

Improved Calculation of Rotational Diffusion and Intrinsic Viscosity of Bead Models for Macromolecules and Nanoparticles

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Received: July 27, 2006; In Final Form: October 10, 2006

The conventional Kirkwood–Riseman calculation of the hydrodynamic properties of bead models gives abnormal results for rotational quantities and the intrinsic viscosities for models with a few beads or when one bead is dominant. The reason is that beads are treated as point sources of friction. This can be remedied by introducing terms that are neglected in the conventional treatment of orders 0 and -3 in interbead distances. An alternative strategy is the cubic substitution in which each bead is replaced by a cubic array of minibeads. These procedures require a computational overload that, in the case of the intrinsic viscosity, can be avoided using an estimate of the correction due to the nonzero volume of the beads. We have found how such a correction can be estimated from the geometry of the model, and its application yields results that are within the range of typical experimental errors.

1. Introduction

The calculation of solution properties of macromolecules in solution using bead models was first proposed by Kirkwood and Riseman (KR),^{1–3} who modeled linear (flexible or rodlike) polymers as strings of identical beads. Bloomfield et al. envisioned the possibility of using models with equal or unequal beads to model the complex, peculiar shapes of biological macromolecules using initially approximate forms of the KR theories⁴ for the treatment of hydrodynamic interactions (HIs).^{5,6} The procedures were later improved employing more advanced descriptions of HI and computational algorithms.^{7–11} Nowadays, the methodology is implemented in a simple, public-domain computer program, HYDRO,¹² and is being widely used to calculate solution properties of macromolecules and nanoparticles.

Bloomfield et al.^{11,13,14} realized that a direct application of the KR treatment would give abnormal results for the rotational coefficients and intrinsic viscosity when applied to models with a few beads or, in general, when the size of one bead is not much smaller than the overall size of the particle. The reason is that, in the KR theory, bead sizes are needed to express the bead's frictional coefficient, but beads are regarded as pointlike sources of friction with frictional forces acting at their centers while friction actually takes place at the spherical bead surface. A similar situation arises in a simpler case, namely, the calculation of radius of gyration, R_g , which for an array of N elements is usually evaluated as

$$R_g^2(\text{uncorr}) = \sum_{i=1}^N f_i s_i^2 = \frac{1}{N} \sum_{i=1}^N s_i^2 = \frac{1}{2N^2} \sum_{i=1}^N \sum_{j=1}^N r_{ij}^2 \quad (1)$$

where f_i and s_i^2 are, respectively, the volume fraction and the distance from the center of element i to the center of mass. The third and fourth terms in eq 1 are for identical elements, where r_{ij} is the distance between i and j . For a particle consisting of a pair of identical, touching spheres of radius a , these formulas

give $R_g^2(\text{uncorr}) = a^2$ when the correct result (obtained from the moments of inertia of this particle) is $R_g^2 = 8a^2/5$. If it is applied to just a single sphere, then eq 1 gives the unphysical result $R_g^2(\text{uncorr}) = 0$ rather than $R_g^2 = 3a^2/5$. Actually, the correct expression for the radius of gyration of a multisubunit structure is¹⁵

$$R_g^2 = R_g^2(\text{uncorr}) + \sum_{i=1}^N f_i^2 R_{g,i}^2 \quad (2)$$

where f_i and $R_{g,i}^2$ are respectively the volume fraction and square radius of gyration ($R_{g,i}^2 = 3a_i^2/5$ for spherical beads) of the i th subunit.

As we describe in the next section, the standard KR treatment presents a similar problem, which may produce rather unphysical results for the rotational coefficients and the intrinsic viscosity. Among the various hydrodynamic properties, rotational quantities are the most sensitive to the size and shape of the particles and play an essential role in advanced instrumental methods like dynamic fluorescence and NMR relaxation. The intrinsic viscosity, which is also sensitive to particle geometry (except for globular particles), is a classically studied property which is again increasingly used, thanks to instrumental advances¹⁶ and its use with a separation technique like multiple-detection size exclusion chromatography. Thus, in this work, we consider the various strategies available to avoid the mentioned difficulties and propose effective and computationally efficient procedures.

2. Theory and Methods

2.1. Shell Model and Cubic Substitution. As an alternative to what we may call bead models in the strict sense, Bloomfield and Filson^{13,17} proposed the strategy of shell modeling in which the particle surface is modeled as a shell of many small, identical “minibeads”; thus, friction takes place throughout the particle surface (a similar concept was later employed in the so-called boundary element approach^{18–22} in which the elements are platelets that pave the particle surface). Originally,^{13,17} the

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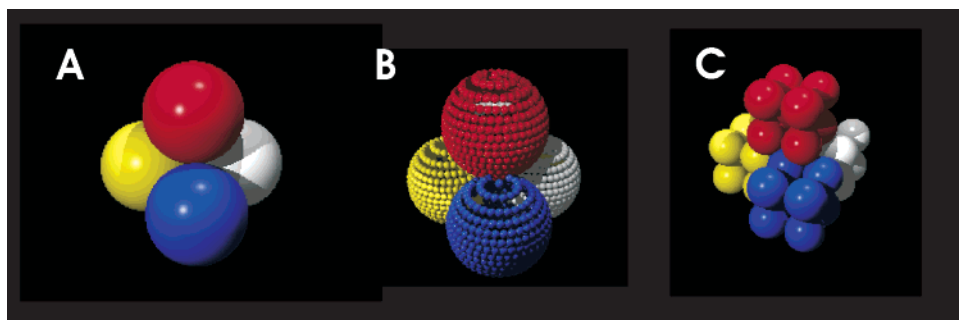


Figure 1. Illustration of the various kinds of modeling for a tetrahedral tetramer. (A) Bead model (in the strict sense). (B) SHM. (C) CBS.

hydrodynamic properties of the shell of beads were calculated with the approximate versions of the KR theory (double-sum formulas),⁴ and presently, the calculation can be done with the more rigorous treatment of HI.²³ In the shell-model (SHM) procedure, calculations are repeated for decreasing minibead sizes (with increasing resolution and an increasing number of beads), and the results are extrapolated to the SHM limit of a zero minibead radius. It has been shown that this strategy gives very accurate results for particles whose hydrodynamic properties are quasi-exactly known from fluid mechanics;^{23,24} some examples will be presented here. Bead models, in a strict sense, can be converted into SHMs by replacing each bead by a spherical shell. The computer program HYDROSUB²⁵ or HYDROPIX²⁶ can be used for this purpose. The drawback of this procedure is that the computing time for rigorous HI calculations is proportional to N^3 ; thus, for instance, if a bead model of 20 beads is replaced by a shell of 2000 minibeads, then the computing time is 10^6 times longer. Still, the typical SHM calculations, as implemented in the programs of the HYDRO suite,^{25–28} take only a few minutes of central processing unit time in a personal computer. This can be easily afforded for calculations of just one or a few structures, but it may become prohibitive in situations where many structures have to be considered, as in conformational searches or Monte Carlo simulations.^{29,30}

The cubic substitution^{11,14,31} (CBS) is another strategy with the same scope. Each bead, with radius σ_i in the primary bead model, is replaced by a cubic array of eight beads. As in SHMs, although not so perfectly, the friction is moved from the center of the bead toward its surface. The radius of the beads in the cube is $0.4668\sigma_i$; this choice nearly makes the volume and rotational friction of the cubic array very close to those of the primitive sphere. Now, the number of beads is 8-fold increased, and the computing time increases by a factor of $8^3 = 512$. Again, this cost may be important for cases requiring multiple calculations. Examples of the SHM and CBS model are displayed in Figure 1.

2.2. Extended Kirkwood–Riseman Treatments: Volume Corrections and Higher-Order HIs. The basic relationship governing the frictional and Brownian behavior of a rigid particle is the generalized Einstein equation,³²

$$\begin{pmatrix} \mathbf{D}^{\text{tt}} & \mathbf{D}^{\text{tr,T}} \\ \mathbf{D}^{\text{tr}} & \mathbf{D}^{\text{rr}} \end{pmatrix} = kT^{-1} \begin{pmatrix} \Xi^{\text{tt}} & \Xi^{\text{tr,T}} \\ \Xi^{\text{tr}} & \Xi^{\text{rr}} \end{pmatrix} \quad (3)$$

The left-hand side of eq 3 is the generalized, 6×6 diffusion matrix, containing 3×3 blocks for translation, \mathbf{D}^{tt} , rotation, \mathbf{D}^{rr} , and translation rotation coupling, \mathbf{D}^{tr} . This is derived from the inverse of the 6×6 generalized friction matrix whose 3×3 blocks are the translation, rotation, and translation rotation coupling friction tensors, Ξ^{tt} , Ξ^{rr} , and Ξ^{tr} . The Ξ 's are obtained

from a $6N \times 6N$ grand resistance matrix, ζ_{6N} . Of the $6N$ coordinates, $3N$ corresponds to the components of linear velocities and the frictional forces, and $3N$ corresponds to the components of the angular velocities and frictional torques.

The grand resistance matrix is, in turn, the inverse of the $6N \times 6N$ grand mobility matrix, $\zeta_{6N} = \mu_{6N}^{-1}$. These grand matrices can be partitioned into four $3N \times 3N$ matrices corresponding to translation, rotation, and translation rotation coupling. Each of them has $N \times N$ blocks of dimension 3×3 corresponding to each pair (i,j) of frictional elements in the model. Thus,

$$\mu_{6N} = \begin{pmatrix} \mu^{\text{tt}} & \mu^{\text{tr}} \\ \mu^{\text{rt}} & \mu^{\text{rr}} \end{pmatrix} = \begin{pmatrix} \mu_{11}^{\text{tt}} & \cdots & \mu_{1N}^{\text{tt}} & \mu_{11}^{\text{tr}} & \cdots & \mu_{1N}^{\text{tr}} \\ \vdots & \cdots & \vdots & \vdots & \cdots & \vdots \\ \mu_{N1}^{\text{tt}} & \cdots & \mu_{NN}^{\text{tt}} & \mu_{N1}^{\text{tr}} & \cdots & \mu_{NN}^{\text{tr}} \\ \mu_{11}^{\text{rt}} & \cdots & \mu_{1N}^{\text{rt}} & \mu_{11}^{\text{rr}} & \cdots & \mu_{1N}^{\text{rr}} \\ \vdots & \cdots & \vdots & \vdots & \cdots & \vdots \\ \mu_{N1}^{\text{rt}} & \cdots & \mu_{NN}^{\text{rt}} & \mu_{N1}^{\text{rr}} & \cdots & \mu_{NN}^{\text{rr}} \end{pmatrix} \quad (4)$$

and ζ_{6N} has the same form. In its most general form, the procedure would start from the calculation of the μ_{ij}^{tt} , μ_{ij}^{tr} , and μ_{ij}^{rr} tensors and end up with the obtention of \mathbf{D}^{tt} , \mathbf{D}^{tr} , and \mathbf{D}^{rr} .

In the conventional KR treatment, the rotation and coupling supermatrices are ignored, that is, $\Xi^{\text{tr}} = 0$ and $\Xi^{\text{rt}} = 0$; in eq 4, there are no rr and tr blocks. The rotational friction is attributed exclusively to the torques corresponding to frictional forces produced by the rotational displacement of the beads, and bead self-rotation is ignored. Indeed, the contribution of the latter is negligible when bead sizes are much smaller than the overall particle size. The conventional KR treatment assigns to μ^{tt} the first term of the order of r_{ij}^{-1} in a series expansion in powers of the reciprocal interbead distance r_{ij} ,

$$\mu_{ij}^{\text{tt}} = \delta_{ij} \zeta_i^{-1} \mathbf{I} + (1 - \delta_{ij}) \mathbf{T}_{ij} \quad (5)$$

where δ_{ij} is Kronecker's delta ($=1$ for $i = j$, $=0$ for $i \neq j$) and \mathbf{I} is the 3×3 identity matrix. The Stokes law friction coefficient of bead i is given by $\zeta_i = 6\pi\eta_0\sigma_i$, η_0 being the solvent viscosity and \mathbf{T}_{ij} being the well-known Oseen tensor

$$\mathbf{T}_{ij} = \frac{1}{8\pi\eta_0 r_{ij}} \left(\mathbf{I} + \frac{r_{ij} \mathbf{r}_{ij}}{r_{ij}^2} \right) \quad (6)$$

In a more rigorous and consistent formalism, if one includes terms up to order -1 in μ^{tt} , then the 0th-order term in μ^{rr} should not be ignored; this term has nonzero ii blocks given by

$$\mu_{ii}^{\text{rr}} = \delta_{ij} (8\pi\eta_0\sigma_i^3)^{-1} \mathbf{I} \quad (7)$$

When this term is included, the rotational friction tensor is changed, taking the form^{33,34}

$$\Xi^{\text{r}}(\text{corr}) = \Xi^{\text{r}}(\text{uncorr}) + 6V_{\text{m}}\eta_0\mathbf{I} \quad (8)$$

where $\Xi^{\text{r}}(\text{uncorr})$ is the result from the primitive KR treatment and the correcting term is related to the total volume of the bead model, $V_{\text{m}} = (4/3)\pi\sum_i\sigma_i^3$.

The 0th-order correction can also be formulated for the intrinsic viscosity. The theoretical description for this property is more complex and lengthy, and the reader is referred to the original publication³⁴ for details. The final result is a correction to the primitive KR-type result:

$$[\eta](\text{corr}) = [\eta](\text{uncorr}) + f_{\eta}\frac{5N_{\text{A}}V_{\text{m}}}{2M} \quad (9)$$

where $f_{\eta} = 1$ (vide infra) and M is the molecular weight. Note that for a model of identical beads of radius σ , each representing a molecular weight $M_1 = M/N$, the volume correction for the viscosity (the second term on the right-hand side of eq 9) reduces to $[\eta]_1 = 10\pi\sigma^3/3M_1$, which is the Einstein law intrinsic viscosity of a single bead. This correction was hinted to (although not formally deduced) and introduced by other workers in the calculation of intrinsic viscosity of short (oligomeric) polymer chains.^{35,36}

The f_{η} factor does not appear in the original formulation³⁴ (or, what is the same, $f_{\eta} = 1$). Examination of the results from the original treatment indicates that, in most instances, while the uncorrected viscosity falls always below exact results, EXA (or more precise results), the inclusion of the volume correction overestimates them. It seems that an intermediate correction would be more appropriate, and indeed, one of the purposes of the present paper is to consider such a possibility, trying to estimate the adequate f_{η} in terms of the structure of the bead model. Actually, if we have a reference value for the intrinsic viscosity, denoted as $[\eta](\text{ref})$, then the correction that, added to the uncorrected value, would match the reference is

$$f_{\eta} = \frac{[\eta](\text{ref}) - [\eta](\text{uncorr})}{[\eta](\text{corr}, f_{\eta} = 1) - [\eta](\text{uncorr})} = \frac{[\eta](\text{ref}) - [\eta](\text{uncorr})}{5N_{\text{A}}V_{\text{m}}/2M} \quad (10)$$

With or without volume correction, the KR treatments described so far involve, as the main computational step, the inversion of the $3N \times 3N$ μ^{t} matrix.

As reviewed elsewhere,³⁷ higher-order approximations are available for the mobility tensors in eq 4; for instance, the theory of Reuland et al.³⁸ includes terms of order up to r_{ij}^{-9} . Another theory is that described by Goldstein,³⁹ which has simpler expressions but has been found to be rather accurate.³⁷ Actually, we have chosen the Goldstein treatment as that representative of the higher-order approaches.

The formalism is much simpler, but still with accurate results, if we take only terms up to that of r_{ij}^{-3} . In the third-order approximation, the translational tensors are given by eq 5, replacing the Oseen tensor by the García de la Torre–Bloomfield tensor⁸ (which is a generalization for unequal beads of the Rotne–Prager–Yamakawa tensor^{40,41} for identical beads):

$$\mathbf{T}_{ij} = (8\pi\eta_0 R_{ij})^{-1} \left(\mathbf{I} + \frac{\mathbf{R}_{ij}\mathbf{R}_{ij}}{R_{ij}^2} + \frac{\sigma_i^2 + \sigma_j^2}{R_{ij}^2} \left(\frac{1}{3}\mathbf{I} - \frac{\mathbf{R}_{ij}\mathbf{R}_{ij}}{R_{ij}^2} \right) \right) \quad (11)$$

It was suggested that a KR calculation using this modification of the Oseen tensor would improve the calculation of translational properties (particularly, by removing singularities in the supermatrix). This procedure, which will be denoted as KRM, does not improve the calculation of rotational properties because it still neglects terms of order -3 in the rotation and coupling tensors (and, therefore, this approach is not to be confused with the full third-order procedure, later denoted as 3RD) and the volume correction or alternative approaches are needed. Instead, one could accept those terms that are given by

$$\mu_{ij}^{\text{r}} = \delta_{ij} \frac{1}{8\pi\eta_0\sigma_i^3} \mathbf{I} + (1 - \delta_{ij}) \frac{1}{16\pi\eta_0 r_{ij}^3} \left(3 \frac{\mathbf{r}_{ij}\mathbf{r}_{ij}}{r_{ij}^2} - \mathbf{I} \right) \quad (12)$$

and

$$\mu_{ij}^{\text{rt}} = -(1 - \delta_{ij})(8\pi\eta_0 r_{ij}^3)^{-1} \epsilon \cdot \mathbf{r}_{ij} \quad (13)$$

where

$$\epsilon \cdot \mathbf{r}_{ij} = \begin{pmatrix} 0 & z_{ij} & -y_{ij} \\ -z_{ij} & 0 & x_{ij} \\ y_{ij} & -x_{ij} & 0 \end{pmatrix} \quad (14)$$

When third-order HIs (3RD) or higher-order HIs (GOL) are considered, one has to invert the full $6N \times 6N$ supermatrix in eq 4 instead of the $3N \times 3N$ supermatrix μ^{t} in the KR treatment, and computing time increases by a factor of $2^3 = 8$, which is much less important than the computing overhead required by the CBS model or the SHM. Unfortunately, the inclusion of third- and higher-order terms makes the formalism for the intrinsic viscosity extremely complicated; therefore, these treatments are limited to the calculation of translational and rotational properties, and the possibility of improving the calculation of the viscosity should be, by now, restricted to an ad hoc modification of the volume correction, as we shall describe in next section.

3. Numerical Results

The different approaches for the calculation of properties to be compared are coded as: EXA, exact results; SHM, shell model; CBS, cubic substitution; 3RD, third-order HI; GOL, higher-order HI;³⁹ KRM, Kirkwood–Riseman with modified García de la Torre–Bloomfield HI but without volume correction; and KRV, the same as KRM but with full volume correction.

The properties that we consider (depending on the cases) are the translational diffusion coefficient, D_t ; the intrinsic viscosity, $[\eta]$; the component of the rotational diffusion tensor corresponding to an axis perpendicular to the main axis of the particle, D_r^{\perp} , which is the one usually observed for elongated or planar conformations; the component of the rotational diffusion tensor corresponding along the main axis of the particle, D_r^{\parallel} ; and the rotational diffusion coefficient, D_r , evaluated as the trace of the rotational diffusion tensor, $D_r = D_r^{\parallel} + 2D_r^{\perp}$, related to the so-called rotational correlation time.

A simple, immediate, and relevant comparison is that of the results of the procedures involving bead models with the exact Einstein values for a spherical particle. Table 1 shows such a comparison for the two advanced bead/shell modeling approaches. It is clear that both the SHM and the CBS yield very accurate results, deviating typically 1% from the EXA, with the only exception of the CBS result for $[\eta]$, whose deviation is still within the usual range of experimental errors for this

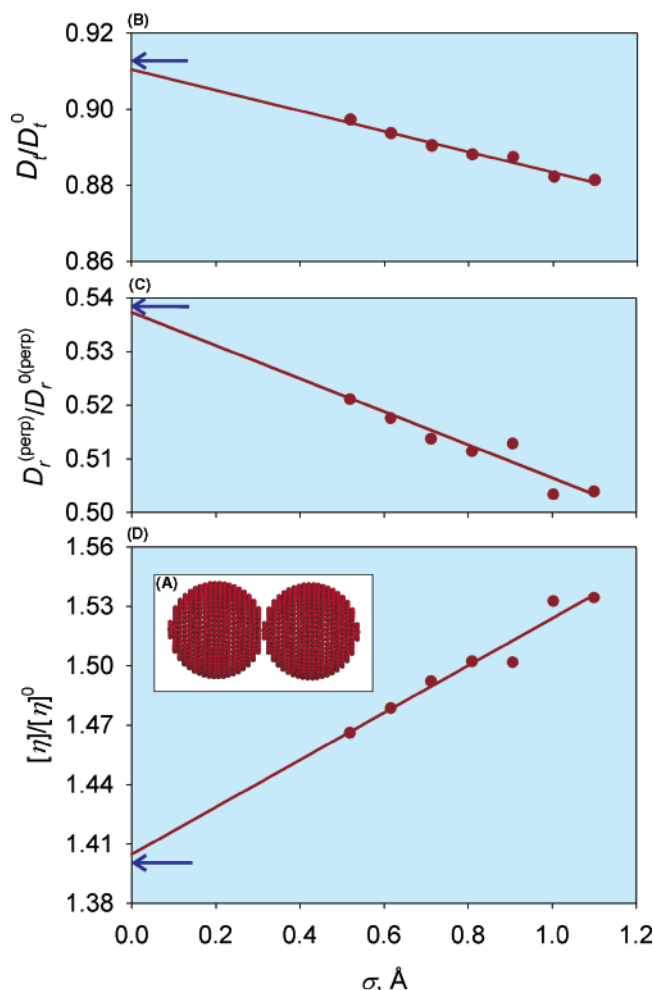
TABLE 1: Percent Deviations of the Results from the SHM and CBS Methods from the EXA for a Spherical Particle

percent deviation	method	
	SHM	CBS
in D_t	0.9	-0.8
in D_r	0.7	-0.9
in $[\eta]$	-1.1	-3.8

property. For completeness, we recall that the KRM method without volume correction gives quite a good estimate of D_t (deviation of only 3%³⁷), but as described above, it fails badly predicting zero friction and zero $[\eta]$ with EXA for the volume corrected KRV and the GOL theories.

In the following tables where we present the results for a variety of structures, numerical values of the properties will be given for only one of the methods, taken as the reference, except for a few cases when a more exact result is available. In order to make the results more general, independent of the actual size of the particle, what we report are the ratios of each property for the particle to the value for a sphere of the same volume, given by $D_t^0 = k_B T / 6\pi\eta_0 a_V$ and $D_r^0 = k_B T / 8\pi\eta_0 a_V^3$ where $a_V = (3V_m/4\pi)^{1/3}$ is the radius of the sphere with equal volume and $[\eta]^0 = 5V_m N_A / 2M$ (Figure 2). For the remaining methods, we list in the tables their percent deviation from the reference; in this way, the presentation of results is more compact, and specifically, we focus on the differences among the various procedures.

Table 2 presents results for dimers consisting of two touching (tangent) spheres with radii σ_1 and σ_2 . Codes for these structures

**Figure 2.** (A) SHM for D1 and (B–D) SHM extrapolations. EXA are indicated for comparison with the SHM results.**TABLE 2: Results for Dimers of Touching Spheres with Ratios of Radii $\sigma_1/\sigma_2 = 1$ (D1), 2 (D2), and 3 (D3)**

	D1	EXA ^a	SHM	GOL	3RD	CBS	KRM	KRV
D_t		0.907	0.6	1.0	2.3	0.0	4.4	4.4
D_r^+		0.532	1.6	0.9	1.4	-0.9	25	-19
$D_r^ $		1.125	-1.9	-0.4	0.1	0.3		-11
D_r		0.730	-0.2	0.2	0.7	-0.3		-15
$[\eta]$		1.38	-1.0			2.7	-54	19

	D2	EXA	SHM ^a	GOL	3RD	CBS	KRM	KRV
D_t			0.956	-0.5	0.2	-1.9	1.5	1.5
D_r^+			0.694	-0.2	0.8	-2.4	74.0	-21.0
$D_r^ $			1.058	-0.1	0.1	-0.7		-5.5
D_r			0.816	-0.2	0.5	-1.6		-34
$[\eta]$			1.177			-0.3	-69	17

	D3	EXA	SHM ^a	GOL	3RD	CBS	KRM	KRV
D_t			0.981	-0.5	-0.1	-2.0	0.5	0.5
D_r^+			0.837	0.0	1.1	-2.4	145	-20
$D_r^ $			1.031	-0.7	-0.6	-1.6		-3.0
D_r			0.902	-0.3	0.5	-2.1		-13
$[\eta]$			1.072			-0.1	-80	13

^a Reference values (relative to those of the sphere of the same volume, see text) with respect to which we give the percent deviation of the other methods in the following columns.

are D_x , where $x = \sigma_1/\sigma_2$. In the case of the dimer with identical beads, practically exact results are available in the literature for translational and rotational diffusion^{42–46} and for the viscosity.^{47–49} For the dimers of unequal spheres, as well as for most of the remaining structures to be considered hereafter, we shall take the SHM results for the reference since, among the most accurate methods, this is the one that allows the calculation of $[\eta]$ along with that of D_t and D_r . It is clear that our 3RD and CBS methods yield excellent results for all of the properties in the three cases. We also note how the results with the volume correction (KRV) improve the uncorrected ones (KRM); still, they deviate somewhat from the reference, particularly for the case of identical beads.

In addition to the single sphere and the pair of spheres, we have analyzed the performance of the bead modeling methods for a variety of arrangements of touching spheres, including the following (1–6).

(1) Polyhedral arrays: tetrahedral (P4), octahedral (P6), cubic (P8), icosahedral (P12), and dodecahedral (P20).

(2) Regular polygons (rings) with N beads (RN).

(3) Linear strings (rods) of N beads (LN).

(4) “Lollipops” consisting of a bead of radius σ_1 followed by a linear array of N beads with radius $\sigma_2 = \sigma_1/5$ (CN).

(5) Three-dimensional dendritic arrays with 3, 4, 5, and 6 generations. The first generation, G1, is a V-shaped trimer with an internal angle of 120°. Successive generations are constructed by attaching two beads to the external beads of the previous generation, making an angle of 120° and avoiding bead overlapping. The numbers of beads are 15, 31, 63, and 127 for G3–G6, respectively. These dendrimers are instantaneous conformations (here considered of rigid structures) of flexible dendritic polymers simulated by a Monte Carlo procedure in an ongoing project.⁵⁰

(6) Models A_n and B_n consisting of seven beads of decreasing radii: 100, 90, 80, 70, 60, 50, and 40 Å for $n = 1$; 100, 80, 60, 40, 20, 10, and 5 Å for cases with $n = 2$; and 100, 95, 90, 85, 80, and 75 Å for $n = 3$. Note that the disparity of bead sizes

TABLE 3: Results for the Translational and Rotational Diffusion Coefficient, Presented as in Table 2

structure	D_t				D_r^+				
	SHM ^a	CBS ^b	3RD (%)	KRM (%)	SHM ^a	CBS ^b	3RD (%)	KRM (%)	KRV (%)
L3	0.84	-0.2	1.7	2.9	0.34	-3.1	-1.5	20	-14
L4	0.78	0.1	1.8	2.8	0.23	-3.4	-1.7	12	-11
L6	0.70	0.2	1.7	2.4	0.13	-4.7	-2.8	4.6	-8.1
L8	0.65	0.4	1.7	2.2	0.09	-5.4	-3.4	1.6	-6.6
L10		0.60	1.2	1.6		0.06	2.2	6.1	-0.01
L13		0.55	1.1	1.4		0.04	2.3	5.1	1.0
L20		0.47	0.9	1.1		0.02	2.2	4	1.9
P4	0.89	-0.8	2.0	8.8	0.64	-0.9	6.3	32	-29
P6	0.90	-1.1	1.5	9.2	0.69	-1.1	8.8	30	-31
P8	0.86	-0.1	1.9	7.6	0.61	0.3	9.6	33	-26
P12		0.88	3.0	8.3		0.67	12	39	-28
P20		0.79	3.0	6.6		0.48	9.9	31	-20
R3	0.89	-0.8	2.3	6.8	0.72	-1.7	2.3	57	-26
R4	0.87	-0.1	2.9	7.0	0.64	-0.1	3.5	21	-33
R6	0.81	-0.6	2.6	5.8	0.5	-2.1	1.7	36	-19
R8		0.74	2.9	5.7		0.37	3.6	30	-12
G3		0.64	1.9	3.1		0.19	2.9	9.9	-8.8
G4		0.60	1.6	2.8		0.17	3	8.2	-8.8
G5		0.60		2.5		0.18		7.4	-10
G6		0.63		2.2		0.22		6.8	-14
C2	1.0	-2	-0.5	-0.3	0.94	-2.2	1	340	-15
C4	0.95	-1.8	-0.5	-0.2	0.67	-1.6	0.6	101	-14
C6	0.89	-1.6	-0.3	0.05	0.45	-2	-0.04	48	-11
C11		0.77	1.2	1.5		0.18	1.8	18	-2.9
C16		0.68	1.1	1.4		0.09	1.9	10	0.0
A1	0.80	-1.2	1.0	2.5	0.39	-2.4	0.5	17	-20
A2	0.85	-1.1	1.9	3.3	0.4	-3.0	-0.2	17	-21
A3	0.79	-1.2	0.8	2.6	0.43	-1.8	1.9	20	-21
B1	0.71	0.0	1.5	2.2	0.13	3.9	-1.8	5.1	-7.6
B2	0.81	-0.4	1.5	2.3	0.26	2.1	0.5	14	-12
B3	0.68		1.5	2.1	0.11		-2.7	3.4	-7.1
normalized rms % difference			1.7	4.4	rms % difference			4.3	68
									17

^a Reference values (relative to those of the sphere of the same volume, see text) with respect to which we give the percent deviation of the other methods in the following columns. ^b Same as *a* or percent.

varies from one case to another. In A structures, the 100 Å bead is surrounded by the other ones in a rather compact arrangement, while in B structures, the successive beads are colinear, on one side of the first one, presenting an elongated, extended conformation.

Results for the translational and rotational coefficients are reported in Table 3, and those of the intrinsic viscosity are reported in Table 4. As for the dimer with unequal beads, the supposedly most accurate result taken as reference is that of the SHM. In a few cases with an appreciably large number of beads ($N > 10$), shell modeling presents some practical difficulties, and then the CBS results are used for reference. As an indicator of the typical errors of the method for each property, we report the root-mean-square percent difference between the result of that method and the reference.

For translational diffusion, the conventional and simplest KRM procedure gives quite acceptable results for most cases (errors of about 4%), except for the compact polyhedral and dendrimeric structures. The CBS and 3RD methods perform very well in all cases; for the latter, typical errors are about 2%.

For rotational diffusion, the volume correction in KRV decreases the large errors found in many cases with the uncorrected KRM calculation (68% reduced to 17%). Again, the performance of the CBS and 3RD methods is very good. Particularly, the 3RD method presents an excellent compromise between computational cost and error (about 4%). The observable properties related to rotational diffusion, like the decays of electric birefringence and fluorescence anisotropy or NMR relaxation, depend in a complex way on the rotational diffusion tensor. The components or eigenvalues of this tensor of a given

particle are affected differently by the hydrodynamic approximations and corrections. As the 3RD procedure provides a rather accurate evaluation of the full tensor, we recommend it as the best choice for the calculation of rotation of bead models.

In Table 4, we notice the previously commented performance of the volume correction of the intrinsic viscosity. For the ample collection of bead models that we have considered, the neglect of the correction (KRM) produces an under-estimation of $[\eta]$ of typically 32%. On the other hand, when the correction is included (KRV), $[\eta]$ is generally over-estimated with a typical deviation of 23%. This deviation is smaller than that found when the correction is neglected; therefore, it can be concluded that, indeed, the volume correction produces an overall improvement, but the general impression is not satisfactory. While in various cases it gives a substantial improvement, for others, particularly for the more globular structures (polyhedral and dendritic oligomers), the volume correction does not improve or worsen the results.

However, the fact that results are too small without correction and too large with it suggests, as anticipated above, that a fractional correction would give better results than the full correction or its absence. Thus, for each structure, we have evaluated the f_η factor that would give a result identical to the reference (eq 10). These adjusted values of f_η are reported in Table 4.

We notice a dependence of f_η on the geometry of the bead model. For models in which one bead or a few beads are dominant and account for a large fraction of the total volume so that the model has a small degree of fragmentation, the introduction of the correction is important, and the f_η values

TABLE 4: Results for Intrinsic Viscosity, Presented as in Tables 2 and 3

structure	SHM ^a	CBS ^b	KRM (%)	KRV (%)	f_η	S	A	$f_\eta(\text{calc})$	KRI (%)
L3	1.83	-0.5	-40	15	0.73	1.50	0.20	0.73	1.2
L4	2.39	-0.54	-31	11	0.74	2.00	0.24	0.74	0.8
L6	3.67	0.33	-20	7.1	0.74	3.00	0.29	0.74	0.6
L8	5.24	0.05	-15	4.1	0.79	4.00	0.33	0.79	-0.3
L10		7.05	-12	2.4	0.83	5.00	0.35	0.83	-0.6
L13		10.17	-8.8	1.0	0.89	6.50	0.37	0.89	-0.9
L20		19.31	-5.5	-0.4	1.07	10.00	0.41	1.07	-1.2
P4	1.41	-0.12	-30	41	0.42	2.00	0.01	0.42	6.7
P6	1.36	2.3	-24	49	0.33	3.00	0.00	0.33	7.5
P8	1.56	-2.5	-24	40	0.38	4.00	0.00	0.38	0.4
P12		2.15	-23	47	0.42	6.00	0.00	0.42	-0.2
P20		2.02	-17	32	0.49	10.00	0.00	0.49	-3.9
R3	1.40	-0.32	-38	34	0.52	1.50	0.12	0.52	10.9
R4	1.55	-2.0	-33	31	0.52	2.00	0.15	0.52	9.3
R6	1.92	1.2	-27	25	0.52	3.00	0.18	0.52	6.0
R8		2.46	-25	16	0.60	4.00	0.19	0.60	0.5
G3		3.78	-13	13	0.50	7.50	0.11	0.50	-1.6
G4		4.41	-9.9	13	0.44	15.50	0.09	0.44	-1.7
G5		4.43	-8.2	14	0.36	31.50	0.07	0.36	-1.3
G6		3.95	-7.0	18	0.27	63.50	0.05	0.27	-0.4
C2	1.02	-0.77	-90	8.3	0.92	0.51	0.01	0.92	-12.2
C4	1.19	-0.37	-75	9.1	0.89	0.55	0.07	0.89	-7.2
C6	1.48	0.00	-60	8.1	0.88	0.62	0.11	0.88	-4.4
C11		2.71	-33	4.1	0.89	0.91	0.19	0.89	-2.7
C16		4.68	-20	1.9	0.91	1.36	0.25	0.91	-2.3
A1	1.96	2.6	-21	30.	0.41	2.06	0.11	0.41	10.1
A2	1.69	2.0	-33	26	0.55	1.15	0.18	0.55	12.8
A3	1.98	2.5	-20	31	0.40	2.82	0.05	0.40	5.2
B1	3.64	0.21	-20	7.1	0.74	2.06	0.30	0.74	1.7
B2	2.10	0.05	-34	13	0.73	1.15	0.23	0.73	4.2
B3	4.25		-17	6.1	0.74	2.82	0.31	0.74	1.0
rms % difference			32	23		rms % difference			5.5

^a Reference values (relative to those of the sphere of the same volume, see text) with respect to which we give the percent deviation of the other methods in the following columns. ^b Same as *a* or percent.

are close to unity. We have found that the degree of fragmentation can be quantified as follows. Suppose that the beads are indexed so that $i = 1$ is the largest one, $i = 2$ is the second largest, and so on. Let ϕ_i be the volume fraction of the bead (in reference to the total volume). Define the cumulative amounts (which form a monotonously increasing series) $S_i = \sum_{k=1}^i \phi_k$ and $S_N = N$. Then, the quantity defined as

$$S = N - \sum_{i=1}^N \frac{S_i + S_{i-1}}{2} \equiv N + \frac{1}{2} - \sum_{i=1}^N S_i \quad (15)$$

has the following properties: (a) for a model with identical beads, it takes the maximum value, $S = N/2$, and (b) in a model in which the largest bead is dominant, with $\phi_1 \approx 1$, $S \approx 0$. In the latter case, the volume correction is essential, while in the former one it may not be so important.

Looking at the values of f_η for models of identical beads (structures PN, RN, LN, and GN), we notice that they are smaller for compact shapes (polygonal and dendrimeric structures) and larger ($f_\eta \approx 1$) for elongated ones, like rods. Among the various possibilities, a convenient one in our case, to express particle's anisometry is the anisotropy of the translational diffusion tensor, \mathbf{D}_t , which is an intermediate result in the calculation of the translational diffusion coefficient. If $D_t^{(1)}$, $D_t^{(2)}$, and $D_t^{(3)}$ are the eigenvalues of \mathbf{D}_t , the tensor anisotropy can be expressed by

$$\Delta = [(D_t^{(1)})^2 + (D_t^{(2)})^2 + (D_t^{(3)})^2 - D_t^{(1)}D_t^{(2)} - D_t^{(1)}D_t^{(3)} - D_t^{(2)}D_t^{(3)}]^{1/2} \quad (16)$$

and a relative anisotropy can be formulated as $A = \Delta/D_t$ where

$D_t = (1/3)(D_t^{(1)} + D_t^{(2)} + D_t^{(3)})$ is the translational diffusion coefficient. For a globular, approximately isometric particle, $\Delta \approx 0$, and $A \approx 0$. At another extreme, for a long rod, $D_t^{(1)} = D_t^{(2)} = 1/2 D_t^{(3)}$, and $\Delta = 3/4$.

Thus, one can expect some correlation between the f_η factor and the quantities A and S . In other words, from the A and S values of a bead model, a proper f_η value could be estimated. With a large body of results obtained in this work, we have found an expression of the type $f_\eta(A, S)$ that fits well all of the data obtained by nonlinear least-squares fitting

$$f_\eta = 1 - 0.542S/(S + 1) - 0.289S^2/(S + 1)^2 + 1.50AS/(S + 1) \quad (17)$$

This kind of function is not the only one, and by no means the best, which fits the data set; simply, it represents the data adequately and is useful to estimate, for a given model, the f_η factor corresponding to an intermediate volume correction.

Then, instead of ignoring the volume correction ($f_\eta = 0$) or including it fully ($f_\eta = 1$), we propose an alternative procedure with an estimated, intermediate correction, denoted as KRI in Table 4. In this procedure, we estimate f_η from eq 17, using the easily calculated S and A values (values denoted as $f_\eta(\text{calc})$ in Table 4), and this estimate is employed for the evaluation of a corrected $[\eta]$ from eq 9. The performance of this procedure is quantified by the root-mean-square value of the difference from the result calculated and that taken as reference (reported in the last column of Table 4); therefore, that can be used as a typical percent error of the procedure. We find that this error is only 5.2%; this is much smaller than what we find, in a similar way, for $f_\eta = 0$ in the absence of correction (40%) or for a full correction with $f_\eta = 1$ (22%). Furthermore, such a small

difference is of the same magnitude as the typical errors in intrinsic viscosity measurements.

4. Computer Methods

Our public-domain HYDRO program worked with the KRV method. A new version of the program, called HYDRO++, is now available from our web site <http://leonardo.fcu.um/macromol>. This new program includes various options: the volume corrections can be neglected or included or, what we indeed recommend, the program can find the optimum f_η value for an adjusted, intermediate (KRI) correction. The adjusted correction does not increase computing time. The program also implements the third-order calculation (that is recommended for the obtention of rotational properties) and the CBS method.

Acknowledgment. The possibility of an intermediate correction for the intrinsic viscosity was suggested to us by Prof. S. Harding (Nottingham University, U.K.). This work was supported by Grant CTQ2006-06831 (including FEDER funds) from *Ministerio de Educacion y Ciencia* (MEC), which also provided a predoctoral grant to G.R.E. A.O. acknowledges a postdoctoral grant from the same source and *Fundacion Cajamurcia*.

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