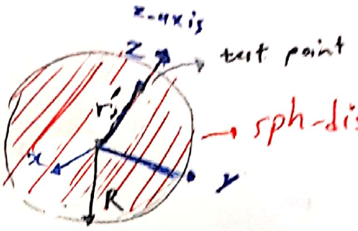
Q1. Consider a spherical matter distribution ($\rho(r)$)

We calculate Gravitational potential due to that distribution in different points:



sph-distribution and a test point.

Case I: if matter extends to a R distance then

We have total matter of:

$$M = \int dm = \int \rho \, dv = \int \rho(r) r^2 dr \overbrace{\int \sin\theta \, d\theta \int d\phi}^{4\pi}$$

$$= 4\pi \int_0^R r^2 \rho(r) \, dr$$

In this case the test point is in the $\underbrace{\text{sph-dist}}_{\text{spherical-distribution}}$ matter and $r < R$

We align z-axis such that it intercepts test point and find potential:

$$\Phi = -G \frac{Mm}{r} \quad \begin{cases} M = \rho(r) \, dv = \rho(r) r^2 \sin\theta \, dr \, d\theta \, d\phi \\ r = \text{Distance between test point and differential volume...} \\ = |\vec{r}' - \vec{r}| = (r^2 + r'^2 - 2r.r' \cos\theta)^{1/2} \end{cases}$$

$$\text{so} \rightarrow \Phi = -Gm \int_0^R \rho(r) r^2 dr \underbrace{\int_0^\pi \frac{\sin\theta \, d\theta}{(r^2 + r'^2 - 2rr'\cos\theta)^{1/2}}}_{\text{Let's calculate it}} \underbrace{\int_0^{2\pi} d\phi}_{2\pi}$$

$\cos\theta = u$

$du = -\sin\theta \, d\theta \rightarrow$

$\Big|_0^\pi \rightarrow \Big|_1^{-1} \rightarrow$ changing Limits of Integration by minus of

$$= \int_{-1}^{1} \frac{du}{(r^2 + r'^2 - 2rr'u)^{1/2}} = ?$$

Note: $\int ds \, (a+bs)^{-1/2} = \frac{2}{b} (a+bs)^{1/2}$

$$= -\frac{2}{2rr'} (r^2 + r'^2 - 2rr'u) \Big|_{-1}^{1} = -\frac{1}{rr'} \left(\sqrt{(r-r')^2} - \sqrt{(r+r')^2} \right)$$

$$= -\frac{1}{rr'} \left(|r-r'| - (r+r') \right) = \frac{1}{rr'} \left(r + r' - |r-r'| \right)$$

plug onto above...

$$= -2\pi G m \int_0^R \rho(r) \frac{r}{r'} \left(r + r' - |r-r'| \right) dr$$

Note that in this case where matter extends beyond test point we should evaluate absolute value carefully.

Let's divide the integration to 2 values...

$$-2\pi G m \left(\int_0^{r'} \rho(r) \times \frac{r}{r'} \underbrace{(r+r' - |r-r'|)}_{\text{it's Negative}} dr + \int_{r'}^{R} \rho(r) \frac{r}{r'} \underbrace{(r+r' - |r-r'|)}_{\text{positive}} dr \right.$$

$$r + \cancel{r'} + r - \cancel{r'} = 2r \qquad \cancel{r} + r' - \cancel{r} + r' = 2r'$$

$$= -\underbrace{\frac{4\pi G m}{r'} \left(\int_0^{r'} \rho(r) r^2 \, dr \right)}_{} - \underbrace{\frac{4\pi G m}{1} \int_{r'}^{R} r \rho(r) \, dr}_{}$$

Note that $4\pi \int_0^{r'} \rho(r) r^2 \, dr$ is mass inside of sphere of radius r' $\Rightarrow$ Call it $M(r')$

then

$$\Phi(r') = -\frac{G m M(r')}{r'} + \rightarrow$$

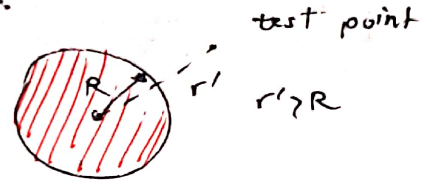
$\downarrow$

As you Can see potential is like potential between a mass m and a mass $M(r')$ both Concentrated in a Point at r' distance ... ▨

In evaluating this, Consider the fact that Force depends on $\frac{\partial}{\partial r'}$ of this Integral and it does not vanishes, this term is present because,

there are masses around test point and it's $r = |r - r'| = 0 \rightarrow$ potential is about ∞ so they serve as some kind of singularity.

We can delete this terms by such a matter distribution.



But there's nothing scary about it.

For the second part of Question :

Now if we evalute the integral only Second part remains and $(r>r')$

$$\phi_2 = -4\pi G m \int_{R_1}^{R_2} r \rho(r) \, dr$$

to evaluate force we should take $\partial r' \rightarrow$ but ϕ_2 is not a function of r'

and so the Force vanishes $\rightarrow$ this means no force exerted by outer masses.

All is done! ▨

Q2:

 and  galaxies?

- As Roos said in Table I → typical diameter of galaxy groups are $1-5\ \mathrm{Mpc}$
- Wikipedia also says that typical diameter of galaxies are: $10^3 - 10^5\ \mathrm{pc}$

$\sigma_{cl} = 10^3 \frac{km}{s}$ $\qquad N_g = 10^3$ $\qquad$ According to Roos ⇒ galaxy number density is $0.01 \frac{h^3}{\mathrm{Mpc}^3}$

But here we need a cluster's number density.

→ For sake of simplicity. Consider all galaxies to be disc shape so Area of a galaxy would be → $\pi r_g^2 \sim \pi (10^4)^2\ pc^2$

→ the Volume swept by a galaxy is: Area × time × speed of galaxy

$$= \pi \times 10^8\ pc^2 \times \underbrace{14\times10^9}_{year} \times \underbrace{\pi\times10^7}_{to\ sec} \times 10^3 \frac{km}{s} \times \frac{1}{3\times10^{13}} \frac{pc}{km}$$

$$\simeq 4\times10^{15}\ pc^3 \sim 4\times10^{-3}\ \mathrm{Mpc}^3$$

Number of Collision $=$ $\overset{\text{cluster's}}{\vee}$ galaxy density × Swept volume

Consider a cluster has typically 5×10^6 pc length

and a galaxy is about 10^4 pc → There is $\frac{5\times10^6}{10^4} = 500$ galaxies in

a cluster and it's number density is $\frac{5\times10^2}{(5\times10^4 pc)^3} = \frac{1}{25}\times10^{-16}\frac{1}{(pc)^3}$

so number of incidents are:

$$4\times10^{15}\ pc^3 \times \frac{1}{25}\times10^{-16}\frac{1}{(pc)^3} \sim 0.016 \text{ incidents!}$$

Mean free path is: → $14\times10^9 \times \pi\times10^7\ s \times 10^3 \times \frac{km}{s} \sim 4\times10^{20} km \sim 1.4\times10^7\ pc$

divided by collisions → mean free path $\simeq 9\times10^8$ pc

mean free time is: $\frac{MFP}{\sigma_{cl}} \sim \frac{9\times3\times10^8\times10^{13}\ km}{10^3 \frac{km}{s}} = 2.7\times10^{19} s \simeq 7\times10^{11}$ year!

We can play this time in poisson distribution and get.

$$pr(k=1,\ T=7\times10^{11} yr) = \frac{\lambda^k e^{-\frac{t}{T}}}{k!} \simeq \frac{1.4\times10^{10}}{7\times10^{11}} \times e^{-\frac{1.4\times10^{10}}{7\times10^{11}}} \sim 1.4\times10^{-2} e^{-1.4\times10^{-2}}$$

where t is 1.4×10^{10} yr

$$\sim 0.02 = 2\%$$

That's really impossible, probably there hasn't been any collision.

It seems that no collisions has been occured and we can't infer anything

from it → So the proposed idea is wrong based on my calculation.

Q3. The flux formula is, $\frac{L}{4\pi r^2}$

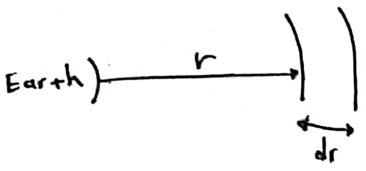
■ Let's first Calculate R_{max} (the radius of which galaxies overlap eachother an no new lights enters Earth after that radius),

As we now from olber's calculation $\longrightarrow$ $R_{max} = \frac{1}{An}$ where A is typical area of a galaxy $\rightarrow$ $A \simeq \pi \times 10^{8}\,pc^2$ $\longrightarrow$ $R_{max} \sim \frac{1}{\pi \times 10^{-4} \times 10^{-8}} \sim 10^{11}\,Mpc$

$= \pi \times 10^{-4}\,Mpc^2$

So R_{max} exceeds Hubble's radius and we should consider galaxies in all the sky,

Flux in a thick shell is $\frac{\text{number of galaxies} \times \text{Luminosity}}{4\pi r^2}$



But $dn = \underbrace{4\pi r^2\,dr}_{dV_{shell}} \times n \rightarrow dF = \cancel{4\pi r^2}\,dr \times n \times \frac{L}{\cancel{4\pi r^2}}$

Integrating $\rightarrow$ $F_{tot} = \int dF = \int_0^{r_H} nL\,dr = r_H n L$

and Flux of sun [solar flux] is, $\frac{L_\odot}{4\pi r_e^2}$ wher $r_e = 8'15''$ $\overset{\text{دقیقه نور}}{1m}$ $\simeq 4.8 \times 10^{-12}\,Mpc$

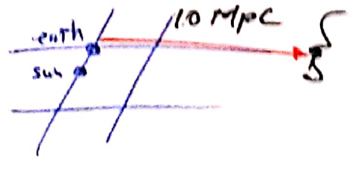
$r_H = c \times t_H = 3\times10^{8} \times 1.5\times10^{7} \times \pi\times10^{7} \times \frac{1}{3\times10^{16}} \simeq 4700\ Mpc$

$\Rightarrow$ $F_{gal} = r_H\, n \times L = 4700 \times 10^{-3} \times 10^{11} L_\odot \simeq 4.7\times10^{11} L_\odot\,(Mpc)^{-2}$

But, $\frac{F_{gal}}{F_\odot} = \frac{4.7\times10^{11}\cancel{L_\odot}}{\frac{\cancel{L_\odot}}{4\pi r_e^2}} = 4\pi \times 4.7\times10^{11} \times (4.8\times10^{-12})^2 \cancel{(Mpc)^2} \times \cancel{(Mpc)^{-2}} \sim 10^{-10}$

So solar flux is 10 order greater than the F_{gal} $\rightarrow$ thats because $\rightarrow$

$n \sim 10^{-3}\frac{1}{(Mpc)^3}$ roughly means in $10^3\,(Mpc)^3$ box is a galaxy or in each 10 Mpc (distance) we can see a galaxy $\Rightarrow$ But $F \propto \frac{1}{r^2}$ and $r \sim Mpc$ $\rightarrow$ So the galactic flux is Completely negilible,



■

Q4 Note that we know all physical lenghts would undergo a contraction or expansion

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, so is λ (wave length) → so the relation between λ_o, λ_e is:

observ emitted

$$\frac{\lambda_o}{a(t_o)} = \frac{\lambda_e}{a(t_e)}$$ [that's true because we normalize length to scale factor in order to get the Coordination length or Comoving length]

$$\Rightarrow \frac{\lambda_o}{\lambda_e} = \frac{a(t_o)}{a(t_e)}$$

expanding a → $a(t_o) = a(t_e) + \dot{a}(t_e)(t_o - t_e) + O(t_o - t_e)^2$ → dividing by $a(t_e)$

suppose $|t_o - t_e| \ll H_o^{-1}$

$$\frac{a(t_o)}{a(t_e)} = 1 + \frac{\dot{a}(t_e)}{a(t_e)}(t_o - t_e) = \frac{\lambda_o}{\lambda_e} \Rightarrow$$

equality

$$\frac{\lambda_o}{\lambda_e} - 1 = \frac{\lambda_o - \lambda_e}{\lambda_e} = \frac{\Delta\lambda_{o-e}}{\lambda_e} \Rightarrow \frac{\Delta\lambda_{o-e}}{\lambda_e} = \frac{\dot{a}(t_e)}{a(t_e)}(t_o - t_e)$$

now notice that → $r = c(t_o - t_e)$ → $(t_o - t_e) = r/c$ → distance to our specified point

and we know that $H(t) = \frac{\dot{a}(t)}{a(t)}$ and → by definition of red shift (z) → $1 + z = \frac{\lambda_o}{\lambda_e}$

or $z = \frac{\Delta\lambda_{o-e}}{\lambda_e}$ ⇒ putting all this together:

$$\frac{\Delta\lambda_{o-e}}{\lambda_e} = \frac{\dot{a}(t_e)}{a(t_e)}(t_o - t_e) \Longrightarrow z = H(t_e) \times r/c \rightarrow cz = H(t_e)\, r$$

Note that we know from doppler effect, that the speed of that point

is $cz = v$ → (you can also expand $\sqrt{\frac{1+\beta}{1-\beta}}$ in relativistic mechanics to

get that $z = v/c = \beta$ → $v = zc$)

so → $v = H(t_e)\, r$ → Hubble's law

ب) Derivating with respect to t_0 →

$$\frac{dz}{dt_0} = \frac{da(t_0)}{dt_0} \times \frac{1}{a(t)} - a(t_0) \times \frac{1}{a^2(t)} \frac{da(t)}{dt} \frac{dt}{dt_0}$$

$$\frac{dz}{dt_0} = \frac{\dot a(t_0)}{a(t)} - \frac{a\cancel{(t_0)}}{a^2(t)} \times \frac{\cancel{a(t)}}{\cancel{a(t_0)}} \times \dot a(t) =$$

$$\frac{dz}{dt_0} = \frac{\dot a(t_0)}{a(t)} - \underbrace{\frac{\dot a(t)}{a(t)}}_{H(t)} = \underbrace{\frac{\dot a(t_0)}{a(t_0)}}_{H(t_0)} \underbrace{\frac{a(t_0)}{a(t)}} - H(t)$$

→ putting all together

$$\times \frac{a(t_0)}{a(t_0)}$$

$$\underbrace{\frac{a(t_0)}{a(t)} = 1+z}_{\text{our main relation}}$$

$$\frac{dz}{dt_0} = H_0(1+z) - H(t) \rightarrow H(t) = H_0(1+z) - \frac{dz}{dt_0}$$