Effect of Temperature on the Standard Transformed Thermodynamic Properties of Biochemical Reactions with Emphasis on the Maxwell Equations

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When the standard transformed Gibbs energy ($\Delta_f G'^\circ$) of formation of a reactant (sum of species) in an enzyme-catalyzed reaction can be expressed as a function of temperature, pH, and ionic strength, all the other standard transformed properties of the reactant ($\Delta_f S'^\circ$, $\Delta_f H'^\circ$, C_{Pm}°, and $\bar{N}_H$) can be calculated as functions of temperature, pH, and ionic strength by taking partial derivatives. Since there is very little data on heat capacities of species in biochemical reactions, the calculations presented here are based on the assumption that the heat capacities of species are equal to zero at zero ionic strength. The most basic data on a biochemical reactant consists of $\Delta_f G^\circ$, $\Delta_f H^\circ$, charge number, and number of hydrogen ions for the species involved. These data can be used to derive functions of temperature, pH, and ionic strength that give the standard transformed thermodynamic properties of reactants, such as ATP, ADP, H_2O, and inorganic phosphate. This makes it possible to obtain the functions that represent $\Delta_r G'^\circ$, $\Delta_r S'^\circ$, $\Delta_r H'^\circ$, $\Delta_r C_P'^\circ$, and $\Delta_r N_H$ for enzyme-catalyzed reactions, such as ATP + H_2O = ADP + P_i. The calculations can be checked by testing that $\Delta_r G'^\circ = \Delta_r H'^\circ - T\Delta_r S'^\circ$ and the Maxwell equation connecting $\Delta_r S'^\circ$ with $\Delta_r N_H$ is obeyed.

The preceding article[1] dealt primarily with effects of temperature on the standard transformed Gibbs energies of biochemical reactants and reactions. It did not emphasize the fact that when the standard transformed Gibbs energy of a reactant can be represented by a function of T, pH, and ionic strength, all the other standard transformed thermodynamic properties can be calculated by taking partial derivatives of this function. This includes the average number of hydrogen atoms in the reactant as a function of T, pH, and ionic strength. The standard transformed heat capacity can be calculated by taking the second partial derivative of the standard transformed Gibbs energy of a reactant with respect to T and multiplying it by $-T$. The previous article also did not deal with the three Maxwell equations that provide relationships between the standard transformed properties. The function of T, pH, and ionic strength for a reactant like adenosine triphosphate is quite complicated, and its partial derivatives are even more complicated.

When the pH is treated as an independent variable in studying enzyme-catalyzed reactions in dilute aqueous solutions, the criterion for spontaneous change and equilibrium is provided by the transformed Gibbs energy defined by[2,3]

$$G' = G - n_c(H)\mu(H^+) \tag{1}$$

The amount of the hydrogen component $n_c(H)$ is given by $\sum N_H(j)n_j$, where $N_H(j)$ is the number of hydrogen atoms in species j and n_j is the amount of species j. The properties G, G', and $\mu(H^+)$ are taken to be functions of the ionic strength so that expressions for apparent equilibrium constants K' of biochemical reactions can be written in terms of sums of concentrations. This Legendre transform leads to the following expression for the transformed chemical potential μ'_j of a species:

$$\mu'_j = \mu_j - N_H(j)\mu(H^+) \tag{2}$$

In making calculations it is customary to use Gibbs energies of formation rather than chemical potentials, and so the standard

transformed Gibbs energy of formation $\Delta_f G'^\circ_j$ of species j at a specified pH and ionic strength is given by

$$\Delta_f G'^\circ_j(\text{pH},I) = \Delta_f G^\circ_j(I{=}0) + N_H(j)RT \ln(10)\text{pH} - \\ RT\alpha(z_j^2 - N_H(j))I^{1/2}/(1 + BI^{1/2}) \tag{3}$$

where α and B are from the extended Debye−Hückel equation.

$$\ln \gamma_j = -\alpha z_j^2 I^{1/2}/(1 + BI^{1/2}) \tag{4}$$

The pH in eq 3 is defined by $-\log[H^+]$. Experimental pH values have to be adjusted to obtain the pH values used here; the adjustments are given at various temperatures and ionic strengths in the preceding article.[1]

The preceeding article discussed the fact that the α in eqs 3 and 4 is a function of temperature. Clarke and Glew[4] have calculated the Debye−Hückel slopes for water between 0 and 150 °C. The coefficient $RT\alpha$ in eq 3 for the standard transformed Gibbs energy of formation of a species is given by

$$RT\alpha = 9.204\,83 \times 10^{-3}T - 1.284\,67 \times 10^{-5}T^2 + \\ 4.951\,99 \times 10^{-8}T^3 \tag{5}$$

When a reactant involves two or more species, the standard transformed Gibbs energy of formation of the reactant (sum of species) is calculated using

$$\Delta_f G'^\circ(\text{iso}) = -RT \ln\left\{\sum_{i=1}^{N_{\text{iso}}} \exp[-\Delta_f G'^\circ_i/RT]\right\} \tag{6}$$

where N_{iso} is the number of pseudoisomers in the reactant. Equations 3 and 6 were first used to study the ATP series of reactions.[5,6] Since eq 5 is substituted in eq 3, which is then substituted in eq 6, it is evident that $\Delta_f G'^\circ$ for a reactant with

two or more species is an extremely complicated function of T, pH, and ionic strength.

Standard Transformed Properties of a Reactant at a Specified pH and Temperature

The fundamental equation for the transformed Gibbs energy G' for a system containing a single reactant is

$$dG' = -S'dT + VdP + \mu'dn' + RT\ln(10)n_c(H)d\text{pH} \quad (7)$$

where S' is the transformed entropy of the system and n' is the amount of the reactant, which is the sum of the amounts of the species making up the reactant.

Equation 7 is very important because, as is often stated, if G' of a system is known as a function of its natural variables T, P, n', and pH, then all the other thermodynamic properties of the reactant can be calculated. But since there is no way to determine G' directly, what is done is to measure apparent equilibrium constants K', which are written in terms concentrations of reactants (sums of species) without $[H^+]$ in the expression for K' explicitly. This makes it possible to obtain $\Delta_f G'^\circ$ values for reactants by using $\Delta_f G'^\circ = -RT\ln K' = \sum \nu_i' \Delta_f G_i'^\circ$, where the ν_i' is the stoichiometric number for the ith reactant and $\Delta_f G_i'^\circ$ is the standard transformed Gibbs energy of formation of the ith reactant. The use of eq 7 to obtain the other thermodynamic properties of a reactant is best described in terms of its Maxwell equations. Since the effect of pressure has not been studied, three Maxwell equations are of interest. The first Maxwell equation is

$$-\left(\frac{\partial S'}{\partial n'}\right)_{T,P,\text{pH}} = \left(\frac{\partial \mu'}{\partial T'}\right)_{P,\text{pH},n'} \quad (8)$$

This can be written as

$$-\Delta_f S' = \left(\frac{\partial \Delta_f G'}{\partial T}\right)_{P,\text{pH}} \quad (9)$$

For ideal solutions, eq 9 can be written as

$$-(\Delta_f S'^\circ - R\ln c) =$$
$$\left(\frac{\partial(\Delta_f G'^\circ + RT\ln c)}{\partial T}\right)_{P,\text{pH}} \quad \text{or} \quad \Delta_f S'^\circ = \left(\frac{\partial \Delta_f G'^\circ}{\partial T}\right)_{P,\text{pH}} \quad (10)$$

where c is the molar concentration of the reactant. This equation is used to calculate the standard transformed entropy of formation of a reactant when $\Delta_f G'^\circ$ can be expressed as a function of T, pH, and ionic strength. The second Maxwell equation is

$$\left(\frac{\partial \mu'}{\partial \text{pH}}\right)_{T,P,n'} = RT\ln(10)\left(\frac{\partial n_c(H)}{\partial n'}\right)_{T,P,\text{pH}} \quad (11)$$

This can be written as

$$\left(\frac{\partial \Delta_f G'^\circ}{\partial \text{pH}}\right)_{T,P} = RT\ln(10)\bar{N}_H \quad (12)$$

where $\bar{N}_H$ is the average number of hydrogen atoms in a molecule of reactant. This equation is used to calculate the average number of hydrogen atoms in a reactant when $\Delta_f G'^\circ$ is known as a function of pH. The third Maxwell equation is

$$-\left(\frac{\partial S'}{\partial \text{pH}}\right)_{T,P,n'} = R\ln(10)\left(\frac{\partial\{Tn_c(H)\}}{\partial T}\right)_{P,\text{pH},n'} \quad (13)$$

When the system contains a mole of reactant, this equation can be written as

$$\left(\frac{\partial \Delta_f S'^\circ}{\partial \text{pH}}\right)_{T,P} = -R\ln(10)\left(\frac{\partial\{T\bar{N}_H\}}{\partial T}\right)_{P,\text{pH}} \quad (14)$$

This equation is interesting because it shows that it is the change in average binding of hydrogen ions with temperature that determines how the standard transformed entropy of formation of a reactant changes with pH. This equation does not yield a new thermodynamic property, but it does provide a useful check on calculations of $\Delta_f S'^\circ$ and $\bar{N}_H$. These last two Maxwell equations arise because of the last term in eq 7, which is a consequence of treating the pH as an independent variable.

The remaining standard transformed thermodynamic properties of a reactant can also be calculated from $\Delta_f G'^\circ$ by taking partial derivatives. The standard transformed enthalpy of formation is obtained using the Gibbs−Helmholtz equation:

$$\Delta_f H'^\circ = -T^2\left(\frac{\partial(\Delta_f G'^\circ/T)}{\partial T}\right)_{P,\text{pH}} \quad (15)$$

The standard molar transformed heat capacity at constant pressure is obtained using

$$C'^\circ_{Pm} = -T\left(\frac{\partial^2 \Delta_f G'^\circ}{\partial T^2}\right)_{P,\text{pH}} \quad (16)$$

As explained in the previous paper,[1] when the standard enthalpy of formation of a species is independent of temperature, the standard Gibbs energies of the species can be calculated by use of

$$\Delta_f G_j^\circ(T) = (T/298.15)\Delta_f G_j^\circ(298.15) +$$
$$(1 - T/298.15)\Delta_f H_j^\circ(298.15) \quad (17)$$

Substituting this equation into eq 3 yields an even more complicated function of T, pH, and ionic strength for $\Delta_f G'^\circ$ for a reactant. *Mathematica* is very convenient for expressing standard transformed Gibbs energies of formation of reactants as functions of T, pH, and ionic strength and for carrying out partial differentiations.[7] Basic thermodynamic data on species of reactants in enzyme-catalyzed reactions are stored in a *Mathematica* package at *MathSource*.[8] These data come from classical thermodynamic tables and a variety of current sources.[9−11] When C°_{Pm} values are known for all the species in a reactant, eq 17 can be replaced with the corresponding equation when C°_{Pm} is independent of temperature.

The *Mathematica* program (calcthprops) given in the Appendix derives the functions of T, pH, and ionic strength that give $\Delta_f G'^\circ$, $\Delta_f S'^\circ$, $\Delta_f H'^\circ$, C'°_{Pm}, and $\bar{N}_H$ for a reactant at any temperature in the range 273.15 −313.15 K, any pH in the range 5−9, and any ionic strength in the range 0−0.35 M. The input is a matrix for the reactant that gives the standard Gibbs energy of formation at 298.15 K, the standard enthalpy of formation at 298.15 K, the electric charge, and the number of hydrogen atoms in each species. There is a row in the matrix for each species in the reactant. *Mathematica* is very well suited to these calculations, and it is quite remarkable that all the functions of T, pH, and ionic strength for a reactant can be calculated using a *Mathematica* program with 12 lines (see Appendix).

The standard thermodynamic properties properties of species of ATP, ADP, H_2O, and P_i used in the calculations discussed here are given in Table 1. The values for ATP and ADP are

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TABLE 1: Standard Thermodynamic Properties of Species at 298.15 K and Zero Ionic Strength[7]

	$\Delta_f G°$/kJ mol^{-1}	$\Delta_f H°$/kJ mol^{-1}	z	N_H
ATP^{4-}	−2768.10	−3619.21	−4	12
HATP^{3-}	−28.11	−3612.91	−3	13
H$_2$ATP^{2-}	−2838.18	−3627.91	−2	14
ADP^{3-}	−1906.13	−2626.54	−3	12
HADP^{2-}	−1947.10	−2620.94	−2	13
H$_2$ADP^{-}	−1971.98	−2638.54	−1	14
HPO$_4^{2-}$	−1096.1	−1299.00	−2	1
H$_2$PO$_4^{-}$	−1137.3	−1302.60	−1	2
H$_2$O	−237.19	−285.83	0	2

different from those in the preceding article because because Boerio-Goates and co-workers[11] have determined the standard molar entropy of adenosine in dilute aqueous solution at 298.15 K so that it is not necessary to use the convention that its standard thermodynamic properties are zero. However, these changes do not affect thermodynamic properties calculated for reactions. It is important to understand that the information in the table is in differences between values. An uncertainty of 0.01 kJ mol^{-1} corresponds approximately with a 1% uncertainty in an equilibrium constant.

Calculations of Standard Transformed Thermodynamic Properties of Reactants

The transformed thermodynamic properties calculated for ATP using calcthprops are given in Table 2. Readers might like to see the functions of T, pH, and ionic strength that represent these five properties for ATP, but they are too complicated to reproduce here. The equations for $\Delta_f S'°$(ATP) and $\Delta_f H'°$(ATP) each occupy a page of *Mathematica* output. The equation for $C'°_{Pm}$(ATP) occupies three and a half pages of *Mathematica* output, and the equation for $\bar{N}_H$ (ATP) occupies half a page of *Mathematica* output. These equations are given in the Supplementary Information.

Since $\Delta_f H°$ of species are assumed to be independent of temperature, it is perhaps surprising that $\Delta_f H'°$ of ATP does depend a little on temperature. This is in agreement with the fact that $C'°_{Pm}$ is not equal to zero, even though $C°_{Pm}$ is taken to be zero for all the species. There are two reasons for this. The dependence of the transformed heat capacity of a species on ionic strength is given by[1]

$$C'°_{P_i}(I) = C°_{P_i}(I{=}0) + RT[2(\partial\alpha/\partial T)_P + \\ T(\partial^2\alpha/\partial T^2)_P](z_i^2 - N_H(i))I^{1/2}/(1 + BI^{1/2}) \quad (18)$$

Thus the transformed heat capacity of a species can be positive or negative at a finite ionic strength. We also have to remember that the transformed heat capacity of a reactant involving two or more species is not simply a weighted average, but is given by the following equation, which applies to any isomer group or pseudoisomer group.[12]

$$C'°_P(\text{iso}) = \\ \sum_{i=1}^{N_{iso}} r_i C'°_{P_i}(I) + (1/RT)\{\sum_{i=1}^{N_{iso}} r_i(\Delta_f H'°_i)^2 - (\Delta_f H'°(\text{iso}))^2\} \quad (19)$$

The second term in this equation is always positive because the weighted average of the squares of the individual standard transformed enthalpies of formation is always greater than the square of the weighted average enthalpy of formation. This is

TABLE 2: Standard Transformed Thermodynamic Properties of ATP at Three Temperatures, Five pH's, and 0.25 M Ionic Strength

	$\Delta_f G'°$/kJ mol^{-1}				
T/K	pH 5	pH 6	pH 7	pH 8	pH 9
273.15	−2536.26	−2468.70	−2403.56	−2340.35	−2277.54
298.15	−2437.46	−2363.76	−2292.50	−2223.44	−2154.88
313.15	−2378.18	−2300.78	−2225.87	−2153.33	−2081.30

	$\Delta_f S'°$/kJ K^{-1} mol^{-1}				
T/K	pH 5	pH 6	pH 7	pH 8	pH 9
273.15	−3.95303	−4.19781	−4.44323	−4.67723	−4.90748
298.15	−3.95189	−4.19806	−4.44202	−4.67563	−4.90584
313.15	−3.95201	−4.19845	−4.44135	−4.67460	−4.90476

	$\Delta_f H'°$/kJ K^{-1} mol^{-1}				
T/K	pH 5	pH 6	pH 7	pH 8	pH 9
273.15	−3616.03	−3615.34	−3617.23	−361793	−3618.02
298.15	−3615.72	−3615.41	−3616.89	−3617.48	−3617.55
313.15	−3615.75	−3615.53	−3616.68	−3717.16	−3617.22

	$C'°_{Pm}$/kJ K^{-1} mol^{-1}				
T/K	pH 5	pH 6	pH 7	pH 8	pH 9
273.15	0.02533	0.000841	0.01410	0.016604	0.016812
298.15	0.002276	−0.006228	0.13596	0.019990	0.020749
313.15	−0.006820	−0.009528	0.013654	0.022231	0.023310

	$\bar{N}_H$				
T/K	pH 5	pH 6	pH 7	pH 8	pH 9
273.15	13.07	12.73	12.20	12.20	12.20
298.15	13.02	12.75	12.22	12.03	12.03
313.15	13.01	12.75	12.23	12.02	12.03

	(d$\Delta_f S'°$/dpH)/kJ K^{-1} mol^{-1}				
T/K	pH 5	pH 6	pH 7	pH 8	pH 9
273.15	0.237359	0.248877	0.238999	0.231026	0.228972
298.15	0.242225	0.247704	0.238092	0.230919	0.229861
313.15	0.243932	0.246877	0.237313	0.230807	0.229849

in accord with Le Châtelier's principle that as the temperature is raised the equilibrium shifts in the direction that absorbs heat.

One check on the values calculated in Table 2 is that $\Delta_f G'° = \Delta_f H'° - T\Delta_f S'°$. This is true to a machine error of about 10^{-13} kJ mol^{-1}. Another check is provided by the third Maxwell equation (eq 14). The values of the derivative (d$\Delta_f S'°$/dpH) for ATP are given in Table 2, and the same values are obtained by calculating $-R \ln(10)(\text{d}\{T \bar{N}_H\}/\text{d}T)$.

Calculation of the Standard Transformed Properties of an Enzyme-Catalyzed Reaction at a Specified pH and Temperature

The fundamental equation for G' a system containing N' reactants is

$$dG' = -S'dT + VdP + \sum_{i=1}^{N'} u'_i dn'_i + RT \ln(10)n_c(\text{H})\text{dpH} \quad (20)$$

When the system involves a single enzyme-catalyzed reaction, this equation can be written as

$$dG' = -S'dT + VdP + (\sum_{i=1}^{N'} \nu'_i u'_i) \, d\xi' + RT \ln(10)n_c(\text{H})\text{dpH} \quad (21)$$

where ξ' is the extent of the biochemical reaction. It is convenient to use the symbol for the transformed Gibbs energy of reaction defined by

$$\Delta_r G' = \left(\frac{\partial G'}{\partial \xi'}\right)_{T,P,\text{pH}} = \sum_{i=1}^{N'} \nu_i' \mu_i' \qquad (22)$$

Thus the fundamental equation for G' can be written as

$$dG' = -S'dT + VdP + \Delta_r G' d\xi' + RT \ln(10) n_c(\text{H}) d\text{pH} \qquad (23)$$

There is no way to determine G' directly, but $\Delta_r G'^\circ$ can be determined for a reaction as a function of T, pH, and ionic strength, so that other standard transformed thermodynamic properties can be calculated by use the Maxwell equations for eq 23. The first of the three Maxwell equations of interest is

$$-\left(\frac{\partial S'^\circ}{\partial \xi'}\right)_{T,P,\text{pH}} = \left(\frac{\partial \Delta_r G'^\circ}{\partial T}\right)_{P,\xi',\text{pH}} = -\Delta_r S'^\circ \qquad (24)$$

where $\Delta_r S'^\circ$ is the standard transformed reaction entropy. This equation is used to calculate the standard transformed entropy of a reaction when $\Delta_r G'^\circ$ can be obtained as a function of temperature. The second Maxwell equation is

$$\left(\frac{\partial_r G'^\circ}{\partial \text{pH}}\right)_{T,P} = RT \ln(10)\left(\frac{\partial n_c(\text{H})}{\partial \xi'}\right)_{T,P,\text{pH}} = RT \ln(10)\Delta_r N_\text{H} \quad (25)$$

Thus the change in binding of hydrogen ions in the biochemical reaction is given by

$$\Delta_r N_\text{H} = \frac{1}{RT\ln(10)}\left(\frac{\partial \Delta_r G'^\circ}{\partial \text{pH}}\right)_{T,P,\xi'} \qquad (26)$$

The third Maxwell equation is

$$-\left(\frac{\partial \text{S}'}{\partial \text{pH}}\right)_{T,P,\xi'} = R \ln(10)\left(\frac{\partial \{T n_c(\text{H})\}}{\partial \text{T}}\right)_{P,\text{pH},\xi'} \qquad (27)$$

This equation can be written in terms of reaction properties as follows:

$$\left(\frac{\partial \Delta_r S'^\circ}{\partial \text{pH}}\right)_{T,P} = -R \ln(10)\left(\frac{\partial \{T\Delta_r N_\text{H}\}}{\partial \text{T}}\right)_{P,\text{pH}} \qquad (28)$$

This equation is interesting because it shows that it is the change in binding of hydrogen ions in a biochemical reaction that determines how the standard transformed entropy of reaction changes with pH. It is largely the increase in $\Delta_r S'^\circ$ in the hydrolysis of ATP that causes $\Delta_r G'^\circ$ to become rapidly more negative as the pH increases above pH 7. This equation does not yield a new thermodynamic property, but it provides a useful check on calculations of $\Delta_r S'^\circ$ and $\Delta_r N_\text{H}$.

The remaining standard transformed thermodynamic properties of a reaction can be also be calculated from the function for $\Delta_r G'^\circ$ for a reaction. The standard transformed enthalpy of formation is calculated using the Gibbs–Helmholtz equation:

$$\Delta_r H'^\circ = -T^2\left(\frac{\partial(\Delta_r G'^\circ/T)}{\partial \text{T}}\right)_{P,\text{pH}} \qquad (29)$$

The change in standard molar transformed heat capacity at constant pressure in a reaction can be calculated using

$$\Delta_r C_p'^\circ = -T\left(\frac{\partial^2 \Delta_r G'^\circ}{\partial T^2}\right)_{P,\text{pH}} \qquad (30)$$

Calculation of the Standard Transformed Thermodynamic Properties of Biochemical Reactions

The *Mathematica* program (calcthpropsrx) given in the Appendix was written to derive the functions of T, pH, and ionic strength that give $\Delta_r G'^\circ$, $\Delta_r S'^\circ$, $\Delta_r H'^\circ$, $\Delta_r C_p'^\circ$, and $\Delta_r N_\text{H}$ for a reaction at any temperature in the range 273.15–313.15 K, any pH in the range 5–9, and any ionic strength in the range 0–0.35 M. The input is the sum of the functions for the standard transformed Gibbs energies of formation of the reactants, each multiplied by the appropriate stoichiometric number. Note that the steps in calcthpropsrx for the calculation of $\Delta_r S'^\circ$, $\Delta_r H'^\circ$, $\Delta_r C_P'^\circ$, and $\Delta_r N_\text{H}$ are of the same form as in calctheprops, as required by eqs 24, 29, 30, and 26. Table 3 gives the standard transformed thermodynamic properties for the hydrolysis of ATP at three temperatures, five pH's, and ionic strength 0.25 M.

These values are more interesting than those in Table 2 because some of the dependencies of properties of reactants on temperature and ionic strength will cancel in a reaction. The rapid decrease in $\Delta_f G'^\circ$ above pH 7 is especially striking, and it is due to the rapid increase in $\Delta_f S'^\circ$, which is attributable to the production of more H$^+$ at pH's above 7. Since heat is evolved in this reaction, the apparent equilibrium constant decreases at higher temperatures, as shown in Table 3. The last section of Table 3 gives K' that is calculated using $K' = \exp(-\Delta_f G'^\circ/RT)$. The $\Delta_f C_P^\circ$ are all negative, perhaps because of the immobilization of H$_2$O by the ions produced.

One check on the values calculated in Table 3 is that $\Delta_f G'^\circ = \Delta_f H'^\circ - T\Delta_f S'^\circ$. This is true to a machine error of about 10^{-13} kJ mol^{-1}. Another check is provided by the third Maxwell equation (eq 27). The values of the derivative $(d\Delta_r S'^\circ/d\text{pH})$ for the reaction are given in Table 3, and the same values are obtained by calculating $-R \ln(10)(d\{T\Delta_r N_\text{H}\}/dT)$.

Discussion

A thermodynamic potential like the transformed Gibbs energy of a reaction system as a function of its natural variables contains all the thermodynamic information about the system, and so the most straightfoward way to obtain these other properties is to take partial derivatives of the transformed Gibbs energy. Although the transformed Gibbs energy cannot be measured directly, equivalent thermodynamic information can be obtained by measuring apparent equilibrium constants K' as a function of temperature or apparent equilibrium constants at a single temperature and heats of reaction. These calculations are illustrated with ATP, ADP, P$_i$, and H$_2$O and the reaction ATP + H$_2$O = ADP + P$_i$ on the basis of the assumption that the enthalpies of formation of the ions involved are independent of temperature in the range 273.15–313.15 K. A short computer program calcthprops has been written to derive functions for all of the transformed thermodynamic properties for reactants (sums of species). A second computer program calcthpropsrx has been written to use the sum of the functions for reactants, each multiplied by its stoichiometric number, to derive equations for all the transformed thermodynamic properties for an enzyme-catalyzed reaction.

A new feature of these calculations is the use of the three Maxwell equations of the fundamental equation for the transformed Gibbs energy. The third Maxwell equation is used as a check, and it provides a new insight into the dependence of the

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TABLE 3: Standard Transformed Thermodynamic Properties of the Reaction ATP + H$_2$O = ADP + P$_i$ at Three Temperatures, Five pH's, and 0.25 M Ionic Strength

	$\Delta_r G'^\circ$/kJ mol^{-1}				
T/K	pH 5	pH 6	pH 7	pH 8	pH 9
273.15	−32.03	−32.58	−34.94	−39.43	−44.57
298.15	−32.56	−33.22	−36.04	−41.07	−46.70
313.15	−32.87	−33.58	−36.69	−42.05	−47.97

	$\Delta_r S'^\circ$/kJ K^{-1} mol^{-1}				
T/K	pH 5	pH 6	pH 7	pH 8	pH 9
273.15	0.02169	0.02605	0.04386	0.06615	0.08584
298.15	0.02073	0.02515	0.04347	0.06542	0.08850
313.15	0.01984	0.02456	0.044319	0.06493	0.08450

	$\Delta_r H'^\circ$/kJ K^{-1} mol^{-1}				
T/K	pH 5	pH 6	pH 7	pH 8	pH 9
273.15	−26.11	−25.46	−22.96	−21.36	−21.12
298.15	−26.38	−25.72	23.08	−21.57	−21.35
313.15	−26.66	−25.90	−23.16	−21.72	−21.51

	$\Delta_r C'^\circ_{Pm}$/kJ K^{-1} mol^{-1}				
T/K	pH 5	pH 6	pH 7	pH 8	pH 9
273.15	−0.004958	−0.009138	−0.003964	−0.007300	−0.008250
298.15	−0.015977	−0.011434	−0.005182	−0.009318	−0.01025
313.15	−0.022025	−0.01279	−0.005994	−0.010589	−0.01155

	$\Delta_r N_H$				
T/K	pH 5	pH 6	pH 7	pH 8	pH 9
273.15	−0.033	−0.226	−0.697	−0.956	−0.995
298.15	−0.038	−0.249	−0.742	−0.965	−0.996
313.15	−0.040	−0.262	−0.765	−0.969	−0.997

	$(d\Delta_r S'^\circ/dpH)$/kJ K^{-1} mol^{-1}				
T/K	pH 5	pH 6	pH 7	pH 8	pH 9
273.15	0.00258	0.00910	0.02363	0.02048	0.01929
298.15	0.00151	0.00994	0.02342	0.02028	0.01927
313.15	0.00137	0.011058	0.02323	0.02016	0.01926

	K'				
T/K	pH 5	pH 6	pH 7	pH 8	pH 9
273.15	1.3×10^6	1.7×10^6	4.8×10^6	3.5×10^7	3.3×10^8
298.15	0.51×10^6	0.66×10^6	2.1×10^6	1.6×10^7	1.5×10^8
313.15	0.30×10^6	0.40×10^6	1.3×10^6	1.0×10^7	1.0×10^8

standard transformed entropy of formation on pH and the temperature dependence of the average number of hydrogen atoms in a reactant. Although the calculations are based on the assumption that C_P° of species are equal to zero, the calculated changes in standard transformed reaction heat capacities $\Delta_f C_P'^\circ$ do depend on temperature. The accuracy of these transformed thermodynamic properties can be improved by including the dependence on pMg and pCa, and when more values of C_{Pm}° of species become available, their effects can be readily included in eq 17.

Currently data is available in BasicBiochemData2 to make calculations of this type on about 70 biochemical reactants.[8] This number of reactants can be increased by use of calorimetric data and equilibrium measurements in the literature, but work is required to analyze these data.[13] These calculations show the need for more calorimetric data on biochemical reactants and reactions.

Appendix

```
calcthprops[speciesmat_] := Module[{dGzero, dGzeroT, dHzero, zi, nH, gibbscoeff,
pHterm, istermG, gpfnsp,dGfn,dSfn,dHfn,dCPfn,dnHfn},(*This program uses the basic
data on the species of a reactant to derive the fuctions of T, pH, and ionic strength (is)
that give the standard transformed Gibbs energies, entropies, enthalpies, heat capacities,
and average numbers of hydrogen atoms for reactants in enzyme-catalyzed reactions.
The assignment operation can then be used to calculate values at temperatures in the
range 273.15 to 313.15 K, pHs 5 to 9, and ionic strengths 0 to 0.35 M.*)
{dGzero, dHzero, zi, nH} = Transpose[speciesmat];
gibbscoeff = (9.20483*t)/10^3 - (1.284668*t^2)/10^5 +(4.95199*t^3)/10^8;
dGzeroT = (dGzero*t)/298.15 + dHzero*(1 - t/298.15);
pHterm = (nH*8.31451*t*Log[10^(-pH)])/1000;
istermG = (gibbscoeff*(zi^2 - nH)*is^0.5)/(1 + 1.6*is^0.5);
gpfnsp = dGzeroT - pHterm - istermG;
dGfn=-((8.31451*t*Log[Plus @@(E^(-(gpfnsp/((8.31451*t)/1000))))])/1000);
dSfn=-D[dGfn,t];
dHfn=-t^2*D[(dGfn/t),t];
dCPfn=-t*D[dGfn,{t,2}];
dnHfn=D[dGfn,pH]/((8.31451*t/1000)*Log[10]);
{dGfn,dSfn,dHfn,dCPfn,dnHfn}]
calcthpropsrx[eq_] := Module[{dGfnrx,dSfnrx,dHfnrx,dCPfnrx,dnHfnrx},(*This
program uses the function of T, pH, and ionic strength (is) that gives the standard
transformed Gibbs energy of an enzyme-catalyzed reaction to derive the functions for all
the other standard transformed thermodynamic properties. The reaction is written in the
form adpGT+piGT-atpGT-h2oGT. The assignment operation can then be used to
calculate values at temperatures in the range 273.15 to 313.15 K, pHs 5 to 9, and ionic
strengths 0 to 0.35 M.*)
dGfnrx=eq;
dSfnrx=-D[dGfnrx,t];
dHfnrx=-t^2*D[(dGfnrx/t),t];
dCPfnrx=-t*D[dGfnrx,{t,2}];
dnHfnrx=D[dGfnrx,pH]/((8.31451*t/1000)*Log[10]);
{dGfnrx,dSfnrx,dHfnrx,dCPfnrx,dnHfnrx}]
```

Supporting Information Available: A *Mathematica* notebook containing the calculations described in this article. This notebook shows the functions of *T*, pH, and ionic strength that yield all the transformed thermodynamic properties of ATP. It also gives all the transformed thermodynamic properties for the reaction ATP + H$_2$O = ADP + P$_i$. This material is available free of charge via the Internet at http://pubs.acs.org.

References and Notes

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