## **Introduction to General Relativity - HW4 - 401208729**

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$$\begin{array}{c|c} Q1. & ? \\ \hline \nabla_{\mathbf{g}} g \stackrel{?}{=} 0 \end{array}$$

By definition of covariant derivative on forms:

We find the second term, the third term is the

Same with a po exchange, since both Tru and gry

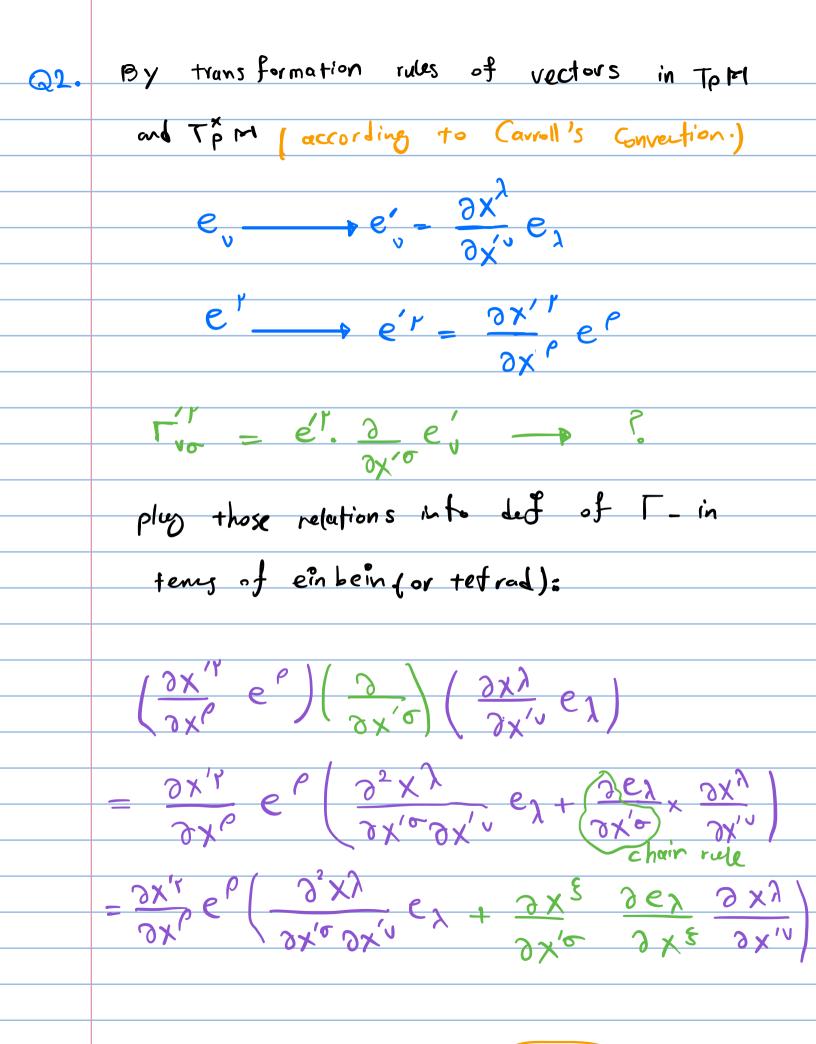
are symmetric under parov.

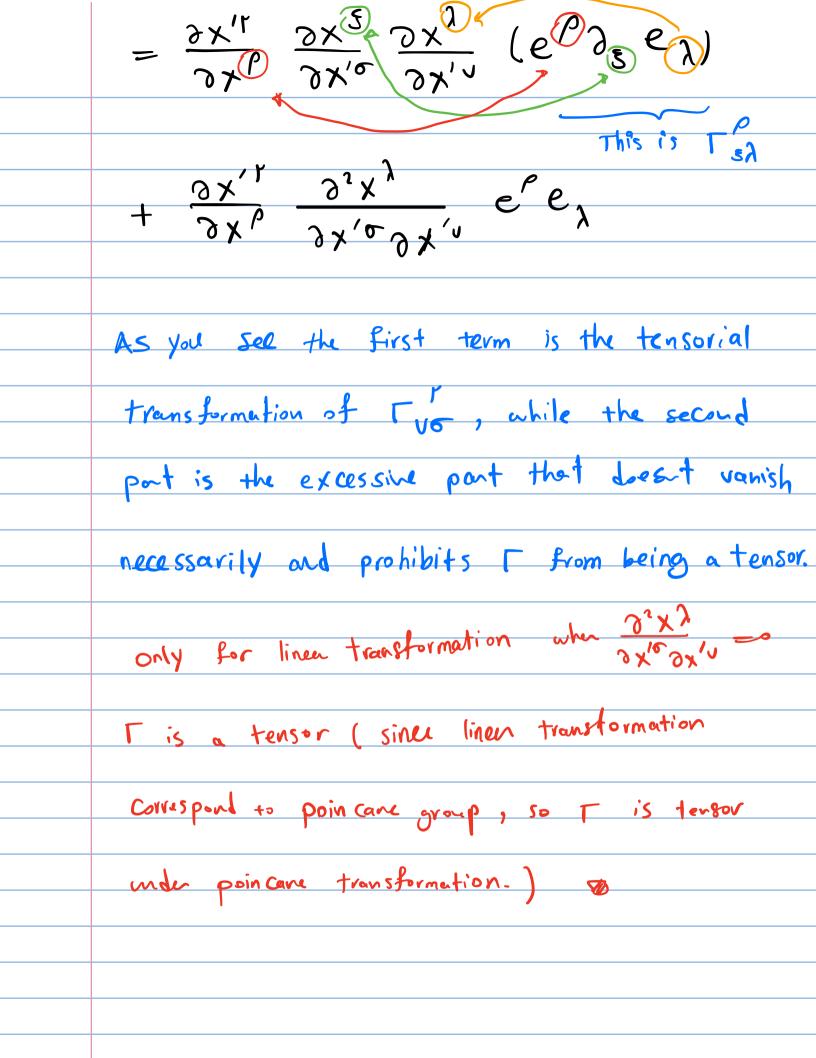
$$\frac{1}{1} \frac{1}{1} \frac{1}$$

$$\Gamma_{\beta\alpha}^{\lambda} g_{\lambda\sigma} = \frac{1}{2} \delta_{\alpha}^{\beta} (\partial_{\beta} g_{\alpha\beta} + \partial_{\alpha} g_{\beta\beta} - \partial_{\beta} g_{\alpha\beta})$$

Now since 
$$\sigma_{\gamma}(\delta_{p}^{\alpha}) = 0$$
 (sing it's constant tensor) we can conclude that examination =  $\frac{1}{2}$  ( $\frac{$ 

1) since 9 por = 90B, these terms Cancel. 2) similarly for these tems. 3) they add up to op gao which exactly cancels the first line ∇β 9ao = 0 Ø





Q3. Consider 
$$\begin{cases} x = u + V \\ y = u - V \quad \text{in 3D flat space (IR}^3). \end{cases}$$

$$Z = 2uv - w$$

In usual Catesian Cordinates, tangent vectors ane

$$g^{n} = \frac{3x}{3} \frac{3n}{3x} + \frac{3\lambda}{3} \frac{3\lambda}{3\lambda} + \frac{35}{3} \frac{3\pi}{35}$$

In verse transformation 
$$\begin{cases} u = \frac{1}{2}(x + y) \\ v = \frac{1}{2}(x - y) \end{cases}$$

$$\frac{\partial u = \partial x + \partial y + (x - y) \partial z}{\partial w = -\partial z}$$

Replace (du, dv, dw) by (eu, ev, ew) and (dx, dy, dz) by (ex, ex, ez)

to Find the relation in the questions convention.

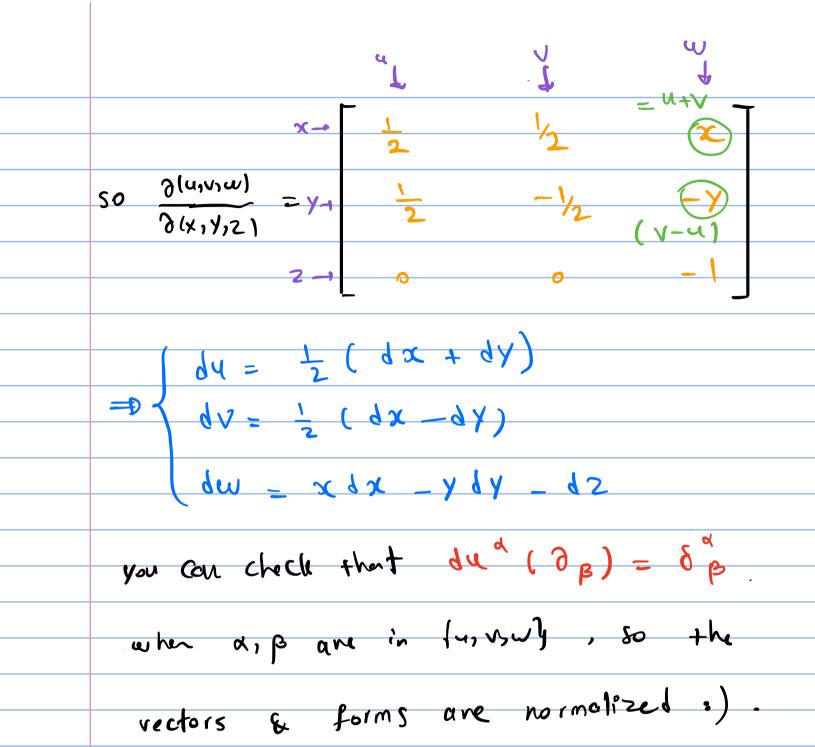
For dual basis

so we need inverse transformations,

And Z = 24v -w =

$$\omega = 2 \times \frac{1}{4} (\alpha - \gamma) (\alpha + \gamma) - Z$$

$$= 0 \quad \omega(\alpha 1 \gamma / 2) = \frac{1}{2} (\alpha^2 \gamma^2) - Z$$



Get's look at 
$$G_{ij} = \frac{1}{2}g^{0d}(g_{acij} + g_{ajj} - g_{ajj})$$

since the metric is biogened as  $d = 6$ 

and since non of metric's components depend on  $\phi$ 

explicitly, the calculation is very easy!

$$G_{ij} = \frac{1}{2} \times g^{0}$$

$$G_{$$

$$\Gamma_{ij}^{\dagger} = \frac{1}{2} g^{\dagger a} \left( g_{ai,j} + g_{aj,i} - g_{ij,a} \right)$$

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$$\Gamma_{ij}^{\dagger} = \frac{1}{2} g^{\dagger a} \left( g_{ai,j} + g_{aj,i} - g_{aj,i} - g_{aj,i} - g_{aj,i} \right)$$

$$\Gamma_{ij}^{\dagger} = \frac{1}{2} g^{\dagger a} \left( g_{ai,j} - g_{aj,i} - g_{aj,i}$$

152 de 2 + 52 6 162 (6) SO simple!  $S = \begin{cases} y_{2} \\ y_{1} \end{cases} \left( \left( \frac{qy}{q\theta(y)} \right)_{5} + \sum_{5} \theta \left( \frac{qy}{q\phi(y)} \right)_{5} \right)_{5}$ when X/(X)=(O(X)) is a cure parametrized by > E( >01 >1). To exterimize the length, we variate B(N) \$ (1) by  $\begin{cases} \theta(\lambda) \longrightarrow \theta(\lambda) + \delta\theta(\lambda) & \text{with} & \delta\theta(\lambda) \\ \phi(\lambda) \longrightarrow \phi(\lambda) + \delta\phi(\lambda) & \text{with} & \delta\phi(\lambda) \\ \delta\phi(\lambda) & \delta\phi(\lambda) & \delta\phi(\lambda) \end{cases} = 0$ 84(x)/x, x,=0 when at the onl, the geolesic path results from usur Eden Lagne equations.  $\frac{d}{d\lambda} \frac{\partial L}{\partial \lambda} = \frac{\partial L}{\partial \lambda} + \frac{\partial L$ ( (de) 2 + sin 2 + (de) 2) is like the Lagragian, and I pavameter is like time. So the situation why NOT takin is quite similar to Classical Mechanics. *i*+5

Span root into d 2 
$$\frac{1}{2}$$
  $\frac{1}{2}$   $\frac{1}{$ 

when 
$$\frac{d}{d\lambda} \sin^2 \theta = \frac{10}{d\lambda} \frac{\partial}{\partial \theta} \left( \sin^2 \theta \right) + \frac{10}{d\lambda} \frac{\partial}{\partial \theta} \left( \sin^2 \theta \right)$$

$$= 2 \sin \theta \cos \theta \times \frac{10}{d\lambda}$$

$$\frac{1}{2\sqrt{3}}\left(\frac{\partial L}{\partial \lambda}\right) = \left(4 \sin \theta \cos \theta\right) \frac{d\theta}{d\lambda} \frac{d\theta}{d\lambda}$$

$$+ 2 \sin^2 \theta \frac{d^2\theta}{d\lambda^2}$$

And of also vanishes, the fore

4 sind Co so 1/2 / 25in2 0 1/4 = 0

Divide by 25in 20 to Set:

$$\frac{d^2d}{dx^2} + 2 \cot \theta \frac{d\theta}{dx} = 0$$

Note: Lagran was l- ((do)2+sin2 (dp)2)2

actually, but since zeros of 81# and	
equel to zeros of 8# 50 we doesn't	
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t with surchs!	
need to carry square root with surrely	
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