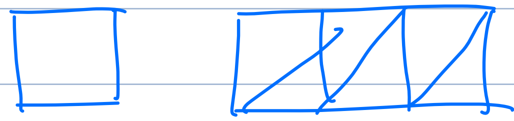
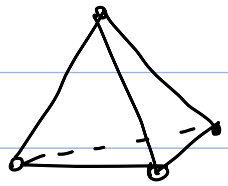
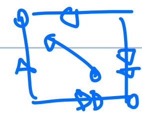
$H_\bullet(D^3)$

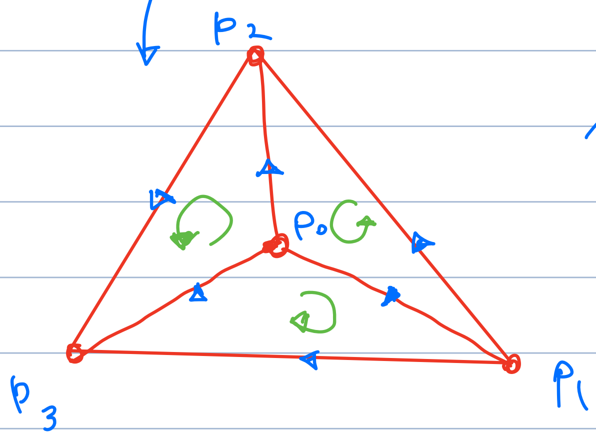
$H_\bullet(\text{Mobius})$ ×



 $\cong S^2$ 

$K = \{ p_0, p_1, p_2, p_3, p_0p_1, p_1p_2, p_2p_3, p_1p_3, p_0p_3, p_0p_2,$

$p_0p_1p_2, p_0p_1p_3, p_0p_2p_3, p_1p_2p_3 \}$



$\phi \underbrace{(p_0p_1p_2p_3)}_{\text{3-complex}} \to D^2$

$H_2 = \dfrac{\ker \partial_2}{\operatorname{Im} \partial_3}$

$H_1(D^2) = H_1(S^2)$

$H_1 = \dfrac{Z_1(K)}{B_1(K)} = \dfrac{\ker \partial_1}{\operatorname{Im} \partial_2}$

$C_1 \xrightarrow{\partial_1} C_0$

6 $p_0p_1 \ldots$ ↓ 4 $p_0 \ldots p_3$

$\partial_1 (\overset{a}{p_0p_1} \quad \overset{b}{p_1p_2} \quad \overset{c}{p_2p_3} \quad \overset{d}{p_1p_3} \quad \overset{e}{p_0p_3} \quad \overset{f}{p_0p_2})$

$a(p_1 - p_0)$

$b(p_2 - p_0)$

$P_0\,(-e - a - f)$

$P_1\,(a - b - d)$

$P_2\,(b - c + f)$

$P_3\,(c + d + e)$

(∂_1) سطرا × ستون

$$(\partial_1)_{4\times 6} = \begin{array}{c|cccccc} & a & b & c & d & e & f \\ \hline P_0 & -1 & 0 & 0 & 0 & -1 & -1 \\ P_1 & 1 & -1 & 0 & -1 & 0 & 0 \\ P_2 & 0 & 1 & -1 & 0 & 0 & 1 \\ P_3 & 0 & 0 & 1 & 1 & 1 & 0 \end{array}$$

Gauss

$Z_1 = ?\quad \ker(\partial_1) \longrightarrow (\partial_1)_{4\times 6}\begin{pmatrix} x \\ - \\ - \\ - \end{pmatrix}_{6\times 1} = \begin{pmatrix} 0\\0\\0\\0 \end{pmatrix}_{4\times 1}$

$$\begin{array}{cccccc} a & b & c & d & e & f \end{array}$$
$$\begin{pmatrix} -1 & -1 & 0 & 0 & 0 & 1 \\ -1 & -1 & -1 & 0 & 1 & 0 \\ 0 & -1 & -1 & 1 & 0 & 0 \end{pmatrix}$$

$\left\langle \begin{matrix} f - a - b, \\ e - a - b - c \\ d - c - b \end{matrix} \right\rangle = \ker \partial_1$

$\partial_2\,(\overset{A}{P_0P_1P_2}\quad \overset{B}{P_0P_1P_3}\quad \overset{C}{P_0P_2P_3}\quad \overset{D}{P_1P_2P_3})$

$A:\ P_1P_2 - P_0P_2 + P_0P_1$

$C:\ P_2P_3 - P_0P_3 + P_0P_2$

$B:\ P_1P_3 - P_0P_3 + P_0P_1$

$D:\ P_2P_3 - P_1P_3 + P_1P_2$

(∂_2) سطرا × ستون

$$\begin{array}{cc|cccc} & & \overset{A}{P_0P_1P_2} & B & C & D \\ \hline a & P_0P_1 & 1 & 1 & 0 & 0 \\ b & P_1P_2 & 1 & 0 & 0 & 1 \\ c & P_2P_3 & 0 & [illegible] & 1 & 1 \end{array}$$

$$(\partial_2)_{6\times 4} = \begin{array}{cc} d & P_1P_3 \\ e & P_0P_3 \\ f & P_0P_2 \end{array} \begin{pmatrix} 0 & 1 & 0 & -1 \\ 0 & -1 & -1 & 0 \\ -1 & 0 & 1 & 0 \end{pmatrix}$$

RR $\rightarrow$

$$\begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \\ & & & \\ & & & \\ & & & \end{pmatrix}$$

3 columns

$$\mathrm{Im}(\partial_2) = \langle (a+b-f), (a+d-e), (c-e+f) \rangle$$

$$\langle f-a-b,\ e-a-b-c,\ d-c-b \rangle = \ker \partial_1$$

$(P_0 \dots P_3)$

$$\cancel{a+b}-\cancel{a}-\cancel{b} = b \qquad \cancel{a+d}-\cancel{a}-b-c$$

$$H_1(D^2) = H_1(S^2) = \frac{\langle \cancel{f-a-b},\ e-a-b-c,\ d-c-b \rangle}{\langle \underbrace{\cancel{a+b-f}}_{=\{0\}},\ \underbrace{a+d-e}_{e=a+d},\ c-e+f \rangle}$$

$f = a+b$

$$= \frac{\langle d-b-c \rangle}{\langle c-e+f \rangle} \rightarrow c = e-f$$

$a+d \quad a+b$

$$\begin{cases} c = \cancel{a}+d - \cancel{a} - b \\ c = d-b \end{cases}$$

$$= \langle \cancel{A} - \cancel{b} - \cancel{A} + \cancel{b} \rangle = \{0\}$$

$$\boxed{H_1(D^2) = H_1(S^2) = \{e\}}$$

$$H_2(S^2) \rightarrow \frac{\text{Ker } \partial_2}{\text{Im}(\partial_3)} \overset{!}{=} \frac{(\dots)}{\{e\}} = \mathbb{Z}$$

$$H_2(D^2) = \frac{\ker \partial_2}{\text{Im}(\partial_3)}$$

$$\ker \partial_2 = \langle \begin{array}{cccc} A & B & C & D \\ (-1 & 1 & -1 & 1) \end{array} \rangle = \langle D - C + B - A \rangle$$

$$\partial_3 : \overset{C_3}{C_3} \longrightarrow C_2 \qquad \partial_3(1 p_0 p_1 p_2 p_3))$$

$$(p_0 p_1 p_2 p_3) \longrightarrow \text{2-smplx} \qquad = \overset{D}{\underbrace{p_1 p_2}} p_3 - p_0 \overset{C}{\overbrace{p_2 p_3}}$$

$$+ \underbrace{p_0 p_1 p_3}_{B} - \underbrace{p_0 p_1 p_2}_{A}$$

$$(\partial_3)_{4\times 1} = \begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} p_0 p_1 \\ \begin{pmatrix} -1 \\ +1 \\ -1 \\ +1 \end{pmatrix} \end{array}$$

$$\{0\}$$

$$H_2(D^2) = \frac{\langle D+B-C-A \rangle}{\langle D+B-C-A \rangle} = \{e\},$$

$$H_k(S^n) = \begin{cases} \mathbb{Z} & k=0,\ k=n \\ \{e\} & \text{others} \end{cases}$$

n-s

$$H_k(D^n) = \begin{cases} \mathbb{Z} & k=1 \\ \{e\} & \text{others} \end{cases}$$

S^{n-1}

$\{p_0, \dots, p_n,\ p_0p_1,\ p_0p_2, \dots, p_0p_n \mid p_1p_2, \dots, p_1p_n \mid p_2p_3 \dots \mid$ $\binom{n+1}{2}$

$n+1$

$(\dots \mid p_{n-1}p_n,$

$p_0p_1p_2, \dots p_0p_1p_n \mid p_0p_2p_3 \dots p_0p_2p_n \mid \dots \mid \dots$

$p_{n-2}p_{n-1}p_n \mid$ $\binom{n+1}{3}$

$\binom{n+1}{n} = (n+1) \dots (n-1)$-simplex $\rightarrow (p_0 \dots p_{n-1}), (p_1 \dots p_n)$

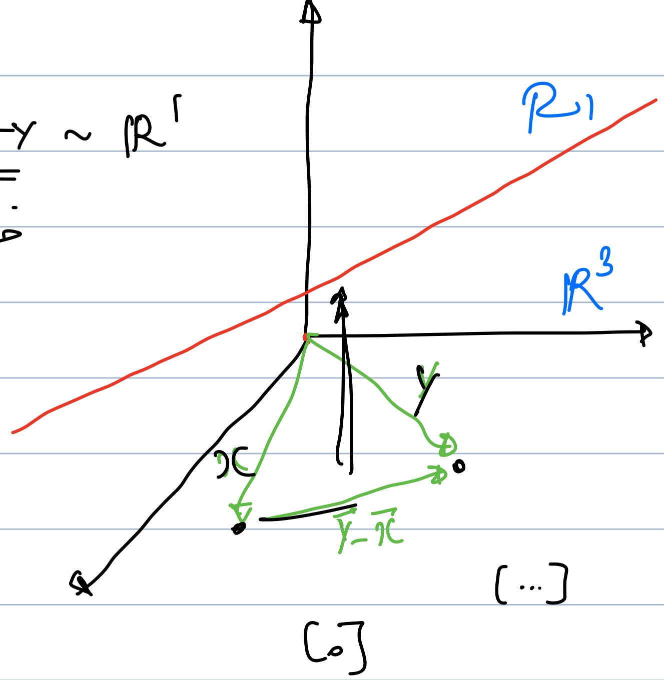
1 D^n: n-s $(p_0 \dots p_n)$

$n=3$

$$\binom{n+1}{1} + \binom{n+2}{2} + \dots + \binom{n+1}{n} = 2^{n+1} - 2$$

$2^4 - 2 = 16 - 2 = 14$

$\dfrac{\mathbb{R}^3}{\mathbb{R}^1} \rightarrow \underline{\underline{x \sim y}}$ if $\underset{\text{خطی}}{\underline{\underline{x-y}}} \sim \mathbb{R}^1$



$\underbrace{\dfrac{R^3}{R^1}}$ 2D - subspace.

$\pi : R^3 \longrightarrow \dfrac{\mathbb{R}^3}{\mathbb{R}}$ $\quad \pi(\vec{r}) = \underline{\underline{[\vec{r}]}}$

$\pi(n_1) = [n_1] = \{n_1 + n \mid n \in R_1\}$, $\quad n' \in R_1 \quad [o] = \{\vec{o} + \vec{n} \mid n \in R_1\}$

$\dfrac{\mathbb{R}^3}{\mathbb{R}^1} \leftarrow \quad \dfrac{G}{H} \quad [g] = gH = \{g + h \mid h \in H\}$

$\dfrac{\mathbb{R}^3}{\mathbb{R}} \ni [\vec{r}] = \{\vec{r} + \vec{n} \mid n \in \overset{\text{ضرایب}}{R^1}\}$

$T : V_1 \longrightarrow V_2$

$\dim(V_1) - \ker T = \dim(\operatorname{Im} T)$

$\underbrace{\dim(\mathbb{R}^3)}_{3} - \underbrace{\dim \ker \pi}_{=1} = \dim\left(\operatorname{Im}\left(\dfrac{\mathbb{R}^3}{\mathbb{R}}\right)\right)$

$= \dim\left(\operatorname{Im} \dfrac{\mathbb{R}^3}{\mathbb{R}}\right) = 3 - 1 = 2$

$\dfrac{V_1}{V_2} \rightarrow$ خارج‌قسمت $\quad \dfrac{G}{H}$ ✓

$\dfrac{R_1}{T}$

$$\frac{\mathbb{R}^3}{\mathbb{R}^2} \simeq \mathbb{R}_1$$

dim (فضا) $-$ dim ker T = dim تصویر

به‌عنوان تمرین ← $\frac{\mathbb{R}^3}{\mathbb{R}^1}$, $\left(\frac{V_1}{V_2}\right)$ عمل + و ضرب اسکالرش خوش تعریفه

$[v_1], [v_2] \in \frac{\mathbb{R}^3}{\mathbb{R}^1}$

v_1'

$[v_1] + [v_2] = \underline{[v_1 + v_2]}$

$r_1[v_1] = \ ? \longrightarrow [r_1 v_1]$

$(g_1)H \ (g_2)H = (g_1 g_2)H$

$(g_1'H)(g_2H) = g_1' g_2 H$

$(g_q \times g_q)$ $\quad H_1$ $\quad$ حذف

van Kampen

$g = \dots$ $\quad$ sequence Mayer-Vietoris

Let M be a connected topological manifold of dimen $n \geq 3$,

$x \in M$: $\pi_1(M) \cong \pi_1(M \setminus \{x\})$ $\quad n \geq 3$

seifert van Kampen: $\pi_1(X) \longrightarrow$

$X = U_1 \cup U_2$ $\quad U_1 \cap U_2 \neq \emptyset$

open subset

+ path connected

$x_0 \in U_1 \cap U_2$ as base point.

$$\pi_1(X) = \pi_1(U_1) *_{\pi_1(U_1\cap U_2)} \pi_1(U_2)$$

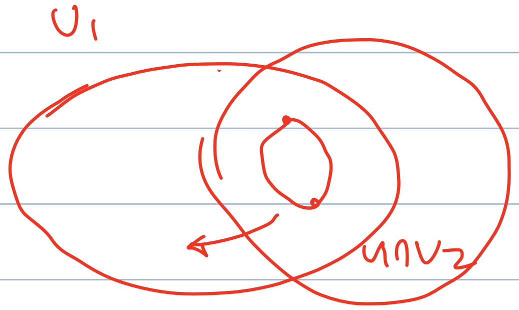
$$= \frac{\pi_1(U_1) * \pi_1(U_2)}{\sim \overline{\overline{F(\pi_1(U_1\cap U_2))}}}$$

$G_1 = \langle a_1, b_1, c_1 \mid r_1, r_2 \rangle$

$G_2 \ \langle a_2, b_2 \mid \tilde{r}_1, \vec{r}_2 \rangle$

$G_1 * G_2 = \langle a_1, b_1, c_1, a_2, b_2 \mid r_1, r_2, \tilde{r}_1, \vec{r}_2 \rangle$

$$\begin{tikzcd}
 & \pi_1(U_1) \arrow[dr] \arrow[drr, "k_*"] & & \\
\pi_1(U_1\cap U_2) \arrow[ur, "i_*"] \arrow[r, dashed, "F"] \arrow[dr, "j_*"'] & \pi_1(U_1) * \pi_1(U_2) \arrow[rr, "\Phi"] & & \pi_1(X) \\
 & \pi_1(U_2) \arrow[ur] \arrow[urr, "l_*"'] & &
\end{tikzcd}$$



$$U_1\cap U_2 \xhookrightarrow{i} U_1$$

$$F(\gamma) = \underbrace{(i_*\gamma)}_{\pi_1(U_1)}{}^{-1} \underbrace{(j_*\gamma)}_{\pi_1(U_2)}$$

$F: \pi_1(U_1 \cap U_2) \longrightarrow \pi_1(U_1) * \pi_1(U_2)$

$$ncl_G(S) = \bigcap_{S \subseteq N \trianglelefteq G} N$$

Fix $x \in M$, topological manifold, $\forall x \in M$ U homeomorphic $\mathbb{R}^n$.

$V = M \setminus \{x\}$

U open ✓ path connected

V open $\to$ is path connected ?

$V \xrightarrow{\text{Homeomorphic}} \mathbb{R}^n - \{p\}$

$\mathbb{R}^n - \{p\}$ $\underset{?}{\longrightarrow}$ $n \geq 3$ $\qquad \mathbb{R}^n - \{p\} \xrightarrow[\text{retracts}]{\text{deformation}} S^{n-1}$

$H = \underbrace{(\mathbb{R}^n - \{p\})}\times[0,1] \longrightarrow S^{n-1}$ $\quad S^1$

$$H(x,t) = (1-t)\vec{x} + t\frac{\vec{x}}{\|x\|}$$

$\|x\| = 1 \to S^{n-1}$

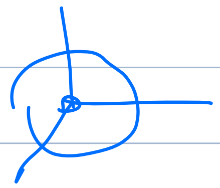
$U \cap V \overset{?}{=} U \setminus \{x\}$

$$\pi_1(M) = \frac{\pi_1(U) * \pi_1(V)}{?\quad \{e\}} = \underbrace{\pi_1(U)}_{\{e\}} * \pi_1(V)$$

$$\overline{\mathcal{F}(\underbrace{\pi_1(U\cap V)}_{\mathbb{R}^n-\{p\}})}\quad \{e\}$$

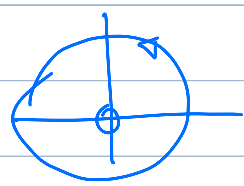
$$= \pi_1(\mathbb{R}^n - \{p\})$$

$$= \pi_1(M - \{p\})$$



$n \geq 3$

تمرین اول ... √



π_1

show that S^{n-1} is <u>not</u> retract D^n !

$r : D^n \longrightarrow S^{n-1}$ ✓

$$\begin{tikzcd} S^{n-1} \arrow[r, hook, "i"] \arrow[rr, bend right, dashed, "id"'] & D^n \arrow[r, "r"] & S^{n-1} \end{tikzcd}$$

$\downarrow \; Hom_{H}(-,-)$

$$\begin{tikzcd} H_q(S^{n-1}) \arrow[r, "i_*"] \arrow[rr, bend right, "id_*"'] & \underbrace{H_q(D^n)} \arrow[r, "r_*"] & H_q(S^{n-1}) \end{tikzcd}$$

$\times$

$q = n-1 \qquad H_q(S^{n-1}) = H_{n-1}(S^{n-1}) = \mathbb{Z}$

$$\begin{tikzcd} \mathbb{Z} \arrow[r, "i_*"] \arrow[rr, bend right, "id_*"] & \{0\} \arrow[r, "r_*"] & \mathbb{Z} \end{tikzcd}$$

$id(\mathbb{Z}) : \mathbb{Z} \to \mathbb{Z}$

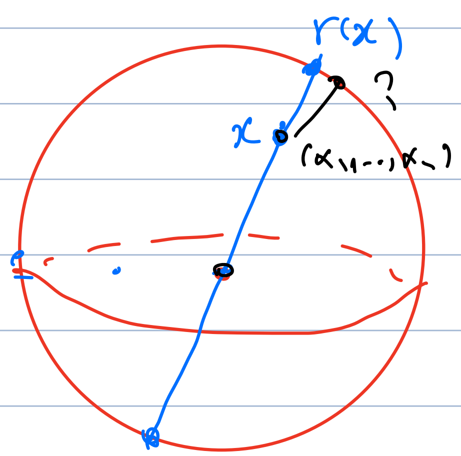
$r_* \circ i_* \neq id_*$

$i_*(?) = 0$

$r_*(0) = \mathbb{Z}$

Brouwer

Fixed point thm: $f : D^n \to D^n$ پیوسته $\quad x_0 \in D^n$, $f(x_0) = x_0$



ایده: فرض خلف کنید.

$x \in D^n$

$r(x)$

$x \in \partial D^n \to r(x) = x$

پس تعریف دیم $S^{n-1} = r(x)|_{\partial D^n}$ پس شرط retraction را دارد.

$\cancel{D^n \to S^{n-1}}$, $r(x)$ پیوسته است.

Σ : genus g

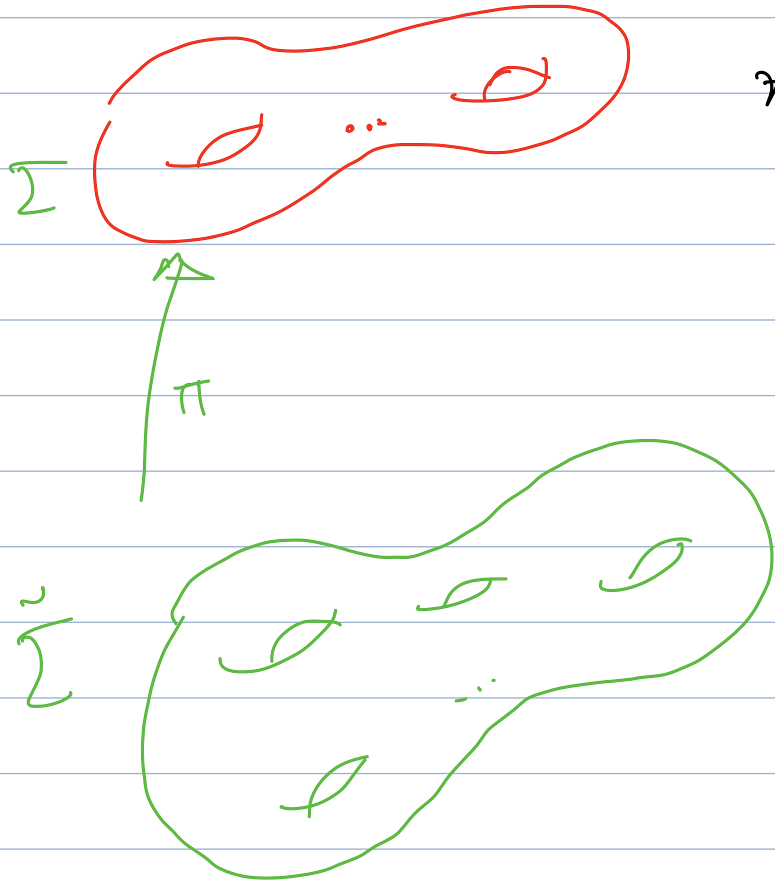
$\tilde{\Sigma}$: d-fold cov. Σ.

الف) $\chi(\tilde{\Sigma}) = d\,\chi(\Sigma)$

ب) $\tilde{g} = dg - d + 1$

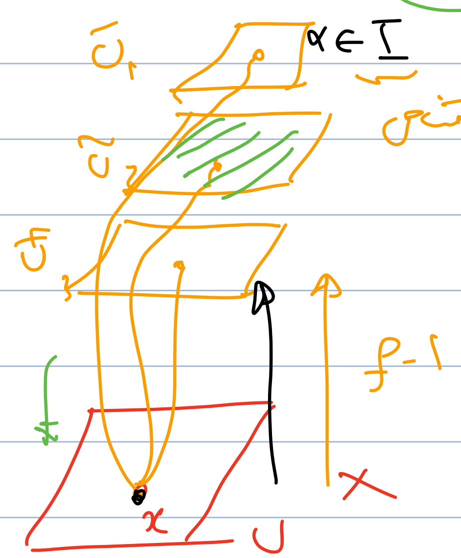
$f : \tilde{X} \longrightarrow X$

فضای کاور   فضای اصلی



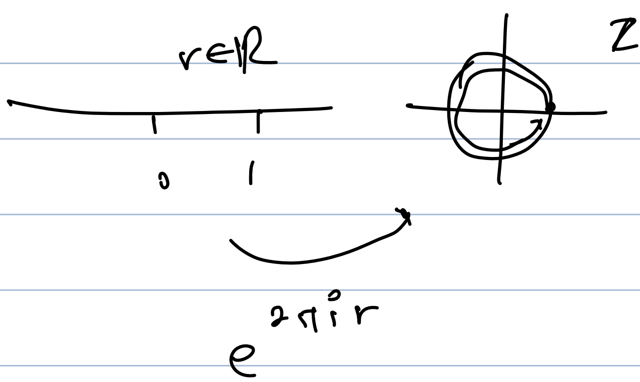
$\forall x \in X \quad \exists U$ neighborhood of x

$f^{-1}(U) = \bigsqcup_{\alpha \in I} \tilde{U}_\alpha$



$f|_{\tilde{U}_\alpha} : \tilde{U}_\alpha \longrightarrow U$

is homeomorphism.



$\Sigma : (V; E, F)$

$\{v_1, \dots, v_V\}$

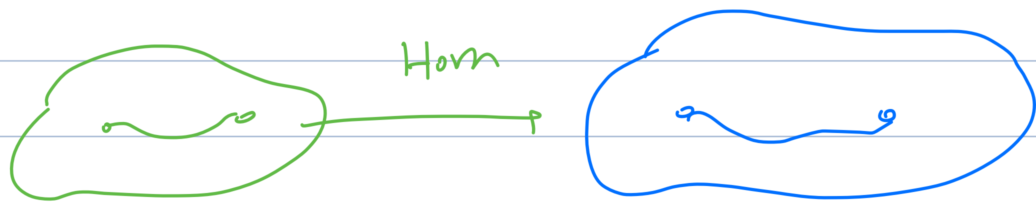
$\{e_1, \dots, e_E\}$

$\chi(K) = V - E + F$

$\pi : \tilde{\Sigma} \longrightarrow \Sigma$

$\pi^{-1}(v_i) = \{ v'_{i_1}, \dots, v'_{i_d} \} \quad 1 \leq i \leq V$

homomorphism $\pi(e_j) = \{ e'_{1,j}, \dots, e'_{d,j} \} \quad 1 \leq j \leq E$



$\pi(f_k) = \{ f_{,k}, \dots, \} \quad 1 \leq k \leq F$

$\# \tilde{V} = dV$

$\# \tilde{E} = dE \longrightarrow \chi(\tilde{\Sigma}) = d\underbrace{(V - E + F)} = d\chi(\Sigma)$

$\# \tilde{F} = dF$

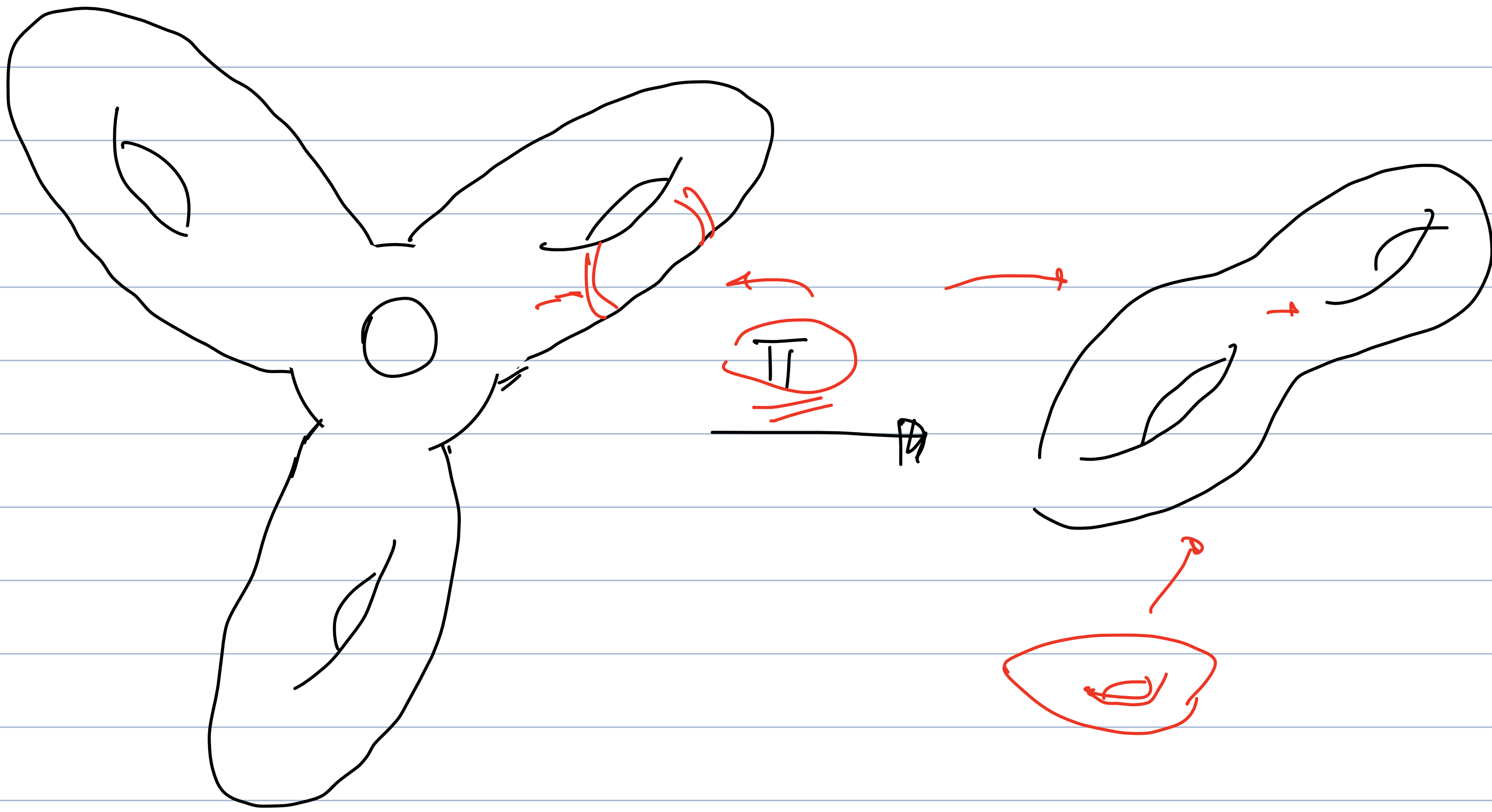
$\tilde{g} = ? \qquad \chi(\Sigma) = 2 - 2g$

$\tilde{g} = 1 - \frac{1}{2}\chi(\tilde{\Sigma}) = 1 - \frac{d}{2}(2 - 2g)$

$= 1 - d + dg$

$4 = 1 - 3 + \underbrace{3 \times 2}_{6}$ ✓

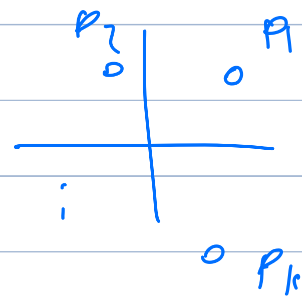
genus 4 — 3-fold cover $g = 2$



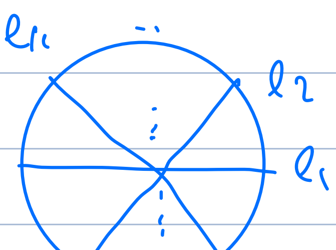
prove that X_1, X_2, X_3 are homotopy equivalent.

$X_1 = \bigvee_{i=1}^{k} S^1$

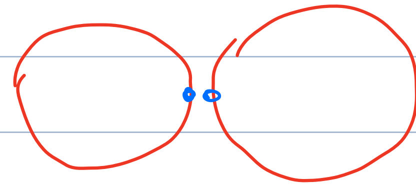
$X_2 = \mathbb{R}^2 - \{p_1, \dots, p_k\}$



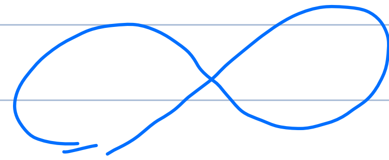
$X_3 =$

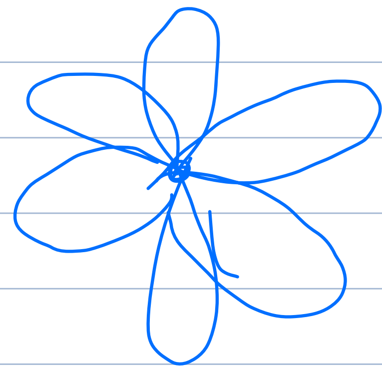


$X_1 \vee X_2 = \dfrac{X_1 \sqcup X_2}{\cdot \sim}$



$\bigvee_{i=1}^{k} S^1$ = k-bouquet





Fundamental group $\mathbb{R}^n - \{k \text{ points}\}$ $\quad n \geq 2,\ k \geq 0$

van Kampen.

$n \geq 3 \longrightarrow S^2$ ✓

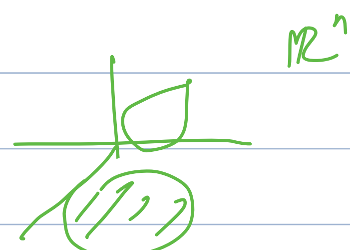
$\pi_1 = (\mathbb{R}^n - \{k \text{ points}\}) = G_{n,k}$

$k=0 \longrightarrow n \geq 2$ $\quad \mathbb{R}^n$ contractibel ...

$n \geq 2$

$\downarrow$

$G_{n,0} = \{e\}$ $\quad \cdot\ n+1,\ n+2,\ \ldots$



$n \geq 2$

$k=1$ $\quad \mathbb{R}^n - \{p\} \xhookleftarrow[\text{retn}]{\text{def.}} S^{n-1}$

$$H : (\mathbb{R}^{n\geq 2} - \{p\}) \times [0,1] \longrightarrow S^{n-1}$$

$$H(x,t) = (1-t)\vec{x} + t\frac{\vec{x}}{\|x\|}$$

$$G_{n,1} = \pi_1(S^{n-1}) = \begin{cases} \mathbb{Z} & n=2 \\ \{0\} & n \geq 3 \end{cases}$$

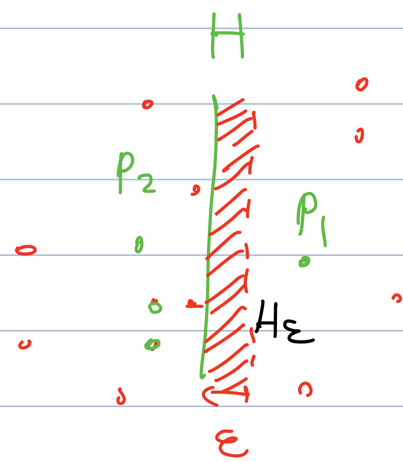
$G_{n,k}$ $(k \geq 2)$: $\mathbb{R}^n$, H, be hyperplane with does not contain $\{p_1, \dots, p_k\}$

WLOG $H = \{x_1 = 0\}$

$(x_1, \dots, x_n)$

$p_1 \in \{x_1 > 0\}$

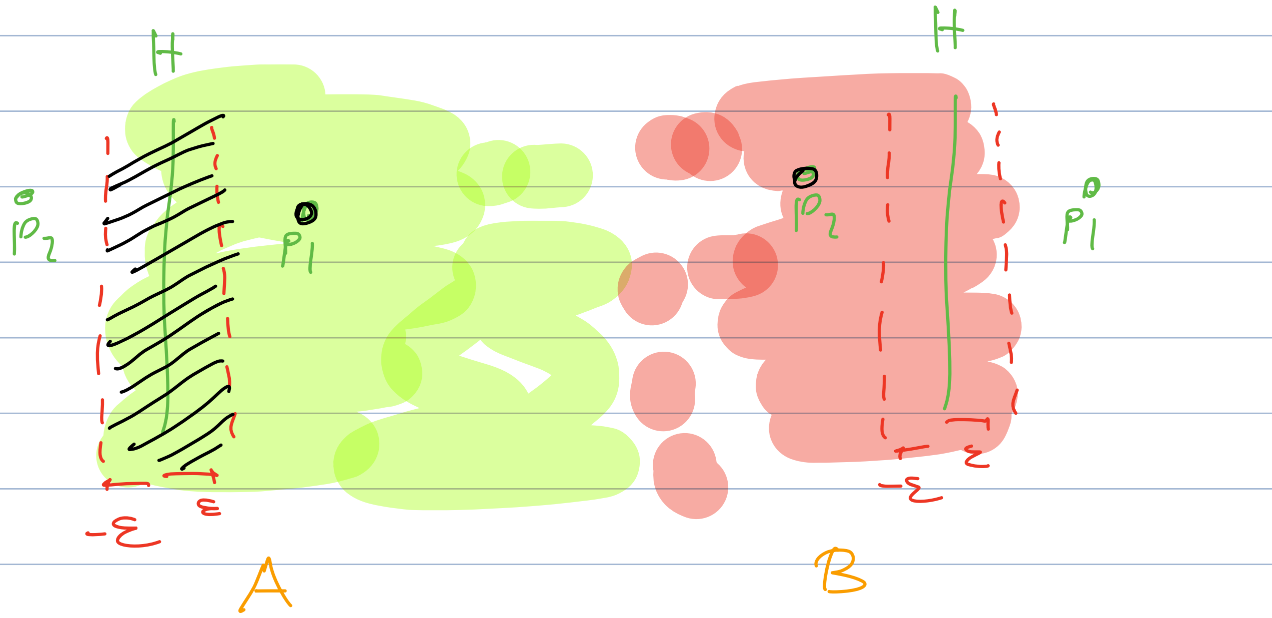
$p_2 \in \{x_1 < 0\}$



$\exists \varepsilon > 0$: $\{(x_1, \dots, x_n) \in \mathbb{R}^n \mid |x_1| < \varepsilon\} = H_\varepsilon$

$A := \{(x_1, \dots, x_n) \in \mathbb{R}^n \mid x_1 > -\varepsilon\}$

$B = \{ \quad \text{"} \quad \mid x_1 < \varepsilon \}$



$A \cup B = \mathbb{R}^n - \{k \text{ نقطه}\} \longrightarrow$ path-connected.

$A \cap B = H_\varepsilon$ path connected

$k_A \longrightarrow$ # points in A

$k_B \longrightarrow$ # " " B.

since $A \cap B = H_\varepsilon$, $p_i \notin H_\varepsilon$

$k_A + k_B = k$

$1 \leq k_A \leq k$

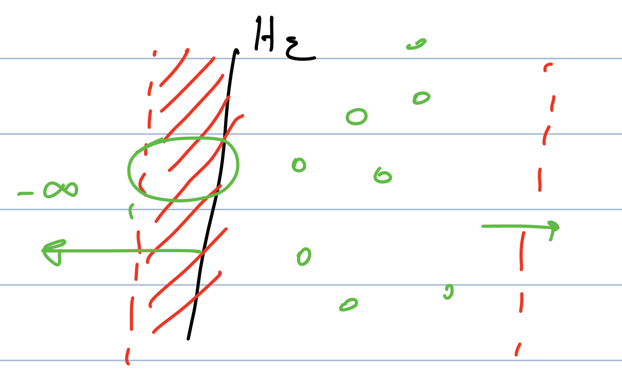
$1 \leq k_B \leq k$

$p_1 \in A$

$p_2 \in B$

$A \xrightarrow{\text{هومئومورفیسم}} \mathbb{R}^n - \{k_a \text{ نقطه}\}$

$B \xrightarrow{\quad " \quad} \mathbb{R}^n - \{k_b \text{ نقطه}\}$



(#)

$\pi_1(\mathbb{R}^n - \{k_A \text{ نقطه}\})$

$\pi_1(A) \simeq G_{n,k_A}$

$\pi_1(B) \simeq G_{n,k_B}$

$G_{2,k} = \pi_1(\mathbb{R}^2 - \{k \text{ نقطه}\}) = \dfrac{\pi_1(A) * \pi_1(B)}{F(\pi_1(H_\varepsilon))} =$

$\{e\}$

$\sim \mathbb{R}^n$

$G_{2,k} = \pi_1(A) * \pi_1(B) = G_{n,k_A} * G_{n,k_B}$ $\qquad k_A + k_B = k$

$G_{n,k_A^1} * G_{n,k_A^2}$ - -

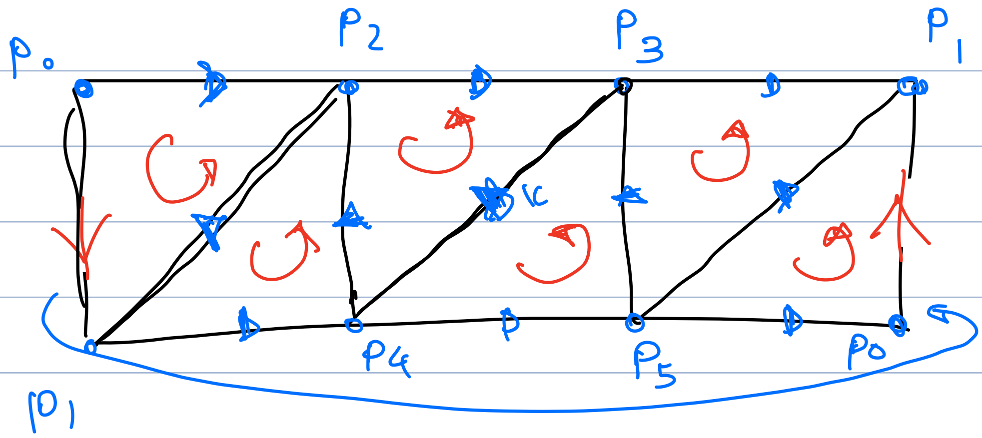
$G_{n,1} \qquad G_{n,1}$

$n = 2 \quad \mathbb{Z} \longrightarrow \bigoplus \mathbb{Z}$ (K)

$n \geq 3 \rightarrow 0 \quad \{0\} \quad \emptyset$

H_0, H_1 ([illegible]) $= \mathbb{Z}$

$H_? (\quad) = \{e\}$



CW-Complex

A. Hatcher

0-Σ $p_0, \dots, p_5$

$$H_1 = \frac{\ker \partial}{\operatorname{Im} \partial_2}$$

∂_1

a	b	c	d	e	f	g
(p_0p_1)	(p_0p_2)	(p_2p_3)	(p_3p_1)	(p_1p_4)	(p_4p_5)	(p_5p_0)

h	i	j	k	l
(p_4p_2)	(p_5p_3)	(p_1p_2)	(p_4p_3)	(p_5p_1)

$$(\partial_1)_{6\times 12} = \begin{array}{c|cccccccccccc} & a & b & c & d & e & f & g & h & i & j & k & l \\ \hline p_0 & -1 & -1 & & & & & & & & & & \\ p_1 & 1 & 0 & & & & & & & & & & \\ p_2 & & 1 & & & & & & & & & & \\ p_3 & & & & & & & & & & & & \\ p_4 & & & & & & & & & & & & \\ p_5 & & & & & & & & & & & & \end{array}$$

6x12

12x1

onb

Null Space [...]

$$
= \begin{array}{c} \\ \\ \end{array}
\begin{array}{cccccccccccc} a & b & c & d & e & f & g & h & i & j & k & l \\ \end{array}
$$

$$
\begin{pmatrix} 0 & 0 & 0 & 0 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & -1 \\ & & \vdots & & & & & & & & & \\ & & & & & & & & & & & \end{pmatrix}
$$

7×12

$\langle -(+e+f+l),\ -a+b+c-e-k,\ -a+b-j,$

$-a+b+c-e-f-i,\ -a+b-e-h,$

$-a-e-f-g,\ a-b-c-d \rangle$ ker

$Im(\partial_2) \to$

$\alpha\ \ P_0P_1P_2 \xrightarrow{\partial_2} \overset{j}{P_1P_2} - \overset{-b}{P_0P_2} + \overset{+a}{P_0P_1} \longrightarrow \langle\ j-b+a$

$\beta\ \ P_1P_4P_2 \to h - j + e$

$\gamma\ \ P_2P_4P_3 \to \overset{k}{P_4P_3} - \overset{-c}{P_2P_3} + \overset{-h}{P_2P_4}$

$k = f + i$ $\quad l = d + 2$

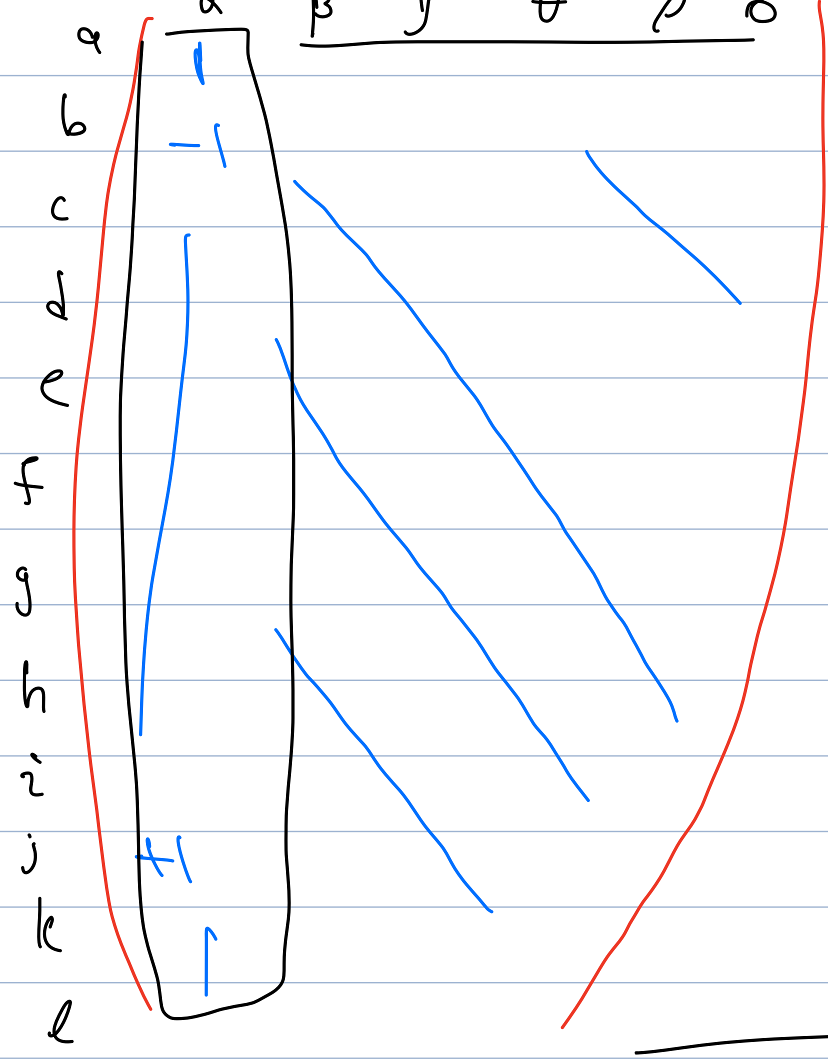
$\theta\ \ P_4P_5P_3 \to i - k + f \to$

Im

$\rho\ \ P_3P_5P_1 \to l - d - i = 0$

$\sigma\ \ P_5P_0P_1 \to a - l + g$ $\quad a - l + g\ \rangle$ Im abcdefghijk

$(\partial_2)_{12\times 6} =$



$$\longrightarrow \text{RowReduce}(\ldots) \longrightarrow \left[\begin{array}{c} I_{6\times 6} \\ \hline 0_{6\times 6} \end{array}\right]$$

$$H_1 = \frac{\ker \partial_1}{\operatorname{Im} \partial_2} =$$

$\mathbb{Z}$ $\quad \mathbb{Z}_n$

$$H_1(\text{Mobius}) = \mathbb{Z} = \frac{\langle 7 \rangle}{\langle 6 \rangle} = \mathbb{Z}_n$$

$H(\mathbb{R}P^2) = \mathbb{Z}_2$

H^1

H^0 [illegible]

H^2 [illegible]