Pricing of the Callable Range Accruals using Longstaff Schwartz algorithm

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March 26, 2019

Abstract

The Callable Range Accrual is one of the most widely used derivatives. The aim of this paper is to model this product under the Libor Market Model.

The first step of the work is the mathematical modeling of the Callable Range Accrual. To do this, we used the method of Longstaff Schwartz method (Longstaff-Schwartz) to determine the sequence of optimal stopping times.

The second step of the work consists of using both stock price and rates as the underlyings of Callable Range Accrual. To do so, we will use Black Scoles model for stock prices and LMM model for LIBOR rates. We will then calibrate the LMM to the market data and then implementing the C ++ program allowing the pricing of the model. To do so, we used the financial open source library Quantlib.

In the last step of this work, we will adress the application of a corrective measure to the model in order to reduce the gap between the market price and the model price and to correct the relative pricing error due to the calibration.

1 INTRODUCTION

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1.1 Black Scholes formula

The Blac

1.2 libor rate

blablabla

1.3 Libor Market Model

blablaba

1.4 Callable Range Accruals

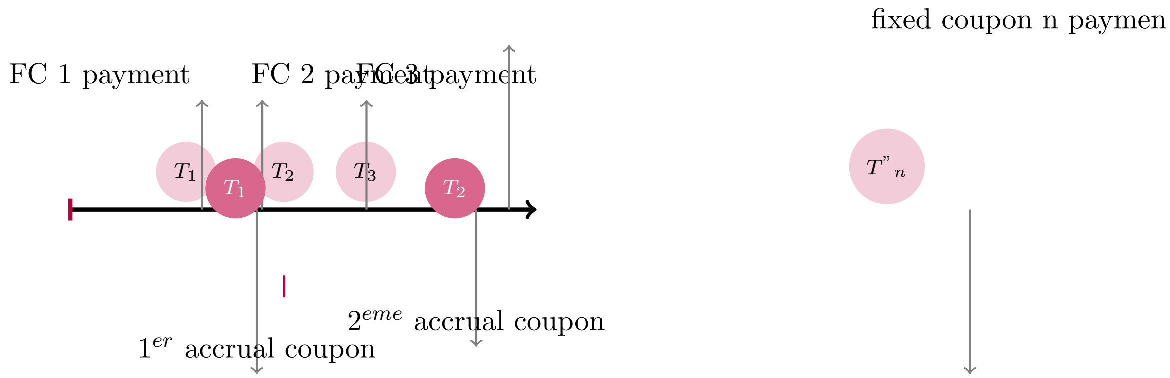
- **Range accruals:**
  A range accrual, or range accrual note, is a type of financial derivative product where one party pays a fixed coupon[1] to a second party and receives a variable coupon from it.
  The variable coupon depends on the value of an index. This index could be an interest rate, currency exchange rate, the price of a commodity, or stock index. If the index value falls within a specified range, the coupon accrues or is credited interest. If the index value falls outside the specified range, the coupon rate does not accumulate.

- **The callable aspect:**
  A callable (or cancellable) instrument gives the exotic interest rate payer (variable leg payer) the option to terminate the contract prior to its maturity, and on predetermined dates.

[1] rate of interest that a bondholder receives

2 Cash flows of a callable range accrual

Figure 1: cash flows



The cash flows Z_i at time T_i of a callable range accrual can be modeled as follows:

$$Z_i = \frac{\delta_{T_i}.C_i - \delta'_{T_i}.F_i}{B(T_i)} \tag{1}$$

C_i is the variable leg coupon paid at date T_i.
F_i are the fixed leg coupons.
$B(T_{i)}$ is the actualization factor, it's equal to e^{-rT_i}.
$[T_1, T_2, ..., T_n]$ are the variable tenor dates: the dates in which we pay the variable leg coupons $F_{ii\in[|1,n|]}$.
$[T_1{}', T_2{}', ..., T_n{}',]$ are the fixed tenor dates: the dates in which we pay the variable leg coupons $C_{ii\in[|1,n|]}$.
$\delta_i = T_{i+1} - T_i, \forall i \in [|1, n|]$
$\delta_i{}' = T_{i+1}{}' - T_i{}', \forall i \in [|1, n|]$

2.1 Fixed leg coupons F_i

2.2 Variable leg coupons C_i

$$C_i = payoff * \frac{\sum_{j=i-1}^{j=i} \mathbb{1}(S_{t_i} \in [S_{min}, S_{max}])}{T_i - T_{i-1}} * \delta(T_{i-1}, T_i)$$

where $\delta(T_1, T_2)$ is the day-count fraction from date T_1 to date T_2:

$$\delta(T_1, T_2) = \frac{T_2 - T_1}{365}$$

when we replace $\delta(T_{i-1}; T_i)$ by its expression, this equation becomes:

$C_i = payoff * \frac{\sum_{j=i-1}^{j=i} \mathbb{1}(S_{t_i} \in [S_{min}, S_{max}])}{T_i - T_{i-1}} * \frac{T_i - T_{i-1}}{365}$

$= payoff * \frac{\sum_{j=i-1}^{j=i} \mathbb{1}(S_{t_i} \in [S_{min}, S_{max}])}{365}$

$$\mathbb{E}(C_i) = payoff * \frac{\sum_{j=i}^{j=i+1} \mathbb{P}(S_{t_i} \in [S_{min}, S_{max}])}{365}$$

$$\mathbb{P}(S_t \in [S_{min}, S_{max}]) = \int_{S_{min}}^{S_{max}} \frac{1}{\sqrt{2\pi}} \frac{1}{s\sigma\sqrt{t}} \exp(-\frac{[\ln(S) - \ln(S_0) - (\mu - \frac{\sigma^2}{2})t]^2}{2\sigma^2 t})ds \tag{2}$$

Let:

$$u = ln(s)$$

So:

$$du = \frac{ds}{s}$$

The equation becomes:

$$\mathbb{P}(S_t \in [S_{min}, S_{max}]) = \int_{ln(S_{min})}^{ln(S_{max})} \frac{1}{\sqrt{2\pi}} \frac{e^{-u}}{\sigma\sqrt{t}} \exp(-\frac{[u - \ln(S_0) - (\mu - \frac{\sigma^2}{2})t]^2}{2\sigma^2 t})e^u du \tag{3}$$

Equation (3) implies:

$$\mathbb{P}(S_t \in [S_{min}, S_{max}]) = \int_{ln(S_{min})}^{ln(S_{max})} \frac{1}{\sqrt{2\pi}} \frac{1}{\sigma\sqrt{t}} \exp(-\frac{[u - \ln(S_0) - (\mu - \frac{\sigma^2}{2})t]^2}{2\sigma^2 t})du \tag{4}$$

$$\mathbb{P}(S_t \in [S_{min}, S_{max}]) = F_Y(S_{max}]) - F_Y(S_{min}]) \tag{5}$$

where $F_X(x)$ is the distribution function of a Gaussian random variable X

$$F_X(x) = \frac{1}{2}[1 + erf(\frac{x - \mu}{\sigma\sqrt{(2)}})]) \tag{6}$$

with:

$$X \sim N(\mu, \sigma)$$

erf is the error function given by:

$$erf(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt \tag{7}$$

in our case,

$$Y \sim N[ln(S_0) + (\mu - \frac{\sigma^2}{2} t); \sigma^2 t]) \tag{8}$$

$\mathbb{P}(S_t \in [S_{min}, S_{max}]) =$
$\frac{1}{2}[1 + erf(\frac{ln(S_{max}) - ln(S_0) - (\mu - \frac{\sigma^2}{2} t)}{\sigma \sqrt{(2t)}})] - \frac{1}{2}[1 + erf(\frac{ln(S_{min}) - ln(S_0) - (\mu - \frac{\sigma^2}{2} t)}{\sigma \sqrt{(2t)}})]$

$\mathbb{P}(S_t \in [S_{min}, S_{max}]) =$
$\frac{1}{2}[erf(\frac{ln(S_{max}) - ln(S_0) - (\mu - \frac{\sigma^2}{2} t)}{\sigma \sqrt{(2t)}})] - erf(\frac{ln(S_{min}) - ln(S_0) - (\mu - \frac{\sigma^2}{2} t)}{\sigma \sqrt{(2t)}})]$

2.3 Exercise value

Is the value of the contract at a certain date
$E(T_i) = \sum_{k=1}^{i} (C_k * e^{-r(T_i - T_k)}) - \sum_{k=1}^{i} (T_f{}' \delta_f{}' e^{-r(T_i{}' - T_k{}')})$
$= payoff * \sum_{k=1}^{i} \frac{\sum_{j=k-1}^{j=k} \mathbb{1}(S_{t_j} \in [S_{min}, S_{max}])}{365} * e^{-r(T_i - T_k)} - \sum_{T_k{}' \in T_1{}', T_p{}'} (T_f{}' \delta_f{}' e^{-r(T_i - T_k{}')})$
where $T_p{}' = \max\{T_j{}' \text{in fixed leg tenor }, T_j' <= T_i\}$

2.4 Hold value

The hold value at a date T_i is the expected future value of the contract if it has not been canceled at T_i, knowing the filtration of information we have today.
$H(T_i) = \mathbb{E}(E(T_i) * e^{r(\tau - T_i)} + \sum_{t \in [T_{i+1}; \tau]} E(t) * e^{-r(t - T_i)} - \sum_{t \in [T_{i+1}{}'; \tau]} (T_f{}' \delta_f{}' e^{-r(T_i - T_k{}')}) | S_{T_i})$

$$\begin{cases} \tau & \text{is the stopping time if the contract is cancelled before its maturity T, and after } T_i \\ \tau & \text{is the maturity T if the contract is held after } T_i \text{ and until it matures} \end{cases}$$

3 CALLABLE RANGE ACCRUAL PRICING

3.1 State of the art

blabla

3.2 Longstaff Schwartz algorithm

blabla

3.3 computing the conditional expectancy

blabla

4 Experiments

4.1 Data

blablabla

4.2 Results

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5 CONCLUSIONS

A conclusion section is not required. Although a conclusion may review the main points of the paper, do not replicate the abstract as the conclusion. A conclusion might elaborate on the importance of the work or suggest applications and extensions.

APPENDIX

Appendixes should appear before the acknowledgment.

ACKNOWLEDGMENT

The preferred spelling of the word acknowledgment in America is without an e after the g. Avoid the stilted expression, One of us (R. B. G.) thanks . . . Instead, try R. B. G. thanks. Put sponsor acknowledgments in the unnumbered footnote on the first page.

References are important to the reader; therefore, each citation must be complete and correct. If at all possible, references should be commonly available publications.

References

[1] G. O. Young, Synthetic structure of industrial plastics (Book style with paper title and editor), in Plastics, 2nd ed. vol. 3, J.