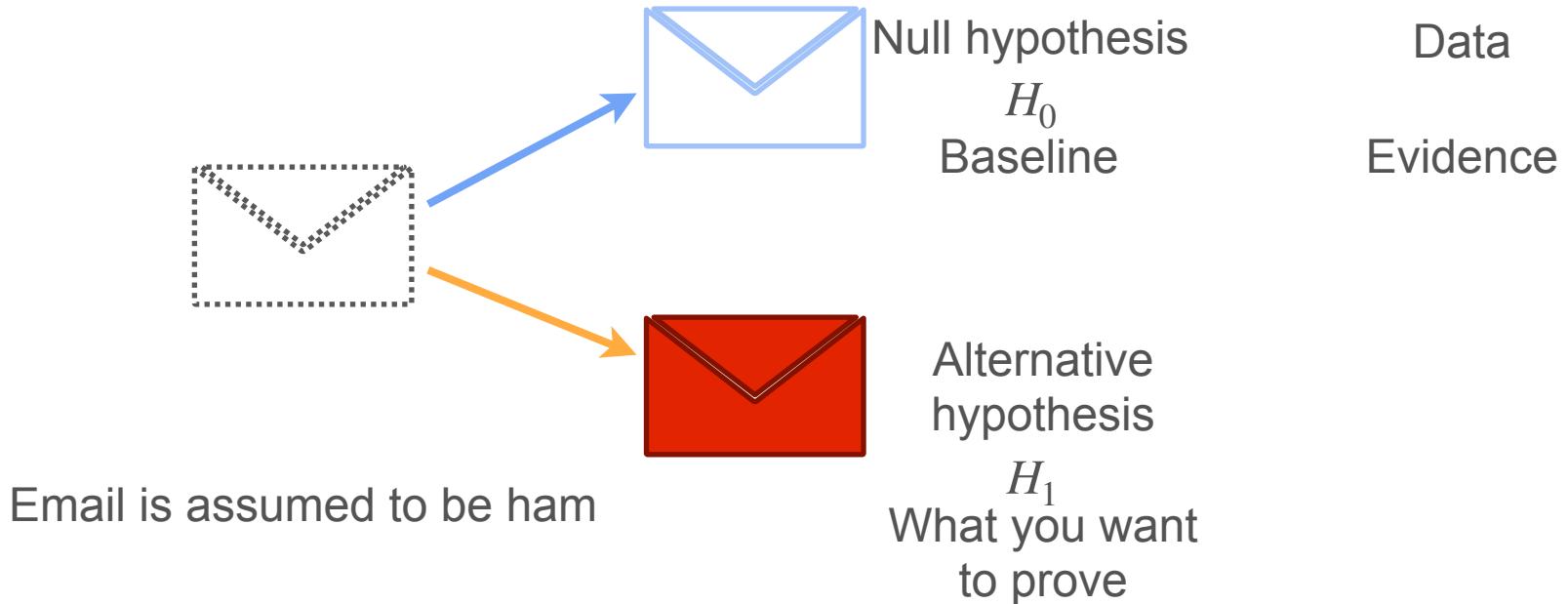
Motivation

Not labeling the email spam, doesn't mean the email is ham!



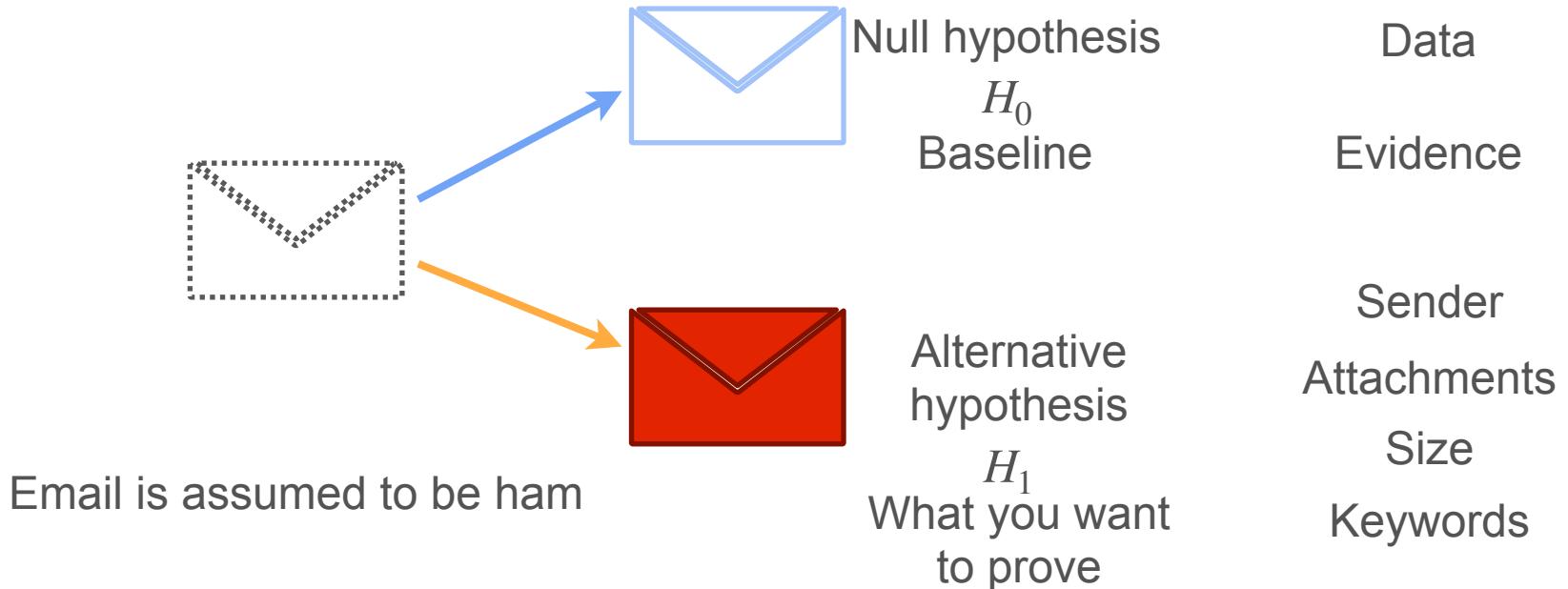
Motivation

Not labeling the email spam, doesn't mean the email is ham!



Motivation

Not labeling the email spam, doesn't mean the email is ham!



How To Determine the Result of the Test

How To Determine the Result of the Test

Plenty of evidence
against H_0



Reject H_0 (and accept H_1)

How To Determine the Result of the Test

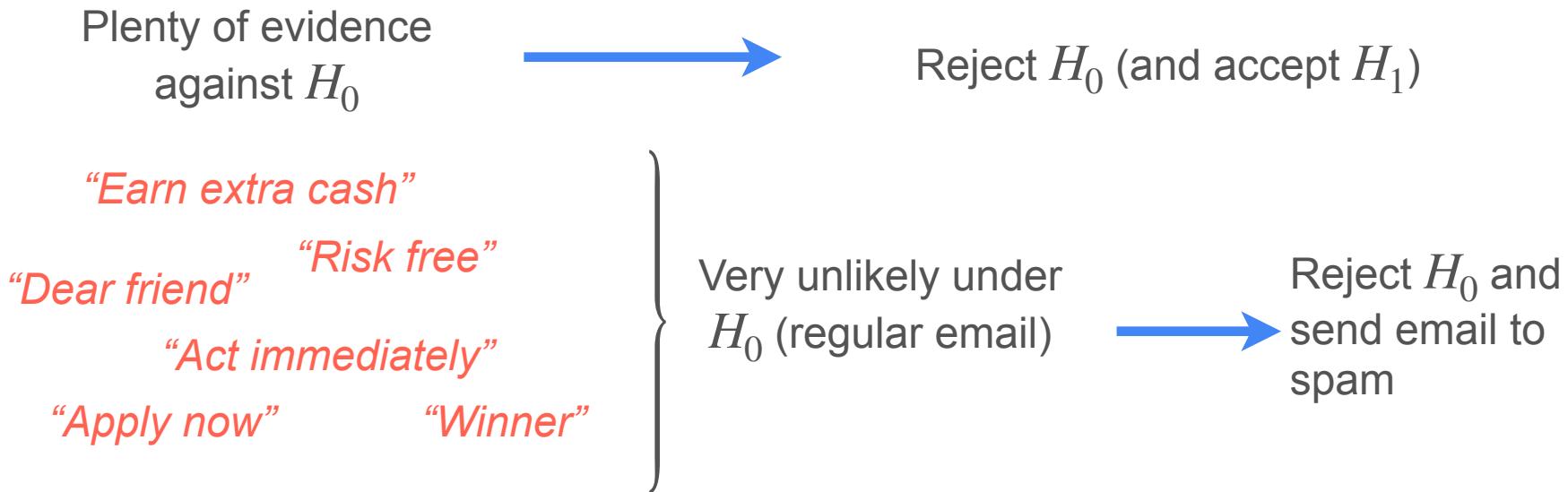
Plenty of evidence
against H_0



Reject H_0 (and accept H_1)

“Earn extra cash”
“Dear friend” *“Risk free”*
“Act immediately”
“Apply now” *“Winner”*

How To Determine the Result of the Test





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Hypothesis Testing

Type I and Type II errors

Sometimes Things Go Wrong...

Sometimes Things Go Wrong...

What if I make the wrong decision?

Sometimes Things Go Wrong...

What if I make the wrong decision?



Type I error
(False positive)

Sometimes Things Go Wrong...

What if I make the wrong decision?



Type I error
(False positive)



Type II error
(False negative)

Sometimes Things Go Wrong...

What if I make the wrong decision?



Type I error
(False positive)



Type II error
(False negative)

Sometimes Things Go Wrong...

What if I make the wrong decision?



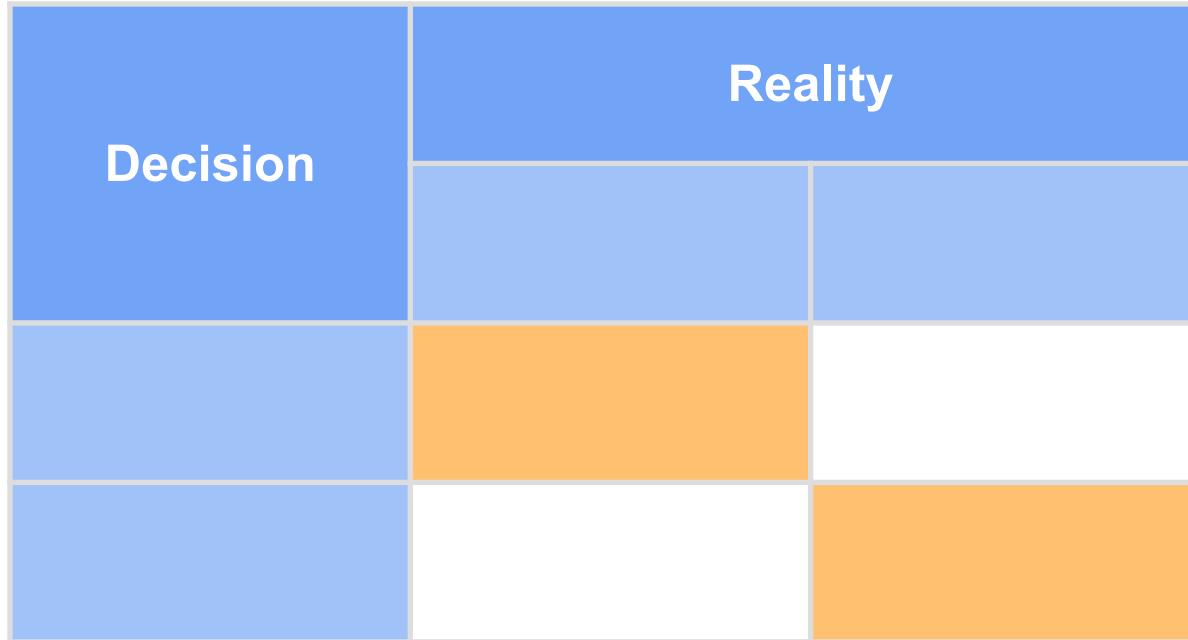
Type I error
(False positive)



Type II error
(False negative)

Type I and Type II Errors

Type I and Type II Errors



Type I and Type II Errors

Decision	Reality	
	H_0 True (Innocent)	
Reject H_0 (Decide Guilty)	Type I error	

Type I and Type II Errors

Decision	Reality	
	H_0 True (Innocent)	
Reject H_0 (Decide Guilty)	Type I error	
Don't reject H_0 (Decide not guilty)	Correct	

Type I and Type II Errors

Decision	Reality	
	H_0 True (Innocent)	H_0 False (Guilty)
Reject H_0 (Decide Guilty)	Type I error	Correct
Don't reject H_0 (Decide not guilty)	Correct	

Type I and Type II Errors

Decision	Reality	
	H_0 True (Innocent)	H_0 False (Guilty)
Reject H_0 (Decide Guilty)	Type I error	Correct
Don't reject H_0 (Decide not guilty)	Correct	Type II error

Significance Level

Significance Level

Type I error

Type II error

Significance Level

The presumption of innocence implies that sending an innocent person to prison is worse than letting a criminal walk

Type I error



Type II error



Significance Level

The presumption of innocence implies that sending an innocent person to prison is worse than letting a criminal walk

Type I error



Type II error



What is the greatest probability of type I error you are willing to tolerate?

Significance Level

What is the greatest probability of type I error you are willing to tolerate?

Type I error



Significance Level

What is the greatest probability of type I error you are willing to tolerate?

Type I error



Innocent person determined guilty

Significance Level

What is the greatest probability of type I error you are willing to tolerate?

Type I error



↓
Significance level

Innocent person determined guilty

Significance Level

What is the greatest probability of type I error you are willing to tolerate?

Type I error



Innocent person determined guilty



Significance level (α)

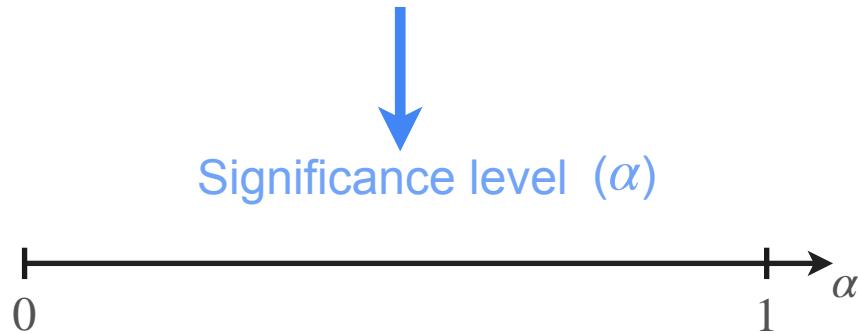
Significance Level

What is the greatest probability of type I error you are willing to tolerate?

Type I error



Innocent person determined guilty



Significance Level

What is the greatest probability of type I error you are willing to tolerate?

Type I error



Innocent person determined guilty



Significance level (α)

0

1

Defendants are
always considered →
'not guilty'



No Type I error

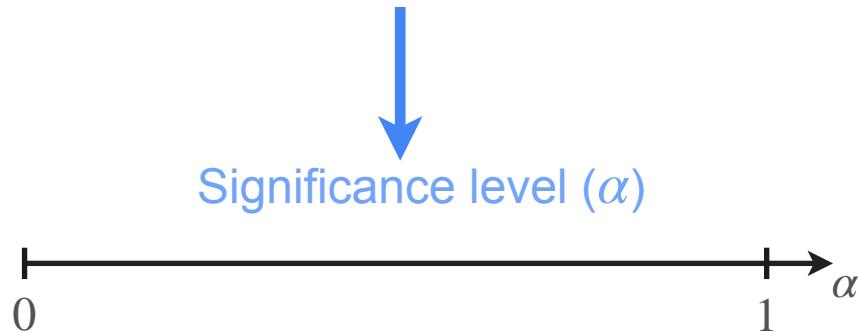
Significance Level

What is the greatest probability of type I error you are willing to tolerate?

Type I error



Innocent person determined guilty



Significance Level

What is the greatest probability of type I error you are willing to tolerate?

Type I error



Innocent person determined guilty you make a Type I error

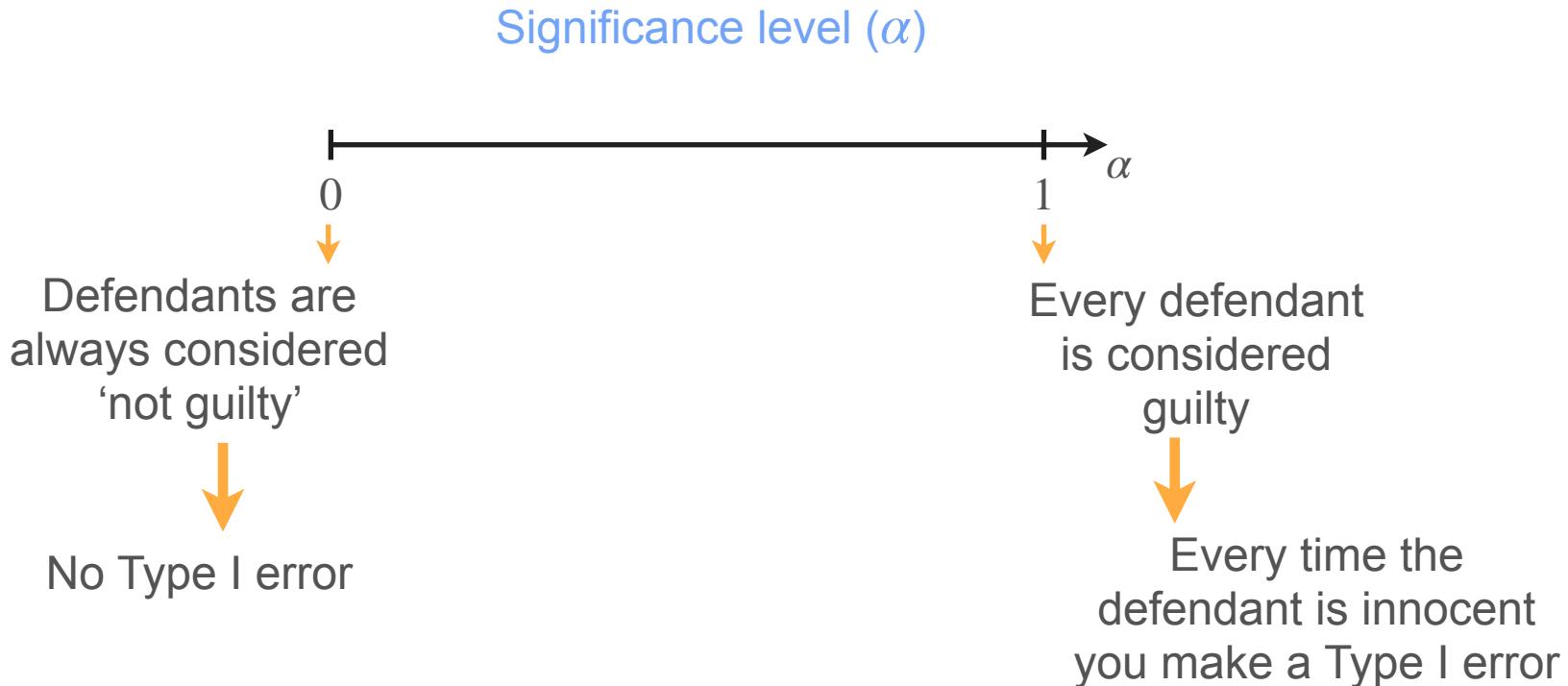


Significance level (α)

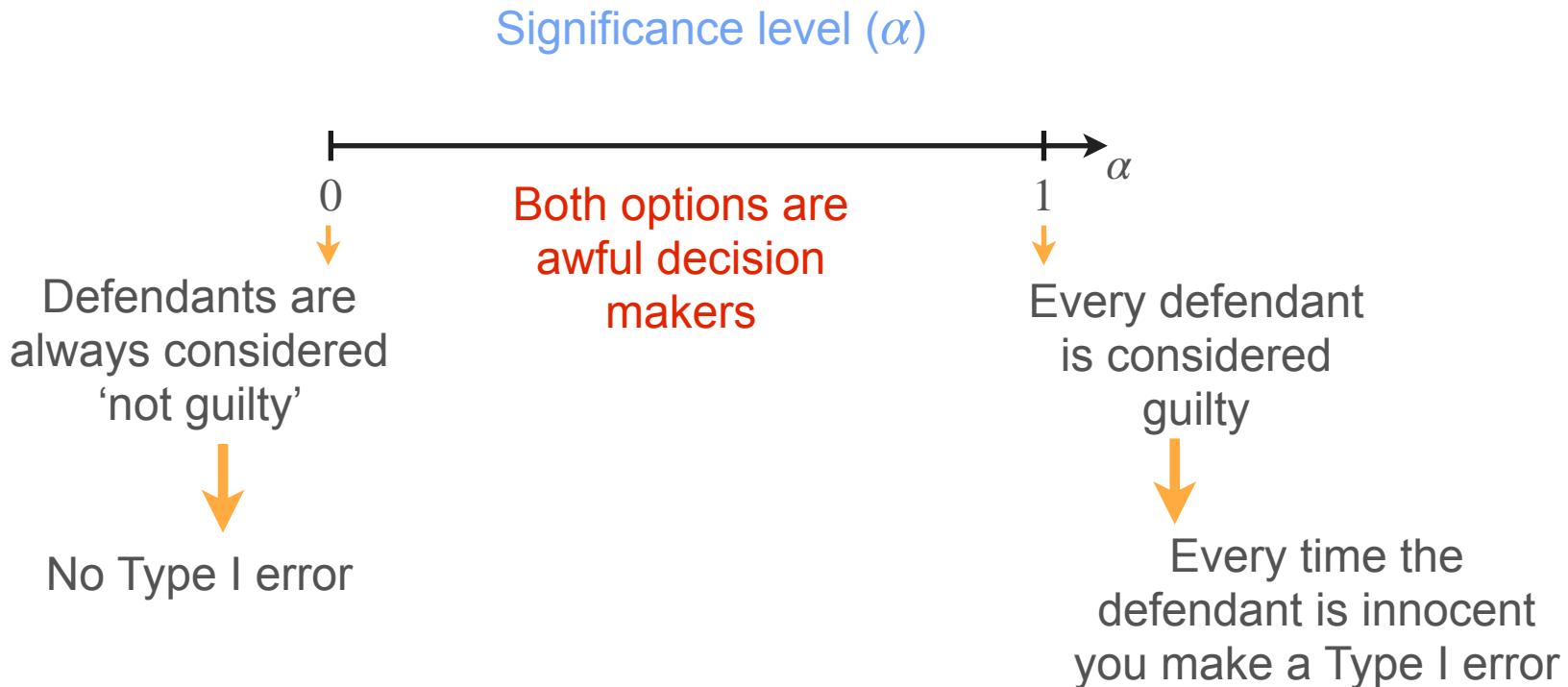
Every time the defendant is innocent

Every defendant is considered guilty

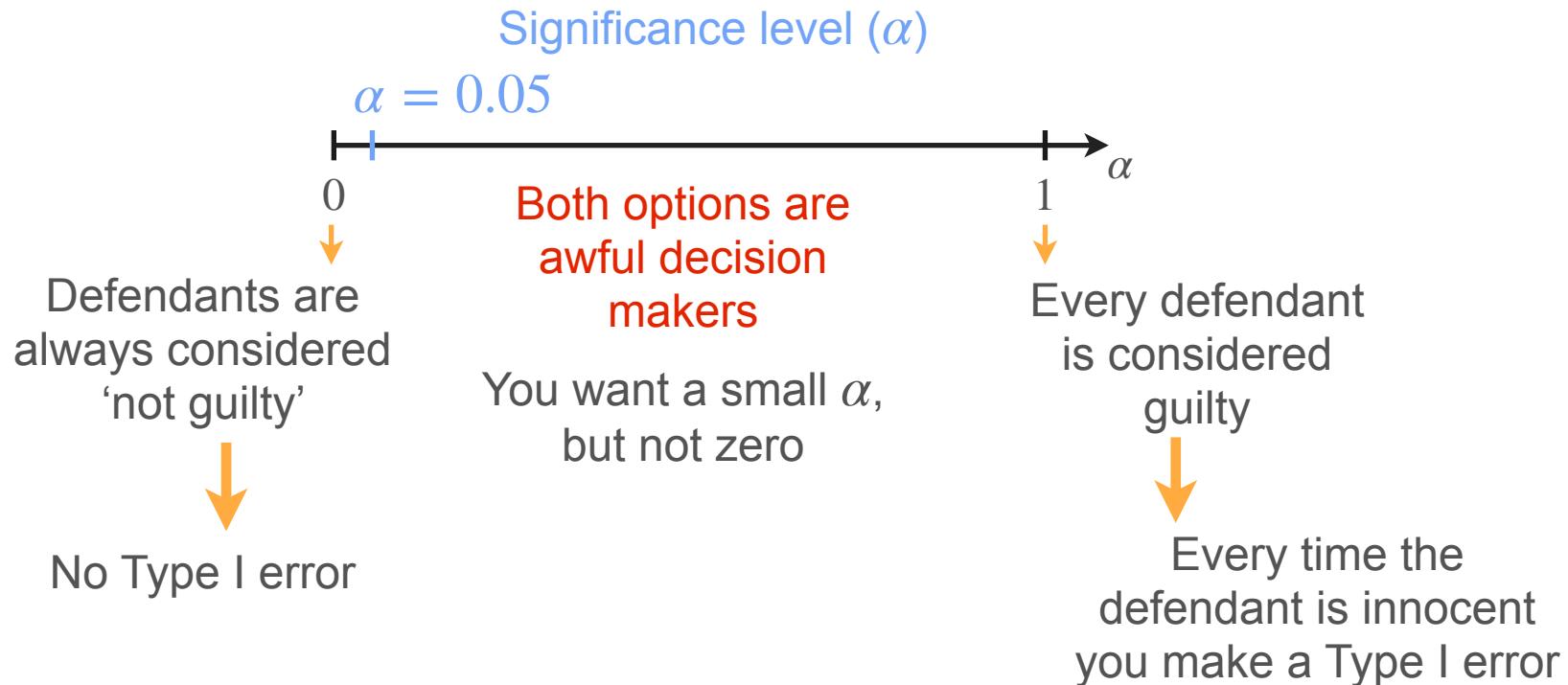
Significance Level



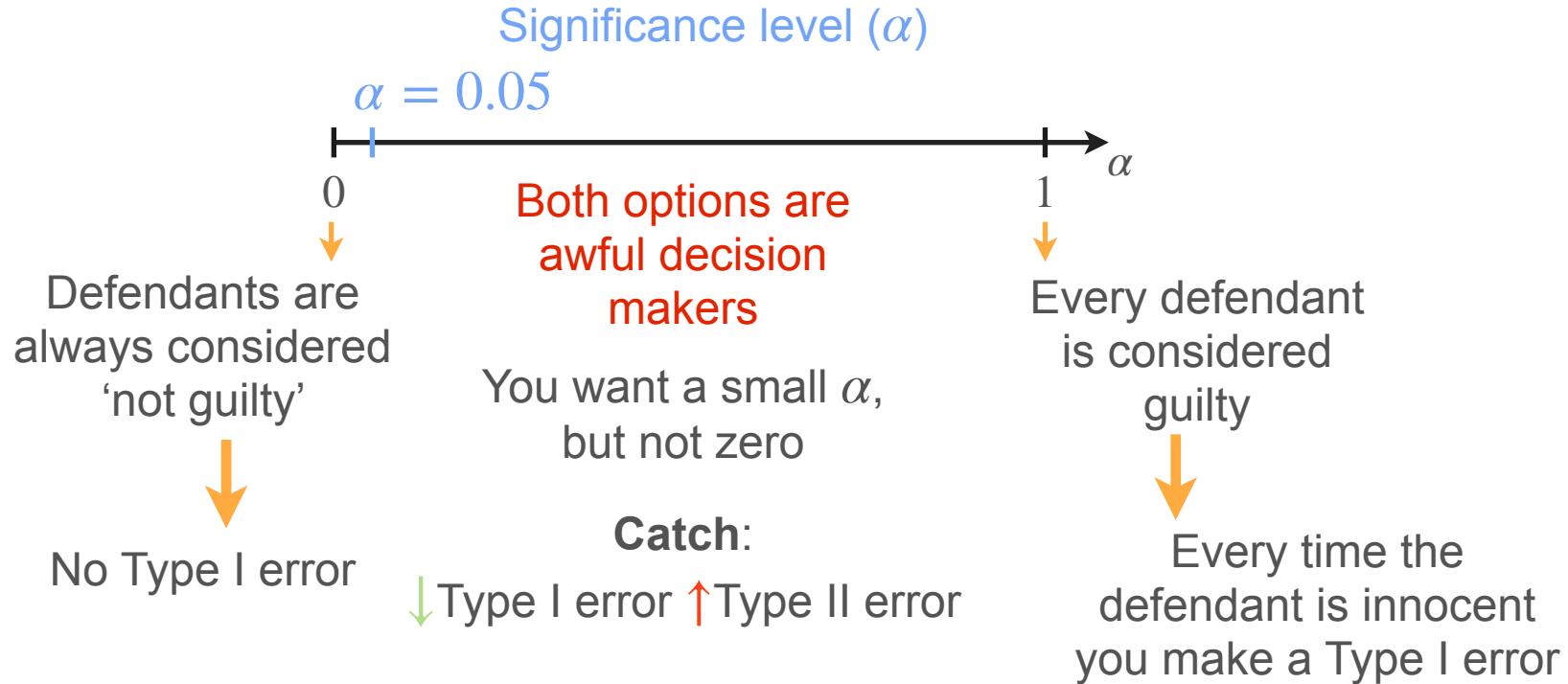
Significance Level



Significance Level



Significance Level



Significance Level

Significance Level

Type I error



Innocent person determined guilty

Significance Level

Type I error



$$\alpha = \max P(\text{Type I error})$$

Innocent person determined guilty

Significance Level

Type I error



$$\begin{aligned}\alpha &= \max \mathbf{P}(\text{Type I error}) \\ &= \max \mathbf{P}(\text{Reject } H_0 | H_0)\end{aligned}$$

Innocent person determined guilty

Significance Level

Type I error



Innocent person determined guilty

$$\begin{aligned}\alpha &= \max \mathbf{P}(\text{Type I error}) \\ &= \max \mathbf{P}(\text{Reject } H_0 | H_0)\end{aligned}$$

The value of α is your criteria for designing your test

Significance Level

Type I error



Innocent person determined guilty

$$\begin{aligned}\alpha &= \max \mathbf{P}(\text{Type I error}) \\ &= \max \mathbf{P}(\text{Reject } H_0 | H_0)\end{aligned}$$

The value of α is your criteria for designing your test

For a given sample, α will determine if you reject H_0 or not



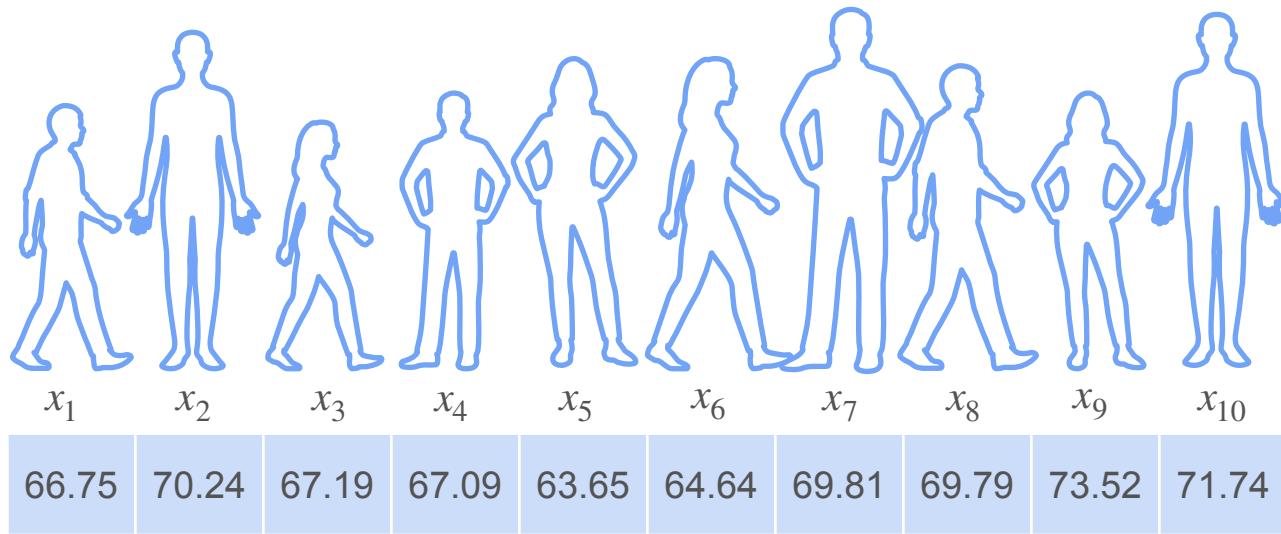
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Hypothesis Testing

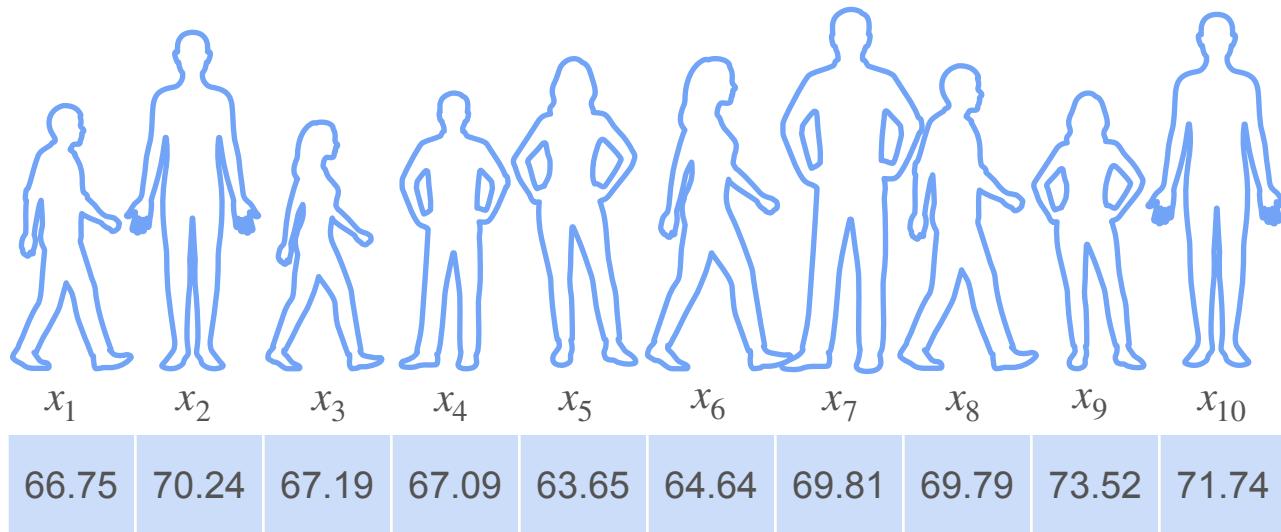
**Right-Tailed, Left-Tailed and
Two-Tailed tests**

Example: Heights

Example: Heights

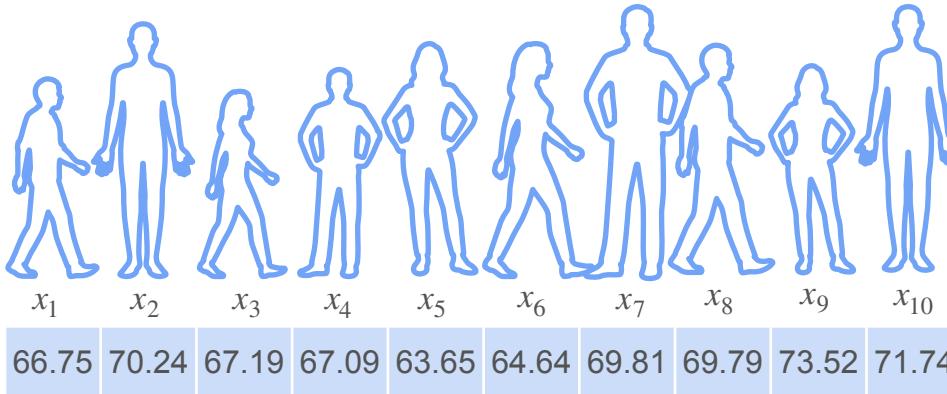


Example: Heights

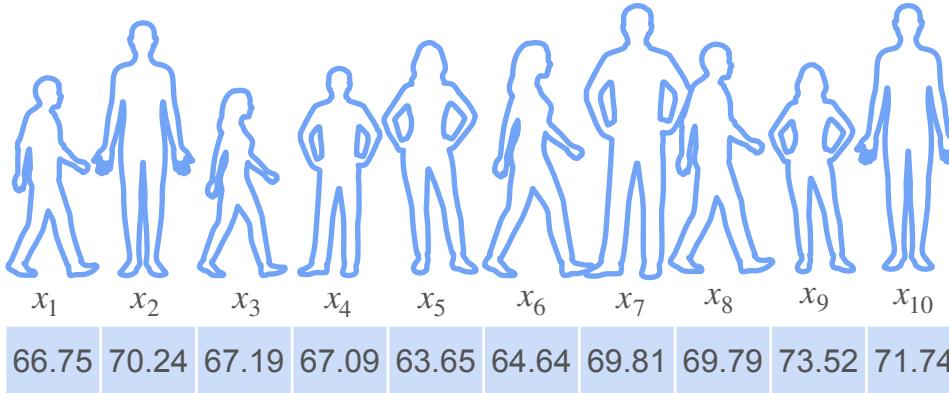


$$\bar{x} = 68.442$$

Data Quality



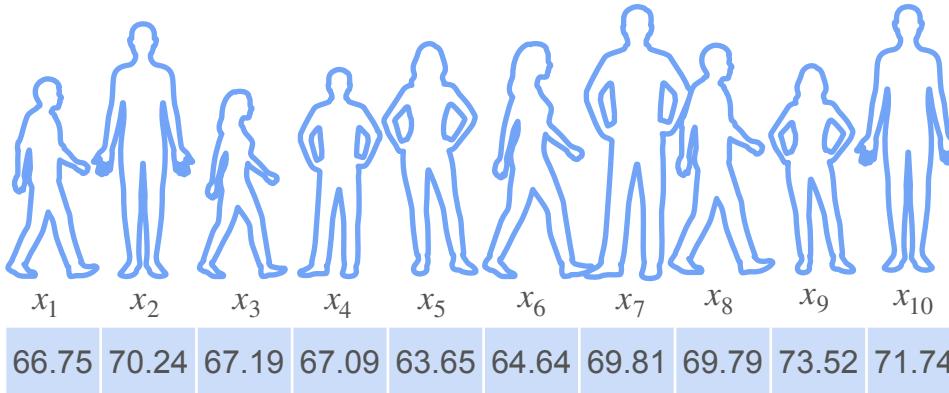
Data Quality



$$\bar{x} = 68.442$$

Reliable

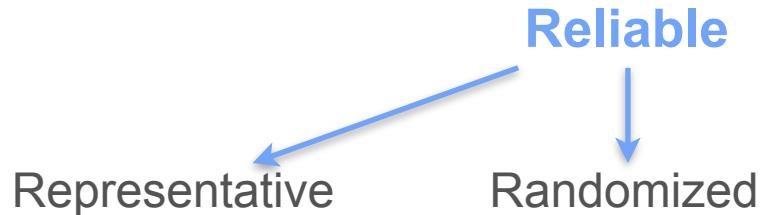
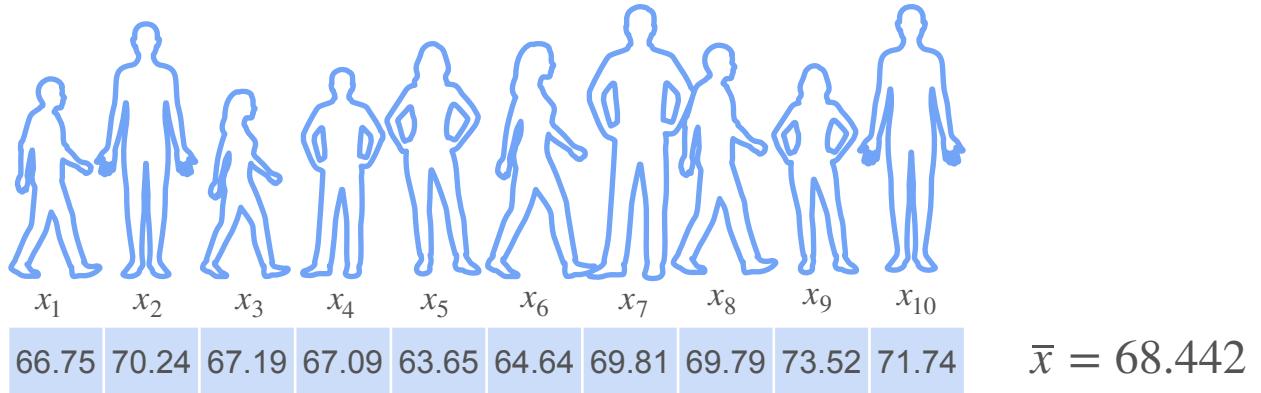
Data Quality



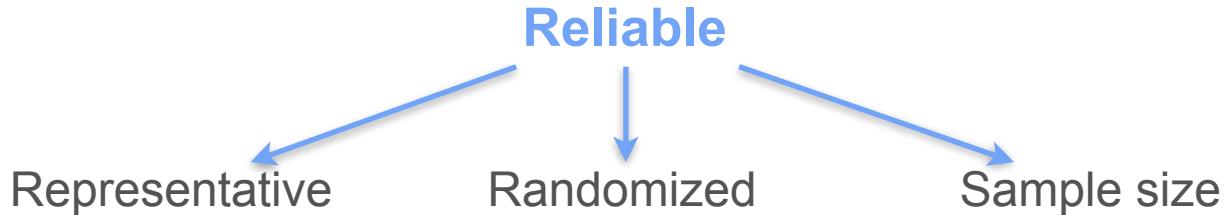
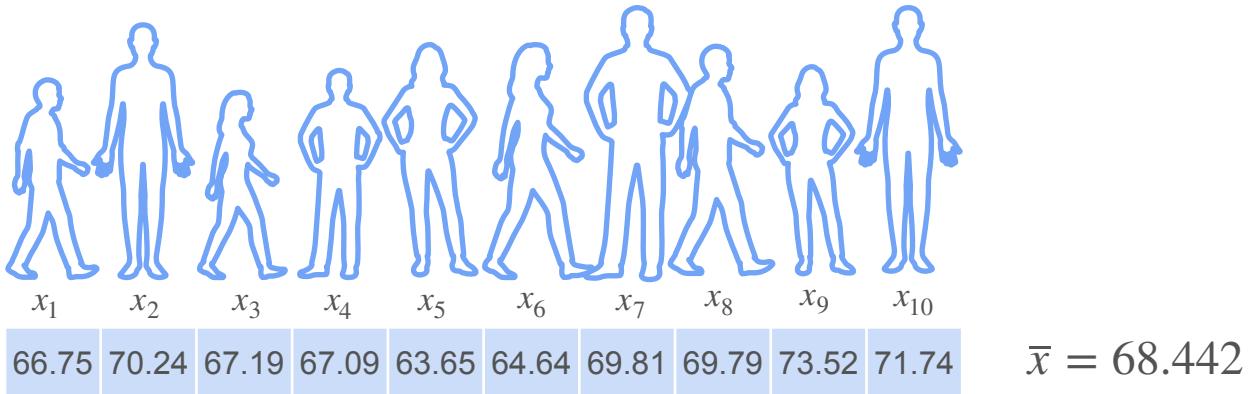
Reliable

Representative

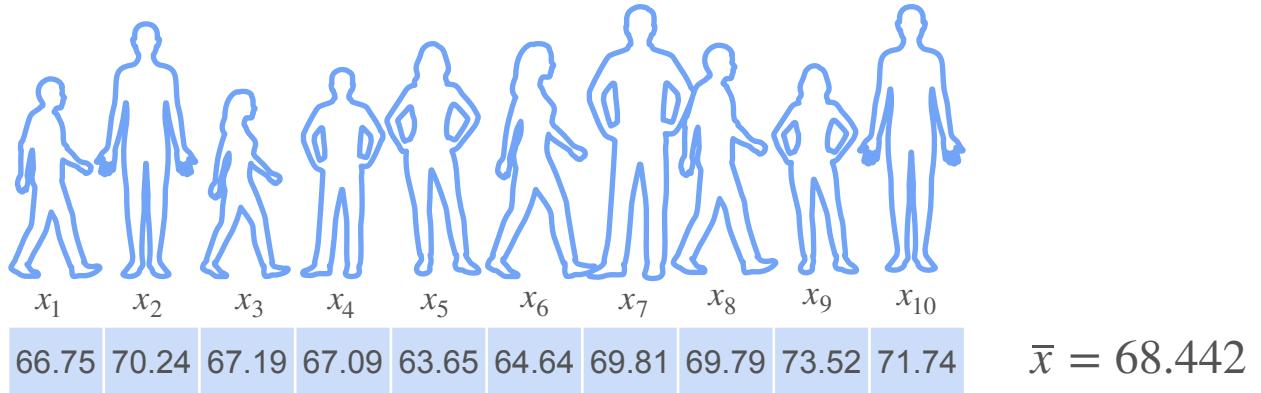
Data Quality



Data Quality

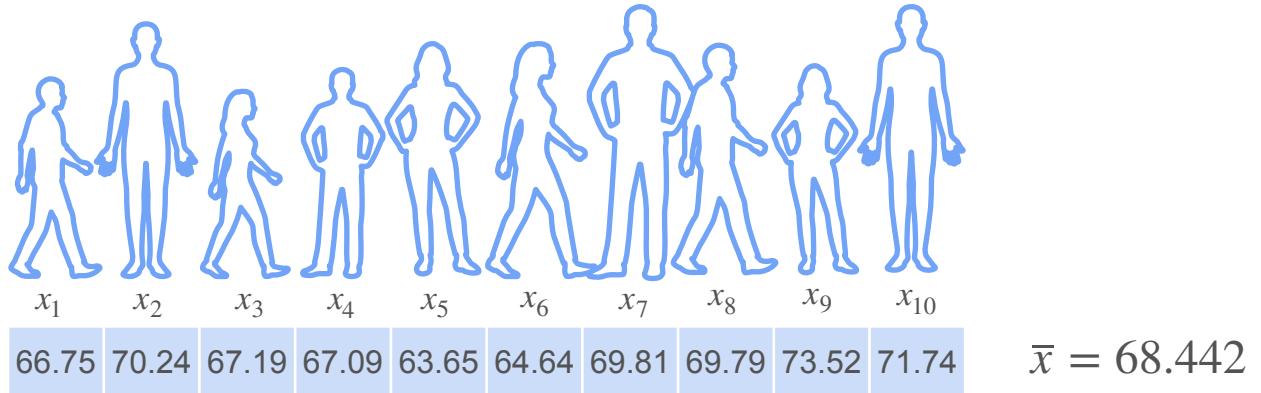


Determining the Hypothesis



Population vs. $H_1 : \mu > 66.7$

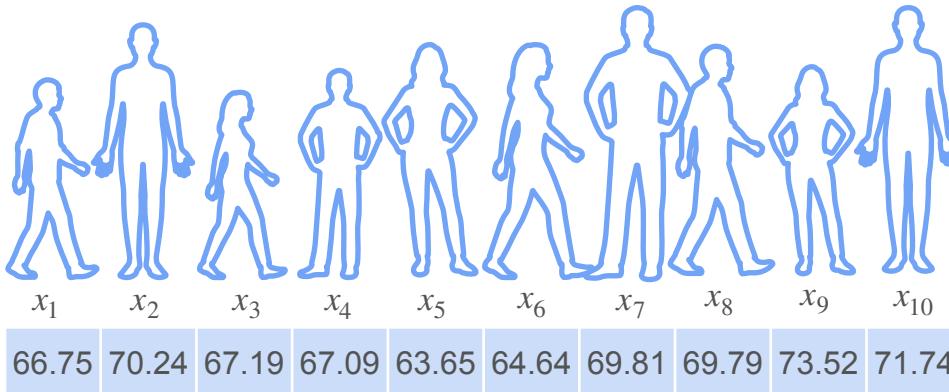
Determining the Hypothesis



The **mean** height for 18 y/o in the US in the 70s was **66.7 in.**

Population $H_0 : \mu = 66.7$ vs. $H_1 : \mu > 66.7$

Test Statistic



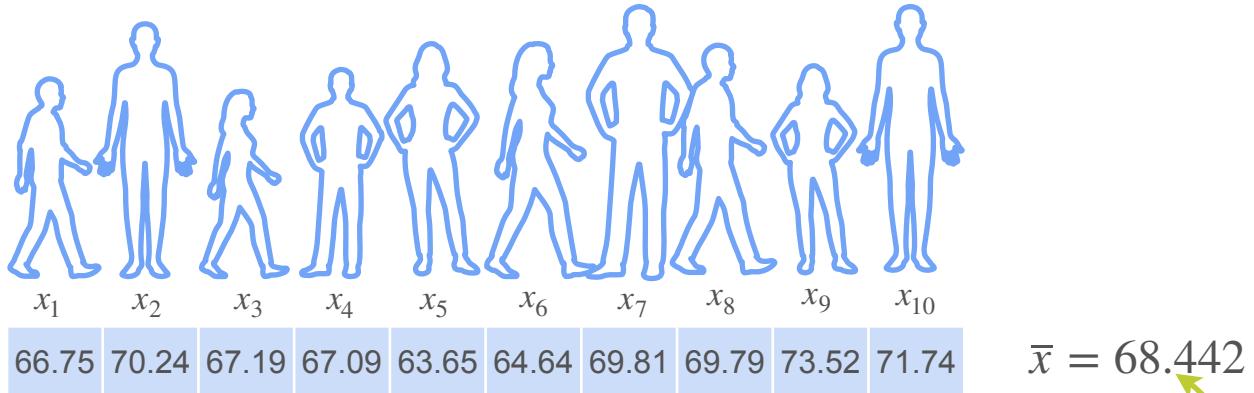
$$\bar{x} = 68.442$$

Observed statistic

$$\text{vs. } H_1 : \mu > 66.7$$

Test statistic $\longrightarrow \bar{X} = \frac{1}{10} \sum_{i=1}^{10} X_i$

Test Statistic



The **mean** height for 18 y/o in the US in the 70s was **66.7 in.**

$$H_0 : \mu = 66.7 \text{ vs. } H_1 : \mu > 66.7$$

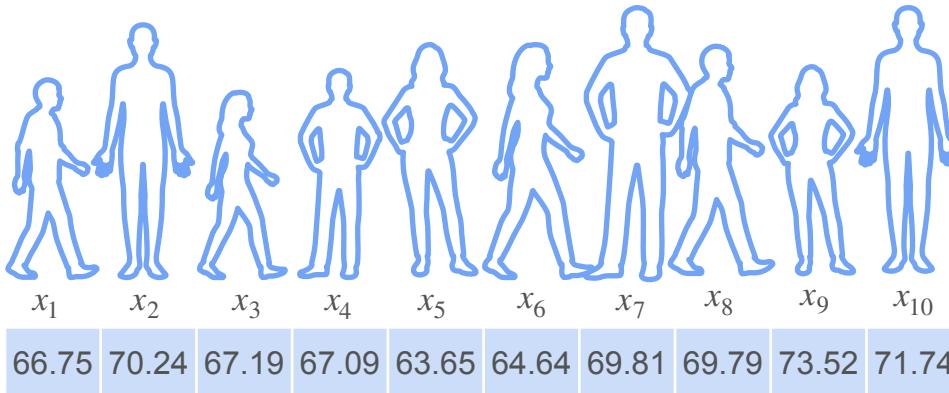
Observed statistic

Test statistic $\longrightarrow \bar{X} = \frac{1}{10} \sum_{i=1}^{10} X_i$

Test Statistic

Test statistic

$$\bar{X} = \frac{1}{10} \sum_{i=1}^{10} X_i$$

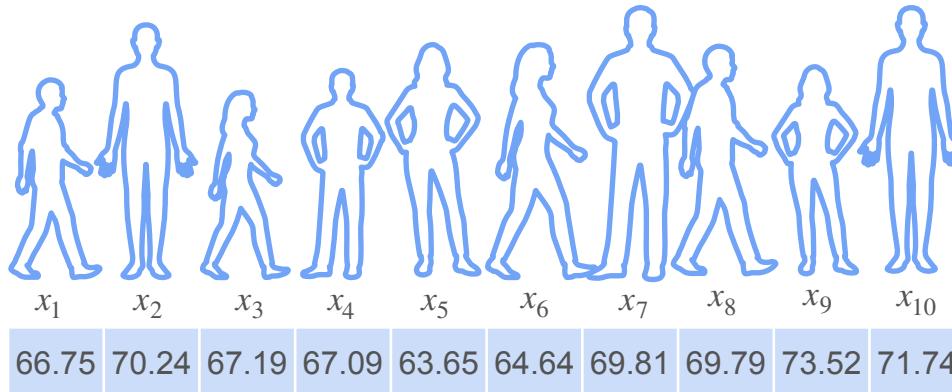


$$\bar{x} = 68.442$$

Test Statistic

Test statistic

$$\bar{X} = \frac{1}{10} \sum_{i=1}^{10} X_i$$



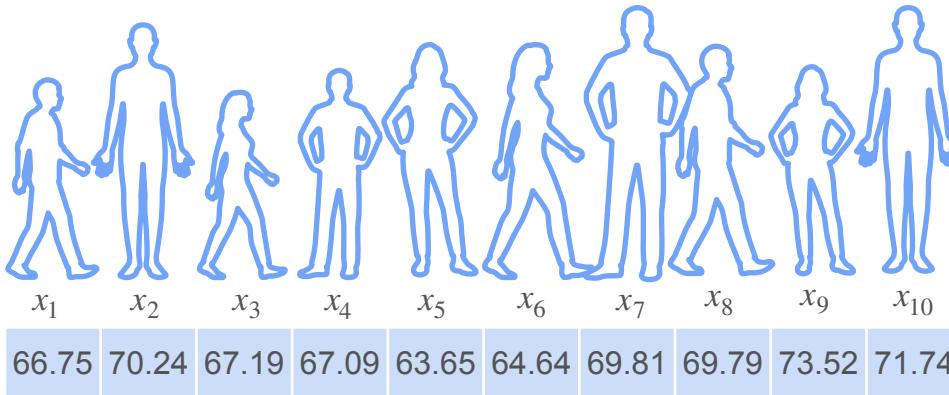
$$\bar{x} = 68.442$$

Test statistic: $T(X)$ $X = (X_1, \dots, X_n)$

Test Statistic

Test statistic

$$\bar{X} = \frac{1}{10} \sum_{i=1}^{10} X_i$$



$$\bar{x} = 68.442$$

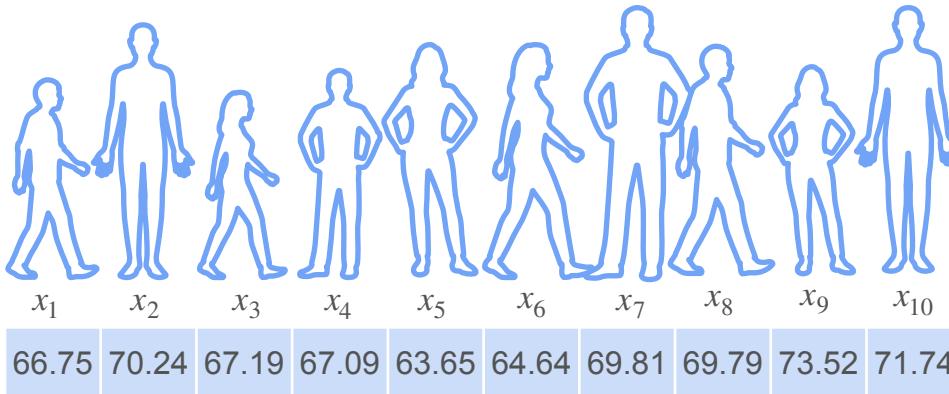
Test statistic: $T(X)$ $X = (X_1, \dots, X_n)$

Information about the population parameter under study

Test Statistic

Test statistic

$$\bar{X} = \frac{1}{10} \sum_{i=1}^{10} X_i$$



$$\bar{x} = 68.442$$

Test statistic: $T(X)$ $X = (X_1, \dots, X_n)$

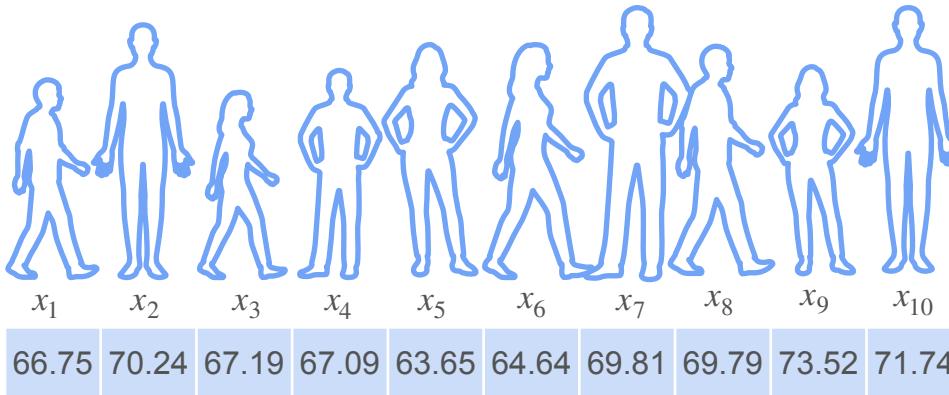
Information about the population parameter under study

$$\mu \rightarrow \bar{X}$$

Test Statistic

Test statistic

$$\bar{X} = \frac{1}{10} \sum_{i=1}^{10} X_i$$



$$\bar{x} = 68.442$$

Test statistic: $T(X)$ $X = (X_1, \dots, X_n)$

Information about the population parameter under study

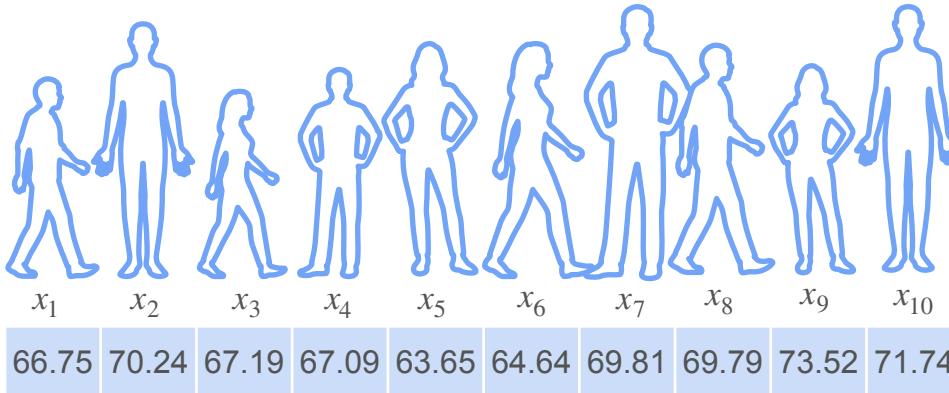
$$\mu \rightarrow \bar{X}$$

$$p \rightarrow \bar{X}$$

Test Statistic

Test statistic

$$\bar{X} = \frac{1}{10} \sum_{i=1}^{10} X_i$$



$$\bar{x} = 68.442$$

Test statistic: $T(X)$ $X = (X_1, \dots, X_n)$

Information about the population parameter under study

$$\mu \rightarrow \bar{X}$$

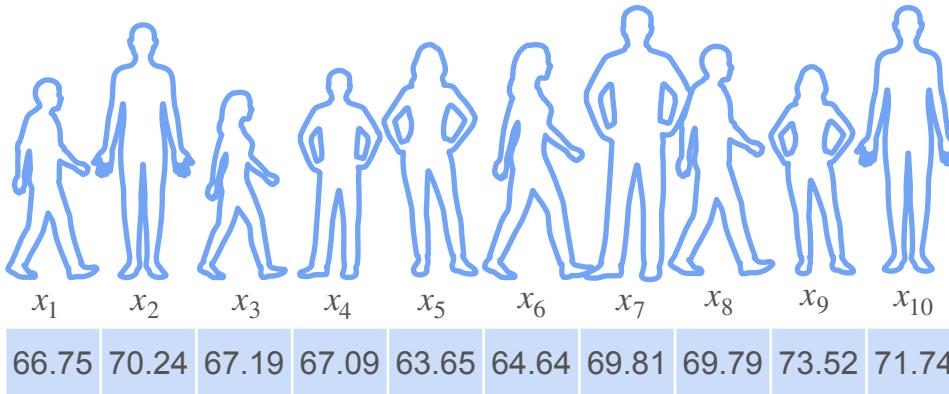
$$p \rightarrow \bar{X}$$

$$\sigma^2 \rightarrow S^2 = \frac{1}{n-1} \sum_{i=1}^n (X_i - \bar{X})^2$$

Test Statistic

Test statistic

$$\bar{X} = \frac{1}{10} \sum_{i=1}^{10} X_i$$



$$\bar{x} = 68.442$$

Test statistic: $T(X)$ $X = (X_1, \dots, X_n)$

Information about the population parameter under study

$$\mu \rightarrow \bar{X}$$

$$p \rightarrow \bar{X}$$

Not unique!

$$\sigma^2 \rightarrow S^2 = \frac{1}{n-1} \sum_{i=1}^n (X_i - \bar{X})^2$$

Example: Heights

Example: Heights

The **mean** height for 18 y/o in the US in the 70s was **66.7 in.**

Example: Heights

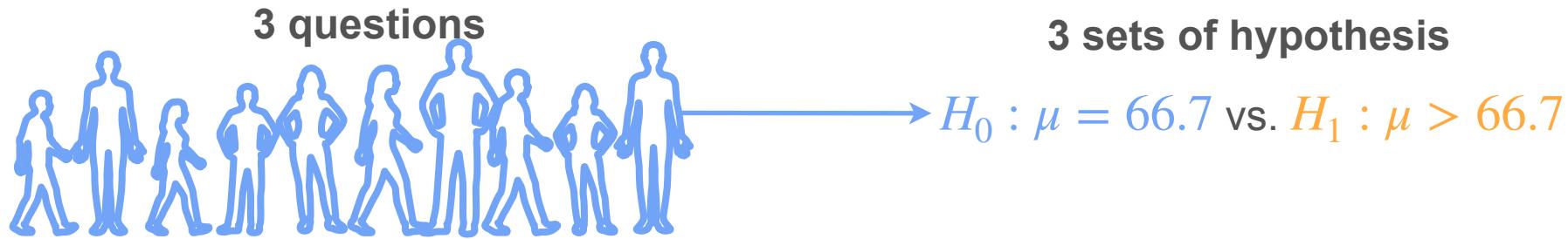
The **mean** height for 18 y/o in the US in the 70s was **66.7 in.**

3 questions

3 sets of hypothesis

Example: Heights

The **mean** height for 18 y/o in the US in the 70s was **66.7 in.**



Example: Heights

The **mean** height for 18 y/o in the US in the 70s was **66.7 in.**

3 questions

Right-Tailed Test

3 sets of hypothesis

$$\longrightarrow H_0 : \mu = 66.7 \text{ vs. } H_1 : \mu > 66.7$$

Example: Heights

The **mean** height for 18 y/o in the US in the 70s was **66.7 in.**

3 questions

Right-Tailed Test

3 sets of hypothesis

$$\longrightarrow H_0 : \mu = 66.7 \text{ vs. } H_1 : \mu > 66.7$$



$$\longrightarrow H_0 : \mu = 66.7 \text{ vs. } H_1 : \mu < 66.7$$

Example: Heights

The **mean** height for 18 y/o in the US in the 70s was **66.7 in.**

3 questions

Right-Tailed Test

3 sets of hypothesis

$$\longrightarrow H_0 : \mu = 66.7 \text{ vs. } H_1 : \mu > 66.7$$

Left Tailed Test

$$\longrightarrow H_0 : \mu = 66.7 \text{ vs. } H_1 : \mu < 66.7$$

Example: Heights

The **mean** height for 18 y/o in the US in the 70s was **66.7 in.**

3 questions

Right-Tailed Test

3 sets of hypothesis

$$H_0 : \mu = 66.7 \text{ vs. } H_1 : \mu > 66.7$$

Left Tailed Test

$$H_0 : \mu = 66.7 \text{ vs. } H_1 : \mu < 66.7$$



$$H_0 : \mu = 66.7 \text{ vs. } H_1 : \mu \neq 66.7$$

Example: Heights

The **mean** height for 18 y/o in the US in the 70s was **66.7 in.**

3 questions

Right-Tailed Test

3 sets of hypothesis

$$\longrightarrow H_0 : \mu = 66.7 \text{ vs. } H_1 : \mu > 66.7$$

Left Tailed Test

$$\longrightarrow H_0 : \mu = 66.7 \text{ vs. } H_1 : \mu < 66.7$$

Two-Tailed Test

$$\longrightarrow H_0 : \mu = 66.7 \text{ vs. } H_1 : \mu \neq 66.7$$

Example: Heights

The **mean** height for 18 y/o in the US in the 70s was **66.7 in.**

$$H_0 : \mu = 66.7 \text{ vs. } H_1 : \mu > 66.7$$

Example: Heights

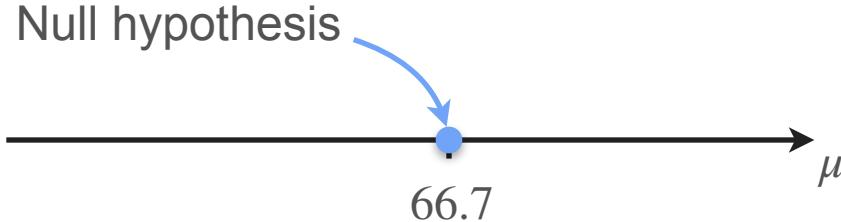
The **mean** height for 18 y/o in the US in the 70s was **66.7 in.**

Right-tailed test  $H_0 : \mu = 66.7$ vs. $H_1 : \mu > 66.7$

Example: Heights

The **mean** height for 18 y/o in the US in the 70s was **66.7 in.**

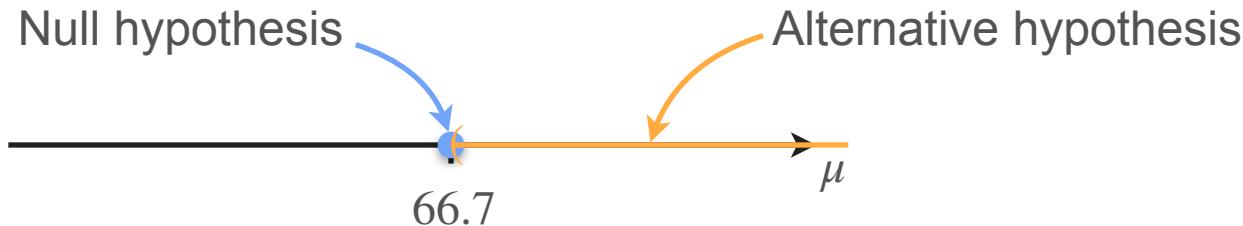
Right-tailed test $\longrightarrow H_0 : \mu = 66.7$ vs. $H_1 : \mu > 66.7$



Example: Heights

The **mean** height for 18 y/o in the US in the 70s was **66.7 in.**

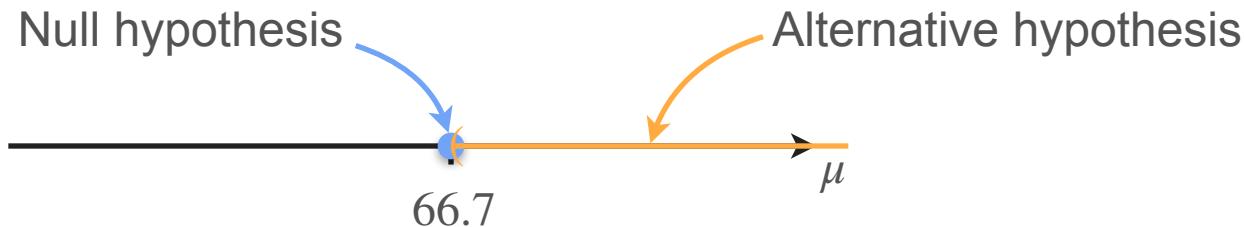
Right-tailed test $\longrightarrow H_0 : \mu = 66.7$ vs. $H_1 : \mu > 66.7$



Example: Heights

The **mean** height for 18 y/o in the US in the 70s was **66.7 in.**

Right-tailed test $\longrightarrow H_0 : \mu \leq 66.7$ vs. $H_1 : \mu > 66.7$

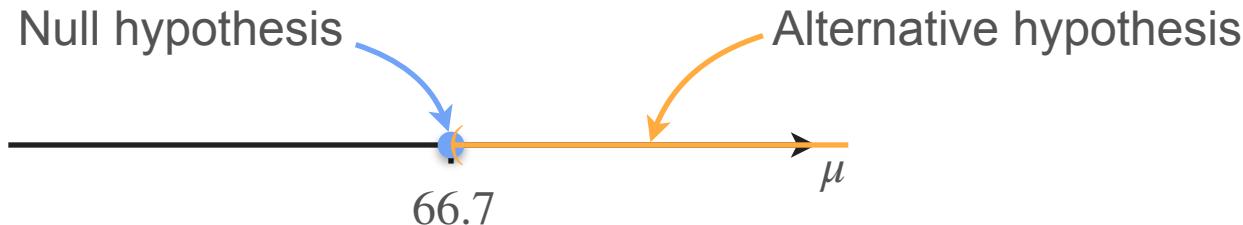


Example: Heights

The mean height for 18 y/o in the US in the 70s was **66.7 in.**

\bar{X} Test statistic

Right-tailed test $\longrightarrow H_0 : \mu = 66.7$ vs. $H_1 : \mu > 66.7$



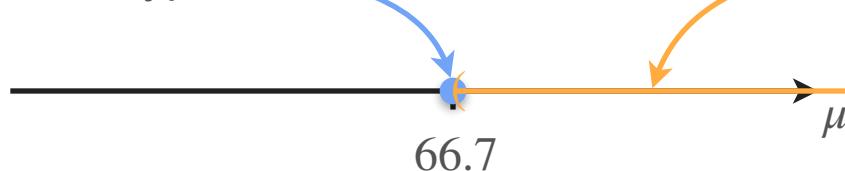
Example: Heights

The mean height for 18 y/o in the US in the 70s was **66.7 in.**

\bar{X} Test statistic

Right-tailed test $\longrightarrow H_0 : \mu = 66.7$ vs. $H_1 : \mu > 66.7$

Null hypothesis Alternative hypothesis



If $\bar{x} \gg 66.7 \Rightarrow$ Reject H_0

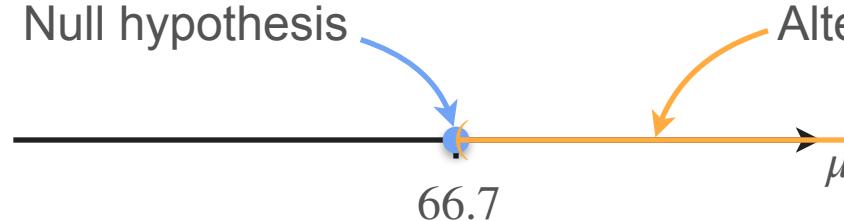
Example: Heights

The mean height for 18 y/o in the US in the 70s was 66.7 in.

\bar{X} Test statistic

Right-tailed test $\longrightarrow H_0 : \mu = 66.7$ vs. $H_1 : \mu > 66.7$

Null hypothesis



Type I error: Determine $\mu > 66.7$, when population mean did not change

If $\bar{x} \gg 66.7 \Rightarrow$ Reject H_0

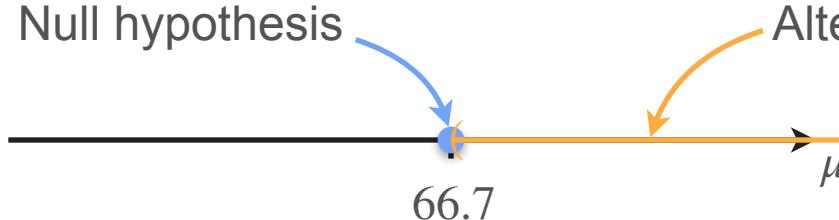
Example: Heights

The mean height for 18 y/o in the US in the 70s was **66.7 in.**

\bar{X} Test statistic

Right-tailed test $\longrightarrow H_0 : \mu = 66.7$ vs. $H_1 : \mu > 66.7$

Null hypothesis



Type I error: Determine $\mu > 66.7$, when population mean did not change

If $\bar{x} \gg 66.7 \Rightarrow$ Reject H_0

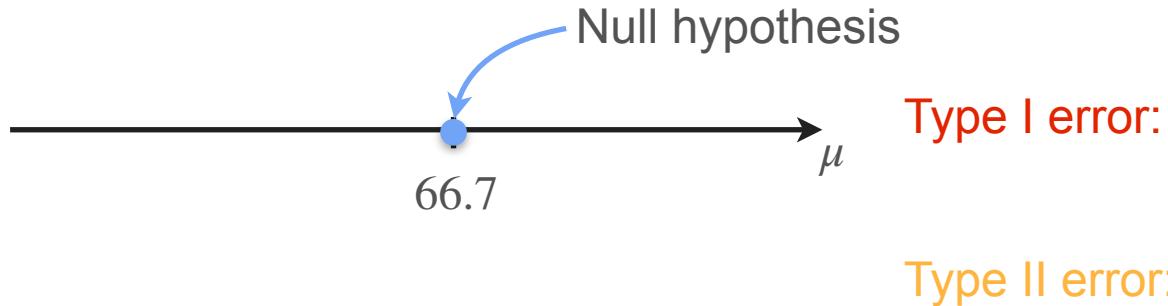
Type II error: Do not reject that $\mu = 66.7$ when in true $\mu > 66.7$

Example: Heights

The mean height for 18 y/o in the US in the 70s was **66.7 in.**

$$\bar{X}$$

$$H_0 : \mu = 66.7 \text{ vs. } H_1 : \mu < 66.7$$



Example: Heights

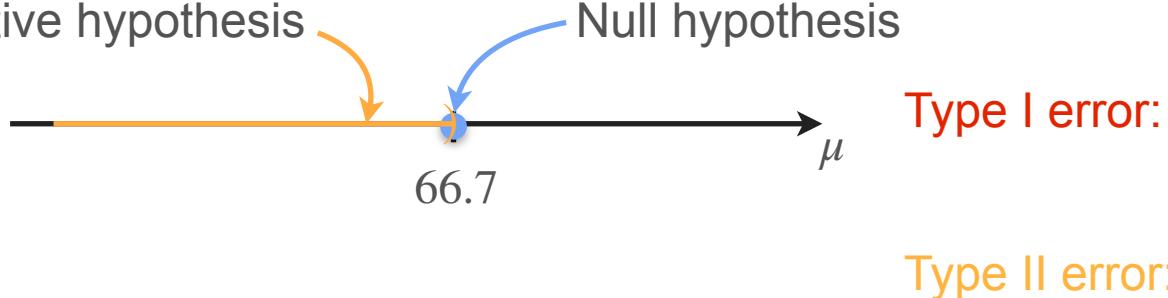
The mean height for 18 y/o in the US in the 70s was **66.7 in.**

$$\bar{X}$$

Left tailed test $\longrightarrow H_0 : \mu = 66.7$ vs. $H_1 : \mu < 66.7$

Alternative hypothesis

Null hypothesis



Example: Heights

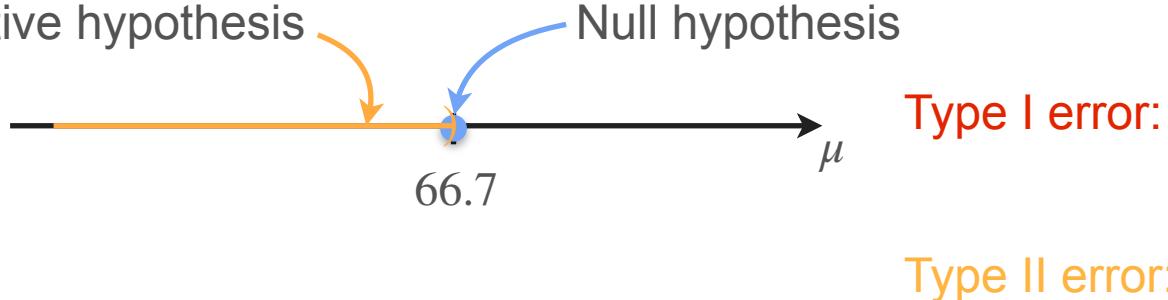
The mean height for 18 y/o in the US in the 70s was **66.7 in.**

$$\bar{X}$$

Left tailed test $\longrightarrow H_0 : \mu \geq 66.7$ vs. $H_1 : \mu < 66.7$

Alternative hypothesis

Null hypothesis



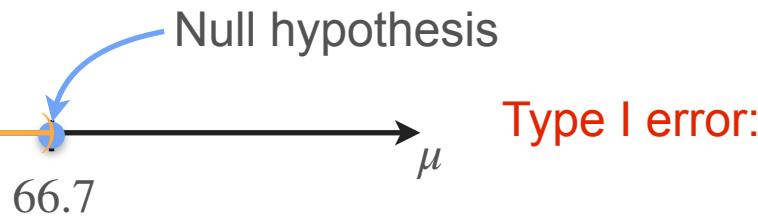
Example: Heights

The **mean** height for 18 y/o in the US in the 70s was **66.7 in.**

$$\bar{X}$$

Left tailed test $\longrightarrow H_0 : \mu = 66.7$ vs. $H_1 : \mu < 66.7$

Alternative hypothesis



Type I error:

If $\bar{x} \ll 66.7 \Rightarrow \text{Reject } H_0$

Type II error:

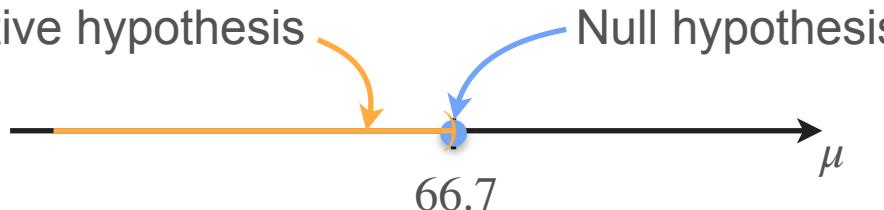
Example: Heights

The **mean** height for 18 y/o in the US in the 70s was **66.7 in.**

$$\bar{X}$$

Left tailed test $\longrightarrow H_0 : \mu = 66.7$ vs. $H_1 : \mu < 66.7$

Alternative hypothesis Null hypothesis



Type I error: Determine $\mu < 66.7$, when population mean did not change

If $\bar{x} \ll 66.7 \Rightarrow$ Reject H_0

Type II error:

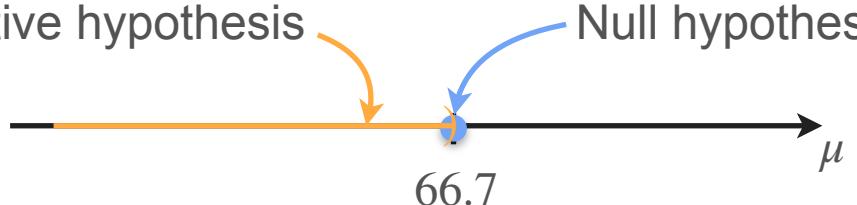
Example: Heights

The **mean** height for 18 y/o in the US in the 70s was **66.7 in.**

$$\bar{X}$$

Left tailed test $\longrightarrow H_0 : \mu = 66.7$ vs. $H_1 : \mu < 66.7$

Alternative hypothesis Null hypothesis



If $\bar{x} \ll 66.7 \Rightarrow$ Reject H_0

Type I error: Determine $\mu < 66.7$, when population mean did not change

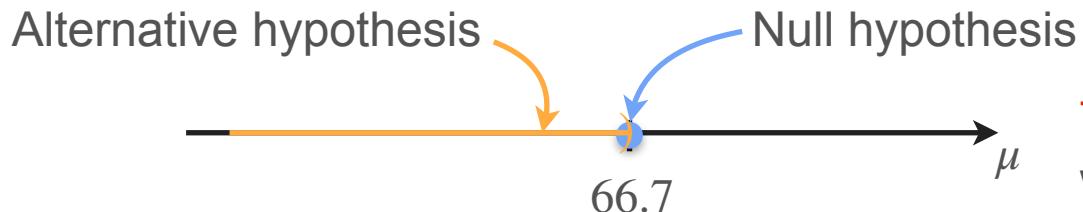
Type II error: Don't reject that $\mu = 66.7$ when true $\mu < 66.7$

Example: Heights

The mean height for 18 y/o in the US in the 70s was 66.7 in.

\bar{X} Test statistic

Left tailed test $\longrightarrow H_0 : \mu = 66.7$ vs. $H_1 : \mu < 66.7$



If $\bar{x} \ll 66.7 \Rightarrow$ Reject H_0

Type I error: Determine $\mu < 66.7$, when population mean did not change

Type II error: Don't reject that $\mu = 66.7$ when true $\mu < 66.7$

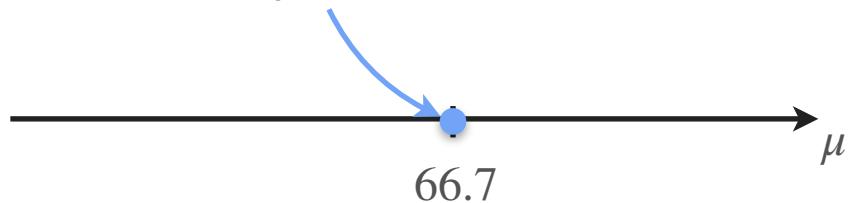
Example: Heights

The **mean** height for 18 y/o in the US in the 70s was **66.7 in.**

$$\bar{X}$$

$$H_0 : \mu = 66.7 \text{ vs. } H_1 : \mu \neq 66.7$$

Null hypothesis



Type I error:

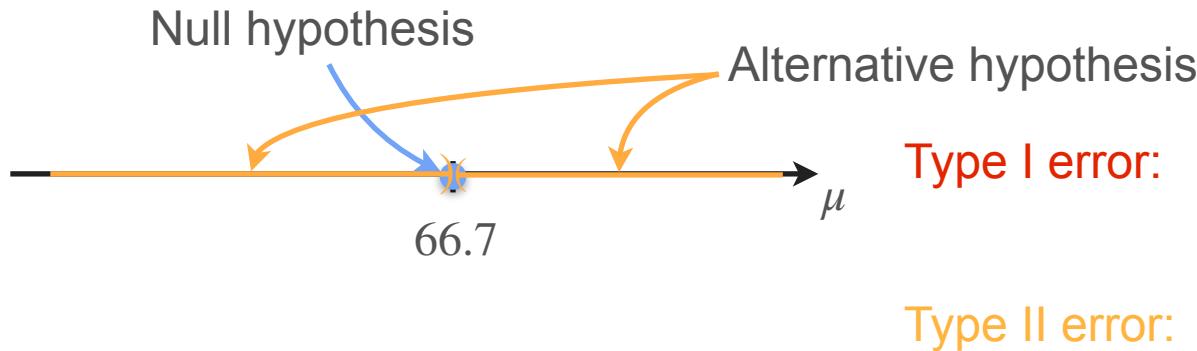
Type II error:

Example: Heights

The mean height for 18 y/o in the US in the 70s was **66.7 in.**

$$\bar{X}$$

Two tailed test $\longrightarrow H_0 : \mu = 66.7$ vs. $H_1 : \mu \neq 66.7$



Example: Heights

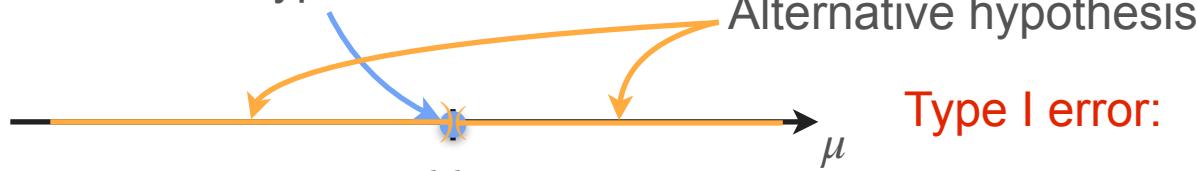
The mean height for 18 y/o in the US in the 70s was **66.7 in.**

$$\bar{X}$$

Two tailed test $\longrightarrow H_0 : \mu = 66.7$ vs. $H_1 : \mu \neq 66.7$

Null hypothesis

Alternative hypothesis



$$\bar{x} \gg 66.7$$

If or \Rightarrow Reject H_0

$$\bar{x} \ll 66.7$$

Type I error:

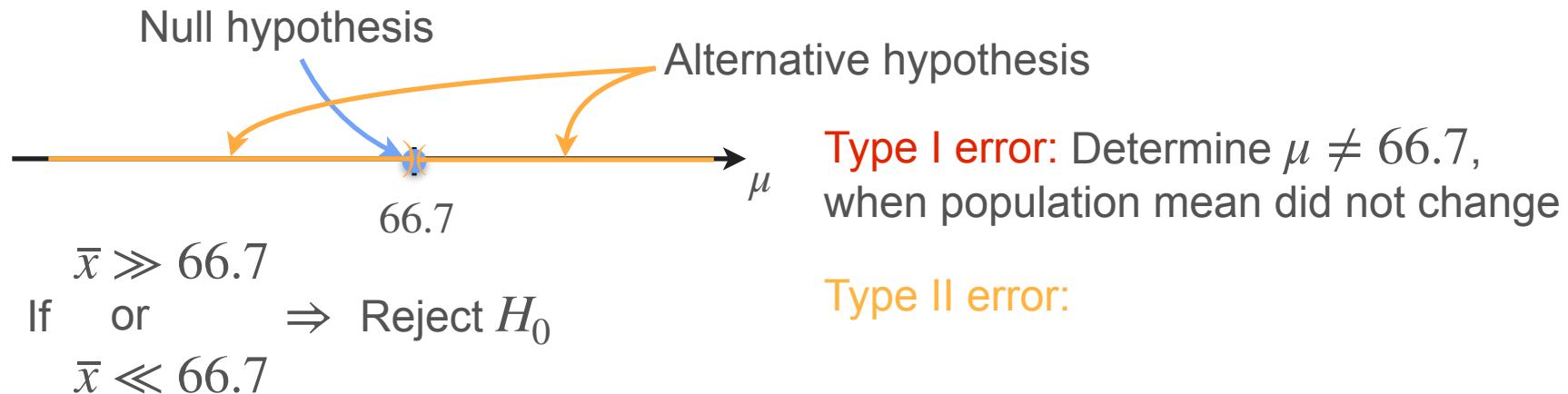
Type II error:

Example: Heights

The mean height for 18 y/o in the US in the 70s was **66.7 in.**

$$\bar{X}$$

Two tailed test $\longrightarrow H_0 : \mu = 66.7$ vs. $H_1 : \mu \neq 66.7$



Type I error: Determine $\mu \neq 66.7$, when population mean did not change

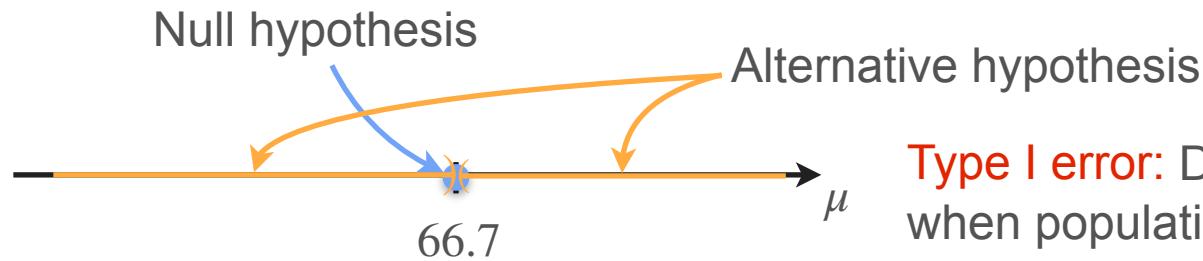
Type II error:

Example: Heights

The mean height for 18 y/o in the US in the 70s was **66.7 in.**

$$\bar{X}$$

Two tailed test $\longrightarrow H_0 : \mu = 66.7$ vs. $H_1 : \mu \neq 66.7$



Type I error: Determine $\mu \neq 66.7$, when population mean did not change

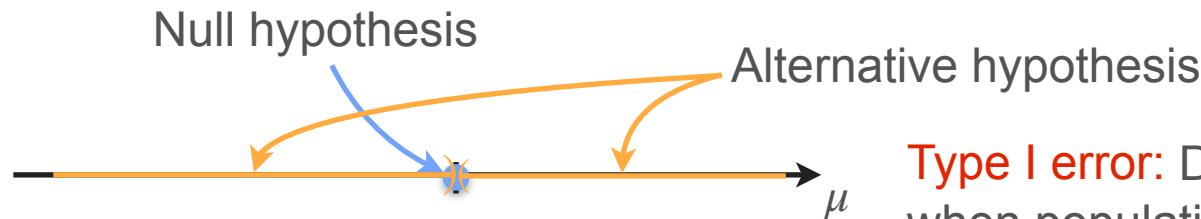
Type II error: Don't reject that $\mu = 66.7$ when true $\mu \neq 66.7$

Example: Heights

The mean height for 18 y/o in the US in the 70s was **66.7 in.**

\bar{X} Test statistic

Two tailed test $\longrightarrow H_0 : \mu = 66.7$ vs. $H_1 : \mu \neq 66.7$



If $\bar{x} \gg 66.7$ or $\bar{x} \ll 66.7 \Rightarrow$ Reject H_0

Type I error: Determine $\mu \neq 66.7$, when population mean did not change

Type II error: Don't reject that $\mu = 66.7$ when true $\mu \neq 66.7$



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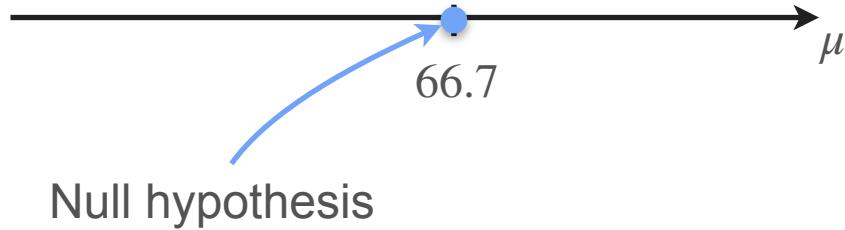
Hypothesis Testing

p -Value

Example: Heights

Example: Heights

The **mean** height for 18 y/o in the US in the 70s was **66.7 in.**

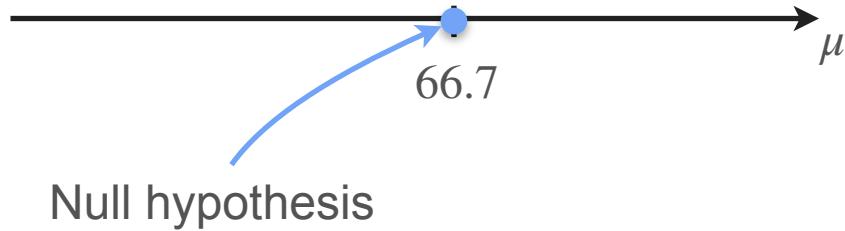


Example: Heights

The **mean** height for 18 y/o in the US in the 70s was **66.7 in.**

$$\sigma = 3$$

$$n = 10$$



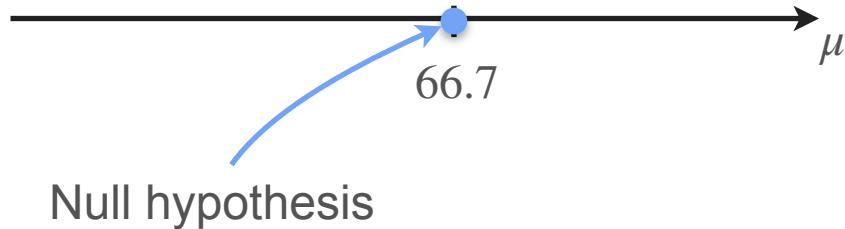
Example: Heights

The **mean** height for 18 y/o in the US in the 70s was **66.7 in.**

$$\sigma = 3$$

$$n = 10$$

If H_0 is true:

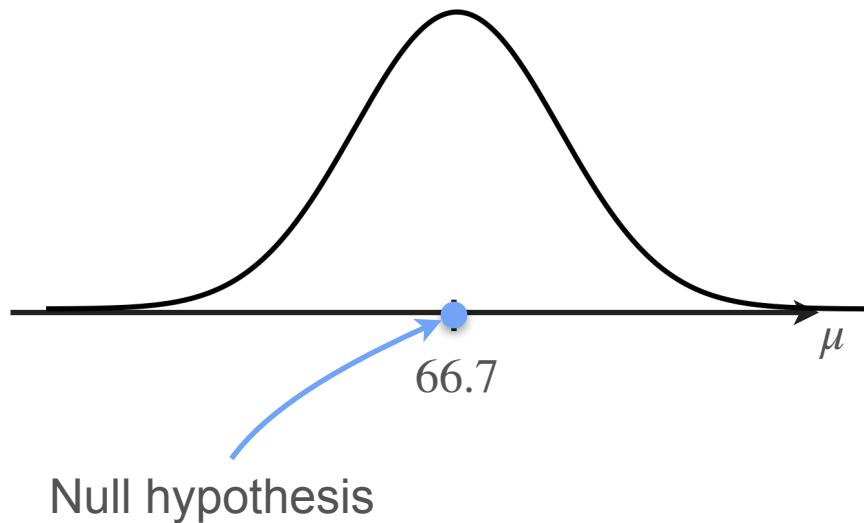


Example: Heights

The mean height for 18 y/o in the US in the 70s was **66.7** in.

$$\sigma = 3$$
$$n = 10$$

If H_0 is true: $\bar{X} \sim \mathcal{N}\left(\quad, \quad \right)$

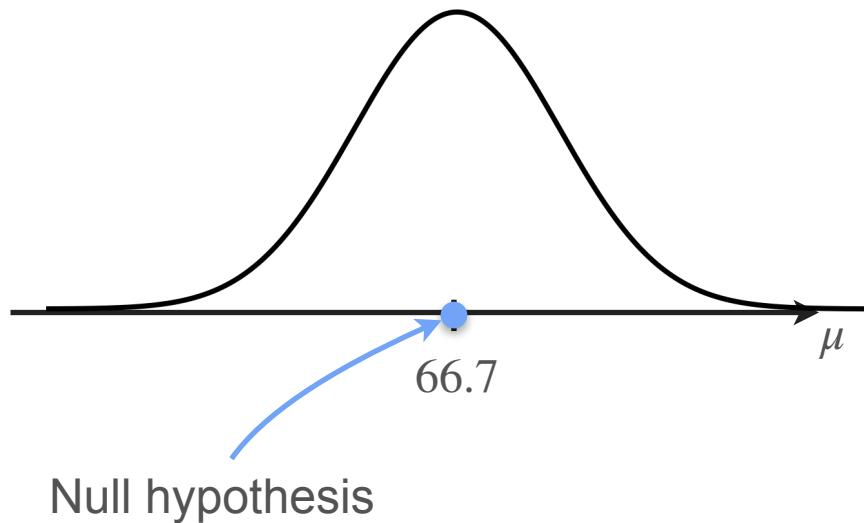


Example: Heights

The mean height for 18 y/o in the US in the 70s was **66.7** in.

$$\begin{aligned}\sigma &= 3 \\ n &= 10\end{aligned}$$

If H_0 is true: $\bar{X} \sim \mathcal{N}\left(66.7, \frac{3^2}{10}\right)$

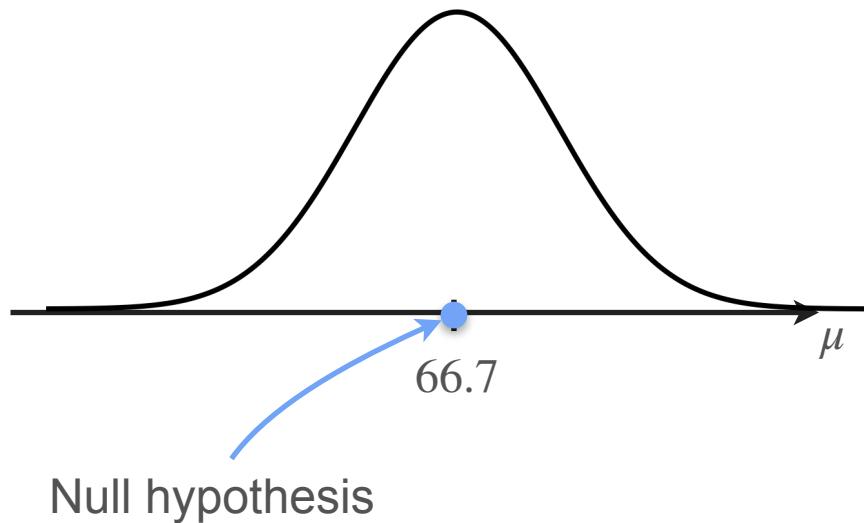


Example: Heights

The mean height for 18 y/o in the US in the 70s was **66.7** in.

$$\begin{aligned}\sigma &= 3 \\ n &= 10\end{aligned}$$

If H_0 is true: $\bar{X} \sim \mathcal{N}\left(66.7, \frac{3^2}{10}\right)$



How likely was your sample if H_0 is true?

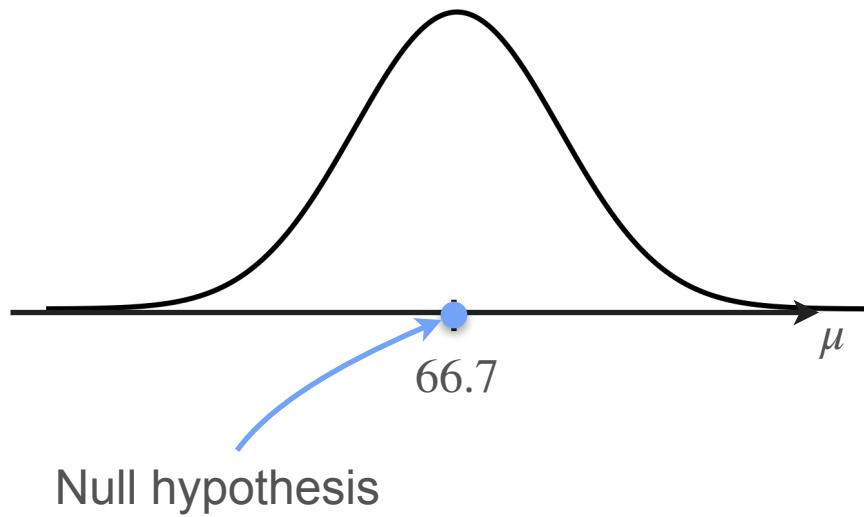
If the answer is very unlikely, then reject H_0

Right-Tailed Test for Gaussian Data (Known σ)

The mean height for 18 y/o in the US in the 70s was **66.7** in.

$$\sigma = 3$$

$$n = 10$$



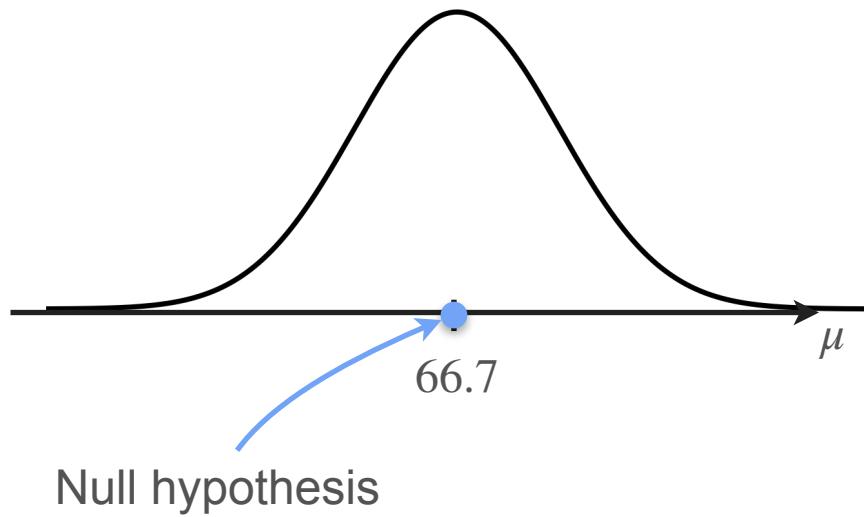
Right-Tailed Test for Gaussian Data (Known σ)

The mean height for 18 y/o in the US in the 70s was **66.7** in.

$$\sigma = 3$$

$$n = 10$$

$$H_0 : \mu = 66.7 \text{ vs. } H_1 : \mu > 66.7$$



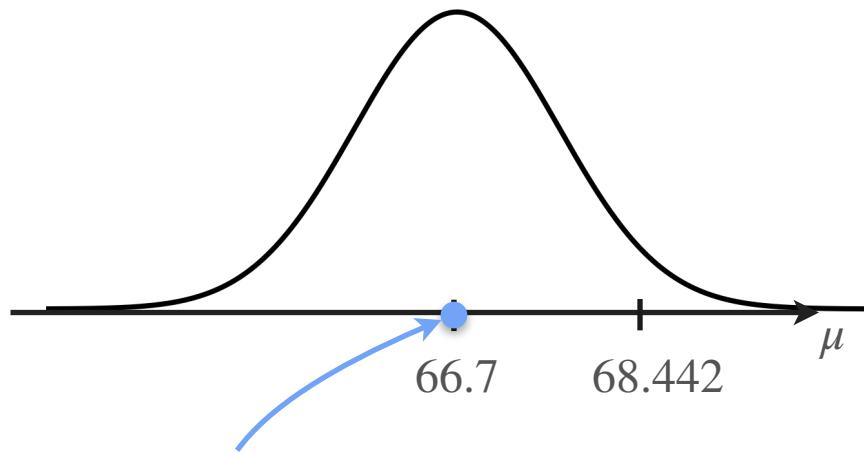
Right-Tailed Test for Gaussian Data (Known σ)

The mean height for 18 y/o in the US in the 70s was **66.7** in.

$$\sigma = 3$$
$$n = 10$$

$$\bar{x} = 68.442$$

$$H_0 : \mu = 66.7 \text{ vs. } H_1 : \mu > 66.7$$



Null hypothesis

Right-Tailed Test for Gaussian Data (Known σ)

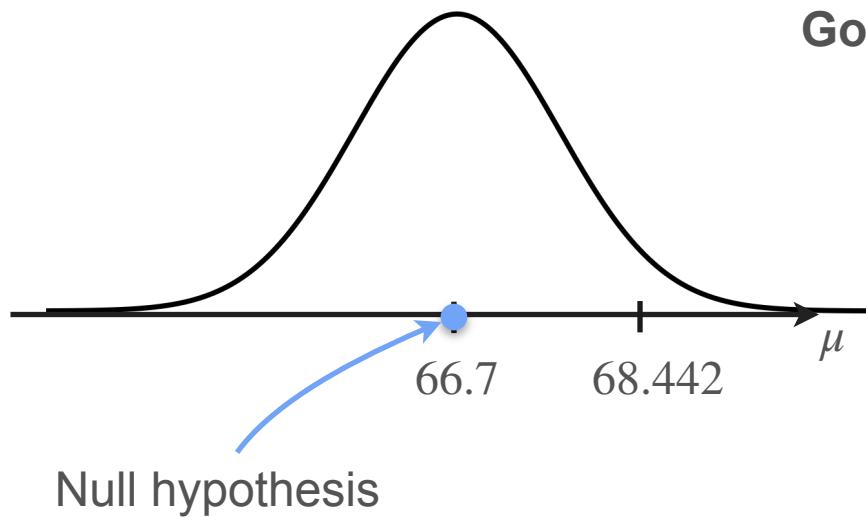
The mean height for 18 y/o in the US in the 70s was **66.7** in.

$$\begin{aligned}\sigma &= 3 \\ n &= 10\end{aligned}$$

$$\bar{x} = 68.442$$

$$H_0 : \mu = 66.7 \text{ vs. } H_1 : \mu > 66.7$$

Goal: Type I error probability $< \alpha = 0.05$



Right-Tailed Test for Gaussian Data (Known σ)

The mean height for 18 y/o in the US in the 70s was **66.7** in.

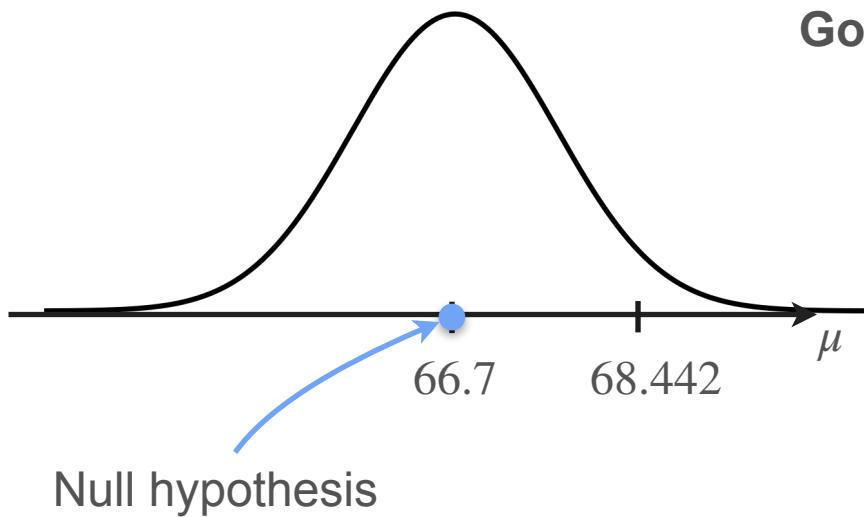
$$\begin{aligned}\sigma &= 3 \\ n &= 10\end{aligned}$$

$$\bar{x} = 68.442$$

$$H_0 : \mu = 66.7 \text{ vs. } H_1 : \mu > 66.7$$

Goal: Type I error probability $< \alpha = 0.05$

Type I error: Determine $\mu > 66.7$,
when population mean did not change



Right-Tailed Test for Gaussian Data (Known σ)

The mean height for 18 y/o in the US in the 70s was **66.7** in.

$$\begin{aligned}\sigma &= 3 \\ n &= 10\end{aligned}$$

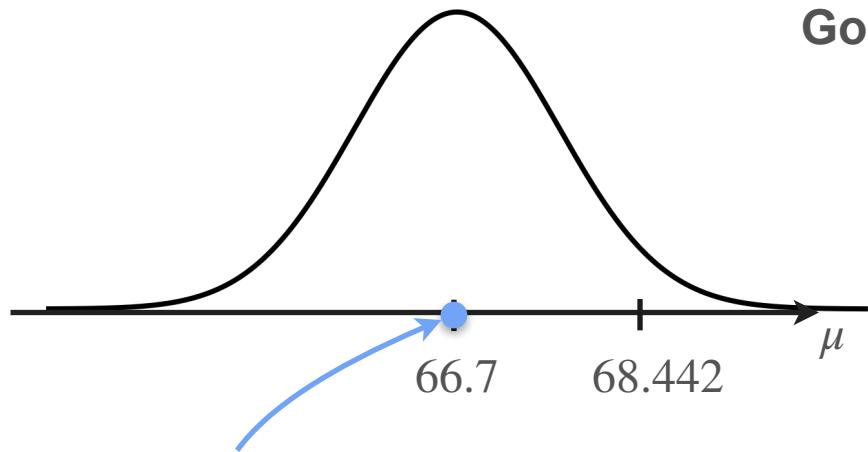
$$\bar{x} = 68.442$$

$$H_0 : \mu = 66.7 \text{ vs. } H_1 : \mu > 66.7$$

Goal: Type I error probability $< \alpha = 0.05$

Type I error: Determine $\mu > 66.7$,
when population mean did not change

$$P(\bar{X} > 68.442 \mid \mu = 66.7) ?$$



Null hypothesis

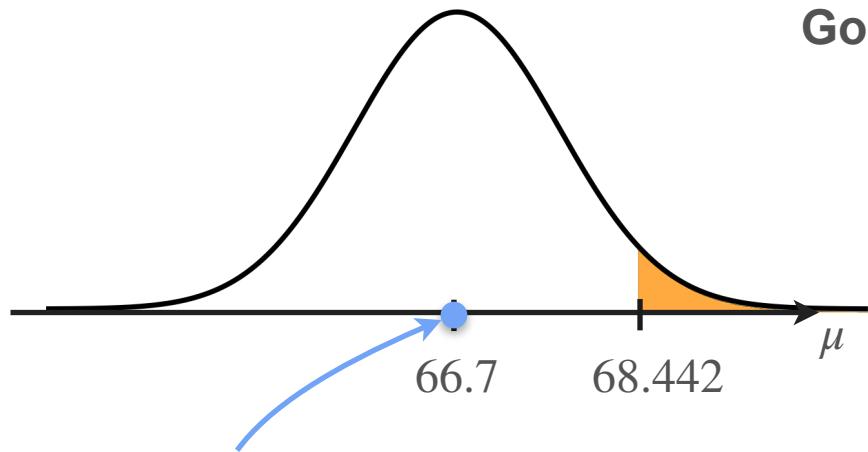
Right-Tailed Test for Gaussian Data (Known σ)

The mean height for 18 y/o in the US in the 70s was **66.7** in.

$$\begin{aligned}\sigma &= 3 \\ n &= 10\end{aligned}$$

$$\bar{x} = 68.442$$

$$H_0 : \mu = 66.7 \text{ vs. } H_1 : \mu > 66.7$$



Goal: Type I error probability $< \alpha = 0.05$

Type I error: Determine $\mu > 66.7$, when population mean did not change

$$\begin{aligned}P(\bar{X} > 68.442 \mid \mu = 66.7) \\ = 0.0407\end{aligned}$$

Null hypothesis

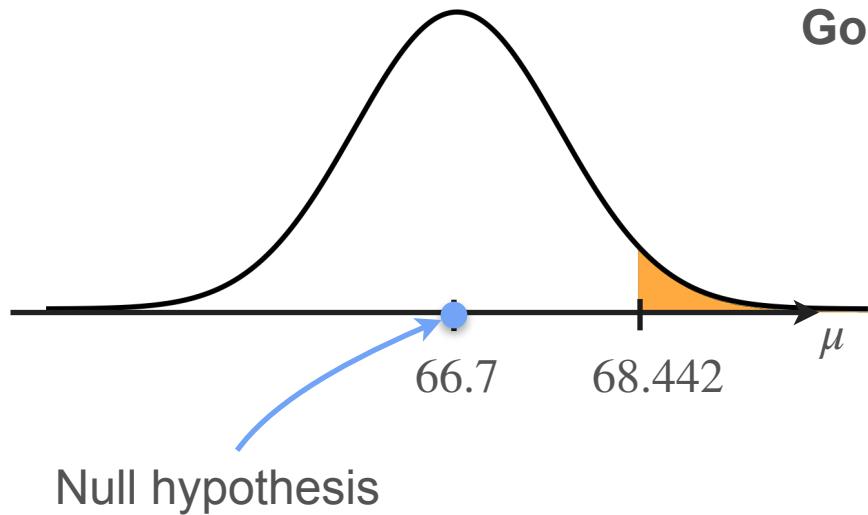
Right-Tailed Test for Gaussian Data (Known σ)

The mean height for 18 y/o in the US in the 70s was **66.7** in.

$$\begin{aligned}\sigma &= 3 \\ n &= 10\end{aligned}$$

$$\bar{x} = 68.442$$

$$H_0 : \mu = 66.7 \text{ vs. } H_1 : \mu > 66.7$$



Goal: Type I error probability $< \alpha = 0.05$

Type I error: Determine $\mu > 66.7$, when population mean did not change

$$\begin{aligned}P(\bar{X} > 68.442 \mid \mu = 66.7) \\ = 0.0407 < \alpha\end{aligned}$$

Conclusion: reject H_0
(with a 5% significance level)

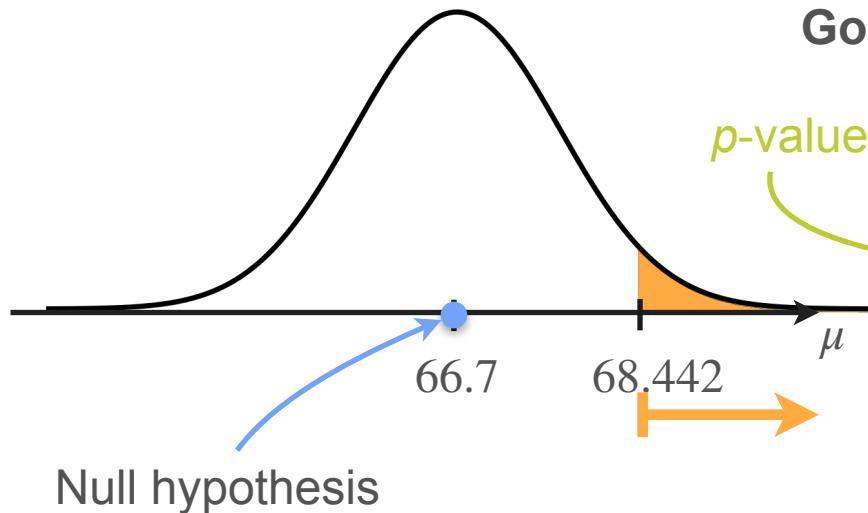
Right-Tailed Test for Gaussian Data (Known σ)

The mean height for 18 y/o in the US in the 70s was **66.7** in.

$$\begin{aligned}\sigma &= 3 \\ n &= 10\end{aligned}$$

$$\bar{x} = 68.442$$

$$H_0 : \mu = 66.7 \text{ vs. } H_1 : \mu > 66.7$$



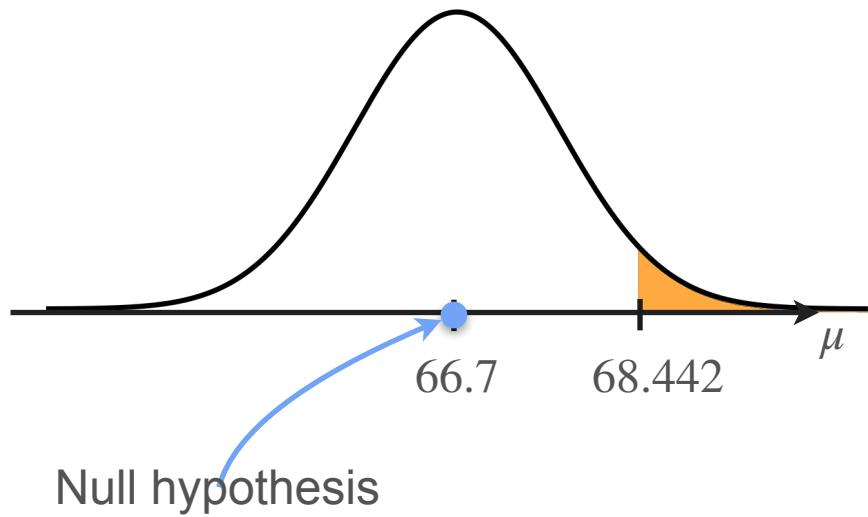
Goal: Type I error probability $< \alpha = 0.05$

Type I error: Determine $\mu > 66.7$, when population mean did not change

$$\begin{aligned}P(\bar{X} > 68.442 \mid \mu = 66.7) \\ = 0.0407 < \alpha\end{aligned}$$

Conclusion: reject H_0
(with a 5% significance level)

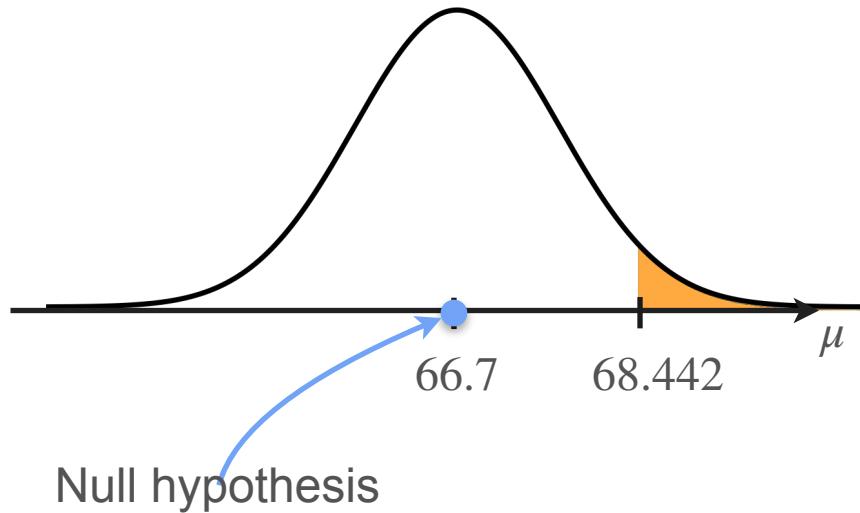
P-Values



$$H_0 : \mu = 66.7 \text{ vs. } H_1 : \mu > 66.7$$

P-Values

A **p-value** is the probability, assuming H_0 is true, that the test statistic takes on a value as extreme as or more extreme than the value observed



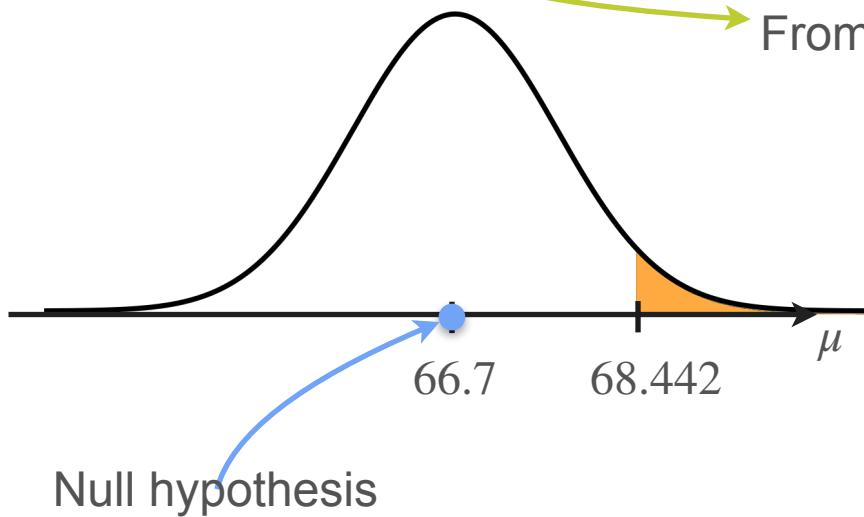
$$H_0 : \mu = 66.7 \text{ vs. } H_1 : \mu > 66.7$$

P-Values

A **p-value** is the probability, assuming H_0 is true, that the test statistic takes on a value **as extreme as or more extreme than** the value observed

From the observed value to the direction of H_1

$$H_0 : \mu = 66.7 \text{ vs. } H_1 : \mu > 66.7$$

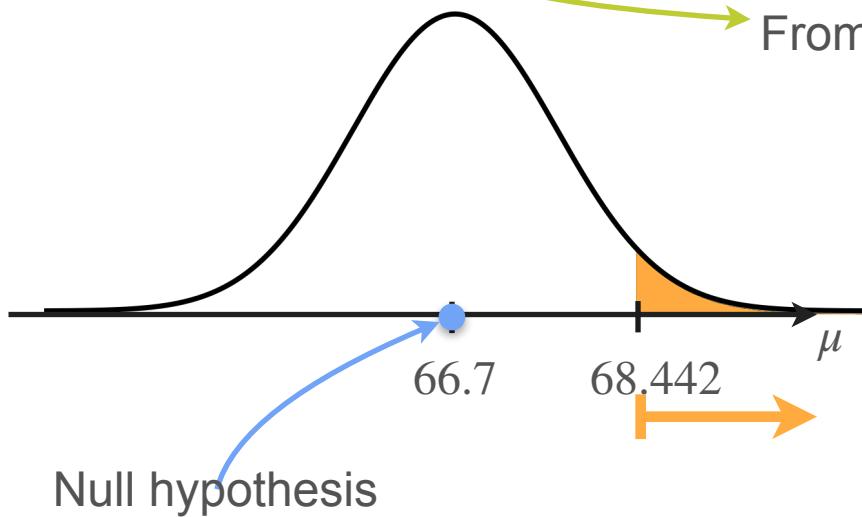


P-Values

A **p-value** is the probability, assuming H_0 is true, that the test statistic takes on a value **as extreme as or more extreme than** the value observed

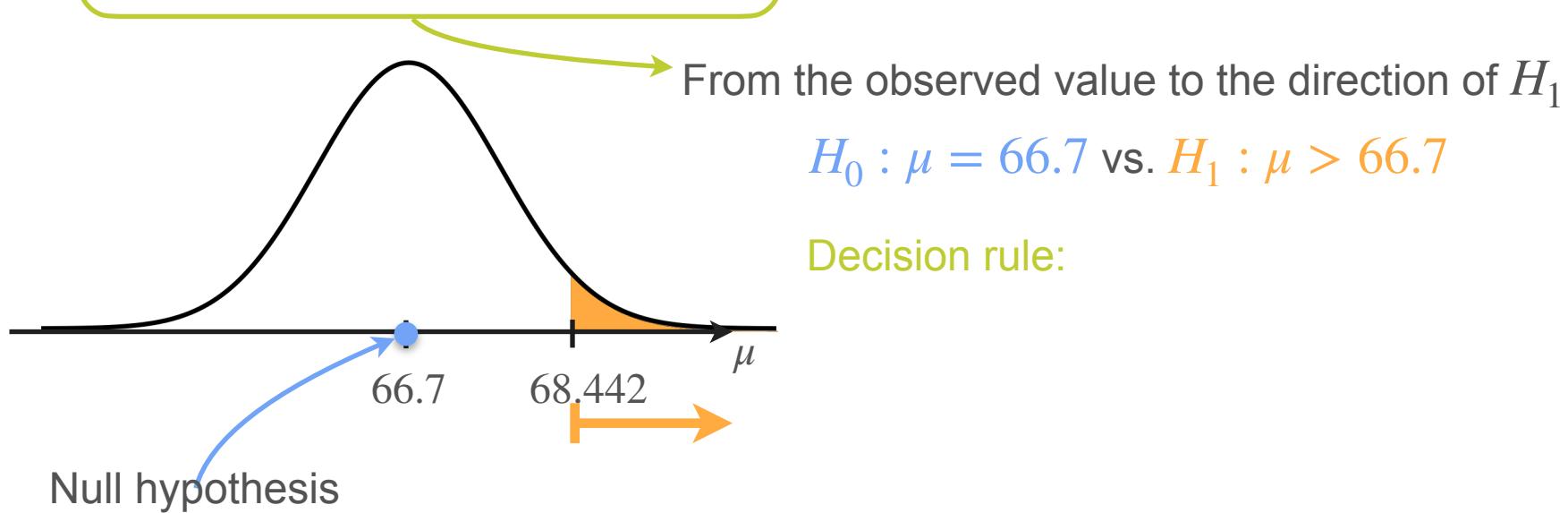
From the observed value to the direction of H_1

$$H_0 : \mu = 66.7 \text{ vs. } H_1 : \mu > 66.7$$



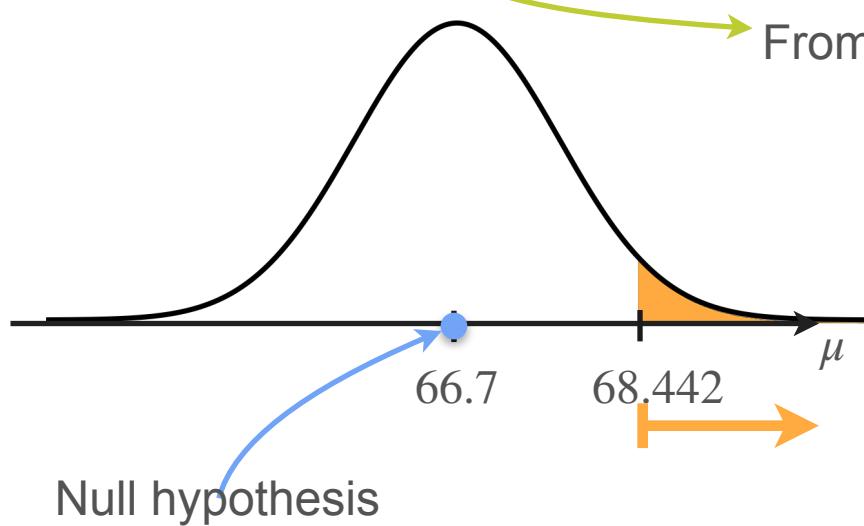
P-Values

A **p-value** is the probability, assuming H_0 is true, that the test statistic takes on a value **as extreme as or more extreme than** the value observed



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A **p-value** is the probability, assuming H_0 is true, that the test statistic takes on a value **as extreme as or more extreme than** the value observed



From the observed value to the direction of H_1

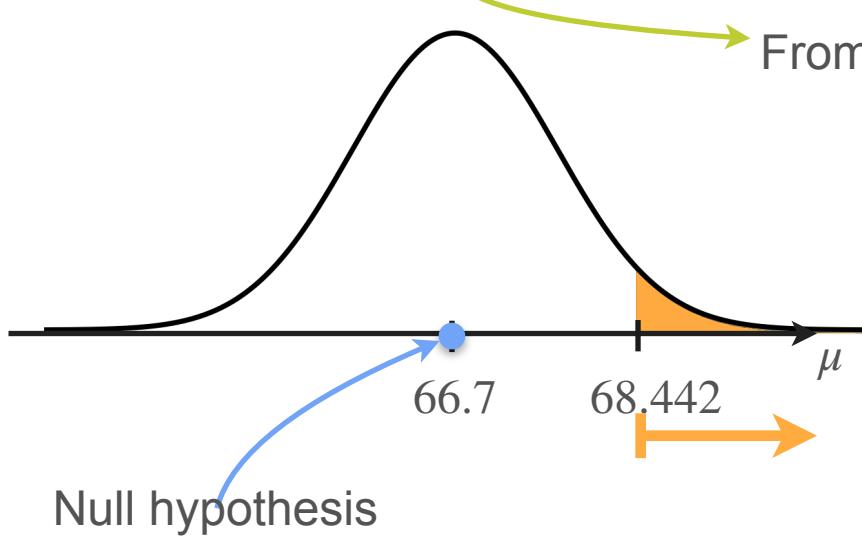
$$H_0 : \mu = 66.7 \text{ vs. } H_1 : \mu > 66.7$$

Decision rule:

If $p\text{-value} < \alpha$ reject H_0 (and accept H_1 as true)

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If $p\text{-value} < \alpha$ reject H_0 (and accept H_1 as true)

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$T(X)$: test statistic t : observed statistic $H_0: \mu = \mu_0$

p-values

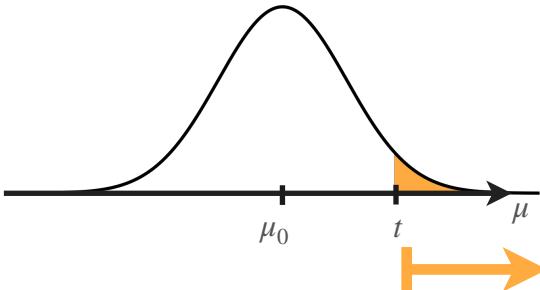
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t : observed statistic

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Right-tailed test



$$\mathbf{P}(T(X) > t | H_0)$$

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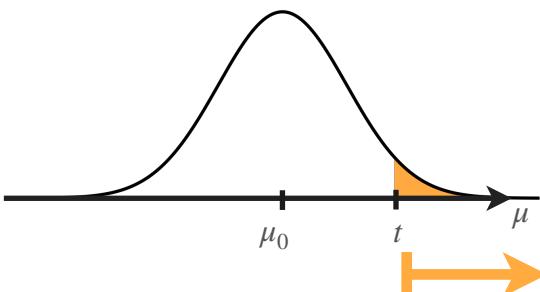
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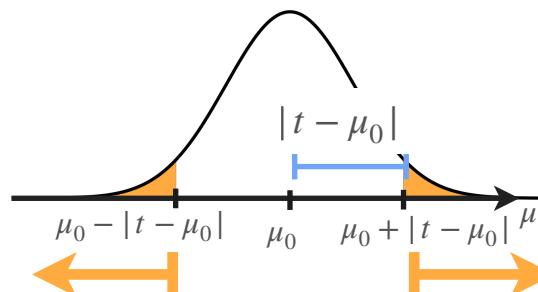
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Two-tailed test



$$\mathbf{P}(|T(X) - \mu_0| > |t - \mu_0| | H_0)$$

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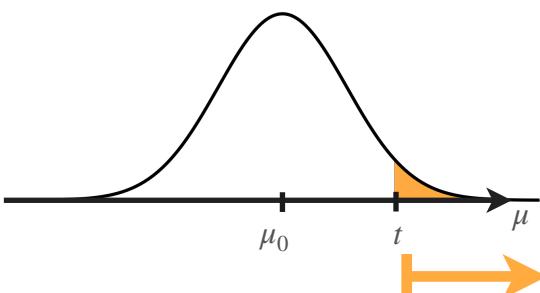
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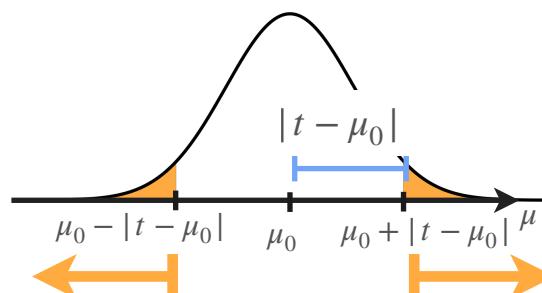
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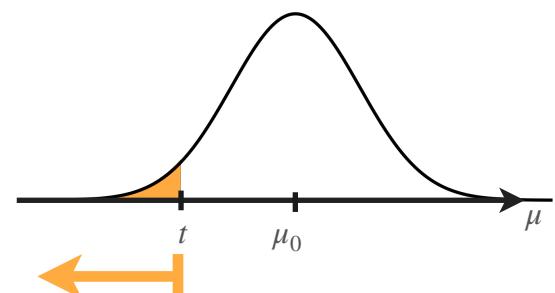
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Left-tailed test



$$\mathbf{P}(T(X) < t | H_0)$$

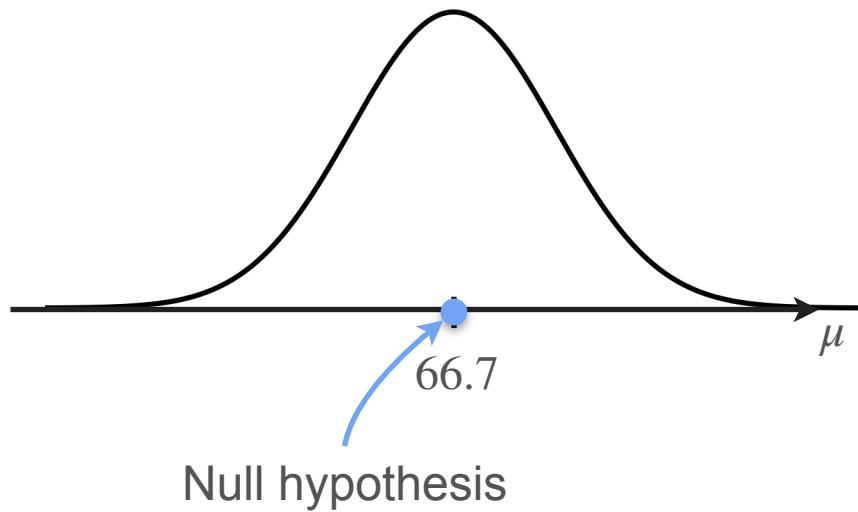
Two-Tailed Test for Gaussian Data (Known σ)

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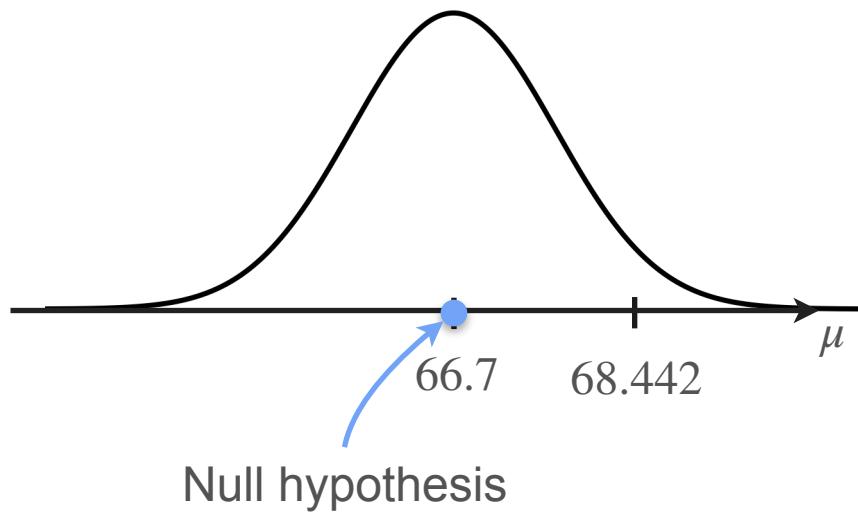
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$$\begin{aligned}\sigma &= 3 \\ n &= 10\end{aligned}$$

$$\bar{x} = 68.442$$

$$H_0 : \mu = 66.7 \text{ vs. } H_1 : \mu \neq 66.7$$



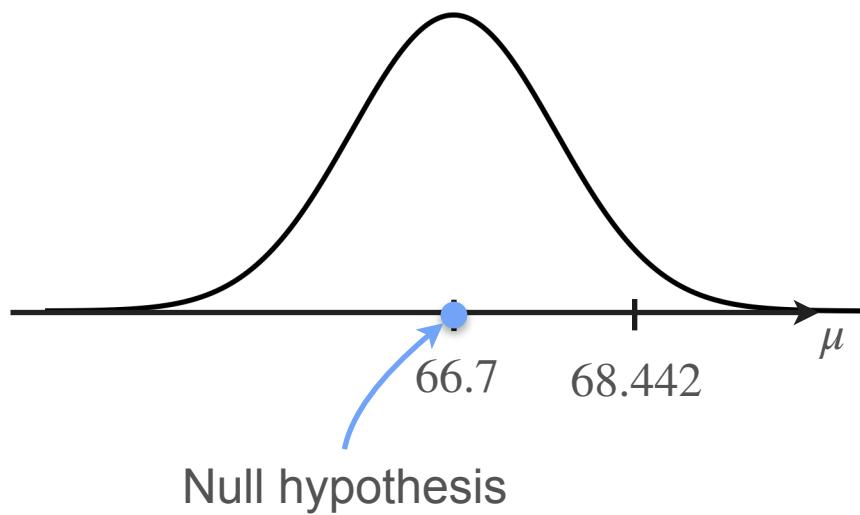
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Type I error: Determine $\mu \neq 66.7$, when population mean did not change

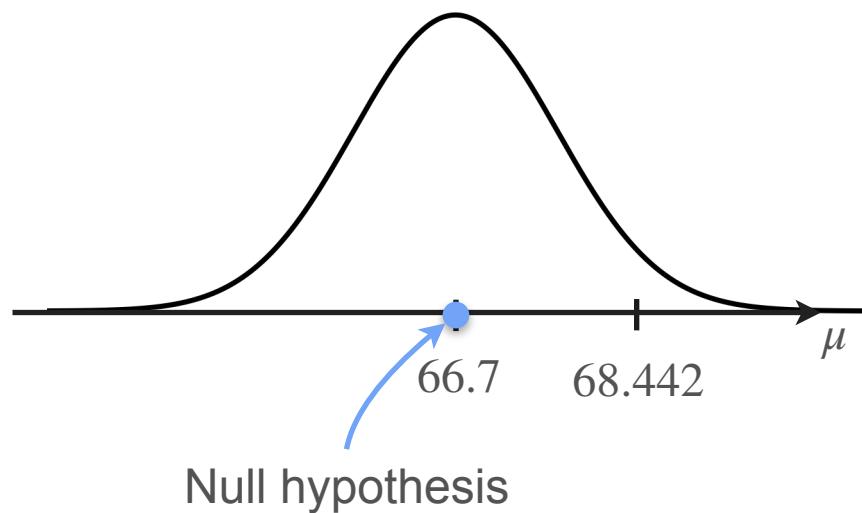
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$$P(|\bar{X} - 66.7| > |68.442 - 66.7| \mid \mu = 66.7)$$

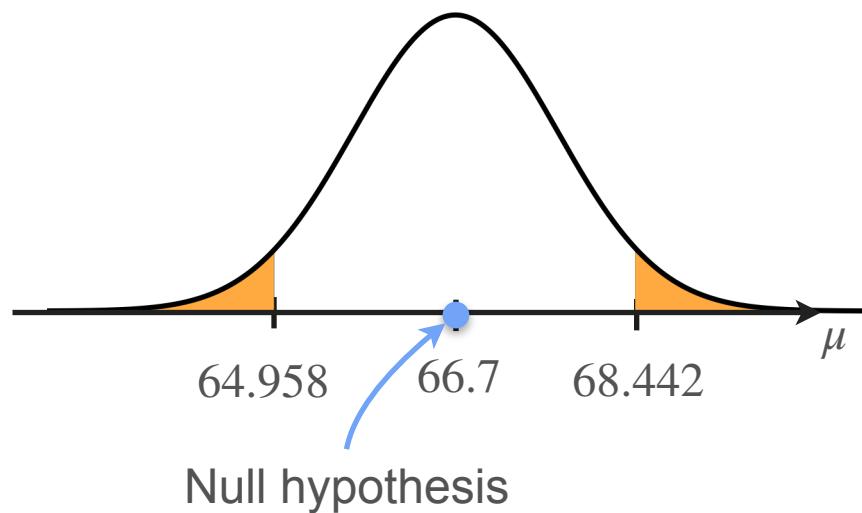
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$$\begin{aligned}P\left(\left|\bar{X} - 66.7\right| > |68.442 - 66.7| \mid \mu = 66.7\right) \\ = 0.082\end{aligned}$$

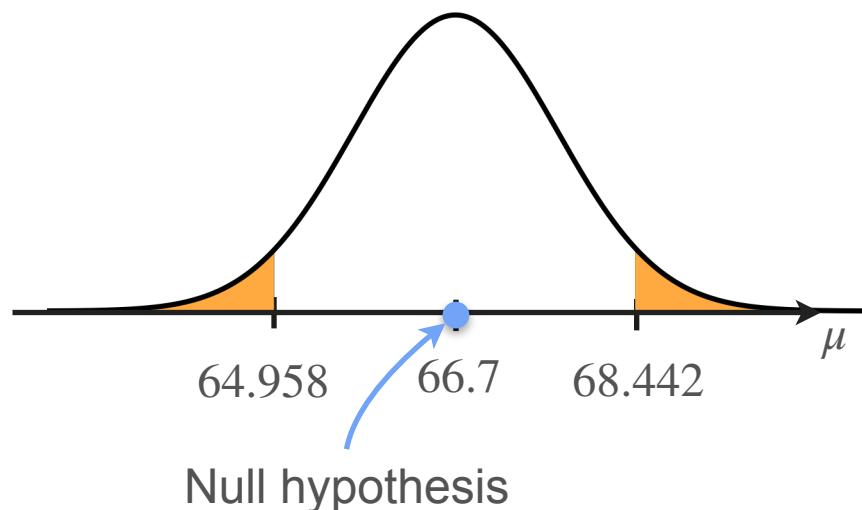
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Conclusion: Do not reject H_0 (with a 5% significance level)

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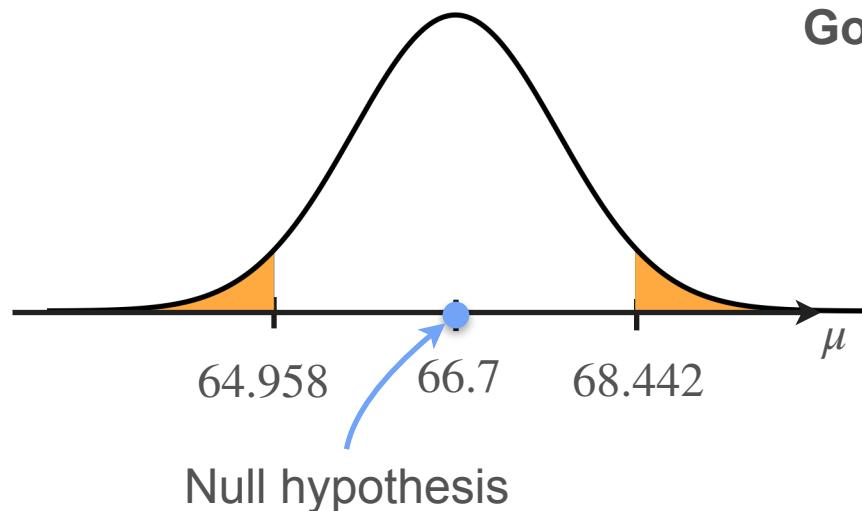
$$\bar{x} = 68.442$$

$$H_0 : \mu = 66.7 \text{ vs. } H_1 : \mu \neq 66.7$$

Goal: Type I error probability $< \alpha = 0.05$

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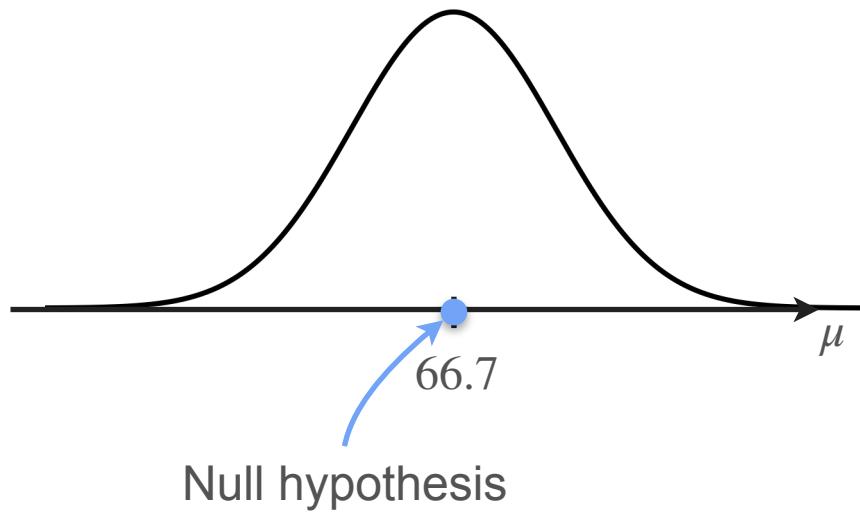
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Left-Tailed Test for Gaussian Data (Known σ)

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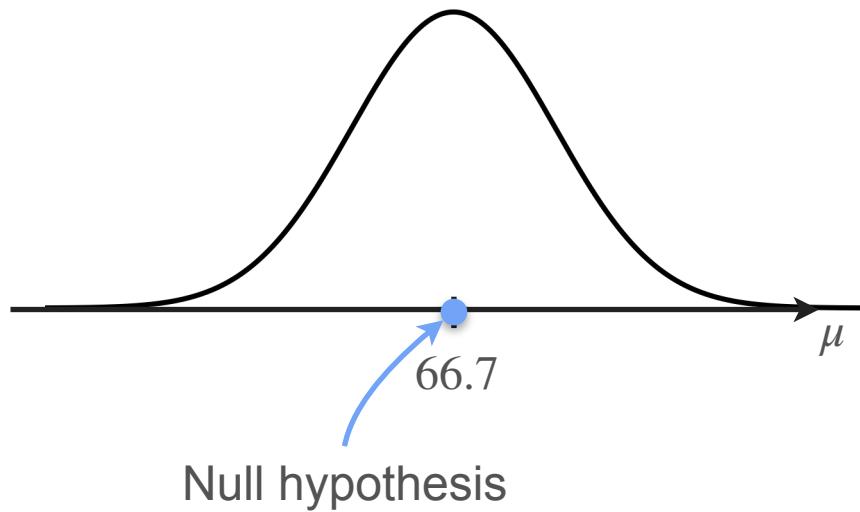
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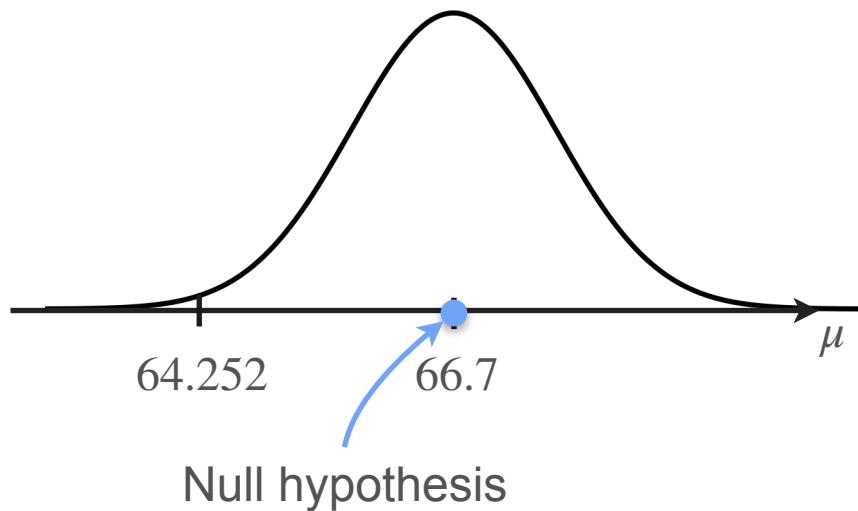
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The mean height for 18 y/o in the US in the 70s was **66.7** in.

$$\sigma = 3$$
$$n = 10$$

$$\bar{x} = 64.252$$

$$H_0 : \mu = 66.7 \text{ vs. } H_1 : \mu < 66.7$$



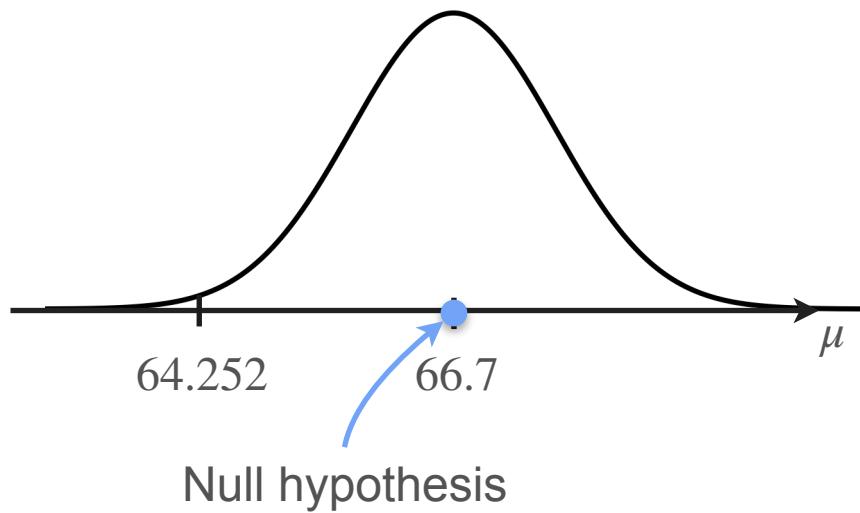
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Type I error: Determine $\mu < 66.7$, when population mean did not change

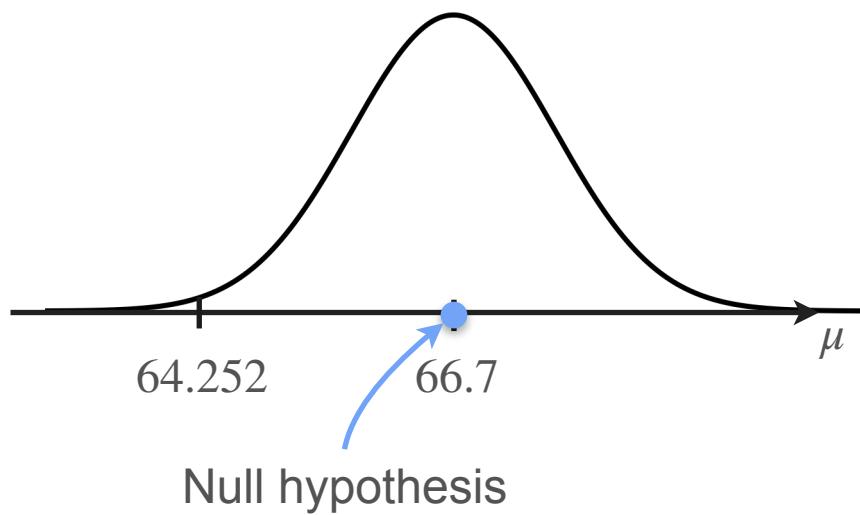
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$$P(\bar{X} < 64.252 \mid \mu = 66.7) ?$$

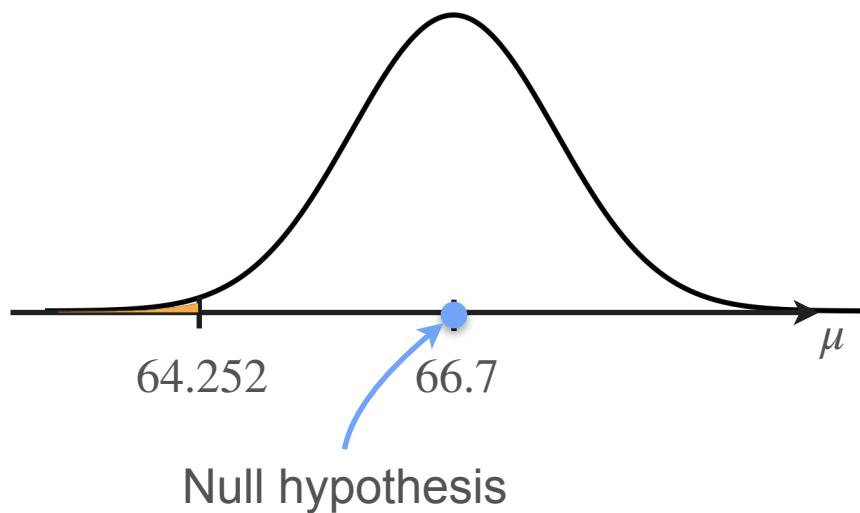
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$$\begin{aligned}P(\bar{X} < 64.252 \mid \mu = 66.7) &: \\ &= 0.0094\end{aligned}$$

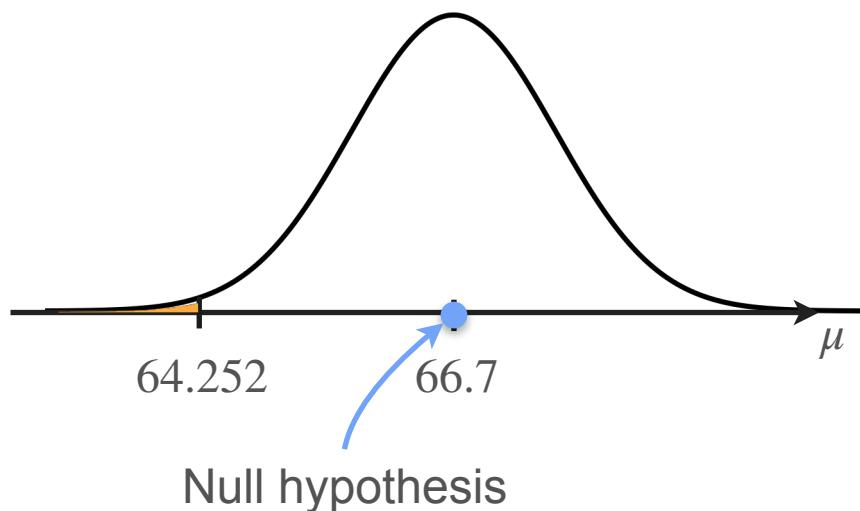
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Type I error: Determine $\mu < 66.7$, when population mean did not change

$$P(\bar{X} < 64.252 \mid \mu = 66.7) : \\ = 0.0094 < \alpha$$

Conclusion: reject H_0
(with a 5% significance level)

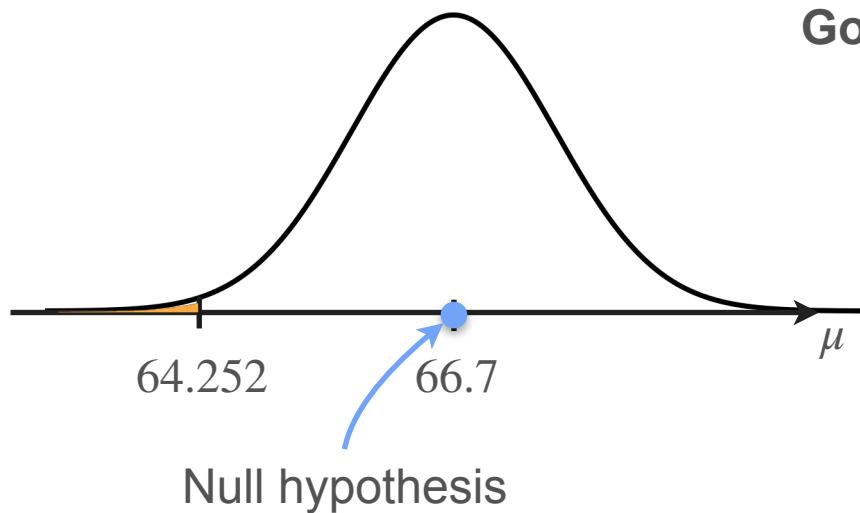
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Applying standardization, you can write equivalent tests using the

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Right-Tailed Test Using the Z Statistic

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The **mean** height for 18 y/o in the US in the 70s was **66.7** in.

$$H_0 : \mu = 66.7 \text{ vs. } H_1 : \mu > 66.7$$

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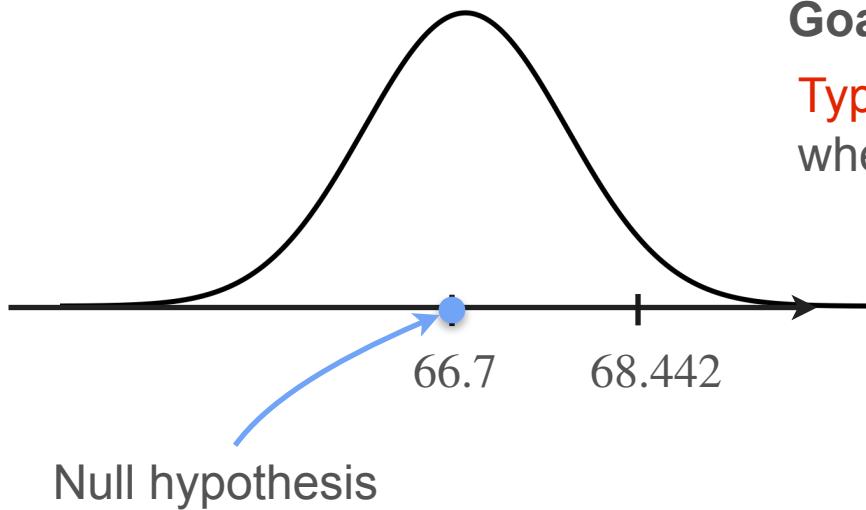
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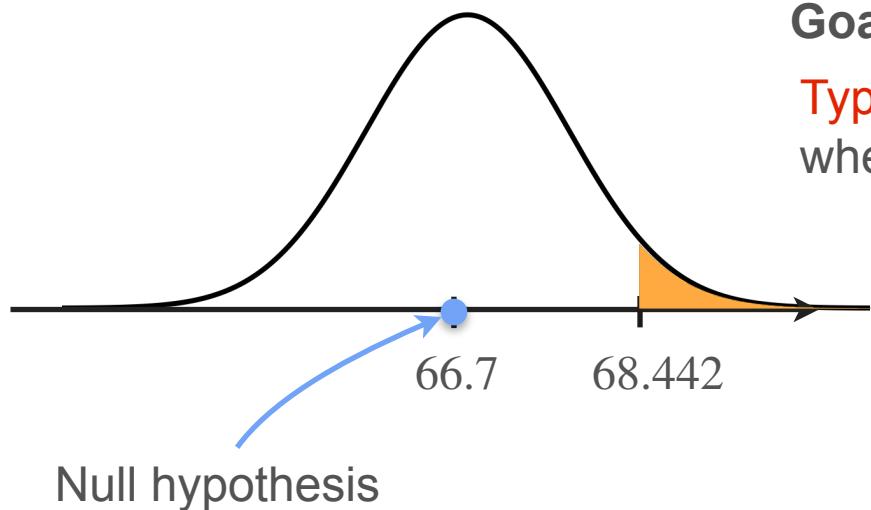
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$$P(\bar{X} > 68.442 \mid \mu = 66.7) \\ = 0.0407 < \alpha$$

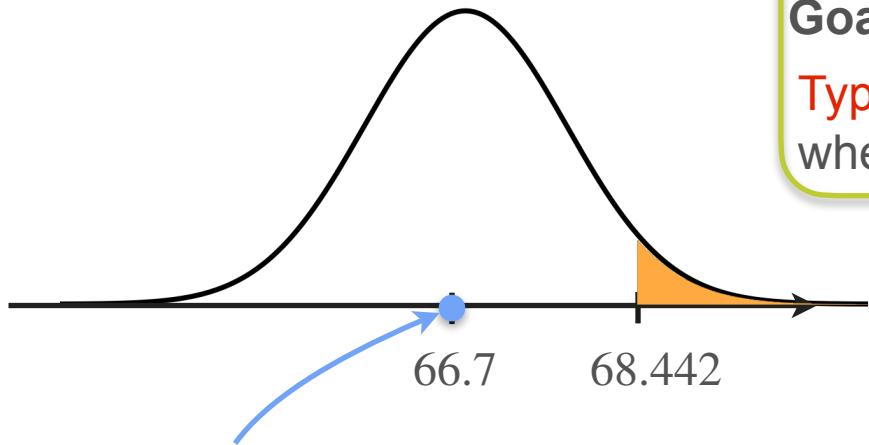
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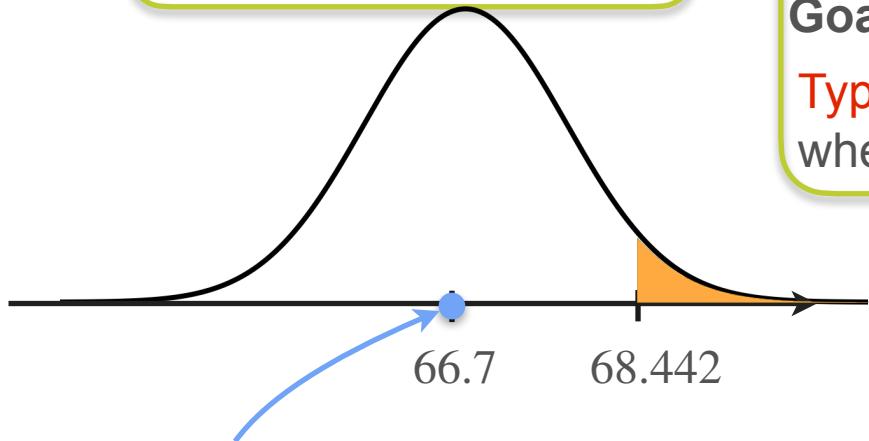
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$$Z = \frac{\bar{X} - \mu_0}{3/\sqrt{10}}$$

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$$Z = \frac{\bar{X} - \mu_0}{3/\sqrt{10}} \rightarrow z = \frac{68.442 - 66.7}{3/\sqrt{10}} = 1.837$$

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$$\frac{\bar{X} - 66.7}{3/\sqrt{10}} > \frac{68.442 - 66.7}{3/\sqrt{10}} = 1.837$$

$$P\left(\quad > \quad \middle| \mu = 66.7 \right) = 0.0407 < \alpha$$

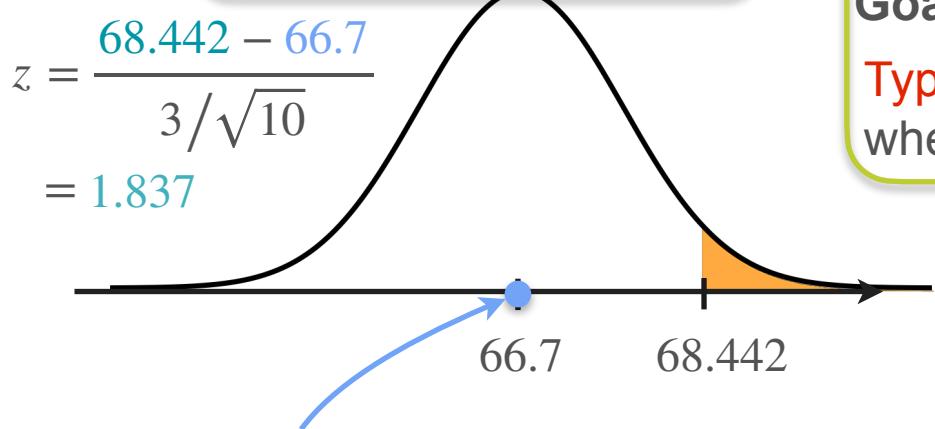
Conclusion: reject H_0
(with a 5% significance level)

Right-Tailed Test Using the Z Statistic

The mean height for 18 y/o in the US in the 70s was 66.7 in.

$$\sigma = 3$$
$$n = 10$$

$$\bar{x} = 68.442$$



$$H_0 : \mu = 66.7 \text{ vs. } H_1 : \mu > 66.7$$

Goal: Type I error probability $< \alpha = 0.05$

Type I error: Determine $\mu > 66.7$, when population mean did not change

$$P\left(\frac{\bar{X} - 66.7}{3/\sqrt{10}} > 1.837 \mid \mu = 66.7\right)$$

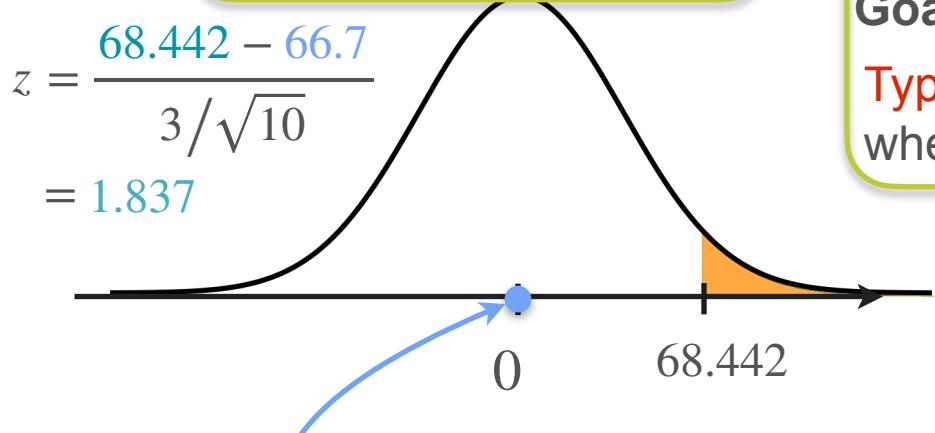
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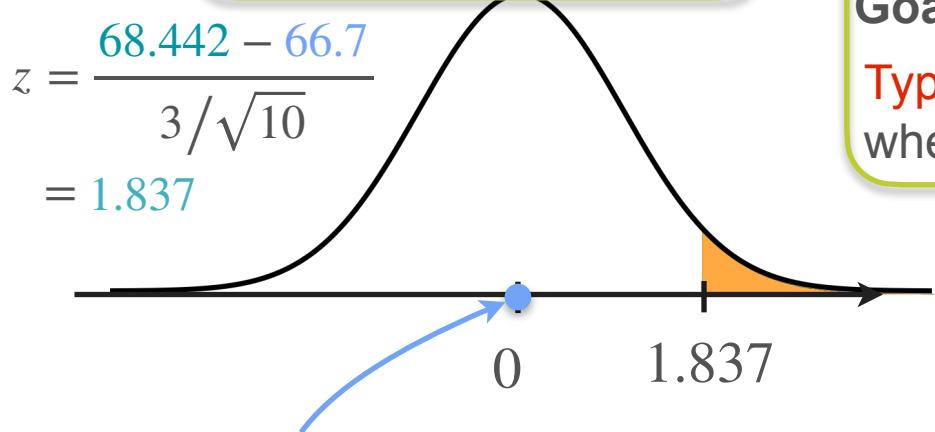
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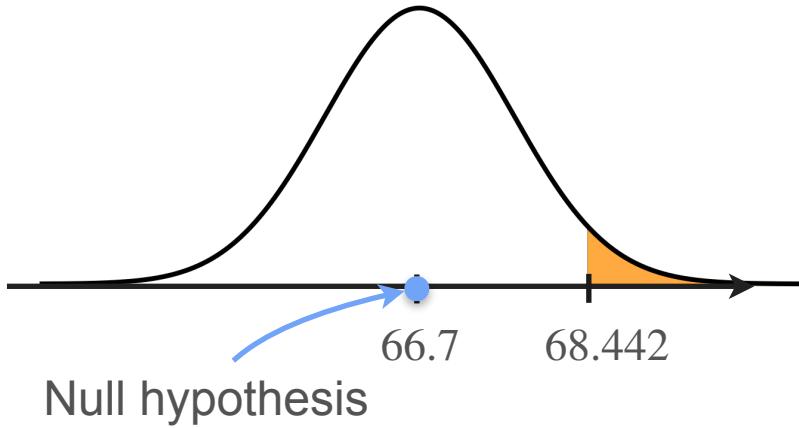
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Hypothesis Testing

Critical Values

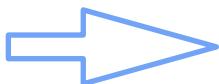
P-Values and Critical Values

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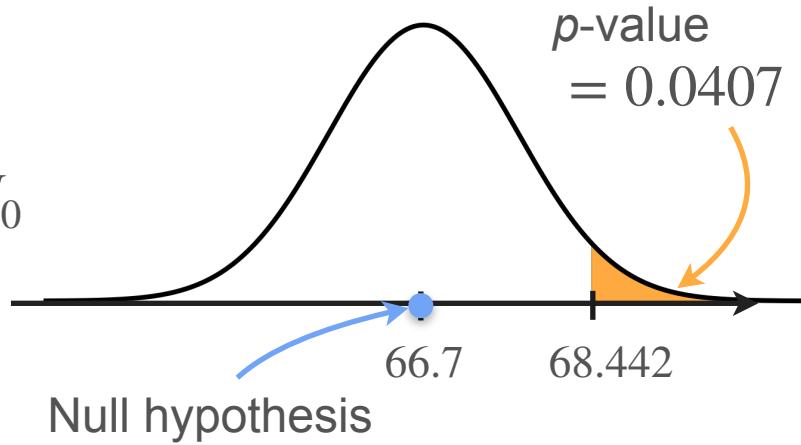


P-Values and Critical Values

If $p\text{-value} < \alpha$

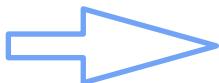


Reject H_0



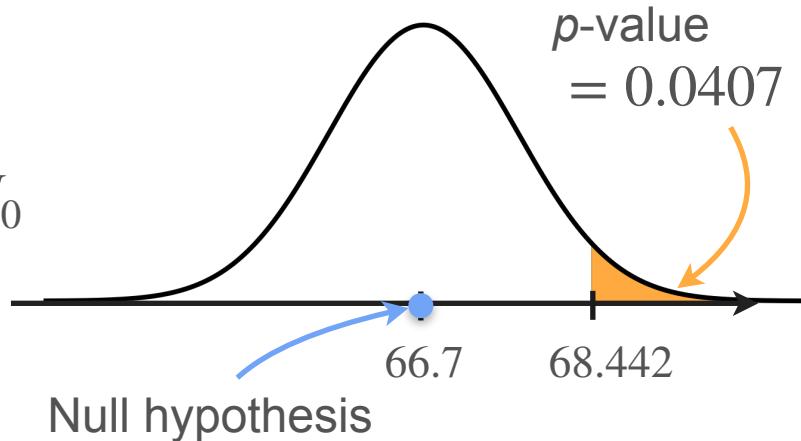
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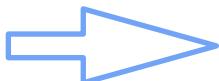
Reject H_0

What is the least extreme sample you could get that you would still reject H_0 ?



P-Values and Critical Values

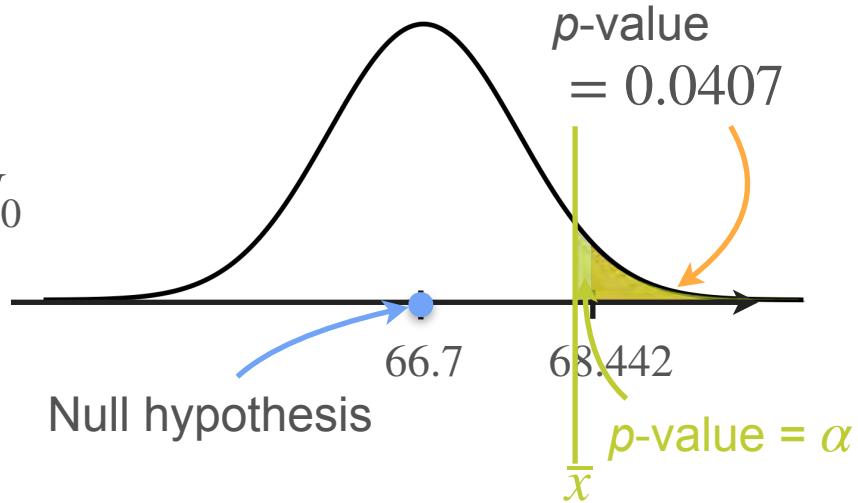
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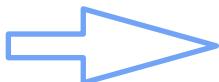
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P-Values and Critical Values

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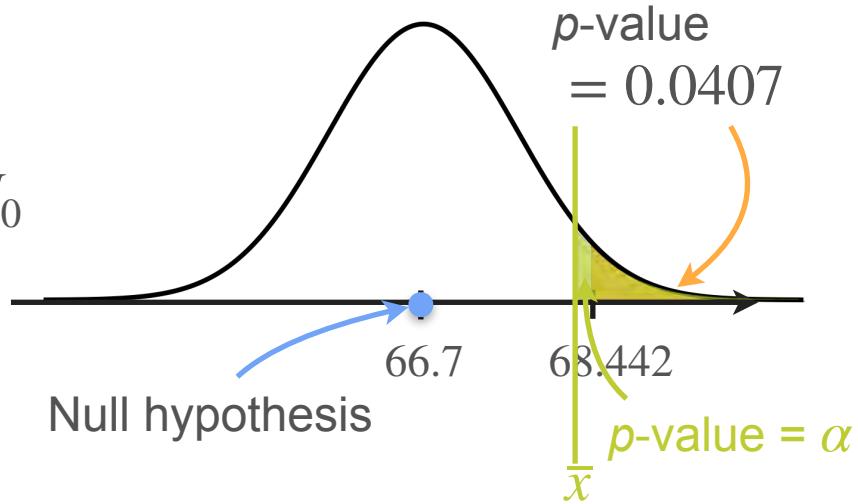


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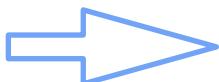
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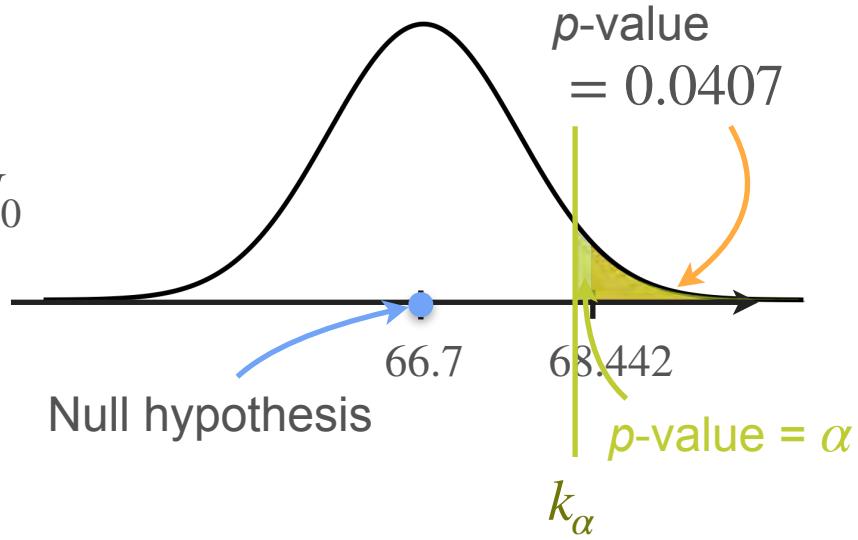


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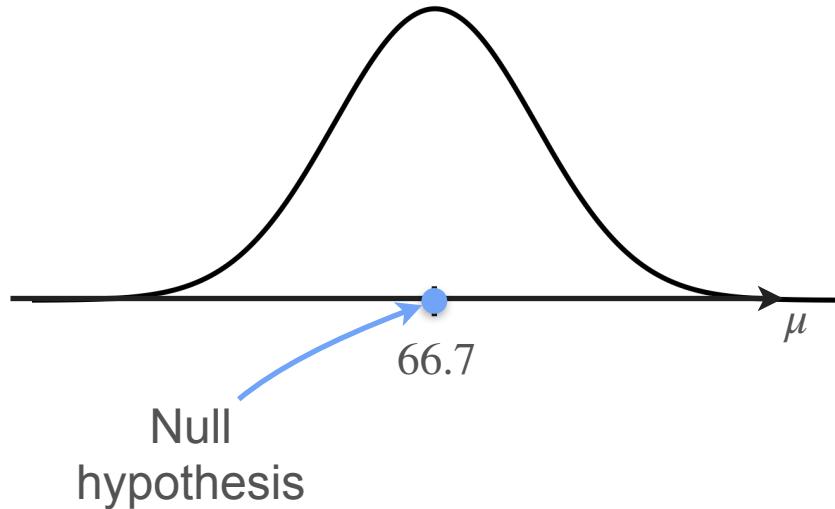


Computing Critical Values

Computing Critical Values

The **mean** height for 18 y/o in the US in the 70s was **66.7 in.**

$$H_0 : \mu = 66.7 \text{ vs. } H_1 : \mu > 66.7$$



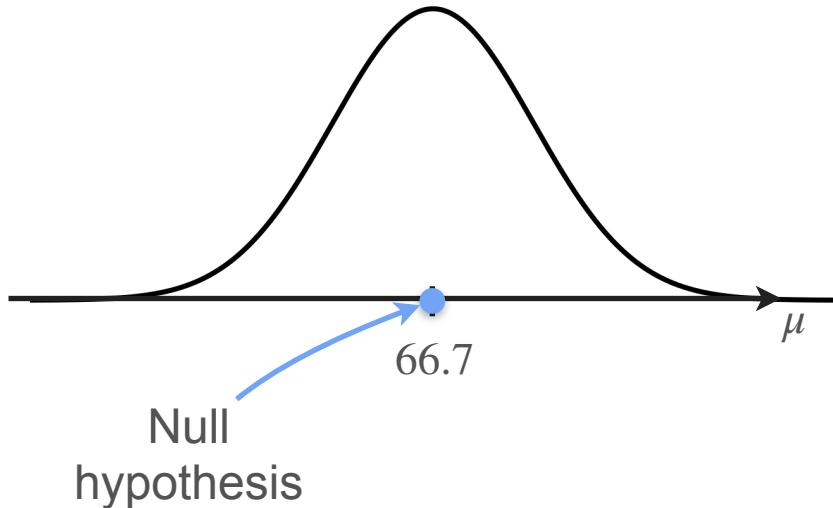
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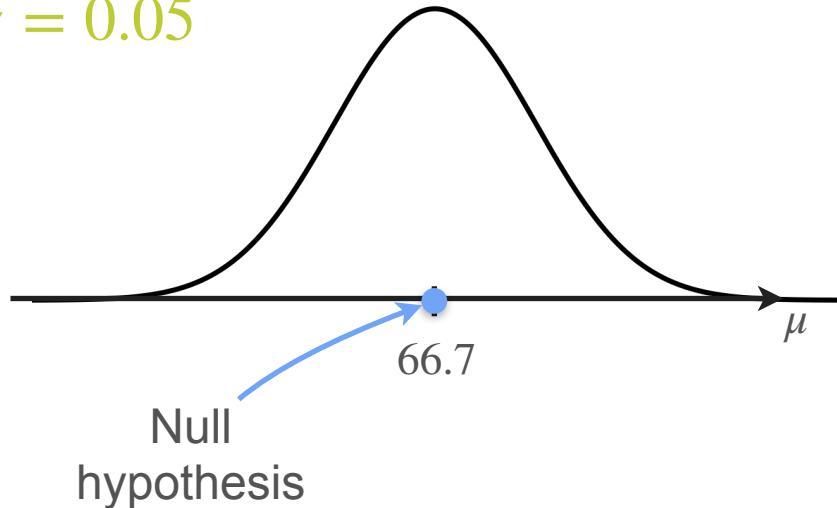
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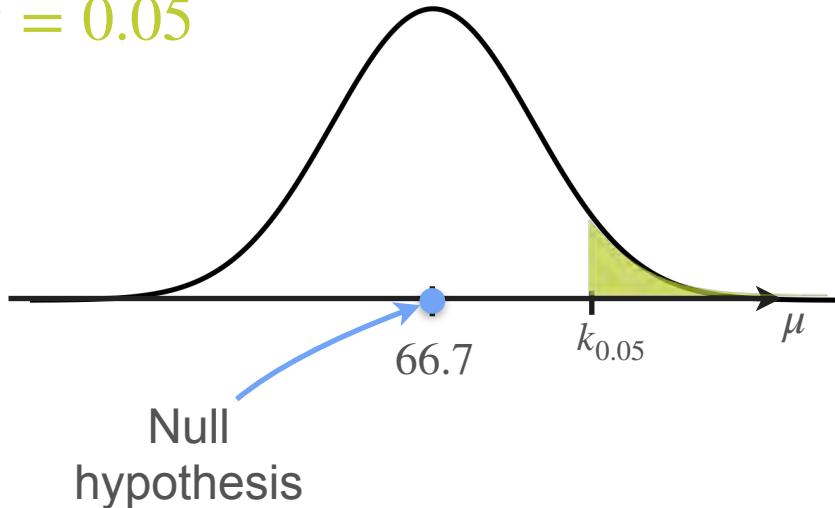
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$$\alpha = 0.05$$

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$$0.05 = P(\bar{X} > k_{0.05} \mid \mu = 66.7)$$



Computing Critical Values

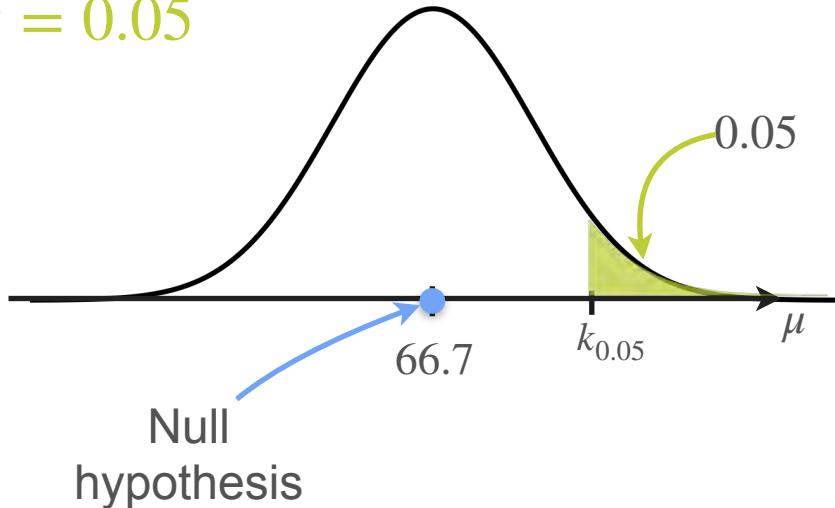
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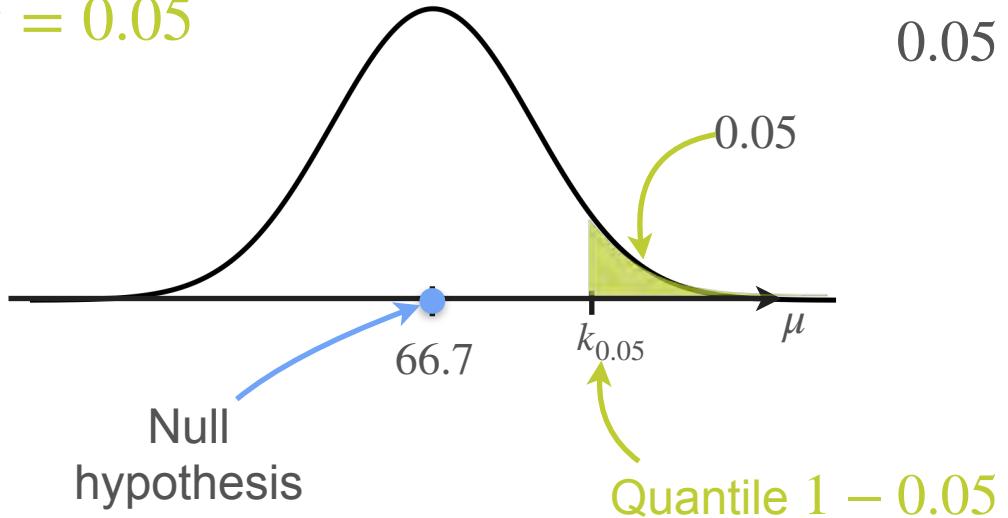
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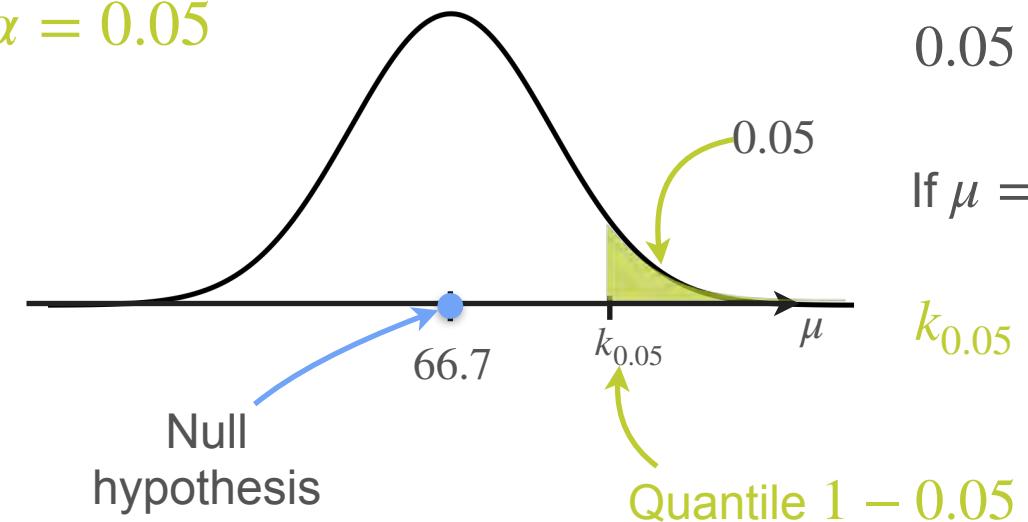


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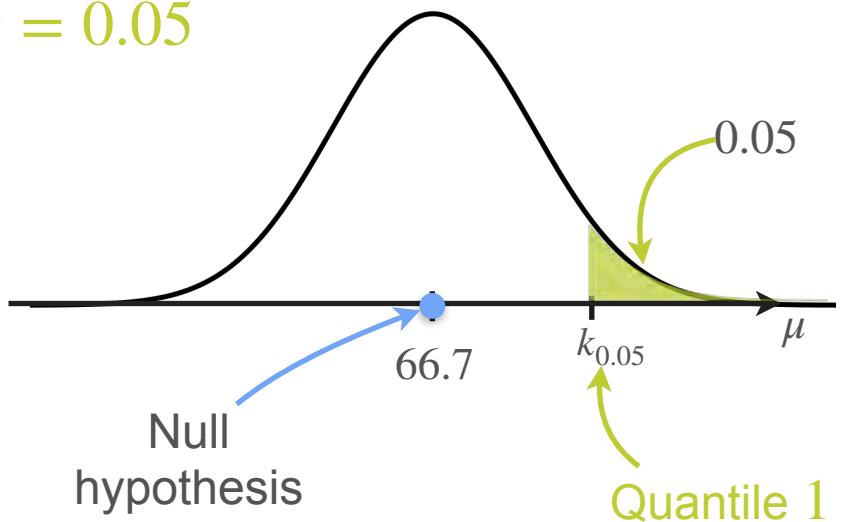
$$k_{0.05} = 68.26$$

Computing Critical Values

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Decision rule: Reject H_0 if $\bar{x} > 68.26$

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The mean height for 18 y/o in the US in the 70s was **66.7 in.** **Reject H_0**

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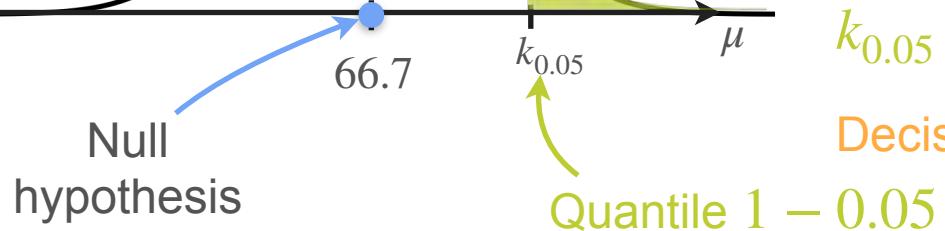
$$n = 10 \quad \sigma = 3 \quad \bar{x} = 68.442$$

$$0.05 = P(\bar{X} > k_{0.05} \mid \mu = 66.7)$$

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Computing Critical Values

The mean height for 18 y/o in the US in the 70s was **66.7 in.** **Reject H_0**

$H_0 : \mu = 66.7$ vs. $H_1 : \mu > 66.7$

$\alpha = 0.01$

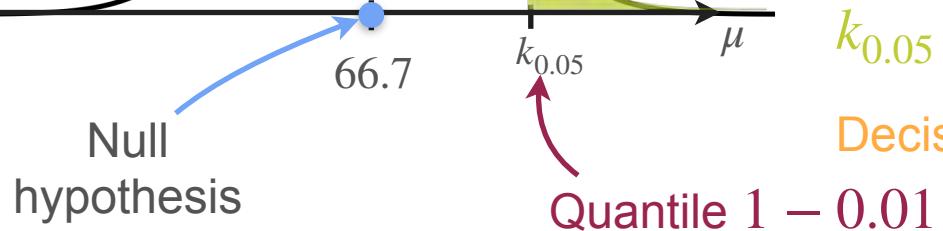
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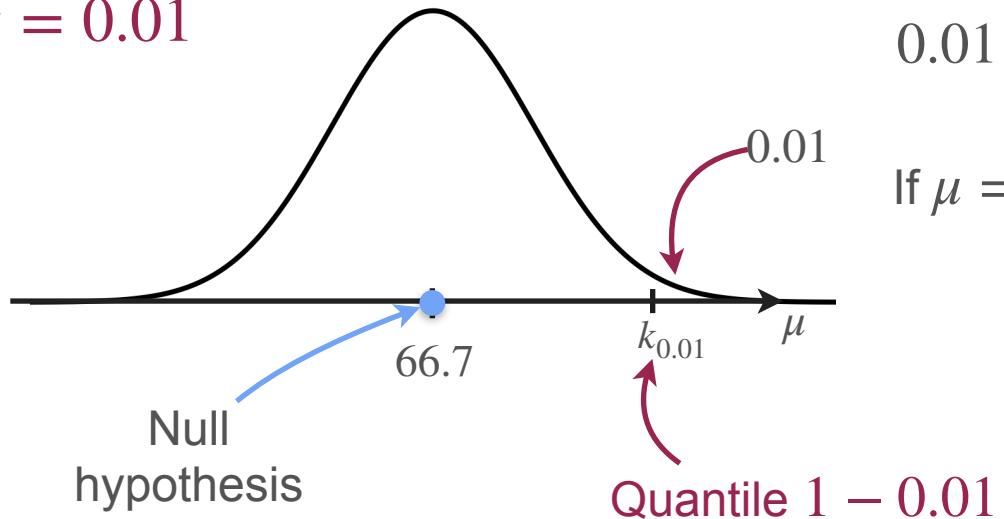


Computing Critical Values

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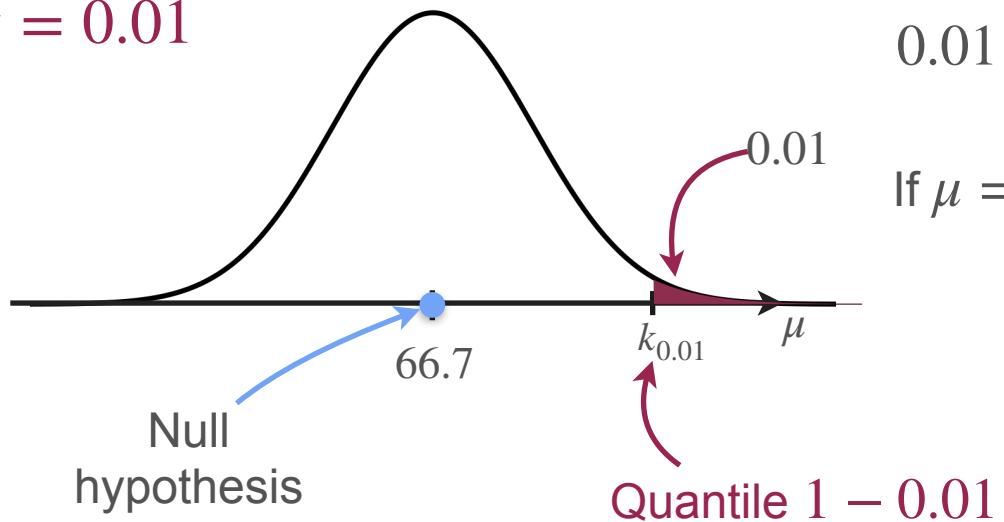
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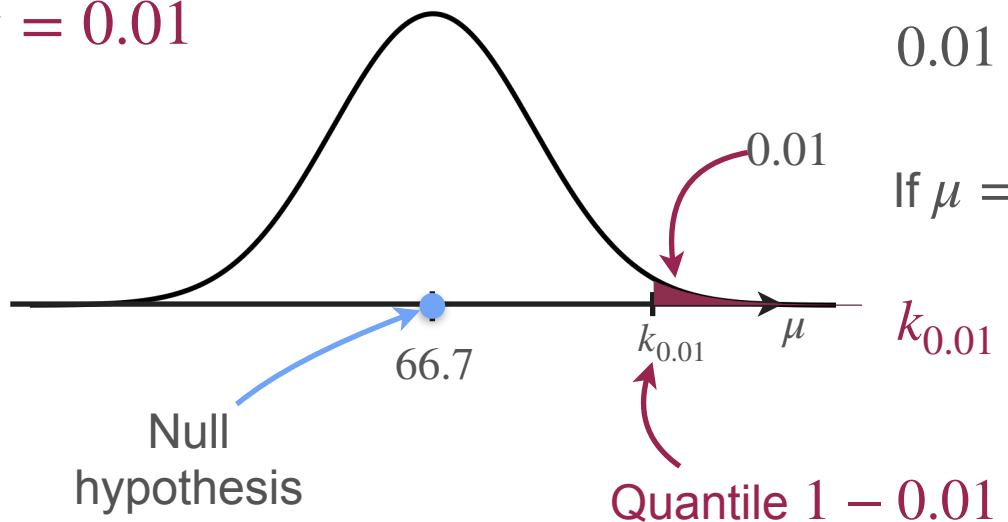
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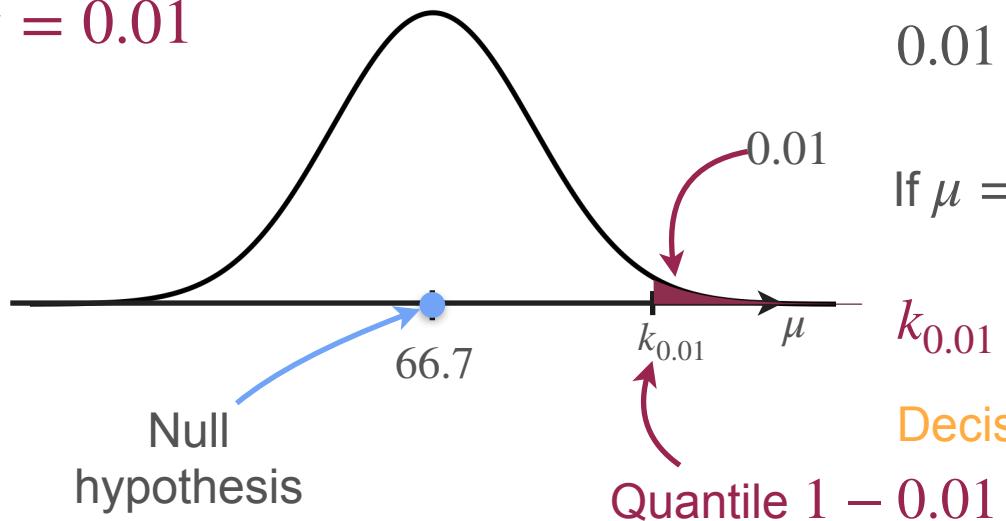
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Decision rule: Reject H_0 if $\bar{x} > 68.91$

Computing Critical Values

The mean height for 18 y/o in the US in the 70s was **66.7 in.**

Do not reject H_0

$H_0 : \mu = 66.7$ vs. $H_1 : \mu > 66.7$

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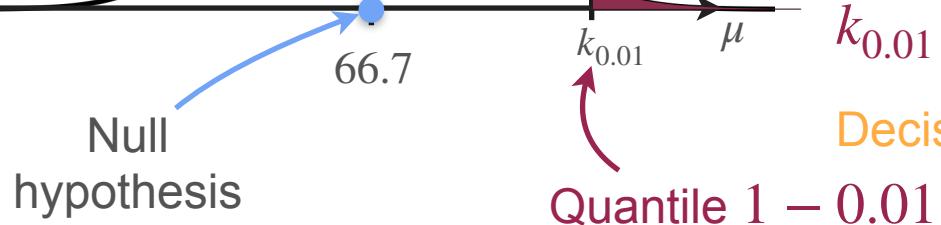
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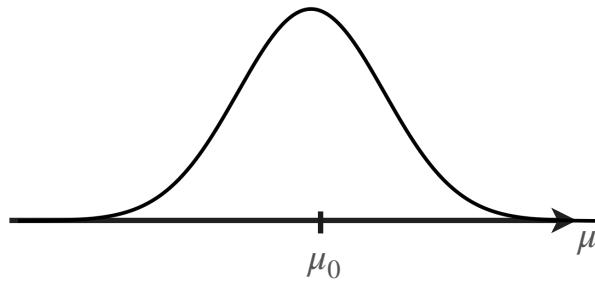
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Critical Values

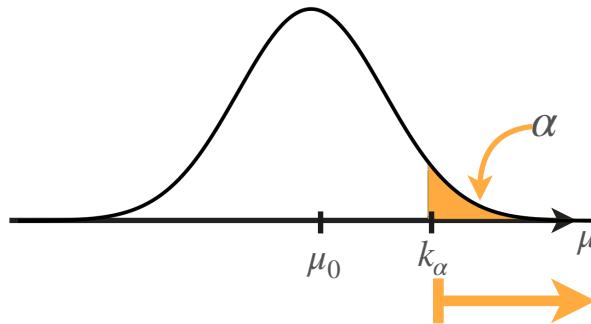
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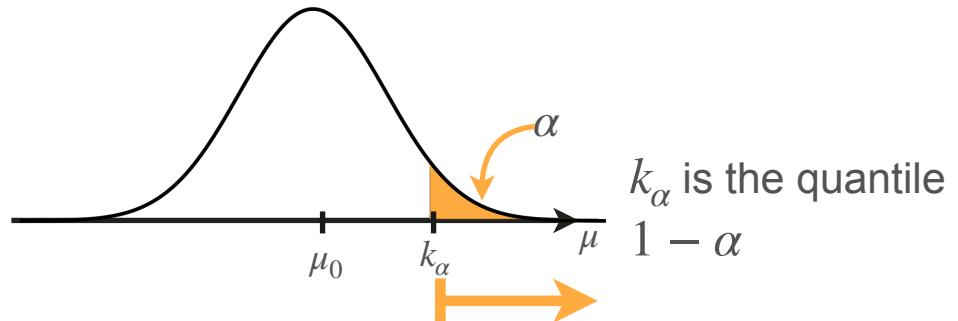
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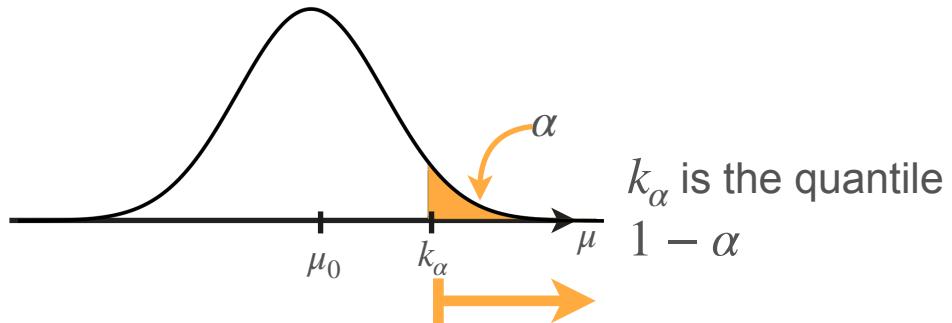


k_α is the quantile
1 - α

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Decision rule: Reject H_0 if $t > k_\alpha$



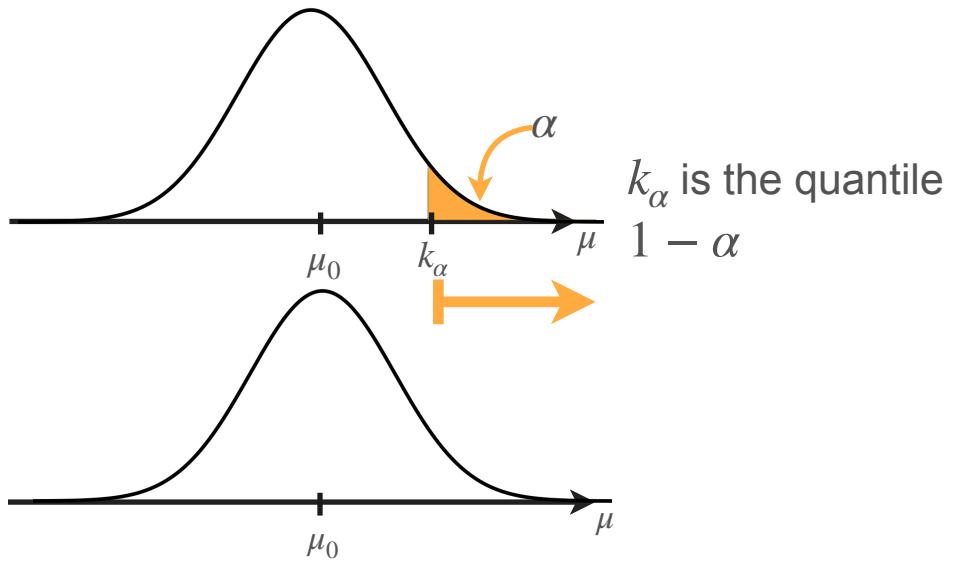
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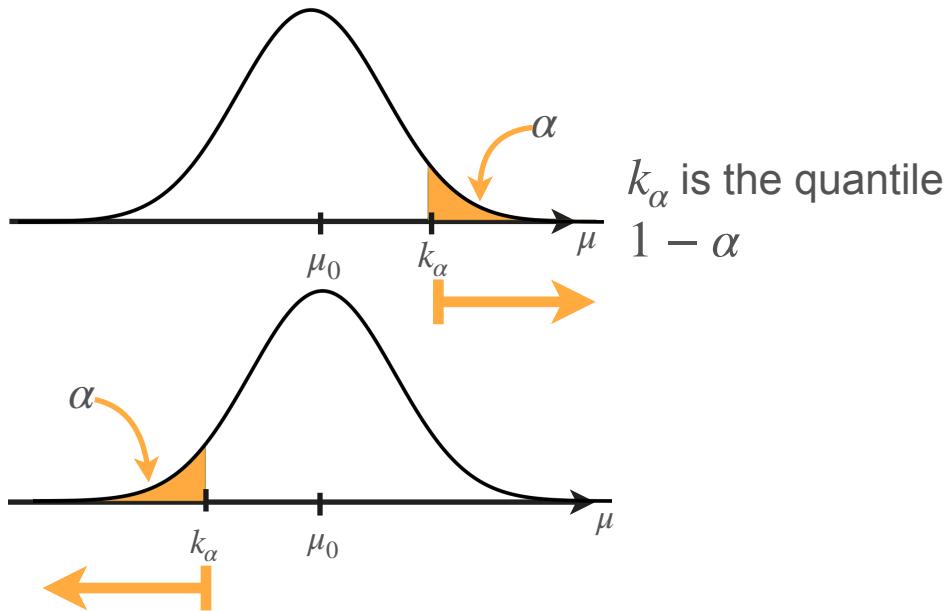


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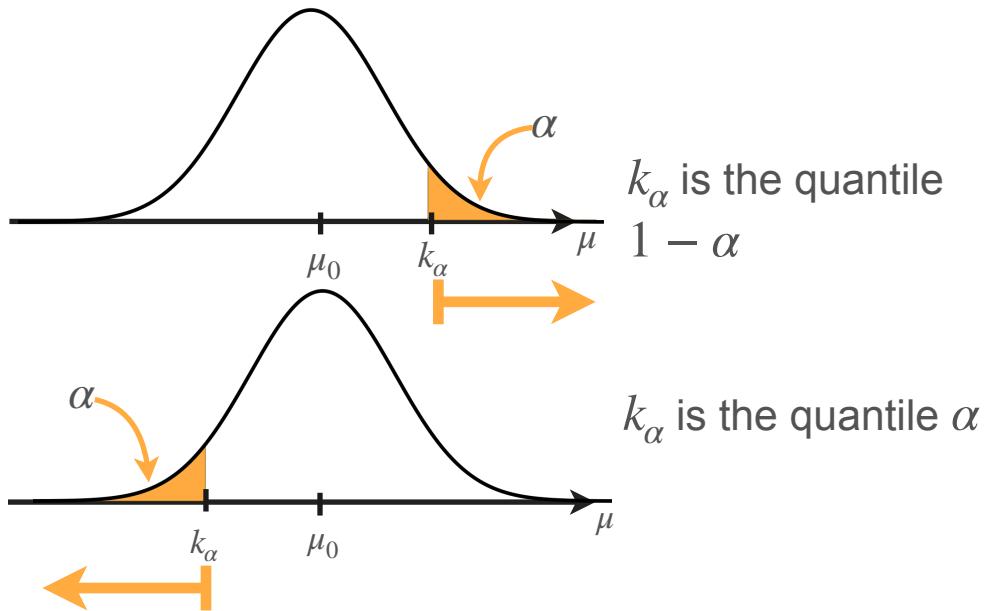


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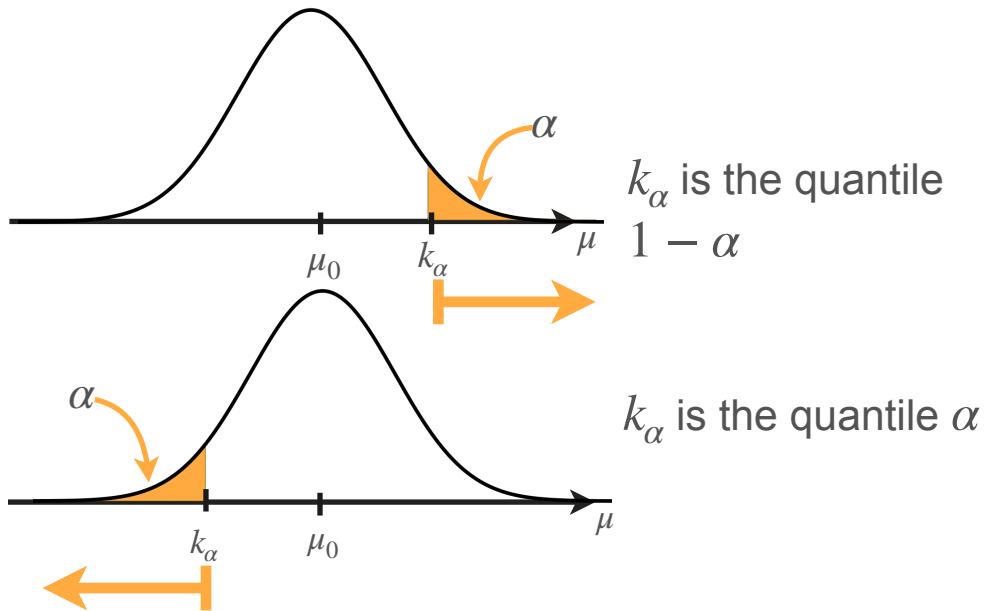
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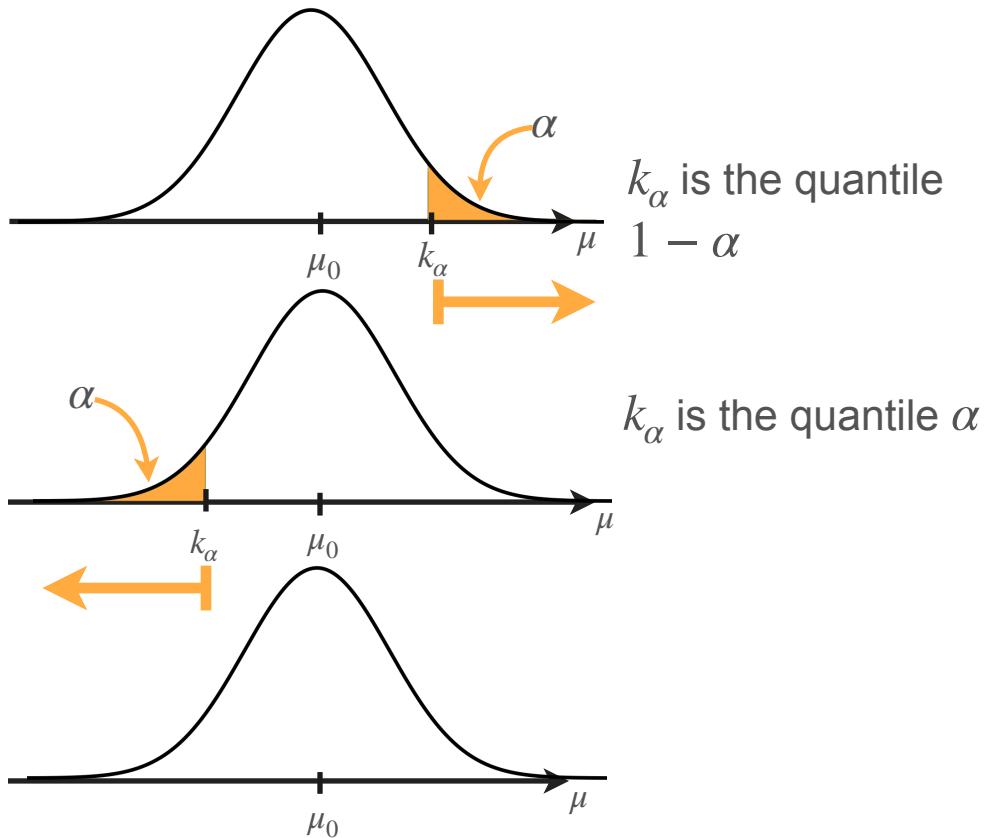
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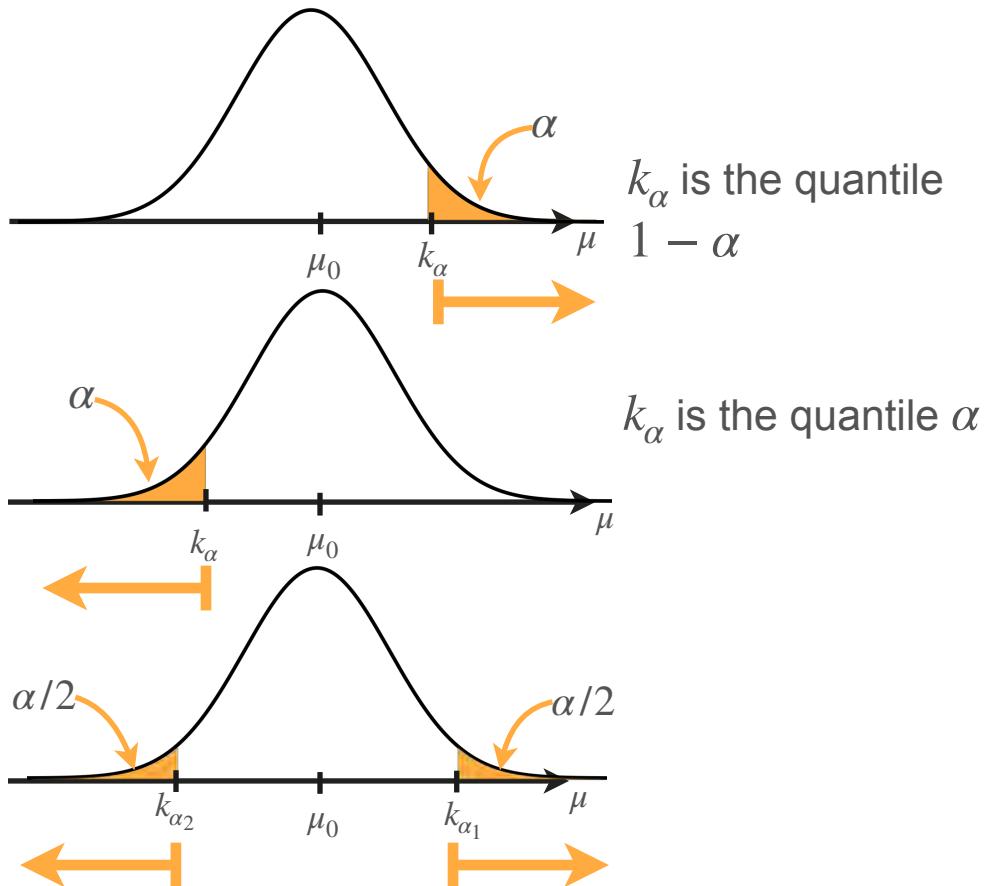
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Critical Values

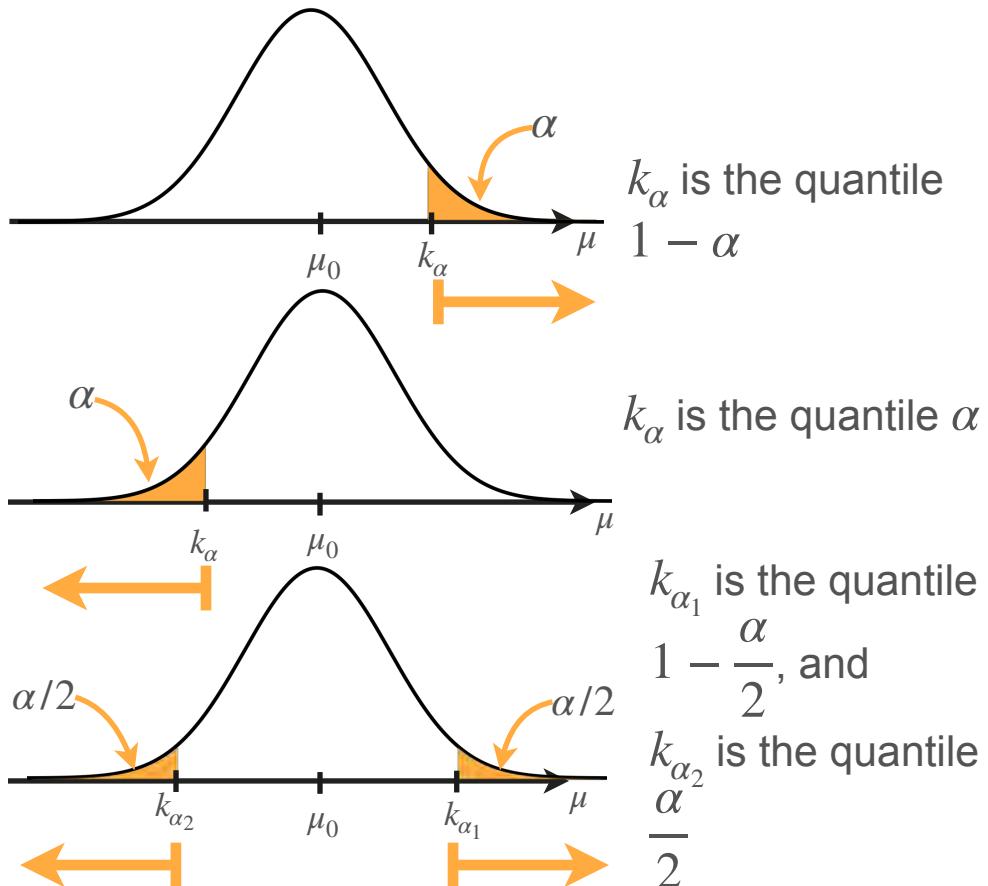
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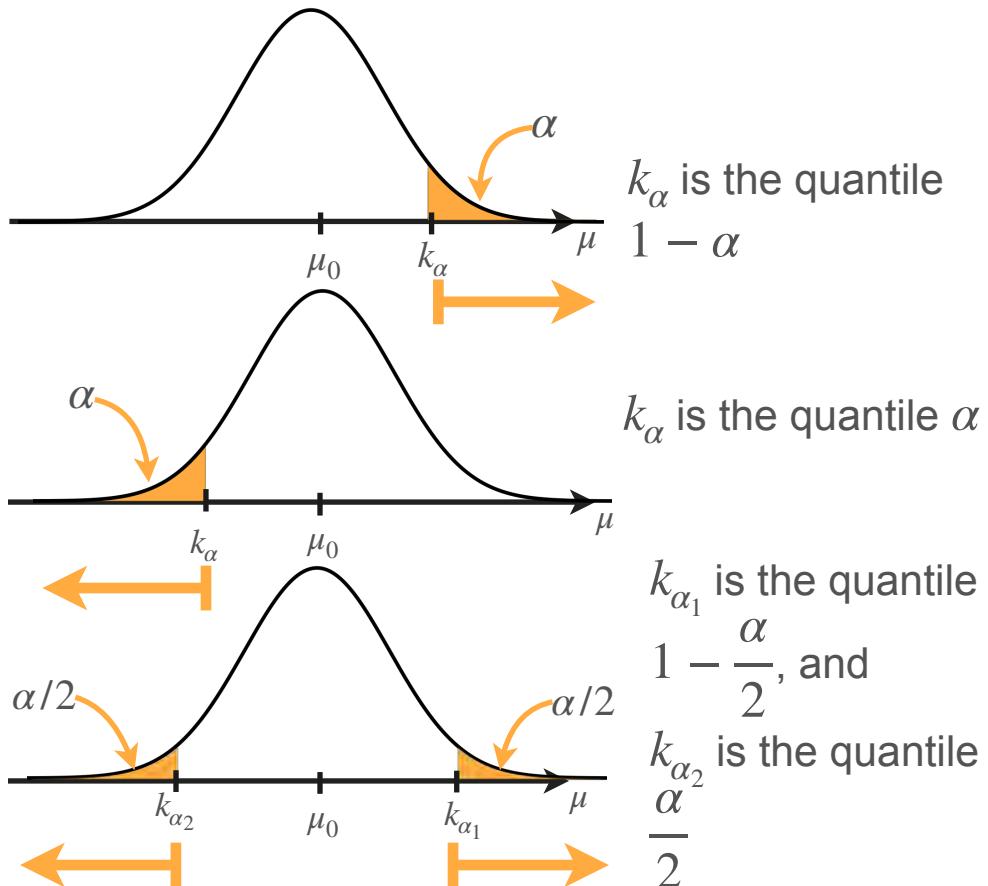
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Decision rule: Reject H_0 if $t < k_\alpha$

$H_0 : \mu = \mu_0$ vs. $H_1 : \mu \neq \mu_0$

Decision rule: Reject H_0 if $t > k_{\alpha_1}$ or
 $t < k_{\alpha_2}$



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DeepLearning.AI

Hypothesis Testing

Power of a test

Type I and Type II Errors

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Decision	Reality	
	H_0 True ($\mu = 66.7$)	H_0 False ($\mu > 66.7$)
Reject H_0 (Decide $\mu > 66.7$)	Type I error	Correct
Don't reject H_0 (Decide $\mu = 66.7$)	Correct	Type II error

Type I and Type II Errors

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	H_0 True ($\mu = 66.7$)	H_0 False ($\mu > 66.7$)
Reject H_0 (Decide $\mu > 66.7$)	Type I error	Correct
Don't reject H_0 (Decide $\mu = 66.7$)	Correct	Type II error

Type I and Type II Errors

The **mean** height for 18 y/o in the US in the 70s was **66.7 in.**

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Finding the Type II Error Probabilities

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Finding the Type II Error Probabilities

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$$H_0 : \mu = 66.7 \text{ vs. } H_1 : \mu > 66.7 \quad n = 10 \quad \sigma = 3$$

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For $\alpha = 0.05$: $k_\alpha = 68.26$

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The **mean** height for 18 y/o in the US in the 70s was **66.7 in.**

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$$\mathbf{P}(\bar{X} < 68.26 | \mu = 70)$$

Finding the Type II Error Probabilities

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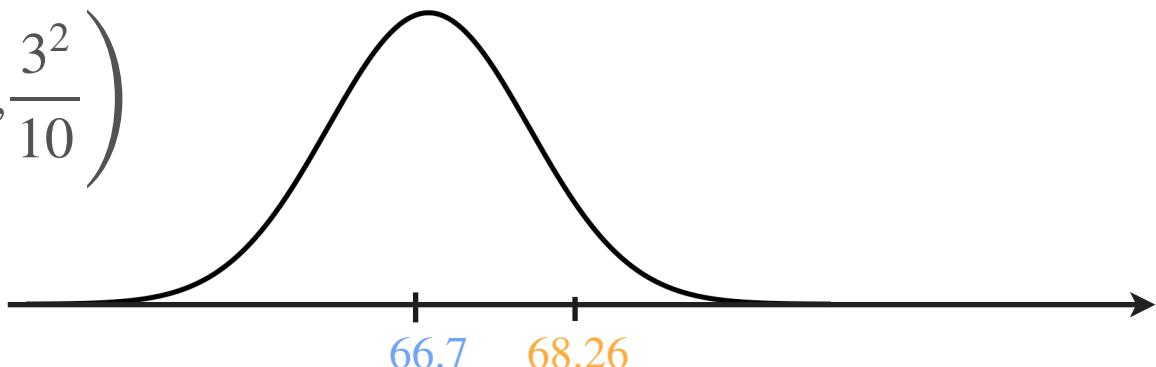
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What is the **Type II error probability** if the true value is $\mu = 70$?

If $\mu = 66.7$ $\bar{X} \sim \mathcal{N}\left(66.7, \frac{3^2}{10}\right)$

$$\mathbf{P}(\bar{X} < 68.26 | \mu = 70)$$



Finding the Type II Error Probabilities

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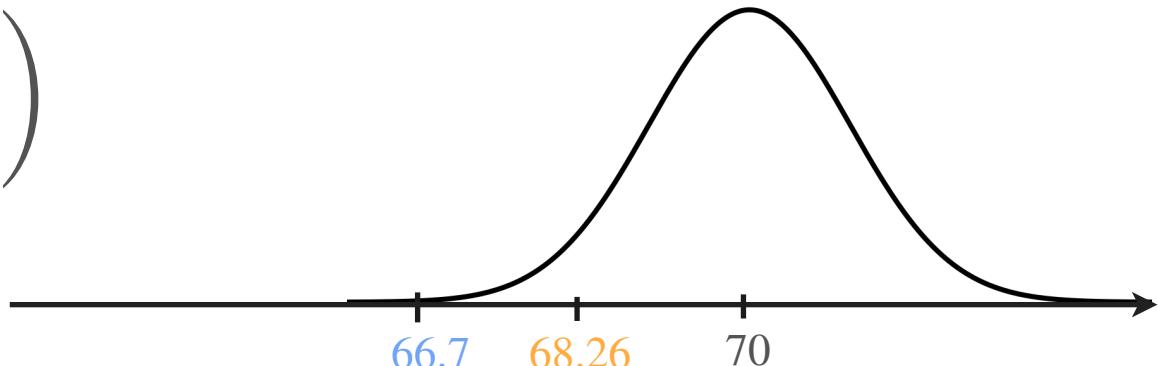
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What is the **Type II error probability** if the true value is $\mu = 70$?

$$\text{If } \mu = 70 \quad \bar{X} \sim \mathcal{N}\left(70, \frac{3^2}{10}\right)$$

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Finding the Type II Error Probabilities

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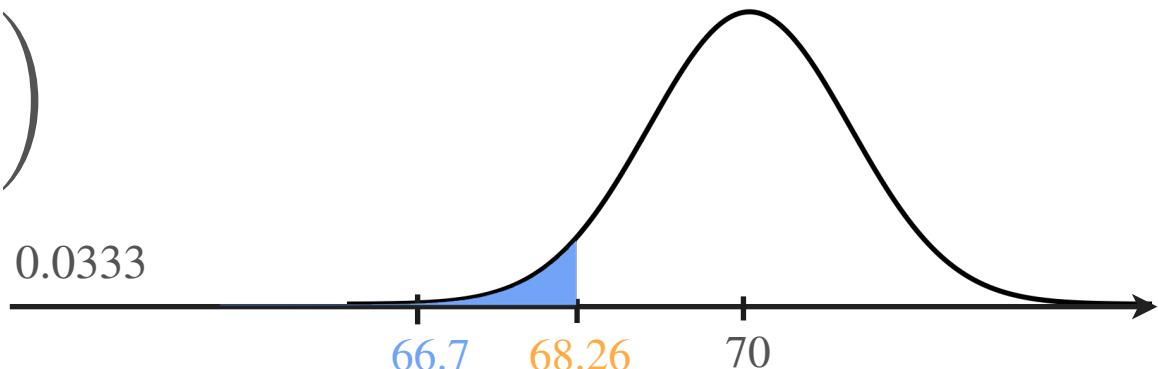
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$$\text{If } \mu = 70 \quad \bar{X} \sim \mathcal{N}\left(70, \frac{3^2}{10}\right)$$

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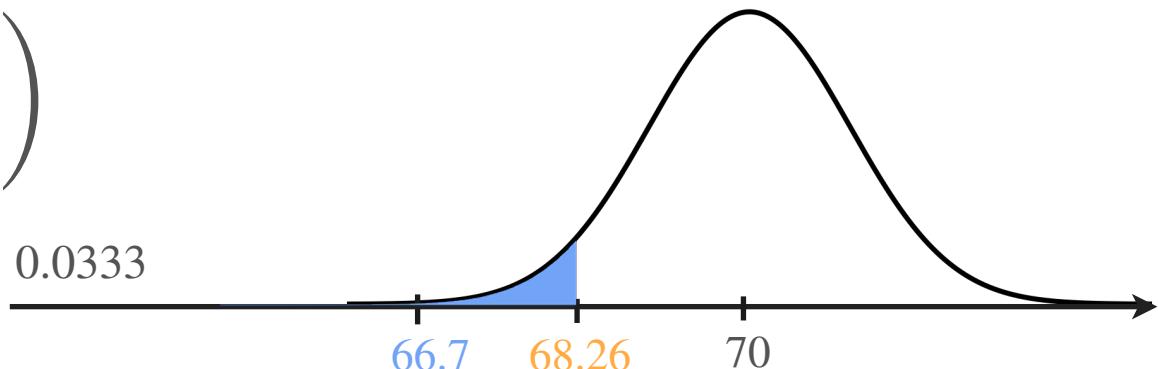
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$$\beta = P(\bar{X} < 68.26 | \mu = 70) = 0.0333$$



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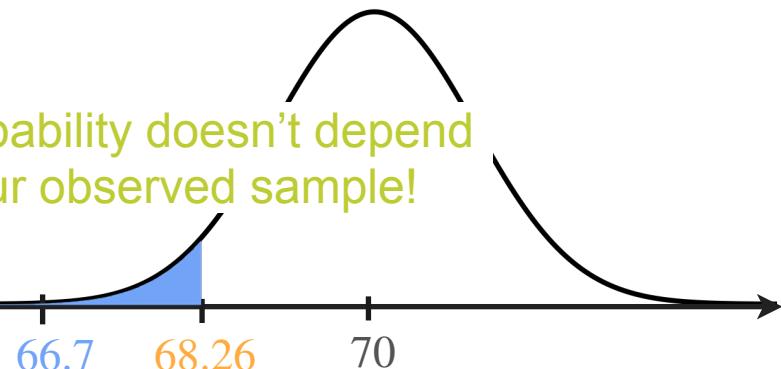
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This probability doesn't depend on your observed sample!



Power of the Test

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$$H_0 : \mu = 66.7 \text{ vs. } H_1 : \mu > 66.7$$

Power of the test

$$\mathbf{P}(\text{Reject } H_0 | \mu \in H_1)$$

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Power of the Test



Power of the Test

Type II error: $\overbrace{\mathbf{P}(\text{Do not reject } H_0 | \mu \in H_1)}$

Power of the Test

Type II error: $\overbrace{\mathbf{P}(\text{Do not reject } H_0 | \mu \in H_1)}^{\beta}$

Power of the test: $\overbrace{\mathbf{P}(\text{Reject } H_0 | \mu \in H_1)}^{1 - \beta}$

Power of the Test

$$\left. \begin{array}{l} \text{Type II error: } \mathbf{P} \left(\text{Do not reject } H_0 \mid \mu \in H_1 \right) \\ \text{Power of the test: } \mathbf{P} \left(\text{Reject } H_0 \mid \mu \in H_1 \right) \end{array} \right\} \overset{\beta}{\overbrace{\phantom{\text{Type II error: } \mathbf{P} \left(\text{Do not reject } H_0 \mid \mu \in H_1 \right)}}^{\text{Complementary probabilities}}}$$

Power of the Test

$$\left. \begin{array}{l} \text{Type II error: } \mathbf{P} \left(\text{Do not reject } H_0 \mid \mu \in H_1 \right) \\ \text{Power of the test: } \mathbf{P} \left(\text{Reject } H_0 \mid \mu \in H_1 \right) \end{array} \right\} \begin{array}{l} \beta \\ 1 - \beta \end{array}$$

Complementary probabilities

Power of the Test

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Complementary probabilities

Power of the test = 1 – Type II error probability

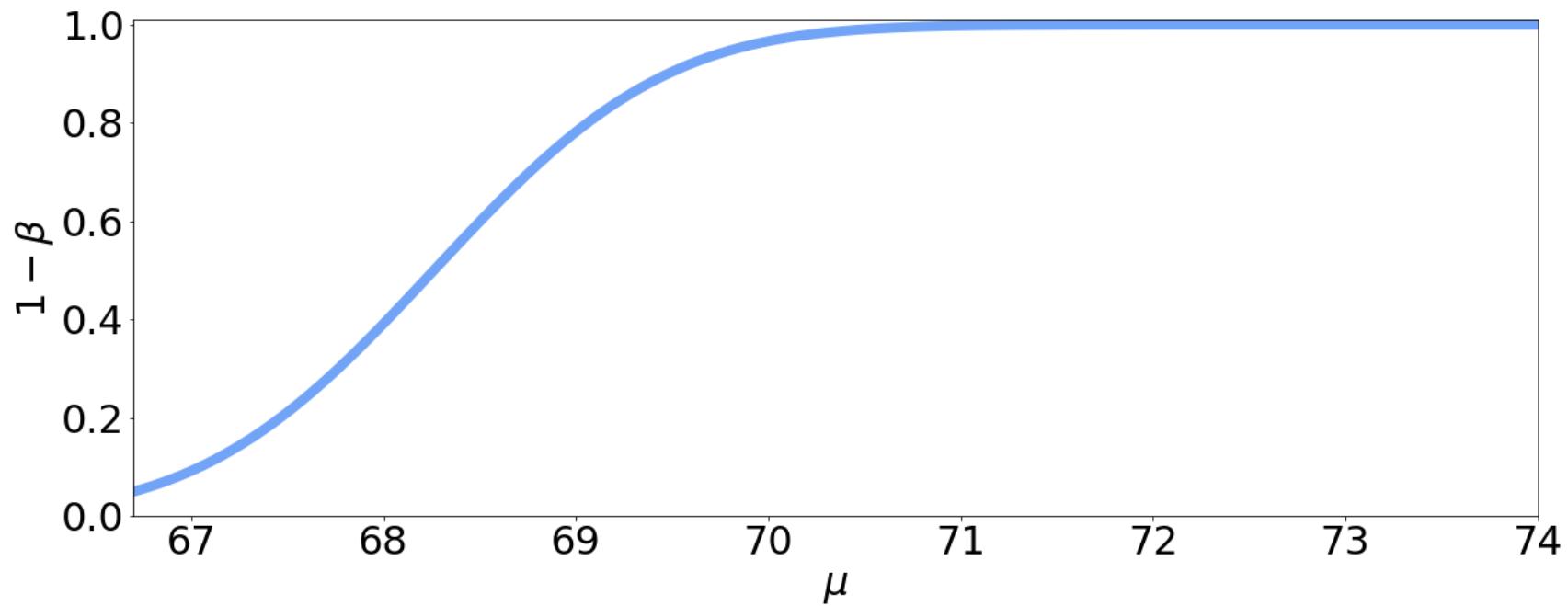
$$= 1 - \mathbf{P} \left(\text{Do not reject } H_0 \mid \mu \in H_1 \right)$$

Power of the Test

$H_0 : \mu = 66.7$ vs. $H_1 : \mu > 66.7$

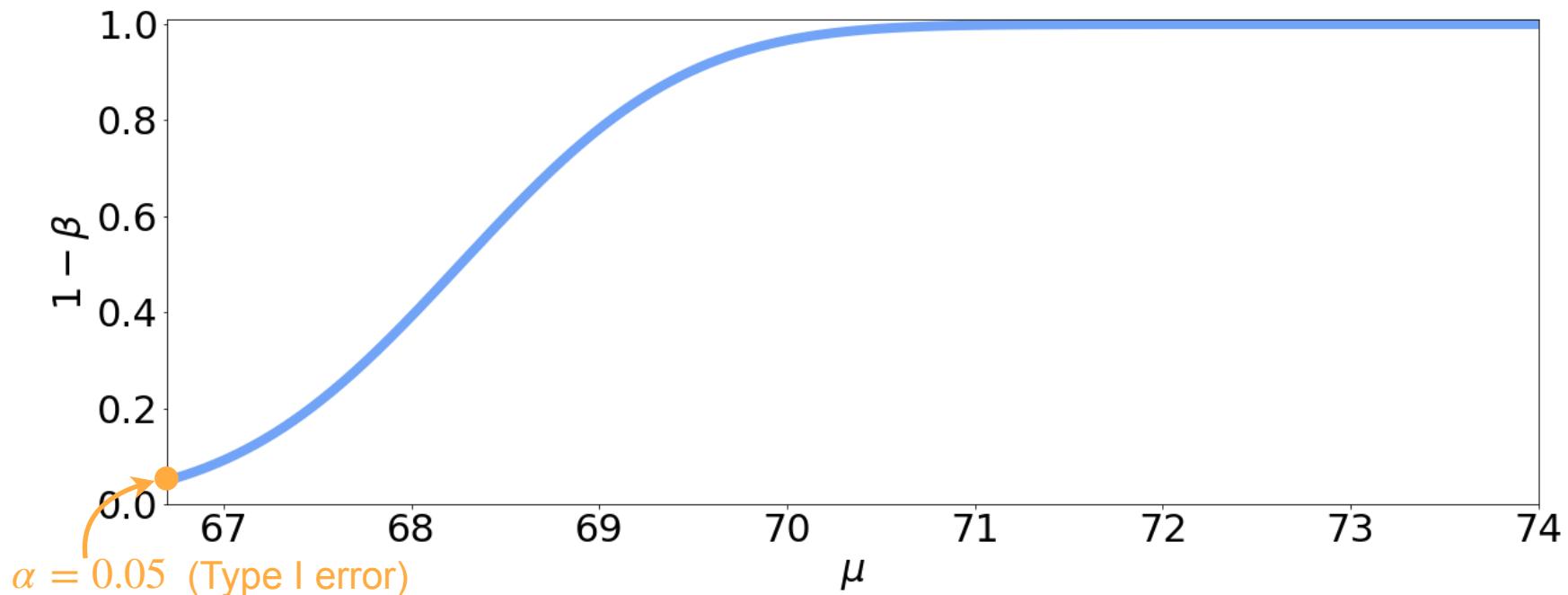
Power of the Test

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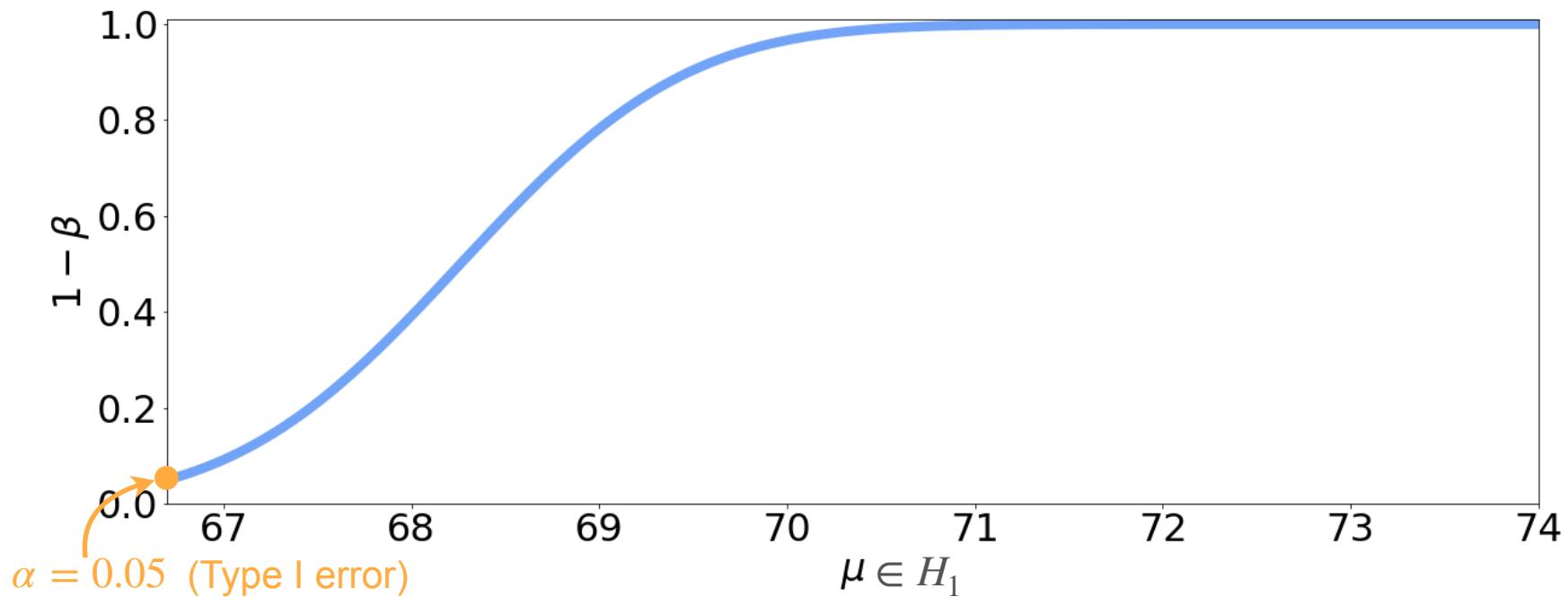
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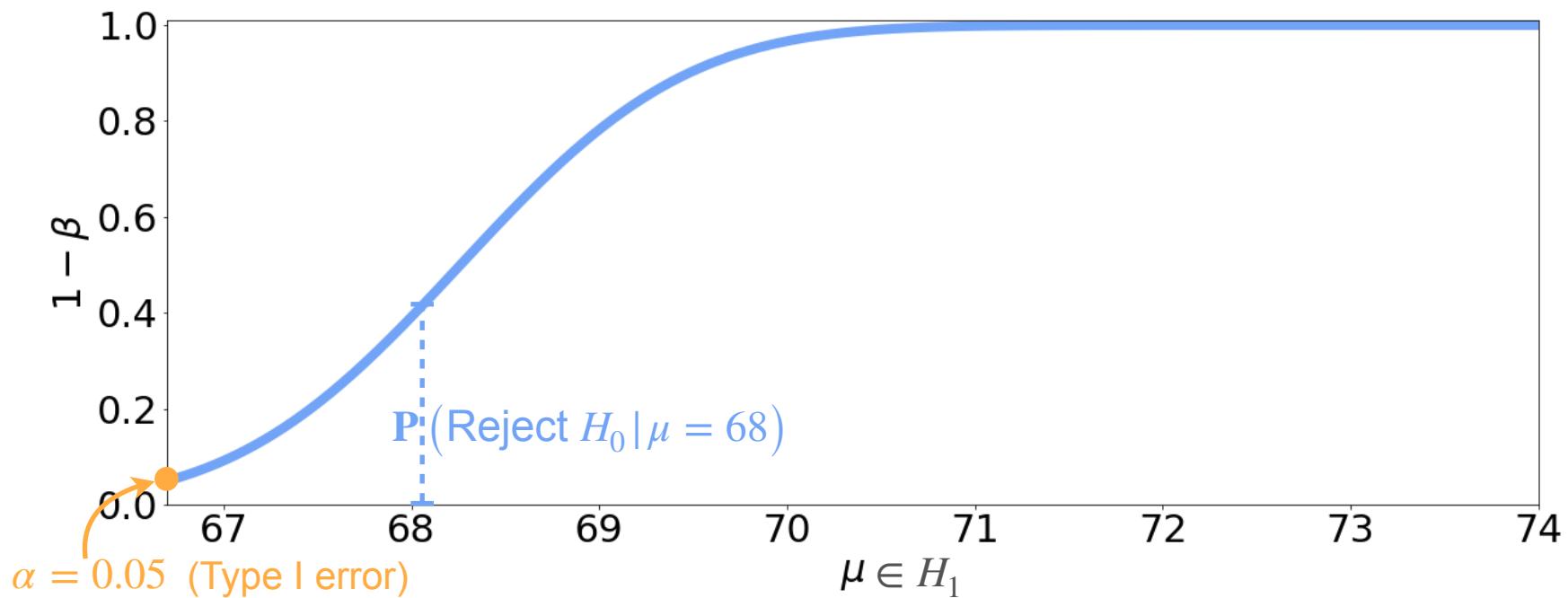
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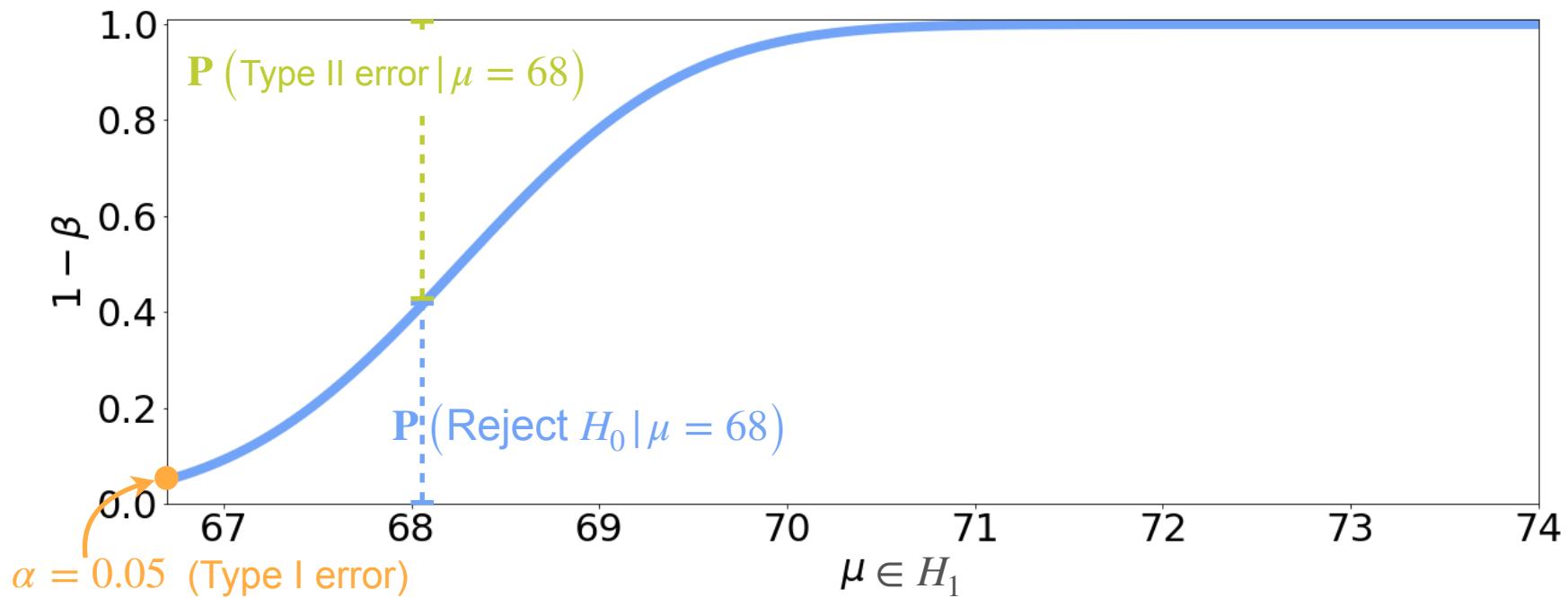
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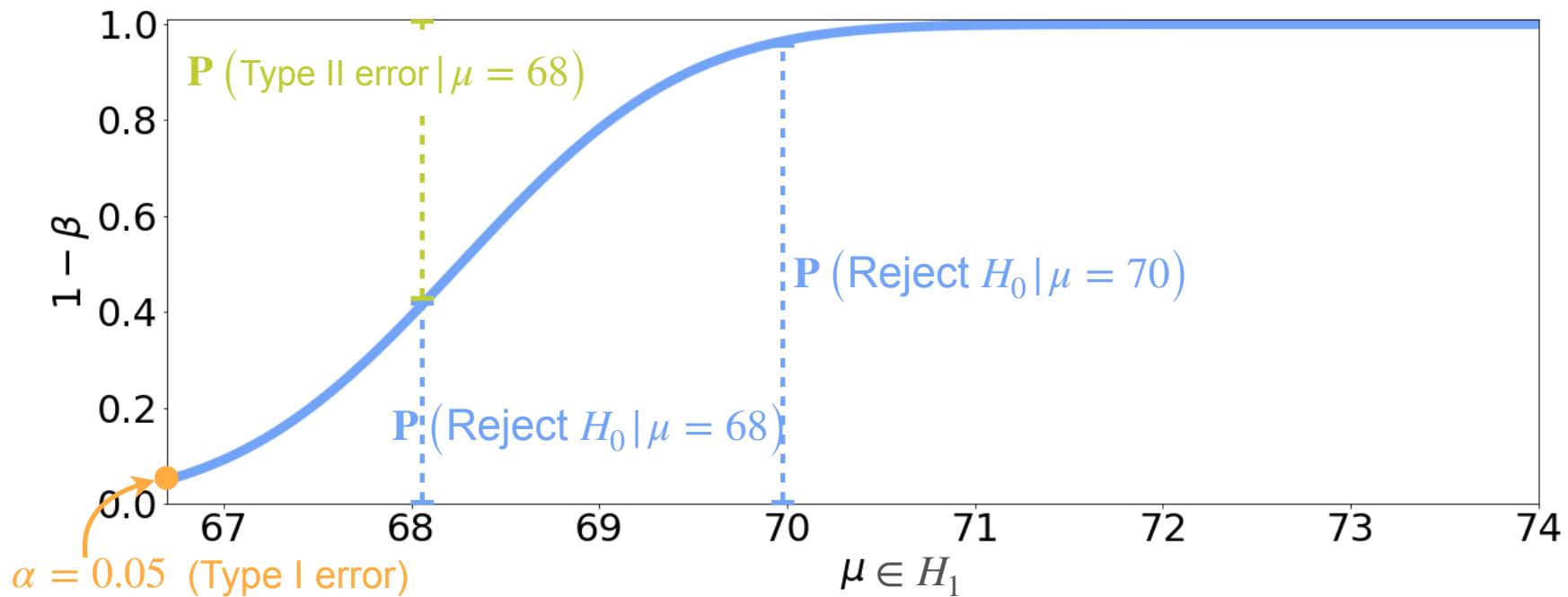
Power of the Test

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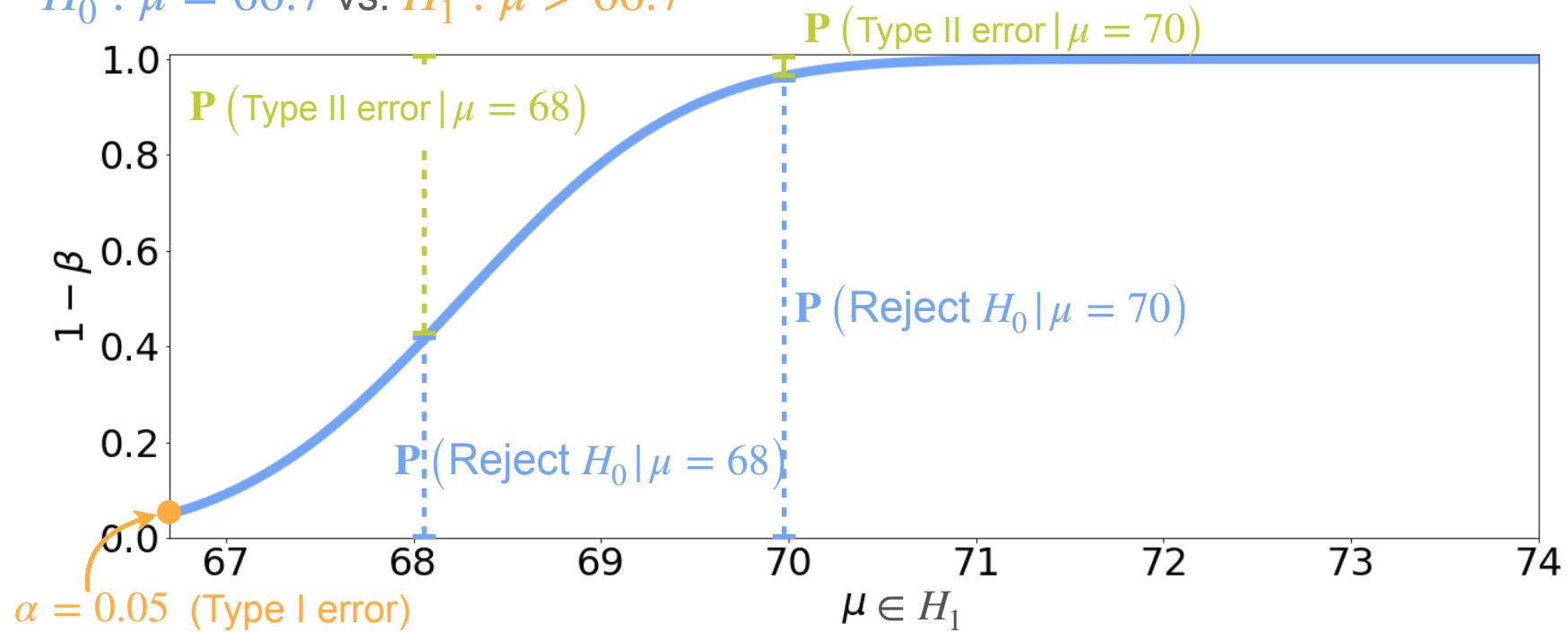
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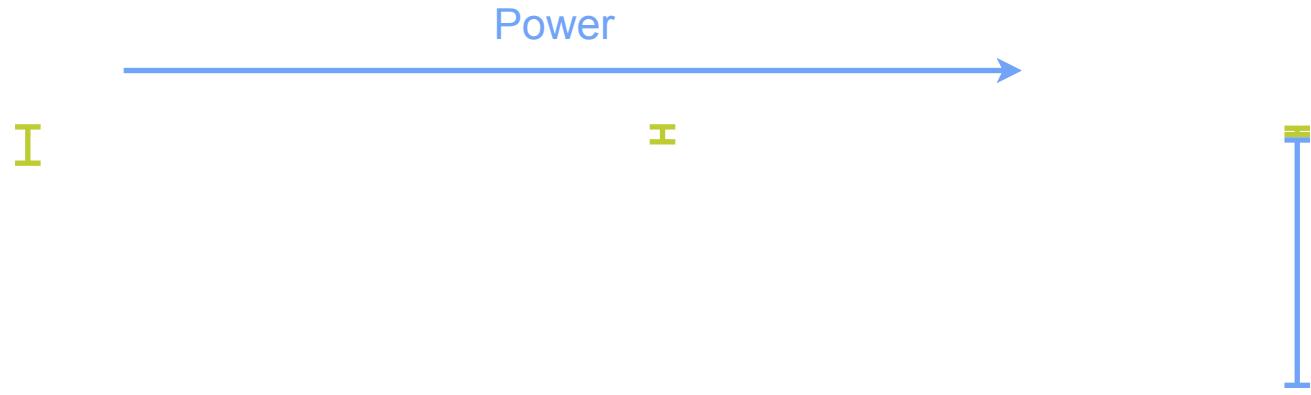


Power of the Test

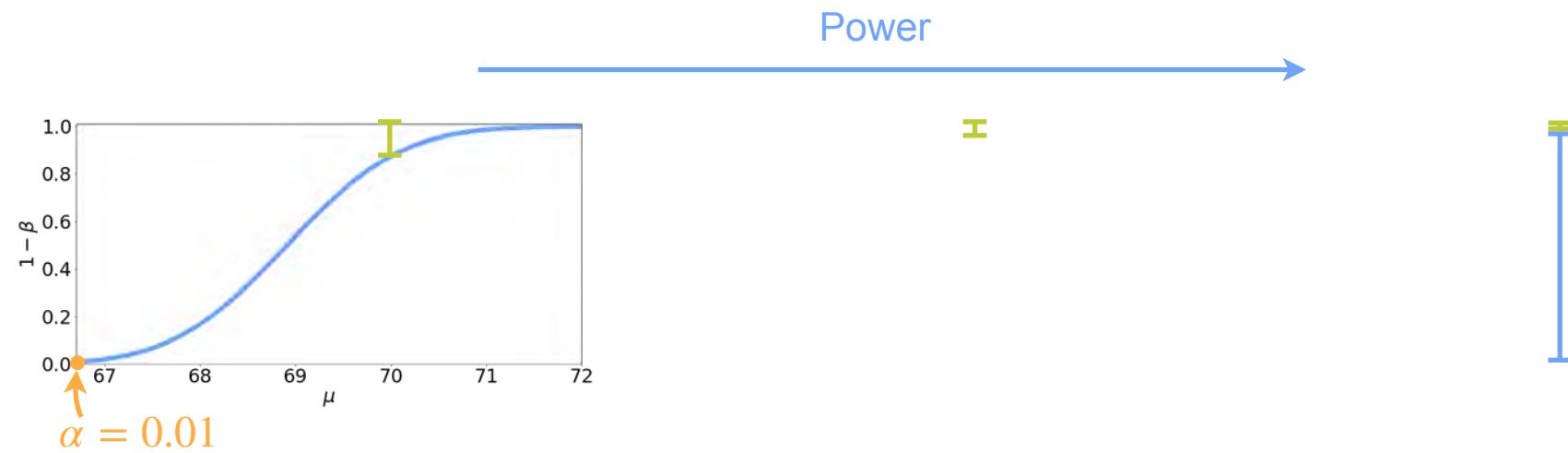
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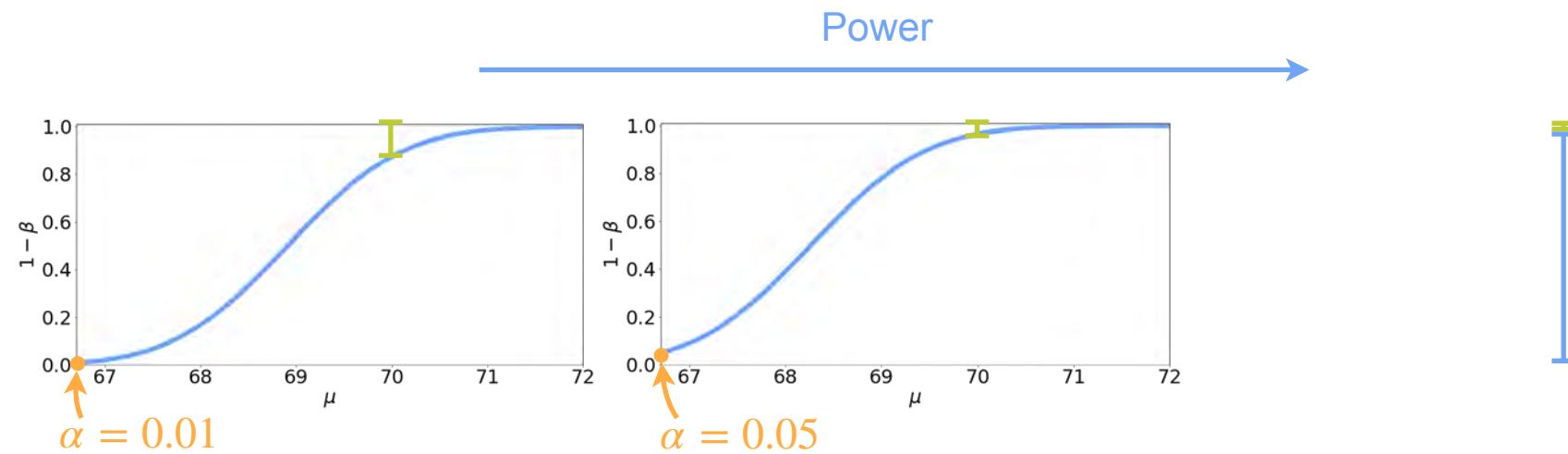
Power of the Test



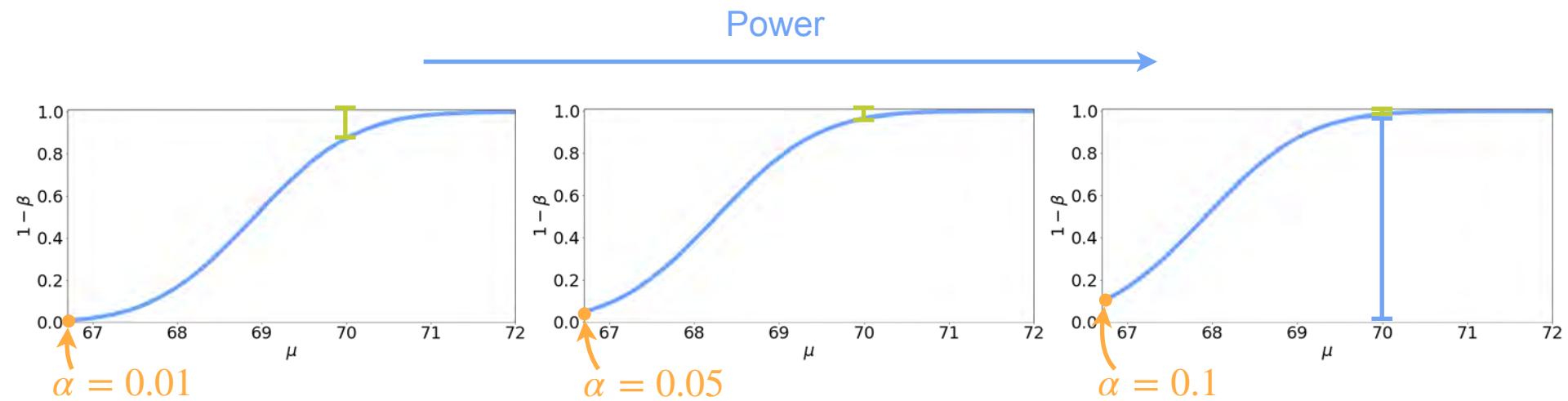
Power of the Test



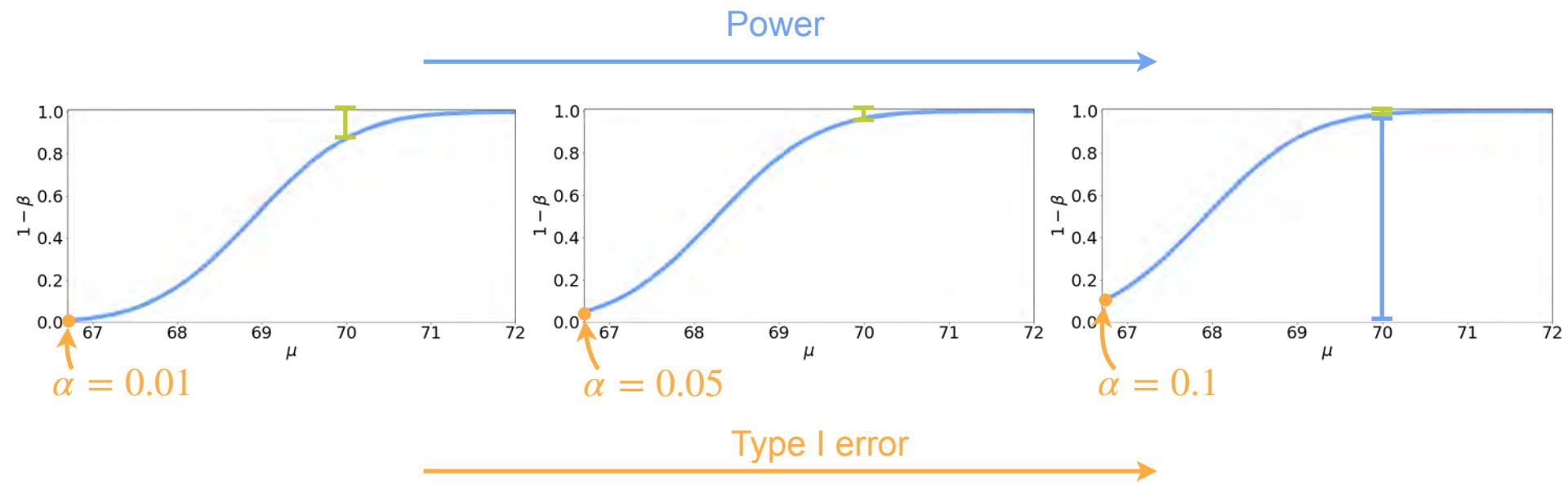
Power of the Test



Power of the Test



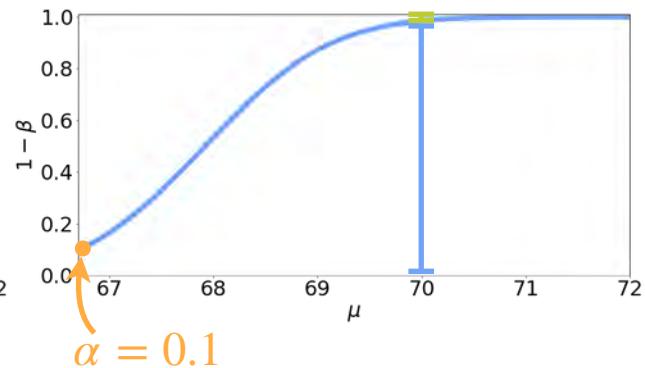
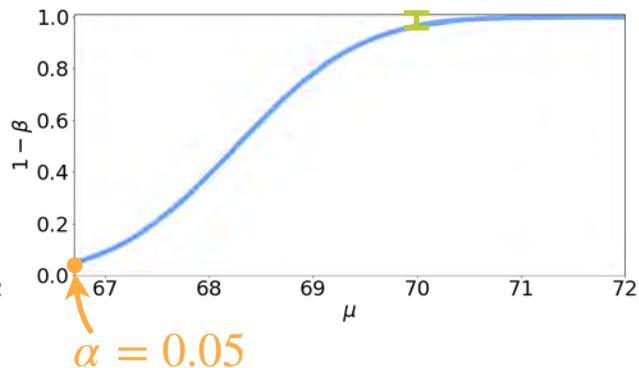
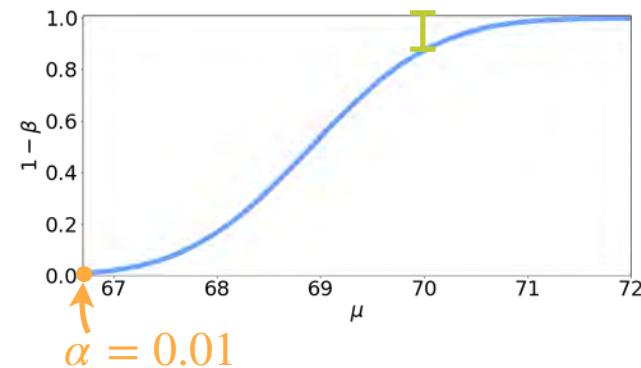
Power of the Test



Power of the Test

$$\mu = 70$$

Power

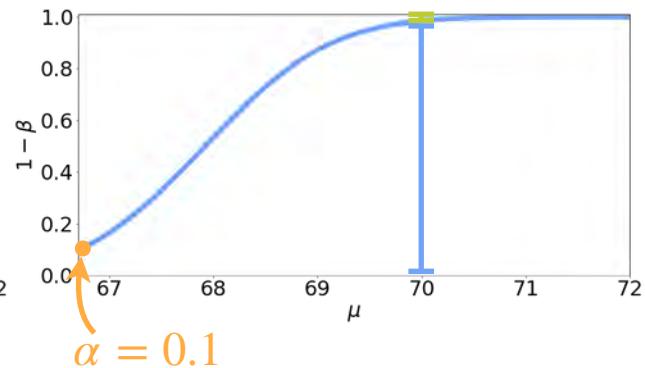
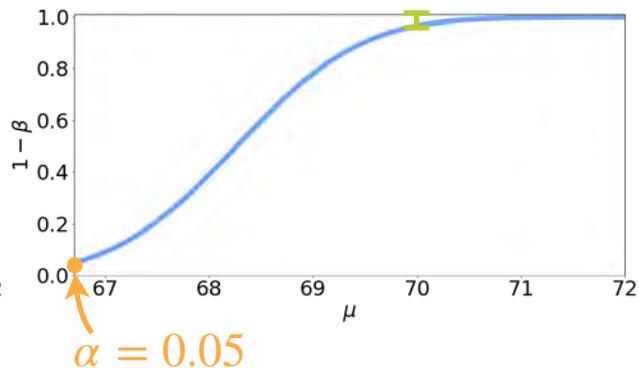
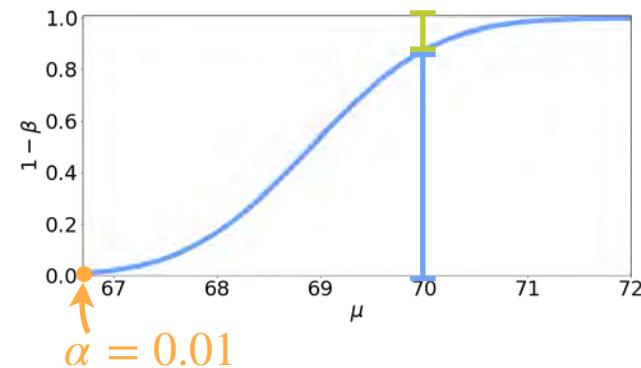


Type I error

Power of the Test

$$\mu = 70$$

Power

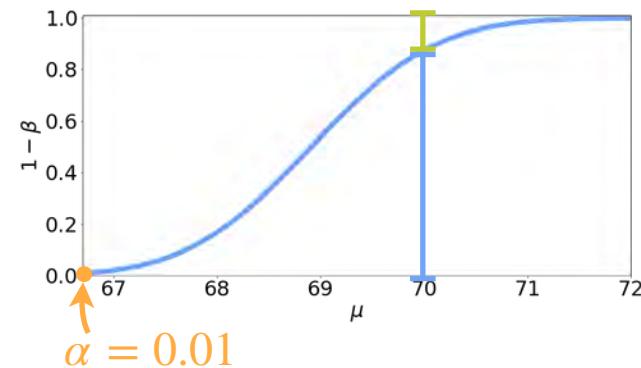


Type I error

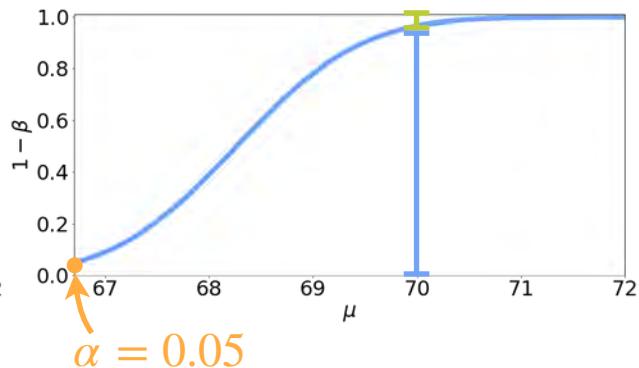
Power of the Test

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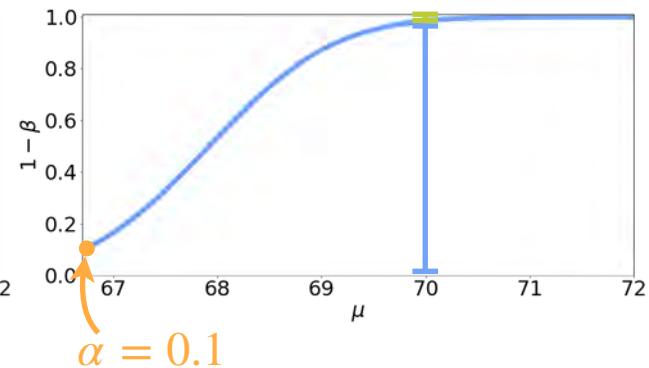
Power



$$\alpha = 0.01$$



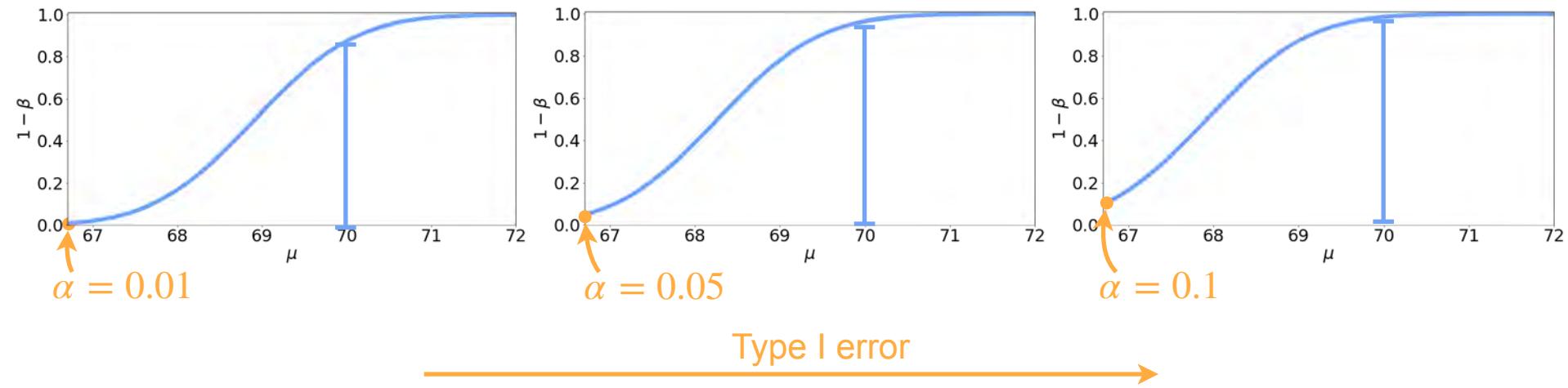
$$\alpha = 0.05$$



Type I error

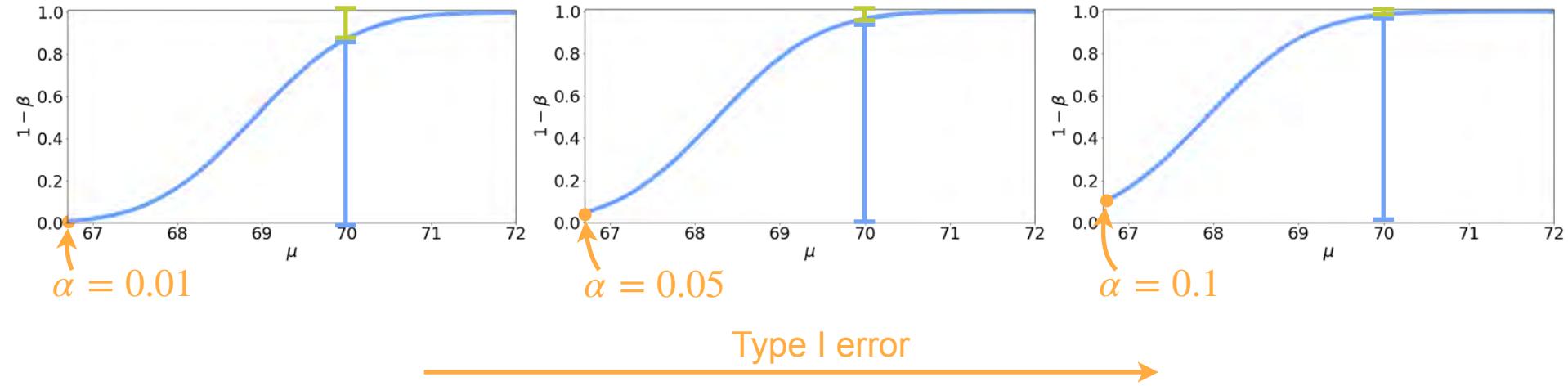
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Power of the Test

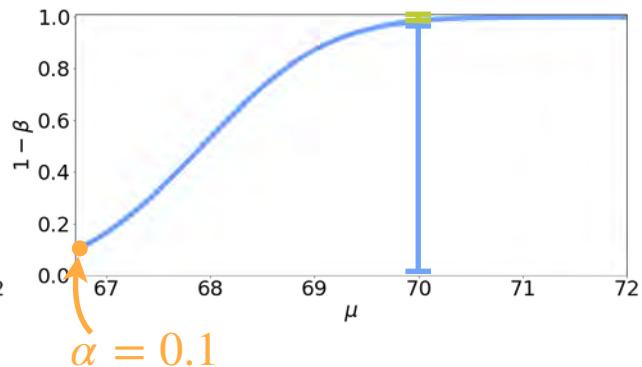
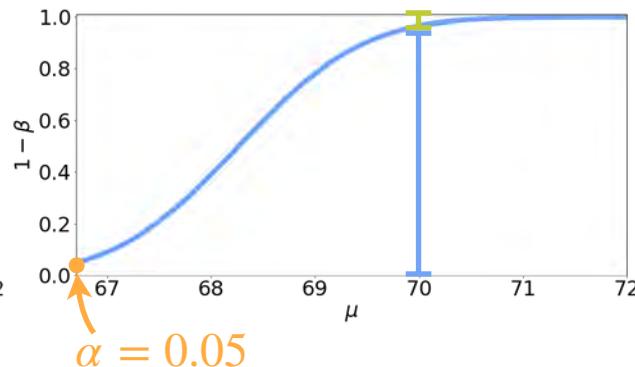
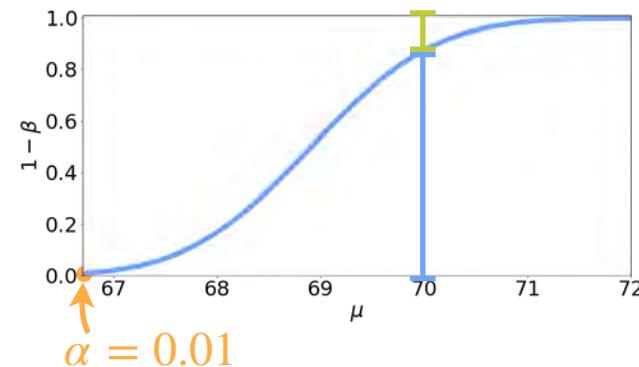
$$\mu = 70$$



Power of the Test

$$\mu = 70$$

Type II error



Type I error



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Hypothesis Testing

Interpreting results

Steps for Performing Hypothesis Testing

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1. State your hypotheses.

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4. Reach a conclusion:

- If the p -value is less than the significance level reject H_0
 $\rightarrow P(\bar{X} > 68.442 | \mu = 66.7) > ? 0.05$

Important Remarks - Interpreting Tests

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- Type I error:
 - Reject null hypothesis when it is true \rightarrow Reject H_0 when $\mu = 66.7$

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Important Remarks - Interpreting Tests

- ***p*-values:**

- If $P(\text{Reject } H_0 | H_0) < \alpha \longrightarrow \text{Reject } H_0 \text{ and then accept } H_1$
- A small *p*-value indicates that the probability of seeing the observed data by chance is small

- **Test conclusions**

- Reject $H_0 \rightarrow H_1$ true
- Do not reject $H_0 \rightarrow H_0$ true



You can only say that there is not enough evidence



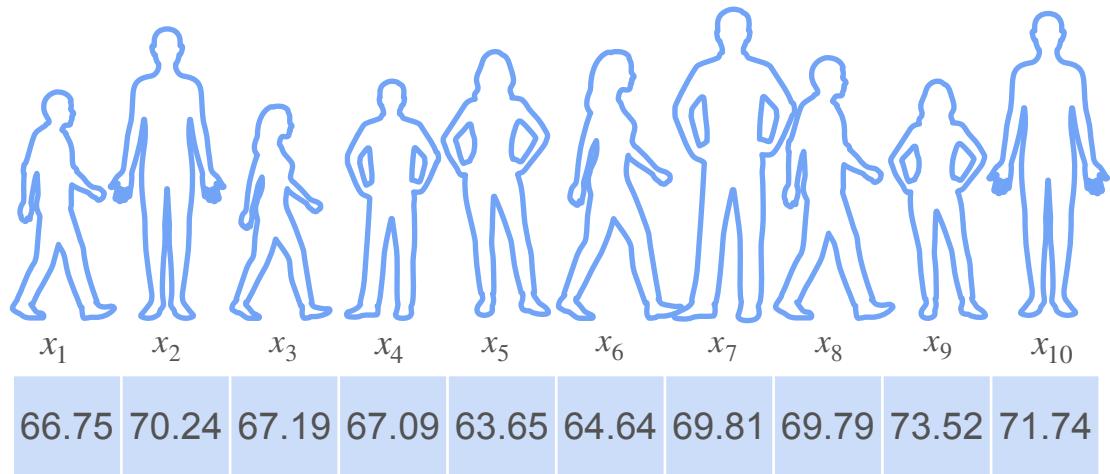
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Hypothesis Testing

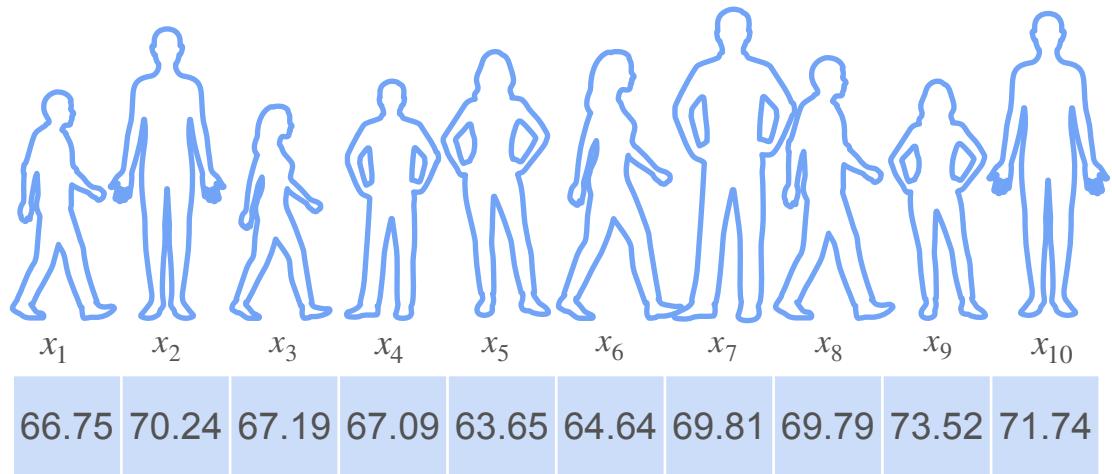
t-Distribution

t -Distribution: Motivation

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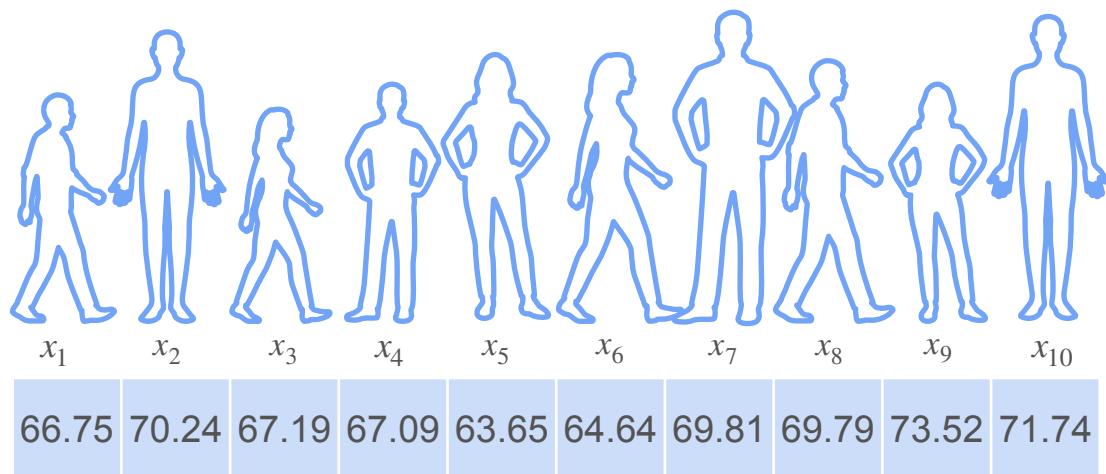


t -Distribution: Motivation



$$X_i \stackrel{i.i.d}{\sim} \mathcal{N}(\mu, \sigma^2)$$

t -Distribution: Motivation

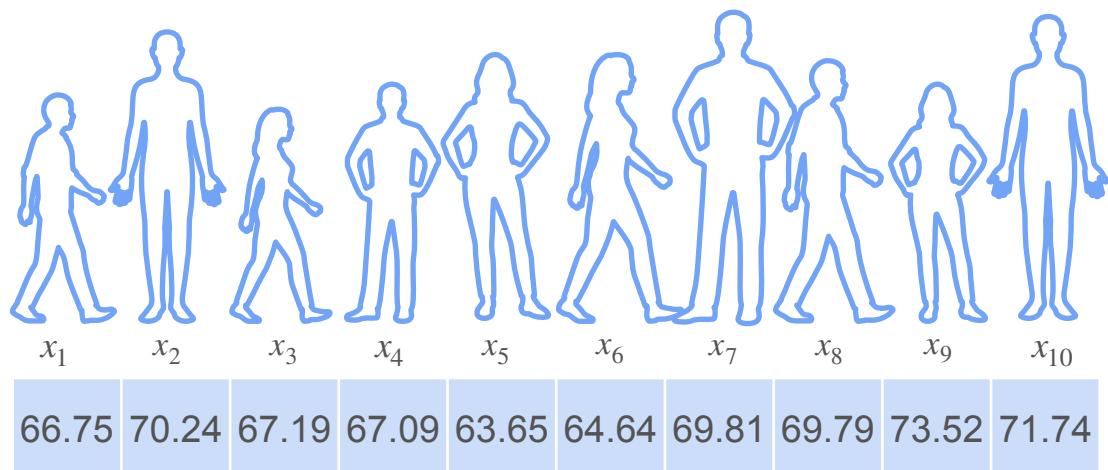


$$X_i \stackrel{i.i.d}{\sim} \mathcal{N}(\mu, \sigma^2)$$

\downarrow

$$\bar{X} = \frac{1}{10} \sum_{i=1}^{10} X_i \sim \mathcal{N}\left(\mu, \frac{\sigma^2}{10}\right)$$

t -Distribution: Motivation



$$X_i \stackrel{i.i.d}{\sim} \mathcal{N}(\mu, \sigma^2)$$

$$\bar{X} = \frac{1}{10} \sum_{i=1}^{10} X_i \sim \mathcal{N}\left(\mu, \frac{\sigma^2}{10}\right)$$

This is fine if you know μ and σ

What if σ is unknown?

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If μ, σ are known

$$\frac{\bar{X} - \mu}{\sigma/\sqrt{10}} \sim \mathcal{N}(0, 1^2)$$

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Z statistic

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What if σ is unknown?

Replace σ with its estimate

$$\frac{\bar{X} - \mu}{S / \sqrt{10}}$$

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T statistic

t -Distribution: Motivation

$$X_i \stackrel{i.i.d}{\sim} \mathcal{N}(\mu, \sigma^2)$$

$$\bar{X} = \frac{1}{10} \sum_{i=1}^{10} X_i \sim \mathcal{N}\left(\mu, \frac{\sigma^2}{10}\right)$$

$$S = \sqrt{\frac{1}{10-1} \sum_{i=1}^{10} (X_i - \bar{X})^2}$$

If μ, σ are known

$$\frac{\bar{X} - \mu}{\sigma / \sqrt{10}} \sim \mathcal{N}(0, 1^2) \text{ (Standardization)}$$

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t -Distribution

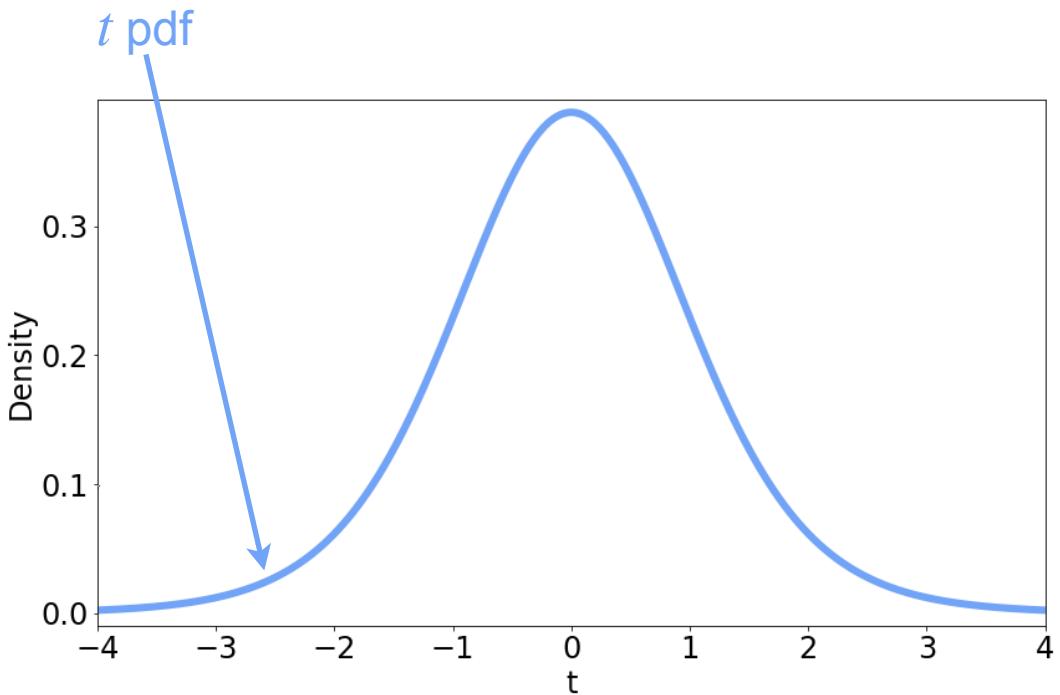
t-Distribution

$\frac{\bar{X} - \mu}{S/\sqrt{10}}$ follows a *t* distribution

t -Distribution

$\frac{\bar{X} - \mu}{S/\sqrt{10}}$ follows a t distribution

What does it look like?

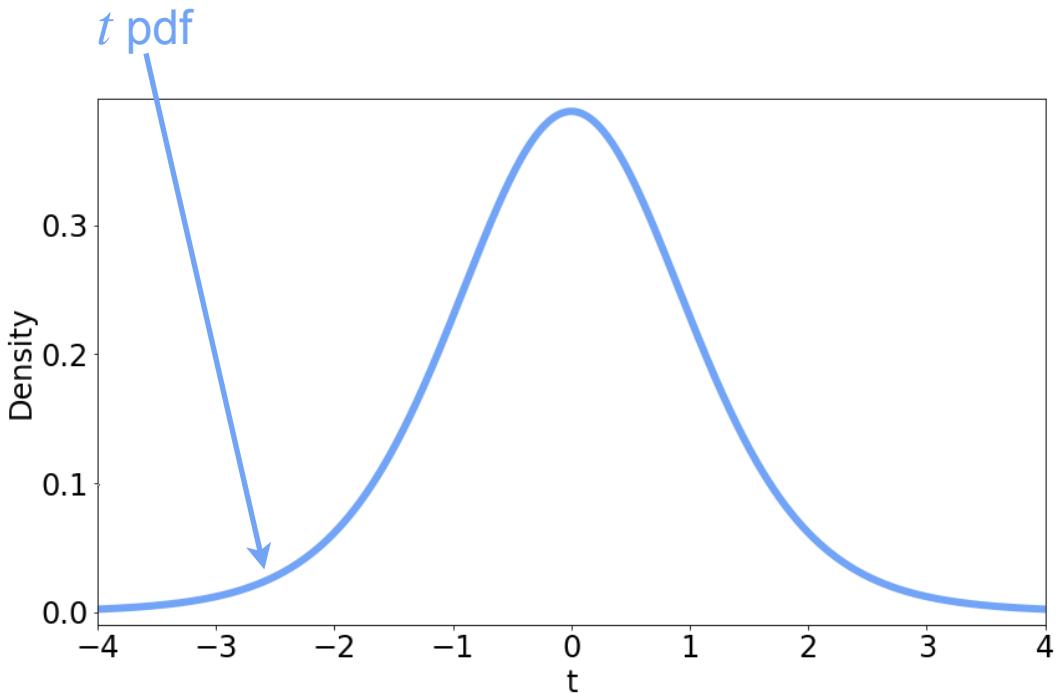


t -Distribution

$\frac{\bar{X} - \mu}{S/\sqrt{10}}$ follows a t distribution

What does it look like?

Still bell-shaped



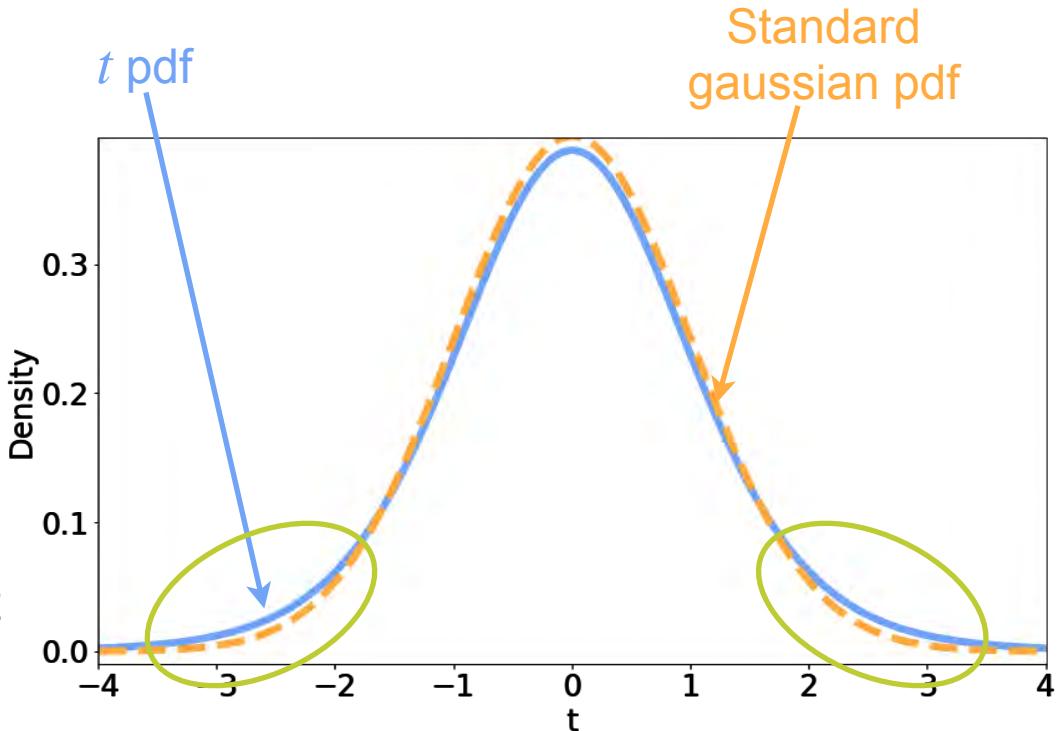
t-Distribution

$\frac{\bar{X} - \mu}{S/\sqrt{10}}$ follows a *t* distribution

What does it look like?

Still bell-shaped

It has heavier tails that account for the uncertainty introduced with the std estimation



t -Distribution

t-Distribution

Parameters:

- Degrees of freedom

t-Distribution

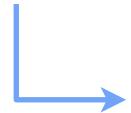
Parameters:

- Degrees of freedom (ν)

t -Distribution

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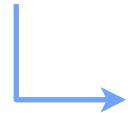


Controls how heavy
the tails are

t -Distribution

Parameters:

- Degrees of freedom (ν)



Controls how heavy
the tails are

$$X \sim t_{\nu}$$

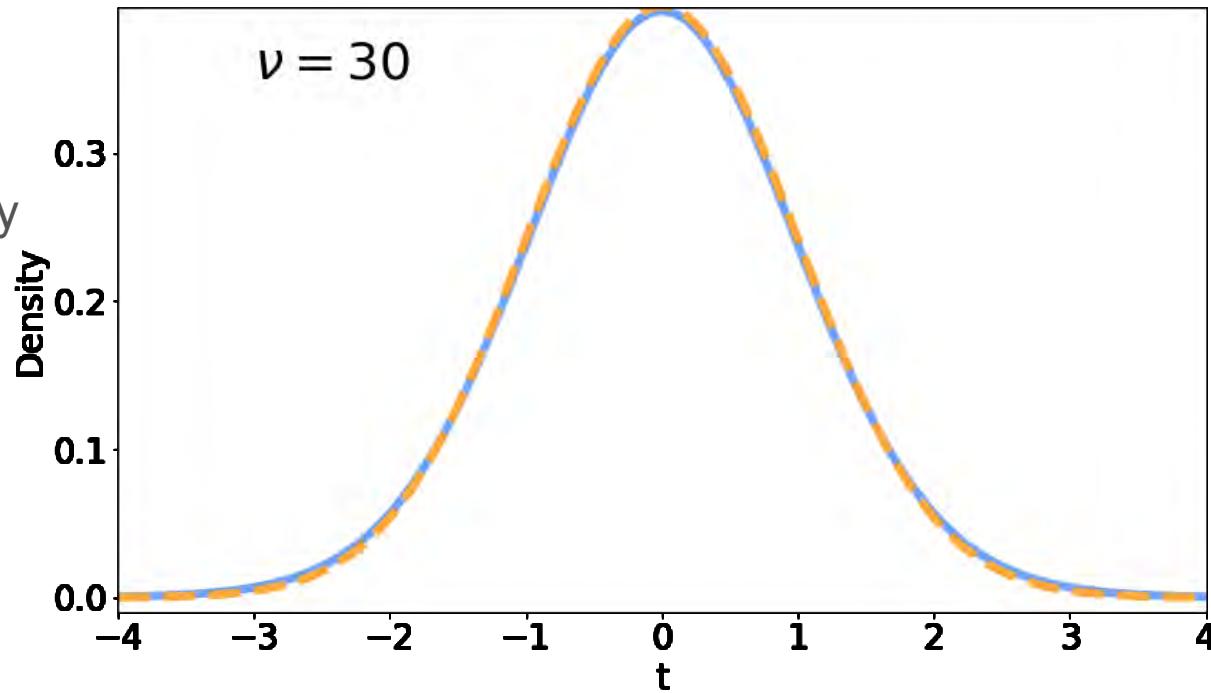
t -Distribution

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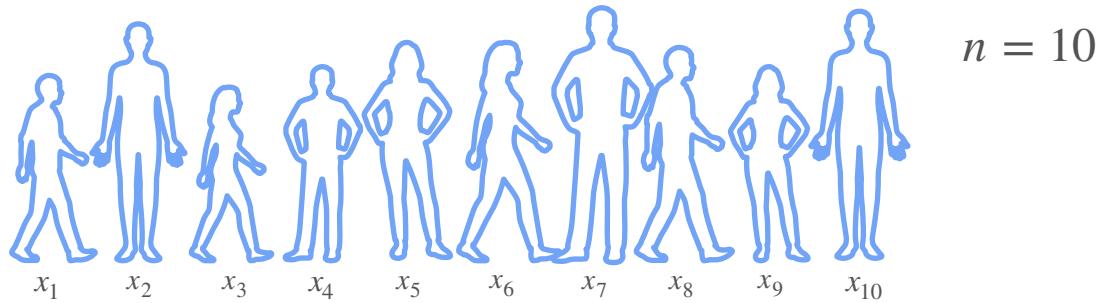
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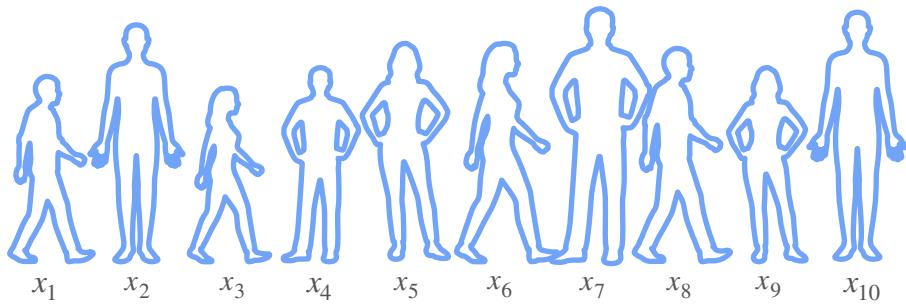


t -Distribution and T -Statistic

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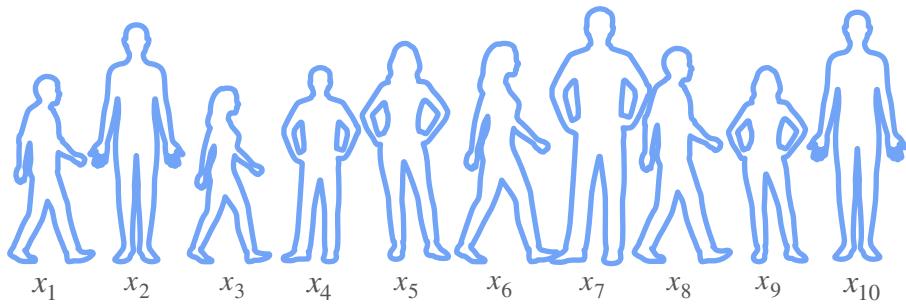
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$$n = 10$$

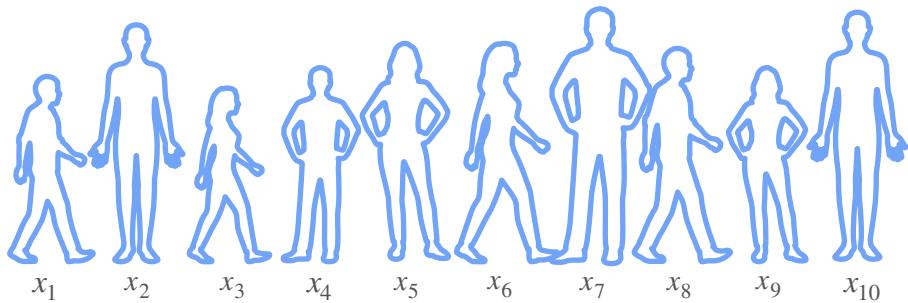
$$T = \frac{\bar{X} - \mu}{S/\sqrt{10}} \sim t_{\nu}$$

t -Distribution and T -Statistic



$$n = 10$$
$$\nu = 10 - 1$$
$$T = \frac{\bar{X} - \mu}{S/\sqrt{10}} \sim t_9$$

t -Distribution and T -Statistic

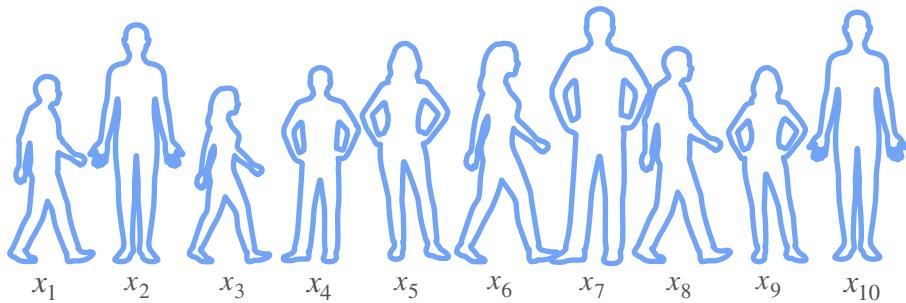


$$n = 10$$
$$T = \frac{\bar{X} - \mu}{S/\sqrt{10}} \sim t_9$$

$\nu = 10 - 1$

Degrees of freedom (ν) = sample size - 1
 $= (n - 1)$

t -Distribution and T -Statistic

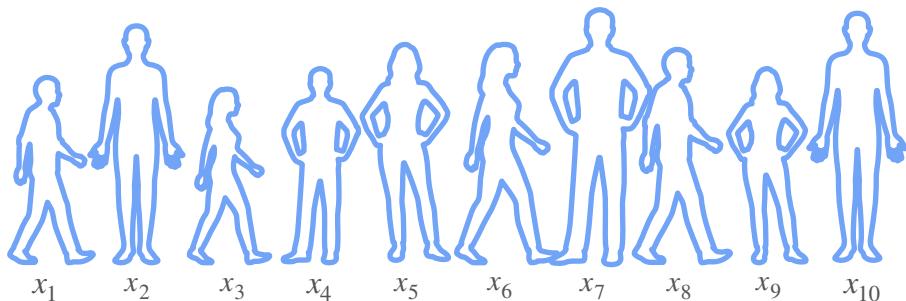


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As n increases, this looks more
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t -Distribution and T -Statistic



$$n = 10$$
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$\nu = 10 - 1$

Degrees of freedom (ν) = sample size - 1
= $(n - 1)$

As n increases, this looks more like a $\mathcal{N}(0, 1^2)$

T -statistic is used when

- The population has a Gaussian distribution
- But you don't know the variance



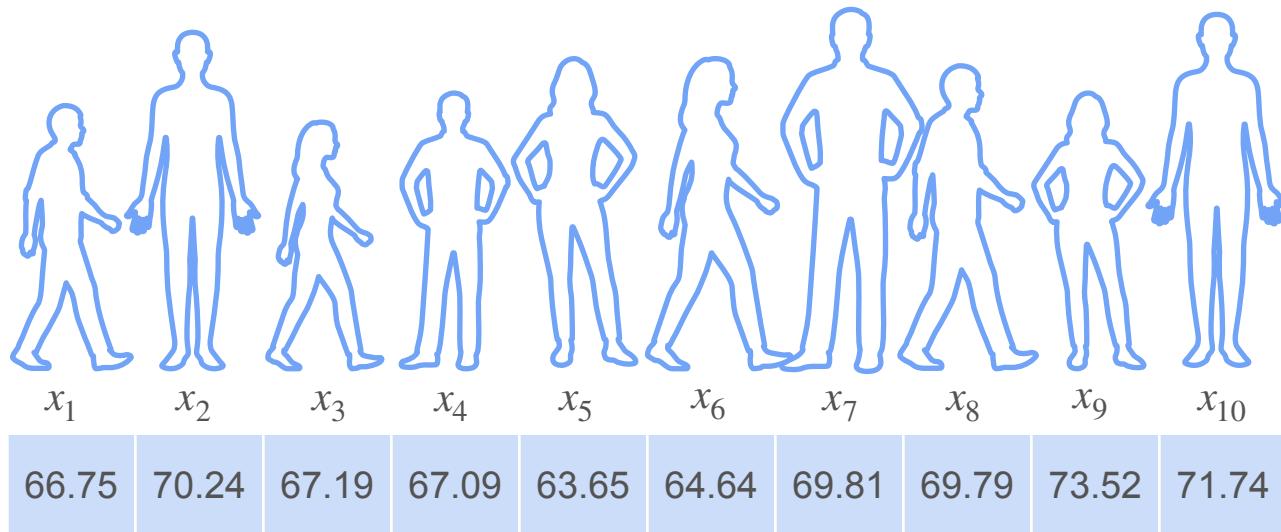
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Hypothesis Testing

t-Tests

Example: Heights

Example: Heights



$$\bar{x} = 68.442$$

Example: Heights

Example: Heights

The **mean** height for 18 y/o in the US in the 70s was **66.7 in.**



3 questions

3 sets of hypothesis

$$H_0 : \mu = 66.7 \text{ vs. } H_1 : \mu > 66.7$$



$$H_0 : \mu = 66.7 \text{ vs. } H_1 : \mu < 66.7$$



$$H_0 : \mu = 66.7 \text{ vs. } H_1 : \mu \neq 66.7$$

Example: Heights

The **mean** height for 18 y/o in the US in the 70s was **66.7 in.**

Example: Heights

The **mean** height for 18 y/o in the US in the 70s was **66.7 in.**

$$\sigma = 3$$

$$n = 10$$

$$H_0 : \mu = 66.7$$

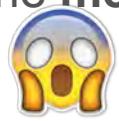
$$\text{If } H_0 \text{ is true: } \bar{X} \sim \mathcal{N}\left(66.7, \frac{3^2}{10}\right)$$

Null hypothesis



Example: Heights

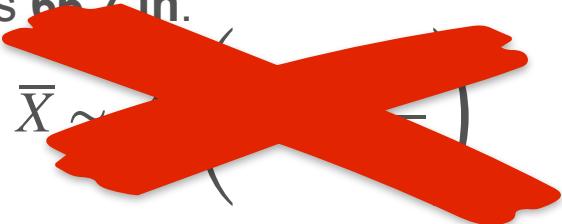
The **mean** height for 18 y/o in the US in the 70s was **66.7 in.**



~~$n = 10$~~

$$H_0 : \mu = 66.7$$

If H_0 is true: $\bar{X} \sim$



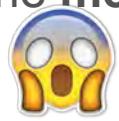
Null hypothesis

66.7

μ

Example: Heights

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~~n = 10~~

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Null hypothesis

66.7

μ

If H_0 is true: $\bar{X} \sim$

$$\text{If } H_0 \text{ is true: } T = \frac{\bar{X} - 66.7}{S/\sqrt{10}} \sim t_9$$

Example: Heights

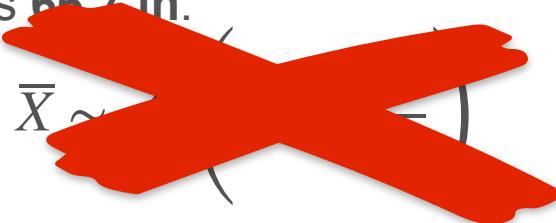
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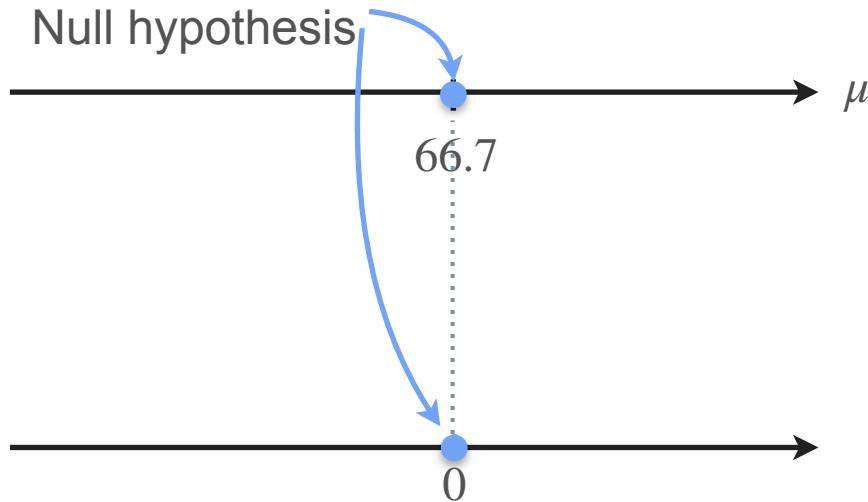
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$$H_0 : \mu = 66.7$$

If H_0 is true: $\bar{X} \sim$



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Example: Heights

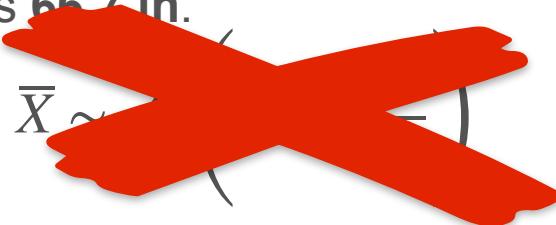
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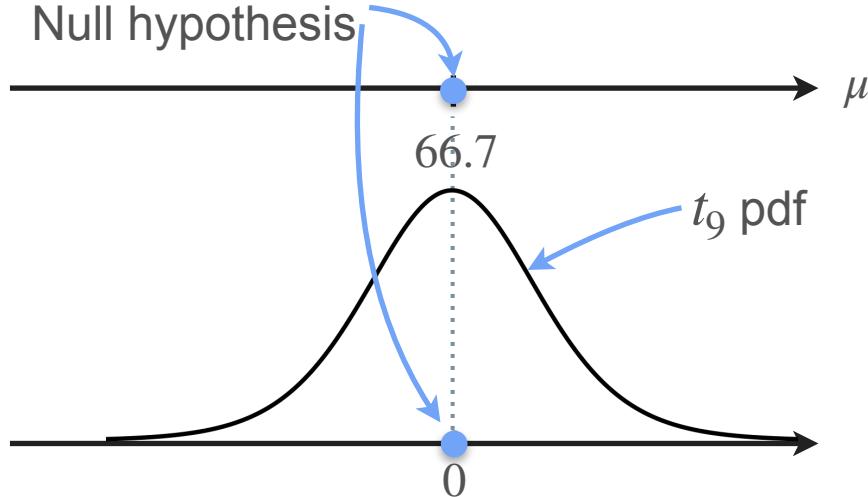
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$$H_0 : \mu = 66.7$$

If H_0 is true: $\bar{X} \sim$



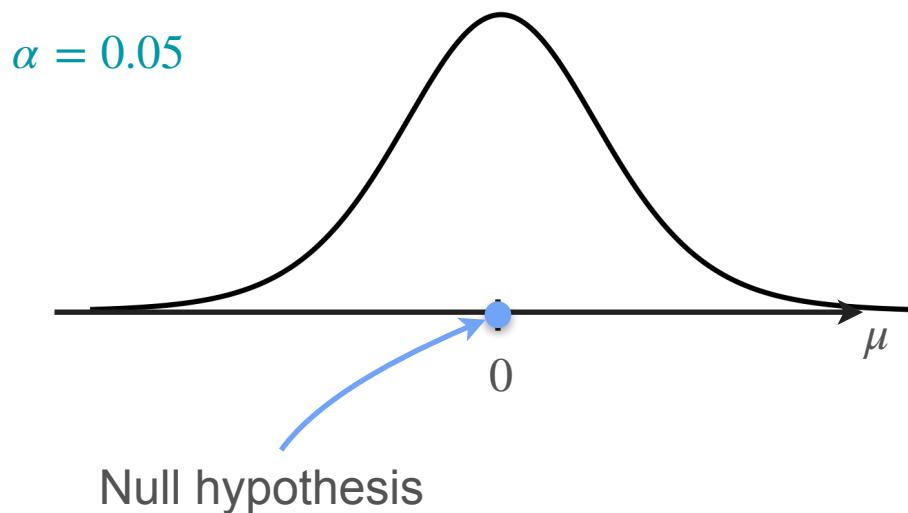
Null hypothesis



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Right-Tailed Test for Gaussian Data (Unknown σ)

The mean height for 18 y/o in the US in the 70s was **66.7 in.**

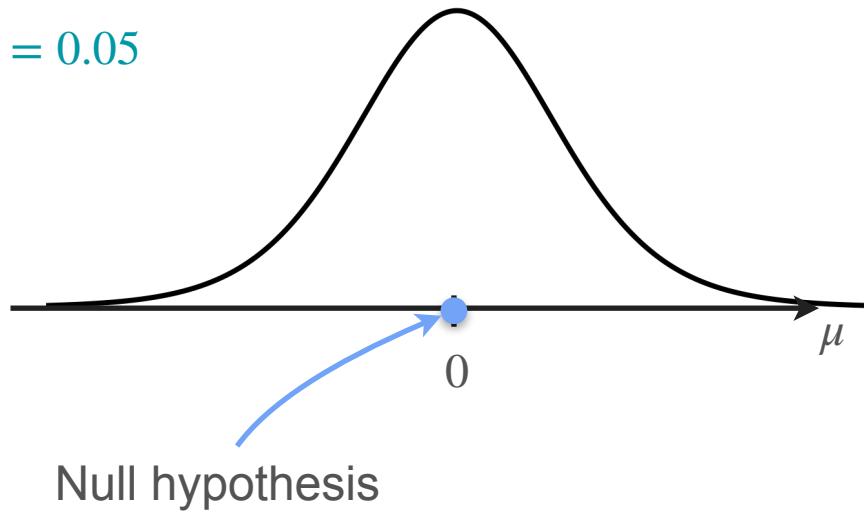


Right-Tailed Test for Gaussian Data (Unknown σ)

The **mean** height for 18 y/o in the US in the 70s was **66.7 in.**

$$H_0 : \mu = 66.7 \text{ vs. } H_1 : \mu > 66.7$$

$$\alpha = 0.05$$



Right-Tailed Test for Gaussian Data (Unknown σ)

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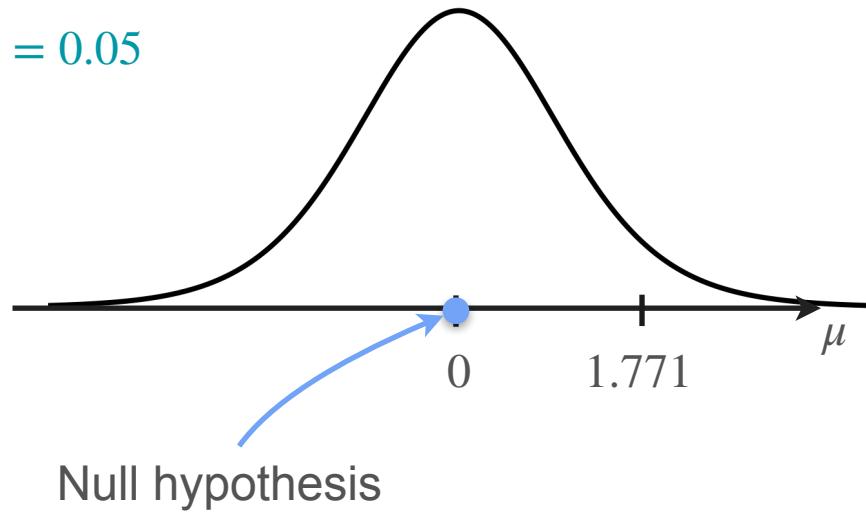
$$H_0 : \mu = 66.7 \text{ vs. } H_1 : \mu > 66.7 \quad n = 10$$

$$\alpha = 0.05$$

$$\bar{x} = 68.442$$

$$s = 3.113$$

$$t = \frac{68.442 - 66.7}{3.113/\sqrt{10}} = 1.771$$



Right-Tailed Test for Gaussian Data (Unknown σ)

The mean height for 18 y/o in the US in the 70s was **66.7** in.

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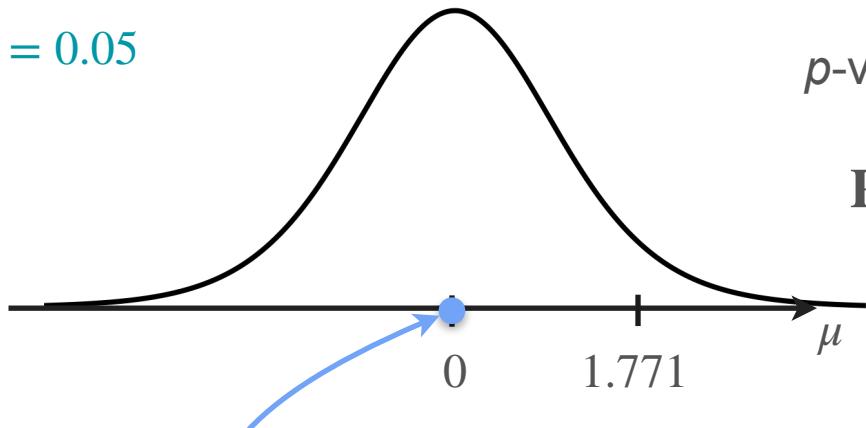
$$s = 3.113$$

$$t = \frac{68.442 - 66.7}{3.113/\sqrt{10}} = 1.771$$

$$\alpha = 0.05$$

p-value:

$$P\left(\frac{\bar{X} - 66.7}{S/\sqrt{10}} > 1.771 \mid \mu = 66.7\right) ?$$



Null hypothesis

Right-Tailed Test for Gaussian Data (Unknown σ)

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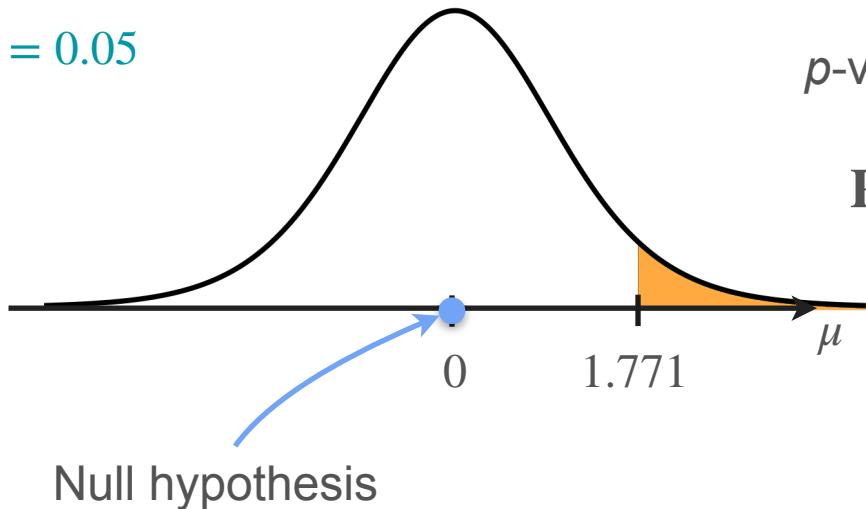
$$s = 3.113$$

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p-value:

$$P\left(\frac{\bar{X} - 66.7}{S/\sqrt{10}} > 1.771 \mid \mu = 66.7\right)$$

$$= 0.0552$$



Right-Tailed Test for Gaussian Data (Unknown σ)

The mean height for 18 y/o in the US in the 70s was **66.7** in.

$$H_0 : \mu = 66.7 \text{ vs. } H_1 : \mu > 66.7 \quad n = 10$$

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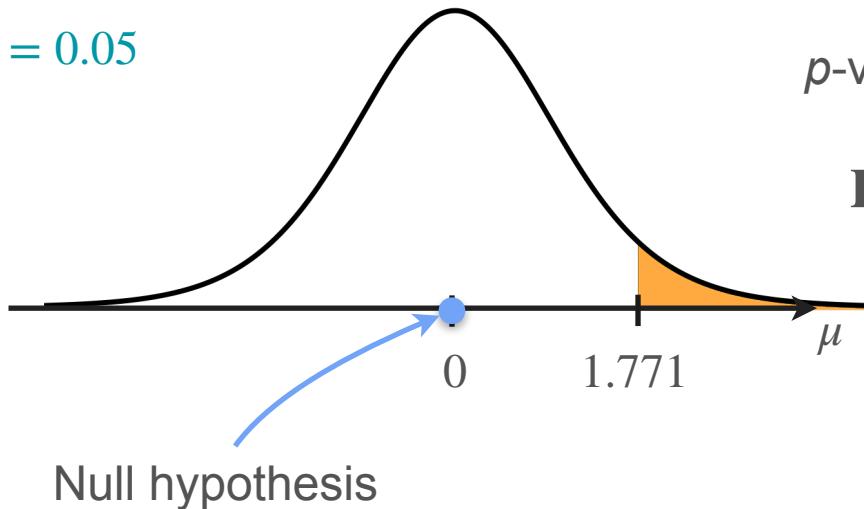
$$\alpha = 0.05$$

p-value:

$$P\left(\frac{\bar{X} - 66.7}{S/\sqrt{10}} > 1.771 \mid \mu = 66.7\right)$$

$$= 0.0552 > \alpha$$

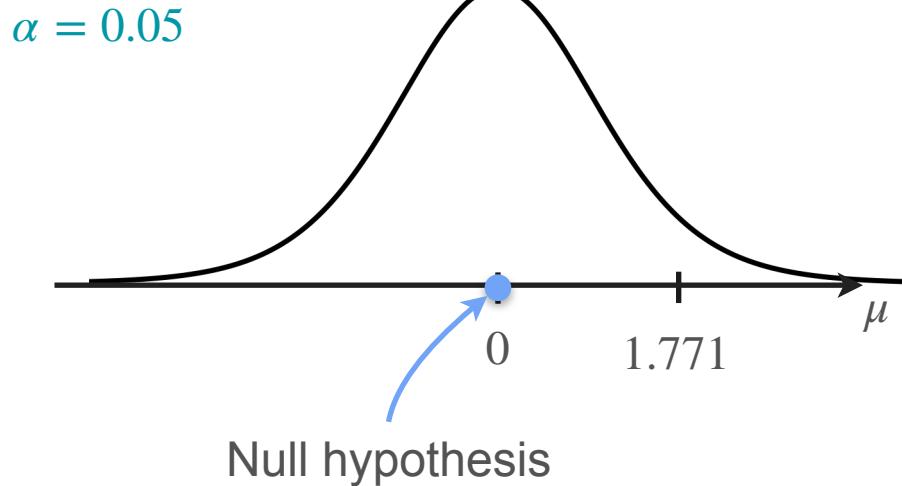
Conclusion: do not reject H_0
(with a 5% significance level)



Two-Tailed Test for Gaussian Data (Unknown σ)

The mean height for 18 y/o in the US in the 70s was **66.7** in.

$$n = 10 \quad \bar{x} = 68.442 \quad t = \frac{68.442 - 66.7}{3.113/\sqrt{10}} = 1.771$$
$$s = 3.113$$



Two-Tailed Test for Gaussian Data (Unknown σ)

The mean height for 18 y/o in the US in the 70s was **66.7** in.

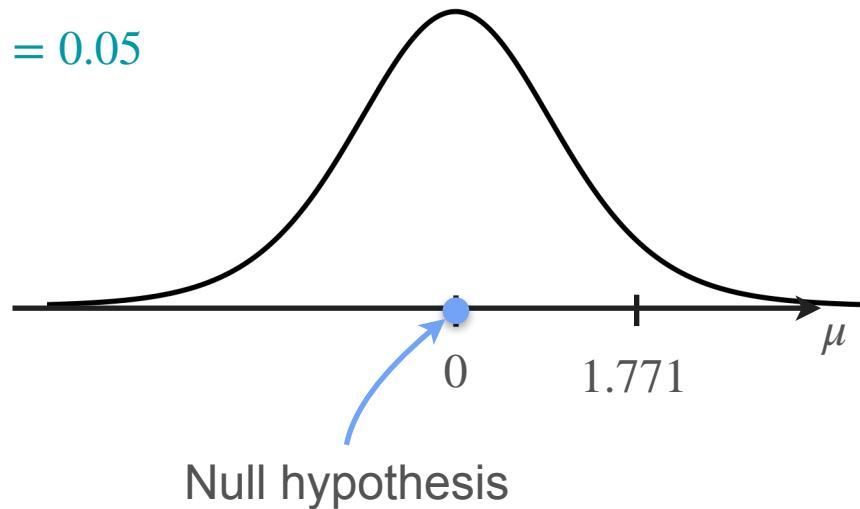
$$H_0 : \mu = 66.7 \text{ vs. } H_1 : \mu \neq 66.7 \quad n = 10$$

$$\bar{x} = 68.442$$

$$s = 3.113$$

$$t = \frac{68.442 - 66.7}{3.113/\sqrt{10}} = 1.771$$

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Two-Tailed Test for Gaussian Data (Unknown σ)

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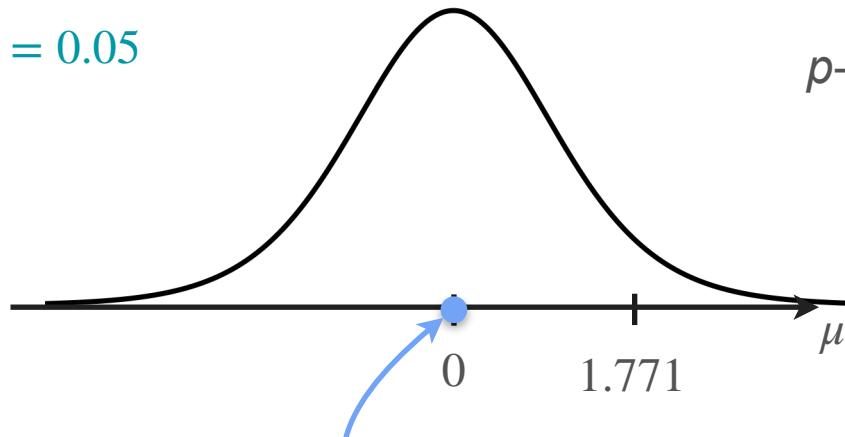
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p-value:

$$P\left(\left|\frac{\bar{X} - 66.7}{S/\sqrt{10}}\right| > 1.771 \mid \mu = 66.7\right)?$$

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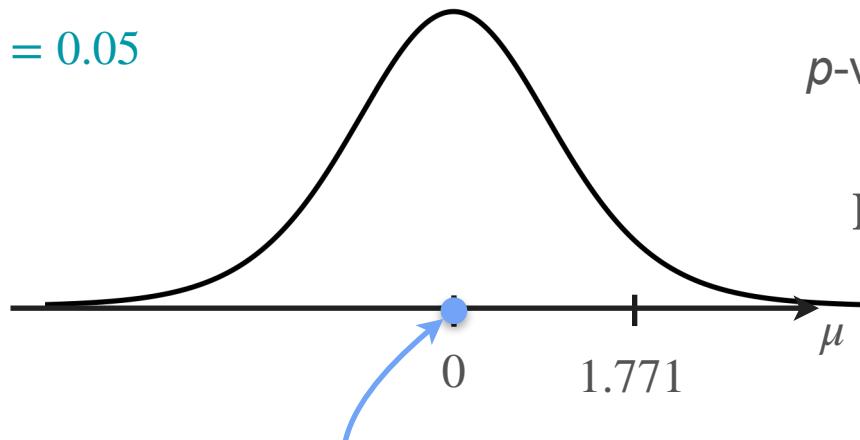
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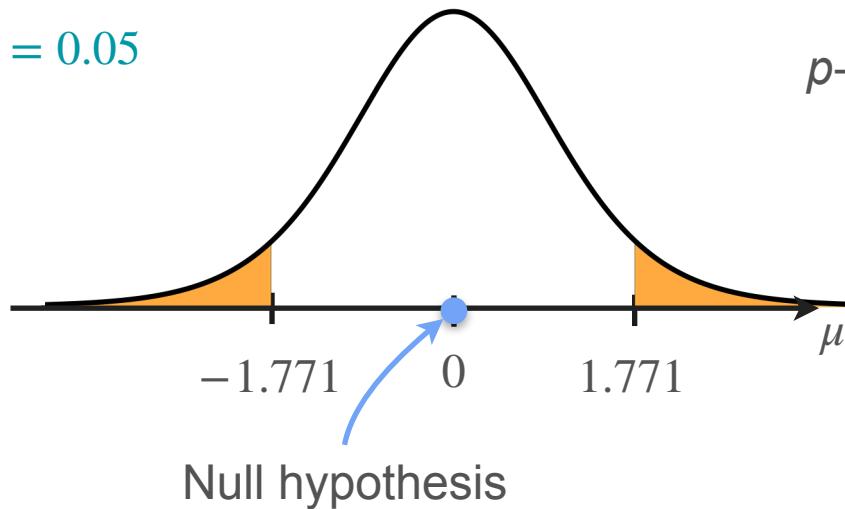
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$$\alpha = 0.05$$



p-value:

$$P\left(\left|\frac{\bar{X} - 66.7}{S/\sqrt{10}}\right| > |1.771| \mid \mu = 66.7\right) = 0.1103$$

Two-Tailed Test for Gaussian Data (Unknown σ)

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$$H_0 : \mu = 66.7 \text{ vs. } H_1 : \mu \neq 66.7$$

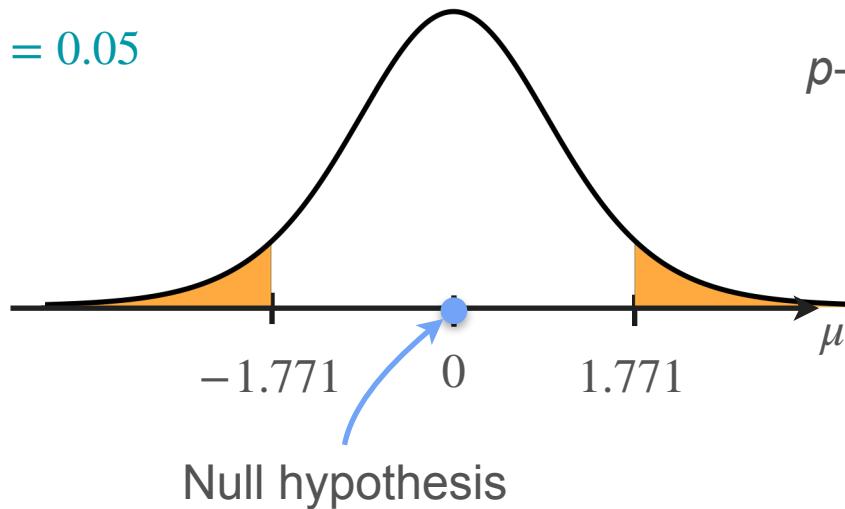
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p-value:

$$P\left(\left|\frac{\bar{X} - 66.7}{S/\sqrt{10}}\right| > |1.771| \mid \mu = 66.7\right) = 0.1103 > \alpha$$

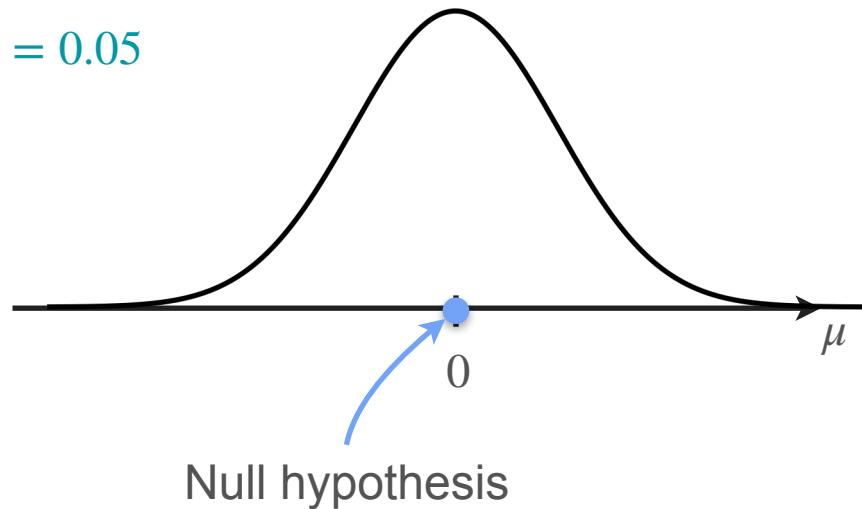
Conclusion: do not reject H_0
(with a 5% significance level)

Left-Tailed Test for Gaussian Data (Unknown σ)

The mean height for 18 y/o in the US in the 70s was **66.7** in.

$$n = 10 \quad \bar{x} = 64.252 \quad t = \frac{64.252 - 66.7}{3.113/\sqrt{10}}$$
$$s = 3.113$$

$$\alpha = 0.05$$



Left-Tailed Test for Gaussian Data (Unknown σ)

The mean height for 18 y/o in the US in the 70s was **66.7** in.

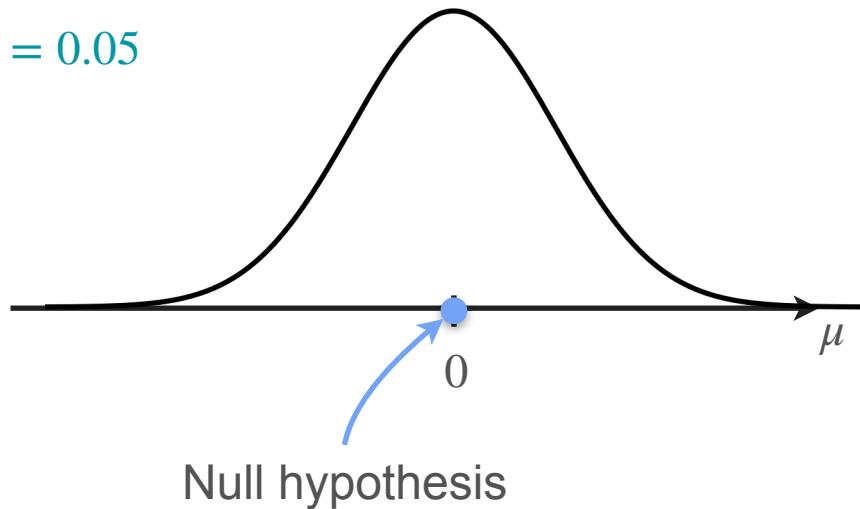
$$H_0 : \mu = 66.7 \text{ vs. } H_1 : \mu < 66.7 \quad n = 10$$

$$\bar{x} = 64.252$$

$$s = 3.113$$

$$t = \frac{64.252 - 66.7}{3.113/\sqrt{10}}$$

$$\alpha = 0.05$$



Left-Tailed Test for Gaussian Data (Unknown σ)

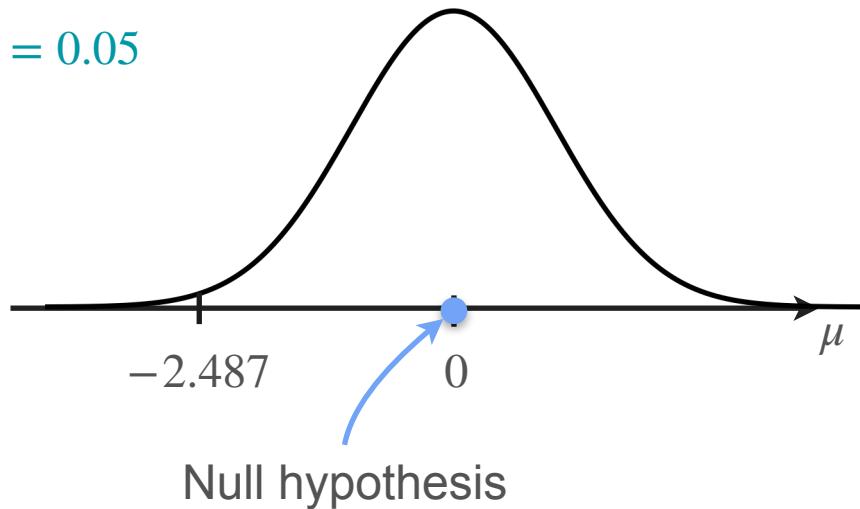
The mean height for 18 y/o in the US in the 70s was **66.7** in.

$$H_0 : \mu = 66.7 \text{ vs. } H_1 : \mu < 66.7 \quad n = 10$$

$$\bar{x} = 64.252 \\ s = 3.113$$

$$t = \frac{64.252 - 66.7}{3.113/\sqrt{10}} = -2.487$$

$$\alpha = 0.05$$



Left-Tailed Test for Gaussian Data (Unknown σ)

The mean height for 18 y/o in the US in the 70s was **66.7** in.

$$H_0 : \mu = 66.7 \text{ vs. } H_1 : \mu < 66.7$$

$$n = 10$$

$$\bar{x} = 64.252$$

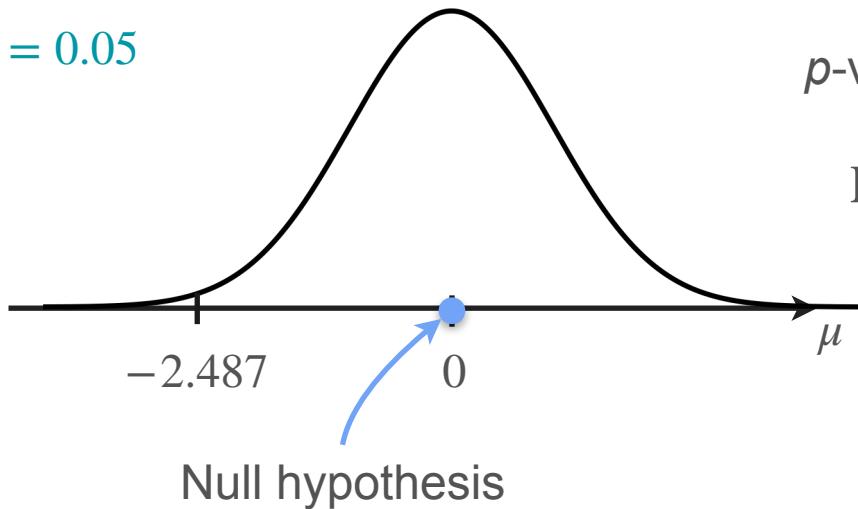
$$s = 3.113$$

$$t = \frac{64.252 - 66.7}{3.113/\sqrt{10}} = -2.487$$

$$\alpha = 0.05$$

p-value:

$$P\left(\frac{\bar{X} - 66.7}{S/\sqrt{10}} < -2.487 \mid \mu = 66.7\right) ?$$

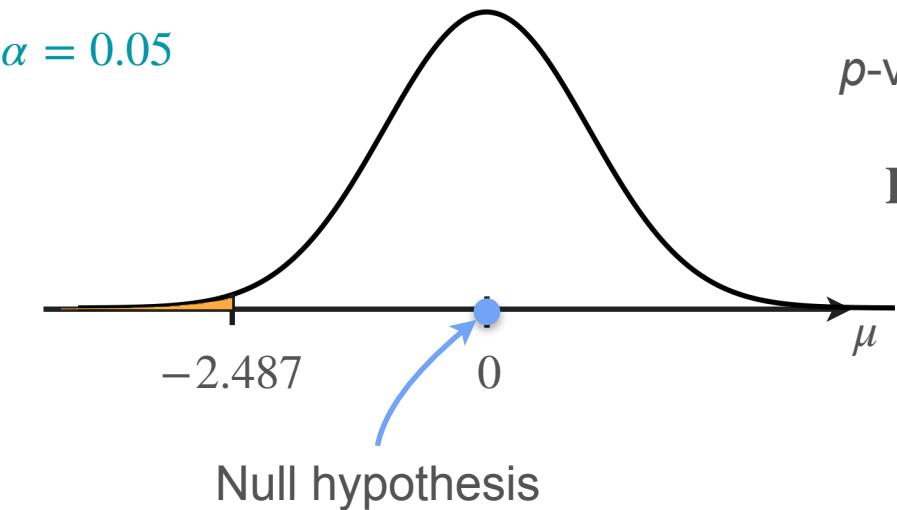


Left-Tailed Test for Gaussian Data (Unknown σ)

The mean height for 18 y/o in the US in the 70s was **66.7** in.

$$H_0 : \mu = 66.7 \text{ vs. } H_1 : \mu < 66.7$$

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$$\bar{x} = 64.252$$
$$s = 3.113$$

$$t = \frac{64.252 - 66.7}{3.113/\sqrt{10}} = -2.487$$

p-value:

$$P\left(\frac{\bar{X} - 66.7}{S/\sqrt{10}} < -2.487 \mid \mu = 66.7\right)$$

$$= 0.0173$$

Left-Tailed Test for Gaussian Data (Unknown σ)

The mean height for 18 y/o in the US in the 70s was **66.7** in.

$$H_0 : \mu = 66.7 \text{ vs. } H_1 : \mu < 66.7$$

$$n = 10$$

$$\bar{x} = 64.252$$

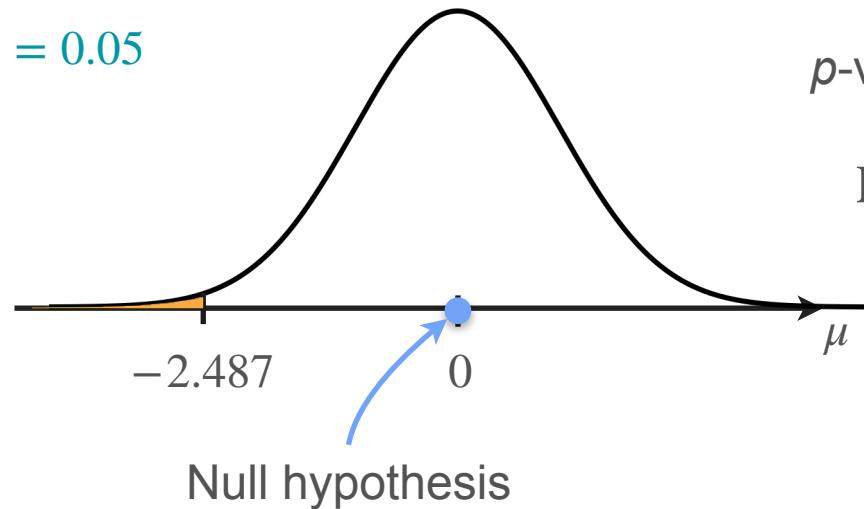
$$s = 3.113$$

$$t = \frac{64.252 - 66.7}{3.113/\sqrt{10}} = -2.487$$

$$\alpha = 0.05$$

p-value:

$$P\left(\frac{\bar{X} - 66.7}{S/\sqrt{10}} < -2.487 \mid \mu = 66.7\right)$$



$$= 0.0173 < \alpha$$

Conclusion: reject H_0
(with a 5% significance level)



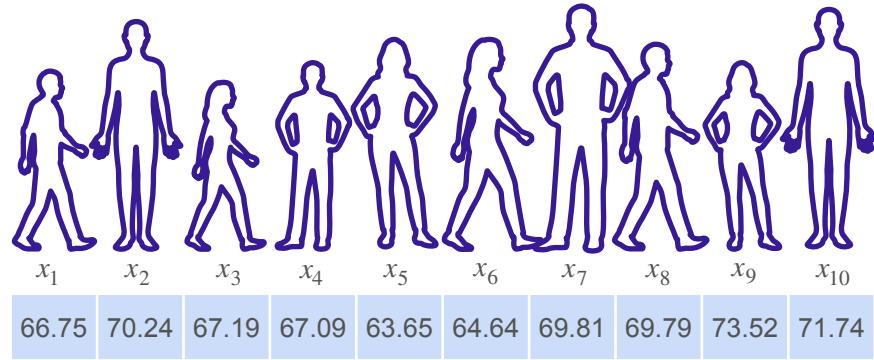
DeepLearning.AI

Hypothesis Testing

Two sample t-test

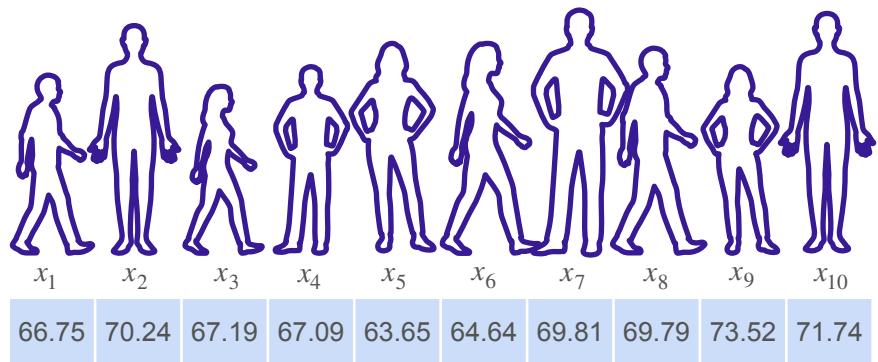
Independent Two-Sample t -Test

Independent Two-Sample t -Test

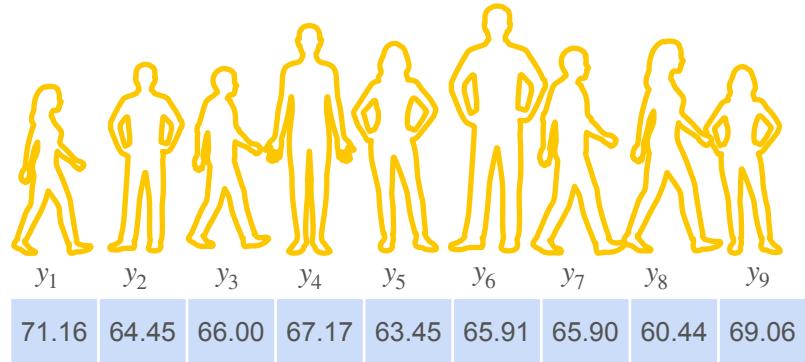


Height of 18 y/o in the US

Independent Two-Sample t -Test

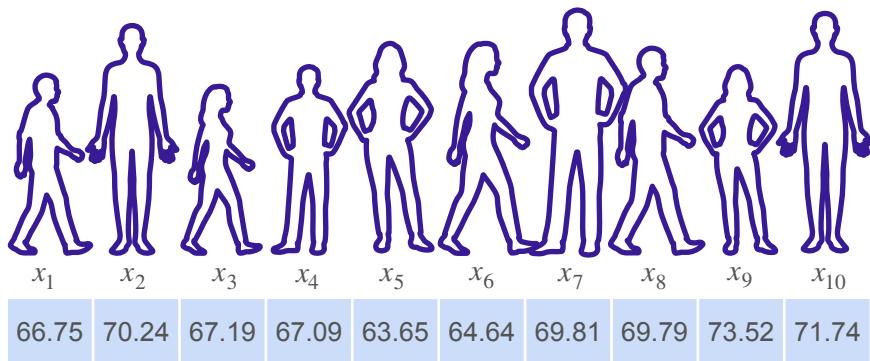


Height of 18 y/o in the US



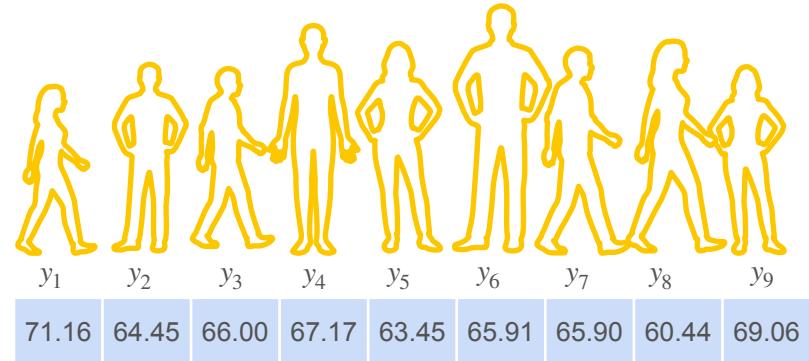
Height of 18 y/o in Argentina

Independent Two-Sample t -Test



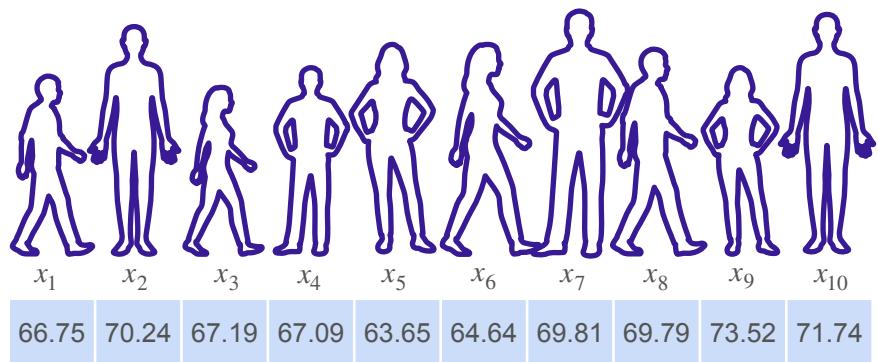
$$n_X = 10 \quad \bar{x} = 68.442$$
$$s_X = 3.113$$

Height of 18 y/o in the US



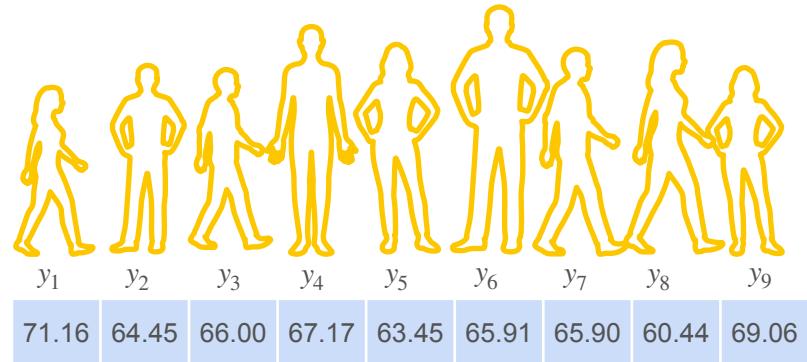
Height of 18 y/o in Argentina

Independent Two-Sample t -Test



$$n_X = 10 \quad \bar{x} = 68.442$$
$$s_X = 3.113$$

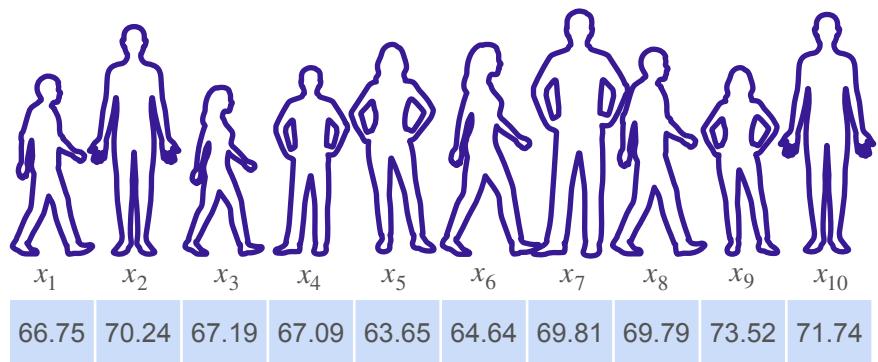
Height of 18 y/o in the US



$$n_Y = 9 \quad \bar{y} = 65.949$$
$$s_Y = 3.106$$

Height of 18 y/o in Argentina

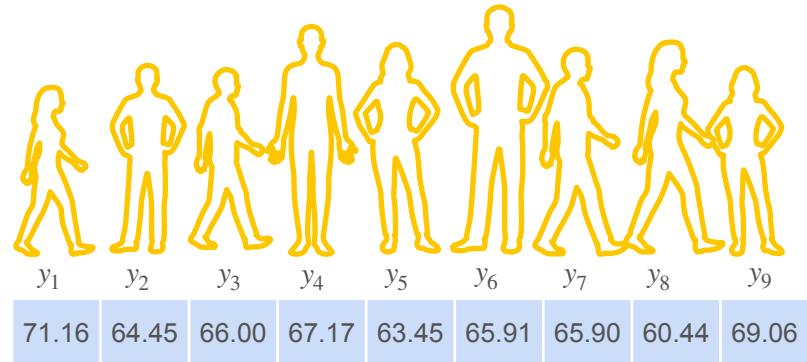
Independent Two-Sample t -Test



$$n_X = 10 \quad \bar{x} = 68.442$$
$$s_X = 3.113$$

Height of 18 y/o in the US

$$\mu_{US} \neq \mu_{Ar}$$

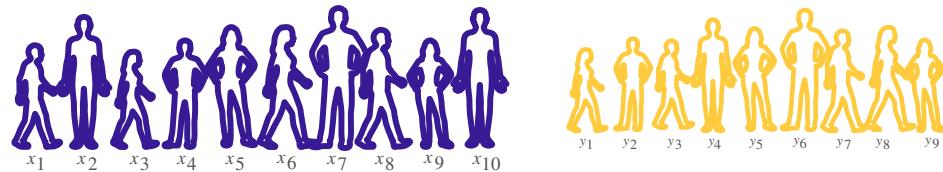


$$n_Y = 9 \quad \bar{y} = 65.949$$
$$s_Y = 3.106$$

Height of 18 y/o in Argentina

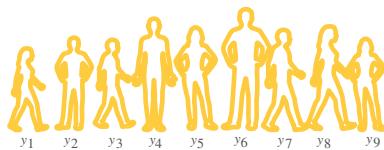
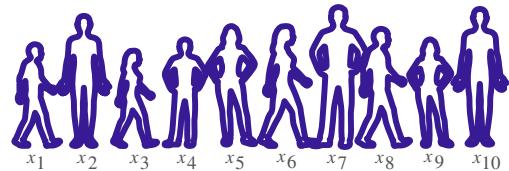
Independent Two-Sample t -Test: Hypothesis

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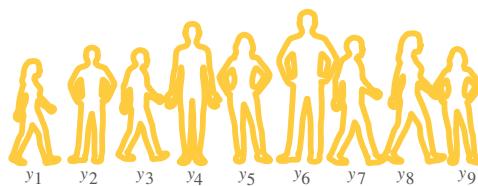
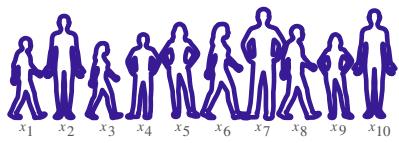


$$H_0 : \mu_{US} = \mu_{Ar} \text{ vs. } H_1 : \mu_{US} > \mu_{Ar}$$

Independent Two-Sample t -Test: Hypothesis

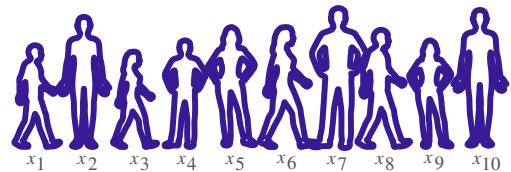


$$H_0 : \mu_{US} = \mu_{Ar} \text{ vs. } H_1 : \mu_{US} > \mu_{Ar}$$

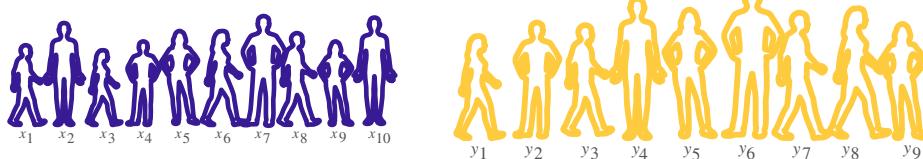


$$H_0 : \mu_{US} = \mu_{Ar} \text{ vs. } H_1 : \mu_{US} < \mu_{Ar}$$

Independent Two-Sample t -Test: Hypothesis



$$H_0 : \mu_{US} = \mu_{Ar} \text{ vs. } H_1 : \mu_{US} > \mu_{Ar}$$

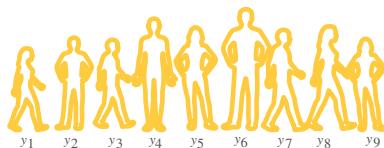
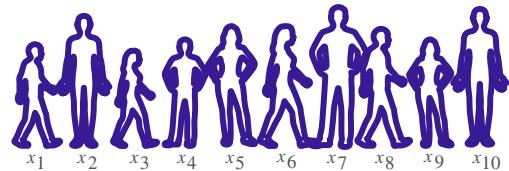


$$H_0 : \mu_{US} = \mu_{Ar} \text{ vs. } H_1 : \mu_{US} < \mu_{Ar}$$

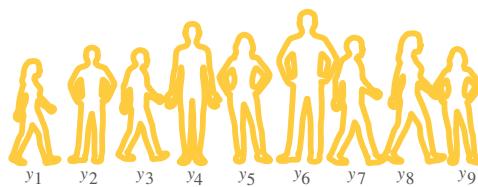
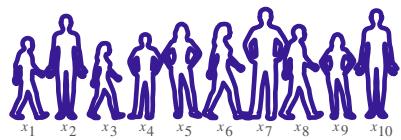


$$H_0 : \mu_{US} = \mu_{Ar} \text{ vs. } H_1 : \mu_{US} \neq \mu_{Ar}$$

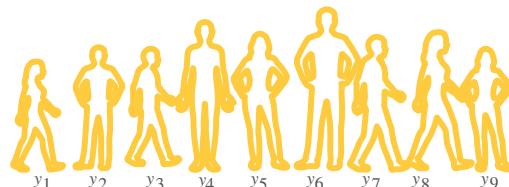
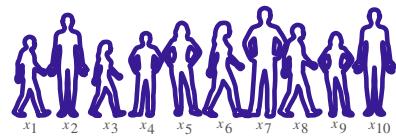
Independent Two-Sample t -Test: Hypothesis



$$H_0 : \mu_{US} - \mu_{Ar} = 0 \text{ vs. } H_1 : \mu_{US} - \mu_{Ar} > 0$$

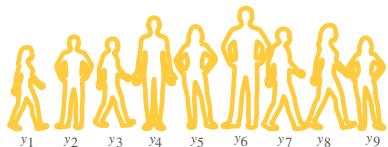
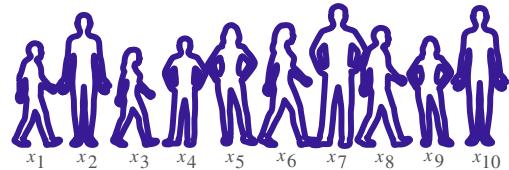


$$H_0 : \mu_{US} = \mu_{Ar} \text{ vs. } H_1 : \mu_{US} < \mu_{Ar}$$

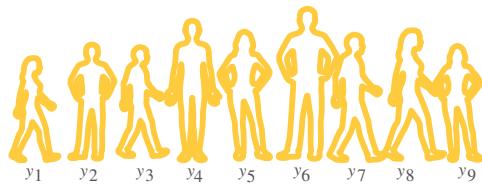
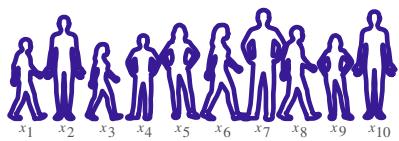


$$H_0 : \mu_{US} = \mu_{Ar} \text{ vs. } H_1 : \mu_{US} \neq \mu_{Ar}$$

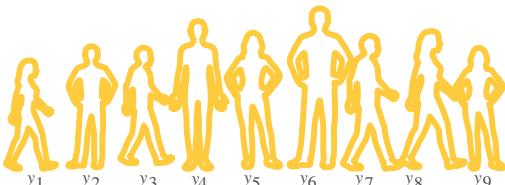
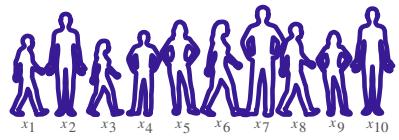
Independent Two-Sample t -Test: Hypothesis



$$H_0 : \mu_{US} - \mu_{Ar} = 0 \text{ vs. } H_1 : \mu_{US} - \mu_{Ar} > 0$$

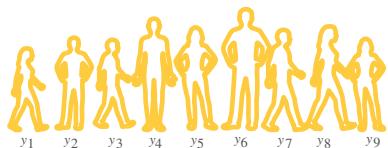
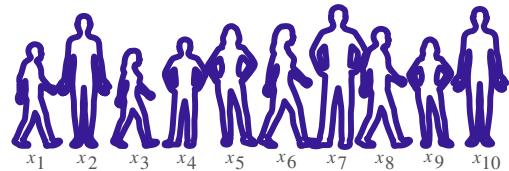


$$H_0 : \mu_{US} - \mu_{Ar} = 0 \text{ vs. } H_1 : \mu_{US} - \mu_{Ar} < 0$$

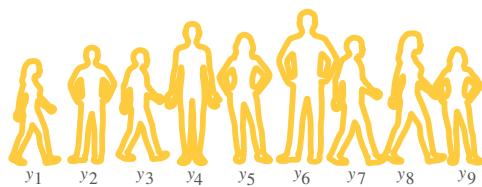
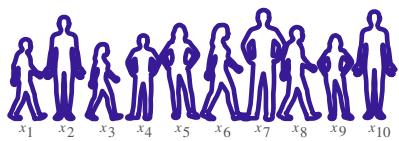


$$H_0 : \mu_{US} = \mu_{Ar} \text{ vs. } H_1 : \mu_{US} \neq \mu_{Ar}$$

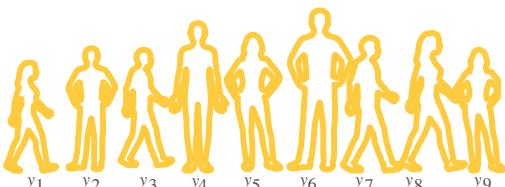
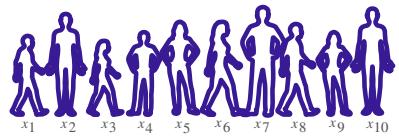
Independent Two-Sample t -Test: Hypothesis



$$H_0 : \mu_{US} - \mu_{Ar} = 0 \text{ vs. } H_1 : \mu_{US} - \mu_{Ar} > 0$$



$$H_0 : \mu_{US} - \mu_{Ar} = 0 \text{ vs. } H_1 : \mu_{US} - \mu_{Ar} < 0$$



$$H_0 : \mu_{US} - \mu_{Ar} = 0 \text{ vs. } H_1 : \mu_{US} - \mu_{Ar} \neq 0$$

Independent Two-Sample t -Test: Assumptions

Independent Two-Sample t -Test: Assumptions

- All people in the sample from the two groups are different

Independent Two-Sample t -Test: Assumptions

- All people in the sample from the two groups are different
- Each person in both samples are independent

Independent Two-Sample t -Test: Assumptions

- All people in the sample from the two groups are different
- Each person in both samples are independent
- Populations are normally distributed

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$$Y \sim \mathcal{N}(\mu_{Arg}, \sigma_{Arg}^2)$$

Independent Two-Sample t -Test: Assumptions

- All people in the sample from the two groups are different
- Each person in both samples are independent
- Populations are normally distributed

$$X \sim \mathcal{N}(\mu_{US}, \sigma_{US}^2) \quad Y \sim \mathcal{N}(\mu_{Arg}, \sigma_{Arg}^2)$$

$$\bar{X} = \frac{1}{10} \sum_{i=1}^{10} X_i$$

$$\bar{Y} = \frac{1}{9} \sum_{i=1}^9 Y_i$$

Independent Two-Sample t -Test: Assumptions

- All people in the sample from the two groups are different
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$$Y \sim \mathcal{N}(\mu_{Arg}, \sigma_{Arg}^2)$$

$$\bar{X} = \frac{1}{10} \sum_{i=1}^{10} X_i$$

$$\bar{X} - \bar{Y} \sim ?$$

$$\bar{Y} = \frac{1}{9} \sum_{i=1}^9 Y_i$$

Independent Two-Sample t -Test: Assumptions

- All people in the sample from the two groups are different
- Each person in both samples are independent
- Populations are normally distributed

$$X \sim \mathcal{N}(\mu_{US}, \sigma_{US}^2)$$

$$Y \sim \mathcal{N}(\mu_{Arg}, \sigma_{Arg}^2)$$

$$\bar{X} = \frac{1}{10} \sum_{i=1}^{10} X_i$$

$$\bar{X} - \bar{Y} \sim \mathcal{N}\left(\quad, \quad \right)$$

$$\bar{Y} = \frac{1}{9} \sum_{i=1}^9 Y_i$$

Independent Two-Sample t -Test: Assumptions

- All people in the sample from the two groups are different
- Each person in both samples are independent
- Populations are normally distributed

$$X \sim \mathcal{N}(\mu_{US}, \sigma_{US}^2)$$

$$Y \sim \mathcal{N}(\mu_{Arg}, \sigma_{Arg}^2)$$

$$\bar{X} = \frac{1}{10} \sum_{i=1}^{10} X_i$$

$$\bar{X} - \bar{Y} \sim \mathcal{N}\left(\mu_{US} - \mu_{Arg}, \right)$$

$$\bar{Y} = \frac{1}{9} \sum_{i=1}^9 Y_i$$

Independent Two-Sample t -Test: Assumptions

- All people in the sample from the two groups are different
- Each person in both samples are independent
- Populations are normally distributed

$$X \sim \mathcal{N}(\mu_{US}, \sigma_{US}^2)$$

$$Y \sim \mathcal{N}(\mu_{Arg}, \sigma_{Arg}^2)$$

$$\bar{X} = \frac{1}{10} \sum_{i=1}^{10} X_i$$

$$\bar{X} - \bar{Y} \sim \mathcal{N}\left(\mu_{US} - \mu_{Arg}, \frac{\sigma_{US}^2}{10} + \frac{\sigma_{Arg}^2}{9}\right)$$

$$\bar{Y} = \frac{1}{9} \sum_{i=1}^9 Y_i$$

Independent Two-Sample t -Test: Assumptions

- All people in the sample from the two groups are different
- Each person in both samples are independent
- Populations are normally distributed

$$X \sim \mathcal{N}(\mu_{US}, \sigma_{US})$$

$$Y \sim \mathcal{N}(\mu_{Arg}, \sigma_{Arg})$$

$$\bar{X} = \frac{1}{10} \sum_{i=1}^{10} X_i$$

$$\bar{Y} = \frac{1}{9} \sum_{i=1}^9 Y_i$$

$$\bar{X} - \bar{Y} \sim \mathcal{N}\left(\mu_{US} - \mu_{Arg}, \sqrt{\frac{\sigma_{US}^2}{10} + \frac{\sigma_{Arg}^2}{9}}\right)$$

Independent Two-Sample t -Test: Statistic

$$X \sim \mathcal{N}(\mu_{US}, \sigma_{US}) \quad Y \sim \mathcal{N}(\mu_{Arg}, \sigma_{Arg})$$

$$\bar{X} - \bar{Y} \sim \mathcal{N}\left(\mu_{US} - \mu_{Arg}, \frac{\sigma_{US}^2}{10} + \frac{\sigma_{Arg}^2}{9}\right)$$

Independent Two-Sample t -Test: Statistic

$$X \sim \mathcal{N}(\mu_{US}, \sigma_{US}) \quad Y \sim \mathcal{N}(\mu_{Arg}, \sigma_{Arg})$$
$$\bar{X} - \bar{Y} \sim \mathcal{N}\left(\mu_{US} - \mu_{Arg}, \frac{\sigma_{US}^2}{10} + \frac{\sigma_{Arg}^2}{9}\right) \xrightarrow{\hspace{1cm}} \frac{(\bar{X} - \bar{Y}) - (\mu_{US} - \mu_{Arg})}{\sqrt{\frac{\sigma_{US}^2}{10} + \frac{\sigma_{Arg}^2}{9}}} \sim \mathcal{N}(0, 1^2)$$

Independent Two-Sample t -Test: Statistic

$$X \sim \mathcal{N}(\mu_{US}, \sigma_{US}) \quad Y \sim \mathcal{N}(\mu_{Arg}, \sigma_{Arg})$$
$$\bar{X} - \bar{Y} \sim \mathcal{N}\left(\mu_{US} - \mu_{Arg}, \frac{\sigma_{US}^2}{10} + \frac{\sigma_{Arg}^2}{9}\right) \rightarrow \frac{(\bar{X} - \bar{Y}) - (\mu_{US} - \mu_{Arg})}{\sqrt{\frac{\sigma_{US}^2}{10} + \frac{\sigma_{Arg}^2}{9}}} \sim \mathcal{N}(0, 1^2)$$

You don't know σ_{US} , σ_{Arg}



Independent Two-Sample t -Test: Statistic

$$X \sim \mathcal{N}(\mu_{US}, \sigma_{US}) \quad Y \sim \mathcal{N}(\mu_{Arg}, \sigma_{Arg})$$
$$\bar{X} - \bar{Y} \sim \mathcal{N}\left(\mu_{US} - \mu_{Arg}, \frac{\sigma_{US}^2}{10} + \frac{\sigma_{Arg}^2}{9}\right) \rightarrow \frac{(\bar{X} - \bar{Y}) - (\mu_{US} - \mu_{Arg})}{\sqrt{\frac{\sigma_{US}^2}{10} + \frac{\sigma_{Arg}^2}{9}}} \sim \mathcal{N}(0, 1^2)$$

You don't know σ_{US} , σ_{Arg}



Replace it with the sample standard deviation

Independent Two-Sample t -Test: Statistic

$$X \sim \mathcal{N}(\mu_{US}, \sigma_{US}) \quad Y \sim \mathcal{N}(\mu_{Arg}, \sigma_{Arg})$$
$$\bar{X} - \bar{Y} \sim \mathcal{N}\left(\mu_{US} - \mu_{Arg}, \frac{\sigma_{US}^2}{10} + \frac{\sigma_{Arg}^2}{9}\right) \rightarrow \frac{(\bar{X} - \bar{Y}) - (\mu_{US} - \mu_{Arg})}{\sqrt{\frac{\sigma_{US}^2}{10} + \frac{\sigma_{Arg}^2}{9}}} \sim \mathcal{N}(0, 1^2)$$

You don't know σ_{US} , σ_{Arg}



Replace it with the sample standard deviation

$$\frac{(\bar{X} - \bar{Y}) - (\mu_{US} - \mu_{Arg})}{\sqrt{\frac{S_X^2}{10} + \frac{S_Y^2}{9}}}$$

Independent Two-Sample t -Test: Statistic

$$X \sim \mathcal{N}(\mu_{US}, \sigma_{US}) \quad Y \sim \mathcal{N}(\mu_{Arg}, \sigma_{Arg})$$
$$\bar{X} - \bar{Y} \sim \mathcal{N}\left(\mu_{US} - \mu_{Arg}, \frac{\sigma_{US}^2}{10} + \frac{\sigma_{Arg}^2}{9}\right) \rightarrow \frac{(\bar{X} - \bar{Y}) - (\mu_{US} - \mu_{Arg})}{\sqrt{\frac{\sigma_{US}^2}{10} + \frac{\sigma_{Arg}^2}{9}}} \sim \mathcal{N}(0, 1^2)$$

You don't know σ_{US} , σ_{Arg}



Replace it with the sample standard deviation

$$T = \frac{(\bar{X} - \bar{Y}) - (\mu_{US} - \mu_{Arg})}{\sqrt{\frac{S_X^2}{10} + \frac{S_Y^2}{9}}} \sim t_{\nu}$$

Independent Two-Sample t -Test: Statistic

$$X \sim \mathcal{N}(\mu_{US}, \sigma_{US}) \quad Y \sim \mathcal{N}(\mu_{Arg}, \sigma_{Arg})$$

$$\bar{X} - \bar{Y} \sim \mathcal{N}\left(\mu_{US} - \mu_{Arg}, \frac{\sigma_{US}^2}{10} + \frac{\sigma_{Arg}^2}{9}\right) \rightarrow \frac{(\bar{X} - \bar{Y}) - (\mu_{US} - \mu_{Arg})}{\sqrt{\frac{\sigma_{US}^2}{10} + \frac{\sigma_{Arg}^2}{9}}} \sim \mathcal{N}(0, 1^2)$$

You don't know σ_{US} , σ_{Arg}

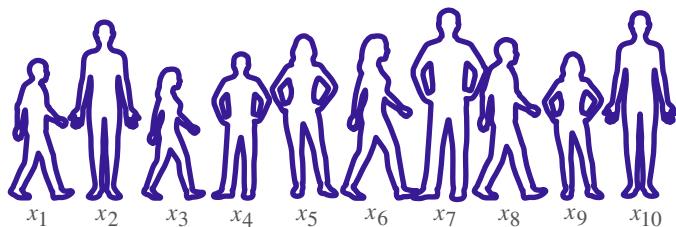


Replace it with the sample standard deviation

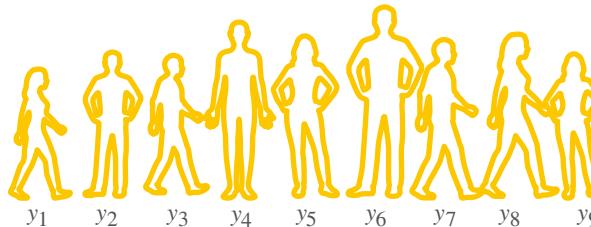
$$T = \frac{(\bar{X} - \bar{Y}) - (\mu_{US} - \mu_{Arg})}{\sqrt{\frac{s_X^2}{10} + \frac{s_Y^2}{9}}} \sim t_{\nu}$$

Degrees of freedom =
$$\frac{\left(\frac{s_X^2}{n_X} + \frac{s_Y^2}{n_Y}\right)^2}{\frac{\left(\frac{s_X^2}{n_X}\right)^2}{n_X - 1} + \frac{\left(\frac{s_Y^2}{n_Y}\right)^2}{n_Y - 1}}$$

Independent Two-Sample t -Test



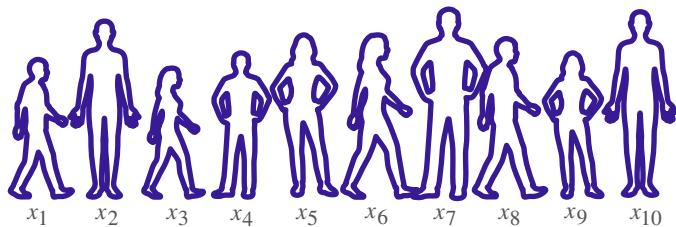
$$n_X = 10 \quad \bar{x} = 68.442$$
$$s_X = 3.113$$



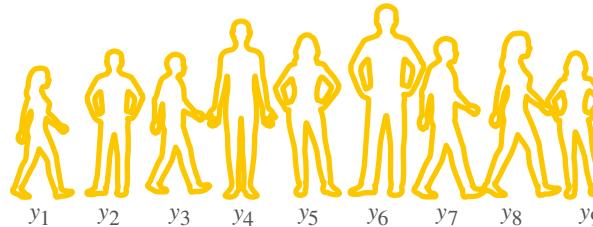
$$n_Y = 9 \quad \bar{y} = 65.949$$
$$s_Y = 3.106$$

Degrees of freedom = $\frac{\left(\frac{n_X - 1}{2} + \frac{n_Y - 1}{2} \right)^2}{\frac{\left(n_X - 1 \right)^2}{n_X - 1} + \frac{\left(n_Y - 1 \right)^2}{n_Y - 1}}$

Independent Two-Sample t -Test



$$n_X = 10 \quad \bar{x} = 68.442$$
$$s_X = 3.113$$



$$n_Y = 9 \quad \bar{y} = 65.949$$
$$s_Y = 3.106$$

Degrees of freedom = $\frac{\overbrace{\left(\frac{2}{3.113} + \frac{2}{3.106} \right)^2}^{16.8}}{\underbrace{\left(\frac{2}{10} \right)^2}_{10-1} + \underbrace{\left(\frac{2}{9} \right)^2}_{9-1}}$

Independent Two-Sample t -Test: Statistic

$$X \sim \mathcal{N}(\mu_{US}, \sigma_{US}) \quad Y \sim \mathcal{N}(\mu_{Arg}, \sigma_{Arg})$$

$$\bar{X} - \bar{Y} \sim \mathcal{N}\left(\mu_{US} - \mu_{Arg}, \sqrt{\frac{\sigma_{US}^2}{10} + \frac{\sigma_{Arg}^2}{9}}\right) \rightarrow \frac{(\bar{X} - \bar{Y}) - (\mu_{US} - \mu_{Arg})}{\sqrt{\frac{\sigma_{US}^2}{10} + \frac{\sigma_{Arg}^2}{9}}} \sim \mathcal{N}(0,1)$$

You don't know σ_{US} , σ_{Arg}



Replace it with the sample standard deviation

$$T = \frac{(\bar{X} - \bar{Y}) - (\mu_{US} - \mu_{Arg})}{\sqrt{\frac{s_X^2}{10} + \frac{s_Y^2}{9}}} \sim t_{16.8}$$

Degrees of freedom = $\frac{\left(\frac{s_X^2}{n_X} + \frac{s_Y^2}{n_Y}\right)^2}{\frac{\left(\frac{s_X^2}{n_X}\right)^2}{n_X - 1} + \frac{\left(\frac{s_Y^2}{n_Y}\right)^2}{n_Y - 1}}$

Independent Two-Sample t -Test: Statistic

$$X \sim \mathcal{N}(\mu_{US}, \sigma_{US})$$

$$Y \sim \mathcal{N}(\mu_{Arg}, \sigma_{Arg})$$

$$\bar{X} - \bar{Y} \sim \mathcal{N}\left(\mu_{US} - \mu_{Arg}, \sqrt{\frac{\sigma_{US}^2}{10} + \frac{\sigma_{Arg}^2}{9}}\right) \rightarrow \frac{(\bar{X} - \bar{Y}) - (\mu_{US} - \mu_{Arg})}{\sqrt{\frac{\sigma_{US}^2}{10} + \frac{\sigma_{Arg}^2}{9}}} \sim \mathcal{N}(0,1)$$

You don't know σ_{US} , σ_{Arg}



Replace it with the sample standard deviation

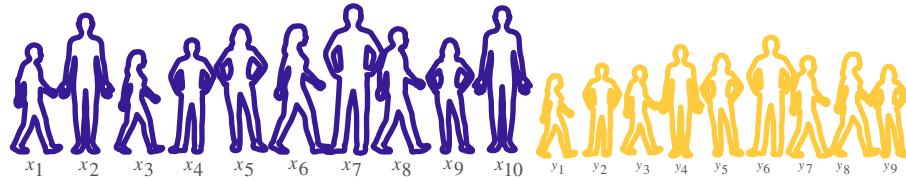
$$T = \frac{(\bar{X} - \bar{Y}) - (\mu_{US} - \mu_{Arg})}{\sqrt{\frac{s_X^2}{10} + \frac{s_Y^2}{9}}} \sim t_{16.8}$$

Degrees of freedom

$$\text{Degrees of freedom} = \frac{\left(\frac{s_X^2}{n_X} + \frac{s_Y^2}{n_Y}\right)^2}{\frac{\left(\frac{s_X^2}{n_X}\right)^2}{n_X - 1} + \frac{\left(\frac{s_Y^2}{n_Y}\right)^2}{n_Y - 1}}$$

Independent Two-Sample t -Test: Right Tailed Test

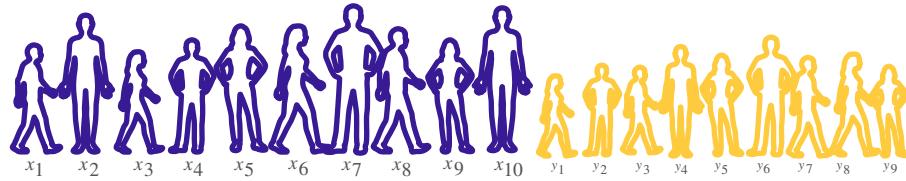
Independent Two-Sample t -Test: Right Tailed Test



$$n_X = 10$$

$$n_Y = 9$$

Independent Two-Sample t -Test: Right Tailed Test



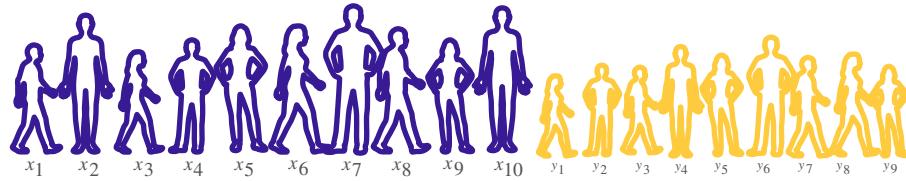
$$\bar{x} = 68.442$$

$$n_X = 10$$

$$s_X = 3.113$$

$$n_Y = 9$$

Independent Two-Sample t -Test: Right Tailed Test



$$\bar{x} = 68.442$$

$$n_X = 10$$

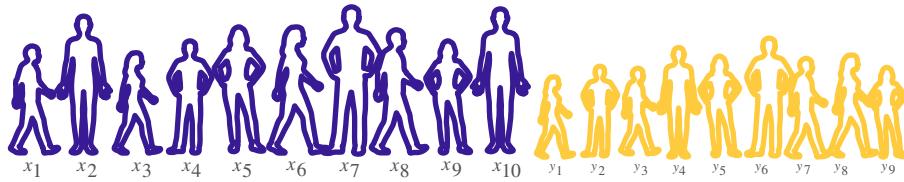
$$\bar{y} = 65.949$$

$$n_Y = 9$$

$$s_X = 3.113$$

$$s_Y = 3.106$$

Independent Two-Sample t -Test: Right Tailed Test



$$\bar{x} = 68.442$$

$$n_X = 10$$

$$\bar{y} = 65.949$$

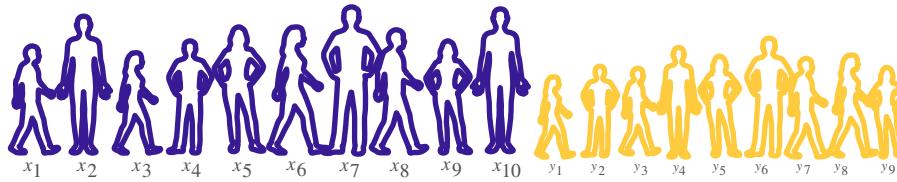
$$n_Y = 9$$

$$s_X = 3.113$$

$$s_Y = 3.106$$

$$H_0 : \mu_{US} - \mu_{Ar} = 0 \text{ vs. } H_1 : \mu_{US} - \mu_{Ar} > 0$$

Independent Two-Sample t -Test: Right Tailed Test



$$\bar{x} = 68.442$$

$$n_X = 10$$

$$\bar{y} = 65.949$$

$$n_Y = 9$$

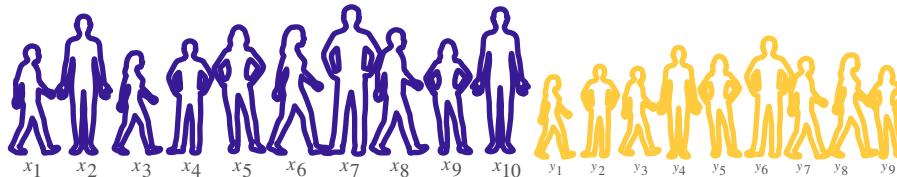
$$s_X = 3.113$$

$$s_Y = 3.106$$

$$H_0 : \mu_{US} - \mu_{Ar} = 0 \text{ vs. } H_1 : \mu_{US} - \mu_{Ar} > 0$$

$$\alpha = 0.05$$

Independent Two-Sample t -Test: Right Tailed Test



$$\bar{x} = 68.442$$

$$s_x = 3.113$$

$$n_X = 10$$

$$\bar{y} = 65.949$$

$$s_Y = 3.106$$

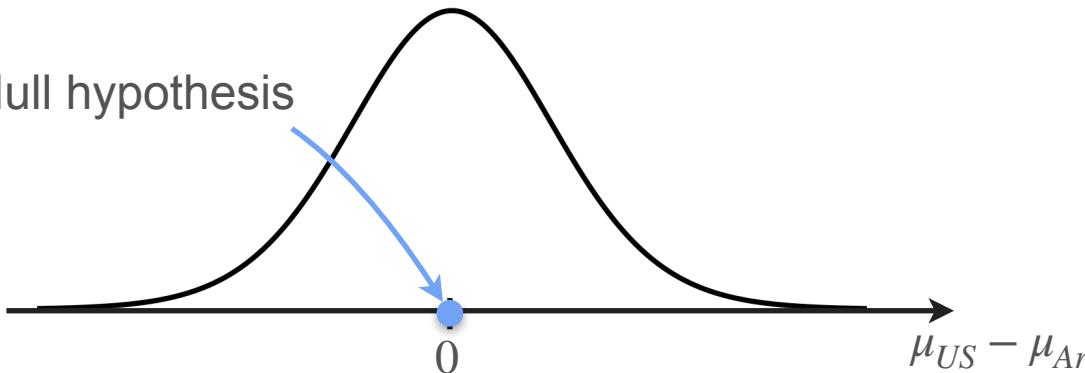
$$n_Y = 9$$

$$H_0 : \mu_{US} - \mu_{Ar} = 0 \text{ vs. } H_1 : \mu_{US} - \mu_{Ar} > 0$$

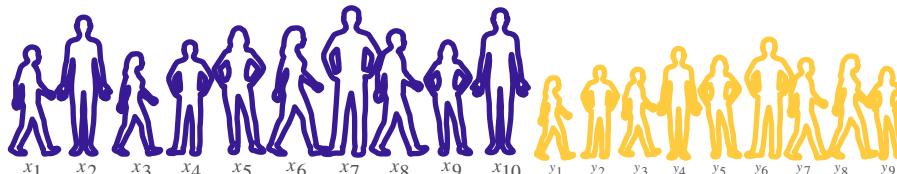
$$\alpha = 0.05$$

$$\text{If } H_0 \text{ is true: } T = \frac{(\bar{X} - \bar{Y}) - 0}{\sqrt{\frac{S_X^2}{10} + \frac{S_Y^2}{9}}} \sim t_{16.8}$$

Null hypothesis



Independent Two-Sample t -Test: Right Tailed Test



$$\bar{x} = 68.442$$

$$s_X = 3.113$$

$$n_X = 10$$

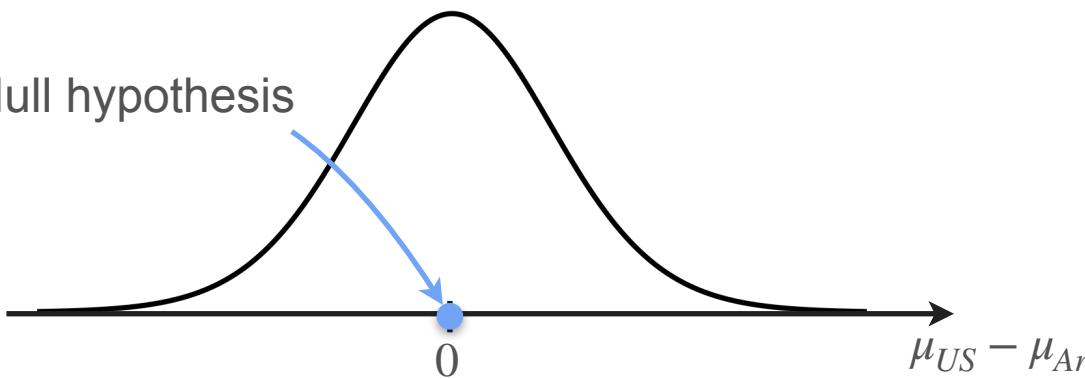
$$\bar{y} = 65.949$$

$$s_Y = 3.106$$

$$n_Y = 9$$

$$t = 1.7459$$

Null hypothesis

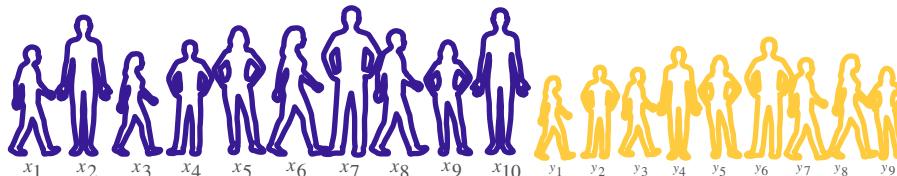


$$H_0 : \mu_{US} - \mu_{Ar} = 0 \text{ vs. } H_1 : \mu_{US} - \mu_{Ar} > 0$$

$$\alpha = 0.05$$

$$\text{If } H_0 \text{ is true: } T = \frac{(\bar{X} - \bar{Y}) - 0}{\sqrt{\frac{s_X^2}{10} + \frac{s_Y^2}{9}}} \sim t_{16.8}$$

Independent Two-Sample t -Test: Right Tailed Test



$$\bar{x} = 68.442$$

$$s_X = 3.113$$

$$n_X = 10$$

$$\bar{y} = 65.949$$

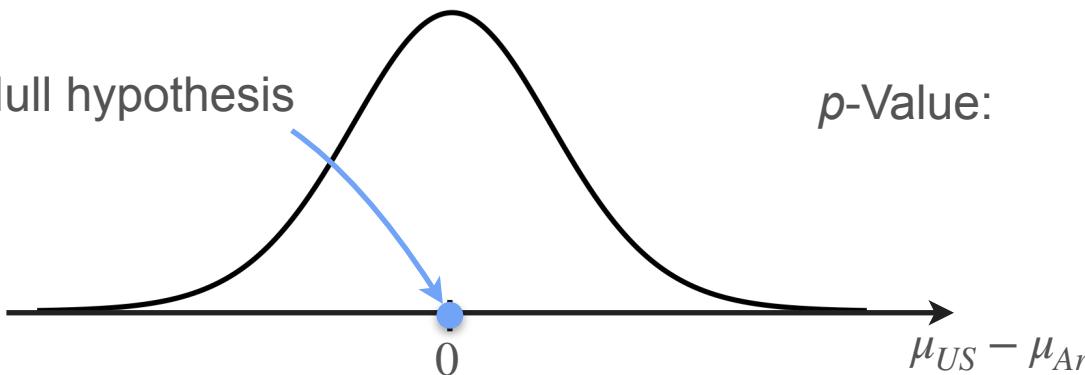
$$s_Y = 3.106$$

$$n_Y = 9$$

$$t = 1.7459$$

Null hypothesis

p -Value:

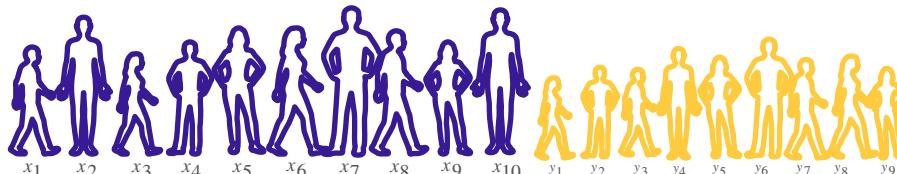


$$H_0 : \mu_{US} - \mu_{Ar} = 0 \text{ vs. } H_1 : \mu_{US} - \mu_{Ar} > 0$$

$$\alpha = 0.05$$

$$\text{If } H_0 \text{ is true: } T = \frac{(\bar{X} - \bar{Y}) - 0}{\sqrt{\frac{S_X^2}{10} + \frac{S_Y^2}{9}}} \sim t_{16.8}$$

Independent Two-Sample t -Test: Right Tailed Test



$$\bar{x} = 68.442$$

$$s_X = 3.113$$

$$n_X = 10$$

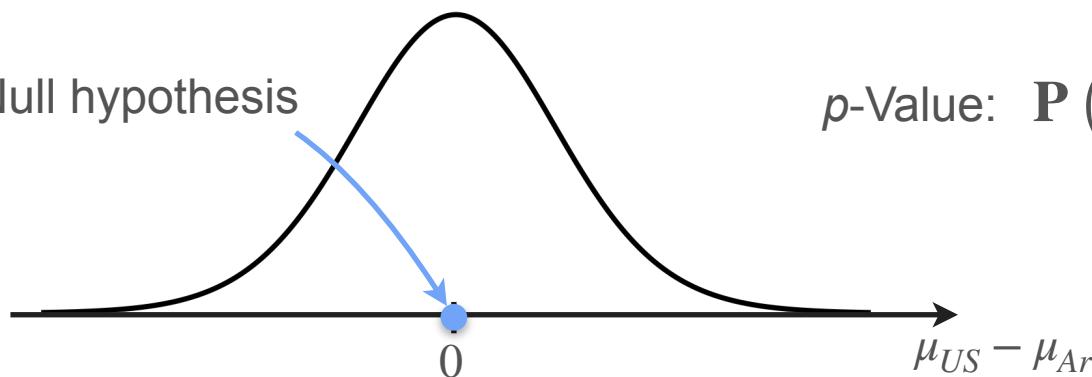
$$\bar{y} = 65.949$$

$$s_Y = 3.106$$

$$n_Y = 9$$

$$t = 1.7459$$

Null hypothesis



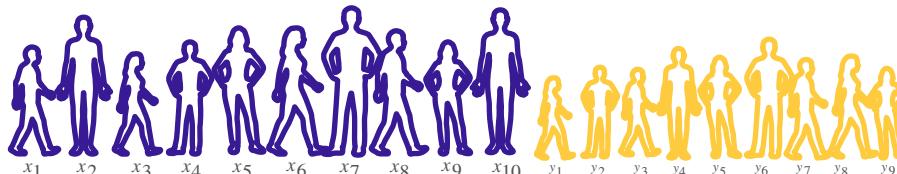
p-Value: $P(T > 1.7459 \mid \mu_{US} - \mu_{Ar} = 0)$

$$H_0 : \mu_{US} - \mu_{Ar} = 0 \text{ vs. } H_1 : \mu_{US} - \mu_{Ar} > 0$$

$$\alpha = 0.05$$

$$\text{If } H_0 \text{ is true: } T = \frac{(\bar{X} - \bar{Y}) - 0}{\sqrt{\frac{s_X^2}{10} + \frac{s_Y^2}{9}}} \sim t_{16.8}$$

Independent Two-Sample t -Test: Right Tailed Test



$$\bar{x} = 68.442$$

$$s_X = 3.113$$

$$n_X = 10$$

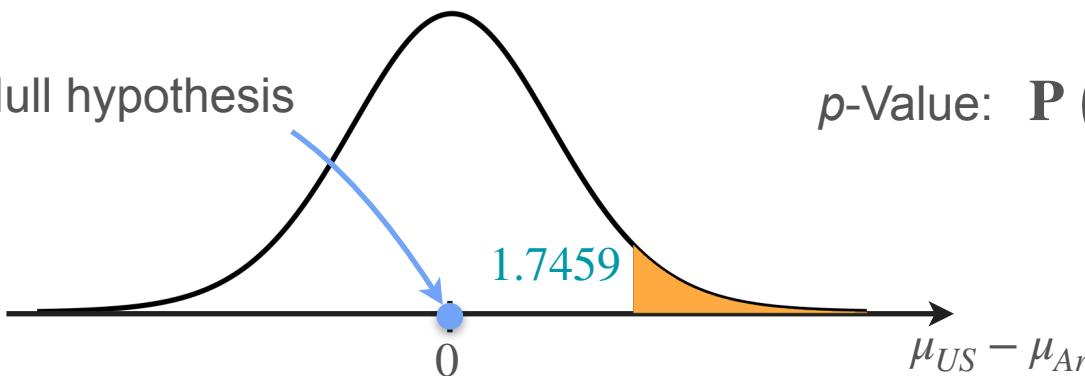
$$\bar{y} = 65.949$$

$$s_Y = 3.106$$

$$n_Y = 9$$

$$t = 1.7459$$

Null hypothesis



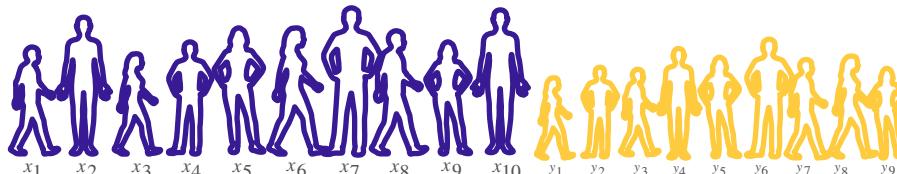
p-Value: $P(T > 1.7459 \mid \mu_{US} - \mu_{Ar} = 0)$

$$H_0 : \mu_{US} - \mu_{Ar} = 0 \text{ vs. } H_1 : \mu_{US} - \mu_{Ar} > 0$$

$$\alpha = 0.05$$

$$\text{If } H_0 \text{ is true: } T = \frac{(\bar{X} - \bar{Y}) - 0}{\sqrt{\frac{s_X^2}{10} + \frac{s_Y^2}{9}}} \sim t_{16.8}$$

Independent Two-Sample t -Test: Right Tailed Test



$$\bar{x} = 68.442$$

$$s_X = 3.113$$

$$n_X = 10$$

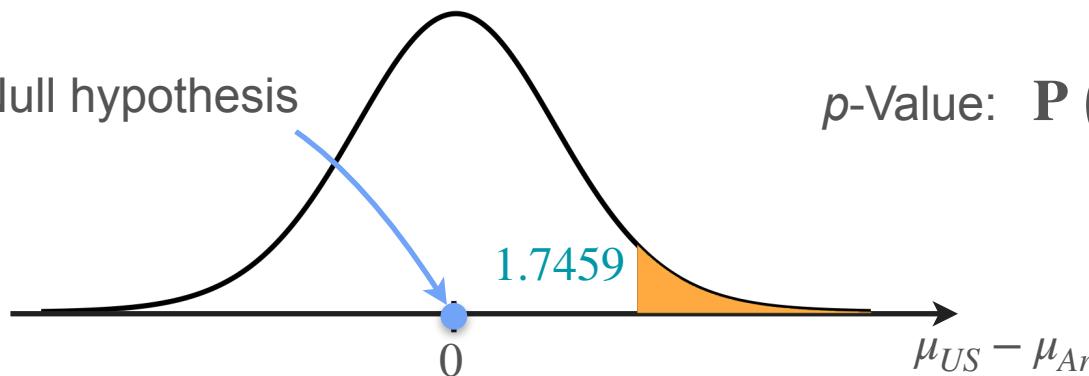
$$\bar{y} = 65.949$$

$$s_Y = 3.106$$

$$n_Y = 9$$

$$t = 1.7459$$

Null hypothesis



p-Value: $P(T > 1.7459 \mid \mu_{US} - \mu_{Ar} = 0)$

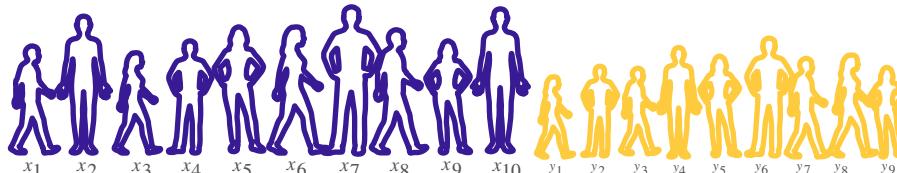
$$= 0.0495$$

$$H_0 : \mu_{US} - \mu_{Ar} = 0 \text{ vs. } H_1 : \mu_{US} - \mu_{Ar} > 0$$

$$\alpha = 0.05$$

$$\text{If } H_0 \text{ is true: } T = \frac{(\bar{X} - \bar{Y}) - 0}{\sqrt{\frac{s_X^2}{10} + \frac{s_Y^2}{9}}} \sim t_{16.8}$$

Independent Two-Sample t -Test: Right Tailed Test



$$\bar{x} = 68.442$$

$$s_x = 3.113$$

$$n_X = 10$$

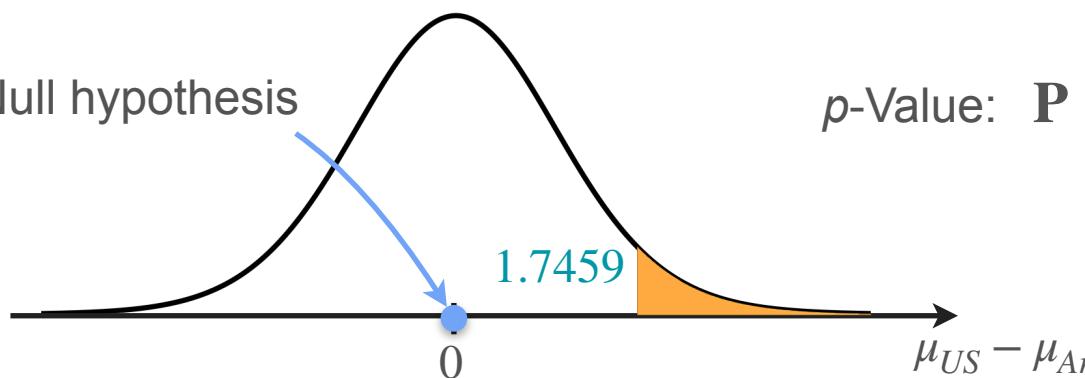
$$\bar{y} = 65.949$$

$$s_Y = 3.106$$

$$n_Y = 9$$

$$t = 1.7459$$

Null hypothesis

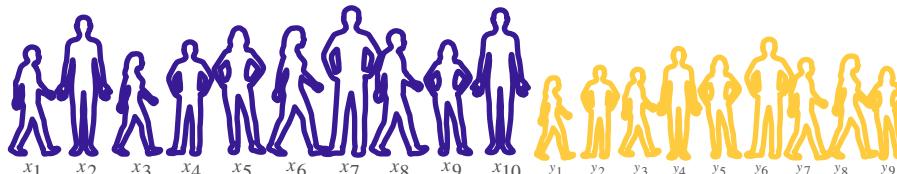


p -Value: $P(T > 1.7459 \mid \mu_{US} - \mu_{Ar} = 0)$

$$= 0.0495 < 0.05$$

\Rightarrow Reject H_0 (and accept H_1)
(with a 5% significance level)

Independent Two-Sample t -Test: Two Tailed Test



$$\bar{x} = 68.442$$

$$s_X = 3.113$$

$$n_X = 10$$

$$\bar{y} = 65.949$$

$$s_Y = 3.106$$

$$n_Y = 9$$

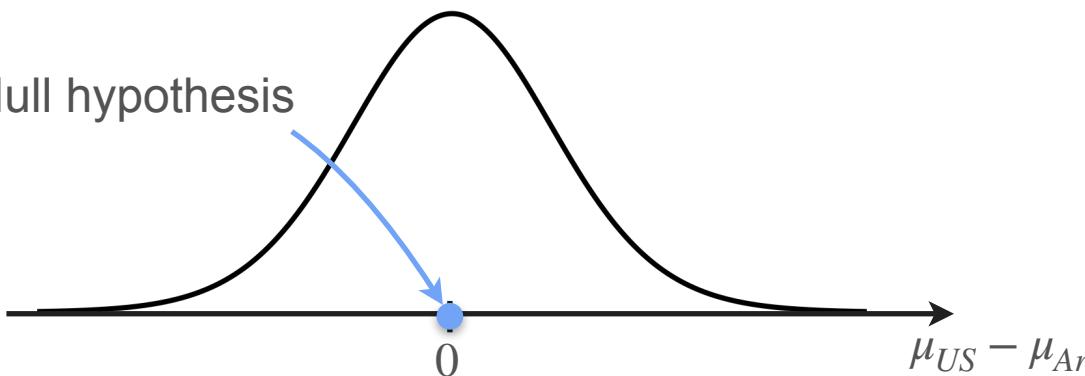
$$t = 1.7459$$

$$H_0 : \mu_{US} - \mu_{Ar} = 0 \text{ vs. } H_1 : \mu_{US} - \mu_{Ar} \neq 0$$

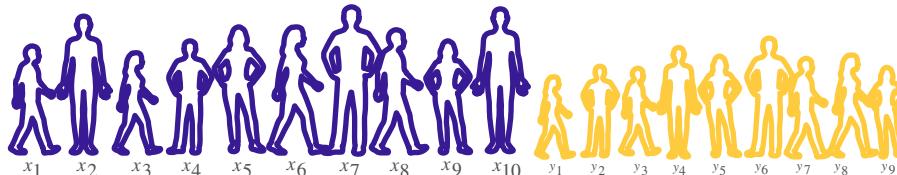
$$\alpha = 0.05$$

$$\text{If } H_0 \text{ is true: } T = \frac{(\bar{X} - \bar{Y}) - 0}{\sqrt{\frac{s_X^2}{10} + \frac{s_Y^2}{9}}} \sim t_{16.8}$$

Null hypothesis



Independent Two-Sample t -Test: Two Tailed Test



$$\bar{x} = 68.442$$

$$s_X = 3.113$$

$$n_X = 10$$

$$\bar{y} = 65.949$$

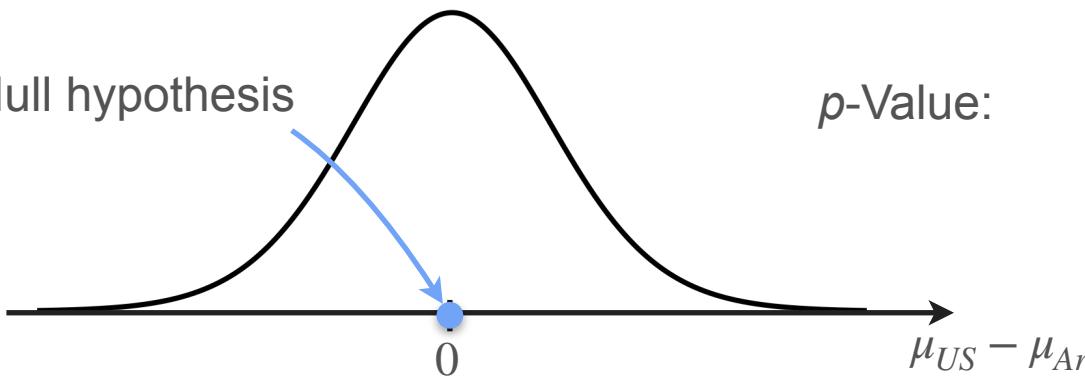
$$s_Y = 3.106$$

$$n_Y = 9$$

$$t = 1.7459$$

Null hypothesis

p -Value:

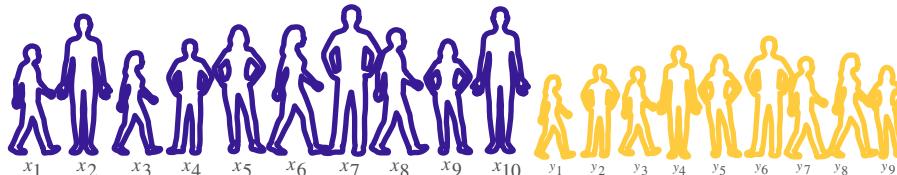


$$H_0 : \mu_{US} - \mu_{Ar} = 0 \text{ vs. } H_1 : \mu_{US} - \mu_{Ar} \neq 0$$

$$\alpha = 0.05$$

$$\text{If } H_0 \text{ is true: } T = \frac{(\bar{X} - \bar{Y}) - 0}{\sqrt{\frac{s_X^2}{10} + \frac{s_Y^2}{9}}} \sim t_{16.8}$$

Independent Two-Sample t -Test: Two Tailed Test



$$\bar{x} = 68.442$$

$$s_X = 3.113$$

$$n_X = 10$$

$$\bar{y} = 65.949$$

$$s_Y = 3.106$$

$$n_Y = 9$$

$$t = 1.7459$$

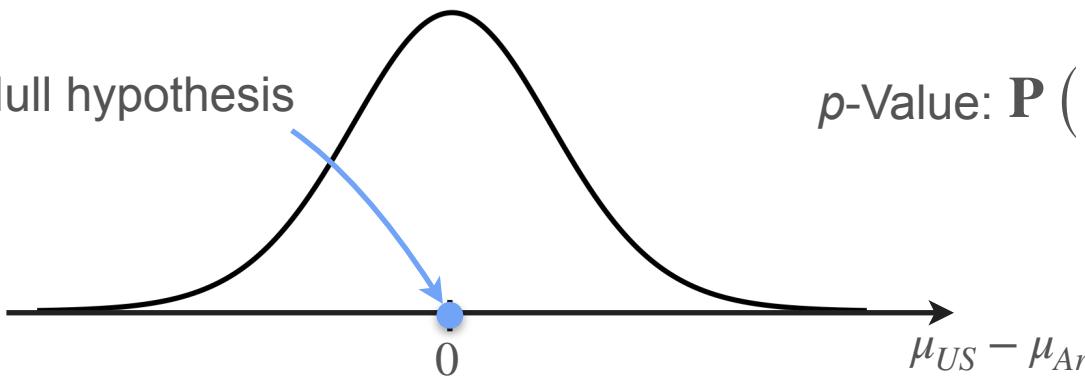
$$H_0 : \mu_{US} - \mu_{Ar} = 0 \text{ vs. } H_1 : \mu_{US} - \mu_{Ar} \neq 0$$

$$\alpha = 0.05$$

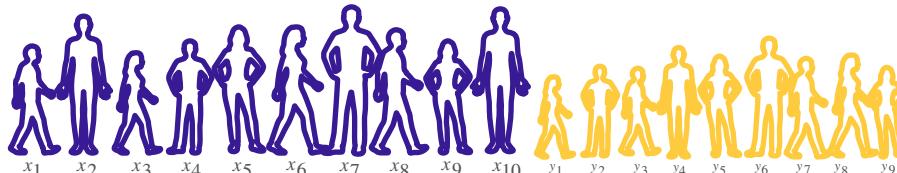
$$\text{If } H_0 \text{ is true: } T = \frac{(\bar{X} - \bar{Y}) - 0}{\sqrt{\frac{s_X^2}{10} + \frac{s_Y^2}{9}}} \sim t_{16.8}$$

Null hypothesis

p -Value: $\mathbf{P}(|T| > 1.7459 | \mu_{US} - \mu_{Ar} = 0)$



Independent Two-Sample t -Test: Two Tailed Test



$$\bar{x} = 68.442$$

$$s_X = 3.113$$

$$n_X = 10$$

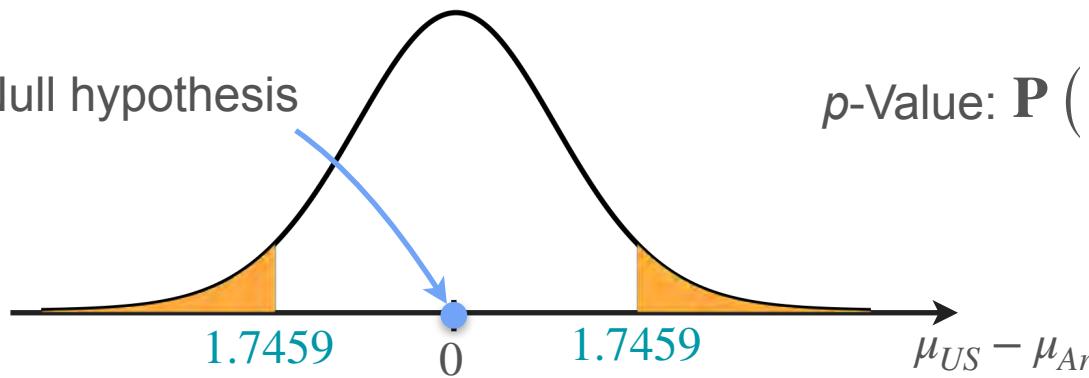
$$\bar{y} = 65.949$$

$$s_Y = 3.106$$

$$n_Y = 9$$

$$t = 1.7459$$

Null hypothesis



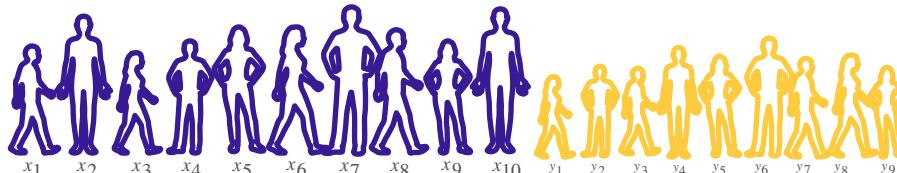
$$H_0 : \mu_{US} - \mu_{Ar} = 0 \text{ vs. } H_1 : \mu_{US} - \mu_{Ar} \neq 0$$

$$\alpha = 0.05$$

$$\text{If } H_0 \text{ is true: } T = \frac{(\bar{X} - \bar{Y}) - 0}{\sqrt{\frac{s_X^2}{10} + \frac{s_Y^2}{9}}} \sim t_{16.8}$$

$$p\text{-Value: } \mathbf{P}(|T| > 1.7459 | \mu_{US} - \mu_{Ar} = 0)$$

Independent Two-Sample t -Test: Two Tailed Test



$$\bar{x} = 68.442$$

$$s_X = 3.113$$

$$n_X = 10$$

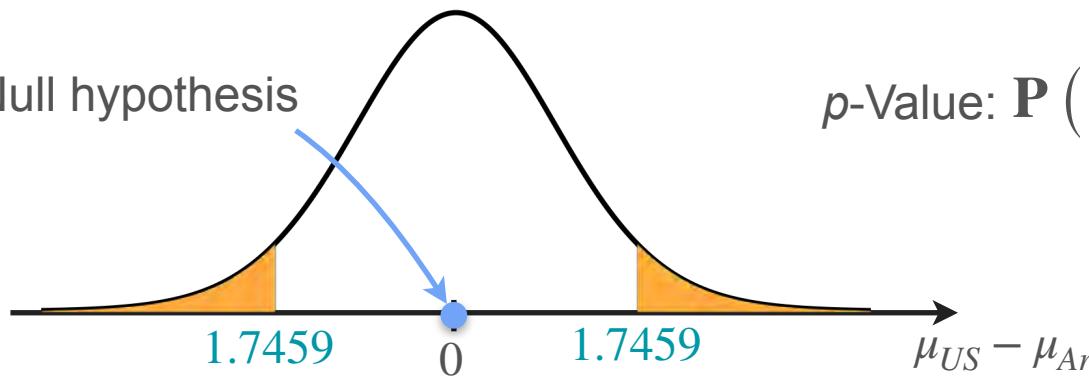
$$\bar{y} = 65.949$$

$$s_Y = 3.106$$

$$n_Y = 9$$

$$t = 1.7459$$

Null hypothesis



$$H_0 : \mu_{US} - \mu_{Ar} = 0 \text{ vs. } H_1 : \mu_{US} - \mu_{Ar} \neq 0$$

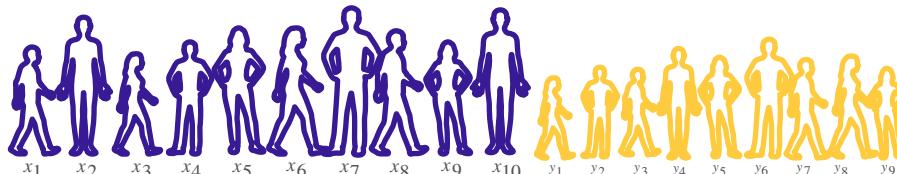
$$\alpha = 0.05$$

$$\text{If } H_0 \text{ is true: } T = \frac{(\bar{X} - \bar{Y}) - 0}{\sqrt{\frac{s_X^2}{10} + \frac{s_Y^2}{9}}} \sim t_{16.8}$$

$$p\text{-Value: } \mathbf{P}(|T| > 1.7459 | \mu_{US} - \mu_{Ar} = 0)$$

$$= 0.0991$$

Independent Two-Sample t -Test: Two Tailed Test



$$\bar{x} = 68.442$$

$$s_X = 3.113$$

$$n_X = 10$$

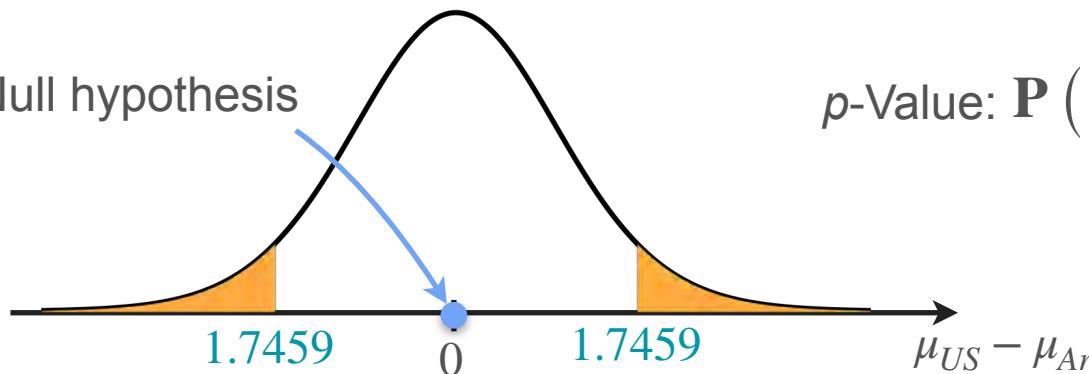
$$\bar{y} = 65.949$$

$$s_Y = 3.106$$

$$n_Y = 9$$

$$t = 1.7459$$

Null hypothesis



$$H_0 : \mu_{US} - \mu_{Ar} = 0 \text{ vs. } H_1 : \mu_{US} - \mu_{Ar} \neq 0$$

$$\alpha = 0.05$$

$$\text{If } H_0 \text{ is true: } T = \frac{(\bar{X} - \bar{Y}) - 0}{\sqrt{\frac{S_X^2}{10} + \frac{S_Y^2}{9}}} \sim t_{16.8}$$

$$p\text{-Value: } \mathbf{P}(|T| > 1.7459 | \mu_{US} - \mu_{Ar} = 0)$$

$$= 0.0991 > 0.05$$

\Rightarrow Do not reject H_0
(with a 5% significance level)



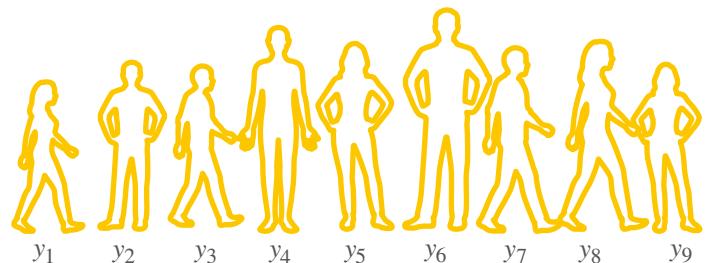
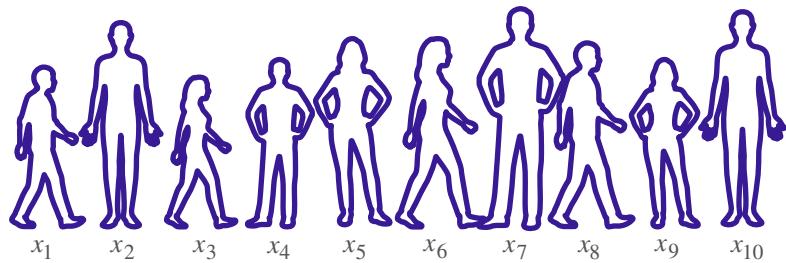
DeepLearning.AI

Hypothesis Testing

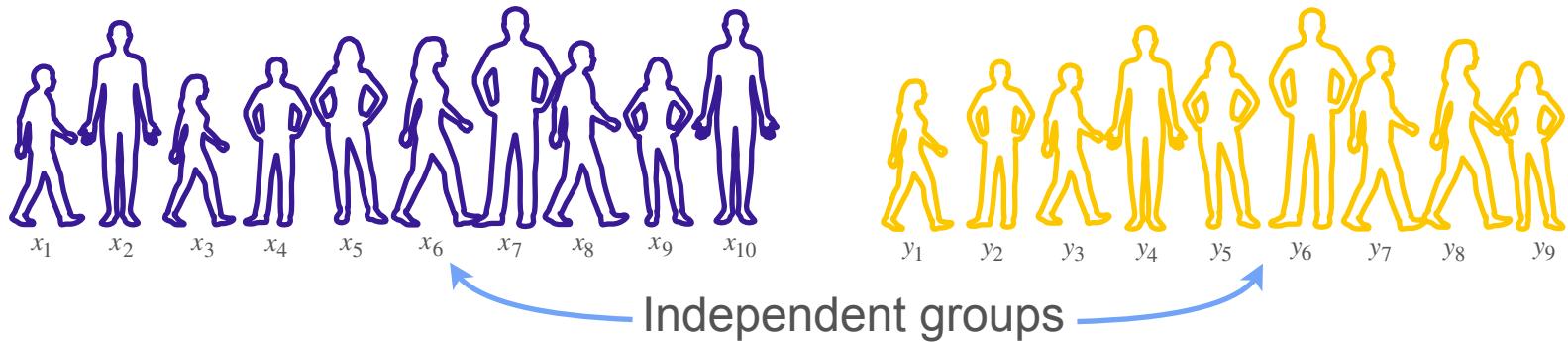
Paired t-test

Paired t -Test and Two-Sample t -Test

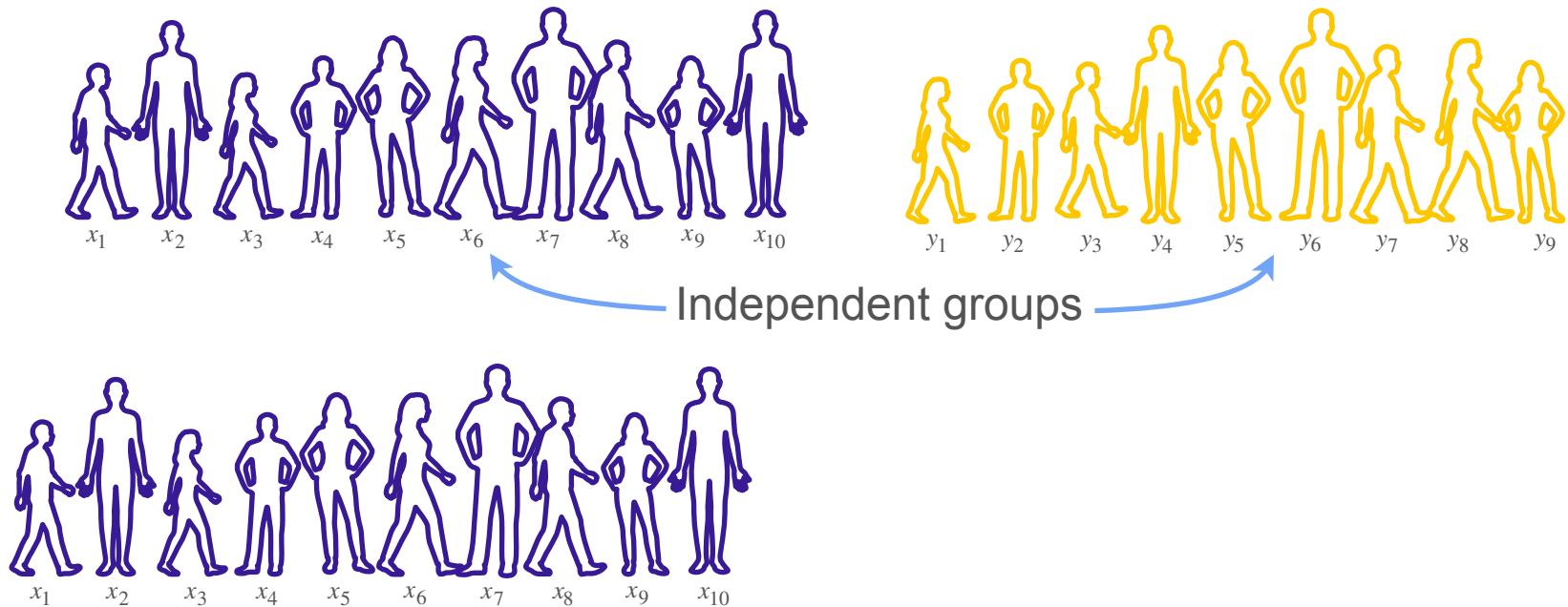
Paired t -Test and Two-Sample t -Test



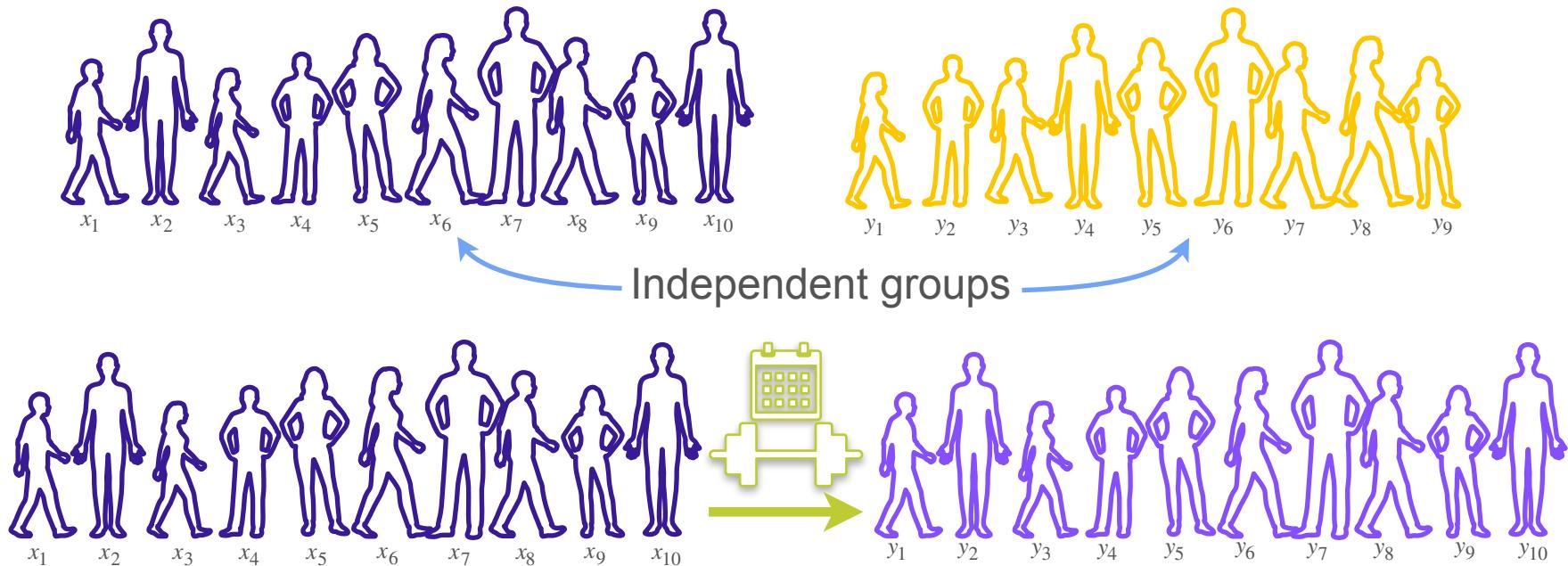
Paired t -Test and Two-Sample t -Test



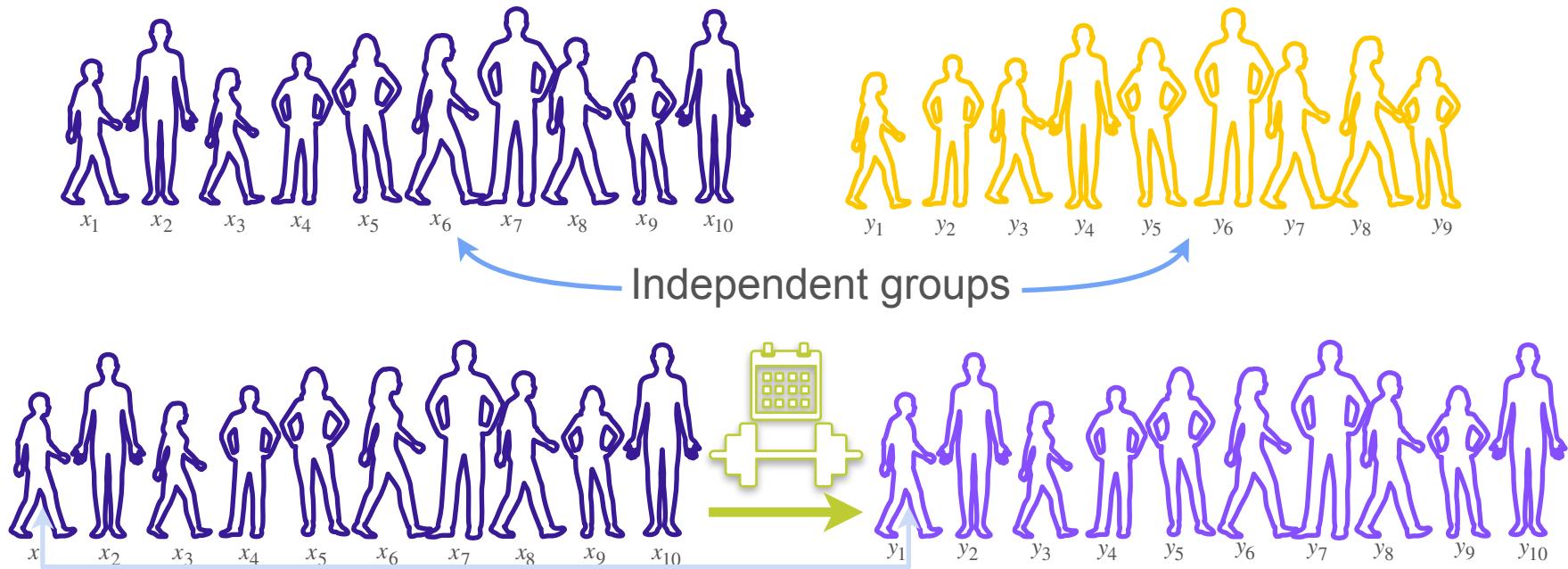
Paired t -Test and Two-Sample t -Test



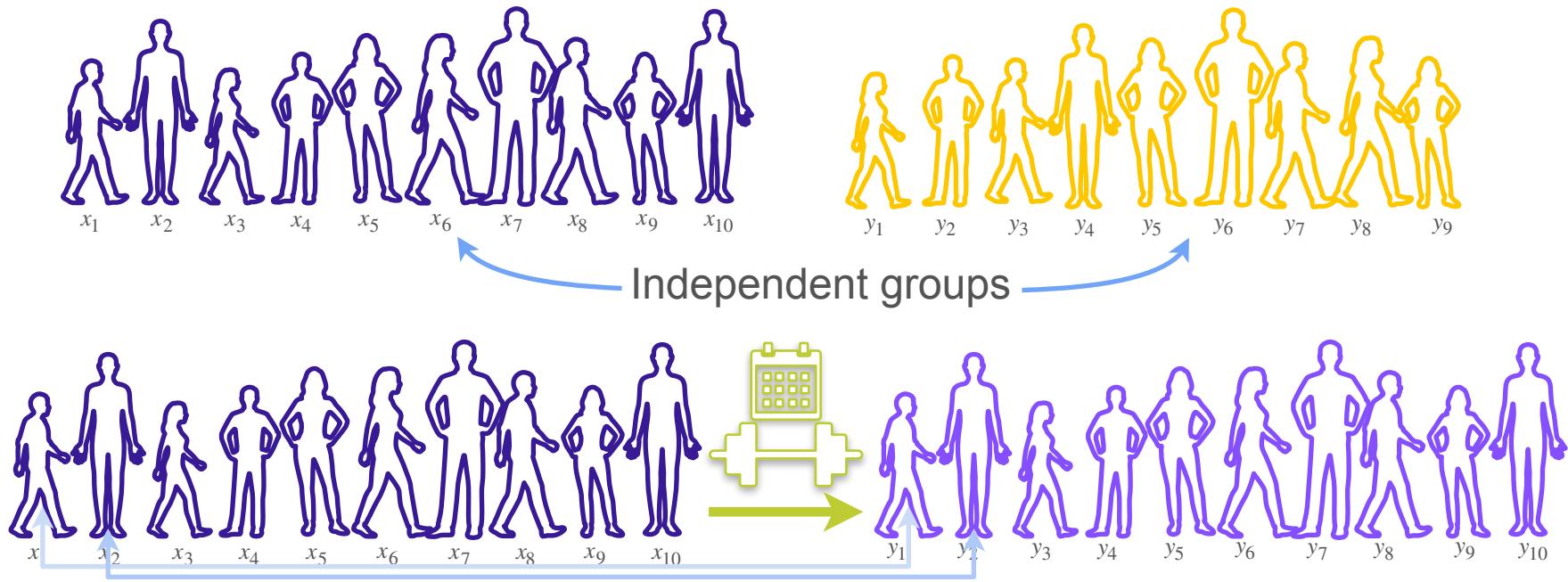
Paired t -Test and Two-Sample t -Test



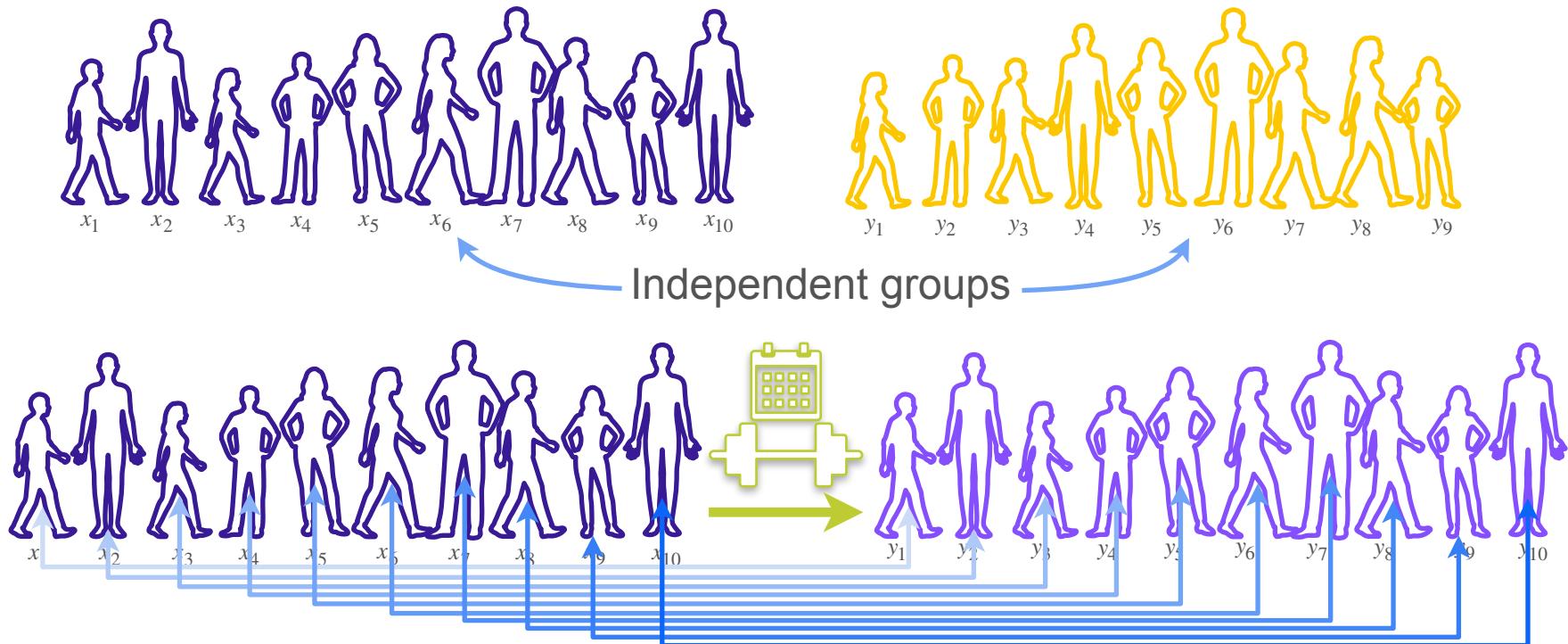
Paired t -Test and Two-Sample t -Test



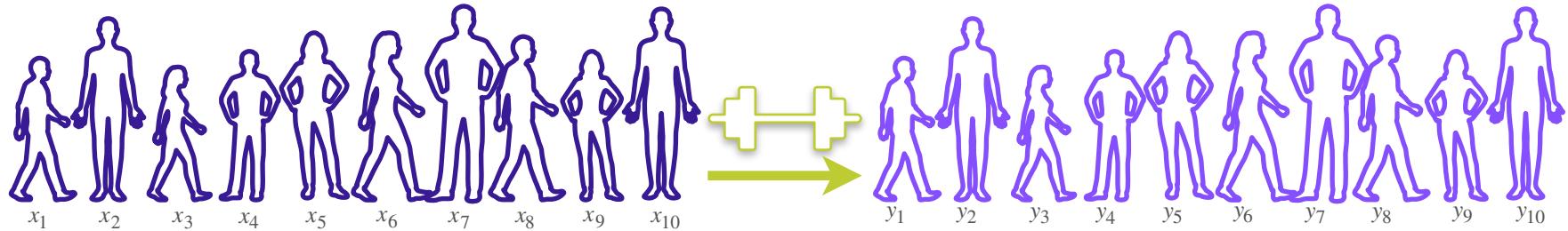
Paired t -Test and Two-Sample t -Test



Paired t -Test and Two-Sample t -Test

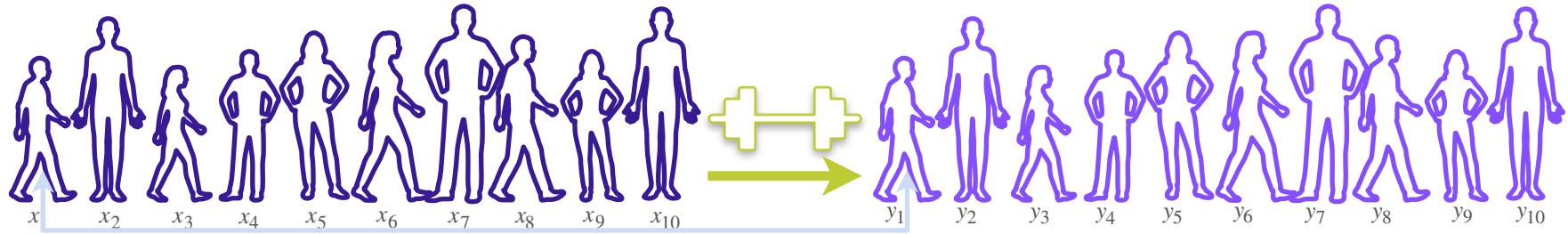


Paired t -Test: Statistic



Now you're interested in the difference between pair of samples

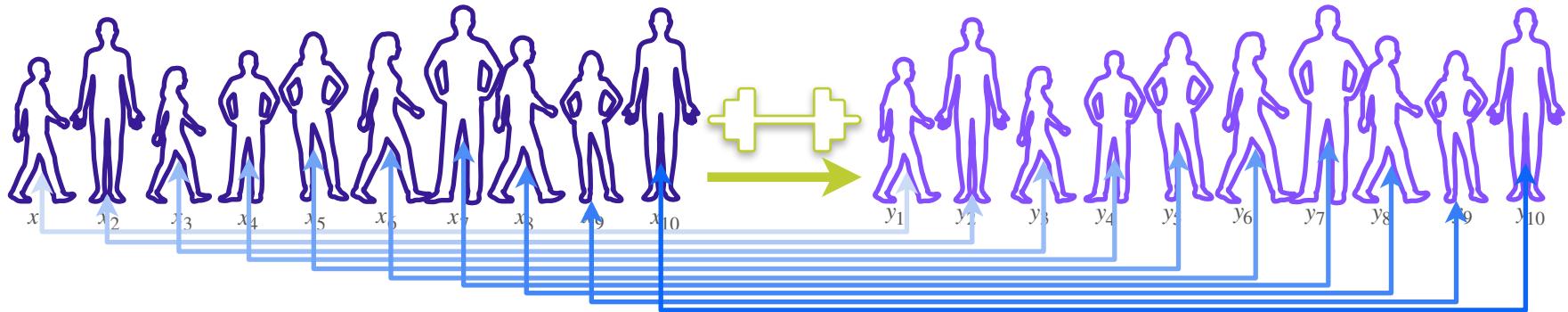
Paired t -Test: Statistic



Now you're interested in the difference between pair of samples

$$(X_1 - Y_1)$$

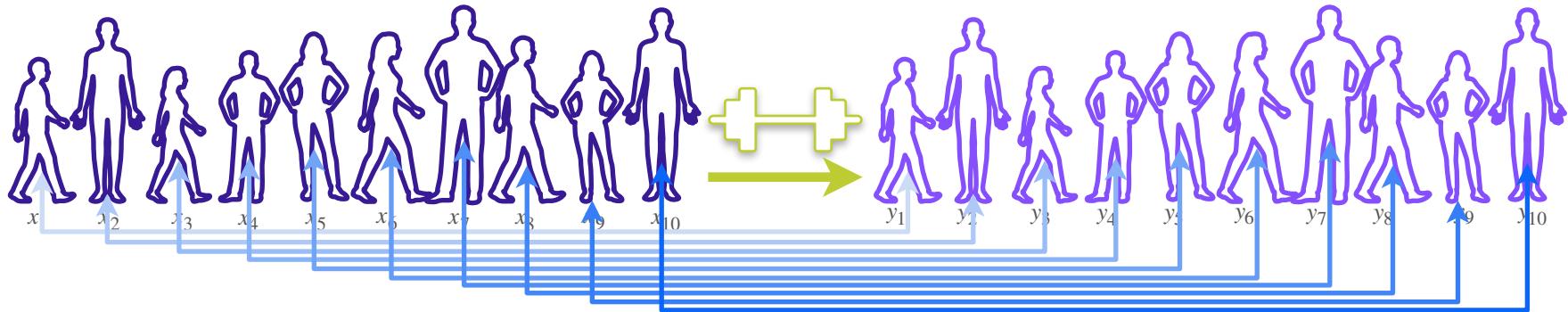
Paired t -Test: Statistic



Now you're interested in the difference between pair of samples

$$(X_1 - Y_1) \quad (X_2 - Y_2) \quad (X_3 - Y_3) \quad (X_4 - Y_4) \quad (X_5 - Y_5) \quad (X_6 - Y_6) \quad (X_7 - Y_7) \quad (X_8 - Y_8) \quad (X_9 - Y_9) \quad (X_{10} - Y_{10})$$

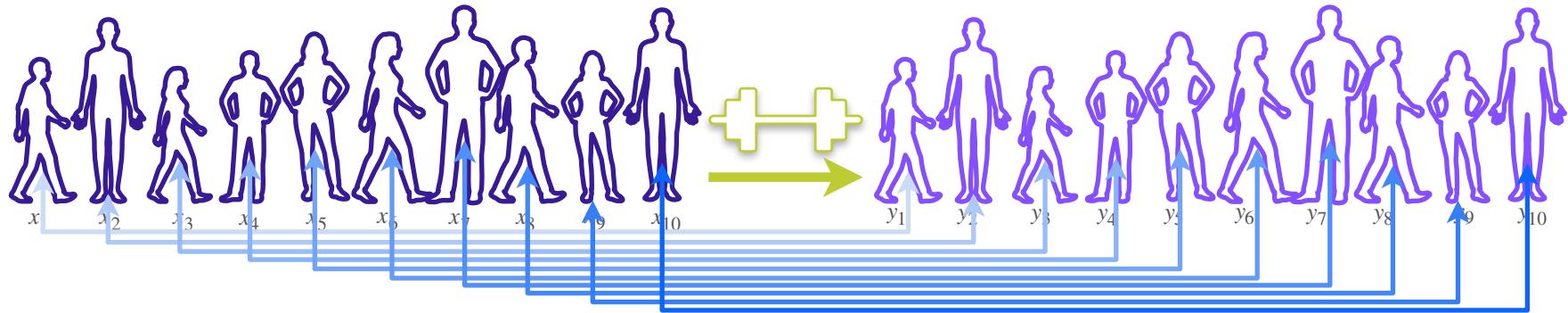
Paired t -Test: Statistic



Now you're interested in the difference between pair of samples

$$\frac{(X_1 - Y_1) + (X_2 - Y_2) + (X_3 - Y_3) + (X_4 - Y_4) + (X_5 - Y_5) + (X_6 - Y_6) + (X_7 - Y_7) + (X_8 - Y_8) + (X_9 - Y_9) + (X_{10} - Y_{10})}{10}$$

Paired t -Test: Statistic

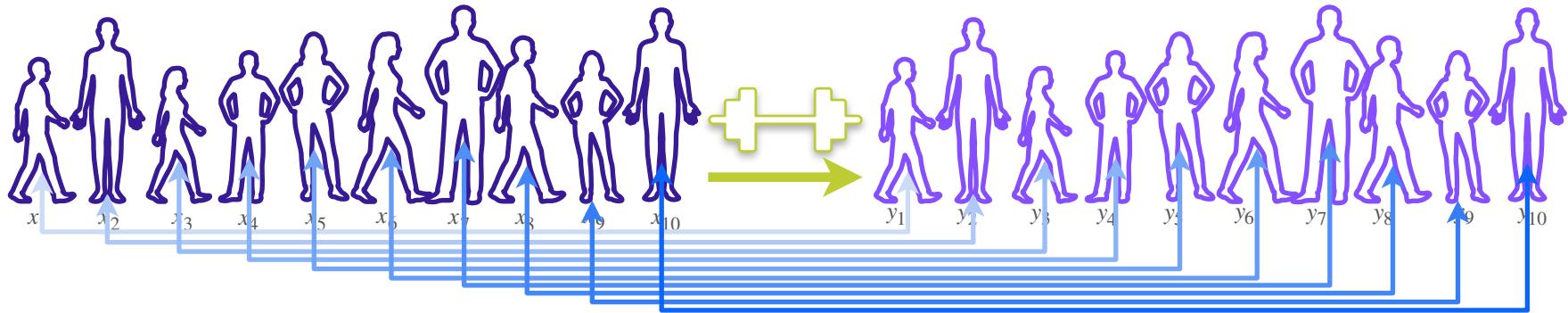


Now you're interested in the difference between pair of samples

$$\frac{(X_1 - Y_1) + (X_2 - Y_2) + (X_3 - Y_3) + (X_4 - Y_4) + (X_5 - Y_5) + (X_6 - Y_6) + (X_7 - Y_7) + (X_8 - Y_8) + (X_9 - Y_9) + (X_{10} - Y_{10})}{10}$$

D_1

Paired t -Test: Statistic



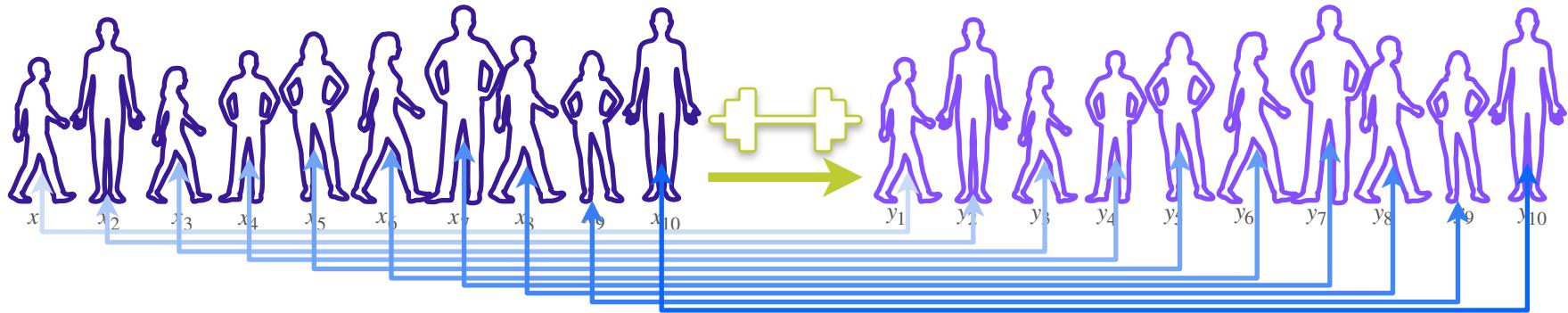
Now you're interested in the difference between pair of samples

$$\frac{(X_1 - Y_1) + (X_2 - Y_2) + (X_3 - Y_3) + (X_4 - Y_4) + (X_5 - Y_5) + (X_6 - Y_6) + (X_7 - Y_7) + (X_8 - Y_8) + (X_9 - Y_9) + (X_{10} - Y_{10})}{10}$$

D_1

D_2

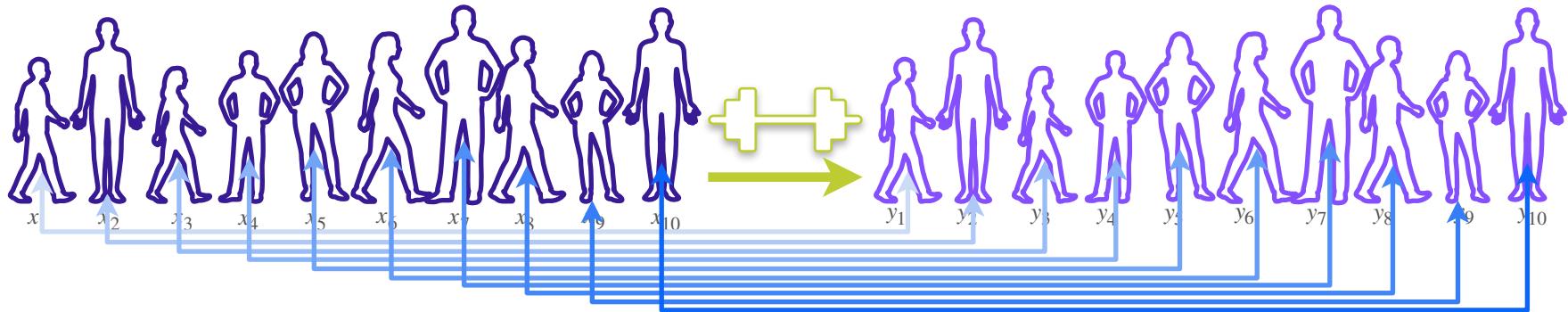
Paired t -Test: Statistic



Now you're interested in the difference between pair of samples

$$\frac{(X_1 - Y_1) + (X_2 - Y_2) + (X_3 - Y_3) + (X_4 - Y_4) + (X_5 - Y_5) + (X_6 - Y_6) + (X_7 - Y_7) + (X_8 - Y_8) + (X_9 - Y_9) + (X_{10} - Y_{10})}{10}$$
$$\frac{D_1 + D_2 + D_3 + D_4 + D_5 + D_6 + D_7 + D_8 + D_9 + D_{10}}{10}$$

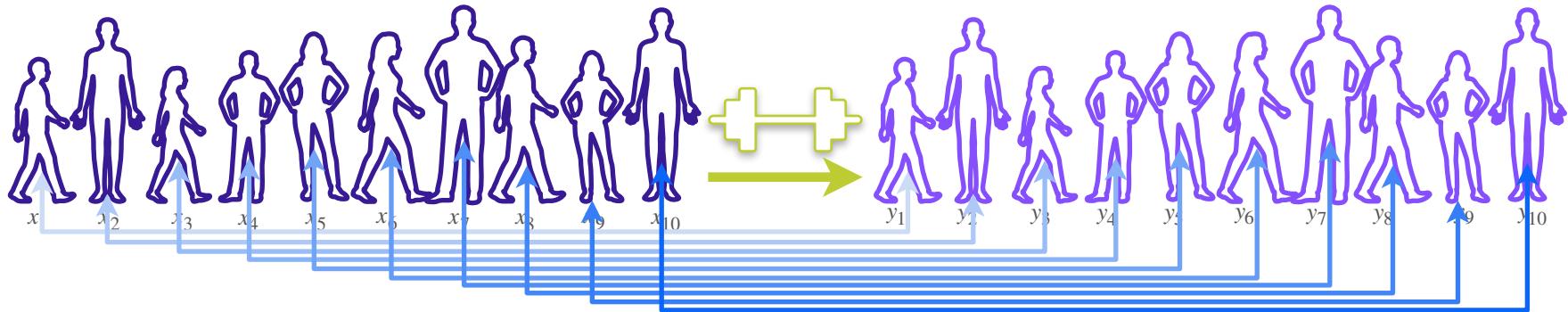
Paired t -Test: Statistic



Now you're interested in the difference between pair of samples

$$\frac{(X_1 - Y_1) + (X_2 - Y_2) + (X_3 - Y_3) + (X_4 - Y_4) + (X_5 - Y_5) + (X_6 - Y_6) + (X_7 - Y_7) + (X_8 - Y_8) + (X_9 - Y_9) + (X_{10} - Y_{10})}{10}$$
$$\frac{D_1 + D_2 + D_3 + D_4 + D_5 + D_6 + D_7 + D_8 + D_9 + D_{10}}{10}$$

Paired t -Test: Statistic



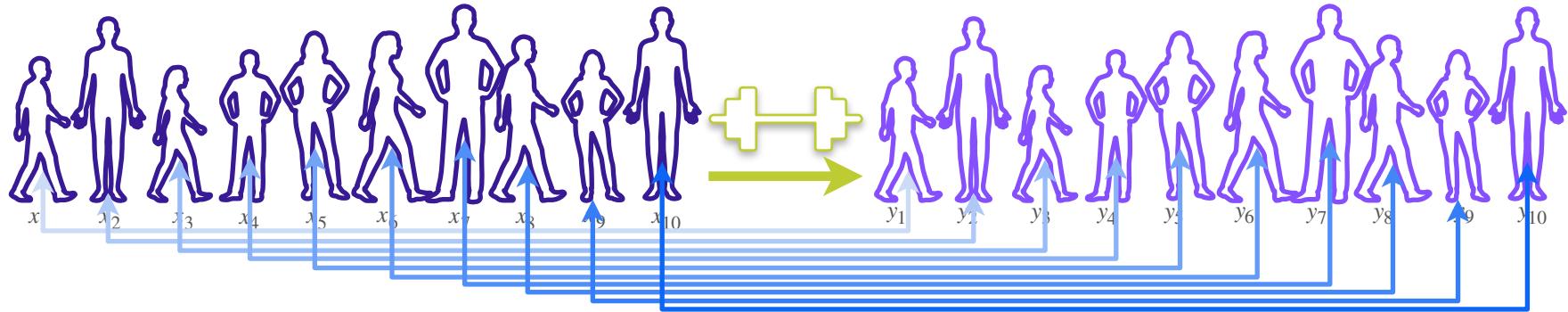
Now you're interested in the difference between pair of samples

$$(X_1 - Y_1) + (X_2 - Y_2) + (X_3 - Y_3) + (X_4 - Y_4) + (X_5 - Y_5) + (X_6 - Y_6) + (X_7 - Y_7) + (X_8 - Y_8) + (X_9 - Y_9) + (X_{10} - Y_{10})$$

10

$$\bar{D} = \frac{D_1 + D_2 + D_3 + D_4 + D_5 + D_6 + D_7 + D_8 + D_9 + D_{10}}{10}$$

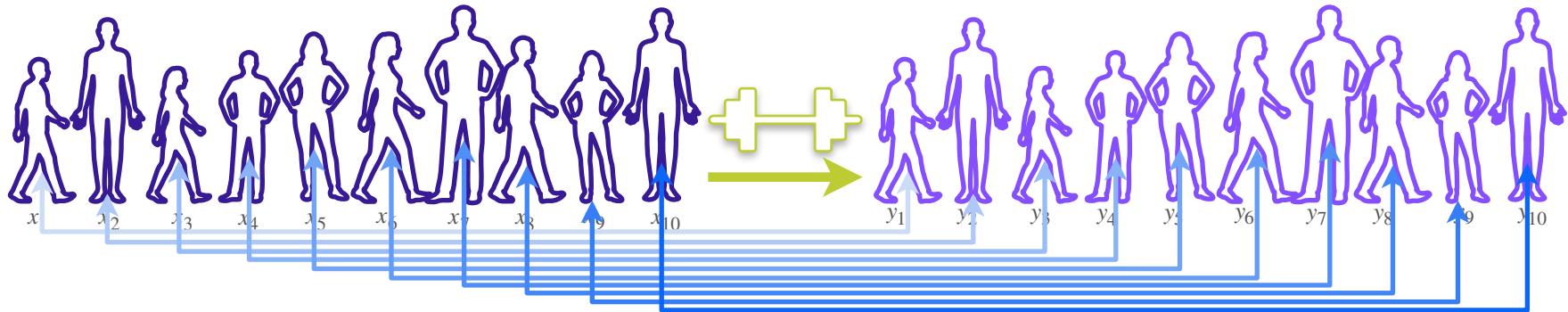
Paired t -Test: Statistic



Now you're interested in the difference between pair of samples

$$\bar{D} = \frac{D_1 + D_2 + D_3 + D_4 + D_5 + D_6 + D_7 + D_8 + D_9 + D_{10}}{10}$$

Paired t -Test: Statistic



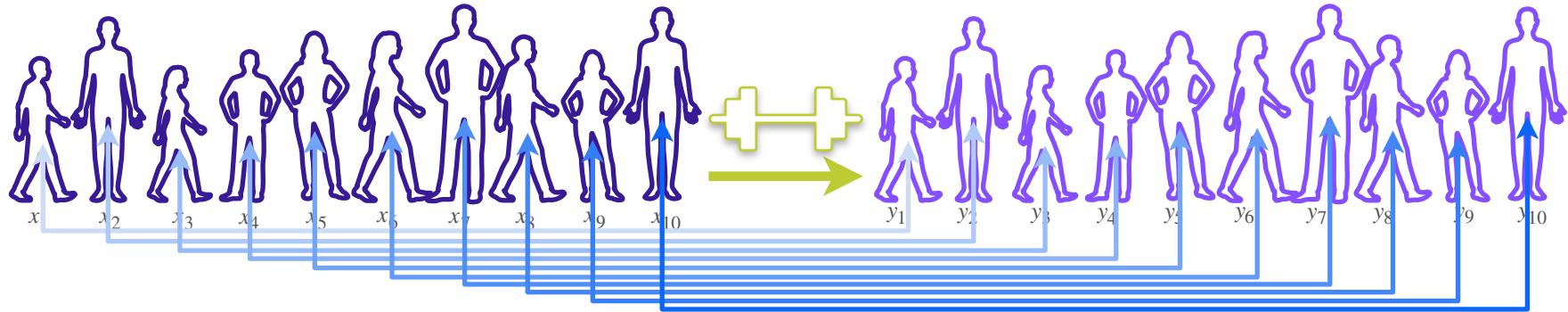
Now you're interested in the difference between pair of samples

$$\bar{D} = \frac{D_1 + D_2 + D_3 + D_4 + D_5 + D_6 + D_7 + D_8 + D_9 + D_{10}}{10}$$

If X_i, Y_i are gaussian $\Rightarrow D_i = X_i - Y_i$ is gaussian.

$$D_i \stackrel{i.i.d}{\sim} \mathcal{N}(\mu_D, \sigma_D^2)$$

Paired t -Test: Statistic

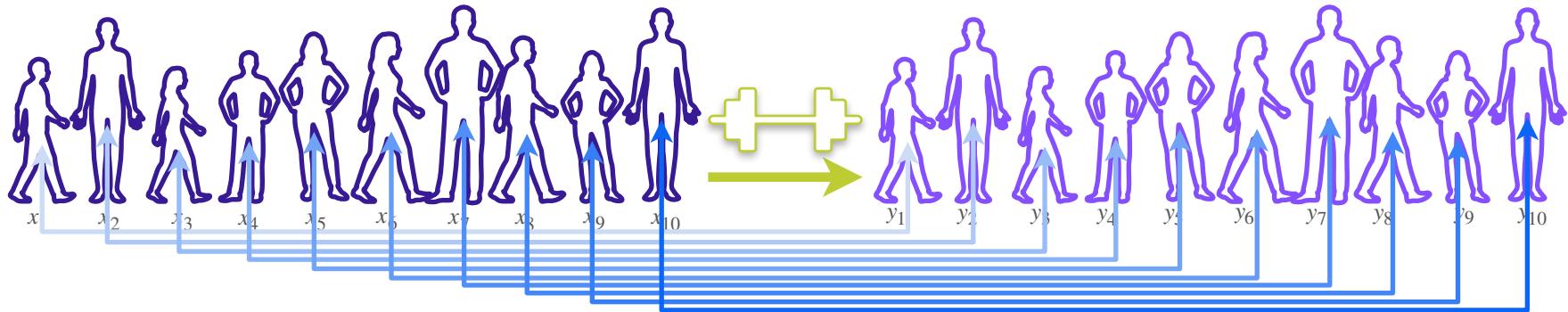


Now you're interested in the difference between pair of samples

$$\bar{D} = \frac{D_1 + D_2 + D_3 + D_4 + D_5 + D_6 + D_7 + D_8 + D_9 + D_{10}}{10}$$

If X_i, Y_i are gaussian $\Rightarrow D_i = X_i - Y_i$ is gaussian.

Paired t -Test: Statistic



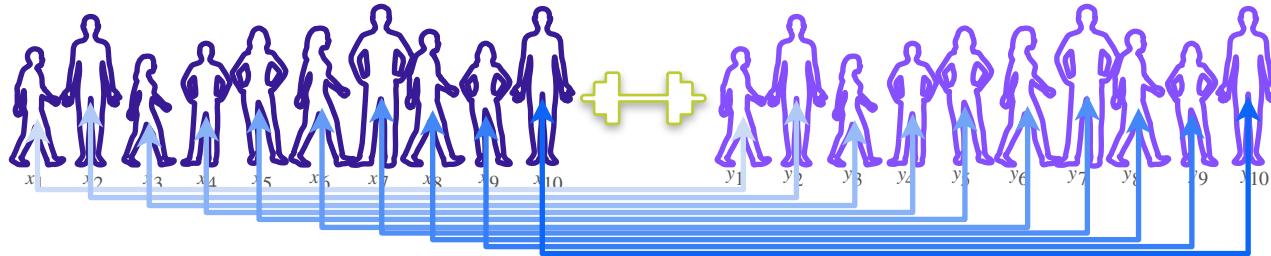
Now you're interested in the difference between pair of samples

$$\bar{D} = \frac{D_1 + D_2 + D_3 + D_4 + D_5 + D_6 + D_7 + D_8 + D_9 + D_{10}}{10}$$

If X_i, Y_i are gaussian $\Rightarrow D_i = X_i - Y_i$ is gaussian.

$$D_i \stackrel{i.i.d}{\sim} \mathcal{N}(\mu_D, \sigma_D^2)$$

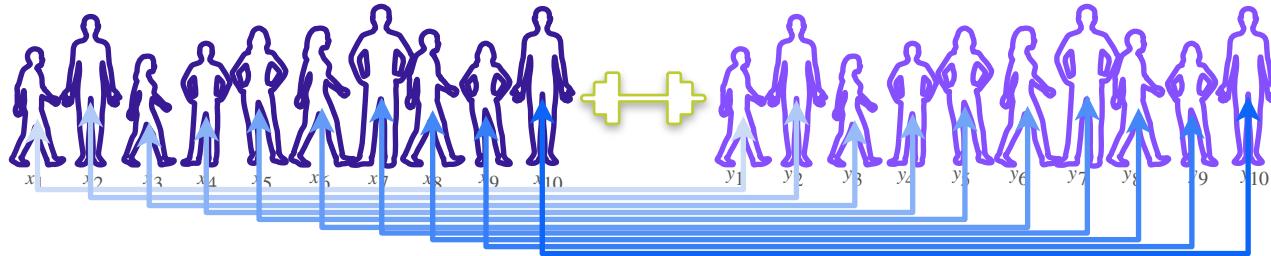
Paired t -Test: Statistic



Now you're interested in the difference between pair of samples

$$\bar{D} = \frac{D_1 + D_2 + D_3 + D_4 + D_5 + D_6 + D_7 + D_8 + D_9 + D_{10}}{10} \quad D_i = X_i - Y_i$$
$$D_i \stackrel{i.i.d}{\sim} \mathcal{N}(\mu_D, \sigma_D^2)$$

Paired t -Test: Statistic

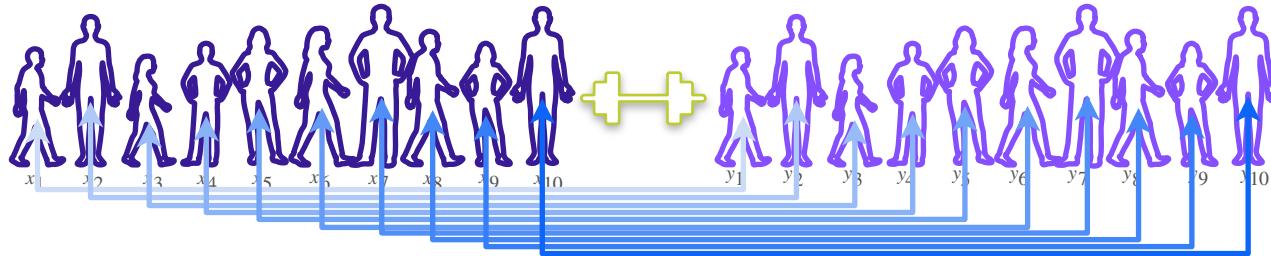


Now you're interested in the difference between pair of samples

$$\bar{D} = \frac{D_1 + D_2 + D_3 + D_4 + D_5 + D_6 + D_7 + D_8 + D_9 + D_{10}}{10} \quad D_i = X_i - Y_i$$
$$D_i \stackrel{i.i.d}{\sim} \mathcal{N}(\mu_D, \sigma_D^2)$$

$$\frac{\bar{D} - \mu_D}{\sigma_D / \sqrt{10}} \sim \mathcal{N}(0, 1^2)$$

Paired t -Test: Statistic



Now you're interested in the difference between pair of samples

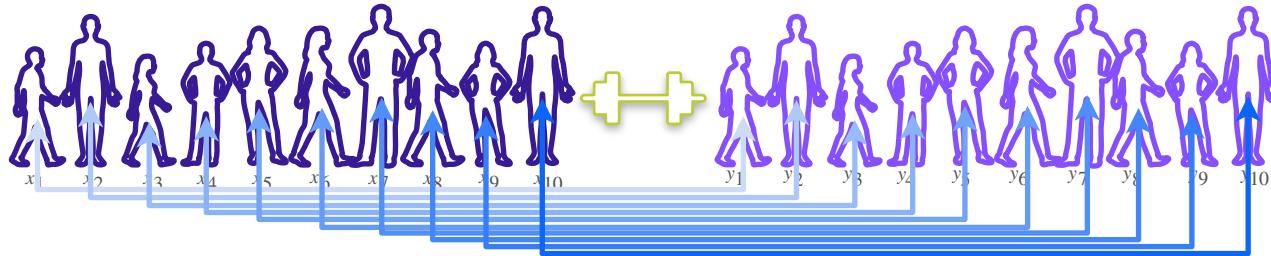
$$\bar{D} = \frac{D_1 + D_2 + D_3 + D_4 + D_5 + D_6 + D_7 + D_8 + D_9 + D_{10}}{10}$$

$$D_i = X_i - Y_i$$
$$D_i \stackrel{i.i.d}{\sim} \mathcal{N}(\mu_D, \sigma_D^2)$$

$$\frac{\bar{D} - \mu_D}{\sigma_D / \sqrt{10}} \sim \mathcal{N}(0, 1^2)$$

But σ_D is unknown

Paired t -Test: Statistic



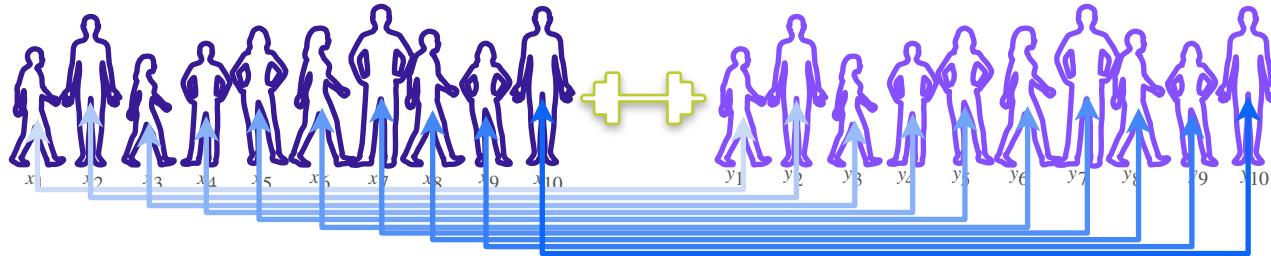
Now you're interested in the difference between pair of samples

$$\bar{D} = \frac{D_1 + D_2 + D_3 + D_4 + D_5 + D_6 + D_7 + D_8 + D_9 + D_{10}}{10} \quad D_i = X_i - Y_i$$
$$D_i \stackrel{i.i.d}{\sim} \mathcal{N}(\mu_D, \sigma_D^2)$$

$$\frac{\bar{D} - \mu_D}{\sigma_D / \sqrt{10}} \sim \mathcal{N}(0, 1^2)$$

But σ_D is unknown $\Rightarrow \sigma_D \rightarrow S_D = \sqrt{\frac{\sum_{i=1}^{10} (D_i - \bar{D})^2}{10 - 1}}$

Paired t -Test: Statistic



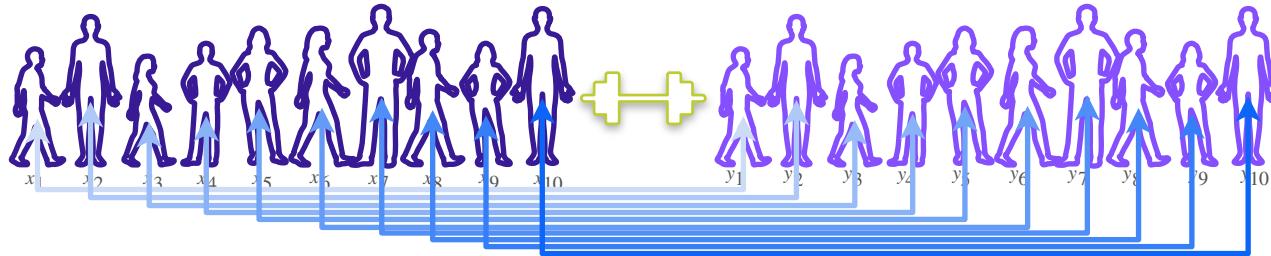
Now you're interested in the difference between pair of samples

$$\bar{D} = \frac{D_1 + D_2 + D_3 + D_4 + D_5 + D_6 + D_7 + D_8 + D_9 + D_{10}}{10} \quad D_i = X_i - Y_i$$
$$D_i \stackrel{i.i.d}{\sim} \mathcal{N}(\mu_D, \sigma_D^2)$$

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Paired t -Test: Statistic



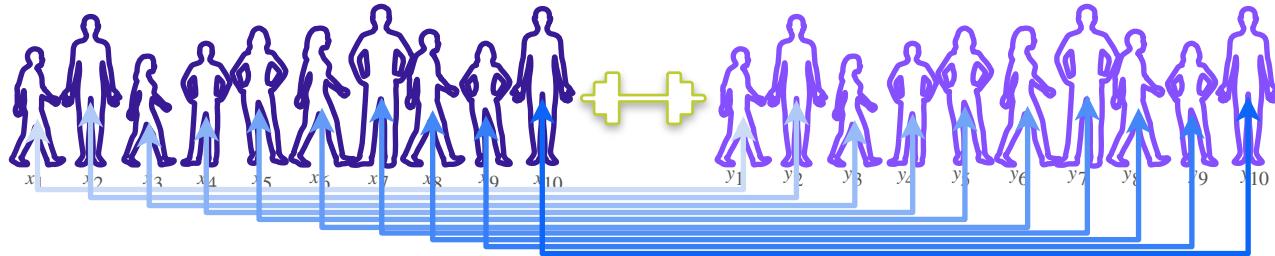
Now you're interested in the difference between pair of samples

$$\bar{D} = \frac{D_1 + D_2 + D_3 + D_4 + D_5 + D_6 + D_7 + D_8 + D_9 + D_{10}}{10} \quad D_i = X_i - Y_i$$
$$D_i \stackrel{i.i.d}{\sim} \mathcal{N}(\mu_D, \sigma_D^2)$$

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But σ_D is unknown $\Rightarrow \sigma_D \rightarrow S_D = \sqrt{\frac{\sum_{i=1}^{10} (D_i - \bar{D})^2}{10 - 1}}$ $\Rightarrow T = \frac{\bar{D} - \mu_D}{S_D / \sqrt{10}} \sim t_{10-1}$

Paired t -Test: Statistic



Now you're interested in the difference between pair of samples

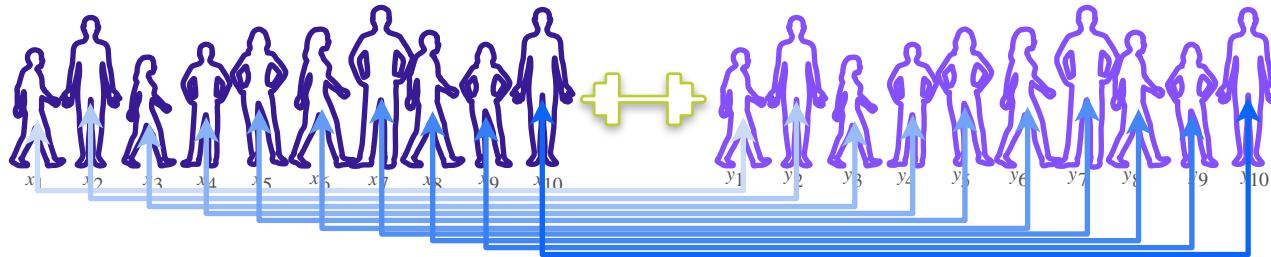
$$\bar{D} = \frac{D_1 + D_2 + D_3 + D_4 + D_5 + D_6 + D_7 + D_8 + D_9 + D_{10}}{10} \quad D_i = X_i - Y_i$$
$$D_i \stackrel{i.i.d}{\sim} \mathcal{N}(\mu_D, \sigma_D^2)$$

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But σ_D is unknown $\Rightarrow \sigma_D \rightarrow S_D = \sqrt{\frac{\sum_{i=1}^{10} (D_i - \bar{D})^2}{10 - 1}}$ $\Rightarrow T = \frac{\bar{D} - \mu_D}{S_D / \sqrt{10}} \sim t_{10-1}$

$$H_0 : \mu_D = 0$$

Paired t -Test: Statistic



Now you're interested in the difference between pair of samples

$$\bar{D} = \frac{D_1 + D_2 + D_3 + D_4 + D_5 + D_6 + D_7 + D_8 + D_9 + D_{10}}{10} \quad D_i = X_i - Y_i$$
$$D_i \stackrel{i.i.d}{\sim} \mathcal{N}(\mu_D, \sigma_D^2)$$

$$\frac{\bar{D} - \mu_D}{\sigma_D / \sqrt{10}} \sim \mathcal{N}(0, 1^2)$$

But σ_D is unknown $\Rightarrow \sigma_D \rightarrow S_D = \sqrt{\frac{\sum_{i=1}^{10} (D_i - \bar{D})^2}{10 - 1}}$ $\Rightarrow T = \frac{\bar{D} - 0}{S_D / \sqrt{10}} \sim t_{10-1}$

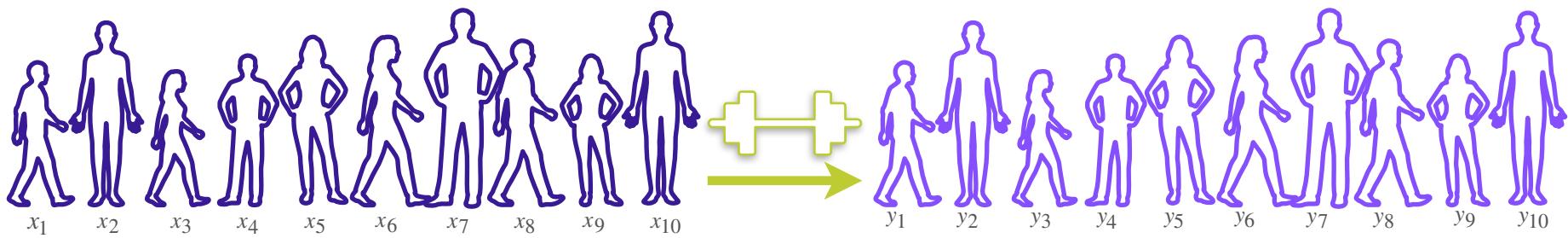
$H_0 : \mu_D = 0$ Test statistic

Paired t -Test: Observations

=

d_i

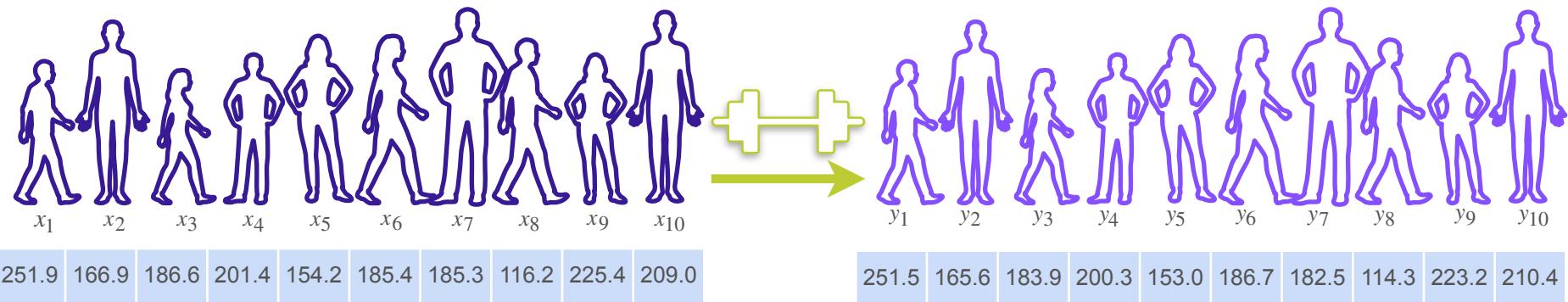
Paired t -Test: Observations



=

$$d_i$$

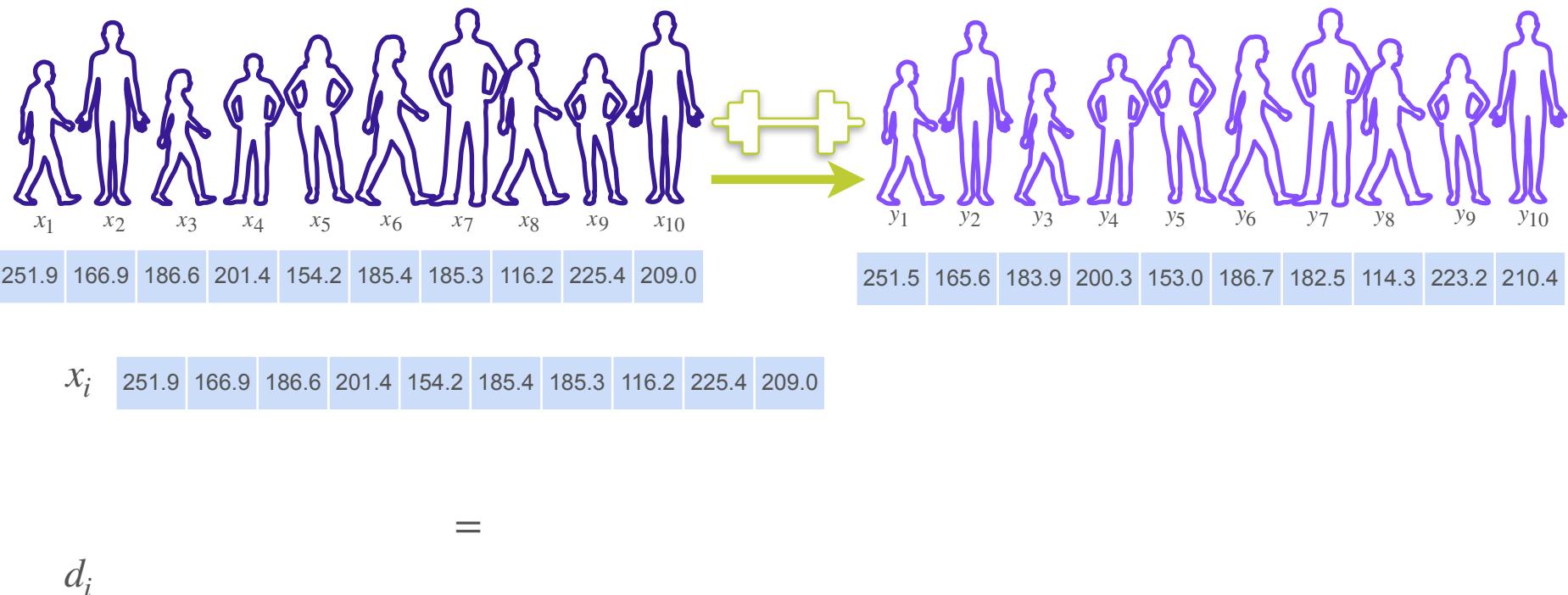
Paired t -Test: Observations



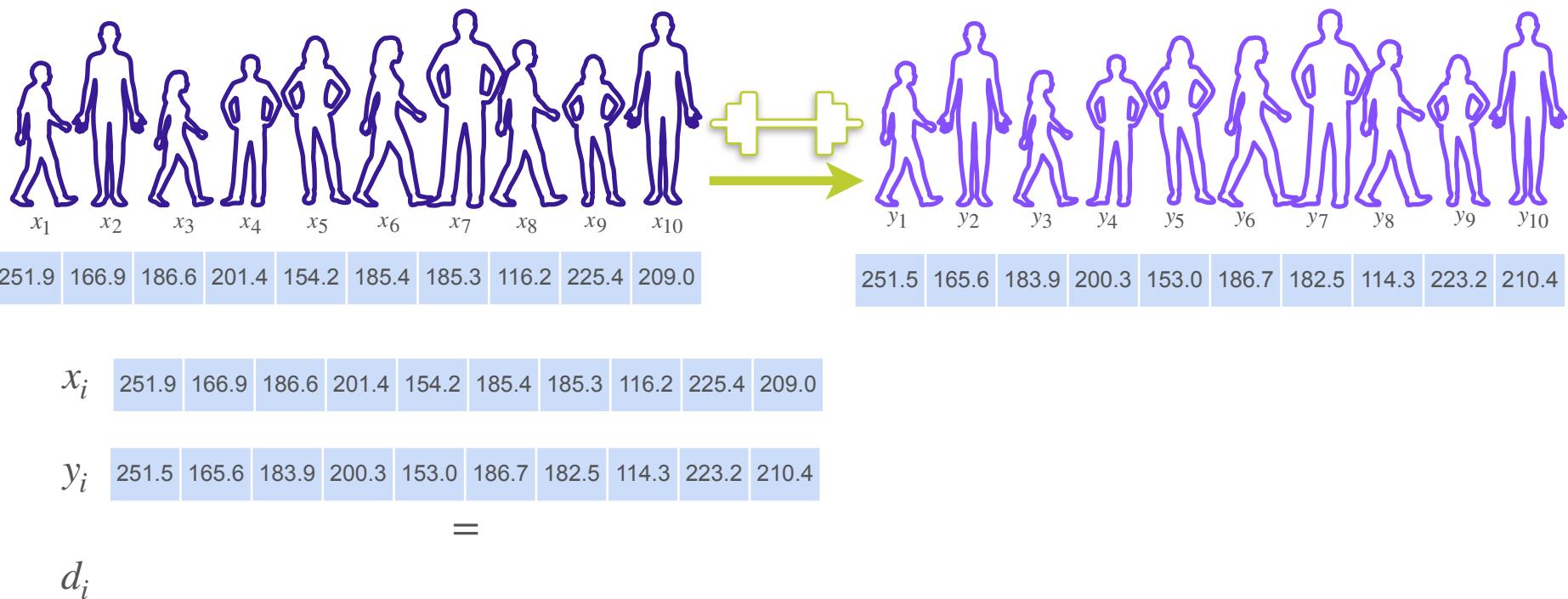
=

d_i

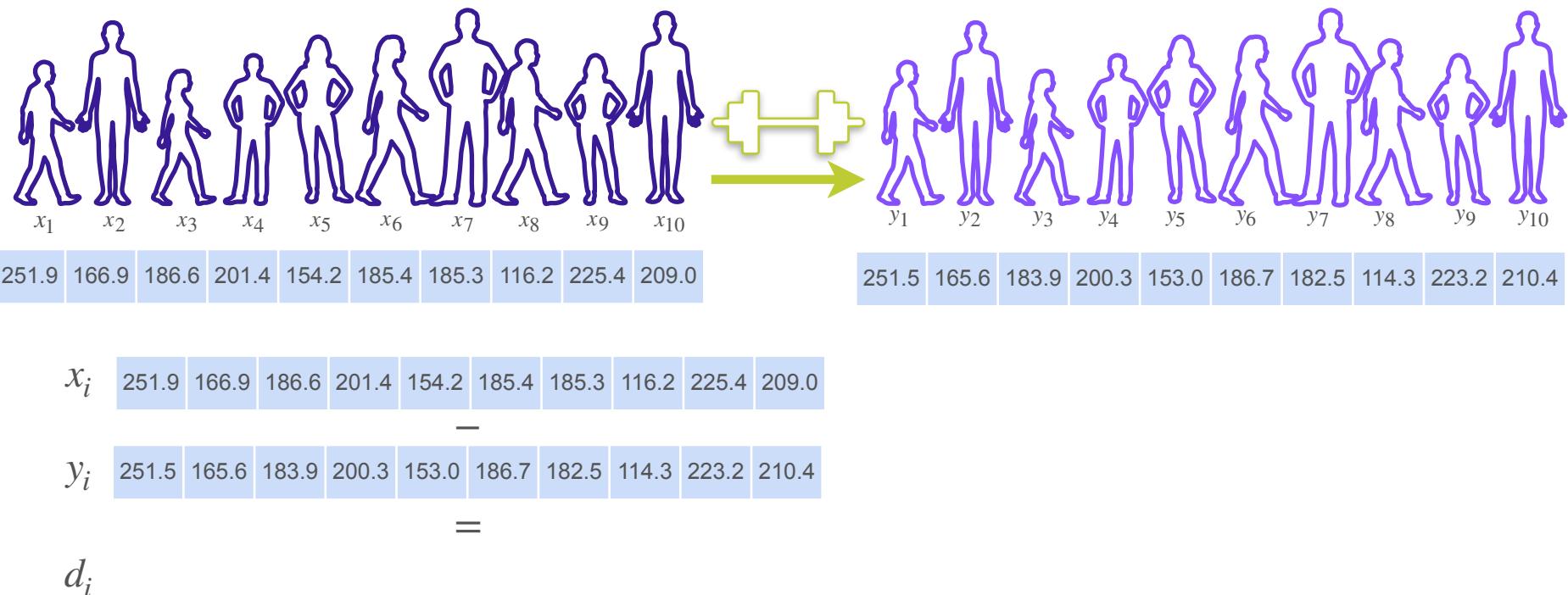
Paired t -Test: Observations



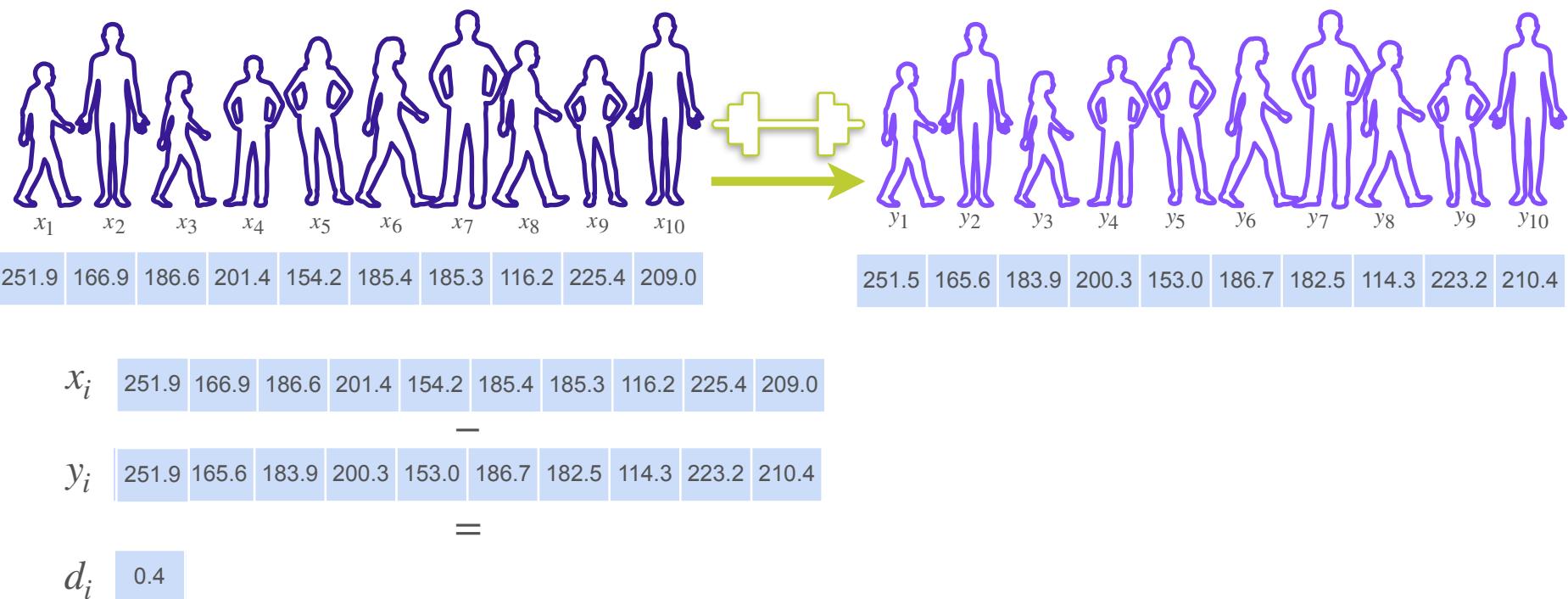
Paired t -Test: Observations



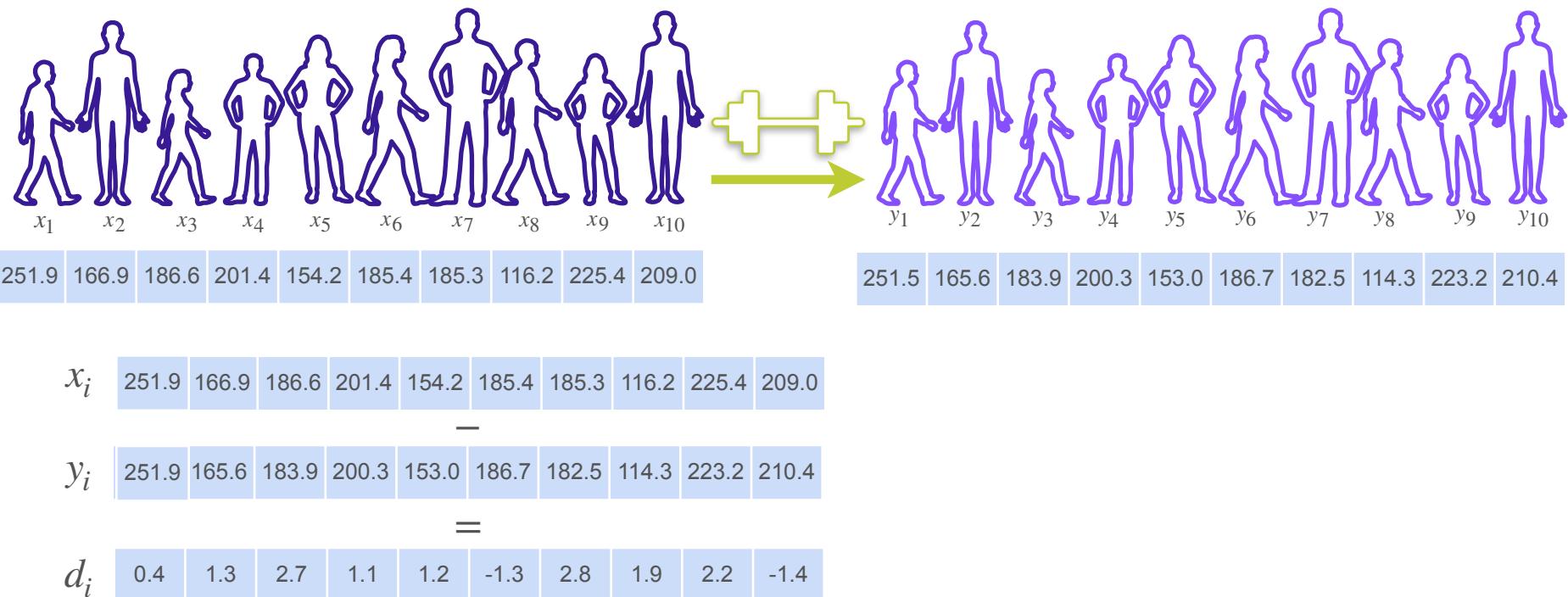
Paired t -Test: Observations



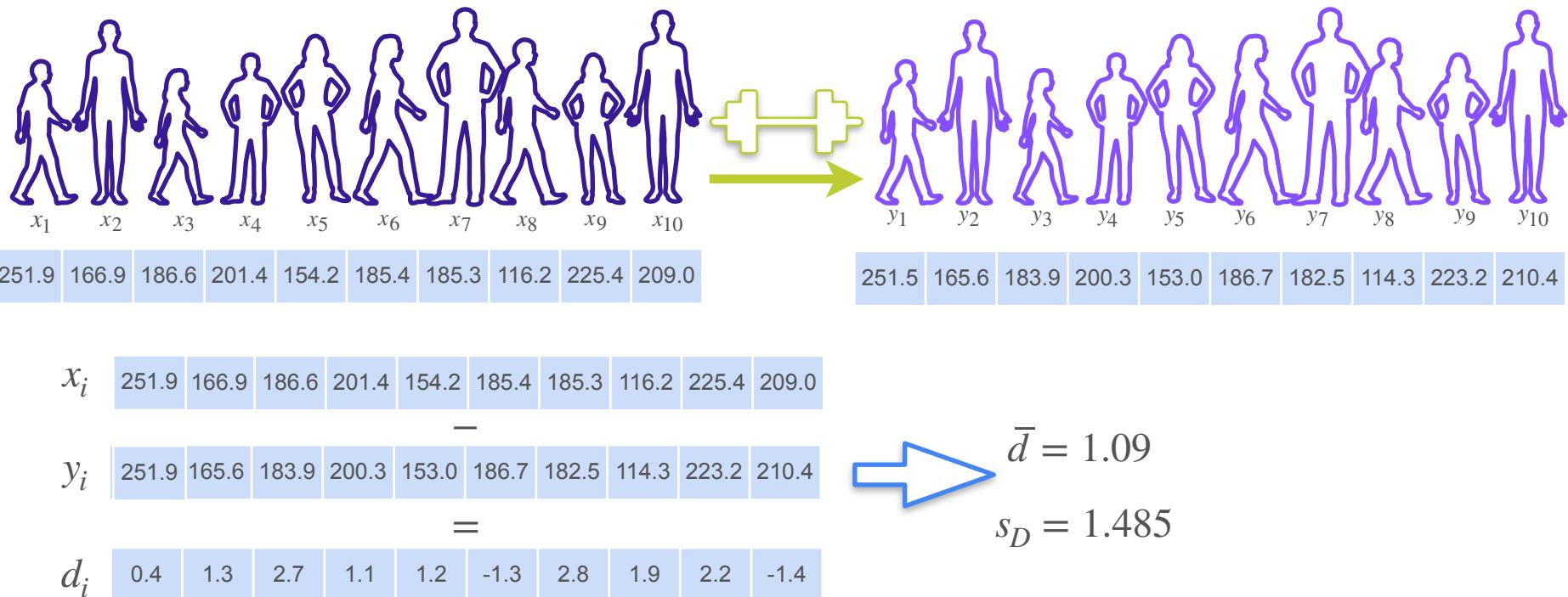
Paired t -Test: Observations



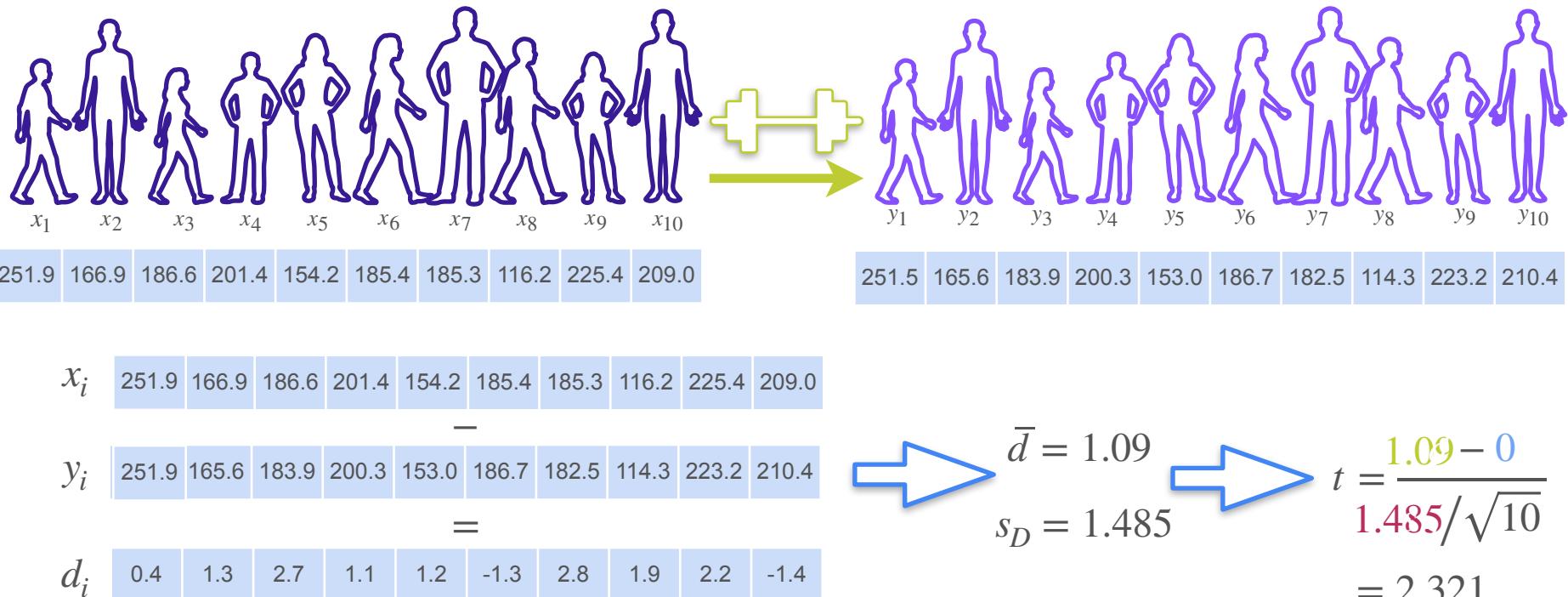
Paired t -Test: Observations



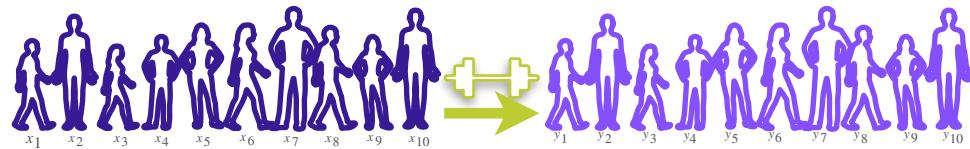
Paired t -Test: Observations



Paired t -Test: Observations



Independent Two-Sample t -Test: Right Tailed Test

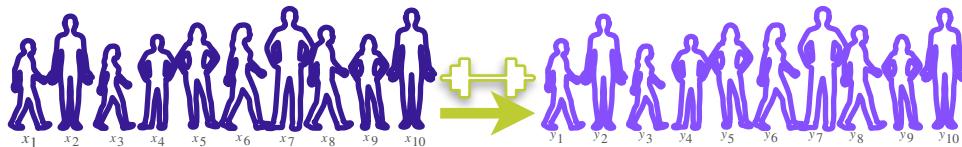


$$\bar{d} = 1.09$$

$$s_D = 1.485$$

$$n = 10$$

Independent Two-Sample t -Test: Right Tailed Test



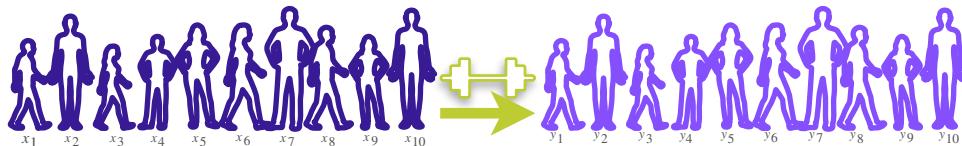
$$\bar{d} = 1.09$$

$$s_D = 1.485$$

$$n = 10$$

$$t = 2.321$$

Independent Two-Sample t -Test: Right Tailed Test



$$H_0 : \mu_D = 0 \text{ vs. } H_1 : \mu_D > 0$$

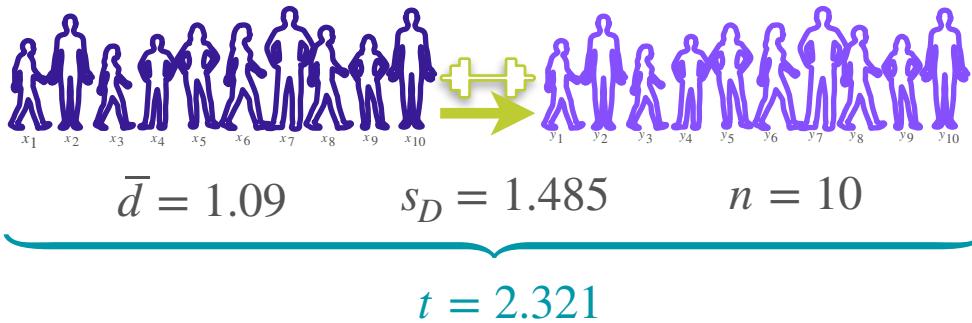
$$\bar{d} = 1.09$$

$$s_D = 1.485$$

$$n = 10$$

$$t = 2.321$$

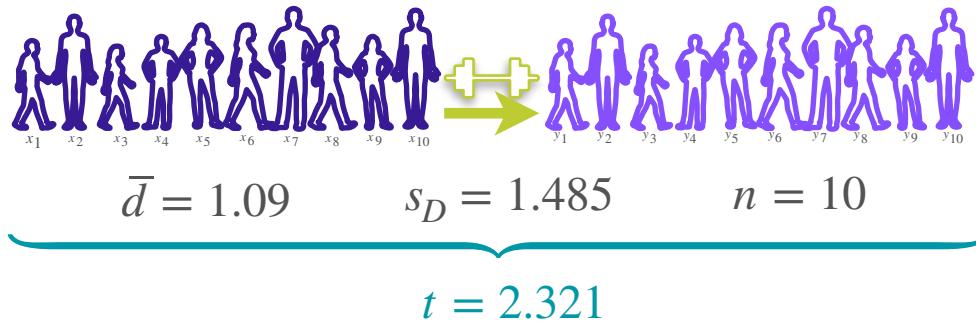
Independent Two-Sample t -Test: Right Tailed Test



$$H_0 : \mu_D = 0 \text{ vs. } H_1 : \mu_D > 0$$

$$\alpha = 0.05$$

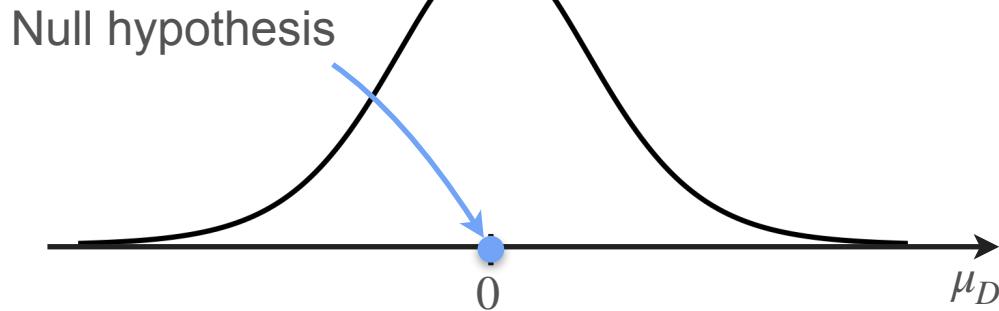
Independent Two-Sample t -Test: Right Tailed Test



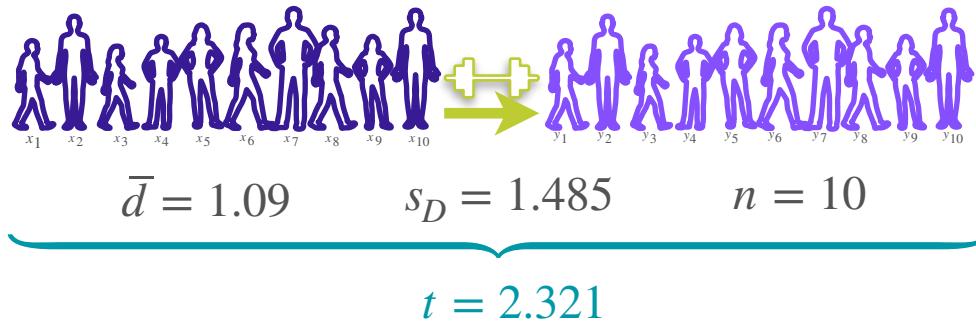
$$H_0 : \mu_D = 0 \text{ vs. } H_1 : \mu_D > 0$$

$$\alpha = 0.05$$

$$\text{If } H_0 \text{ is true: } T = \frac{\bar{D} - 0}{S_D / \sqrt{10}} \sim t_{10-1}$$



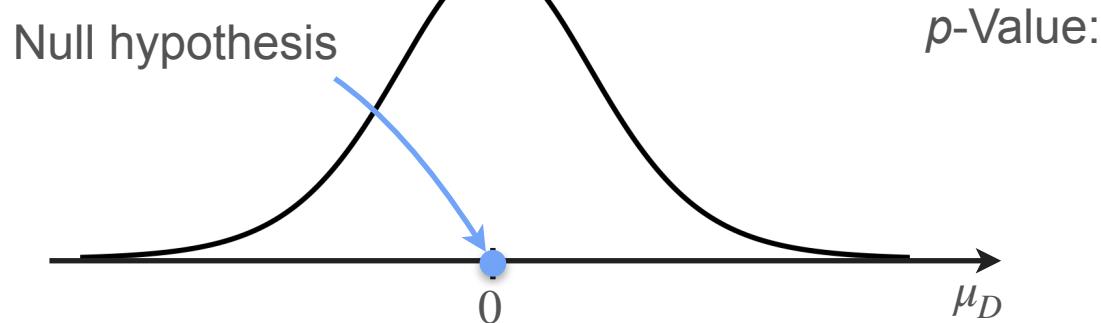
Independent Two-Sample t -Test: Right Tailed Test



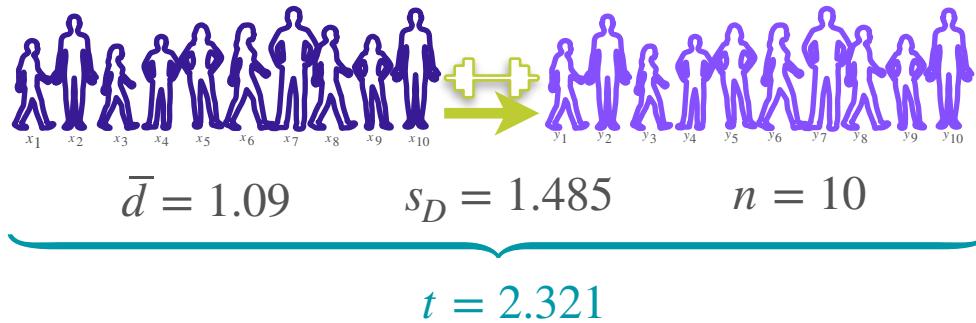
$$H_0 : \mu_D = 0 \text{ vs. } H_1 : \mu_D > 0$$

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Independent Two-Sample t -Test: Right Tailed Test



$$H_0 : \mu_D = 0 \text{ vs. } H_1 : \mu_D > 0$$

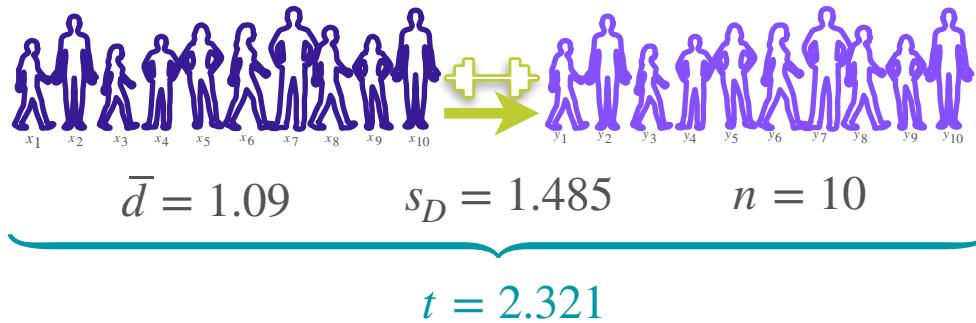
$$\alpha = 0.05$$

$$\text{If } H_0 \text{ is true: } T = \frac{\bar{D} - 0}{S_D / \sqrt{10}} \sim t_{10-1}$$

Null hypothesis

$$p\text{-Value: } P(T > 2.321 \mid \mu_D = 0)$$

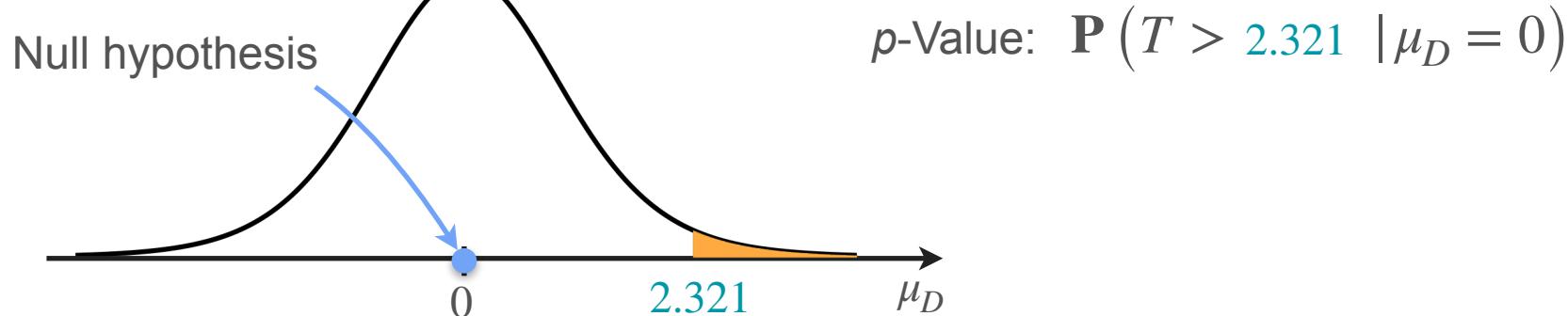
Independent Two-Sample t -Test: Right Tailed Test



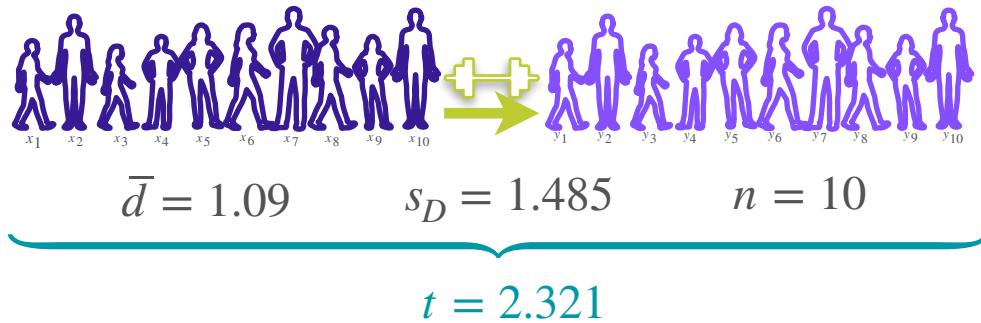
$$H_0 : \mu_D = 0 \text{ vs. } H_1 : \mu_D > 0$$

$$\alpha = 0.05$$

$$\text{If } H_0 \text{ is true: } T = \frac{\bar{D} - 0}{S_D / \sqrt{10}} \sim t_{10-1}$$



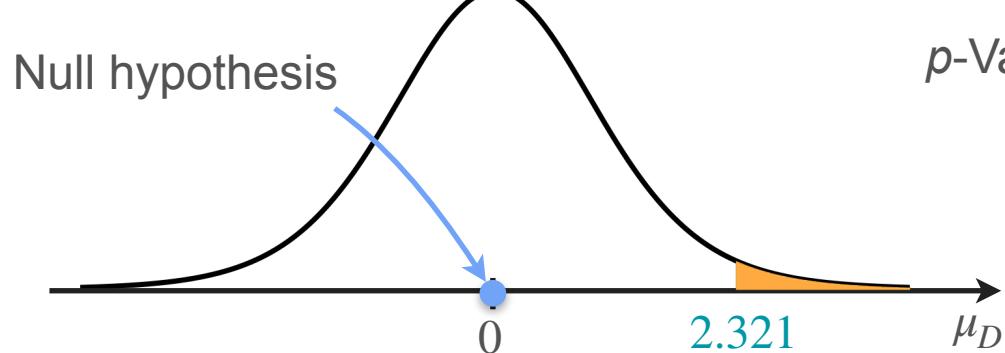
Independent Two-Sample t -Test: Right Tailed Test



$$H_0 : \mu_D = 0 \text{ vs. } H_1 : \mu_D > 0$$

$$\alpha = 0.05$$

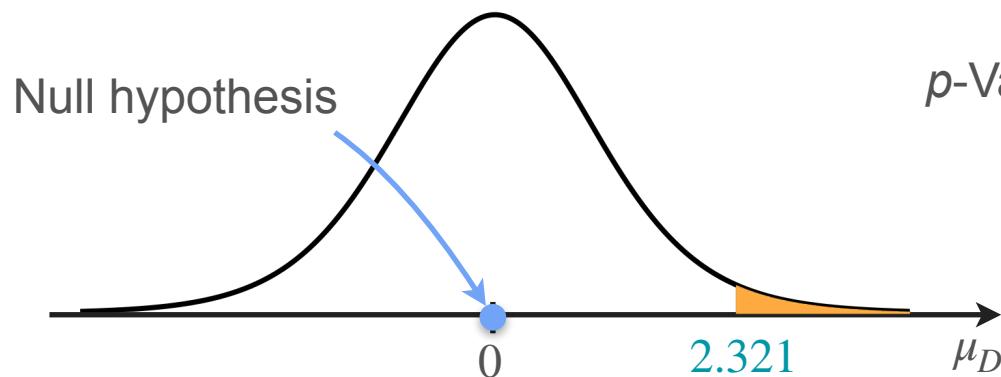
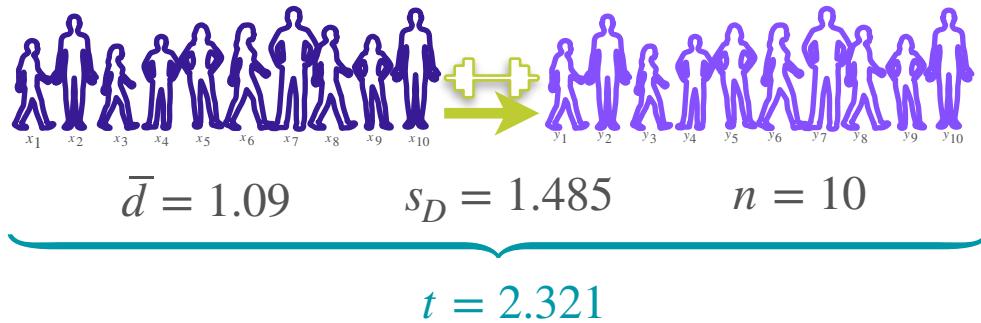
$$\text{If } H_0 \text{ is true: } T = \frac{\bar{D} - 0}{S_D / \sqrt{10}} \sim t_{10-1}$$



$$p\text{-Value: } P(T > 2.321 \mid \mu_D = 0)$$

$$= 0.0227$$

Independent Two-Sample t -Test: Right Tailed Test



$$H_0 : \mu_D = 0 \text{ vs. } H_1 : \mu_D > 0$$

$$\alpha = 0.05$$

$$\text{If } H_0 \text{ is true: } T = \frac{\bar{D} - 0}{S_D / \sqrt{10}} \sim t_{10-1}$$

$$p\text{-Value: } P(T > 2.321 \mid \mu_D = 0)$$

$$= 0.0227 < 0.05$$

\Rightarrow Reject H_0 (and accept H_1)
(with a 5% significance level)



DeepLearning.AI

Hypothesis Testing

ML Application: A/B testing

A/B Testing: Purchase Amount

A/B Testing: Purchase Amount



Design A



Design B

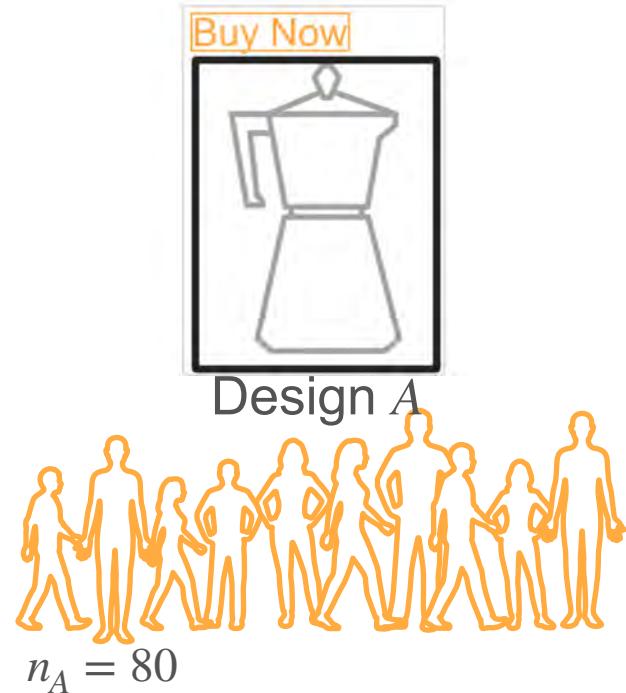
A/B Testing: Purchase Amount



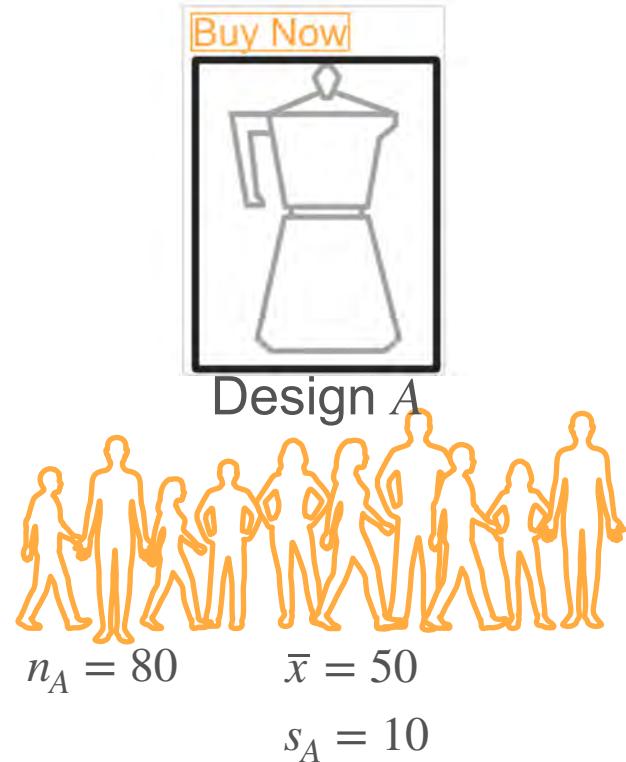
A/B Testing: Purchase Amount



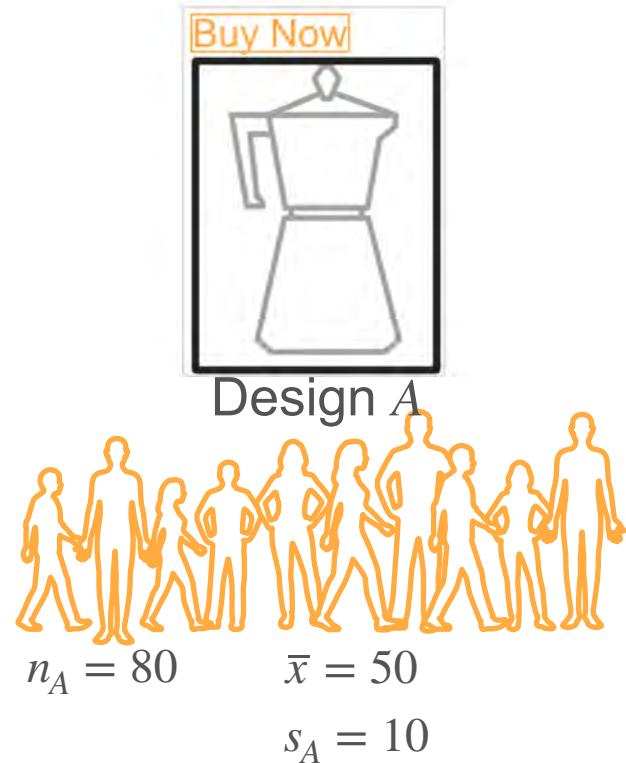
A/B Testing: Purchase Amount



A/B Testing: Purchase Amount



A/B Testing: Purchase Amount



A/B Testing: Purchase Amount



Design A



$$n_A = 80$$

$$\bar{x} = 50$$

$$n_B = 20 \quad \bar{y} = 55$$

$$s_A = 10$$



Design B



$$t = \frac{(\bar{x} - \bar{y}) - 0}{\sqrt{\frac{s_A^2}{n_A} + \frac{s_B^2}{n_B}}}$$

A/B Testing: Purchase Amount



Design A



$n_A = 80$

$\bar{x} = 50$

$n_B = 20$ $\bar{y} = 55$

$s_A = 10$



Design B



$$H_0 : \mu_A = \mu_B \text{ vs. } H_1 : \mu_A < \mu_B$$

$$t = \frac{(\bar{x} - \bar{y}) - 0}{\sqrt{\frac{s_A^2}{n_A} + \frac{s_B^2}{n_B}}}$$

A/B Testing: Purchase Amount



Design A



$$n_A = 80$$

$$\bar{x} = 50$$

$$n_B = 20$$

$$s_A = 10$$



Design B

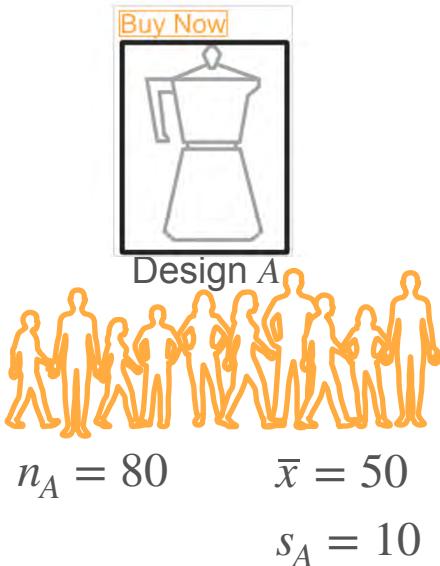


$$s_B = 15$$

$$H_0 : \mu_A - \mu_B = 0 \text{ vs. } H_1 : \mu_A - \mu_B < 0$$

$$t = \frac{(\bar{x} - \bar{y}) - 0}{\sqrt{\frac{s_A^2}{n_A} + \frac{s_B^2}{n_B}}}$$

A/B Testing: Purchase Amount



$$H_0 : \mu_A - \mu_B = 0 \text{ vs. } H_1 : \mu_A - \mu_B < 0$$

$$\alpha = 0.05$$

$$t = \frac{(\bar{x} - \bar{y}) - 0}{\sqrt{\frac{s_A^2}{n_A} + \frac{s_B^2}{n_B}}}$$

A/B Testing: Purchase Amount



Design A



$$n_A = 80$$

$$\bar{x} = 50$$

$$n_B = 20$$

$$s_A = 10$$

$$\bar{y} = 55$$

$$s_B = 15$$



Design B



$$H_0 : \mu_A - \mu_B = 0 \text{ vs. } H_1 : \mu_A - \mu_B < 0$$

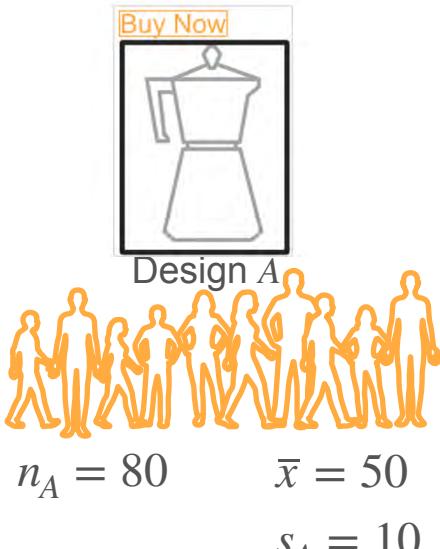
$$\alpha = 0.05$$

$$t = \frac{(\bar{x} - \bar{y}) - 0}{\sqrt{\frac{s_A^2}{n_A} + \frac{s_B^2}{n_B}}}$$

$$X \sim \mathcal{N}(\mu_A, \sigma_A^2)$$

$$Y \sim \mathcal{N}(\mu_B, \sigma_B^2)$$

A/B Testing: Purchase Amount



$$X \sim \mathcal{N}(\mu_A, \sigma_A^2)$$

$$Y \sim \mathcal{N}(\mu_B, \sigma_B^2)$$

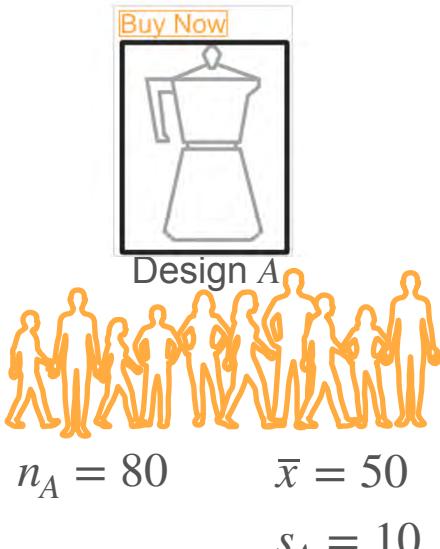
$$H_0 : \mu_A - \mu_B = 0 \text{ vs. } H_1 : \mu_A - \mu_B < 0$$

$$\alpha = 0.05$$

If H_0 is true: $T = \frac{(\bar{X} - \bar{Y}) - 0}{\sqrt{\frac{s_A^2}{10} + \frac{s_B^2}{10}}} \sim t_{23.38}$

$$t = \frac{(\bar{X} - \bar{Y}) - 0}{\sqrt{\frac{s_A^2}{n_A} + \frac{s_B^2}{n_B}}}$$

A/B Testing: Purchase Amount



$$X \sim \mathcal{N}(\mu_A, \sigma_A^2)$$

$$Y \sim \mathcal{N}(\mu_B, \sigma_B^2)$$

$$H_0 : \mu_A - \mu_B = 0 \text{ vs. } H_1 : \mu_A - \mu_B < 0$$

$$\alpha = 0.05$$

$$\text{If } H_0 \text{ is true: } T = \frac{(\bar{X} - \bar{Y}) - 0}{\sqrt{\frac{S_A^2}{10} + \frac{S_B^2}{10}}} \sim t_{23.38}$$

$$t = \frac{(50 - 55) - 0}{\sqrt{\frac{10^2}{80} + \frac{15^2}{20}}}$$

$$-1.482$$

A/B Testing: Purchase Amount



$$X \sim \mathcal{N}(\mu_A, \sigma_A^2)$$

$$Y \sim \mathcal{N}(\mu_B, \sigma_B^2)$$

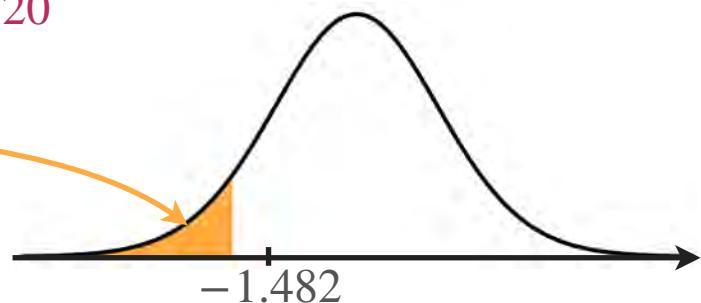
$$H_0 : \mu_A - \mu_B = 0 \text{ vs. } H_1 : \mu_A - \mu_B < 0$$

$$\alpha = 0.05$$

If H_0 is true: $T = \frac{(\bar{X} - \bar{Y}) - 0}{\sqrt{\frac{s_A^2}{n_A} + \frac{s_B^2}{n_B}}} \sim t_{23.38}$

$$t = \frac{(50 - 55) - 0}{\sqrt{\frac{10^2}{80} + \frac{15^2}{20}}} = -1.482$$

$p\text{-Value:}$



A/B Testing: Purchase Amount



$$X \sim \mathcal{N}(\mu_A, \sigma_A^2)$$



$$Y \sim \mathcal{N}(\mu_B, \sigma_B^2)$$

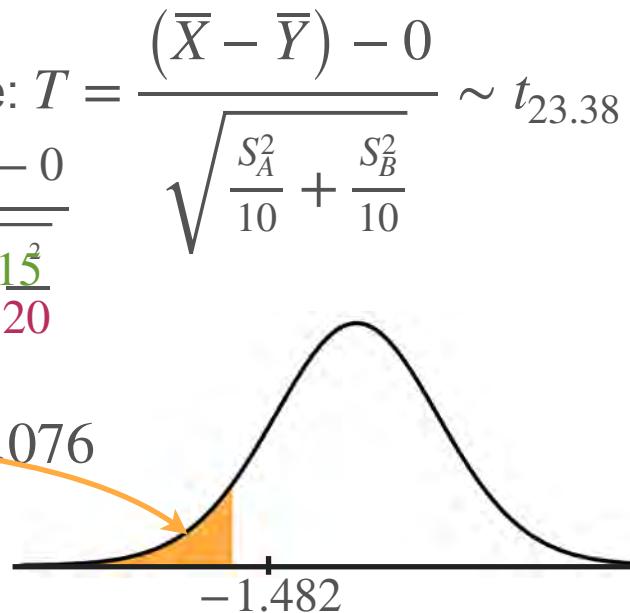
$$H_0 : \mu_A - \mu_B = 0 \text{ vs. } H_1 : \mu_A - \mu_B < 0$$

$$\alpha = 0.05$$

If H_0 is true: $T = \frac{(\bar{X} - \bar{Y}) - 0}{\sqrt{\frac{s_A^2}{n_A} + \frac{s_B^2}{n_B}}} \sim t_{23.38}$

$$t = \frac{(50 - 55) - 0}{\sqrt{\frac{10^2}{80} + \frac{15^2}{20}}} = -1.482$$

p-Value: 0.076



A/B Testing: Purchase Amount



Design A



$$n_A = 80$$

$$\bar{x} = 50$$

$$n_B = 20$$

$$s_A = 10$$

$$X \sim \mathcal{N}(\mu_A, \sigma_A^2)$$

$$Y \sim \mathcal{N}(\mu_B, \sigma_B^2)$$



Design B



$$\bar{y} = 55$$

$$s_B = 15$$

$$H_0 : \mu_A - \mu_B = 0 \text{ vs. } H_1 : \mu_A - \mu_B < 0$$

$$\alpha = 0.05$$

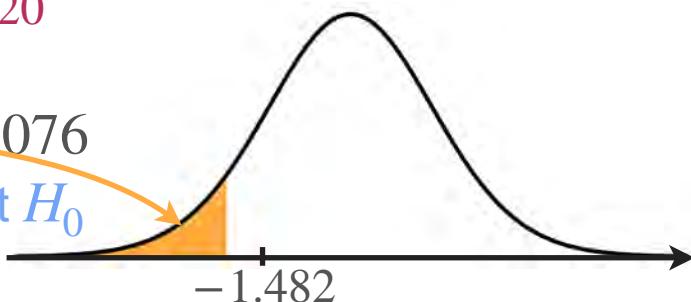
$$\text{If } H_0 \text{ is true: } T = \frac{(\bar{X} - \bar{Y}) - 0}{\sqrt{\frac{S_A^2}{10} + \frac{S_B^2}{10}}} \sim t_{23.38}$$

$$t = \frac{(50 - 55) - 0}{\sqrt{\frac{10^2}{80} + \frac{15^2}{20}}}$$

$$-1.482$$

$$p\text{-Value: } 0.076$$

Don't reject H_0



A/B Testing and t -Tests

A/B Testing and t -Tests

A/B testing is a methodology for comparing two variations (A/B)

A/B Testing and t -Tests

A/B testing is a methodology for comparing two variations (A/B)

Propose variations
(A/B)

A/B Testing and t -Tests

A/B testing is a methodology for comparing two variations (A/B)



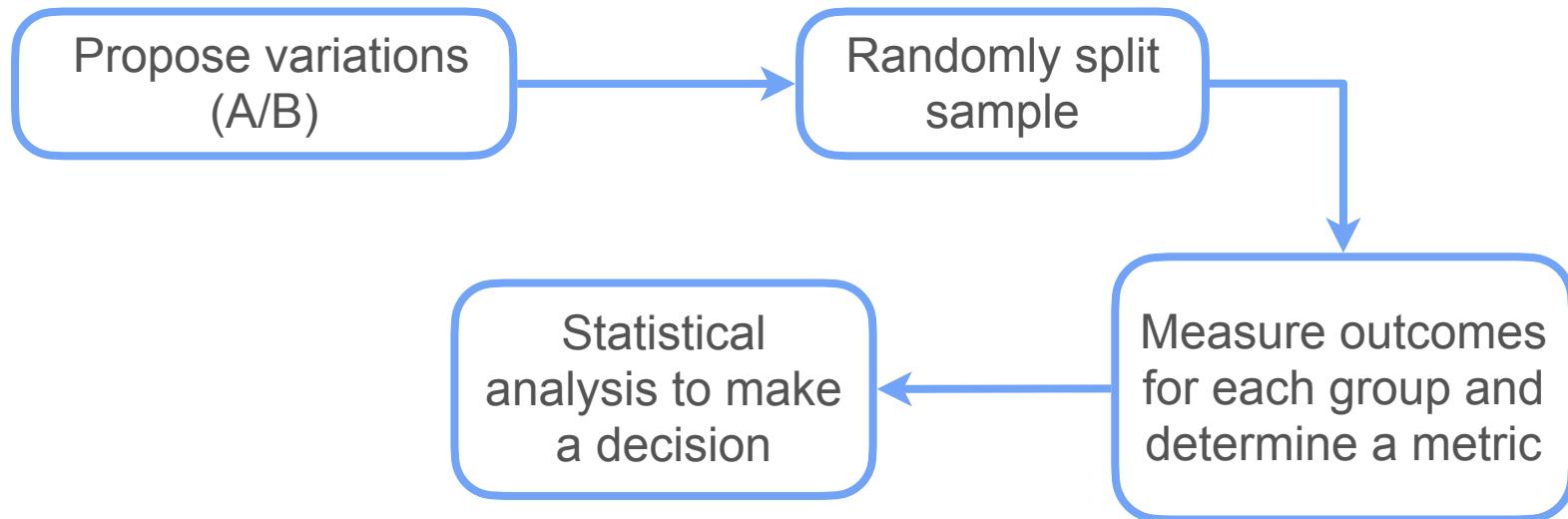
A/B Testing and t -Tests

A/B testing is a methodology for comparing two variations (A/B)



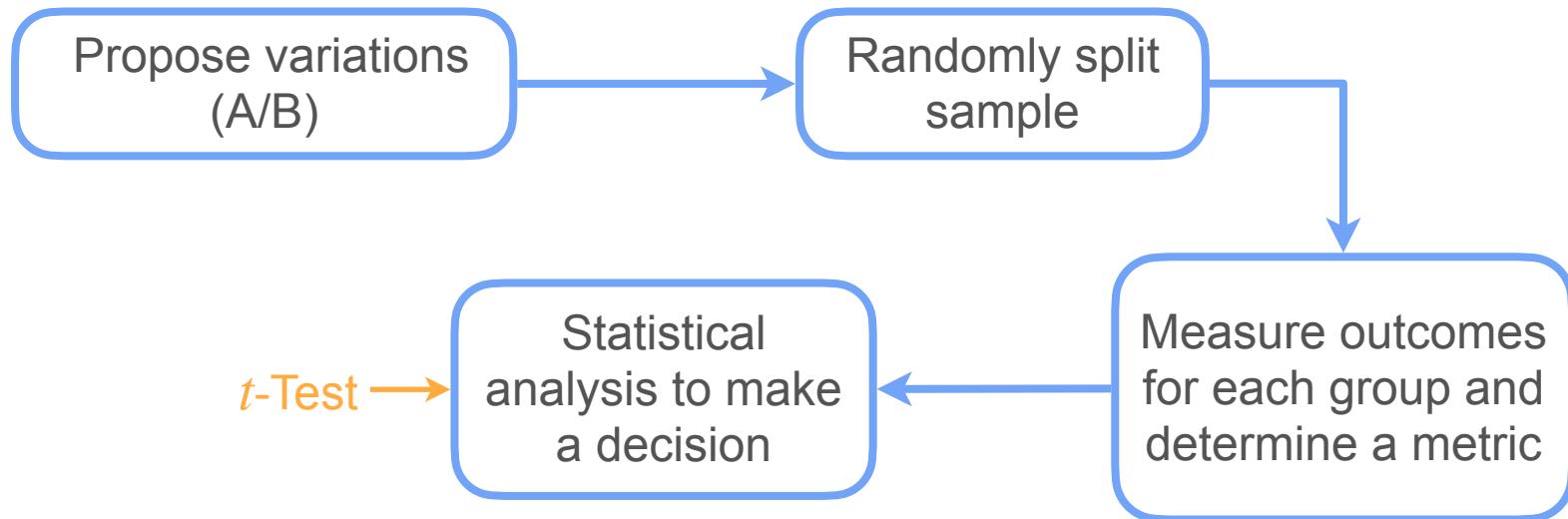
A/B Testing and t -Tests

A/B testing is a methodology for comparing two variations (A/B)



A/B Testing and t -Tests

A/B testing is a methodology for comparing two variations (A/B)



A/B Testing: Conversion Rates

A/B Testing: Conversion Rates



A/B Testing: Conversion Rates



Design A

Higher conversion rates?



Design B

A/B Testing: Conversion Rates



Design A
Higher conversion rates?



Design B



A/B Testing: Conversion Rates



Design A



Design B

A/B Testing: Conversion Rates



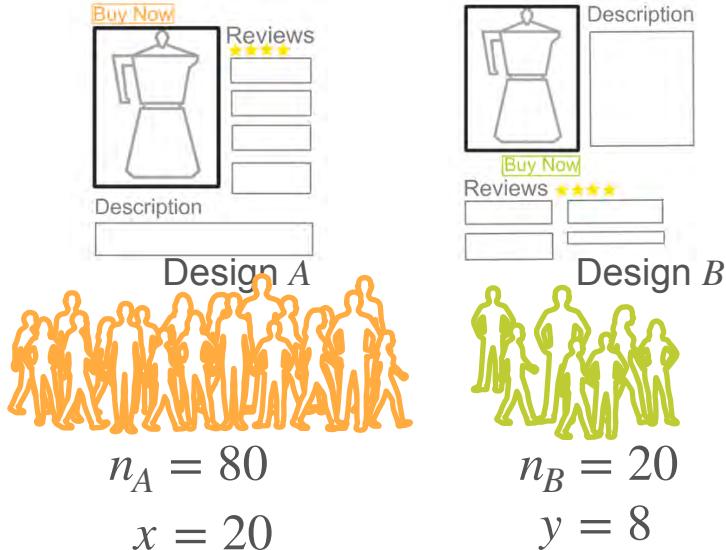
A/B Testing: Conversion Rates



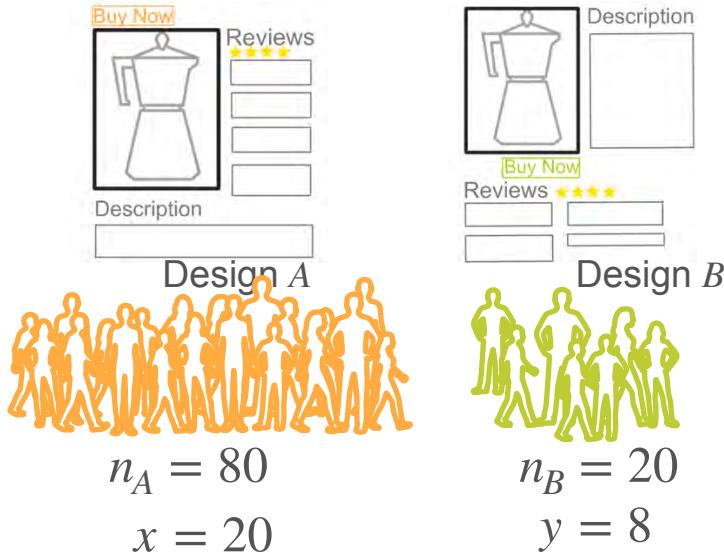
A/B Testing: Conversion Rates



A/B Testing: Conversion Rates



A/B Testing: Conversion Rates



$$H_0 : p_A - p_B = 0 \text{ vs. } H_1 : p_A - p_B < 0$$

p_A = Conversion rate from Design A

p_B = Conversion rate from Design B

A/B Testing: Conversion Rates



$$n_A = 80$$

$$x = 20$$



$$n_B = 20$$

$$y = 8$$

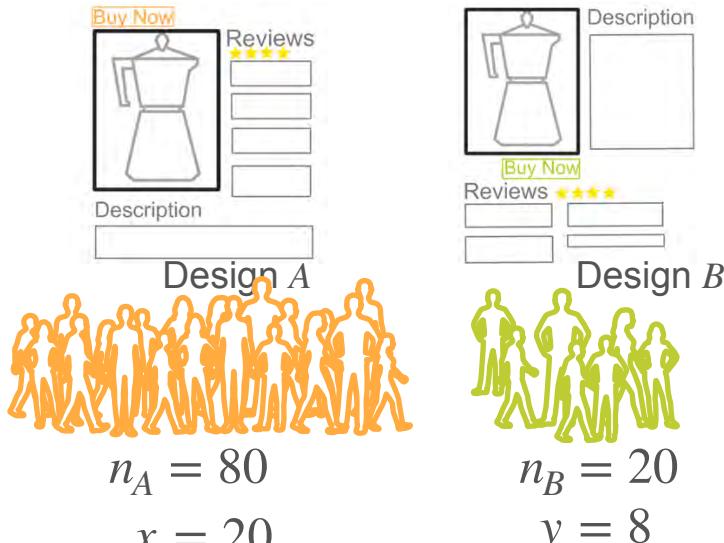
$$H_0 : p_A - p_B = 0 \text{ vs. } H_1 : p_A - p_B < 0$$

p_A = Conversion rate from Design A

p_B = Conversion rate from Design B

$$\alpha = 0.05$$

A/B Testing: Conversion Rates



$$X \sim \text{Binomial}(n_A, p_A) \quad Y \sim \text{Binomial}(n_B, p_B)$$

$$H_0 : p_A - p_B = 0 \text{ vs. } H_1 : p_A - p_B < 0$$

p_A = Conversion rate from Design A

p_B = Conversion rate from Design B

$$\alpha = 0.05$$

A/B Testing: Conversion Rates

A/B Testing: Conversion Rates

Statistic?

A/B Testing: Conversion Rates

Statistic?

Law of large numbers

$$\frac{X}{n_A} \rightarrow p_A$$

$$\frac{Y}{n_B} \rightarrow p_B$$

A/B Testing: Conversion Rates

Statistic?

Law of large numbers

$$\frac{X}{n_A} \rightarrow p_A$$



$$\frac{X}{n_A} \sim \mathcal{N}\left(p_A, \frac{p_A(1-p_A)}{n_A}\right)$$

$$\frac{Y}{n_B} \rightarrow p_B$$

C.L.T.



$$\frac{Y}{n_B} \sim \mathcal{N}\left(p_B, \frac{p_B(1-p_B)}{n_B}\right)$$

A/B Testing: Conversion Rates

Statistic?

$$\frac{X}{n_A} \stackrel{a}{\sim} \mathcal{N} \left(p_A, \frac{p_A(1 - p_A)}{n_A} \right)$$

$$\frac{Y}{n_B} \stackrel{a}{\sim} \mathcal{N} \left(p_B, \frac{p_B(1 - p_B)}{n_B} \right)$$

A/B Testing: Conversion Rates

Statistic?

$$\left. \begin{aligned} \frac{X}{n_A} &\stackrel{a}{\sim} \mathcal{N} \left(p_A, \frac{p_A(1-p_A)}{n_A} \right) \\ \frac{Y}{n_B} &\stackrel{a}{\sim} \mathcal{N} \left(p_B, \frac{p_B(1-p_B)}{n_B} \right) \end{aligned} \right\}$$

A/B Testing: Conversion Rates

Statistic?

$$\left. \begin{array}{l} \frac{X}{n_A} \stackrel{a}{\sim} \mathcal{N} \left(p_A, \frac{p_A(1-p_A)}{n_A} \right) \\ \frac{Y}{n_B} \stackrel{a}{\sim} \mathcal{N} \left(p_B, \frac{p_B(1-p_B)}{n_B} \right) \end{array} \right\} \frac{X}{n_A} - \frac{Y}{n_B} \rightarrow p_A - p_B$$

A/B Testing: Conversion Rates

Statistic?

$$\left. \begin{array}{l} \frac{X}{n_A} \stackrel{a}{\sim} \mathcal{N} \left(p_A, \frac{p_A(1-p_A)}{n_A} \right) \\ \frac{Y}{n_B} \stackrel{a}{\sim} \mathcal{N} \left(p_B, \frac{p_B(1-p_B)}{n_B} \right) \end{array} \right\} \quad \begin{array}{l} \frac{X}{n_A} - \frac{Y}{n_B} \stackrel{a}{\sim} \mathcal{N} \left(p_A - p_B, \frac{p_A(1-p_A)}{n_A} + \frac{p_B(1-p_B)}{n_B} \right) \\ \frac{X}{n_A} - \frac{Y}{n_B} \rightarrow p_A - p_B \end{array}$$

A/B Testing: Conversion Rates

Statistic?

$$\left. \begin{array}{l} \frac{X}{n_A} \stackrel{a}{\sim} \mathcal{N} \left(p_A, \frac{p_A(1-p_A)}{n_A} \right) \\ \frac{Y}{n_B} \stackrel{a}{\sim} \mathcal{N} \left(p_B, \frac{p_B(1-p_B)}{n_B} \right) \end{array} \right\} \frac{X}{n_A} - \frac{Y}{n_B} \rightarrow p_A - p_B \quad \begin{aligned} \frac{X}{n_A} - \frac{Y}{n_B} &\stackrel{a}{\sim} \mathcal{N} \left(p_A - p_B, \frac{p_A(1-p_A)}{n_A} + \frac{p_B(1-p_B)}{n_B} \right) \\ \Downarrow \\ \frac{\left(\frac{X}{n_A} - \frac{Y}{n_B} \right) - (p_A - p_B)}{\sqrt{\frac{p_A(1-p_A)}{n_A} + \frac{p_B(1-p_B)}{n_B}}} &\stackrel{a}{\sim} \mathcal{N} (0, 1^2) \end{aligned}$$

A/B Testing: Conversion Rates

$$\frac{\left(\frac{X}{n_A} - \frac{Y}{n_B} \right) - (p_A - p_B)}{\sqrt{\frac{p_A(1-p_A)}{n_A} + \frac{p_B(1-p_B)}{n_B}}} \stackrel{a}{\sim} \mathcal{N}(0,1)$$

A/B Testing: Conversion Rates

If H_0 is true $\Rightarrow p_A = p_B = p$

$$\frac{\left(\frac{X}{n_A} - \frac{Y}{n_B}\right) - (p_A - p_B)}{\sqrt{\frac{p_A(1-p_A)}{n_A} + \frac{p_B(1-p_B)}{n_B}}} \stackrel{a}{\sim} \mathcal{N}(0,1)$$

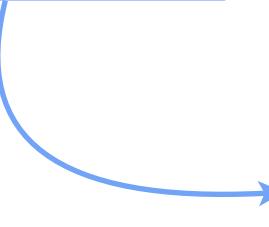
A/B Testing: Conversion Rates

If H_0 is true $\Rightarrow p_A = p_B = p$

$$\frac{\left(\frac{X}{n_A} - \frac{Y}{n_B}\right) - (p - p)}{\sqrt{\frac{p(1-p)}{n_A} + \frac{p(1-p)}{n_B}}} \stackrel{a}{\sim} \mathcal{N}(0, 1^2)$$

A/B Testing: Conversion Rates

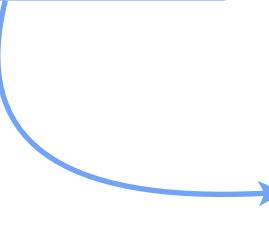
If H_0 is true $\Rightarrow p_A = p_B = p$

$$\frac{\left(\frac{X}{n_A} - \frac{Y}{n_B}\right) - (p - p)}{\sqrt{\frac{p(1-p)}{n_A} + \frac{p(1-p)}{n_B}}} \stackrel{a}{\sim} \mathcal{N}(0, 1^2)$$

$$= p(1-p) \left(\frac{1}{n_A} + \frac{1}{n_B} \right)$$

A/B Testing: Conversion Rates

If H_0 is true $\Rightarrow p_A = p_B = p$

$$\frac{\left(\frac{X}{n_A} - \frac{Y}{n_B}\right) - (p - p)}{\sqrt{\frac{p(1-p)}{n_A} + \frac{p(1-p)}{n_B}}} \stackrel{a}{\sim} \mathcal{N}(0, 1^2)$$



$$= p(1-p) \left(\frac{1}{n_A} + \frac{1}{n_B} \right) = p(1-p)(n_A + n_B) \frac{1}{n_A n_B}$$

A/B Testing: Conversion Rates

If H_0 is true $\Rightarrow p_A = p_B = p$

$$\frac{\left(\frac{X}{n_A} - \frac{Y}{n_B}\right) - (p - p)}{\sqrt{\frac{p(1-p)}{n_A} + \frac{p(1-p)}{n_B}}} \stackrel{a}{\sim} \mathcal{N}(0, 1^2)$$

A/B Testing: Conversion Rates

If H_0 is true $\Rightarrow p_A = p_B = p$

$$\frac{\left(\frac{X}{n_A} - \frac{Y}{n_B}\right) - (p - p)}{\sqrt{\frac{p(1-p)}{n_A} + \frac{p(1-p)}{n_B}}} \stackrel{a}{\sim} \mathcal{N}(0, 1^2)$$


A/B Testing: Conversion Rates

If H_0 is true $\Rightarrow p_A = p_B = p$

$$\frac{\left(\frac{X}{n_A} - \frac{Y}{n_B}\right) - (p - p)}{\sqrt{\frac{p(1-p)}{n_A} + \frac{p(1-p)}{n_B}}} \stackrel{a}{\sim} \mathcal{N}(0, 1^2) \longrightarrow \frac{\left(\frac{X}{n_A} - \frac{Y}{n_B}\right) - 0}{\sqrt{(n_A + n_B)p(1-p)}} \sqrt{n_A n_B} \stackrel{a}{\sim} \mathcal{N}(0, 1^2)$$

A/B Testing: Conversion Rates

If H_0 is true $\Rightarrow p_A = p_B = p$

$$\frac{\left(\frac{X}{n_A} - \frac{Y}{n_B}\right) - (p - p)}{\sqrt{\frac{p(1-p)}{n_A} + \frac{p(1-p)}{n_B}}} \stackrel{a}{\sim} \mathcal{N}(0, 1^2) \longrightarrow \frac{\left(\frac{X}{n_A} - \frac{Y}{n_B}\right) - 0}{\sqrt{(n_A + n_B)p(1-p)}} \sqrt{n_A n_B} \stackrel{a}{\sim} \mathcal{N}(0, 1^2)$$

But you don't know p



Replace it by estimation! $\hat{p} = \frac{X + Y}{n_A + n_B}$

A/B Testing: Conversion Rates

If H_0 is true $\Rightarrow p_A = p_B = p$

$$\frac{\left(\frac{X}{n_A} - \frac{Y}{n_B}\right) - (p - p)}{\sqrt{\frac{p(1-p)}{n_A} + \frac{p(1-p)}{n_B}}} \stackrel{a}{\sim} \mathcal{N}(0, 1^2) \longrightarrow \frac{\left(\frac{X}{n_A} - \frac{Y}{n_B}\right) - 0}{\sqrt{(n_A + n_B)p(1-p)}} \sqrt{n_A n_B} \stackrel{a}{\sim} \mathcal{N}(0, 1^2)$$

But you don't know p

Replace it by estimation! $\hat{p} = \frac{X + Y}{n_A + n_B}$

Test statistic

$$\frac{\left(\frac{X}{n_A} - \frac{Y}{n_B}\right) - 0}{\sqrt{(X + Y)\left(1 - \frac{X + Y}{n_A + n_B}\right)}} \sqrt{n_A n_B} \stackrel{a}{\sim} \mathcal{N}(0, 1^2)$$

A/B Testing: Conversion Rates

A/B Testing: Conversion Rates



A/B Testing: Conversion Rates



Design A



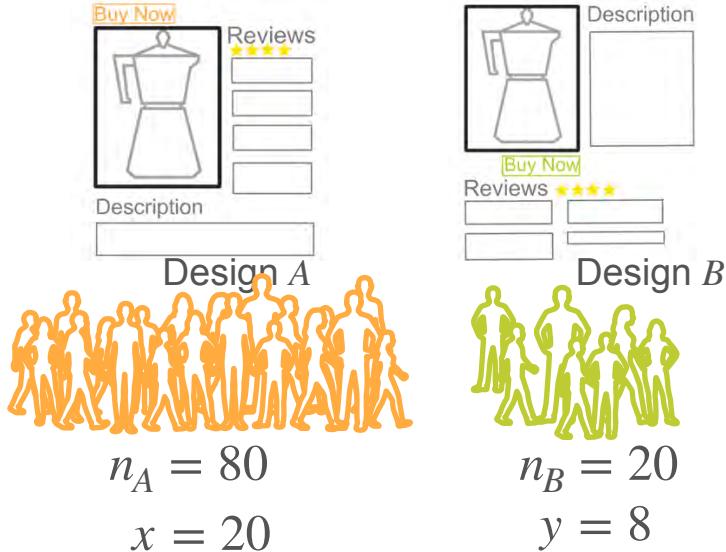
Design B



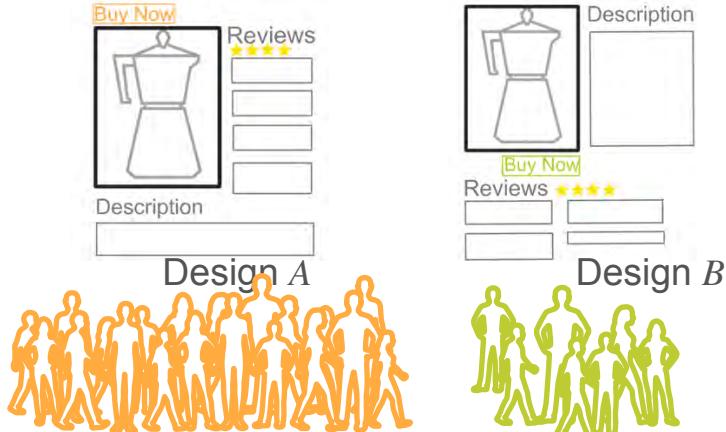
$$n_A = 80$$

$$x = 20$$

A/B Testing: Conversion Rates



A/B Testing: Conversion Rates



$$n_A = 80$$

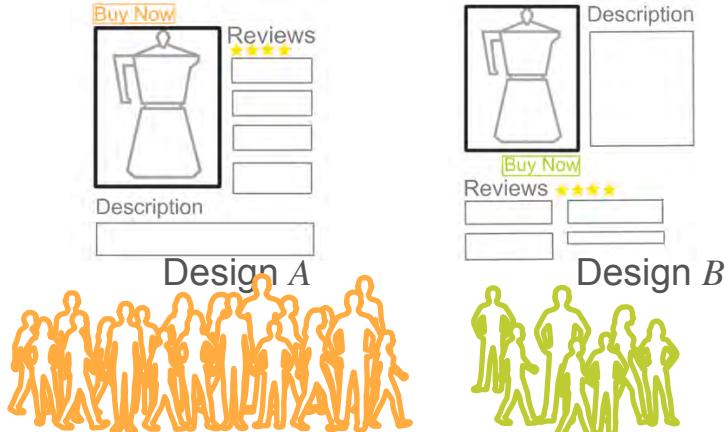
$$x = 20$$

$$n_B = 20$$

$$y = 8$$

$$X \sim \text{Binomial}(n_A, p_A) \quad Y \sim \text{Binomial}(n_B, p_B)$$

A/B Testing: Conversion Rates



$$n_A = 80$$

$$x = 20$$

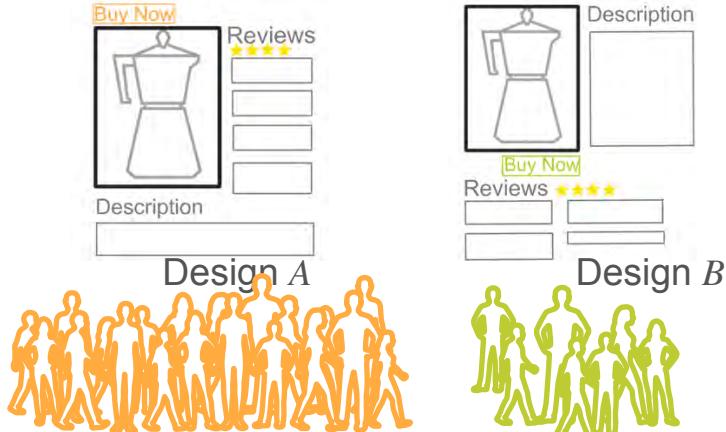
$$n_B = 20$$

$$y = 8$$

$$H_0 : p_A - p_B = 0 \text{ vs. } H_1 : p_A - p_B < 0$$

$$X \sim \text{Binomial}(n_A, p_A) \quad Y \sim \text{Binomial}(n_B, p_B)$$

A/B Testing: Conversion Rates



$$n_A = 80$$

$$x = 20$$

$$n_B = 20$$

$$y = 8$$

$$H_0 : p_A - p_B = 0 \text{ vs. } H_1 : p_A - p_B < 0$$

$$\alpha = 0.05$$

$$X \sim \text{Binomial}(n_A, p_A) \quad Y \sim \text{Binomial}(n_B, p_B)$$

A/B Testing: Conversion Rates



$$n_A = 80$$

$$x = 20$$



$$n_B = 20$$

$$y = 8$$

$$X \sim \text{Binomial}(n_A, p_A) \quad Y \sim \text{Binomial}(n_B, p_B)$$

$$H_0 : p_A - p_B = 0 \text{ vs. } H_1 : p_A - p_B < 0$$

$$\alpha = 0.05 \quad \text{If } H_0 \text{ is true } \Rightarrow p_A = p_B = p$$

$$Z = \frac{\left(\frac{X}{n_A} - \frac{Y}{n_B} \right) - 0}{\sqrt{(X+Y)\left(1 - \frac{X+Y}{n_A+n_B}\right)}} \sqrt{n_A n_B} \sim \mathcal{N}(0, 1^2)$$

A/B Testing: Conversion Rates



$$n_A = 80$$

$$x = 20$$

$$X \sim \text{Binomial}(n_A, p_A) \quad Y \sim \text{Binomial}(n_B, p_B)$$



$$n_B = 20$$

$$y = 8$$

$$H_0 : p_A - p_B = 0 \text{ vs. } H_1 : p_A - p_B < 0$$

$$\alpha = 0.05 \quad \text{If } H_0 \text{ is true } \Rightarrow p_A = p_B = p$$

$$Z = \frac{\left(\frac{X}{n_A} - \frac{Y}{n_B} \right) - 0}{\sqrt{(X+Y)\left(1-\frac{X+Y}{n_A+n_B}\right)}} \sqrt{n_A n_B} \sim \mathcal{N}(0, 1^2)$$

$$z = \frac{\left(\frac{20}{80} - \frac{8}{20} \right) - 0}{\sqrt{(20+8)\left(1-\frac{20+8}{80+20}\right)}} \sqrt{\frac{80}{20}}$$

$$z = -1.336$$

A/B Testing: Conversion Rates



$$X \sim \text{Binomial}(n_A, p_A) \quad Y \sim \text{Binomial}(n_B, p_B)$$

$$H_0 : p_A - p_B = 0 \text{ vs. } H_1 : p_A - p_B < 0$$

$$\alpha = 0.05 \quad \text{If } H_0 \text{ is true } \Rightarrow p_A = p_B = p$$

$$Z = \frac{\left(\frac{X}{n_A} - \frac{Y}{n_B} \right) - 0}{\sqrt{(X+Y)\left(1 - \frac{X+Y}{n_A+n_B}\right)}} \sqrt{n_A n_B} \sim \mathcal{N}(0, 1^2)$$

-2.07

A/B Testing: Conversion Rates



$$n_A = 80$$

$$x = 20$$



$$n_B = 20$$

$$y = 8$$

$$X \sim \text{Binomial}(n_A, p_A) \quad Y \sim \text{Binomial}(n_B, p_B)$$

$$H_0 : p_A - p_B = 0 \text{ vs. } H_1 : p_A - p_B < 0$$

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$$z = -1.336$$



A/B Testing: Conversion Rates



$$n_A = 80$$

$$x = 20$$



$$n_B = 20$$

$$y = 8$$

$$X \sim \text{Binomial}(n_A, p_A) \quad Y \sim \text{Binomial}(n_B, p_B)$$

$$H_0 : p_A - p_B = 0 \text{ vs. } H_1 : p_A - p_B < 0$$

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$$Z = \frac{\left(\frac{X}{n_A} - \frac{Y}{n_B} \right) - 0}{\sqrt{(X+Y)\left(1 - \frac{X+Y}{n_A+n_B}\right)}} \sqrt{n_A n_B} \sim \mathcal{N}(0, 1^2)$$

$$z = -1.336$$

p-value



A/B Testing: Conversion Rates



$$n_A = 80$$

$$x = 20$$

$$X \sim \text{Binomial}(n_A, p_A) \quad Y \sim \text{Binomial}(n_B, p_B)$$



$$n_B = 20$$

$$y = 8$$

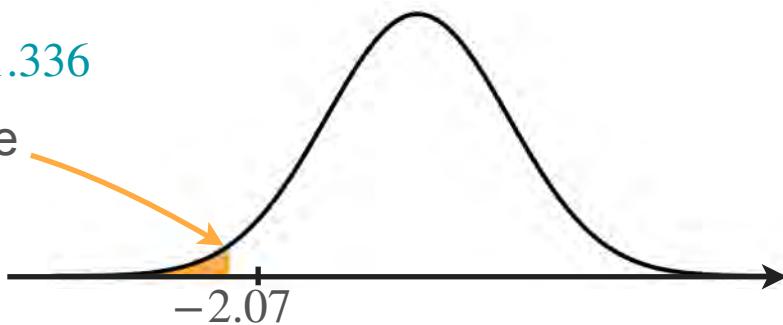
$$H_0 : p_A - p_B = 0 \text{ vs. } H_1 : p_A - p_B < 0$$

$$\alpha = 0.05 \quad \text{If } H_0 \text{ is true } \Rightarrow p_A = p_B = p$$

$$Z = \frac{\left(\frac{X}{n_A} - \frac{Y}{n_B} \right) - 0}{\sqrt{(X+Y)\left(1 - \frac{X+Y}{n_A+n_B}\right)}} \sqrt{n_A n_B} \sim \mathcal{N}(0, 1^2)$$

$$z = -1.336$$

p-value



A/B Testing: Conversion Rates



$$n_A = 80$$

$$x = 20$$

$$X \sim \text{Binomial}(n_A, p_A) \quad Y \sim \text{Binomial}(n_B, p_B)$$



$$n_B = 20$$

$$y = 8$$

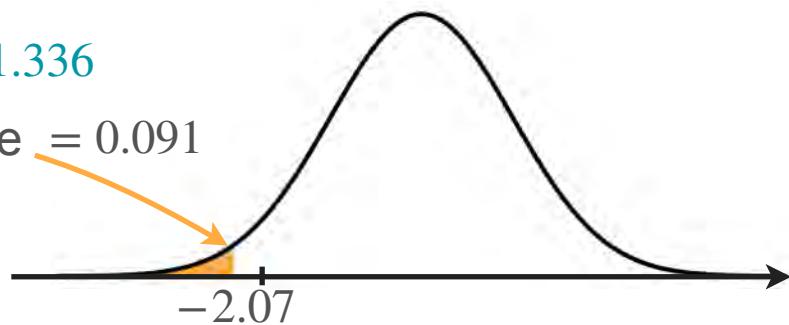
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$$z = -1.336$$

$$p\text{-value} = 0.091$$



A/B Testing: Conversion Rates



$$n_A = 80$$

$$x = 20$$

$$X \sim \text{Binomial}(n_A, p_A) \quad Y \sim \text{Binomial}(n_B, p_B)$$



$$n_B = 20$$

$$y = 8$$

$$H_0 : p_A - p_B = 0 \text{ vs. } H_1 : p_A - p_B < 0$$

$$\alpha = 0.05 \quad \text{If } H_0 \text{ is true} \Rightarrow p_A = p_B = p$$

$$Z = \frac{\left(\frac{X}{n_A} - \frac{Y}{n_B} \right) - 0}{\sqrt{(X+Y)\left(1 - \frac{X+Y}{n_A+n_B}\right)}} \sqrt{n_A n_B} \sim \mathcal{N}(0, 1^2)$$

$$z = -1.336$$

$$p\text{-value} = 0.091$$

Do not reject
 H_0

$$-2.07$$