

CS4023D Artificial Intelligence

Assignment 2

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The question, report and source code can be found here.

[Github Repo](#)

Solution 1

Based on the formula given:

$$g_i(\mathbf{x}) = -\frac{1}{2}(\mathbf{x} - \boldsymbol{\mu}_i)^t \boldsymbol{\Sigma}_i^{-1}(\mathbf{x} - \boldsymbol{\mu}_i) - \frac{d}{2} \ln 2\pi - \frac{1}{2} \ln |\boldsymbol{\Sigma}_i| + \ln P(\omega_i).$$

The function has been defined:

```
def discriminant_function(x, mean, cov, d, P):
    if d == 1:
        output = -0.5*(x - mean) * (1/cov)
        output = output * (x - mean)
        output += -0.5*d*log(2*pi) - 0.5*log(cov)

    else:
        output = np.matmul(-0.5*(x - mean), np.linalg.inv(cov))
        output = np.matmul(output, (x - mean).T)
        output += -0.5*d*log(2*pi) - 0.5*log(np.linalg.det(cov))

    # Adding Prior Probability
    output += log(P)

    return output
```

It also accomdatees the case if only one feature is used, thus using only scalar quantities.

The variables can be configured based on the scenario. Here, it's assumed that prior probabilities are equally distributed and all features are taken:

```
n = len(data)
P = [1/n for i in range(n)]
d = len(data[0][0])
```

The input is the sample dataset, each set separated by the class they belong to as given below:

```
data = [
    # W1
    np.array([
        [-5.01, -8.12, -3.68],
        [-5.43, -3.48, -3.54],
```

```

        [1.08, -5.52, 1.66],
        [0.86, -3.78, -4.11],
        [-2.67, 0.63, 7.39],
        [4.94, 3.29, 2.08],
        [-2.51, 2.09, -2.59],
        [-2.25, -2.13, -6.94],
        [5.56, 2.86, -2.26],
        [1.03, -3.33, 4.33]
    ]),

    # W2
    np.array([
        [-0.91, -0.18, -0.05],
        [1.30, -2.06, -3.53],
        [-7.75, -4.54, -0.95],
        [-5.47, 0.50, 3.92],
        [6.14, 5.72, -4.85],
        [3.60, 1.26, 4.36],
        [5.37, -4.63, -3.65],
        [7.18, 1.46, -6.66],
        [-7.39, 1.17, 6.30],
        [-7.50, -6.32, -0.31]
    ]),

    # W3
    np.array([
        [5.35, 2.26, 8.13],
        [5.12, 3.22, -2.66],
        [-1.34, -5.31, -9.87],
        [4.48, 3.42, 5.19],
        [7.11, 2.39, 9.21],
        [7.17, 4.33, -0.98],
        [5.75, 3.97, 6.65],
        [0.77, 0.27, 2.41],
        [0.90, -0.43, -8.71],
        [3.52, -0.36, 6.43]
    ])
]

```

In order to classify the sample data, we first run the function through our sample dataset, classwise. On each sample, we find the class which gives the maximum output from its discriminant function.

A count and total count is maintained in order to find the success and failure rates.

```

for j in range(n):
    print("\nData classes should be classified as:", j+1)
    total_count, count = 0, 0

    # Taking x as dataset belonging to class j + 1
    for x in data[j]:
        g_values = [0 for g in range(n)]

        # Itering through each class' discriminant function
        for i in range(n):
            g_values[i] = discriminant_function(x, means[i], cov[i], d, P[i])

        # Now to output the maximum result
        result = g_values.index(max(g_values)) + 1

```

```

print(x, "\twas classified as", result)
total_count, count = total_count + 1, (count + 1 if j == result - 1 else count)

print("Success Rate:", (count/total_count)*100,"%")
print("Fail Rate:", 100 - ((count/total_count))*100,"%")

```

Assuming that all classes have an equal prior probability (as per the configuration in the example picture), the following output is produced:

```

> Assg1 main* python Qn1/main.py

Data classes should be classified as: 1
[-5.01 -8.12 -3.68] was classified as 1
[-5.43 -3.48 -3.54] was classified as 1
[ 1.08 -5.52  1.66] was classified as 1
[ 0.86 -3.78 -4.11] was classified as 1
[-2.67  0.63  7.39] was classified as 2
[4.94  3.29  2.08] was classified as 3
[-2.51  2.09 -2.59] was classified as 1
[-2.25 -2.13 -6.94] was classified as 1
[ 5.56  2.86 -2.26] was classified as 3
[ 1.03 -3.33  4.33] was classified as 1
Success Rate: 70.0 %
Fail Rate: 30.0 %

Data classes should be classified as: 2
[-0.91 -0.18 -0.05] was classified as 2
[ 1.3 -2.06 -3.53] was classified as 3
[-7.75 -4.54 -0.95] was classified as 2
[-5.47  0.5  3.92] was classified as 2
[ 6.14  5.72 -4.85] was classified as 2
[3.6  1.26  4.36] was classified as 3
[ 5.37 -4.63 -3.65] was classified as 2
[ 7.18  1.46 -6.66] was classified as 2
[-7.39  1.17  6.3 ] was classified as 2
[-7.5 -6.32 -0.31] was classified as 2
Success Rate: 80.0 %
Fail Rate: 20.0 %

Data classes should be classified as: 3
[5.35 2.26 8.13] was classified as 3
[ 5.12  3.22 -2.66] was classified as 3
[-1.34 -5.31 -9.87] was classified as 3
[4.48 3.42 5.19] was classified as 3
[7.11 2.39 9.21] was classified as 3
[ 7.17  4.33 -0.98] was classified as 3
[5.75 3.97 6.65] was classified as 3
[0.77 0.27 2.41] was classified as 1
[ 0.9 -0.43 -8.71] was classified as 3
[ 3.52 -0.36  6.43] was classified as 3
Success Rate: 90.0 %
Fail Rate: 10.0 %

```

Solution 2

Part (a) and (b)

In order to match the question, the configuration variables are altered.

- `data-1` for `n` indicates that only 2 classes will be considered (the final class would not be considered as its Prior probability is 0, implying that it wouldn't appear.)

- We iterate through `n + 1` in the outer loop as datasets of all 3 classes are being classified. (Although class 3 is fully misclassified.)
- The `d` value is changed to 1, indicating that only 1 feature will be used. (which is x_1)

```
n = len(data) - 1
P = [0.5, 0.5, 0]
d = 1
```

The configuration parameters being passed are also changed.

- `x[0]` indicates that only x_1 will be used.
- `means[i][0]` indicates that we need the mean only for x_1 .
- `cov[i][0][0]` indicates the variance of feature x_1 .

```
for j in range(n + 1):
    print("\nData classes should be classified as:", j+1)
    total_count, count = 0, 0

    # Taking x as dataset belonging to class j + 1
    for x in data[j]:
        g_values = [0 for g in range(n)]      # Array for all discriminant function outputs.

        # Itering through each class' discriminant function
        for i in range(n):
            g_values[i] = discriminant_function(x[0], means[i][0], cov[i][0][0], d, P[i])

        # Now to output the maximum result
        result = g_values.index(max(g_values)) + 1
        print(x, "\twas classified as", result)
        total_count, count = total_count + 1, (count + 1 if j == result - 1 else count)

    print("Success Rate:", (count/total_count)*100,"%")
    print("Fail Rate:", 100 - ((count/total_count)*100,"%")
```

This results in the following output:

```

> Assg1 main* python Qn2/a_b.py

Data classes should be classified as: 1
[-5.01 -8.12 -3.68] was classified as 1
[-5.43 -3.48 -3.54] was classified as 2
[ 1.08 -5.52  1.66] was classified as 1
[ 0.86 -3.78 -4.11] was classified as 1
[-2.67  0.63  7.39] was classified as 1
[4.94  3.29  2.08] was classified as 2
[-2.51  2.09 -2.59] was classified as 1
[-2.25 -2.13 -6.94] was classified as 1
[ 5.56  2.86 -2.26] was classified as 2
[ 1.03 -3.33  4.33] was classified as 1
Success Rate: 70.0 %
Fail Rate: 30.0 %

Data classes should be classified as: 2
[-0.91 -0.18 -0.05] was classified as 1
[ 1.3  -2.06 -3.53] was classified as 1
[-7.75 -4.54 -0.95] was classified as 2
[-5.47  0.5  3.92] was classified as 2
[ 6.14  5.72 -4.85] was classified as 2
[3.6  1.26  4.36] was classified as 1
[ 5.37 -4.63 -3.65] was classified as 2
[ 7.18  1.46 -6.66] was classified as 2
[-7.39  1.17  6.3 ] was classified as 2
[-7.5  -6.32 -0.31] was classified as 2
Success Rate: 70.0 %
Fail Rate: 30.0 %

Data classes should be classified as: 3
[5.35  2.26  8.13] was classified as 2
[ 5.12  3.22 -2.66] was classified as 2
[-1.34 -5.31 -9.87] was classified as 1
[4.48  3.42  5.19] was classified as 2
[7.11  2.39  9.21] was classified as 2
[ 7.17  4.33 -0.98] was classified as 2
[5.75  3.97  6.65] was classified as 2
[0.77  0.27  2.41] was classified as 1
[ 0.9  -0.43 -8.71] was classified as 1
[ 3.52 -0.36  6.43] was classified as 1
Success Rate: 0.0 %
Fail Rate: 100.0 %

```

Part (c)

Here, the configuration parameters are changed slightly.

- `d` is changed to 2, as now we are considering the first and second features.
- The matrix parameters passed now include necessary values for the same reason.

```

n = len(data) - 1
P = [0.5, 0.5, 0]
d = 2

```

This results in the following output:

```
> Assg1 main* python Qn2/c.py

Data classes should be classified as: 1
[-5.01 -8.12 -3.68] was classified as 1
[-5.43 -3.48 -3.54] was classified as 2
[ 1.08 -5.52  1.66] was classified as 1
[ 0.86 -3.78 -4.11] was classified as 1
[-2.67  0.63  7.39] was classified as 2
[4.94  3.29  2.08] was classified as 2
[-2.51  2.09 -2.59] was classified as 2
[-2.25 -2.13 -6.94] was classified as 1
[ 5.56  2.86 -2.26] was classified as 2
[ 1.03 -3.33  4.33] was classified as 1
Success Rate: 50.0 %
Fail Rate: 50.0 %
```

```
Data classes should be classified as: 2
[-0.91 -0.18 -0.05] was classified as 1
[ 1.3  -2.06 -3.53] was classified as 1
[-7.75 -4.54 -0.95] was classified as 2
[-5.47  0.5   3.92] was classified as 2
[ 6.14  5.72 -4.85] was classified as 2
[3.6  1.26  4.36] was classified as 1
[ 5.37 -4.63 -3.65] was classified as 2
[ 7.18  1.46 -6.66] was classified as 2
[-7.39  1.17  6.3 ] was classified as 2
[-7.5  -6.32 -0.31] was classified as 1
Success Rate: 60.0 %
Fail Rate: 40.0 %
```

```
Data classes should be classified as: 3
[5.35 2.26 8.13] was classified as 2
[ 5.12 3.22 -2.66] was classified as 2
[-1.34 -5.31 -9.87] was classified as 1
[4.48 3.42 5.19] was classified as 1
[7.11 2.39 9.21] was classified as 2
[ 7.17 4.33 -0.98] was classified as 2
[5.75 3.97 6.65] was classified as 2
[0.77 0.27 2.41] was classified as 1
[ 0.9 -0.43 -8.71] was classified as 1
[ 3.52 -0.36 6.43] was classified as 1
Success Rate: 0.0 %
Fail Rate: 100.0 %
```

Part (d)

Here again, the configurations are changed in a similar fashion as in (c).

- `d` values is changed to 3 as all three features are now considered.
- The matrix parameters are now passed without slicing as all values are important.

```
n = len(data) - 1
P = [0.5, 0.5, 0]
d = 3
```

The results in the following output:

```

> Assg1 main* python Qn2/d.py

Data classes should be classified as: 1
[-5.01 -8.12 -3.68] was classified as 1
[-5.43 -3.48 -3.54] was classified as 1
[ 1.08 -5.52  1.66] was classified as 1
[ 0.86 -3.78 -4.11] was classified as 1
[-2.67  0.63  7.39] was classified as 2
[4.94  3.29  2.08] was classified as 1
[-2.51  2.09 -2.59] was classified as 1
[-2.25 -2.13 -6.94] was classified as 1
[ 5.56  2.86 -2.26] was classified as 2
[ 1.03 -3.33  4.33] was classified as 1
Success Rate: 80.0 %
Fail Rate: 20.0 %

Data classes should be classified as: 2
[-0.91 -0.18 -0.05] was classified as 2
[ 1.3 -2.06 -3.53] was classified as 2
[-7.75 -4.54 -0.95] was classified as 2
[-5.47  0.5  3.92] was classified as 2
[ 6.14  5.72 -4.85] was classified as 2
[3.6  1.26  4.36] was classified as 1
[ 5.37 -4.63 -3.65] was classified as 2
[ 7.18  1.46 -6.66] was classified as 2
[-7.39  1.17  6.3 ] was classified as 2
[-7.5 -6.32 -0.31] was classified as 2
Success Rate: 90.0 %
Fail Rate: 10.0 %

Data classes should be classified as: 3
[5.35 2.26 8.13] was classified as 1
[ 5.12  3.22 -2.66] was classified as 2
[-1.34 -5.31 -9.87] was classified as 1
[4.48 3.42 5.19] was classified as 1
[7.11 2.39 9.21] was classified as 1
[ 7.17  4.33 -0.98] was classified as 2
[5.75 3.97 6.65] was classified as 1
[0.77 0.27 2.41] was classified as 1
[ 0.9 -0.43 -8.71] was classified as 1
[ 3.52 -0.36  6.43] was classified as 1
Success Rate: 0.0 %
Fail Rate: 100.0 %

```

Part (e)

On comparing the three outputs, using one or three features give more accurate results than using the first and second features.

```

> Assg1 main* python Qn2/a_b.py
Data classes should be classified as: 1
[-5.01 -8.12 -3.68] was classified as 1
[-5.43 -3.48 -3.54] was classified as 2
[ 1.08 -5.52  1.66] was classified as 1
[ 0.86 -3.78 -4.11] was classified as 1
[-2.67  0.63  7.39] was classified as 1
[ 4.94  3.29  2.08] was classified as 2
[-2.51  2.09 -2.59] was classified as 1
[-2.25 -2.13 -6.94] was classified as 1
[ 5.56  2.86 -2.26] was classified as 2
[ 1.03 -3.33  4.33] was classified as 1
Success Rate: 70.0 %
Fail Rate: 30.0 %

Data classes should be classified as: 2
[-0.91 -0.18 -0.05] was classified as 1
[ 1.3 -2.06 -3.53] was classified as 1
[-7.75 -4.54 -0.95] was classified as 2
[-5.47  0.5  3.92] was classified as 2
[ 6.14  5.72 -4.85] was classified as 2
[ 3.6  1.26  4.36] was classified as 1
[ 5.37 -4.63 -3.65] was classified as 2
[ 7.18  1.46 -6.66] was classified as 2
[-7.39  1.17  6.3 ] was classified as 2
[-7.5 -6.32 -0.31] was classified as 2
Success Rate: 70.0 %
Fail Rate: 30.0 %

Data classes should be classified as: 3
[ 5.35  2.26  8.13] was classified as 2
[ 5.12  3.22 -2.66] was classified as 2
[-1.34 -5.31 -9.87] was classified as 1
[ 4.48  3.42  5.19] was classified as 2
[ 7.11  2.39  9.21] was classified as 2
[ 7.17  4.33 -0.98] was classified as 2
[ 5.75  3.97  6.65] was classified as 2
[ 0.77  0.27  2.41] was classified as 1
[ 0.9 -0.43 -8.71] was classified as 1
[ 3.52 -0.36  6.43] was classified as 1
Success Rate: 0.0 %
Fail Rate: 100.0 %

> Assg1 main* |

> Assg1 main* python Qn2/c.py
Data classes should be classified as: 1
[-5.01 -8.12 -3.68] was classified as 1
[-5.43 -3.48 -3.54] was classified as 2
[ 1.08 -5.52  1.66] was classified as 1
[ 0.86 -3.78 -4.11] was classified as 1
[-2.67  0.63  7.39] was classified as 2
[ 4.94  3.29  2.08] was classified as 2
[-2.51  2.09 -2.59] was classified as 2
[-2.25 -2.13 -6.94] was classified as 1
[ 5.56  2.86 -2.26] was classified as 2
[ 1.03 -3.33  4.33] was classified as 1
Success Rate: 50.0 %
Fail Rate: 50.0 %

Data classes should be classified as: 2
[-0.91 -0.18 -0.05] was classified as 1
[ 1.3 -2.06 -3.53] was classified as 1
[-7.75 -4.54 -0.95] was classified as 2
[-5.47  0.5  3.92] was classified as 2
[ 6.14  5.72 -4.85] was classified as 2
[ 3.6  1.26  4.36] was classified as 1
[ 5.37 -4.63 -3.65] was classified as 2
[ 7.18  1.46 -6.66] was classified as 2
[-7.39  1.17  6.3 ] was classified as 2
[-7.5 -6.32 -0.31] was classified as 1
Success Rate: 60.0 %
Fail Rate: 40.0 %

Data classes should be classified as: 3
[ 5.35  2.26  8.13] was classified as 2
[ 5.12  3.22 -2.66] was classified as 2
[-1.34 -5.31 -9.87] was classified as 1
[ 4.48  3.42  5.19] was classified as 2
[ 7.11  2.39  9.21] was classified as 2
[ 7.17  4.33 -0.98] was classified as 2
[ 5.75  3.97  6.65] was classified as 2
[ 0.77  0.27  2.41] was classified as 1
[ 0.9 -0.43 -8.71] was classified as 1
[ 3.52 -0.36  6.43] was classified as 1
Success Rate: 0.0 %
Fail Rate: 100.0 %

> Assg1 main* |

> Assg1 main* python Qn2/d.py
Data classes should be classified as: 1
[-5.01 -8.12 -3.68] was classified as 1
[-5.43 -3.48 -3.54] was classified as 1
[ 1.08 -5.52  1.66] was classified as 1
[ 0.86 -3.78 -4.11] was classified as 1
[-2.67  0.63  7.39] was classified as 2
[ 4.94  3.29  2.08] was classified as 1
[-2.51  2.09 -2.59] was classified as 1
[-2.25 -2.13 -6.94] was classified as 1
[ 5.56  2.86 -2.26] was classified as 2
[ 1.03 -3.33  4.33] was classified as 1
Success Rate: 80.0 %
Fail Rate: 20.0 %

Data classes should be classified as: 2
[-0.91 -0.18 -0.05] was classified as 2
[ 1.3 -2.06 -3.53] was classified as 2
[-7.75 -4.54 -0.95] was classified as 2
[-5.47  0.5  3.92] was classified as 2
[ 6.14  5.72 -4.85] was classified as 2
[ 3.6  1.26  4.36] was classified as 1
[ 5.37 -4.63 -3.65] was classified as 2
[ 7.18  1.46 -6.66] was classified as 2
[-7.39  1.17  6.3 ] was classified as 2
[-7.5 -6.32 -0.31] was classified as 2
Success Rate: 90.0 %
Fail Rate: 10.0 %

Data classes should be classified as: 3
[ 5.35  2.26  8.13] was classified as 1
[ 5.12  3.22 -2.66] was classified as 2
[-1.34 -5.31 -9.87] was classified as 1
[ 4.48  3.42  5.19] was classified as 1
[ 7.11  2.39  9.21] was classified as 1
[ 7.17  4.33 -0.98] was classified as 2
[ 5.75  3.97  6.65] was classified as 1
[ 0.77  0.27  2.41] was classified as 1
[ 0.9 -0.43 -8.71] was classified as 1
[ 3.52 -0.36  6.43] was classified as 1
Success Rate: 0.0 %
Fail Rate: 100.0 %

> Assg1 main* |

```

The reason for this could be because the covariance with the third feature is much higher than the ones associated with the second feature.

```

[[[ 14.38051111  7.69537778  4.12232222]
 [ 7.69537778 14.62312111  3.90684 ]
 [ 4.12232222  3.90684 19.72453778]]

 [[ 36.82933444  9.98092667 -16.36675111]
 [ 9.98092667 13.16855111  0.40905111]
 [-16.36675111  0.40905111 18.42121778]]

 [[ 8.30475667  7.44494667 13.14957778]
 [ 7.44494667  8.56044889 11.60861111]
 [ 13.14957778 11.60861111 47.28728889]]]

```

Part (f)

In order to consider the possible configurations mentioned, the code takes an input vector and goes through all of them.

General Configuration values

```

n = len(data) - 1
P = [0.5, 0.5, 0]
g_values = [0 for i in range(n)]

```

Get input

```

x = list(map(float, input("Enter the input vector: ").strip().split()))

```

Case A

```

d = 1

```



```

print("Case A: Using only feature vector x1")
for i in range(n):
    g_values[i] = discriminant_function(x[0], means[i][0], cov[i][0][0], d, P[i])

result = g_values.index(max(g_values)) + 1
print(x, "\twas classified as", result)

```

Case B

```

d = 2
print("\nCase B: Using only feature vectors x1 and x2")
for i in range(n):
    g_values[i] = discriminant_function(x[0:2], means[i][0:2], cov[i][0:2, 0:2], d, P[i])

result = g_values.index(max(g_values)) + 1
print(x, "\twas classified as", result)

```

Case C

```

d = 3
print("\nCase C: Using all feature vectors")
for i in range(n):
    g_values[i] = discriminant_function(x, means[i], cov[i], d, P[i])

result = g_values.index(max(g_values)) + 1
print(x, "\twas classified as", result)

```

Here are the outputs for the 4 input vectors mentioned in the question:

<pre> > Assg1 main* python Qn2/f.py Enter the input vector: 1 2 1 Case A: Using only feature vector x1 [1.0, 2.0, 1.0] was classified as 1 Case B: Using only feature vectors x1 and x2 [1.0, 2.0, 1.0] was classified as 1 Case C: Using all feature vectors [1.0, 2.0, 1.0] was classified as 2 > Assg1 main* [8s] python Qn2/f.py Enter the input vector: 5 3 2 Case A: Using only feature vector x1 [5.0, 3.0, 2.0] was classified as 2 Case B: Using only feature vectors x1 and x2 [5.0, 3.0, 2.0] was classified as 2 Case C: Using all feature vectors [5.0, 3.0, 2.0] was classified as 1 > Assg1 main* </pre>	<pre> > Assg1 main* python Qn2/f.py Enter the input vector: 0 0 0 Case A: Using only feature vector x1 [0.0, 0.0, 0.0] was classified as 1 Case B: Using only feature vectors x1 and x2 [0.0, 0.0, 0.0] was classified as 1 Case C: Using all feature vectors [0.0, 0.0, 0.0] was classified as 1 > Assg1 main* [8s] python Qn2/f.py Enter the input vector: 1 0 0 Case A: Using only feature vector x1 [1.0, 0.0, 0.0] was classified as 1 Case B: Using only feature vectors x1 and x2 [1.0, 0.0, 0.0] was classified as 1 Case C: Using all feature vectors [1.0, 0.0, 0.0] was classified as 1 > Assg1 main* </pre>
--	--