Delayed Choice Experiment's Realizations

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Delayed Choice Experiment, is one of the striking quantum mechanical experiments that aims to reveal the wave-particle duality, more specifically it aims to find when the system (photon in this case) ”decides” to behave as a ”wave” or ”particle” ... Then by changing the set-up, people try to derive some sort of paradox, but at the end it's quantum mechanics that makes the surprised.

Thanks to my dear friend, Mr.Farokhi, which discussed the theoretical aspects of this experiment, we just discuss the set-ups which are assumed to show this phenomenon.

1 Wheeler's Idea and ”Cosmic interferometer”

It was first just a thought experiment, that proposed by John Archibald Wheeler, and this thought experiment was motivated by ”Complementarity principle”. This principle states that there are some complementary properties of object that can not be measure simultaneously. One of these properties is ”wave-particle behaving” property. The simplest experiment which by Wheeler is something like that:

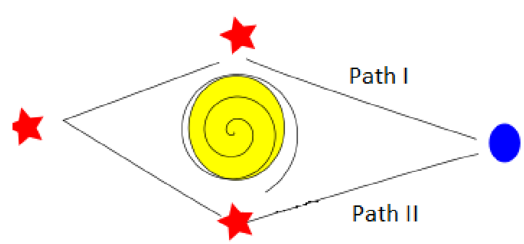

Figure 1: Wheeler's thought experiment for revealing ”particle” nature

In this thought experiment, suppose that a quasar located in many light years distance from Earth, produces a photon, and midway between Earth an the quasar there is a galaxy or some kind of gigantic and massive object. The photon after arriving near this galaxy should decide whether choose a specific path which means it should choose to behave like a ”particle”, or it should behave as a ”wave” which means it should pass both way around, as we expect form double slit experiment.

Note that ”nowadays” this may looks like very natural to expect that the photon will behave simultaneously as particle and wave. But as I mentioned earlier the ”Complementarity principle” force the particle to choose how it should behave.

But what would happen if the photon arrives Earth? The Answer is it depends. One can direct two telescopes along ”path I” and ”path II” as showed in Figure 1. If the photon ”decided” to travel as a particle all those million light years then one of the two telescopes catch the photon. But the experimenters decide to provide another set-up to observe photon, which could be guessed that it reveals the ”wave” nature of the photon. They direct output of the two telescopes into a beam-splitter and observer that the diffraction pattern appears! (You may ask that was a though experiment and how could you conclude that the diffraction pattern appears? The answer is we can find a realization of this thought experiment and find these results.) you can see such a set-up in the figure below.

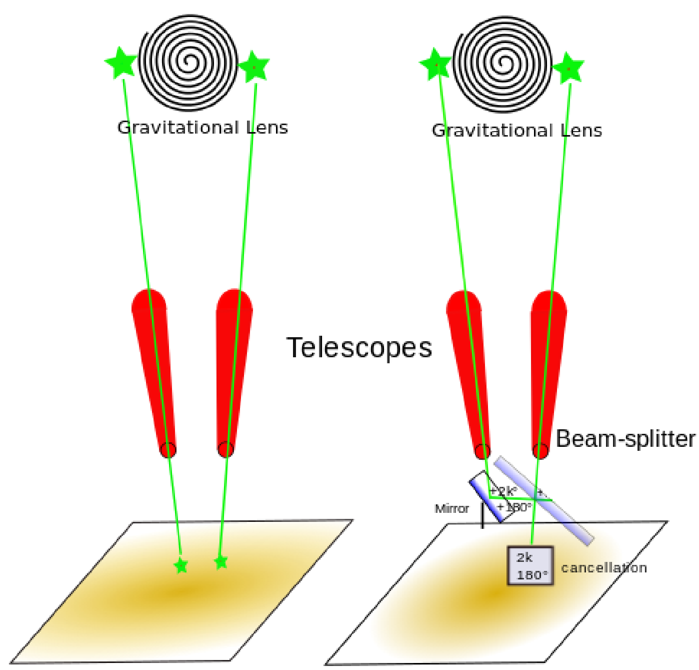

Figure 2: Wheeler’s thought experiment for revealing ”wave” nature

There were amazing and even crazy interpretations of these experiment that would lead to ”past can change future”, which discussed at the theoretical aspects of this experiment.

Before dive into the paper let's learn some basics such as entanglement swapping experiment and Bell states.

2 Entanglement swapping protocol

We work here with qubits and define Bell states to be:

$$\left|\phi^{\pm}\right\rangle = \frac{1}{\sqrt{2}}(|hh\rangle \pm |vv\rangle)$$

$$\left|\psi^{\pm}\right\rangle = \frac{1}{\sqrt{2}}(|hv\rangle \pm |vh\rangle)$$

where $|h\rangle$ and $|v\rangle$ are states that represent horizontal and vertical polarization. Label these states with some vector such as: $\{|\chi_i\rangle\}_{i=1}^{4}$ And beacuse $\sum_{i=1}^{4} |\chi_i\rangle \langle\chi_i| = I_4$.

Then we can choose to do "projective measurment" in this basis, which is called "Bell state measurements".

The the so called "Entanglement swapping" protocol is done like this: Alice

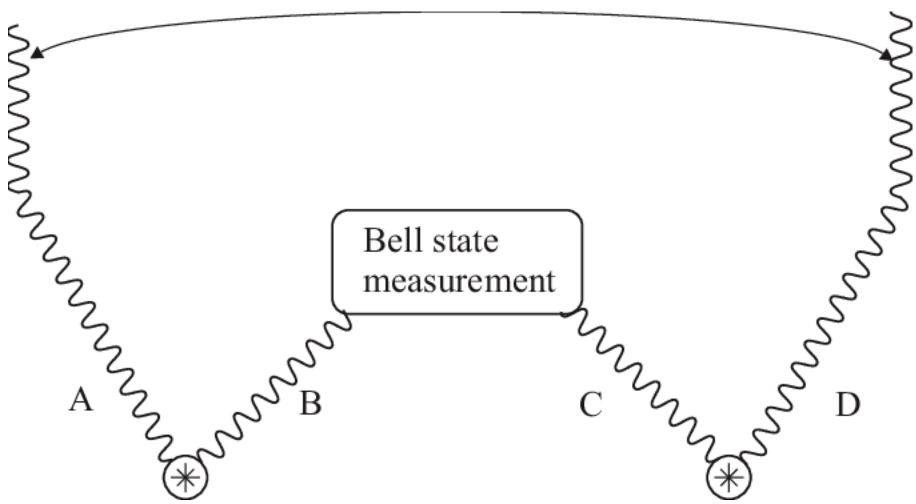

Figure 3: Entanglement swapping protocol's set-up

has a pair of entangled photon in state $|\phi^{+}\rangle$ And Bob also has a pair of entangled photon in state $|\phi^{+}\rangle$ so the total system that consists of four photons and belong to Alice and Bob has is in the state:

$$|\Psi\rangle = \left|\phi^{+}\right\rangle \otimes \left|\phi^{+}\right\rangle = \frac{1}{2}(|hh\rangle + |vv\rangle) \otimes (|hh\rangle + |vv\rangle)$$

so: $|\Psi\rangle = \frac{1}{2}(|hhhh\rangle + |hhvv\rangle + |vvhh\rangle + |vvvv\rangle)$ Now Alice and bob send their second and first ,respectively, qubits to a lab and in this lab, the experimenters perform a Bell state measurement in two qubits. It more instructive to relate basic states to Bell states and then by substituting in the $|\Psi\rangle$ find the final state, which is the state of photon A & D on the Figure 3.

$$\begin{aligned}
|hh\rangle &= \frac{1}{\sqrt{2}}(\left|\phi^+\right\rangle + \left|\phi^-\right\rangle) \\
|vv\rangle &= \frac{1}{\sqrt{2}}(\left|\phi^+\right\rangle - \left|\phi^-\right\rangle) \\
|hv\rangle &= \frac{1}{\sqrt{2}}(\left|\psi^+\right\rangle + \left|\psi^-\right\rangle) \\
|vh\rangle &= \frac{1}{\sqrt{2}}(\left|\psi^+\right\rangle - \left|\psi^-\right\rangle)
\end{aligned}$$

Then:

$$\begin{aligned}
|\Psi\rangle &= \frac{1}{2}(|h\rangle\,|hh\rangle\,|h\rangle + |h\rangle\,|hv\rangle\,|v\rangle + |v\rangle\,|vh\rangle\,|h\rangle + |v\rangle\,|vv\rangle\,|v\rangle) \\
&= \frac{1}{2\sqrt{2}}(|h\rangle \otimes (\left|\phi^+\right\rangle + \left|\phi^-\right\rangle) \otimes |h\rangle + |h\rangle \otimes (\left|\psi^+\right\rangle + \left|\psi^-\right\rangle) \otimes |v\rangle + \\
&|v\rangle \otimes (\left|\psi^+\right\rangle - \left|\psi^-\right\rangle) \otimes |h\rangle + |v\rangle \otimes (\left|\phi^+\right\rangle - \left|\phi^-\right\rangle) \otimes |v\rangle)
\end{aligned}$$

Now a minor reordering the kets, $|A,B,C,D\rangle \to |A,D,B,C\rangle$

$$\begin{aligned}
|\Psi\rangle &= \frac{1}{2}(\frac{1}{\sqrt{2}}(|hh\rangle + |vv\rangle) \otimes \left|\phi^+\right\rangle + \frac{1}{\sqrt{2}}(|hh\rangle - |vv\rangle) \otimes \left|\phi^-\right\rangle + \\
&\frac{1}{\sqrt{2}}(|hv\rangle + |vh\rangle) \otimes \left|\psi^+\right\rangle + \frac{1}{\sqrt{2}}(|hv\rangle - |vh\rangle) \otimes \left|\psi^-\right\rangle) \\
&= \frac{1}{2}(\left|\phi^+\right\rangle \otimes \left|\phi^+\right\rangle + \left|\phi^-\right\rangle \otimes \left|\phi^-\right\rangle + \left|\psi^+\right\rangle \otimes \left|\psi^+\right\rangle + \left|\psi^-\right\rangle \otimes \left|\psi^-\right\rangle)
\end{aligned}$$

The final equation says that, After Bell state measurement of photons B,C in Figure 3, photons A,D will collapse in one of the Bell states, and because Bell states are all entangled states, we can conclude that after the measurement, Entanglement swapped between pairs (B,C) and (A,D).

This was the explanation of the protocol which we need it to discuss the paper.

3 Entanglement swapping between photons that have never Coexisted

3.0.1 An review of this paper

Now we go to cover this paper . As They mention in this paper, people already know entanglement between spatially separated photons and how they can use such things to run teleportation and quantum computation processes. But in this paper they demonstrate how we can entangle two photons that never have coexisted, And there are lots of amazing conclusions in this paper such as, ordering of measurement has no effect of the outcome of the experiment, different time correlations the at first glance may seem paradoxical but they do not cause a paradox, and crazy interpretations of the results of this experiment.

We want now to roughly explain this experiment. This is done by creating two entangled photons (1-2) and then immediately measuring photon 1 (notice that it's nothing but a dummy label on photons, because as we will see later, photons must be indistinguishable) and photon 2 is delayed until a second pair of entangled photon is created (3-4). Then photons 2 and 3 are projected onto a Bell basis (we measure them in Bell basis), this projection swaps the entanglement between photons 1 and 4 (as discussed in Entanglement swapping protocol) but photons 1 no longer exists. Surprisingly enough, show correlations and according to some tests that has been developed for entanglement, They are entangled. 'When photon 1 is measured in a certain basis, it does not "know" That photon 4 is going to be created and in which basis in will be measured', Nevertheless they exhibit quantum correlation, despite the fact that they never coexisted.(See details in Figure 4)

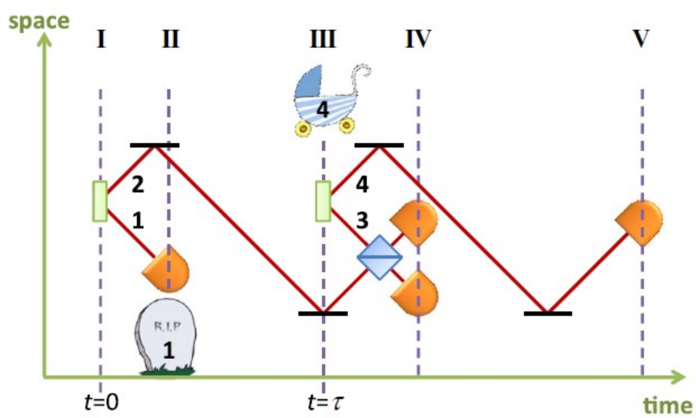

Figure 4: Time line diagram. (I) Birth of photons 1 and 2, (II) detection of photon 1, (III) birth of photons 3 and 4, (IV) Bell projection of photons 2 and 3, (V) detection of photon 4.

As a conclusion, The main difference between this experiment and lots of others that carry the label of "delayed choice", this one is the first that studies entanglement in temporally-separated not spatially-separated. And this paper suggests a set-up and some data of this experiment.

3.1 Theoretical explanation of this experiment

After generating the entangled photons by PDC method, they are in $|\psi^-\rangle$ state and the whole wave-function of the system by including the wave-function of two other photons is:

$$|\psi^-\rangle_{a,b}^{0,0} \otimes |\psi^-\rangle_{a,b}^{\tau,\tau} = \frac{1}{2}(|h_a^0 v_b^0\rangle - |v_a^0 h_b^0\rangle) \otimes (|h_a^\tau v_b^\tau\rangle - |v_a^\tau h_b^\tau\rangle)$$

where subscripts denote spatial mode labels and the superscripts are the time label of photons. Now we should make a Bell measurement of second and third photons, the second photon should be delayed by τ and then the same delay is applied on the second photon of second pair. Then the final state will be(Similar manipulations were done in the Entanglement swapping protocol section except for $|\phi^+\rangle$ state):

$$\begin{aligned} |\psi^-\rangle_{a,b}^{0,\tau} \otimes |\psi^-\rangle_{a,b}^{\tau,2\tau} =& \frac{1}{2}(|\psi^+\rangle_{a,b}^{0,2\tau} |\psi^+\rangle_{a,b}^{\tau,\tau} - |\psi^-\rangle_{a,b}^{0,2\tau} |\psi^-\rangle_{a,b}^{\tau,\tau} \\ & - |\phi^+\rangle_{a,b}^{0,2\tau} |\phi^+\rangle_{a,b}^{\tau,\tau} + |\phi^-\rangle_{a,b}^{0,2\tau} |\phi^-\rangle_{a,b}^{\tau,\tau}) \end{aligned}$$

According to the above discussion (were we conclude that order of time does not matter in measurement), the time labels in the above states are redundant labels. This scenario is somehow similar to the spatially-separated entangled photons. In the standard case, measuring one of the photon instantaneously changes the physical description of another photon, which is known as "action at distance". Here measuring the forth photon affects the discription of the first photon in the past. Another point of view is measuring the first photon is immediately steering the future physical description of the last photon.

3.2 Realization

The scenario discussed above was realized in the setup of figure 5.

Pairs of polarization-entangled photons are created by type-II PDC (parametric down conversion). A pulse of laser with 76 MHz repetition rate is frequency double to 390 nm and a average of 400mW. this beam is focused in 2mm thick BaB_2O_4 crystal. Compensating crystals correct the temporal walk-offs, and a phasor is located to make the state $|\psi^-\rangle$ exactly. HWPs (half wave plate) are some kind of change of basis ($\frac{1}{\sqrt{2}}(|h\rangle \pm |v\rangle)$), they analyze the photons in rotated basis. Two photons are spatially-filtered at source by using 3 nm wide bandpass filters.

The second photon of first pair is delayed until another pulse arrives to crystal and generates another pair of photons. The delaying time is 105 ns so the distance between two mirrors that are shown in Fig 5 by "Delay line" is 31.6 m. These are high-reflecting mirror with transmittance higher than 90% after 10 reflections, and less 10% of signal is sampled to another signal and used as feedback for stabilize the spatial properties of beam.

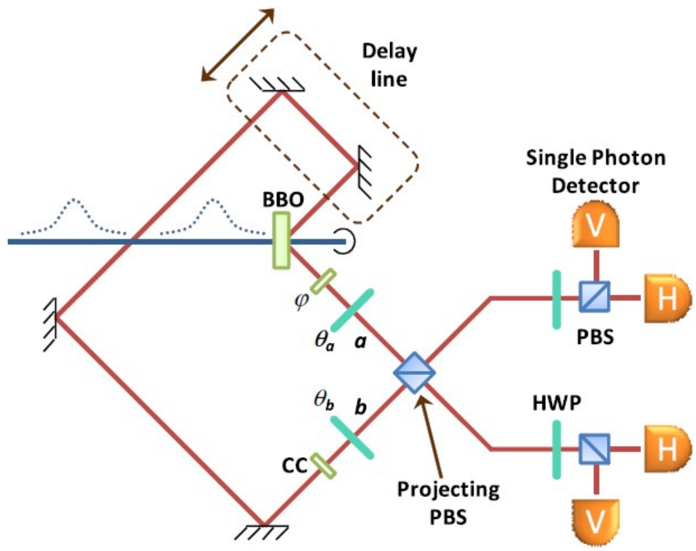

Figure 5: The experimental setup

PBS is polarizing beam splitter, and "projecting PBS" will do a projecting measurement in Bell states and then send each photon on its own path due to its polarization. H and V signs are single photon detectors for measuring polarization. The delay time is about eight consecutive laser pulses in order to provide enough time for measurement of the first photon of the first pair.

Notice that because the optical instruments and BBO polarized-entangled photon generator are not perfect(this source is probabilistic and with a certain probability it produces a pair of photons.) so we should postselect the results of this experiment, which we will discuss in greater detail in the next section.

3.3 Results of this experiment

Quantum tomography procedure is need for characterizing the state of the first and last photon. Generally in such procedure we reconstruct the density matrix of the system (photons 1 and 4) by a set of Measurements. See figure 6 for coefficient of each density matrix (Notice that if $z = f(x, y)$ then the real part of coefficient of $|x\rangle\langle y|$ is z), and one can simply conclude that every Bell states that photons (2,3) collapse in, photons (1,4) will be on that state.

Entanglement is observed when the observed fidelity[1] of the pairs is above 50%

[1]Fidelity is a quantum measure that characterizes the resemblance of two quantum state(generally density matrices). if ρ and σ are two density matrices then the fidelity re-

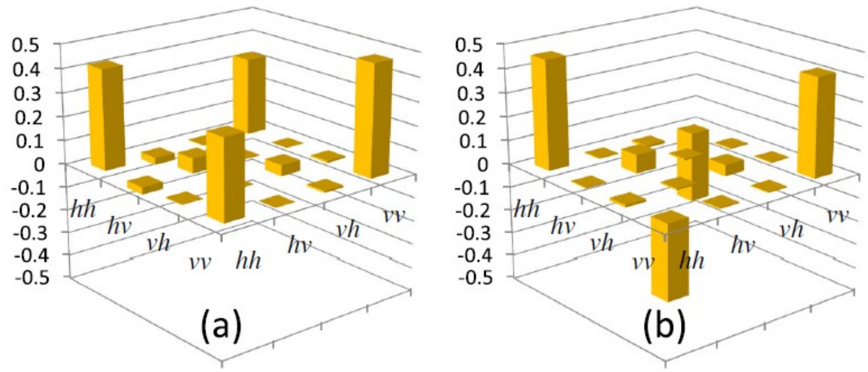

Figure 6: Real part of the density matrices of the first and last photon(a) The two middle photons are projected onto the $|\phi^{+}\rangle$ state, (b) The two middle photons are projected onto the $|\phi^{-}\rangle$ state.The total fourfold count rate was 12 Hz and each polarization setting was integrated over 6 min.

and in this setup the fidelity of theoretical and observed density matrix is about $77\% \pm 1$.

The concurrence[2] of the state is 0.57 ± 0.03. The Peres criterion[3] is -0.28 ± 0.01 and notice that it should be negative for entangled system.

The famous CHSH inequality value in this system reads 2.04 ± 0.04 since it must be less than or equal to 2, thats a rather small violation of Bell inequality. This value together with quantum steering parameter [4] which is 0.70 ± 0.02 ensures the existence of non-local correlations in temporally-separated photons.

3.4 Possible errors and justification

First of all, as they mention in this paper, all the result of the experiment depends on the quality of the polarized-entangled pair of photons that created by BBO crystal and all the optical or measurement instruments that are imple-

lation is $F(\rho, \sigma) = \left(tr\sqrt{\sqrt{\rho}\sigma\sqrt{\rho}}\right)^2$

[2]For a two qubits system (which in this case, polarization of the photons, are a good choice) the concurrence is defined by $\mathcal{C}(\rho) = \max\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\}$ and $\{\lambda_i\}_{i=1}^4$ are the eigenvalues of the following matrix in decreasing order, $R = \sqrt{\sqrt{\rho}\tilde{\rho}\sqrt{\rho}}$ and $\tilde{\rho} = (\sigma_y \otimes \sigma_y)\rho^(*)(\sigma_y \otimes \sigma_y)$. And naturally it should be a positive number between 0 and 1.

[3]This criterion which sometimes called Peres–Horodecki criterion, is a necessary condition, for the joint density matrix ρ of two quantum mechanical systems A and B, to be separable. The condition and its associated relations are easy to express but rather long, so we avoid mentioning it here.

[4]quantum steering is a special kind of non-local correlations, which is intermediate between Bell nonlocality and quantum entanglement.

mented in.

The fidelity of the measured entanglement is not perfect due to two main imperfections in the setup: the entanglement quality of the produced photon pairs and the quality of the projection of the photons 2 and 3 on a Bell state. PDC process produces some distinguishability between photons that cause reduction of the quality of the pairs and in turn causes reduction of quality of the entanglement swapping and projective measurements.

Quantum state tomography also is not a perfect procedure because in the process of ”changing the basis”, is not perfect.The fidelity of the photon pairs further reduced after passing delay line in the Figure 5 by 1 to 2 percent.

All of this imperfections cause to fidelity of the state before projection to be about 93.7%

Authors suggest that this scenario is very likely to be useful in quantum repeaters, when a qubit arrives a node it should be delayed or stored in some kind of memory until a second photon from another entangled pair arrives, and then by using this protocol repeating process will be done.

4 Realization of Delayed choice experiment

And here we want to introduce some famous realization of delayed choice experiment and interpret the results to gain more insight into it.

There have been several realizations and experimental proposals, including neutron interferometers and photons interfrometers. Lets start with pioneering endeavors.

4.1 Hellmuth's realization

Hellmuth and collaborators[5] performed delayed-choice experiments with a low-intensity Mach-Zehnder interferometer (MZI) in the spatial domain as well as timeresolved atomic uorescence in the time domain (Hellmuth et al., 1987). An attenuated picosecond laser (on average less than 0.2 photons per pulse) was used as the light source for the MZI. Two 5 m (20 ns) glass bers were used to delay the input photon. The transit time of the photon through the whole interferometer was about 24 ns. The combination of a Pockels cell (PC) and a polarizer (POL) was placed in the upper arm of the MZI as a shutter. When a half-wave voltage was applied on

You can see the setup of this realization in Figure 7

[5] Hellmuth, T., H. Walther, A. Zajonc, and W. Schleich, 1987, Physical Review A 35, 2532.

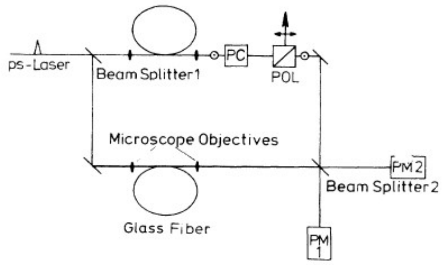

Figure 7: Setup of the delayed-choice experiment. The combination of a Pockels cell (PC) and a polarizer (POL) in the upper arm of the interferometer was used as a shutter.

When a half-wave voltage was applied on the Pockels cell[6], it rotates the polarization of the photons propagating through it, in a way that they were reflected out of the interferometer.

If the shutter in the above arm is closed then interference pattern disappears as the upper path of the interferometer was interrupted and only photons from the lower arm could reach the photomultipliers (PM 1 and PM 2). This provided which-path information, and inform us about the path that photon chosen when it reaches beam splitter 2. On the other hand if the shutter was open upon the arrival of photon, then interference pattern appears because there is no information of the path taken, and this is nothing but the "wave" nature.

But what about the delay or temporal separation in this experiment? Let's look at Figure 7 deeply.

The input photon meets beam splitter 1 first and it's amplitude is splitted between two perpendicular paths, then in the above arm it enters a fiber (one at each path) that last 20s for photon to pass the fiber , during propagation of the photon in fiber, the shutter opens after rise-time of the Pockels cell. After it photon exits the fiber and passes through shutter and meets beam splitter 2.In this case, opening of the shutter was delayed until the photon is inside the fiber and well inside the interferometer(remember the wheeler's idea, when the photon has chosen its behaviour then we change the setup, that's what exactly done in here.) This assures that photon's entry to the interferometer is located in the past light cone of the opening shutter's event.

However we can do this by two "modes", normal mode which the shutter opens when the photon is outside the MZI. But our favor is "delayed choice mode" that is opening shutter after the photon reach's beam splitter 1, while keeping all the arrangement to be the same. The results of this mode was drawn in the

[6]The Pockels effect is the basis of the operation of Pockels cells.(Roughly enough, Pockels effect is changing or producing birefringence in an optical medium induced by an electric field.) Pockels cells may be used to rotate the polarization of a beam that passes through.

Figure 8,

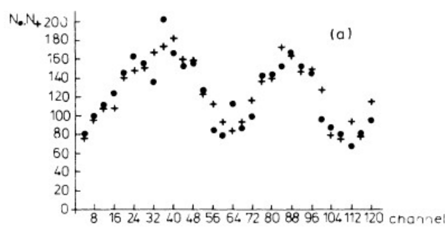

Figure 8: Experimental results of delayed choice experiment.(Vertical axis is the number of counted photons by polarization measurement 1 and vertical axis is the phase difference.) Dotes indicated normal modes and crosses show delayed choice mode, you can see interference and verify the predictions of standard quantum mechanics for this experiment.

and the results for PM 2 show complementary to there in PM 1.(shifted by a π phase with respect to data in the Figure 8).

Although it's a pioneering realization of delayed choice, but here are some problems, first of all, The experiment should be done with a single photon, which in this setup we use a pair of photon and note that there is no real active choice of the state of shutter.

Another experiment was done later by Alley to refine such imperfections and they reported successful experimental demonstration[7]. The setup in basically the same as Hellmuth setup but with delayed random choice of the configuration implemented within. Additionally a photomultiplier was implemented to catch the photons that reflect out of the setup.

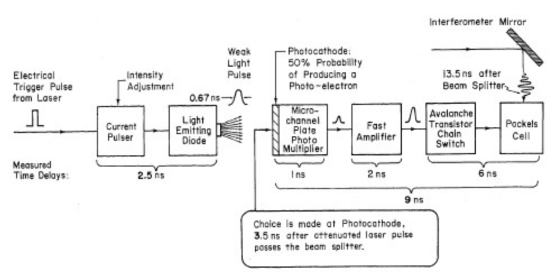

Figure 9: Schematic diagram of the device generating the random choices proposed in Alley realization.

[7]Alley, C. O., O. Jakubowicz, C. A. Steggerda, and W. C. Wickes, 1983, in International Symposium on foundations of Quantum Mechanics in the light of New Technology (Physical Society of Japan, Tokyo).

The random choice is made in photocathode which has 50% probability of producing an photon-electron pair after striking a light pulse of laser to it. Then the photo-electron is amplified and used to switch EOM[8] in the setup. Details of this setup is in Figure 9.

Five experimental runs with different space-time configurations were implemented (figure 10). In runs R, T and U the choice events were not only 3.5 ns delayed with respect to the entry of the photons into the MZI in the laboratory reference frame (event E, the origin point in Fig. 13) but also space-like separated from E. In run Y, the choices were also space-like separated from E but took place 5 ns earlier. In run W, the choices were in the time-like past of E. About 90% interference visibility was obtained when the wave property of the input photons was measured, and no observable interference was obtained when the particle property was measured. The authors conclude: "The predictions of quantum mechanics are confirmed even with the choice of the final configuration being made randomly during the course of the elementary quantum phenomenon".

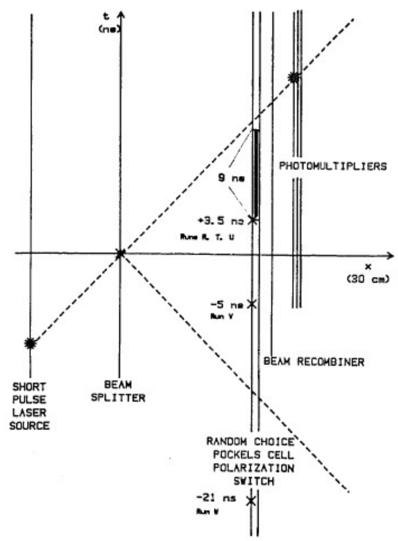

Figure 10: Space-time diagram reported in Alley's realization.

[8]Electro-optic modulator, is an optical instrument that is used to modulate electrical pulses onto light beams. That's a very basic experimental instrument that is used in laboratories. The modulation may be imposed on the phase, frequency, amplitude, or polarization of the beam.

4.2 A pace toward more realistic setup, single particle realization

Hellmuth realization was done by two photons, here we introduce Baldzuhn's[9] realization which they used single photon generated by SPDC[10] method to perform delayed-choice experiment.
The layout of the setup is in figure 11

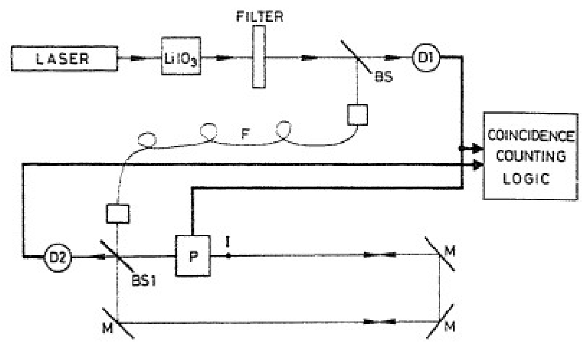

Figure 11: Setup of the delayed-choice experiment reported in Ref. (Baldzuhn et al., 1989); Figure taken therefrom. Photon pairs were produced by parametric down-conversion in the $LiIO_3$ crystal. Photon 1 is a trigger photon for Pockels cell and the second is a signal photon for delayed-choice experiment. The detection at D1 triggered a Pockels cell P in the interferometer through which the signal photons propagated in a clockwise or anti-clockwise path before reaching detector D2.

The upper half of the figure 11 is dedicated to producing photons in SPDC method and the lower half of it is an interferometer, Sagnac Interferometer [11]. One of two photons that created, is used to trigger a Pockels cell(P) located in the interferometer, and the other plays the active role in delayed-choice experiment.
The experiment runs as follows:

"In the clockwise path, the signal photon first passed the Pockels

[9] Baldzuhn, J., E. Mohler, and W. Martienssen, 1989, Z. Phys. B: Cond. Mat. 77, 347.

[10] Spontaneous parametric down-conversion (also known as SPDC) is a nonlinear instant optical process that converts one photon of higher energy, into a pair of photons (even, a signal photon) of lower energy, in accordance with the law of conservation of energy and law of conservation of momentum. It is an important process in quantum optics, for the generation of entangled photon pairs, and of single photons.

[11] Sagnac effect: A beam of light is split and the two beams are made to follow the same path but in opposite directions. On return to the point of entry the two light beams are allowed to exit and undergo interference. The relative phases of the two exiting beams, and thus the position of the interference fringes, are shifted according to the angular velocity of the apparatus. And Sagnac interferometer is a ring interferometer that directs the light in opposite directions and interfere them, as you can see in figure 11.

cell P and then the reference point I. In the anti-clockwise path, however, the situation is reverse. (a) If the Pockels cell was off during the photon's propagation through the whole interferometer, the polarization of the signal photon was not rotated and remained the same for both the clockwise and the anti-clockwise path. (b) Similarly, if the Pockels cell was continuously on, the polarization was rotated in both paths. In both cases (a) and (b) the final polarization state was the same for both paths, leading to interference. If, however, the Pockels cell was switched on at the time when the signal photon arrives at the reference point I and was kept on until after the photon met the beam splitter again, no interference was observed. This is because the polarization of the clockwise path remained unchanged, while the polarization of the counter-clockwise path was rotated. The polarization degree of freedom introduced a distinguishability between the two paths and hence destroyed the possibility of interference.“ (Taken from[12] ...)

So, finally, The signal photons showed wave behavior, if the Pockels cell was continuously left on or off. Particle behavior was revealed if the Pockels cell was switched on at the moment when the signal photons reached the reference point I in the interferometer.

The results of this realizations are shown in figure 12. And the results of the standard quantum mechanics if fully established.

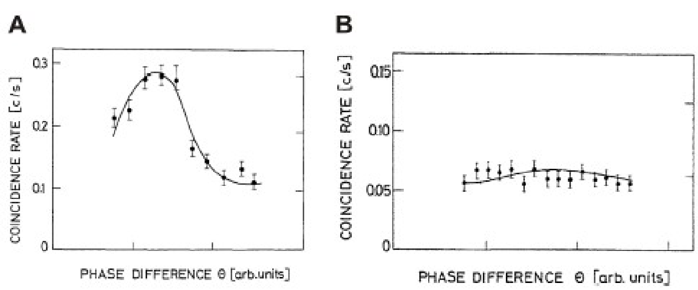

Figure 12: Experimental results of the delayed-choice experiment A: If the Pockels cell was continuously on or off, an interference pattern was observed. B: If the Pockels cell was switched on when the signal photon reached the reference point I, no interference showed up.

“The delayed-choice aspect of this experiment was realized by delaying the signal photon by an optical fiber (labeled ‘F‘ in Fig 11) and varying the time of the application of the voltage on the Pockels

[12] arXiv:1407.2930v3

cell via electronic delays. This allowed to switch the Pockels cell at the time when the photon was at the reference point, i.e. already within the interferometer. Space-like separation between the choice of the performed measurement and the entering of the photon into the interferometer was not implemented in this experiment." (Taken from ...)

4.3 Realizations of entanglement swapping and delayed-choice entanglement swapping

Entanglement swapping is one of the surprising phenomenons in quantum mechanics and generalization of quantum teleportation and can teleport entangled states.

It is of crucial importance in quantum information processing because it is one of the basic building blocks of quantum repeaters, quantum cryptography and other protocols. On the other hand, entanglement swapping also allows experiments on the foundations of quantum physics.
The entanglement swapping protocol itself has been experimentally demonstrated with various physical systems (Barrett et al., 2004; Halder et al., 2007; Kaltenbaek et al., 2009; Matsukevich et al., 2008; Pan et al., 1998; Riebe et al., 2004; Yuan et al., 2008).(should to refer to them at future)
In the light of finding which kind of physical interactions and processes are needed for the production of quantum entanglement, Peres has put forward the radical idea of delayed-choice entanglement swapping (Peres, 2000). Realizations of this proposal are discussed in the following.

4.3.1 Entanglement swapping realization

In (Jennewein et al., 2001), a delayed entanglement swapping experiment was performed. For the conceptual setup see figure 13.

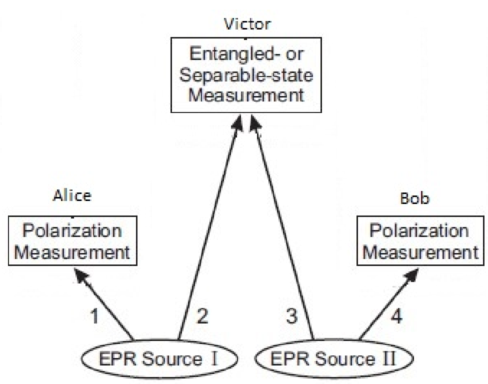

Figure 13: Experimental setup for entanglement swapping protocol.

Detection of photons 2 and 3 by Victor was delayed by two 10 m (about 50 ns) optical

fiber delays after the outputs of the Bell-state analyzer. Alice's and Bob's detectors were located next to each other. The traveling time of photons 1 and 4 from the source to these detectors was about 20 ns. Victor was separated from Alice and Bob by about 2.5 m, corresponding to luminal traveling time of approximately 8 ns between them. Therefore, Victor's measurements were in the time-like future of Alice's and Bob's measurements. The observed fidelity of the measured density matrix of photons 1 and 4 (ρ_{14}) with the ideal singlet state, was around 0.84, both above the classical limit of $\frac{2}{3}$ and the limit of approximately 0.78 necessary to violate Bell's inequality.

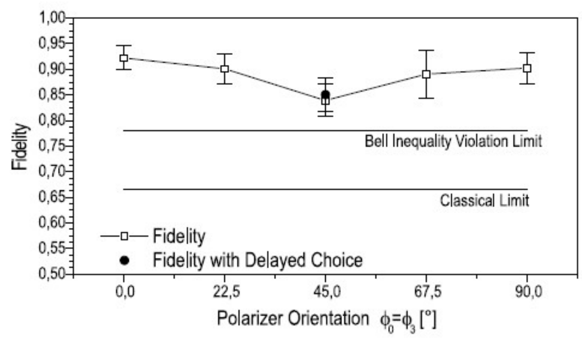

Figure 14: Experimental results of the entanglement swapping reported in Ref. (Jennewein et al., 2001). Data points show fidelity which was observed through correlation measurement ofo photons 1 and 4. Data shown with white square are victor's Bell state measurements that spacelike separated from Alice and Bob and filled black circles are ictor's Bell state measurements that timelike separated from Alice and Bob. As you can see the minimum of fidelity is above the limit achievable by classical swapping methods.

This was the first attempt of the realization of delayed-choice entanglement swapping.

4.3.2 Delayed-choice entanglement swapping

A refined and more realistic realization of Peres' proposal was reported in (Ma ... say the refs[13]). The layout of this experiment is shown in figure 15.

As you can see this setup is much more complicated and uses more optical instruments for realizing the delayed-choice entanglement swapping experiment. It has BBO crystal and PDC method for creating entangled pair of photons. Green

[13] Ma, X.-S., S. Zotter, J. Ko er, R. Ursin, T. Jennewein,C Brukner, and A. Zeilinger, 2012, Nature Phys. 8, 479.

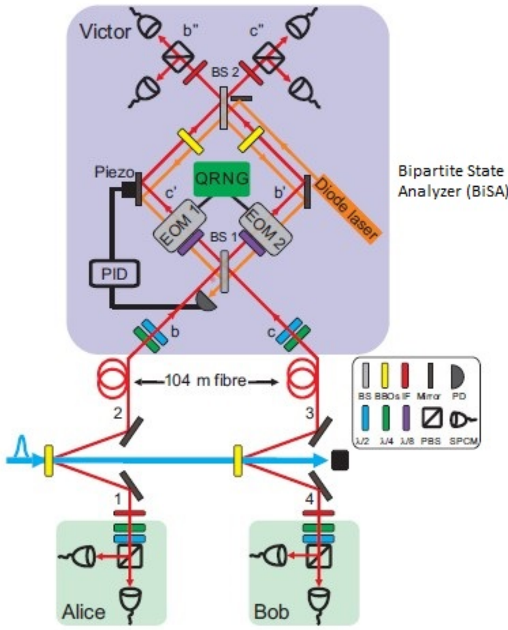

Figure 15: Experimental setup of delayed-choice entanglement swapping reported in (Ma, X.-S., S. Zotter, J. Ko er, R. Ursin, T. Jennewein, C Brukner, and A. Zeilinger, 2012, Nature Phys. 8, 479.) Note that we just need to understand the basic structure of this setup. Some instruments were implemented for stabilizing and compensating of phase noise of photons.

areas are the places where Alice and Bob are and the measurements of Alice and Bob takes place there. The purple box is what we call ”BiSA”, Bipartite State Analyzer which measures photons based on random and delayed-choice. Random measurement between BSM(Bell state measurement) and SSM(Separable-state measurement) done on 2 and 3 photons.The QRNG (Quantum Random Number Generator) was implemented at the heart of the box is used to choose between the BSM or SSM measurements. A fiber of length 104 m was implemented between ”BiSA” and the creating area in order to make enough delay for Victor and to ensure that Alice and Bob had done their measurements after photons 2 and 3 arrive the purple box. ”BiSA” also contains high-speed tunable beam-splitters and other optical instruments[14] [15] [16] [17] that we will mention at footnote.

[14]BS: Beam Splitter

[15]PBS: Polarization Beam Splitter

[16]SPCM: Single Photon Counting Module

[17]$\frac{\lambda}{2}$, $\frac{\lambda}{4}$, $\frac{\lambda}{8}$ are waveplates or retarders. Which are optical devices that alter the polarization state of a light wave travelling through them.

Let's see how this setup works:

Two pair of entangled photons are created By BBO crystals in this state: $|\psi\rangle_{1234} = |\psi^-\rangle_{12} \otimes |\psi^-\rangle_{34}$ and photons 1 and 4 go toward Alice and Bob and they measure it with a PBS (Polarization Beam Splitter).Photons 2 and 3 were send into the red fiber in order to make the delay(which lasts about 520 ns), then Victor actively chose the measurement by a QRNG (Quantum Random Number Generator) and do the measurement.

Both the choice and the measurement of photons 2 and 3 were in the time-like future of the registration of photons1 and 4. This projected the state of the two already registered photons, 1 and 4, onto either an entangled or a separable state.

> "The diagram of the temporal order of the relevant events is shown in figure 16. For each successful run (a 4- fold coincidence count), both Victor's measurement event and his choice were in the time-like future of Alice's and Bob's measurements."(Taken from[18])

And what we can conclude from this realization is:

> "In that experiment, the existence of entanglement was verified by measuring the state fidelities and the expectation values of entanglement witness operators (Refrencing[19]). It was found that whether photons 1 and 4 were entangled or separable only depended on the type of the measurements Victor implement, not on the temporal order." (Taken from[20])

[18]arXiv:1407.2930v3

[19]Guhne, O. and G. Toth, 2009, Phys. Rep. 474, 1.

[20]arXiv:1407.2930v3

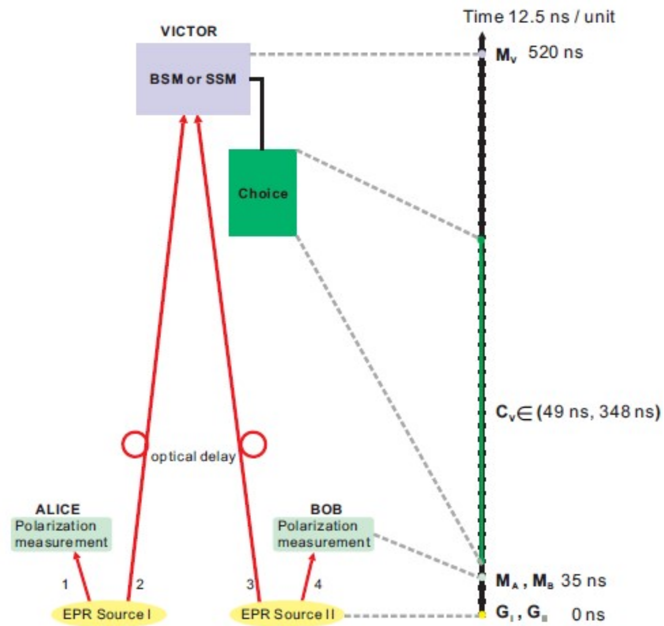

Figure 16: Time diagram of the delayed-choice entanglement swapping experiment reported in Ref. (Ref Ma, X.-S., S. Zotter, J. Ko er, R. Ursin, T. Jennewein,C Brukner, and A. Zeilinger, 2012, Nature Phys. 8, 479.); figure taken therefrom. Two entangled photon pairs (1 & 2 and 3 & 4) were generated by EPR sources I and II (events GI and GII) at 0 ns. Alice and Bob measured the polarization of photons 1 and 4 at 35 ns (events MA and MB). Photons 2 and 3 were delayed and sent to Victor who chose (event CV) to perform a Bell-state measurement (BSM) or a separable-state measurement (SSM) (event MV). Victor's choice and measurement were made after Alice's and Bob's polarization measurements.