Analysis of Curved Free Kick with the Compartment Model

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1 Introduction

The Compartment Model is a type of mathematics model used for analyzing influences of different homogeneous entities in a system. The Compartment Model is popular among various subjects like, finical, biology, physic, etc. One simple instance is the simulation of fish population with a harvesting rate. By studying different influence factors together, we can generate a comprehensive and accurate relation among various parts of the whole system.

In this paper, we prepare to analyze the curved free kick in a soccer match. Specifically, this paper's aim is to study the spinning ball's motion in the air.

2 Background

As the Newton's second law states, a net force leads to an acceleration in the same direction. For a moving object, external forces will cause an offset on its original trajectory [1]. A soccer ball moving in the air experiences at least two forces: the vertically downward gratify force and the air drag force against its moving direction. Thus, we can observe that the ball's trajectory is like a parabola. In some cases, especially free kicks, we saw a horizontally curved parabola-like trajectory [2]. We can quickly conclude that the soccer ball experiences another force that causes the horizontal offset. According to information from the Physics Of Soccer, the third force is called the Magnus Force [3].

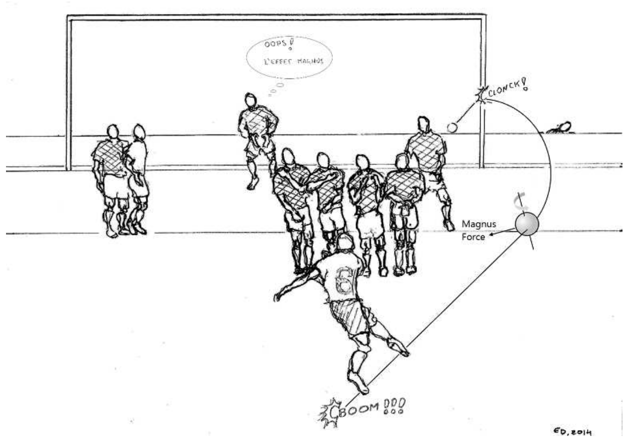

Figure 1: Curved Free Kick [4]

3 Theory

The gravity force, the air drag force, and the Magnus force make up a net force that controls the soccer ball's movement. Unfortunately, it is hard to analyze the net force because its magnitude and direction are changing as the soccer ball moving. Consequently, the compartment model is applied in this situation. As the following Free Boday Diagram shows, three forces work in three different directions.

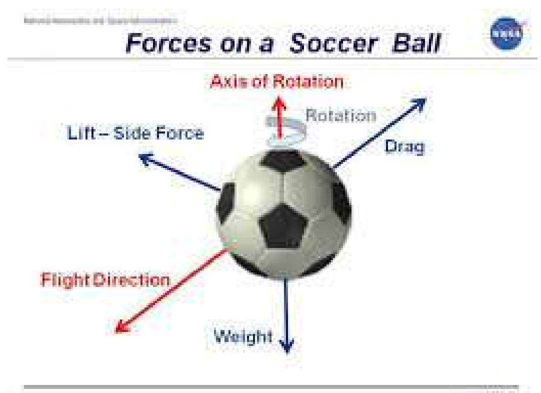

Figure 2: Forces on a Horizontal Move Spinning Soccer Ball [5]

The next step is to calculate the effect of each force and combine them together at the end.

3.1 Gravity

Based on Figure 1, the gravity force only affects the y-axis (vertically downward). Additionally, the magnitude of the gravity acceleration is stationary ($g = 9.8m/s^2$). So, we can generate gravity acceleration equation:

$$A_g(t) = [0, 0, g]$$

3.2 The Drag Force

In fluid dynamic, the drag is a force acting opposite to the relative motion of any object moving in a fluid [6]. For the soccer ball in a curved free kick, the direction of air drag is against the ball's actual moving direction. The Drag equation is

$$F_D = \frac{\rho u^2 A C_D}{2}[7]$$

ρ is the mass density of the air, and u is the velocity of the ball. A is the reference area and C_D is the drag coefficient.

To analyze the air drag effectively, the net air drag force is divided into three component air forces respecting to three axis. Then, we have three air drag equitions:

$$F_{Dx} = \frac{\rho {u_x}^2 A C_D}{2}$$

$$F_{Dy} = \frac{\rho {u_y}^2 A C_D}{2}$$

$$F_{Dz} = \frac{\rho {u_z}^2 A C_D}{2}$$

Next, we transfer them into acceleration functions:

$$A_d(t) = [\frac{F_{Dx}}{M_{ball}}, \frac{F_{Dy}}{M_{ball}}, \frac{F_{Dz}}{M_{ball}}]$$

3.3 The Magnus Force

When an object moves in the stationary air flow, it gives the air flow a speed that equals to its moving velocity. However, if the object is spinning and its angular velocity's direction is different with the moving direction, the object will experience a side force whose direction is perpendicular to the moving direction [4]. The side force is called the magnus force. When the object is spinning in the airflow, it will affect the surrounding airflow's velocity. The velocity of the airflow against the spinning direction is less than that of the airflow in the object's spinning direction. According to Bernoulli's principle, the velocity

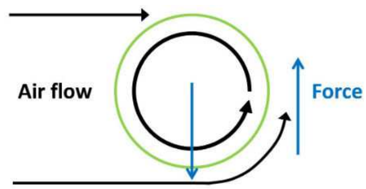

Figure 3: The Magnus Force [8]

difference leads to different pressures: the higher the velocity the lower the pressure. Since different pressures exists, a new force from the high pressure to the low pressure occurs.

According to information from NASA, the Magnus force (lift force) works on a spinning ball can be consider as a collection of lift forces on an infinite number of tiny rotating cylinders. Since the lift force can be calculated by Kutta–Joukowski theorem, the adding up (integration) of forces on those rotation cylinders is the lift force of the spinning ball[9]. Here is the equation:

$$L = Lift = \frac{4}{3}(4\pi^2 r^3 s\rho V)[9]$$

r stands for the radius of the ball, and s stands for the rotating speed in revolution per second, and ρ stands for the density of the air, and V stands for the velocity of the ball. However, this equation is generated for ideal environment, and real life physics needs to consider multiple factors. So, scientists introduce the lift coefficient to reduce those heavy work [10].

$$Lift_{actual} = \frac{4}{3}(4\pi^2 r^3 s\rho V)C_L$$

Unlike the gravity force or the air drag, the lift force depends on the velocity on other axis. Assume that the ball spins respect to the Z axis, the lift force affects the y axis

$$L_y = \frac{4}{3}(4\pi^2 r^3 s\rho V_x)C_L$$

3.4 Combine All Forces

Since all forces are clear, the next step is to combine those forces together and build a differential equation system:

$$X''(t) = -\frac{\rho X'(t)^2 A C_D}{2M_{ball}}$$

$$Y''(t) = -\frac{\rho Y'(t)^2 AC_D}{2M_{ball}} - \frac{4}{3}(4\pi^2 r^3 s\rho X'(t))C_L$$

$$Z''(t) = -\frac{\rho Z'(t)^2 AC_D}{2M_{ball}} - 9.8$$

4 Simulation Tool

The built differential equation system is a no linear second order differential system, and a strict dependency exists in this system. Unfortunately, normal ODE (ordinary differential equation) solvers, like ODE45, ODE23s, even Dsolve, have a hard time finding a solution to this system. According to information from Wolfram, NDSolve in Mathematica is an effective solver to this no linear differential system. Instead of finding the solution to the ordinary equations, NDSolve will find a numerical solution to the system based on input variables. So, NDSolve is the chosen solver in this simulation [11]. Furthermore, Mathematica will show the simulated trajectory in 3D coordinate with the result from NDSolve.

5 Assumption

In this simulation, the airflow's proprieties are essential. Airflow's velocity, density, and humidity have significant influences on the simulation. Besides that, the soccer ball's material also play a significant role in this simulation. Due to the limitation on time and computation skills, this simulation focuses on a simple situation. Here is the list of simulation conditions:

1. The airflow is stationary, which means the velocity of the airflow is zero.
2. The temperature affects the airflow's density. In this simulation, the temperature does not change.
3. The ball's spinning respects to the Z axis, which means that the magnus force affects the Y-axis displacement.
4. No air drag affects the rotating speed. In other words, the rotating speed is constant.

6 Preparation

The previous function system requires parameters about both the airflow and the soccer ball. To ensure the accuracy of this simulation, all data mus from reliable sources. Here is a list of necessary data based on scientific researches and experiments.

1. According to information from International Union of Pure and Applied Chemistry(IUPAC), the density of the dry airflow at 293.15K (20 °C) is $1.2041 kg/m^3$.
2. Different soccer ball have different properties. In this simulation, Brazuca, the soccer ball used in 2014 World Cup, is the experimental soccer ball. All data comes from Mail Online [12].
   - The mass of the soccer ball is 0.437 kg.
   - The Circumference of the soccer ball is 0.69m. Calculated radius of the soccer ball is 0.11m
3. The Drag Coefficient of the soccer ball varies for different materials and surface structures. Based on the research done by University of Tsukuba, the design of Brazuca makes its Drag Coefficient lower and more stable than other soccer balls. Although it is stable, the Drag Coefficient still changes slightly for different Reynolds Number. Consequently, the Drag Coefficient in this simulation is determined as 0.175 to get rid of heavy computing work [13].
4. The Lift Coefficient is a number that aerodynamicists used to model complex factors contributing to the lift force in real life[15]. In other words, scientists generate the Lift Coefficient to modified the ideal lift force produced by the force equation to the lift force in real life. Due to its particular purpose, the Lift Coefficient only can be determined by experiments. NASA applied determined the average value of the Lift Coefficient of kicked ball, Goff, and Carre to be about 0.25 [14].

7 Simulation

7.1 Simplification

Based on the data in last section, the equation system is simplified:

$$X''(t) = -0.0092X'(t)^2$$

$$Y''(t) = -0.0092Y'(t)^2 - 0.0211SX'(t)$$
$$Z''(t) = -0.0092Z'(t)^2 - 9.8$$

7.2 Solution

To solve this differential equation system, the initial condition must be defined. The initial position of the soccer ball is (0, 0, 0) in the 3-D coordinate system. The initial X'(t) is 35, and the initial Y'(t) = 4, and the initial Z'(t) is 6. The spinning speed is 10 revolutions per second. The total travel time is 1. Since all parameters are set up, Mathematica implements NDSolve to generate the numerical solutions for the given system.

Here is the command used to solve this differential system.

```
s=NDSolve[{x''[t] == -0.0092 x'[t]^2,
  y''[t] == -0.211 x'[t] - 0.0092 y[t]^2,
  a''[t] == -9.8 - 0.0092 a'[t]^2,
  x[0] == y[0] == a[0] == 0,
  x'[0] == 35, y'[0] == 4, a'[0] == 6},
  {x, y, a}, {t, 1}]

ParametricPlot3D[Evaluate[{x[t], y[t], a[t]} /. s],
  {t, 0, 1}]
```

X(t) is the displacement function respecting to time in X Axis; Y(t) is the displacement function respecting to time in Y Axis; a(t) is the displacement function respecting to time in Z Axis. The simulation result is:

7.3 Discussion

Figure 3 shows the computed trajectory of the soccer ball based on the equation system and input parameters. The most explicit information from the Figure 3 is the displacement in x axis. Due to the effects of the air drag, the the displacement in X axis i less than 35. To explore the ball's behaviors in other two axis, another views of the trajectory are necessary.

If the soccer ball travels without effects from the air drag and the magnus force, the displacement in Y axis should be 4. However, the Figure 4 shows that the total displacement in Y axis is less 3. Additionally, the trajectory in the Z axis is not a typical parabola.

In this simulation, the magnus effect forces the ball to move 3.5 units toward the negative Y axis. In real life, it will be 3.5 meters toward the center/side of

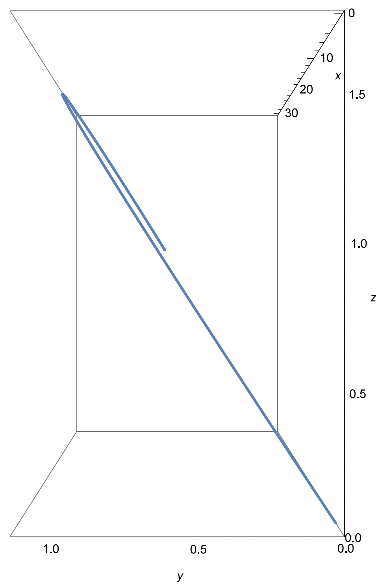

Figure 4: Simulated Trajectory of the Soccer Ball (Kicker's View)

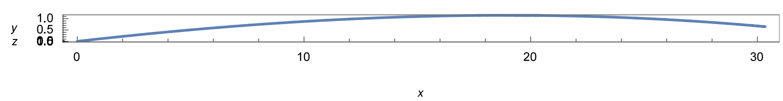

Figure 5: Simulated Trajectory of the Soccer Ball (Top Camera View)

the goal net. Furthermore, this simulation also tried different spinning speeds. The results showed that the higher the spinning speed, the more curved the trajectory is. As a result, Free kickers can increase the spinning speed of the soccer ball to make the path more complicated. This fact makes the goalkeeper hard to figure out the final placement of the ball and save that goal.

8 Reflection

In this simulation, the researcher creates an Ideal environment: no weather effect, no temperature change, only a constant side spinning, etc. However, the actual situation is much more complex than this. To get a more accurate simulation, the researcher needs to consider all related factors. Thus, the future simulation should focus on the external factors' effects on the soccer ball's trajectory. Specifically, the wind, the spinning and other assumption could be changeable in the future simulation.

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