

# Self test number 1 – SPECIMEN

## Task 1

The following vectors are given.

Determine whether these vectors can be basis or generators of a vector space and determine the dimension of this space.

$$v_1 = (-27, -24, -9, -21, 6)$$

$$v_2 = (7, 5, 1, 5, 2)$$

$$v_3 = (-4, -6, -4, -4, 8)$$

### Question 1

What do you know about the linear dependence of these vectors?

- ☐ These vectors are linearly independent
- ☒ These vectors are linearly dependent

### Question 2

Can these vectors be generators?

- ☐ These vectors can be vector space generators with dimension 5
- ☐ These vectors can be vector space generators with dimension 4
- ☐ These vectors can be vector space generators with dimension 3
- ☒ These vectors can be vector space generators with dimension 2
- ☐ These vectors cannot be vector space generators

### Question 3

Can these vectors be basis?

- ☐ These vectors can form a basis of a vector space of dimension 5
- ☐ These vectors can form a basis of a vector space of dimension 4
- ☐ These vectors can form a basis of a vector space of dimension 3
- ☐ These vectors can form a basis of a vector space of dimension 2
- ☒ These vectors cannot form a basis of a vector space

## Task 2

Multiply the following matrices  $A \cdot B$

$$A = \begin{pmatrix} -11 & -18 & 10 \\ -19 & 9 & 19 \\ 1 & 6 & 15 \\ 11 & -10 & -20 \end{pmatrix} \quad B = \begin{pmatrix} 11 & 4 & -5 & -8 \\ -17 & -20 & 6 & -13 \\ -2 & -10 & -16 & 12 \end{pmatrix}$$

The element of the resulting matrix  $C = A \cdot B$  in row 3 and column 1 is a (scalar) product of numbers or vectors - input these vectors' element values below into the suitable answer structure (if some answer structures from below do not correspond with the required operation, fill them with zeros.)

- $\begin{pmatrix} 0 \end{pmatrix} \cdot \begin{pmatrix} 0 \end{pmatrix}$
- $\begin{pmatrix} 1 & 6 & 15 \end{pmatrix} \cdot \begin{pmatrix} 11 & -17 & -2 \end{pmatrix}$
- $\begin{pmatrix} 0 & 0 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} 0 & 0 & 0 & 0 \end{pmatrix}$

Value of the element  $C_{31}$  is  $-121$

The resulting matrix has dimension  $4 \times 4$

### Task 3

A system of linear equations is given

$$\begin{aligned}-7x_1 + 21x_2 - 7x_3 + 28x_4 &= -217 \\ 40x_1 + 15x_2 + 5x_3 + 5x_4 &= -135 \\ 6x_1 - 1x_2 + 2x_3 + 2x_4 &= -76 \\ 24x_1 + 4x_2 + 8x_3 + 4x_4 &= -192\end{aligned}$$

Solve it using the Jordanians method of elimination.

#### Question 1

How many solutions does this system of linear equations have?

- ☐ No solution  
☒ One solution  
☐ Infinitely many solutions

#### Question 2

How many variables are basic?

In this case, there are  basic variables.

#### Question 3

How many variables are non-basic?

In this case, there are  non-basic variables.

#### Question 4

Find the basic solution of the given system?

If it does not exist, write 0

$x_1 =$    $, x_2 =$    $, x_3 =$    $, x_4 =$

#### Question 5

Find another basic solution for the given system?

If it does not exist, write 0

$x_1 =$    $, x_2 =$    $, x_3 =$    $, x_4 =$

#### Question 6

Can we require the  $x_4$  variable to be equal to 22 in the solution?

- ☐ Yes  
☒ No

#### Question 7

What is the solution to the system of equations in such a case?

If it does not exist, write 0

$x_1 =$    $, x_2 =$    $, x_3 =$    $, x_4 =$

# Task 4

Given a matrix  $A$  and a vector  $w$

$$A = \begin{pmatrix} -15 & 40 & 20 \\ 2 & -8 & -4 \\ 4 & 4 & -8 \end{pmatrix}$$

$$w = (100, -32, -44)$$

Find the vector  $v$  for which  
 $Av=w$

Instructions: use the inverse matrix and its property  $A^{-1}Av = E$

## Question 1

Find the vector  $v$ ?

If it does not exist, write 0

$$v = ( \boxed{12}, \boxed{1}, \boxed{12} )$$

# Task 5

Compute the inverse of the following matrix A

$$\begin{pmatrix} 0 & 10 & -14 \\ 8 & 8 & 4 \\ 10 & -15 & 5 \end{pmatrix}$$

Show the step-by-step calculation steps

Attention - Do not change the order of rows and columns when calculating.

After the first step of the calculation, you will get a matrix  
(If it cannot be counted, enter 0)

1	9/4	-5/4	1/8	1/8	0
0	-10	14	-1	0	0
0	-75/2	35/2	-5/4	-5/4	1

FRACTION  
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DECIMAL  
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After the second step of the calculation, you will get a matrix  
(If it cannot be counted, enter 0)

1	0	19/10	-1/10	1/8	0
0	1	-7/5	1/10	0	0
0	0	-35	5/2	-5/4	1

The inverse matrix is  
(If it cannot be counted, enter 0)

1/28	2/35	19/350
0	1/20	-1/25
-1/14	1/28	-1/35

The determinant of the matrix A is equal to 2800.

Since the determinant of the matrix A is  $\neq 0$ , this matrix is  $\neq 0$  and has  $\neq 0$  is the inverse matrix.  
non-null regular