Introduction to General Relativity - HW5 - 401208729

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I show this by two ways, first by the expression

of Rupo in terms of Connection, and second by using

vectorial notations

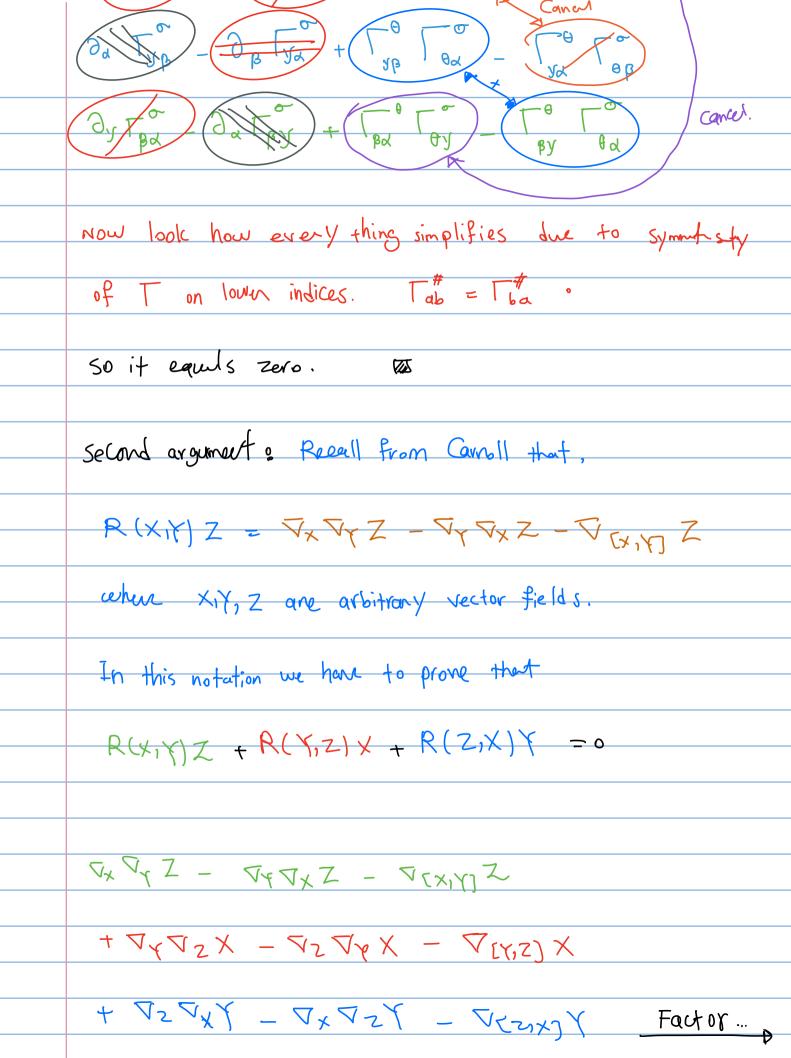
we can multiply by got and raise the r index:

NOTICE that since go is nowher singular, this guarantees that

always Rrapy + Rryap + Rrpy a = 0, other wise we are multiplyn

by zero which ruins on problem.

Rapy + Royas + Ropyx =



Recall that for torsion-free connection (V), we have

$$\frac{[\times, \vee]^r = \times^{\alpha} \partial_{\alpha} \vee^r - \vee^{\alpha} \partial_{\alpha} \times^r = \times^{\alpha} \nabla_{\alpha} \vee^r - \vee^{\alpha} \vee \times \vee^r}{= \nabla_{\alpha} \vee^r - \nabla_{\gamma} \times^r}$$

so rewrite the pavanthes as:

Again use the above rulle.

$$= \left[\times_{1} \left[Y_{1} Z_{1} \right] \right] + \left[Y_{1} \left[Z_{1} X_{1} \right] \right] + \left[Z_{1} \left[X_{1} Y_{1} \right] \right] = 0$$

But this is nothing but Jocobi identity, which vomishes

Using symmetries of Riemann tensor on (3-4 symmetry.)

$$= \frac{1}{3} \left(R_{Y\alpha\beta y} + R_{Y\beta y\alpha} + R_{Yy\alpha\beta} \right) = 0$$

Thanks for the laboring problem! Christoffel symbols: $g_{rv} = (-1, \alpha^2, \alpha^{-2}, \alpha^{-2})$ T. = 1 900 (2.9 + 2.9 ... - 2.9 ...) (or has to be zero, o = 0) $=-\frac{1}{2}(-1) \times \{3+(-1)+3+(-1)\} = 0$ $\frac{1}{5} = \frac{1}{2} \frac{9}{(-1)} \left(\frac{3}{9} \frac{1}{10} + \frac{3}{10} \frac{1}{10} - \frac{3}{10} - \frac{3}{10} \frac{1}{10} \right)$ $= \frac{1}{2} (-1) \left(\frac{3}{10} \frac{1}{10} + \frac{3}{10} \frac{1}{10} - \frac{3}{10} - \frac{3}{10} \frac{1}{10} \frac{1}{10} + \frac{3}{10} \frac{1}{10} \right)$ = aa S; -ik = 1 gio (digor + Ok gjo - Jodile) Note that since alt) is independent of positions for o = i all partial derivatives vanish Γ.j = Γj° = ½ gj° (δ; g.o + δ. gj° - δο gj°) $= \frac{1}{2} \times \frac{1}{\alpha^2} \times \left(\circ + \partial_{+}(\delta_{ji} \alpha^2) - \circ \right)$ = 1 x 2xaa 8; = a si.

other symbols vanish, also check by mathematica. code.

NOW Riemann Tensor:

As you know, then are 20 distinct Components of Riemann tensor at d=4, also symmetries of Riemann tensor help us not to aventle.

let's see them:

Vanishes for 0=0,1,23!

2 (ad 8 ij)

If i=j then

it for 0=1=j

has contribution,

its' zero othwise,

(for i=j)

$$= \dot{\alpha}^2 + \ddot{\alpha}\alpha - \dot{\alpha}^2 = \alpha \dot{\alpha}$$

(for i ≠i)

This gives Roin, Rozoz, Rozoz,

And by Riemann tensor is symmetries,

NOW R' oil could be found by tensovial properites of

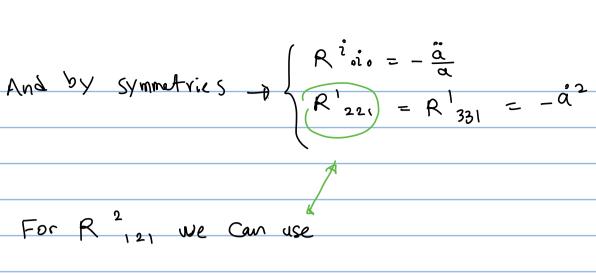
$$=\frac{1}{\alpha^2}\alpha\alpha^2=\frac{\alpha}{\alpha}$$

For R' 112 or R 313 we must ply the relation:

$$R_{21} = \frac{1}{22} - \frac{1}{22} + \frac{1}{22} + \frac{1}{21} + \frac{1}{21} + \frac{1}{22}$$

$$Y = 0$$
No r hos
Contribution

$$= 0 + 0 + qa \times a + 0 = (a)^{2}$$



$$R^{\frac{2}{|2|}} = 9^{2} R_{121} = \frac{1}{a^{2}} R_{2121} = \frac{1}{a^{2}} R_{1221} = \frac{1}{a^{2}}$$

$$\frac{1}{\alpha^2} \frac{9}{\gamma_1} \frac{R^{\frac{7}{221}}}{R^{\frac{2}{112}}} = \frac{9^2}{9^2} \times R^{\frac{1}{221}} = \frac{\alpha^2}{\alpha^2}$$
while $R^{\frac{2}{112}} = -R^{\frac{2}{121}} = -\alpha^2$

By the same logic you can find
$$\begin{cases} R^2_{323} = \mathring{a}^2 \\ R^2_{372} = -\mathring{a}^2 \end{cases}$$

Based on these similar ideas, also

$$\begin{cases} R^3 &= -\alpha^2 \\ R^3 &= \alpha^2 \end{cases}$$

But
$$R^{\frac{3}{232}} = \frac{3}{3} + \frac{3}{22} - \frac{3}{22} + \frac$$

$$=\frac{a^2}{a} \times aa^2 = a^2$$

All other Composits of Riemann tensor vanish, as you can check via morthematica Code.

Since all Rope vanish for P +6 > Rpo is diagonal.

$$R_{11} = R_{101}^{0} + R_{111}^{0} + R_{121}^{2} + R_{131}^{3} =$$

$$a\ddot{a}$$
 $+ o$ $+ \dot{a}^2 + \dot{a}^2 = 2\dot{a}^2 + \alpha\dot{a}$

And Ricci scal is:

$$g^{YV} R_{VV} = -\left(-3\frac{\ddot{a}}{a}\right) + \frac{1}{a^2}\left(6\dot{a}^2 + 3a^{a}\right)$$

$$= \frac{3\ddot{a}}{a} + \frac{6\dot{a}^2 + 3\ddot{a}a}{a^2} = \frac{6\ddot{a}a + 6\dot{a}^2}{a^2}$$

$$=\frac{6}{\alpha^2}\left(\alpha\alpha+\alpha^2\right)$$

Q3. 1 Trv = ? we contract Try with arbitrary X', Y' vector fields, then use the fact that acting Lo on scalar will be like cecting 5-vector on it - By equety both side well do. L3(TrxXxx) = 8PVp(TrxXxxx) Scalor Now use a trick to dy acts like Tr on scot = STp (Tru X M. Y") use the Leibnitz rule of covart derivative. In the seard scenario, act Lg on each tensor, then use the fact that LgX' = [S,X]' = 5 TpX' - XTp5' LS(Tru XYY) = (LSTru) XYY+ Tru(LSX)Y+ TruXILSYV =(L5 Trv) x Y + Trv (5 Tpx Y - x P Tp5 P)

+ Trux"(5 PpY" - YPDP5")

Q4. This question is fairly short!

A tanget vector at a curve x(x) has comports

$$\wedge_{\beta} = \frac{qy}{q} \times_{\beta}(y)$$

Hence, we parallel transport this taget vector, along the

XY(7) Cum itself.

It's a total derivative w. r.t).

$$\frac{97}{97} \wedge \left(\frac{97}{9} \times_{b}\right) = \frac{99}{99} \left(\frac{97}{9} \times_{b}\right) + \frac{1}{2} \times_{b} \frac{97}{9}$$

$$= \frac{32}{6 \lambda^2} \times \beta + \frac{1}{16} \frac{9}{6 \lambda} \frac{9}{6 \lambda} \frac{9}{6 \lambda} = 0$$

So geodeic equession is nothy but parallel trapped of the taget rector to a cure aboy the cure itself.