

A novel 2D graphical representation of DNA sequences and its application

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Abstract

In this paper, we introduce a novel 2D graphical representation of DNA sequences, the W-curve, which is embedded in two unit circles. We associate the W-curves with the classifications of the nucleotides according to their chemical properties. Then we obtain an 8-component vector with entries being the average sums of the abscissa and y-axis of A+C and T+G (A+T and C+G, A+G and T+C), respectively. The introduced vector results in simpler characterizations and comparisons of DNA sequences. The construction of the W-curve has some important advantages: (1) it avoids loss of information and the W-curve standing for DNA doesn't overlap or intersect itself; (2) the space the W-curve occupied is very small, just two unit circles. The utility of the approach can be illustrated by the examination of similarities/dissimilarities among the coding sequences of the first exon of β -globin gene of eleven different species in Table 1.

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1. Introduction

The number of DNA sequences as strings of four nucleotides: A (adenine), C (cytosine), G (guanine), T (thymine) is growing rapidly in the DNA database. But it is difficult to obtain information from DNA sequences directly. Therefore, many kinds of methods have been proposed to characterize DNA sequences.

Some researchers introduced an alternative way to compare DNA sequences, based on a set of invariants of DNA sequences, rather than directly using string comparison. Graphical representation of DNA sequences provides a simple way of viewing, sorting, and comparing various gene structures [1–15]. Nandy [8] presented a graphical representation by assigning A (adenine), G (guanine), T (thymine), and C (cytosine) to four direction $(-x)$, $(+x)$, $(-y)$, $(+y)$, respectively. Such a representation of DNA is accompanied with (1) some loss of visual information associated with crossing and overlapping of the curve with itself; (2) an arbitrary decision with respect to the

choice of the direction for the bases. Recently, several authors have outlined different graphical representations of the DNA sequences based on 2D, 3D or 4D [3–6,10–15], but some representations [10–12] are also accompanied by some loss of information due to overlapping and crossing of the curve with itself. Then they constructed the D/D , L/L and high order matrices to extract the leading eigenvalues as invariants to characterize the DNA sequences, so their computation is very complex. Furthermore, their representations need a large memory if the sequence is long.

Here, we introduce a novel 2D graphical representation embedded in two unit circles and call it the W-curve. It can avoid the loss of information and uniquely denote the DNA sequence. Finally, we give a simple way to numerically characterize the DNA sequences.

2. 2D graphical representation of DNA sequences and its properties

From the knowledge of biology, we know that the four DNA bases can be classified as $R = \{A, G\}$ and $Y = \{C, T\}$, $M = \{A, C\}$ and $K = \{G, T\}$, $W = \{A, T\}$ and $S = \{G, C\}$ according to their chemical properties. Based on the classification of the four nucleotides, we construct two maps ϕ_1 , ϕ_2 between the

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bases of the DNA sequence and the plots in 2D space. If $G = g_1, g_2, \dots, g_n$ is the given DNA sequence, we define functions φ_1, φ_2 as following:

$$\varphi_1(g_i) = \begin{cases} \left(\frac{2}{n}(i-1), \sqrt{1 - \left(1 - \frac{2}{n}(i-1)\right)^2} \right), & \text{if } g_i = \text{A or C}, \\ \left(\frac{2}{n}(i-1), -\sqrt{1 - \left(1 - \frac{2}{n}(i-1)\right)^2} \right), & \text{if } g_i = \text{T or G}, \end{cases}$$

$$\varphi_2(g_i) = \begin{cases} \left(-\frac{2}{n}(i-1), \sqrt{1 - \left(1 - \frac{2}{n}(i-1)\right)^2} \right), & \text{if } g_i = \text{A or C}, \\ \left(-\frac{2}{n}(i-1), -\sqrt{1 - \left(1 - \frac{2}{n}(i-1)\right)^2} \right), & \text{if } g_i = \text{T or G}, \end{cases}$$

where $p_i = \varphi_1(g(i)), q_i = \varphi_2(g(i)) (i = 1, 2, \dots, n)$. Finally, we connect adjacent points and obtain a curve which can better illustrate the characteristic sequence considered. In Fig. 1, the W-curve is a graphical presentation of the first exon of the human β -globin gene.

Property 1. For a given DNA sequence, all the dots obtained from the maps φ_1, φ_2 are on two unit circles, whose centers are $(-1, 0)$ and $(1, 0)$, respectively.

Proof. Given a DNA sequence, the maps φ_1, φ_2 transform the DNA sequence into a series of dots $p_1, p_2, \dots, p_n, q_1, q_2, \dots, q_n$ where $p_i = \left(\frac{2}{n}(i-1), \pm \sqrt{1 - \left(1 - \frac{2}{n}(i-1)\right)^2} \right)$,

$q_i = \left(-\frac{2}{n}(i-1), \pm \sqrt{1 - \left(1 - \frac{2}{n}(i-1)\right)^2} \right)$. So we can calculate

$$\left(\frac{2}{n}(i-1) - 1 \right)^2 + \left(\pm \sqrt{1 - \left(1 - \frac{2}{n}(i-1)\right)^2} \right)^2 = 1,$$

$$\left(1 - \frac{2}{n}(i-1) \right)^2 + \left(\pm \sqrt{1 - \left(1 - \frac{2}{n}(i-1)\right)^2} \right)^2 = 1.$$

$$i = 1, 2, \dots, n$$

So p_i and $q_i (i = 1, 2, \dots, n)$ are on two unit circles $(x-1)^2 + y^2 = 1$ and $(x+1)^2 + y^2 = 1$, and their centers are $(1, 0)$ and $(-1, 0)$, respectively. \square

Property 2. For a given sequence, there is a unique 2D representation corresponding to it.

Proof. From the definition of φ_1, φ_2 , we know that a DNA sequence is transformed into a series of dots (p_i, q_i) uniquely. Let $(x_i^1, y_i^1), (x_i^2, y_i^2)$ be the coordinates of p_i, q_i , respectively. If $y_i^1 = \sqrt{1 - \left(1 - \frac{2}{n}(i-1)\right)^2}$ and $y_i^2 = \sqrt{1 - \left(1 - \frac{2}{n}(i-1)\right)^2}$, then $g_i = \text{A}$; if $y_i^1 = \sqrt{1 - \left(1 - \frac{2}{n}(i-1)\right)^2}$ and $y_i^2 = -\sqrt{1 - \left(1 - \frac{2}{n}(i-1)\right)^2}$, then $g_i = \text{C}$; if $y_i^1 = -\sqrt{1 - \left(1 - \frac{2}{n}(i-1)\right)^2}$ and $y_i^2 = -\sqrt{1 - \left(1 - \frac{2}{n}(i-1)\right)^2}$, then $g_i = \text{T}$; if $y_i^1 = -\sqrt{1 - \left(1 - \frac{2}{n}(i-1)\right)^2}$ and $y_i^2 = \sqrt{1 - \left(1 - \frac{2}{n}(i-1)\right)^2}$, then $g_i = \text{G}$. So every pair of dots $(p_i$ and $q_i)$ corresponds to a nucleotide g_i , and the correspondence is one to one. Therefore, a DNA sequence can be represented uniquely. \square

Property 3. For any $i = 1, 2, \dots, n$, where n is the length of DNA sequence, We denote $k_{p_i q_i}$ is the slope of the line determined by two dots p_i and q_i . If $k_{p_i q_i}$ is equal to zero, g_i belongs to W; If $k_{p_i q_i}$ is not equal to zero, g_i belongs to S.

Proof. Actually

$$k_{p_i q_i} = \frac{\Delta y_i}{\Delta x_i} = \frac{y_i^2 - y_i^1}{x_i^2 - x_i^1}.$$

Δx_i and Δy_i can be calculated from the i th position of the DNA sequence. If the i th residue is A or T, we will get $k_{p_i q_i} = 0$; if the i th residue is G, we find $k_{p_i q_i} < 0$; if the i th residue is C, $k_{p_i q_i} > 0$. So if $k_{p_i q_i}$ is equal to zero, g_i belongs to W; if $k_{p_i q_i}$ is not equal to zero, g_i belongs to S. \square

Property 4. The 2D representation possesses the reflection symmetry.

Proof. Usually the sequence is expressed in the order from 5' to 3'. Suppose that the 2D representation for the DNA sequence

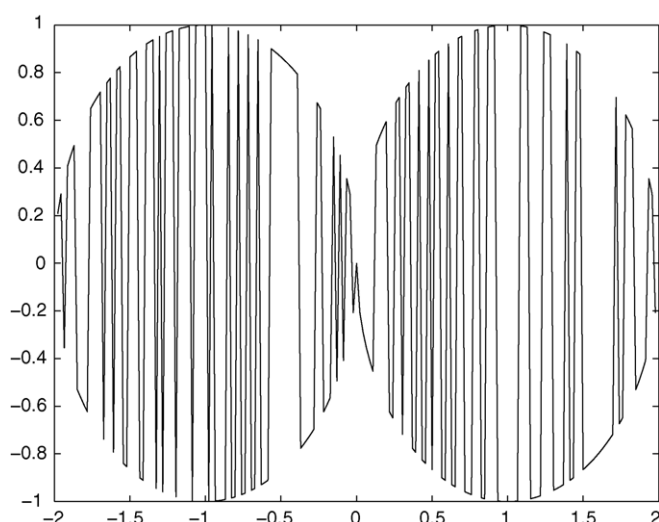


Fig. 1. W-curve of the first exon of human β -globin gene.

is described by the dots $p_i(x_i^1, y_i^1), q_i(x_i^2, y_i^2) i = 1, 2, \dots, n$ on the circles. Suppose that the 2D representation for the reverse sequence, i.e., the same sequence but from 3' to 5' is described by $p_i^*(x_{1i}^1, y_{1i}^1), q_i^*(x_{2i}^2, y_{2i}^2) i = 1, 2, \dots, n$. We obtain

$$\begin{cases} x_{1i}^1 = 2 - x_i^1, \\ y_{1i}^1 = y_i^1. \\ \\ \begin{cases} x_{2i}^2 = -2 - x_i^2, \\ y_{2i}^2 = y_i^2. \end{cases} \end{cases} \quad \square$$

3. Application

3.1. The content relation of the nucleotides A, C, G, T of eleven species in Table 1

Given a sequence with the length of n , if we consider the right part of the W-curve, we know the step length is $\frac{2}{n}$ from the definition of the function φ , the abscissa of the first A or C in the sequence must be 0 and the abscissa of the last A or C in the sequence must be $2 - \frac{2}{n}$. Observing Fig. 1, we can find out that every pair of neighboring nucleotides may be both in $\{A, C\}(\{T, G\})$, or if one is in $\{A, C\}$ and the other is in $\{T, G\}$. First we denote $X_i^{A-C}(X_i^{T-G})$ the abscissa of A or C (T or G) appearing i th times in the given sequence. Let $X_l^{A-C}(X_l^{T-G})$ be the abscissa of the last A or C (T or G). Then we can calculate $\Delta X^{A-C} = X_i^{A-C} - X_{i-1}^{A-C}, i = 2, 3, \dots, n$. If ΔX^{A-C} equals $\frac{2}{n}$, they must be the neighboring nucleotides; if ΔX^{A-C} is greater than $\frac{2}{n}$, it is easy to imagine that T or G must

appear between X_{i-1}^{A-C} and X_i^{A-C} , and the number of T and G appearing between X_{i-1}^{A-C} and X_i^{A-C} is $\Delta X^{A-C} \frac{n}{2} - 1$. If the abscissa of the first A or C is zero, A or C must be the first nucleotides; otherwise there must be T or G appearing before the X_1^{A-C} , and the number of T+G appearing before X_1^{A-C} is $X_1^{A-C} \frac{n}{2} - 1$. If the abscissa of the last A or C is equal to $2 - \frac{2}{n}$, it means that A or C is the last nucleotides of the considered sequence; Otherwise there must be T or G behind the last A or C, and the number of T+G behind the last A or C is $(2 - \frac{2}{n} - x_l^{A-C}) \frac{n}{2}$. So the total number of T+G is

$$\begin{aligned} & \sum_{i=2}^l ((X_i^{A-C} - X_{i-1}^{A-C}) \frac{n}{2} - 1) + (2 - \frac{2}{n} - X_l^{A-C}) \frac{n}{2} \\ & + (X_1^{A-C} \frac{n}{2} - 1). \end{aligned}$$

Following the same method, we can compute the numbers of A+C, A+G, A+T, G+C, and C+T, which are listed in Table 2. Taking a closer look at Table 2, we will find the interesting relation, which may give us more information about their evolution. In Table 3, we list the relation of the content of the nucleotides of the sequences in Table 1. Meanwhile we can observe that the longer the ΔX^{A-C} is, the more the T and G are contained in this section of the sequence. Therefore, we can find out the longest section of the sequence contains only T or G. Let $\Delta X_m^{A-C} = \max \{X_i^{A-C} - X_{i-1}^{A-C}, 2 - \frac{2}{n} - X_l^{A-C}, X_1^{A-C}\}, i = 2, 3, \dots, n$ denote the longest T and G section.

Observing Table 3, we find most results are similar to the results in Bo Liao et al. [5]. Although there are some difference in Gallus, Opossum, Bovine and Chimpanzee. After validation, our result is correct.

Table 1
The coding sequences of the first exon of β -globin gene of different species

Species	Coding sequence
Human	ATGGTGCACCTGACTCCTGAGGAGAAGTCTGCCGTACTGCCCTGTGGGGCAAGGTGAACGTGG ATGAAGTTGGTGGTGAGGCCCTGGGCAG
Goat	ATGCTGACTGCTGAGGAGAAGGCTGCCGTCACCGCTTCTGGGGCAAGGTGAAAGTGGATGAAG TTGGTGCTGAGGCCCTGGGCAG
Opossum	ATGGTGCACCTGACTTCTGAGGAGAAGAAGTGCATCACTACCATCTGGTCTAAGGTGCAGGTTG ACCAGACTGGTGGTGAGGCCCTTGGCAG
Gallus	ATGGTGCACCTGGACTGCTGAGGAGAAGCAGCTCATCACCGCCTCTGGGGCAAGGTCAATGTGG CCGAATGTGGGGCCGAAGCCCTGGCCAG
Lemur	ATGACTTTGCTGAGTGCTGAGGAGAATGCTCATGTCACCTCTCTGTGGGGCAAGGTGGATGTAG AGAAAGTTGGTGGCGAGGCCTTGGGCAG
Mouse	ATGGTTGCACCTGACTGATGCTGAGAAGTCTGCTGTCTCTTGCCCTGTGGGCAAAGGTGAACCCC GATGAAGTTGGTGGTGAGGCCCTGGGCAGG
Rabbit	ATGGTGCATCTGTCCAGTGAGGAGAAGTCTGCCGTACTGCCCTGTGGGGCAAGGTGAATGTGG AAGAAGTTGGTGGTGAGGCCCTGGGC
Rat	ATGGTGCACCTAACTGATGCTGAGAAGGCTACTGTTAGTGGCCTGTGGGGAAAGGTGAACCCTG ATAATGTTGGCGCTGAGGCCCTGGGCAG
Gorilla	ATGGTGCACCTGACTCCTGAGGAGAAGTCTGCCGTTACTGCCCTGTGGGGCAAGGTGAACGTGG ATGAAGTTGGTGGTGAGGCCCTGGGCAGG
Bovine	ATGCTGACTGCTGAGGAGAAGGCTGCCGTCACCGCCTTTTGGGGCAAGGTGAAAGTGGATGAA GTTGGTGGTGAGGCCCTGGGCAG
Chimpanzee	ATGGTGCACCTGACTCCTGAGGAGAAGTCTGCCGTACTGCCCTGTGGGGCAAGGTGAACGTGG ATGAAGTTGGTGGTGAGGCCCTGGGCAGGTTGGTATCAAGG

Table 5

The similarity/dissimilarity matrix for the coding sequences of Table 1 based on the angle between the eight-component vectors

Species	Human	Goat	Opossum	Gallus	Lemur	Mouse	Rabbit	Rat	Gorilla	Bovine	Chimpanzee
Human	0	0.0540	0.1322	0.1028	0.0658	0.0571	0.0593	0.0820	0.0330	0.0614	0.0301
Goat		0	0.1652	0.1161	0.0628	0.1039	0.0290	0.0719	0.0313	0.0242	0.0353
Opossum			0	0.1140	0.1567	0.0863	0.1711	0.1271	0.1548	0.1749	0.1503
Gallus				0	0.1528	0.1004	0.1391	0.1238	0.1172	0.1371	0.1205
Lemur					0	0.0968	0.0424	0.0629	0.0522	0.0468	0.0460
Mouse						0	0.1067	0.0926	0.0837	0.1110	0.0782
Rabbit							0	0.0695	0.0366	0.0168	0.0316
Rat								0	0.0746	0.0737	0.0691
Gorilla									0	0.0327	0.0170
Bovine										0	0.0359
Chimpanzee											0

component vectors and the correlation angle of the eight-component vectors.

Observing Tables 4 and 5, we find gallus (the only non-mammal among them) and Opossum (the most remote species from the remaining mammals) show larger entries among these species. This is in agreement with the results of Li et al. [15], Liao et al. [5]. Human–Chimpanzee, Human–Mouse, Goat–Bovine, Mouse–Rat, Gorilla–Chimpanzee, Rabbit–chimpanzee have smaller entries, so they are more similar species pairs. This is not an accident, but shows they have close evolutionary relationship.

4. Conclusion

A comparison of DNA sequences even with even fewer than a hundred bases is quite difficult [16]. And as pointed in [14], a direct comparison of the sequences using computer codes is somewhat less straightforward due to the fact that the sequences have different lengths.

In this paper, we have given a novel 2D graphical representation of DNA sequences and similarity analysis of the coding sequences of the first exon of β -globin gene of 11 species. First we outline the novel 2D graphical presentation. Then we give a simple way to extract an 8-component vector as an invariant to characterize the DNA sequence. At last we analyze the similarity and dissimilarity among the eleven different species. Our graphical representation of DNA

sequences is visual and direct, without overlapping or intersection, and avoids loss of information.

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References

- [1] E. Hamori, J. Ruskin, J. Biol. Chem. 258 (1983) 1318.
- [2] E. Hamori, BioTechniques 7 (1989) 710.
- [3] M. Randić, A.T. Balaban, J. Chem. Inf. Comput. Sci. 43 (2003) 532.
- [4] R. Chi, K. Ding, Chem. Phys. Lett. 407 (2005) 63–67.
- [5] B. Liao, M. Tan, K. Ding, Chem. Phys. Lett. 402 (2005) 380–383.
- [6] B. Liao, T. Wang, J. Mol. Struct.: Theochem. 681 (2004) 209–212.
- [7] M.A. Gates, J. Theor. Biol. 119 (1986) 319.
- [8] A. Nandy, Curr. Sci. 66 (1994) 309.
- [9] A. Nandy, Curr. Sci. 66 (1994) 821.
- [10] X. Guo, A. Nandy, Chem. Phys. Lett. 369 (2003) 361.
- [11] Y. Wu, A.W. Liew, H. Yan, M.S. Yang, Chem. Phys. Lett. 367 (2003) 170.
- [12] A. Nandy, P. Nandy, Chem. Phys. Lett. 368 (2003) 102.
- [13] M. Randić, Chem. Phys. Lett. 386 (2004) 468–471.
- [14] M. Randić, M. Vračko, N. Lerš, Plavšić, Chem. Phys. Lett. 371 (2003) 202.
- [15] C. Li, J. Wang, Comb. Chem. High Throughput Screening 6 (2003) 795–799.
- [16] X. Guo, M. Randić, S.C. Bassk, Chem. Phys. Lett. 350 (2001) 106.