Session 10 of group theory

Ladies and gentlemen, it's now time to do some physics!

First glance of symmetry in particle physics (Gell-Mann's geniuty.)

$m_p \approx 938 \frac{MeV}{c^2}$	$m_{\pi^+} \approx 140 \frac{MeV}{c^2}$	$m_{\Delta^{++}} \approx 1231\ MeV/c^2$
$m_n \approx 939 \frac{MeV}{c^2}$	$m_{\pi^0} \approx 135\ MeV/c^2$	$m_{\Delta^+} \approx 1232\ MeV/c^2$
	$m_{\pi^-} \approx 140 \frac{MeV}{c^2}$	$m_{\Delta^0} \approx 1233\ MeV/c^2$
		$m_{\Delta^-} \approx 1235\ MeV/c^2$

The notion of "isospin" was first introduced by Heisenberg in 1932. He was puzzled by the fact that p_s and n_s have nearly equall masses, and apart from difference in electrical charge, most of their properties are much alike!

He thought with himself : « Like electron that has two possible spin states, and the rest of their properties are alike, It has to be something like spin "ISOSPIN" which p & n are diffent states of it! »

That's mesmorising how he thught about this apparent similarity.

Thus, the nucleons form a doublet, just like e^- that show a doublet structure as a consequence of two-state spin orientations.

Later on, Gell-mann found that particles with nearly equal masses can always be arranged in isospin multiplet.

$(p, n) \rightarrow$ isospin doublet

$(\pi_+, \pi_0, \pi_-) \longrightarrow$ isospin triplet

$(\Delta_{++}, \Delta_+, \Delta_0, \Delta_-) \longrightarrow$ isospin quadruplet

particles within a multiplet have "equal masses" but diffent charges.

But isospin symmetry is actully approximation! since the masses are not exactly equal!

Isospin symmetries are internal degree of freedom.

I mean, we only transform internal d.o.fs with some rules!

E.x: $SU(3)$ on triplets $(\vec{\phi})$

$$\begin{pmatrix} \alpha_1 & \alpha_2 & \alpha_3 \\ \beta_1 & \beta_2 & \beta_3 \\ \theta_1 & \theta_2 & \theta_3 \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix} \longrightarrow \sum_i \begin{pmatrix} \alpha_i \phi_i \\ \beta_i \phi_i \\ \theta_i \phi_i \end{pmatrix}$$

They are diffent from the so called "spacetime symmetries", which act on $x^\mu = (t, \vec{x})$ Coordinates!

These states transform according to an irrep of the symmetry group.

Just like the action of rotation on 3D vector, $SU(3)$ acts on quark field which it's bases represent diffent quarks $(u, d, s, \dots)$

Concrete examples:

Consider decay of $\Delta^{++} \longrightarrow p\,\pi^+$

under isospin matrices (analogeous to pauli-matrices)

$I_3^{(\Delta)} |\Delta^{++}\rangle = \frac{3}{2} |\Delta^{++}\rangle$ $\qquad$ I_3^{Δ} on $|\Delta^+, \Delta^0, \Delta^-\rangle$ has $\frac{1}{2}, -\frac{1}{2}, -\frac{3}{2}$ eigenvalues respectively

$I_3^{(\pi)} |\pi^+\rangle = |\pi^+\rangle$ I_3^{π} on $|\pi^0, \pi^-\rangle$ has 0 and (−1) eigenvalue.

$I_3^{(p)} |p\rangle = \frac{1}{2} |p\rangle$ $I_3^{(p)}$ on $|n\rangle$ has $-\frac{1}{2}$ eigenvalue.

so that $\underbrace{I_3}_{I_3^{(p)} \otimes \mathbb{1}^{(\pi)} + \mathbb{1}^{(p)} \otimes I_3^{(\pi)}} |p\rangle \otimes |\pi^+\rangle = I_3^{(p)} |p\rangle \otimes \mathbb{1}^{(\pi)} |\pi^+\rangle + \mathbb{1}^{(p)} |p\rangle \otimes I_3^{(\pi)} |\pi^+\rangle$

$= \frac{1}{2} |p\rangle \otimes |\pi^+\rangle + |p\rangle \otimes |\pi^+\rangle = \frac{3}{2} |p\rangle \otimes |\pi^+\rangle$

Remark: The index 3 in $I_{③}$ stands for third generator of isospin group.

Just like $\sigma_3 = \sigma_z$ acts on $|m = \pm \frac{1}{2}\rangle$ gives $\pm\frac{1}{2} |m = \pm\frac{1}{2}\rangle$.

so the isospin quantum number is conserved during this reaction.

Remark: $I_\pm$ raise and lower isospin quantum number, like

$I_-^{(\Delta)} |\Delta^{++}\rangle = I_- |\frac{3}{2}, \frac{3}{2}\rangle = \sqrt{3} |\frac{3}{2}, \frac{1}{2}\rangle = \sqrt{3} |\Delta^+\rangle$

Let's work out a mysterious physical fact, only by representation & group theory!

It's been observed that Δ predominantly decays into nucleon + pion.

By charge conservation, Δ^{++} just can decay like $\Delta^{++} \to p\pi^+$

For other Δ-particles, the situation is more complicated!

Let's start the analysis by looking into six different nucleon-pion states:

$I_3^{(p)} \otimes \mathbb{1}^{(\pi)} + \mathbb{1}^{(p)} \otimes I_3^{(\pi)} \leftarrow I_3 \left(|p\rangle |\pi^+\rangle \right) = \frac{3}{2} |p\rangle |\pi^+\rangle$

$I_3 \left(|p\rangle |\pi^0\rangle \right) = \frac{1}{2} |p\rangle |\pi^0\rangle$

$I_3 \left(|p\rangle |\pi^-\rangle \right) = -\frac{1}{2} |p\rangle |\pi^-\rangle$

$$I_3(|n\rangle|\pi^+\rangle) = \tfrac{1}{2}|p\rangle|\pi^+\rangle$$
$$I_3(|n\rangle|\pi^0\rangle) = -\tfrac{1}{2}|p\rangle|\pi^0\rangle$$
$$I_3(|n\rangle|\pi^-\rangle) = -\tfrac{3}{2}|p\rangle|\pi^-\rangle$$

$\pm 3/2$ eigenvalus occurs only once.

$\pm 1/2$ " " twice!

Similaries with "angular momentum addition" is STRIKING.

the above repetition of eigenvalues means that in product space of $|p,n\rangle \otimes |\pi^{\pm}, \pi^0\rangle$ one can have $3/2$ and $1/2$ (total isospin) values!

$$1 \otimes \tfrac{1}{2} = \tfrac{1}{2} \oplus \tfrac{3}{2}.$$

Let's see what decay of Δ^{++} teachs us, and apply it to other Δ-particles decay.

$$\Delta^{++} \longrightarrow p\,\pi^-$$

By isospin invarian of particle physics (in ordinary regime's) we have

$$\Delta^{++} \longrightarrow \underbrace{p\ \pi^-}$$

$\downarrow I_-$ $\quad\quad\quad\quad\quad\quad \downarrow I^-$

$$\sqrt{3}\,|\Delta^+\rangle \longrightarrow |n\rangle|\pi^+\rangle + \sqrt{2}\,|p\rangle|\pi^0\rangle$$

$\Downarrow$

$$|\Delta^+\rangle \longrightarrow \frac{1}{\sqrt{3}}|n\rangle|\pi^+\rangle + \sqrt{\frac{2}{3}}\,|p\rangle|\pi^0\rangle$$

Thus, the decay of Δ^+ should be this!

The theoretical calculation teaches us that the ratio of occurence of $p\pi^0$ decay to $n\pi^+$ decay must be 2.

This is in Complete agreement with observational data.

Apply once more I_- ... to get:

$$|\Delta^0\rangle \longrightarrow \frac{1}{\sqrt{3}}|p\rangle|\pi^-\rangle + \sqrt{\frac{2}{3}}|n\rangle|\pi^0\rangle \quad ✓$$

So, the 2:1 ratio of particle's decay could somehow related to its internal symmetries!

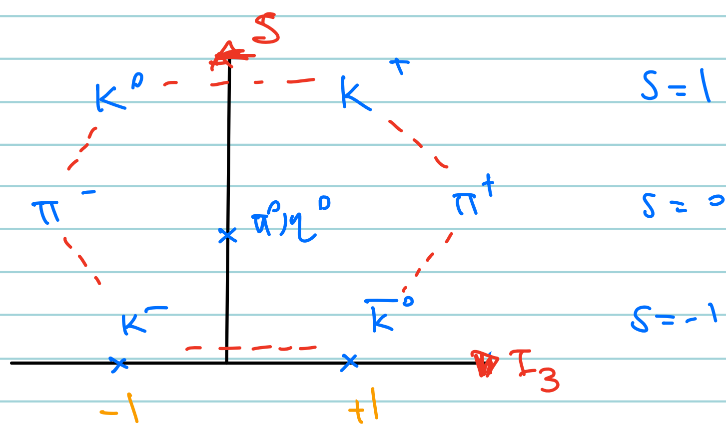
Various isospin multiplet + masses!

	isospin_multiplet	spin	isospin	mass
pion	$\pi^\pm, \pi^0$	0	1	139.6, 135
kaon	K^0, K^+	0	1/2	497.7, 493.7
$\overline{\text{kaon}}$	$K^-, \overline{K^0}$	0	1/2	493.7 497.7
eta, η^0		0	0	547.3
eta', η'		0	0	960
nucleons	n, p	1/2	1/2	939.5 938.3
delta	$\Delta^-, \Delta^0, \Delta^+, \Delta^{++}$	3/2	3/2	$\approx$ 1235
sigma	$\Sigma^-, \Sigma^0, \Sigma^+$	1/2	1	1197.4 1192 1189
Sigma *	Σ^{*-} Σ^{*0} Σ^{*+}	3/2	1	$\approx$ 1385
lambda	Λ^0	1/2	0	

ksi	Ξ^-, Ξ^0	$1/2$	$1/2$
ksi*	Ξ^{*-}, Ξ^{*0}	$3/2$	$1/2$
omega	Ω^-	$3/2$	0

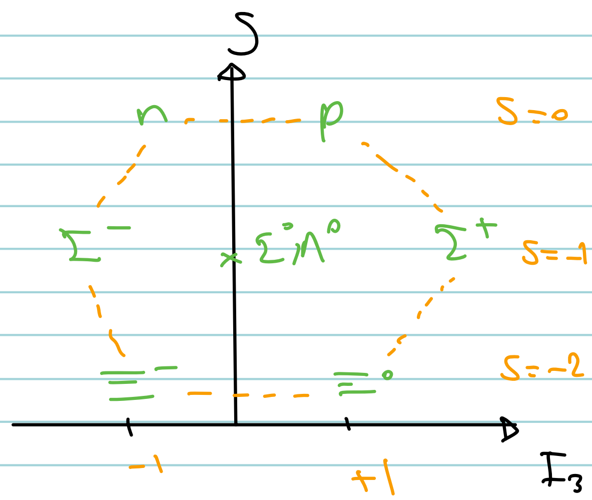
SU(3) and its role in particle physics are also magical!

Meson octet.



with 0-spins!

Baryons octet.



The rep. thry of SU(3) gives a magical strucfure to all known particles! It embeds them to known reps of SU(3).