$D = \{1, x, x^2, x^3, x^4, x^5, y, xy, x^2y, x^3y, x^4y, x^5y \mid x^6 = 1, y^2 = 1, yx = x^5y\}$

الف) $\{1, y\}$ $\{1, xy\}$ $\{1, x^2y\}$ $\{1, x^3y\}$ $\{1, x^4y\}$ $\{1, x^5y\}$ $\{1, x^3\}$ مرتبه 2

$\{1, x^2, x^4\}$ مرتبه 3

$\{1, x, x^2, x^3, x^4, x^5\}$ مرتبه 6

$H = \{1, x^2, x^4\}$

ب)

روش ساده کردن مثلث ها:

معادل رابطه مقابل $\Leftarrow yx^2 = (yx)x$

$x^5(yx) = x^{10}y = \underbrace{x^6}_{e} x^4 y = x^4 y$

$1H = H = x^2H = x^4H$

$xH = \{x, x^3, x^5\} = x^3H = x^5H$

$yH = \{y, \underbrace{yx^2}_{x^4y}, \underbrace{yx^4}_{x^2y}\} = \{y, x^4y, x^2y\} = x^2yH = x^4yH$

$xyH = \{xy, \underbrace{xyx^2}_{x^5y}, \underbrace{xyx^4}_{x^3y}\} = \{xy, x^5y, x^3y\} = x^3yH = x^5yH$

مثال دیگر $xyx^4 = x\underbrace{x^5y}x^3 \cdots = x\underbrace{x^{20}}_{e}y = x^{21}y = (x^6)^3 x^3 y = x^3y$

ج)

$H(1) = H = Hx^2 = Hx^4$

$Hx = \{x, x^3, x^5\} = Hx^3 = Hx^5$

$Hy = \{y, x^2y, x^4y\} = Hx^2y = Hx^4y$

$Hxy = \{xy, x^3y, x^5y\} = Hx^3y = Hx^5y$

مثلا اعضای xH و Hy کاملا برابرند.

د) $\forall g \in D,\ gH = Hg \Rightarrow$ H نرمال است

ه)

$I = \{1, x^3, y, x^3y\}$

$(1)I = I = x^3I = yI = x^3yI$

$xI = \{x, x^4, xy, x^4y\} = x^4I = xyI = x^4yI$

$x^2I = \{x^2, x^5, x^2y, x^5y\} = x^5I = x^2yI = x^5yI$

$$I(1) = \{1, x^3, y, x^3y\} = Ix^3 = Iy = Ix^3y$$

$$Ix = \{x, x^4, yx, x^3yx\} = Ix^4 = Ix^2y = Ix^5y$$

$$yx = x^5y \qquad x^3yx = x^2y$$

$$Ix^2 = \{x^2, x^5, yx^2, x^3yx^2\} = Ix^5 = Ix^4y = Ixy$$

$$yx^2 = x^4y \qquad x^3yx^2 = xy$$

مثال: I جابجایی نیست.

$$Ix \neq xI$$

$$xyx^2 \in xI \cdot x^2I$$

$$xyx^2 = xx^5yx = xx^{10}y = x^5y \Rightarrow x^5y \in xI \cdot x^2I$$

$$x^5y \notin x^3I$$

$$\Rightarrow x^3I \neq xI \cdot x^2I$$

تمرین ۱