Algorithms
Dynamic Programming Group Project (75 pts)

This project requires you to first theoretically solve the dynamic programming problem below and then write a program that implements your solution[1]. **You are not allowed to use the internet or consult any references. The only people you can work with on this project are your group members.**

1. *Problem Description*:

   A small business is faced with the following algorithmic problem. Their customers are located in k cities across the United States. It costs the small business $c_{i,m}$ to operate out of city i in month m. They can relocate their operations from city i to city j for a fixed cost of f_{ij}. They are not limited on number of relocations, but must stay in a city for the entire month. Given a time horizon of n months, a plan is a sequence of n locations such that the i^{th} location indicates the city in which they will operate in the i^{th} month. The cost of the plan is the sum of the operating costs for each of the n months, plus the moving costs for each relocation. The plan can begin in any of the k cities.

   The optimization problem you have to solve is: given operating costs $c_{i,m}$ and relocation costs f_{ij} for a time horizon of n months, find the minimum cost operating plan.

   Example: In the example below where $k = 2$, $n = 4$, $f_{12} = f_{21} = 10$, and operating costs for each city in month i given in the table.

	Months			
City	1	2	3	4
1 (NY)	1	3	20	30
2 (LA)	50	20	2	4

   The optimal solution is to spend Months 1 and 2 operating out of City 1 (NY), and Months 3 and 4 operating out of City 2 (LA), for a total cost of $1 + 3 + 2 + 4 + 10 = 20$.

2. *Deliverables*: Please submit all of the items requested below in a single PDF file on Canvas.

   (a) [40 pts] Theory: Devise an efficient dynamic programming algorithm that finds an optimal solution. Demonstrate each of the following dynamic programming steps:

      i. Describe the main idea of your approach including how you break the problem into smaller recursive problems. The best way to do this is to develop notation and provide a recurrence relation [20 pts].

      ii. Write pseudocode for your dynamic programming algorithm [10 pts].

      iii. Develop a traceback algorithm that returns an optimal operating location plan [10 pts].

   (b) [10 pts] Theory: Derive the complexity of your algorithm in terms of k and n.

   (c) [15 pts] Implementation: Implement your algorithm and include a listing of your code. If you were unable to get your code to compile/run, please state this clearly. Although we don't plan to run everybody's code, we might choose a few groups randomly and ask them to demonstrate that their code works.

   (d) [10 pts] Implementation: Demonstrate that your code works correctly by showing its results for the following instance:

[1] The problem has been adapted from the text by Kleinberg and Tardos

Operating Costs:

City	Months 1	2	3	4	5	6	7	8	9	10	11	12
1 (NY)	8	3	10	43	15	48	5	40	20	30	28	24
2 (LA)	18	1	35	18	10	19	18	10	8	5	8	20
3 (DEN)	40	5	8	13	21	12	4	27	25	10	5	15

Relocation costs:

City	1 (NY)	2 (LA)	3 (DEN)
1 (NY)	0	20	15
2 (LA)	20	0	10
3 (DEN)	15	10	0

Your output should be formatted in two lines. The first line gives the total cost for your plan and the second line lists the sequence of cities. For the small example given at the beginning, the output would be:

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20
NY NY LA LA
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(e) Your group effort percentages will be submitted separately on Canvas.