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**Generalized Belief Propagation of Potentials**

1. **Simple Belief Propagation**

Recently, Leo noted that the message-passing method can be formulated in terms of potentials [1]. The total energy is given by



In this framework, the messages in belief propagation is given by



Assume that the messages have the forms



Then



Minimization equation:

 ⇒ 

Quadratic term in *Fj*(*μi*)



Linear term in *Fj*(*μi*)





The recursion relation becomes

 and 

**2. Factor-to-Node Message**

Consider generalized belief-propagation. A factor collects factor-to-node messages from factors connected from nodes 2, 3 and 4 neighboring node *j* and region-to-factor messages from regions *p* and *q* feeding the factor. The factor then sends a message from node *j* to node *i*.

***i***

**2**

**4**

***j***

***p***

***q***

**3**

Then the generalized belief-propagation equation becomes



*g* is inserted for checking. It will be set to *g* = 1 at the end. Assume that the messages have the forms





Minimization equation:





Quadratic term in *Fj*(*μi*) 







Linear term in *Fj*(*μi*)





The recursion relation becomes

 and



1. **Region-to-Factor Messages**

Consider a region *p* consisting of factors *a*, *b*, *c* and *d*. It collects 4 factor-to-node messages from factors connected from nodes 3 and 4, 7 and 8 to nodes 5 and 6 respectively, and 3 region-to-factor messages feeding respectively nodes 2 and 5, 5 and 6, 6 and 1. The region then sends a message to factor 1 and 2.

**7**

**8**

**4**

**61**

**25**

**56**

**12**



**1**

**2**

**5**

**3**

**6**

Then the generalized belief-propagation equation becomes







Minimization equation:









In matrix form,





Solution:

 where





Neglecting constant terms,











where 

Coefficient of 

Coefficient of 

Coefficient of 

Coefficient of = *c*1



Coefficient of = *c*2



 ⇒ 

The recursion relation becomes







 and  where





[1] P. Rebeschini and S. Tatikonda. “A new approach to Laplacian solvers and flow problems”,

arXiv:1611.07138 (2016).