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In order to drive the σ^2 for this problem, first we know that:

$$< x(t) >= Nl(p-q) \tag{1}$$

As it calculated in Lecture Notes of Random Walk . Next we should Find $< x^2(t) >$:

$$< x^2(t) >= \left\langle l\left(\sum_{i=1}^{N} a_i\right) l\left(\sum_{j=1}^{N} a_j\right)\right\rangle = l^2\left\langle \sum_{i=1}^{N}\sum_{j\neq i}^{N} a_i a_j + \sum_{i=1}^{N} a_i^2\right\rangle = l^2\left\langle \sum_{i=1}^{N}\sum_{j\neq i}^{N} a_i a_j\right\rangle + l^2\left\langle \sum_{i=1}^{N} a_i^2\right\rangle$$
$$l^2\sum_{i=1}^{N}\sum_{j\neq i}^{N} < a_i a_j > + l^2\sum_{i=1}^{N} < a_i^2 >$$

Now, Since a_i is a random variable that has values 1 and -1 with specific probablities p and q, a_i^2 is always equal to 1 and since a_i is independent of a_j for $i \neq j$ it's obvious that we can write $< a_i a_j >$ $= < a_i >< a_j >$. By using this we can write:

$$l^2\left(\sum_{i=1}^{N}\sum_{j\neq i}^{N} < a_i a_j > + \sum_{i=1}^{N} < a_i^2 >\right) = l^2\left(\sum_{i=1}^{N}\sum_{j\neq i}^{N} < a_i >< a_j > + \sum_{i=1}^{N} 1\right)$$

For each i, j is summed over N-1 times (j=i is neglected) and recall that $< a_i >= p - q$ so we can rewrite the whole first term as:

$$l^2\left(\sum_{i=1}^{N}\sum_{j\neq i}^{N} < a_i >< a_j > + \sum_{i=1}^{N} 1\right) = l^2(N(N-1)(p-q)^2 + N)$$

So by : $\sigma^2 =< x^2(t) > - < x(t) >^2$, We write:

$$\sigma^2 = l^2(N^2(p-q)^2 - N(p-q)^2 + N) - N^2 l^2 (p-q)^2 = Nl^2(1-(p-q)^2) = Nl^2(1+p-q)(1-p+q)$$

But $p + q = 1$:

$$\sigma^2 = Nl^2(1+p-q)(1-p+q) = Nl^2(2p)(2q) = 4Nl^2pq$$

Which is like the equation that mentioned in lecture note but with a little deiffrence: I replaced $\frac{t}{\tau}$ with N which is the number of steps. So I prove that formula for the standard variance in Random Walk.