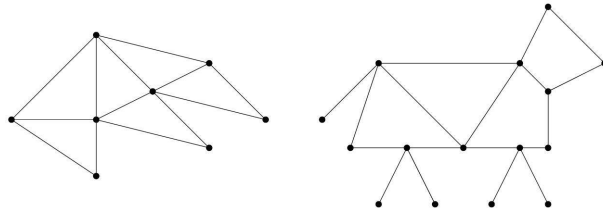


**7.** (20 points) Let  $G$  be a planar graph drawn on a plane with no edge crossing. If all the vertices of  $G$  lie on the unbounded face in this case, then  $G$  is *outerplanar*. The following are two examples of outerplanar graphs.



- (a) Give an example of a planar graph that is not outerplanar. [2 marks]
- (b) Prove that any outerplanar graph is 3-colorable. [8 marks]
- (c) Suppose that  $G$  has  $n \geq 2$  vertices and  $m$  edges. Determine the maximum value of  $m$  and prove your claim. [10 marks]