Influences of Periodic Mechanical Deformation on Spiral Breakup in Excitable Media

Jiang-Xing Chen,*,† Jiang-Rong Xu,† Xiao-Ping Yuan,† and He-Ping Ying‡

Department of Physics, HangZhou Dianzi University, Hangzhou 310018, China, and Zhejiang Institute of Modern Physics and Department of Physics, Zhejiang University, Hangzhou 310027, China

Received: July 31, 2008; Revised Manuscript Received: November 6, 2008

Influences of periodic mechanical deformation (PMD) on spiral breakup that results from Doppler instability in excitable media are investigated. We present a new effect: a high degree of homogeneous PMD is favored to prevent the low-excitability-induced breakup of spiral waves. The frequency and amplitude of PMD are also significant for achieving this purpose. The underlying mechanism of successful control is also discussed, which is believed to be related to the increase of the minimum temporal period of the meandering spiral when the suitable PMD is applied.

I. Introduction

Spiral waves of excitation play an important role in the dynamics of two-dimension (2D) active media, such as Belousov−Zhabotinsky (BZ) reactions,[1] CO oxidation on platinum surfaces,[2] cardiac muscle tissue,[3] aggregating slime mold cells,[4,5] etc. Recent studies on animal hearts show clear evidence that transitions from spiral waves to defect-mediated turbulence are responsible for life-threatening situations such as tachycardia and fibrillation.[3,6−11] Spiral breakup and subsequent evolution into defect-mediated turbulence induced by Doppler instability is proposed as a plausible mechanism for fibrillation,[8,12] it is quite important to deepen the insight into this transition and prevent such events from a practical point of view. So far as we know, different approaches have been considered to control spiral breakup, for examples, feedback,[13,14] decreasing intercellular conductance,[15] periodic forcing,[16−18] and local modulation of transport properties,[19] etc.

On the other hand, oscillating gels undergoing the BZ reaction provide ideal mediums for investigating nonlinear dynamical phenomena that arise from a coupling of mechanical and chemical energy[20] in addition to their practical utility used as microactuators[21] for pulsatile drug delivery.[22] To study how cardiac muscle contraction affects the behavior of spiral waves, Muñuzuri et al.[23] designed an elastic excitable medium by incorporating the BZ reaction into a polyacrylamide−silica gel to investigate the influence of mechanical deformation on spiral dynamics. They observed drift of spiral when the frequency of periodic mechanical deformation (PMD) is equal to that of the spiral. Zhang et al.[24] explained the drift on the basis of an approximate formula. They[25] also found that PMD of media can induce the breakup of spiral, which was proposed as a new mechanism of transition from spiral to spatiotemporal chaos. Recently, complicated spiral turbulence may be quenched due to enhancing the drift of spiral tips induced by PMD.[26] Yashin and Balazs[27] developed an approach to simulate chemoresponsive gels that can exhibit large variations in volume and alterations in shape. More recently, Panfilov et al.[28] investigated deformation-induced breakup of spiral waves as well as drift by applying a RDM model. However, the investigations of effects induced by deformation on spiral dynamics are still open due to their complex interaction, which may arouse the attention of investigation and render the subject to study new interesting scenarios. A topic worth looking into is to clarify the influences of the different degrees of homogeneous PMD on spiral dynamics. It is generally believed that homogeneous PMD of medium may be helpful to propagate waves, whereas inhomogeneous PMD may break propagating waves in a realistic excitable media, e.g., cardiac tissues. Thus, in contrast to breakup of spiral induced by deformation,[25,28] a significant question is: whether or not the breakup of spiral can be prevented by PMD with different degrees of homogeneity?

In this paper, the breakup of spiral wave resulting from low-excitability-induced Doppler instability is investigated under the influence of PMD. An interesting effect of PMD for preventing spiral breakup will be studied. The influences of different degrees of homogeneous PMD on spiral behavior will be considered. It will be shown that both the frequency and amplitude of PMD are significant for reach the goal. The underlying mechanism of successful control is discussed.

II. Model

The excitable medium is described by a modified FitzHugh-Nagumo model[16,29,30] (the Bär model):

$$\frac{\partial u}{\partial t} = g(u,v) + \nabla^2 u, \quad \frac{\partial v}{\partial t} = f(u) - v \tag{1}$$

Here variables u and v represent the concentrations of the reagents; In this model $g(u, v) = (1/\varepsilon)u(1 - u)[u - (v + b)/a]$. $f(u)$ describes a delayed production of the inhibitor with $f(u) = 0$ for $0 \le u < {}^1/_3$, $f(u) = 1 - 6.75u(u - 1)^2$ for ${}^1/_3 \le u \le 1$, and $f(u) = 1$ for $u > 1$. The excitability of a system can be indicated by the inverse of ε. Upon increase of ε the ability of an excitable medium to propagate waves is usually lost. To ensure the medium is excitable, the spatiotemporal dynamics is investigated by fixing $a = 0.84$ and $b = 0.07$. We model the mechanical deformation with a typical operation where any fixed point r is changed to r'. A simple oscillation by varying the size of the grid is considered in our simulation: $r' = x[1 + F\cos(\omega_F t + \phi_x)]i + y[1 + F\cos(\omega_F t + \phi_y)]j$, which is similar to the manner used in refs 23−26. Then, eq 1 is modified to[24−26]

* Corresponding author. Electronic address: jxchen@hdu.edu.cn.
† HangZhou Dianzi University.
‡ Zhejiang University.

10.1021/jp806811j CCC: $40.75
Published on Web 12/29/2008

$$\frac{\partial u}{\partial t} = g(u,v) + \frac{1}{[1 + F\cos(\omega_F t + \varphi_x)]^2}u_{xx} + \frac{1}{[1 + F\cos(\omega_F t + \varphi_y)]^2}u_{yy},$$
$$\frac{\partial v}{\partial t} = f(u) - v \qquad (2)$$

The computations are performed by the explicit Euler method with no-flux boundary conditions, using the five-point approximation of the Laplacian on a medium $L_x = L_y = 256\Delta x \times 256\Delta y$ with a grid spacing $\Delta x = \Delta y = 0.3906$ and time steps $\Delta t = 0.02$.

III. Results and Discussion

In our simulations, the different degrees of homogeneity of PMD are realized by varying the phase shift $\Phi = |\phi_x - \phi_y|$ in two directions while keeping the amplitude and frequency in two directions identical. The difference of deformation in two directions, i.e., $\delta = |\Delta L_x - \Delta L_y|$, indicates the different degrees of deformation of the medium. The relation $\delta = A \sin(\Phi/2)|\sin(\omega_F t + \phi_0)|$ (here, $A = 2L_xF$ and $\phi_0 = (\phi_x + \phi_y)/2$) shows an oscillation with amplitude determined by phase shift Φ. Then, we define $Ud = |1 - \Phi/\pi|$ ($\Phi \in [0, \pi]$) as a quantitative measurement of different degrees of homogeneity. $Ud = 1.0$ (corresponding to $\Phi = 0$, i.e., $\delta = 0$) means homogeneous PMD and $Ud = 0$ (corresponding to $\Phi = \pi$, that is, δ with the largest amplitude) means the highest degree of inhomogeneous PMD. Suitable initial conditions of equation (1) lead to solutions of steadily rotating spiral waves within the range $0.01 < \varepsilon < 0.06$. At $\varepsilon = 0.06$, the spirals start to meander undergoing a supercritical Hopf bifurcation.[29−32] When the parameter $\varepsilon > \varepsilon_c = 0.0715$, spiral waves will break up without the modulation of PMD and the system will quickly evolve into a chaotic state due to the pronounced Doppler effect for low excitability.[12,14,29,30]

Now, we concentrate on the influences of PMD on the process of spiral breakup. A developed meandering spiral wave at $\varepsilon = 0.071$ (just below ε_c) is selected as the initial condition (see Figure 1a). In Figure 1b, the corresponding power spectra obtained from the FFT method is shown. The peaks at ω_{s_1} and ω_{s_2} show the primary and secondary frequency of the meandering spiral. The excitability of the medium is decreased by increasing the value of ε with step $\Delta\varepsilon = 0.0001$. After every skip of ε, the system runs for a long time $t = 500$ ($t = 2000$ is also checked for some fixed parameters to ensure that the time is long enough to absolutely relax the system) to receive the next skip of $\Delta\varepsilon$. In this process, the PMD is applied to the system all the time with fixed Ud, frequency ω_F, and amplitude F. In Figure 2, sequences of patterns under the action of homogeneous PMD with increasing ε are presented. In Figure 2a spiral at $\varepsilon = 0.071 < \varepsilon_c = 0.0715$ is shown. From Figure 2b, one can see that the spiral breakup can be successfully prevented by PMD with $Ud = 1.0$ when the excitability is decreased across ε_c. The effectiveness of preventing spiral breakup is maintained to $\varepsilon_{max} = 0.088$ (see Figure 2c,d), which is a lot larger than $\varepsilon_c = 0.0715$. Then, the range of ε supporting a meandering spiral is extended almost two-and-a-half-times, i.e., from [0.06, 0.071] to [0.06, 0.088]. Across $\varepsilon_{max} = 0.088$, the growing Doppler effect results in losing control of the meandering spiral. Consequently, the system evolves into a turbulent state again (see Figure 2e).

The degree of homogeneity of PMD is significant to prevent spiral breakup. The $\varepsilon - Ud$ phase diagram is shown in Figure 3. An evident indication is that the controllable regions above ε_c are increased with Ud. The plotted values of ε_{max} increase linearly with Ud. At about $Ud = 0.50$, with an abrupt jump, the controllable region vanishes. Thus, we believe that the increase of degree of homogeneous PMD is beneficial to prevent the spiral breakup at low excitability. Contrary to the positive effects, inhomogeneous PMD of medium, e.g., the cases at $Ud < 0.50$, may lead to the breakup of spiral even at high excitability $\varepsilon < \varepsilon_c$.[25]

On successful control of spiral breakup with $Ud > 0.50$, the frequency of PMD plays a vital role. In Figure 4, we plot a phase diagram in the $\omega_F/\omega_{s_1} - \varepsilon$ plane. A striking characteristic in the phase diagram is that the controllable region mostly falls in the vicinity of regions with $\omega_F/\omega_{s_1} = 1.0$ and $\omega_F/\omega_{s_1} = 2.0$. In our simulations, no controllable regimes have been observed around $\omega_F/\omega_{s_1} = 3.0$, 4.0, and so on. From Figure 4, one can see that small frequencies of PMD do not work until $\omega_F/\omega_{s_1} = 0.89$ accompanied with a sudden enhance of ε_{max}. Then, the values of ε_{max} decrease gradually with increase of ω_F/ω_{s_1}. At the point $\omega_F = 2\omega_{s_1}$, the ε_{max} abruptly bursts again. The deviation from $\omega_F/\omega_{s_1} = 2.0$ results in the reduction of controllable region again. Beyond $\omega_F/\omega_{s_1} = 2.7$, the homogeneous PMD cannot prevent spiral breakup any longer. It is found that the values of ε_{max} around $\omega_F/\omega_{s_1} = 1.0$ are similar to that around $\omega_F/\omega_{s_1} = 2.0$.

Moreover, homogeneous PMD with appropriate frequency still may break the spiral[26,28] if the deformation is strong enough, which requires that the amplitude F cannot be too high (see Figure 5a). On the other hand, too weak PMD cannot to modulate the behavior of spiral in low excitability (see Figure 5b). Therein, the amplitude F should be selected suitably (see Figure 5c). In Figure 5d, the controllable region depending on ε and F with fixed proper frequency is summarized. The values of ε_{max} decrease rapidly at the edge of the controllable regime, and collapse beyond the critical amplitude $F_c = 0.03$ or $F_{c_2} = 0.11$.

IV. Underlying Mechanism

Since the breakup of spiral results from Doppler instability, we explore the influences of PMD on Doppler effects, which is believed to underpin the corresponding mechanism. In Figure 6a,b, we present the radii of tip orbits at different frequencies with homogeneous PMD ($Ud = 1.0$). The radii decrease when the frequency departures from multiple frequency, i.e., $\omega_F/\omega_{s_1} = 1.0$ or 2.0. Two main scenarios are observed in the successful controllable regimes when the frequency is varied: (1) Spiral wave becomes sparse near the multiple frequency (comparing Figure 6c,d) accompanied with expansion of meandering region of tip (see Figure 6f). (2) Spiral wave becomes dense (see Figure 6e) when the frequency of PMD departures from multiple frequency accompanied with shrinkage of meandering region of tip (see Figure 6h), which makes the configuration of spiral in Figure 6e more homogeneous.

No matter whether the meandering spiral is sparse or dense, an identical characteristic is that the minimum temporal period T_{min} is enhanced in both cases. In Figure 7a,b, we present two examples of the evolution of temporal period T with time. The fluctuation of T induced by Doppler effect shows a band, which introduces a minimal temporal period T_{min} (see the horizontal line in Figure 7a). To further clarify the corresponding mechanism, we present the frequency of spiral under the influence of homogeneous PMD in Figure 7c. One can see the both primary frequency ω_1 and second frequency ω_2 are monotonously increased by PMD with ω_F/ω_{s_1} around 1.0 and 2.0.

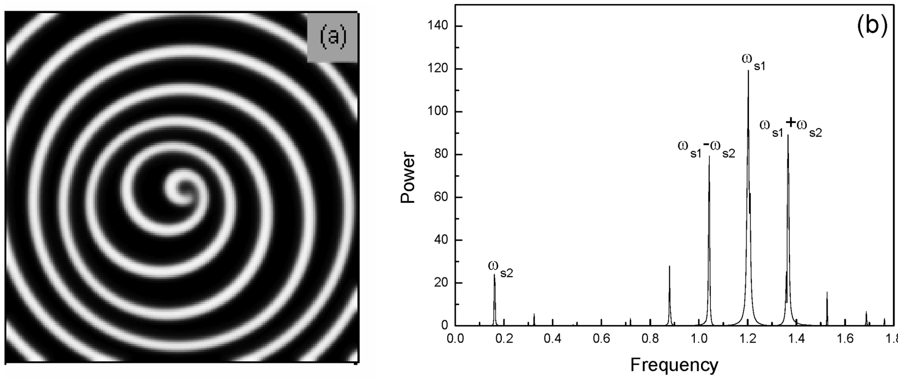

Figure 1. (a) Initial free spiral at $\varepsilon = 0.071$; (b) corresponding power spectra from site (10, 10) shows primary frequency $\omega_{s_1} = 1.20$ and second frequency $\omega_{s_2} = 0.18$.

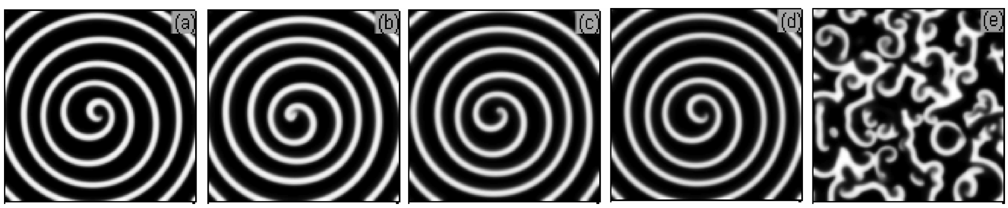

Figure 2. Patterns under the influence of homogeneous PMD ($Ud = 1.0$) with frequency $\omega_F/\omega_{s_1} = 1.1$ and amplitude $F = 0.1$. (a) $\varepsilon = 0.071$, (b) $\varepsilon = 0.075$, (c) $\varepsilon = 0.080$, (d) $\varepsilon = 0.085$, (e) $\varepsilon = 0.09$. Note that $\varepsilon_c = 0.0715$, beyond which the spiral will breakup without control.

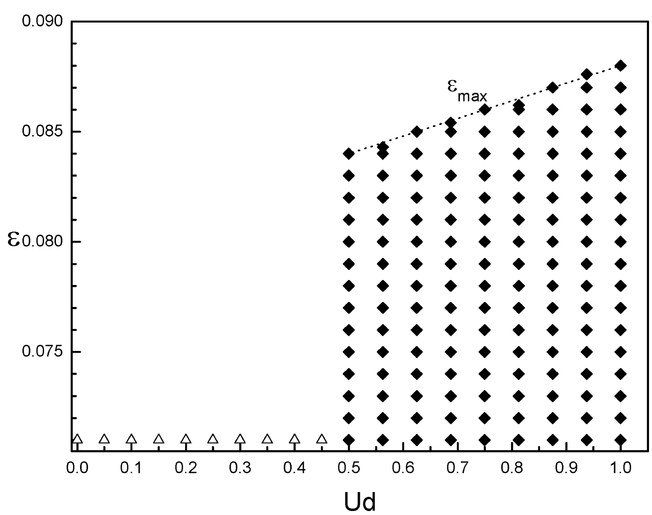

Figure 3. Effect of different degrees PMD ($F = 0.05$, $\omega_F/\omega_{s_1} = 1.0$) on preventing spiral wave breakup at the $Ud-\varepsilon$ plane. The dotted curve shows the controllable maximum value of ε at fixed Ud. The squares show the controllable region and the triangles shows the breakup of spiral.

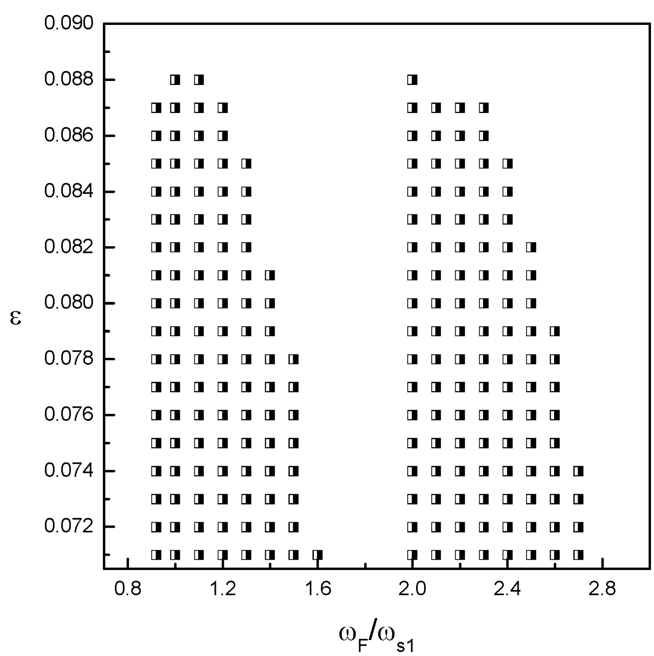

Figure 4. Phase diagram spanned by ratio of frequency ω_F/ω_{s_1} and ε with $F = 0.1$ and $Ud = 1.0$.

Comparing with the initial values ω_{s_1} and ω_{s_2} (indicated by horizontal line in Figure 7c, the values also can be found in Figure 1b) without control, one can find the PMD first induces a reduction of ω_1 and ω_2, e.g., from $\omega_{s_1} = 1.20$ to $\omega_1 = 1.01$ and from $\omega_{s_2} = 0.18$ to $\omega_2 = 0.15$ at $\omega_F/\omega_{s_1} = 1.0$. Next, the values of ω_1 and ω_2 are increased across ω_{s_1} and ω_{s_2} by increasing ω_F/ω_{s_1}. The value of ω_{s_1} determines mean period T_{mean} measured from fixed point in the system, that is, $T_{mean} = 2\pi/\omega_{s_1}$.[34,35] Therefore, the decrease (increase) of ω_{s_1} (to ω_1) by PMD will enhance (reduce) the value of T_{mean}. From Figure 7a,b, one can see this point clearly. ω_{s_1} is decreased to ω_1 in Figure 7a with $\omega_F/\omega_{s_1} = 1.0$ and is increased in Figure 7b where $\omega_F/\omega_{s_1} = 1.5$. Correspondingly, T_{mean} is increased in Figure 7a (the band is moved up) and is decreased in Figure 7b (the band is move down) after PMD is applied at $t = 100$. The Doppler effect is introduced and characterized by the emergence of ω_2 from Hopf bifurcation.[31,32] From Figure 6a,b and Figure 6f−h, it is observed that the decrease (increase) of ω_2 will expand (shrink) the meandering region of tip, which makes the Doppler instability more (less) pronounced. Accordingly, the oscillation of period measured from fixed point is strengthened (weakened). Consequently, the band of measured temporal period in Figure 7a is widened and that in Figure 7b becomes narrow.

Now, let us consider the T_{min} in Figure 7a,b. For the case in Figure 7a, the decrease of ω_{s_1} (to corresponding ω_1) enhances the T_{mean} and consequently enhances T_{min} (the band is moved

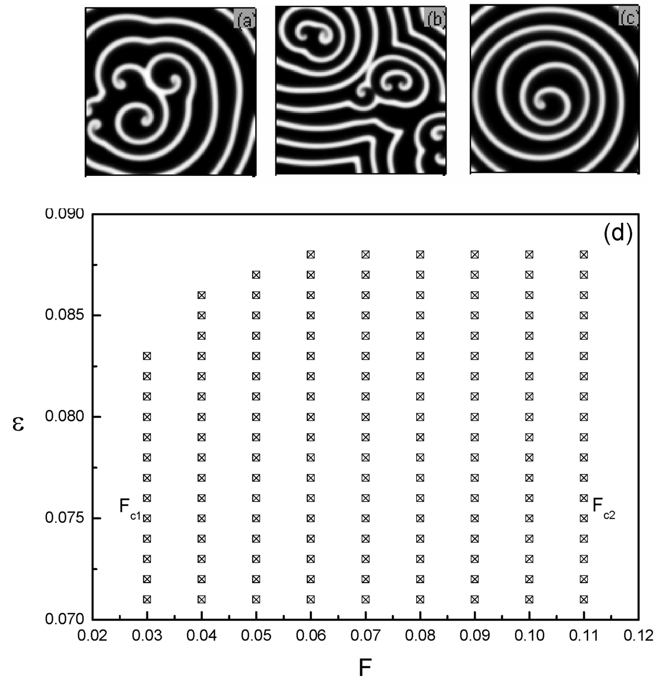

Figure 5. (a) Pattern with strong amplitude $F = 0.12$, $\varepsilon = 0.071$. (b) Pattern with weak amplitude $F = 0.02$, $\varepsilon = 0.071$. (c) Spiral with suitable amplitude $F = 0.07$, $\varepsilon = 0.086$. (d) Dependence of controllable region on amplitude F of PMD and excitability parameter ε. Other parameters: $\omega_F/\omega_{s_1} = 1.0$ and $Ud = 1.0$.

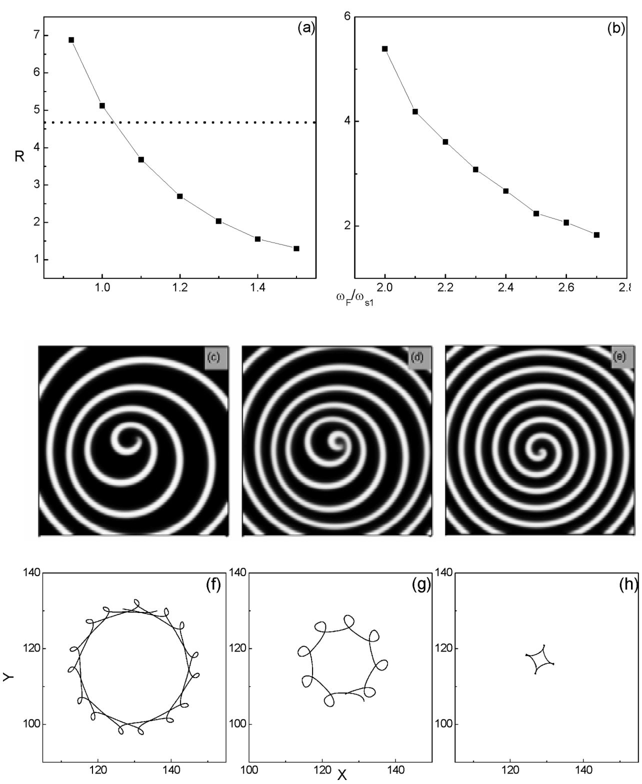

Figure 6. (a), (b) Radii of tip orbit as a function of frequency under the control of PMD with $Ud = 1.0$ and $F = 0.1$. The horizontal dotted line shows the value of radii of tip orbit without control. Spiral waves at $\varepsilon = 0.071$, $Ud = 1.0$ (c) PMD with $\omega_F/\omega_{s_1} = 0.92$, (d) without control, (e) PMD with $\omega_F/\omega_{s_1} = 1.5$. (f)−(h) corresponding trajectories plotted with same size.

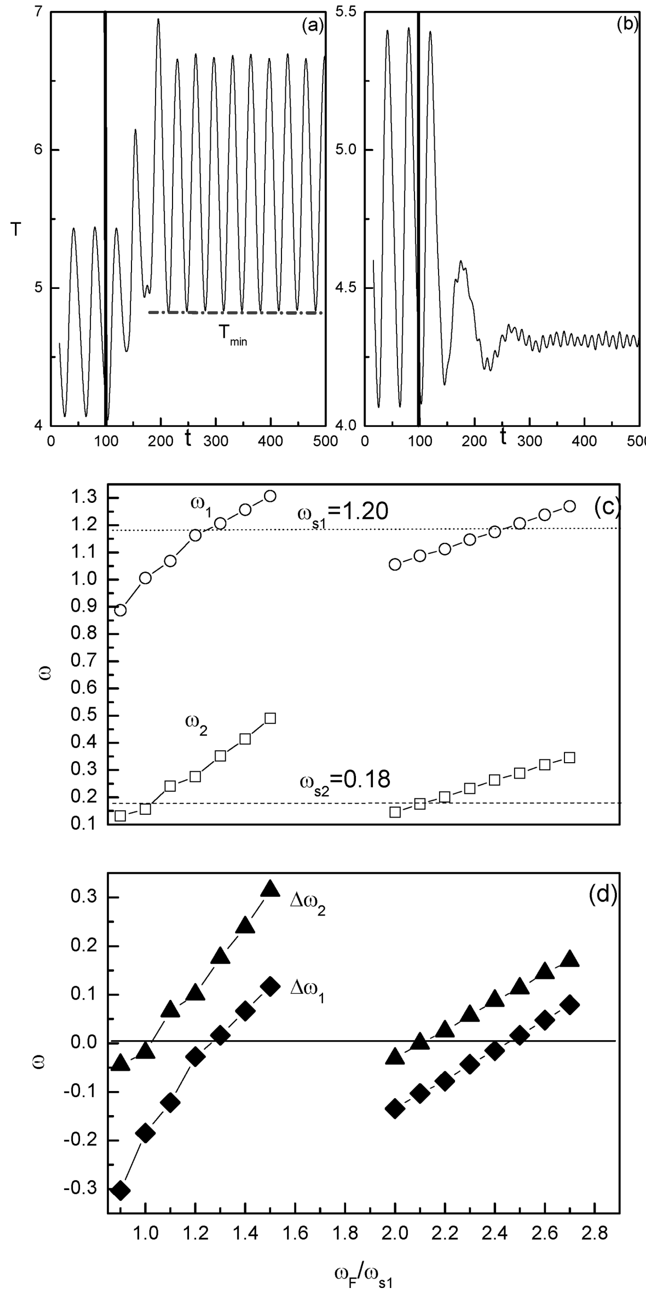

Figure 7. Evolution of temporal period T before and after the PMD is applied at $t = 100$ with $Ud = 1.0$ and $F = 0.1$ and (a) $\omega_F/\omega_{s_1} = 1.0$, (b) $\omega_F/\omega_{s_1} = 1.5$. The horizontal line in (a) shows the value of T_{min}. (c) Frequencies of spiral at $\varepsilon = 0.071$ influenced by PMD. Open circles give the primary frequency ω_1 and open squares give second frequency ω_2 of spiral under the influence of PMD with different ω_F/ω_{s_1}. The horizontal dotted line and dashed line show the two frequencies of spiral at $\varepsilon = 0.071$ without control (see Figure 1b, $\omega_{s_1} = 1.20$ and $\omega_{s_2} = 0.18$). (d) Full triangles show corresponding values of $\Delta\omega_2 = (\omega_2 - \omega_{s_2})$ and full diamonds show $\Delta\omega_1 = (\omega_1 - \omega_{s1})$.

up). However, the decrease of ω_{s_2} (to corresponding ω_2) widens the band, which consequently reduces T_{min}. So, whether the T_{min} is increased or not depends on what factor plays a dominant role. From Figure 7d, it is found that $\Delta\omega_1 < \Delta\omega_2 < 0$. Thereby, the considerable decrease of ω_{s_1} plays a decisive role that eventually increases T_{min}. In Figure 7b, the condition is $0 < \Delta\omega_1 < \Delta\omega_2$ (see Figure 7d) at $\omega_F/\omega_{s_1} = 1.50$. That is to say, the drastically increase of ω_{s_2} dominates the dynamical behavior. Accordingly, the band narrows so far, which ultimately increases T_{min} although the band is moved down due to increased $\Delta\omega_1$.

Thus, the underlying mechanism of preventing spiral breakup in low excitability due to PMD can be rationalized as follows, on the basis of the corresponding instability mechanism

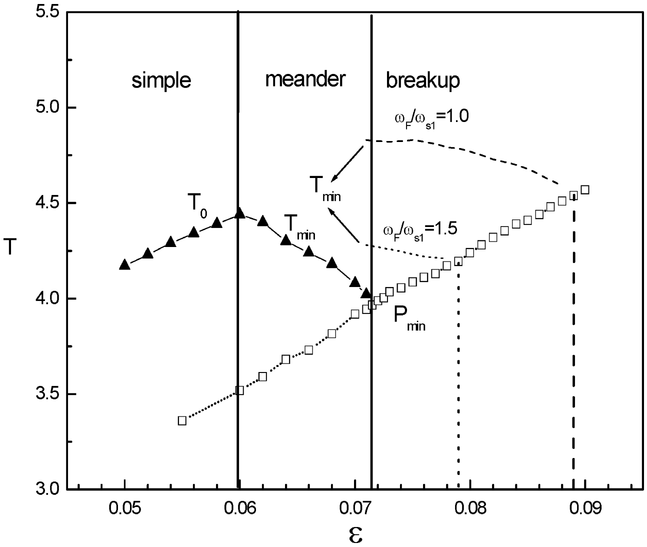

Figure 8. Bifurcation diagram of the spiral period as a function of the excitability ε. The regimes are divided by the vertical solid line: simple spiral $\varepsilon < 0.06$, meandering regime ($0.06 \leq \varepsilon < 0.0715$) (the minimum temporal period $T_{\min}$ is measured at the point near the spiral center where the breakup first occurs at $\varepsilon = \varepsilon_c = 0.0715$), and breakup regime ($\varepsilon > \varepsilon_c$). The open squares give minimum periods $P_{\min}$ of one-dimensional wave trains. The minimum temporal period $T_{\min}$ in the region $\varepsilon > \varepsilon_c$ is plotted after the PMD with $Ud = 1.0$ and $F = 0.1$ applied. The dotted and dashed vertical lines show the value when the $T_{\min}$ merges with $P_{\min}$.

proposed by Bär et al.[29,30] and observed by Ouyang et al.[12] in experiment: the behavior of a spiral wave in an excitable medium is governed by a dispersion relation that relates the speed to the period of the traveling waves. There exists a minimum period $P_{\min}$ below which the system cannot recover itself to the excitable state. And, therefore, the traveling waves fail to propagate;[25] that is, $T_{\min}$ of spiral should be larger than $P_{\min}$ of the propagating wave. In Figure 8, we present the period T_0 of the spiral and the minimum period $P_{\min}$ of traveling wave (by shortening wave trains in one-dimensional system, according to ref 33). At the rigid rotating regime $\varepsilon < 0.06$, it is obvious that the condition $T_{\min} = T_0 > P_{\min}$ is fulfilled. After the onset of meandering, a large variation of the period shows maximum and minimum temporal period. Correspondingly, the plotted minimum temporal period $T_{\min}$ shows a clear knee at this point in Figure 8. Due to the pronounced Doppler effect, the minimum temporal period $T_{\min}$ decreases as ε grows.[12,29,30] Ultimately, the $T_{\min}$ becomes less than $P_{\min}$ at $\varepsilon = 0.0715$, rendering the local waves unstable. Accordingly, at the same time the system undergoes the transition from spirals to defect-mediated turbulence. Once the suitable PMD affects the system, the minimum temporal period $T_{\min}$ is modulated with considerable increase. The cases at $\omega_F/\omega_{s_1} = 1.0$ and $\omega_F/\omega_{s_2} = 1.50$ in Figure 8 clarify this point. The gap of period $\Delta T_{\min} \approx 0.76$ at $\omega_F/\omega_{s_1} = 1.0$ and $\Delta T_{\min} \approx 0.21$ at $\omega_F/\omega_{s_1} = 1.5$. Naturally, the breakup of spiral is efficiently inhibited. Thus, the parameter space maintaining a spiral wave in low excitability is extended so far. The $P_{\min}$ grows while the $T_{\min}$ decreases by further decreasing the excitability. As a result, the spiral breakup is inevitable when the curves of $T_{\min}$ crosses the $P_{\min}$ line again at $\varepsilon = 0.089$ as $\omega_F/\omega_{s_1} = 1.0$ and at $\varepsilon = 0.078$ as $\omega_F/\omega_{s_1} = 1.50$, respectively.

V. Conclusion

In conclusion, we studied the influence of mechanical deformation on spiral breakup due to Doppler instability in excitable media. It is found that PMD with a high degree of homogeneity is significant to prevent the breakup of spiral waves. Our simulations showed that frequencies of PMD should fall in the vicinity of equal or double that of the free spiral. Suitable amplitudes of PMD are also necessary in the controlling process. Three corresponding phase diagram displaying successful control, i.e., $\varepsilon - F$, $\varepsilon - \omega_F/\omega_{s_1}$ and $\varepsilon - Ud$, are presented. Dense and sparse spirals have been observed with the same effect of increase of minimum period of the meandering spiral. The deeper mechanism was explored by discussing the changes of frequency after the PMD is applied. Because the heart is stretching and contracting all the time, the investigation on effects of PMD are essential for understanding the transition from regular waves to a state of defect-mediated turbulence that is responsible for life-threatening situations. Also, we hope that our results can be observed in experiments, e.g., BZ reaction.

Acknowledgment. We thank Hong Zhang for useful discussions. This work was supported by the National Nature Science Foundation of China (No. 10747120) and Natural Science Foundation of Zhejiang Province (No. Y607525).

Note Added after Print Publication. This paper was published on the Web on December 29, 2008, and in the January 22, 2009, issue. Equations 1 and 2 were corrected. The electronic version was corrected and reposted to the Web issue on April 16, 2009. An Addition and Correction appears in the April 23, 2009, issue (Vol. 113, No. 16).

References and Notes

(1) *Chemical Waves and Patterns*; Kapral, R., Showalter, K., Eds.; Kluwer Academic: Dordrecht, 1995.
(2) Jakubith, S.; Rotermund, H. H.; Engel, E.; Oertzen, A. V.; Ertl, G. *Phys. Rev. Lett.* **1990**, *65*, 3013.
(3) Davidenko, J. M.; Pertsov, A. V.; Salomonsz, R.; Baxter, W.; Jalife, J. *Nature (London)* **1992**, *355*, 349.
(4) Gerish, G. *Naturwissenschaften* **1983**, *58*, 430.
(5) Devreotes, P. *Science* **1989**, *245*, 1045.
(6) Winfree, A. T. *Science* **1994**, *266*, 1003.
(7) Gray, R. A.; Jalife, J.; Panfilov, A. V.; Baxter, W. T.; Cabo, C.; Davidenko, J. M.; Pertsov, A. M. *Science* **1995**, *270*, 1222.
(8) Gray, R. A.; Pertsov, A. M.; Jalife, J. *Nature (London)* **1998**, *392*, 675.
(9) Holden, A. V. *Nature (London)* **1998**, *392*, 20.
(10) Glass, L. *Phys. Today* **1996**, *49*, 40.
(11) Witkowski, F. X.; Leon, L. J.; Penkoske, P. A.; Giles, W. R.; Spano, M. L.; Ditto, W. L.; Winfree, A. T. *Nature (London)* **1998**, *39*, 78.
(12) Ouyang, Q.; Swinney, H. L.; Li, G. *Phys. Rev. Lett.* **2000**, *84*, 1047.
(13) Rappel, W. J.; Fenton, F.; Karma, A. *Phys. Rev. Lett.* **1999**, *83*, 456.
(14) Xiao, J. H.; Hu, G.; Zhang, H.; Hu, B. *Europhys. Lett.* **2005**, *69*, 29.
(15) Panfilov, A. V. *Phys. Rev. Lett.* **2002**, *88*, 118101.
(16) Zhang, H.; Chen, J. X.; Li, Y. Q.; Xu, J. R. *J. Chem. Phys.* **2006**, *125*, 204503.
(17) Wu, Y. B.; Qiao, C.; Ouyang, Q.; Wang, H. L. *Phys. Rev. E* **2008**, *77*, 036226.
(18) Alonso, S.; Sagués, F.; Mikhailov, A. S. *Science* **2003**, *299*, 1722.
(19) Bergero, P. E.; Irurzun, I. M.; Mola, E. E. *Chem. Phys. Lett.* **2007**, *445*, 157.
(20) Shibayama, M.; Tanaka, T. *Adv. Polym. Sci.* **1993**, *109*, 1.
(21) Howse, J. R.; et al. *Nano Lett.* **2006**, *6*, 73.
(22) Dhanarajan, A. P.; Mistra, G. P.; Siegel, R. A. *J. Phys. Chem. A* **2002**, *106*, 8835.
(23) Muñuzuri, A. P.; Innocenti, C.; Flesselles, J. M.; Gilli, J. M.; Agladze, K. I.; Krinsky, V. I. *Phys. Rev. E* **1994**, *50*, R667.
(24) Zhang, H.; Hu, B.; Hu, G.; Xiao, J. H. *J. Chem. Phys.* **2003**, *119*, 4468.
(25) Zhang, H.; Ruan, X. S.; Hu, B.; Ouyang, Q. *Phys. Rev. E* **2004**, *70*, 016212.
(26) Zhang, H.; Li, B. W.; Sheng, Z. M.; Cao, Z.; Hu, G. *Europhys. Lett.* **2006**, *76*, 1109.

(27) Yashin, V. V.; Balazs, A. C. *Science* **2006**, *314*, 798.

(28) Panfilov, A. V.; Keldermann, R. H.; Nash, M. P. *Proc. Natl. Acad. Sci. U.S.A.* **2007**, *104*, 7922.

(29) Bär, M.; Eiswirth, M. *Phys. Rev. E* **1993**, *48*, R1635.

(30) Bär, M.; Or-Guil, M. *Phys. Rev. Lett.* **1999**, *82*, 1160.

(31) (a) Barkley, D.; Kness, M.; Tuckerman, L. S. *Phys. Rev. A* **1990**, *42*, 2489. (b) Barkley, D. *Phys. Rev. Lett.* **1992**, *68*, 2090.

(32) Li, G.; Ouyang, Q.; Petrov, V.; Swinney, H. L. *Phys. Rev. Lett.* **1996**, *77*, 2105.

(33) Winfree, A. T. *Chaos* **1991**, *1*, 303.

(34) (a) Braune, M.; Schrader, A.; Engel, H. *Chem. Phys. Lett.* **1994**, *222*, 358. (b) Schrader, A.; Braune, M.; Engel, H. *Phys. Rev. E* **1995**, *52*, 98.

(35) Zykov, V. S.; Steinbock, O.; Müller, C. *Chaos* **1994**, *4*, 509.

JP806811J