Informed Search

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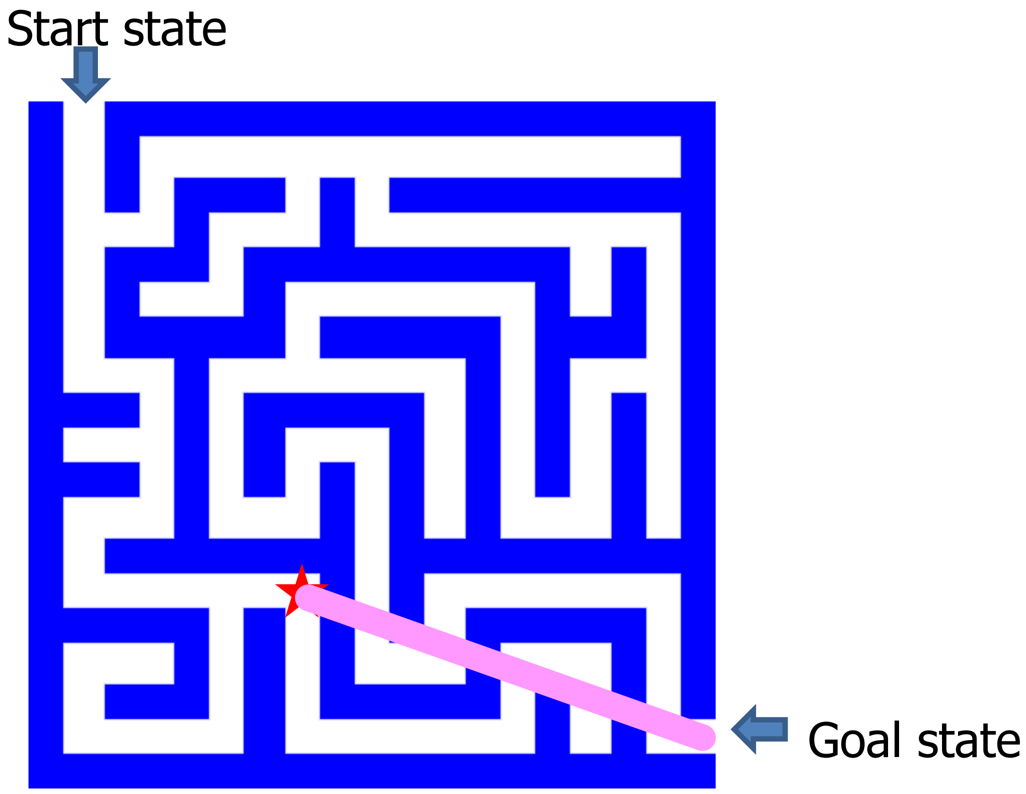
Informed search

- Idea: give the algorithm “hints” about the desirability of different states
  - Use an *evaluation function* to rank nodes and select the most promising one for expansion

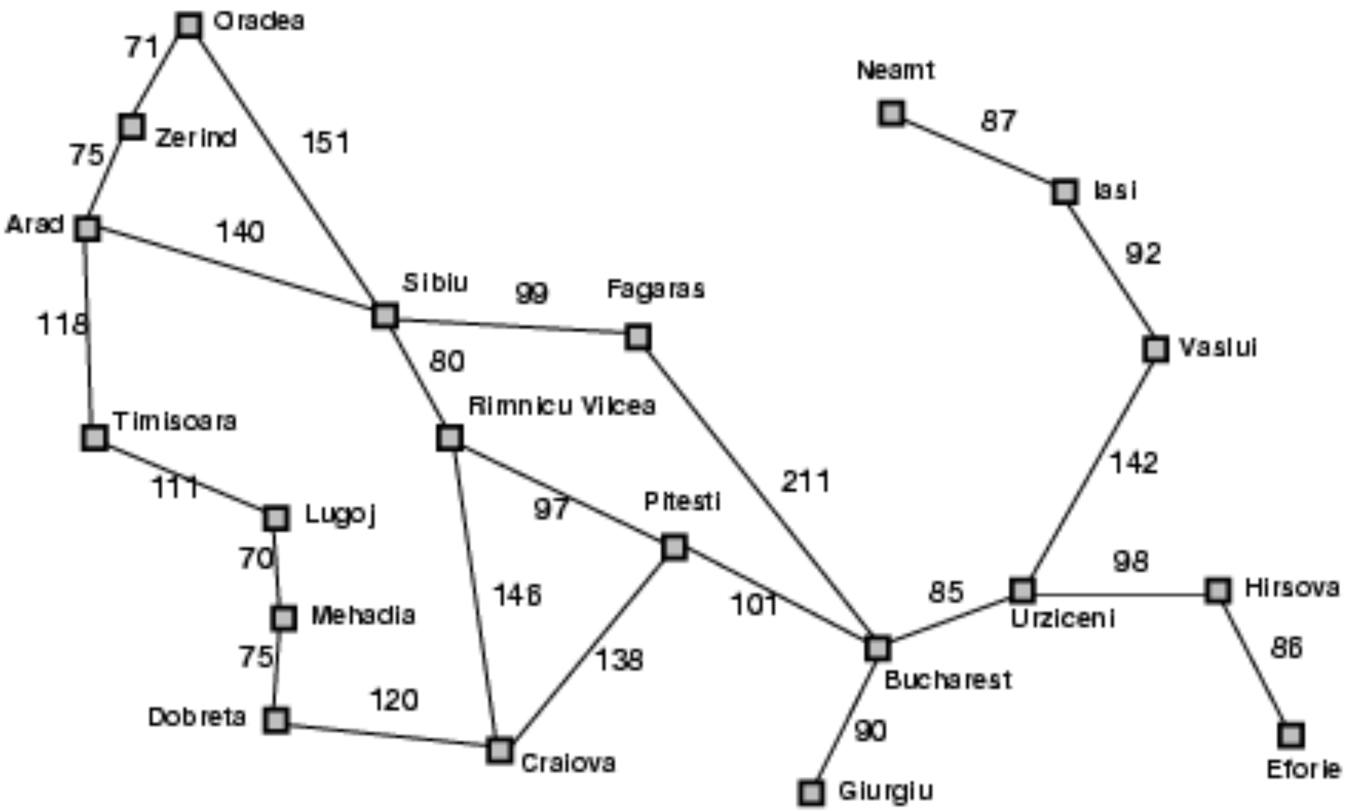
- Greedy best-first search
- A* search

Heuristic function

- Heuristic function $h(n)$ estimates the cost of reaching goal from node n
- Example:



Heuristic for the Romania problem



Straight–line distance to Bucharest

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Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	176
Giurgiu	77
Hirsova	151
Iasi	226
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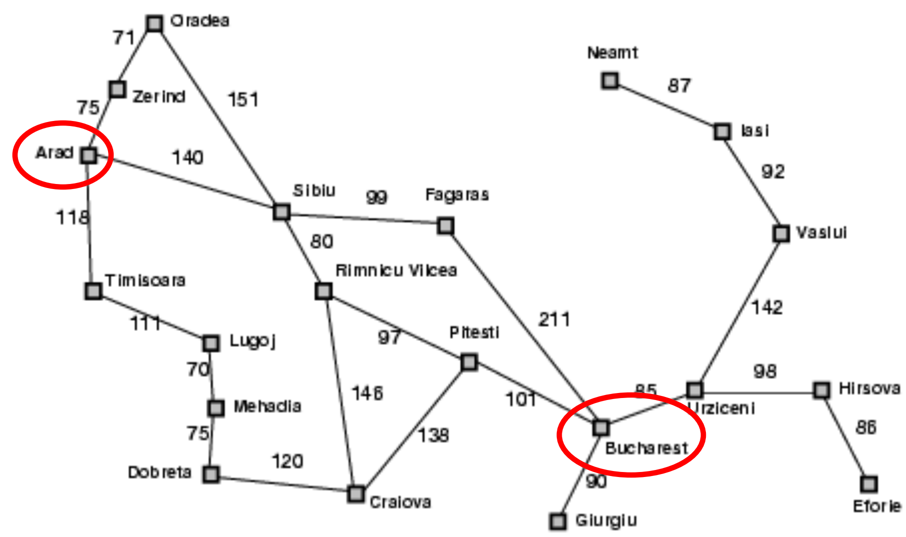
Zerind	374

Greedy best-first search

- Expand the node that has the lowest value of the heuristic function $h(n)$

Greedy best-first search example



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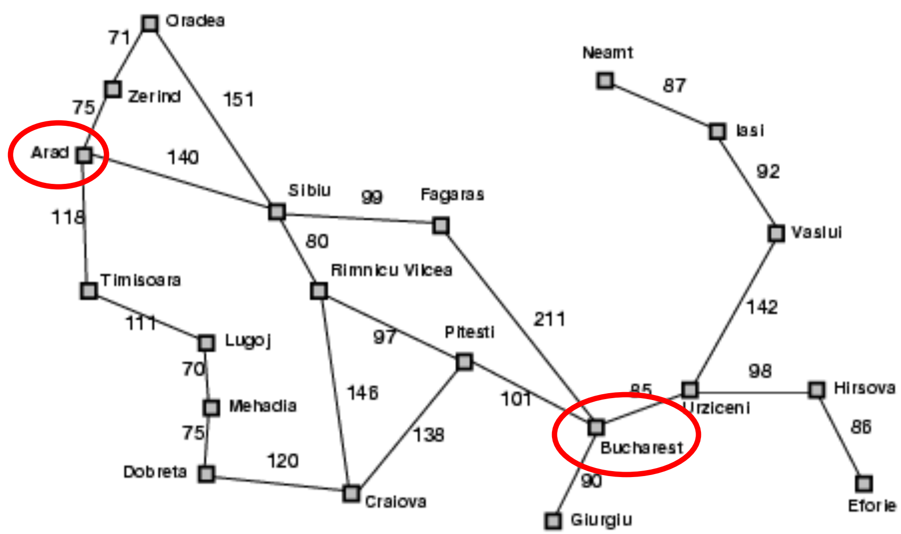

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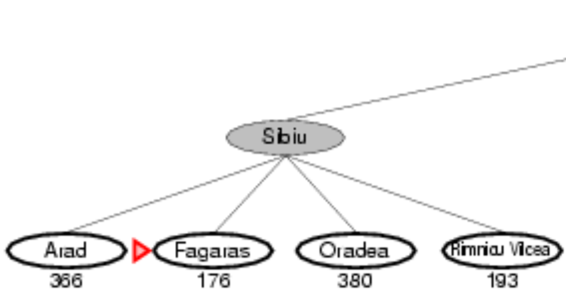

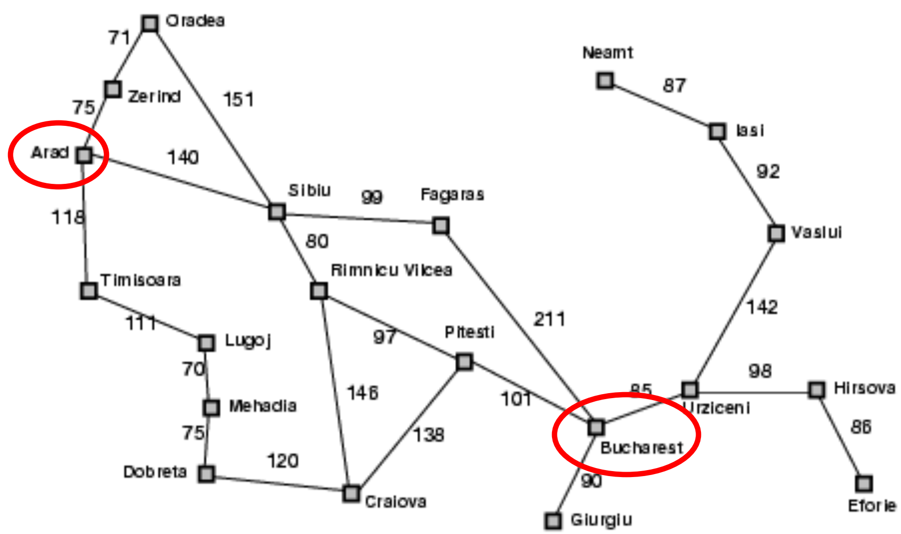

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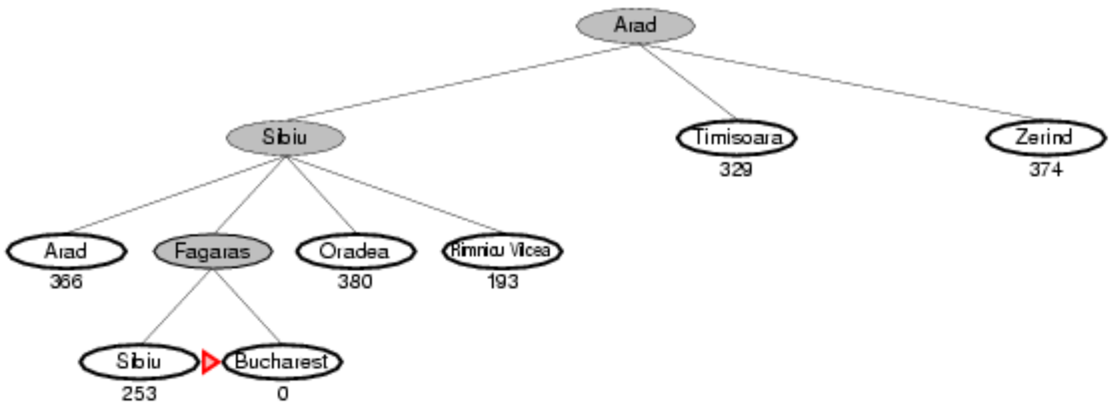

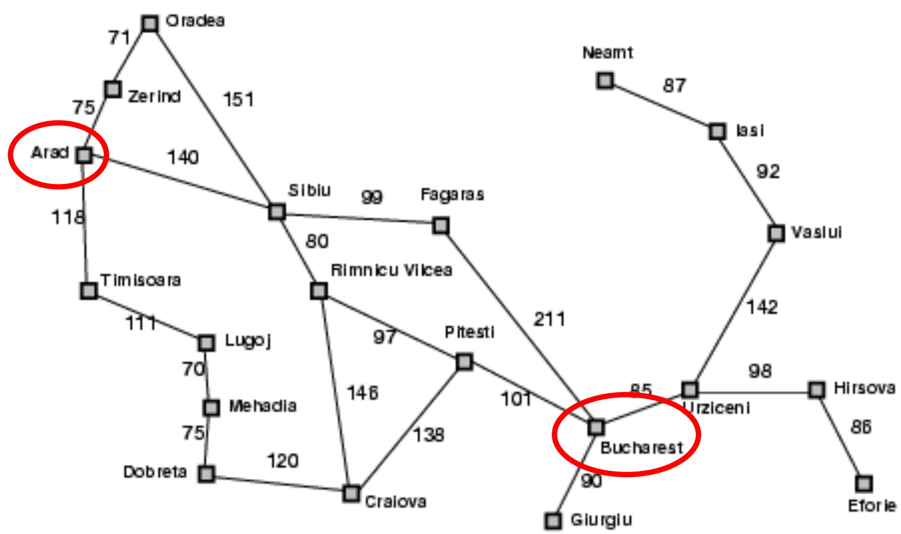

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Properties of greedy best-first search

- **Complete?**

  No – can get stuck in loops

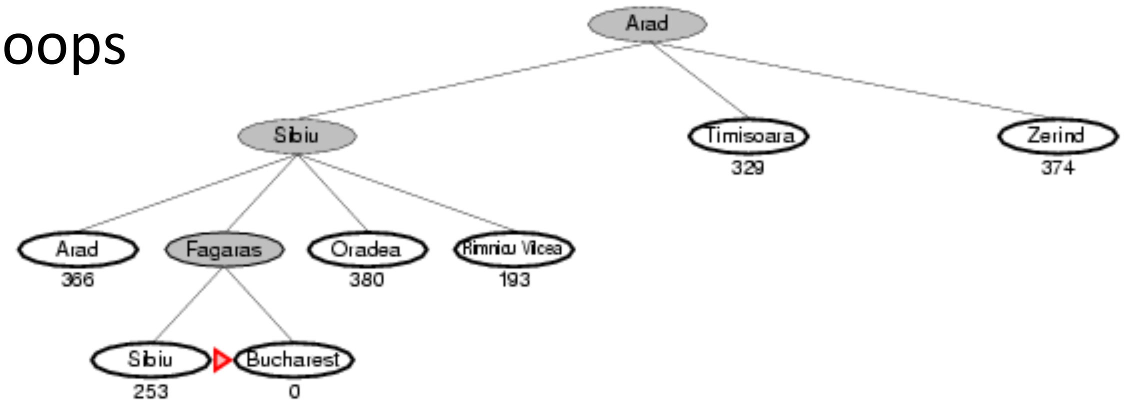
Properties of greedy best-first search

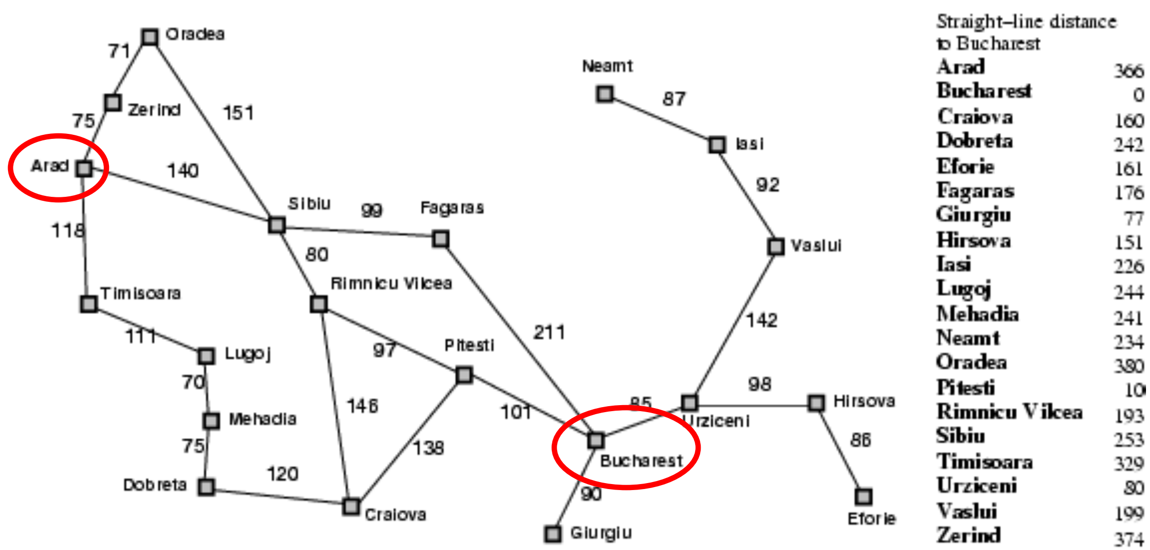
- **Complete?**

  No – can get stuck in loops

- **Optimal?**

  No





Properties of greedy best-first search

- **Complete?**

  No – can get stuck in loops
- **Optimal?**

  No
- **Time?**

  Worst case: $O(b^m)$

  Best case: $O(bd)$ – If $h(n)$ is 100% accurate
- **Space?**

  Worst case: $O(b^m)$

How can we fix the greedy problem?

- Add another parameter to evaluate nodes!?

A^* search

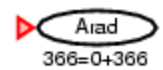
- Idea: avoid expanding paths that are already expensive
- The evaluation function $f(n)$ is the estimated total cost of the path through node n to the goal:

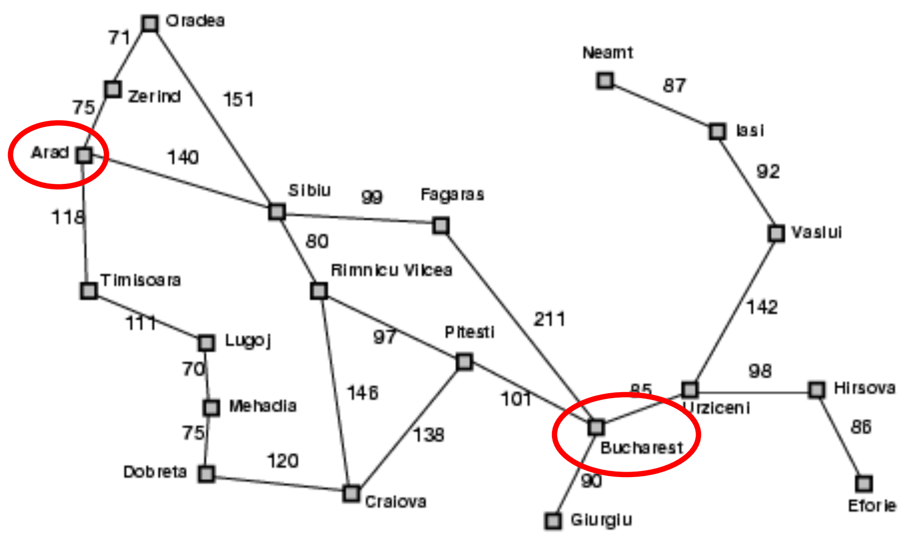
$$f(n) = g(n) + h(n)$$

$g(n)$: cost so far to reach n (path cost)

$h(n)$: estimated cost from n to goal (heuristic)

A* search example





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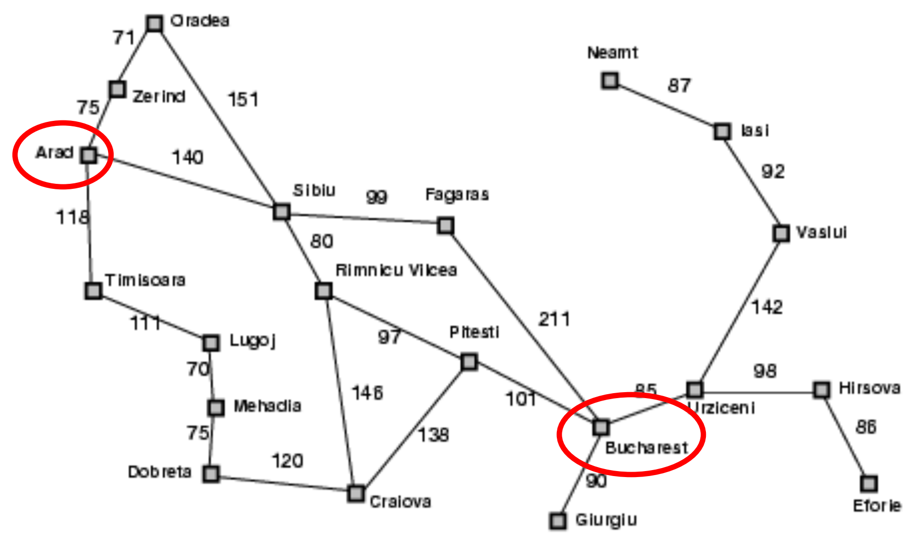

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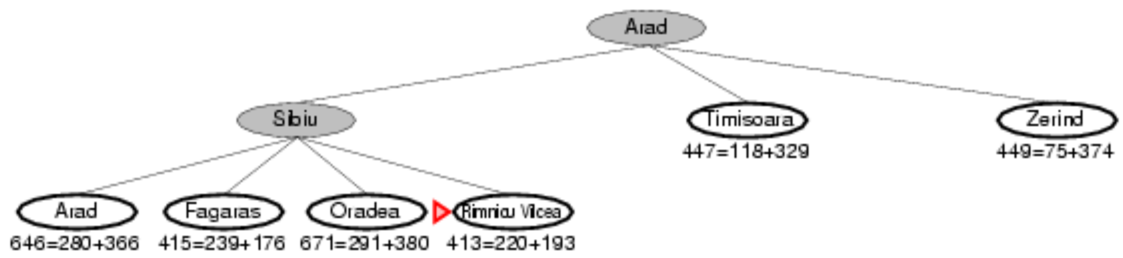

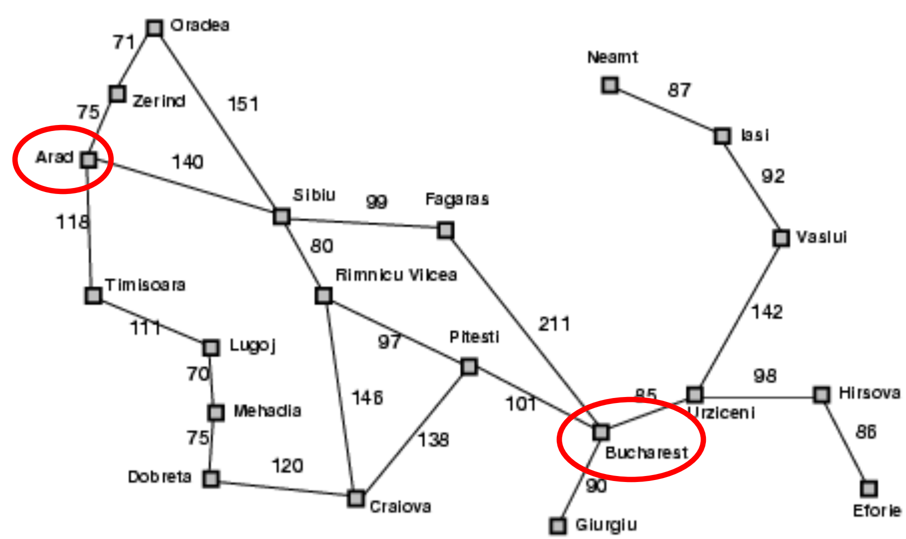

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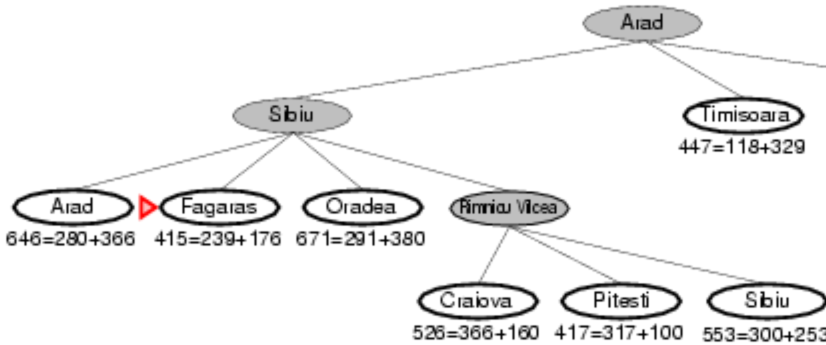

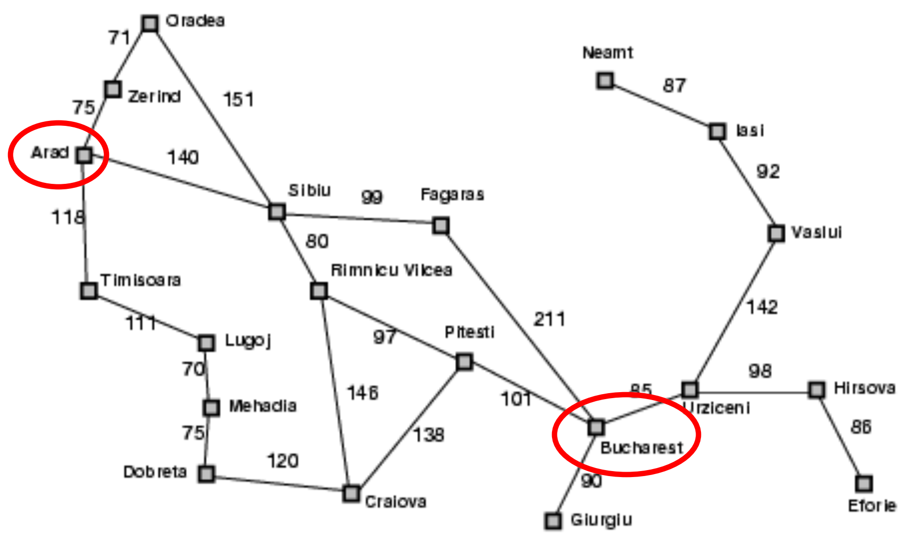

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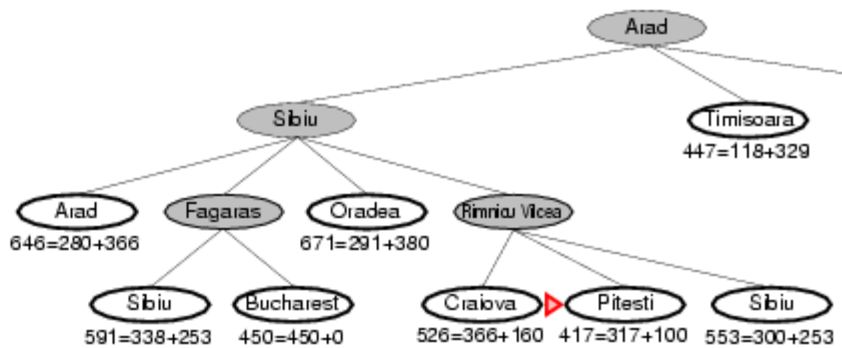

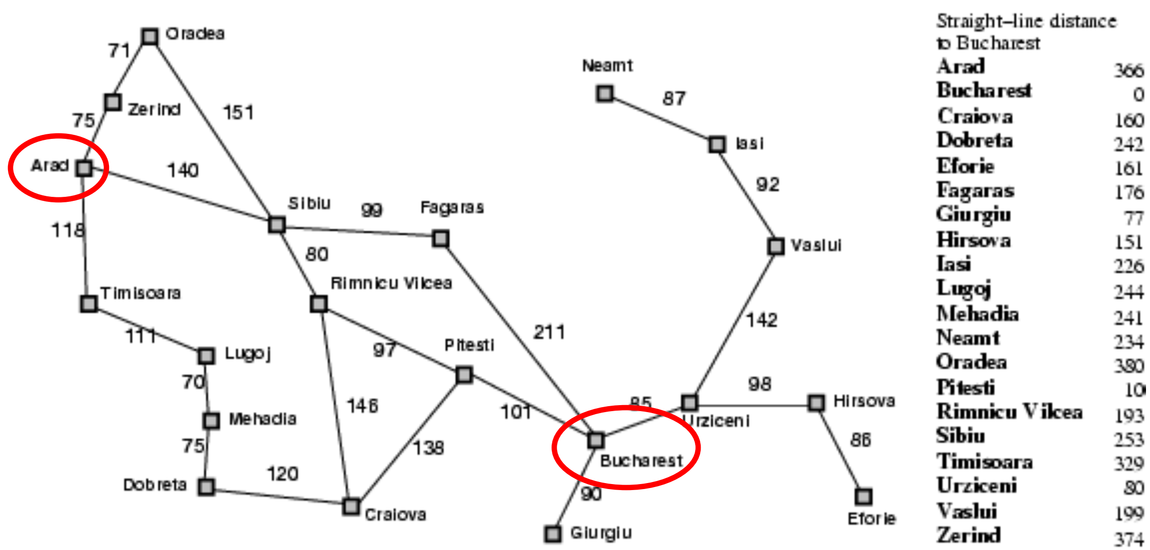

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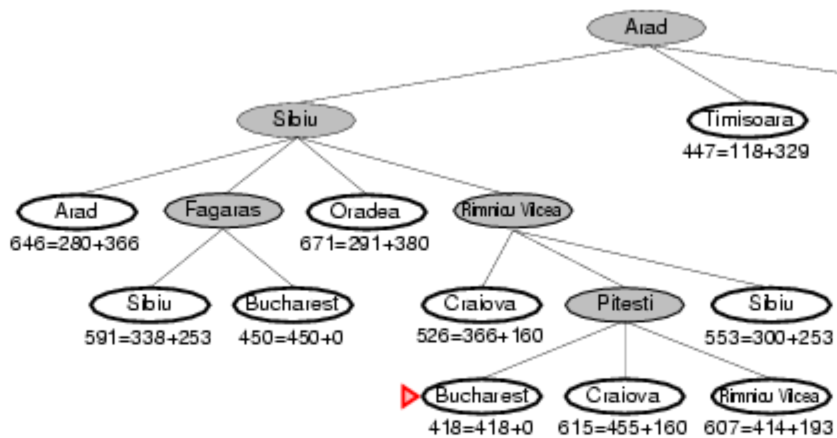

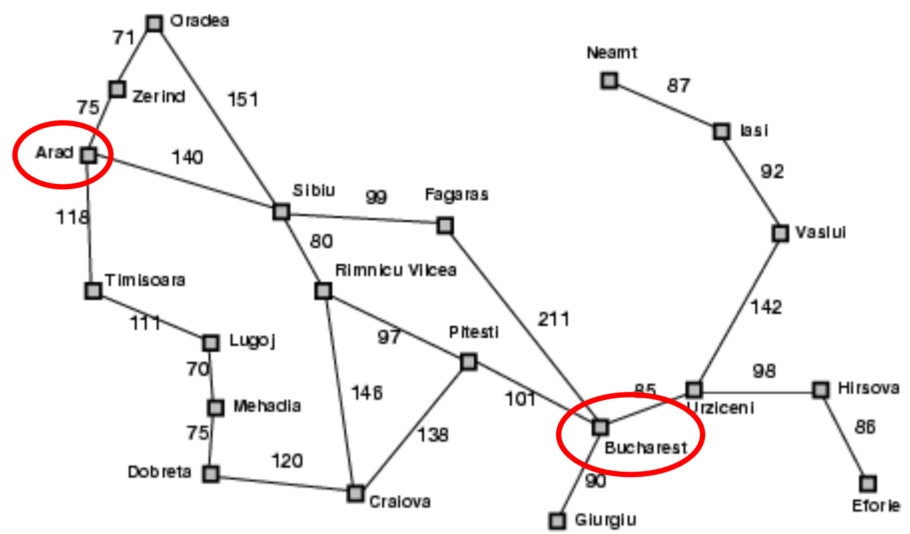

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Admissible heuristics

- An admissible heuristic never overestimates the cost to reach the goal, i.e., it is optimistic
- A heuristic $h(n)$ is admissible if for every node n, $h(n) \leq h^*(n)$, where $h^*(n)$ is the true cost to reach the goal state from n
- Example: straight line distance never overestimates the actual road distance
- **Theorem:** If $h(n)$ is admissible, A^* is optimal

Optimality of A*

- A* is *optimally efficient* – no other tree-based algorithm that uses the same heuristic can expand fewer nodes and still be guaranteed to find the optimal solution
  - any algorithm that does not expand all nodes in the contours between the root and the goal contour runs the risk of missing the optimal solution

Properties of A*

- **Complete?**

  Yes – unless there are infinitely many nodes with $f(n) \leq C^*$
- **Optimal?**

  Yes
- **Time?**

  Number of nodes for which $f(n) \leq C^*$ (exponential)
- **Space?**

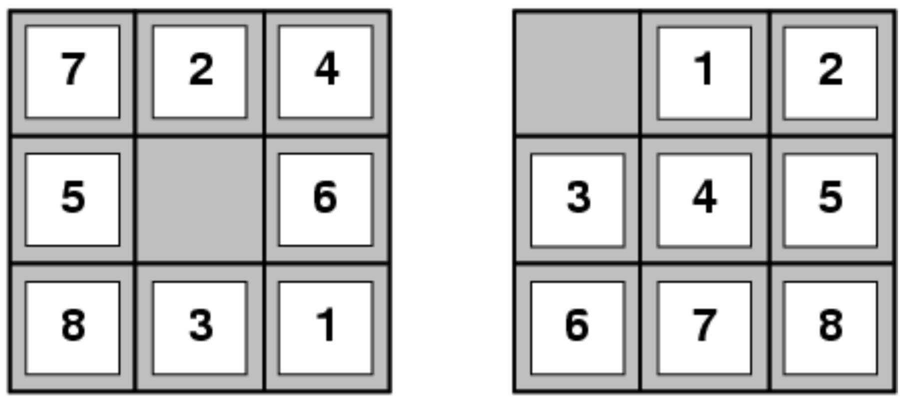
  Exponential

Designing heuristic functions

- Heuristics for the 8-puzzle

  $h_1(n)$ = number of misplaced tiles

  $h_2(n)$ = total Manhattan distance (number of squares from desired location of each tile)



Start State Goal State

$$h_1(\text{start}) = 8$$

$$h_2(\text{start}) = 3+1+2+2+2+3+3+2 = 18$$

- Are h_1 and h_2 admissible?

Dominance

- If h_1 and h_2 are both admissible heuristics and $h_2(n) \geq h_1(n)$ for all n, (both admissible) then h_2 dominates h_1
- Which one is better for search?
  - A* search expands every node with $f(n) < C^*$ or $h(n) < C^* - g(n)$
  - Therefore, A* search with h_1 will expand more nodes, so h_2 is better

C^* - optimal cost

Heuristics from relaxed problems

- A problem with fewer restrictions on the actions is called a relaxed problem
- The cost of an optimal solution to a relaxed problem is an admissible heuristic for the original problem
- If the rules of the 8-puzzle are relaxed so that a tile can move anywhere, then $h_1(n)$ gives the shortest solution
- If the rules are relaxed so that a tile can move to any adjacent square, then $h_2(n)$ gives the shortest solution

Dominance

- Typical search costs for the 8-puzzle (average number of nodes expanded for different solution depths):

- *d*=12 IDS = 3,644,035 nodes
  $A^*(h_1)$ = 227 nodes
  $A^*(h_2)$ = 73 nodes

- *d*=24 IDS ≈ 54,000,000,000 nodes
  $A^*(h_1)$ = 39,135 nodes
  $A^*(h_2)$ = 1,641 nodes

Combining heuristics

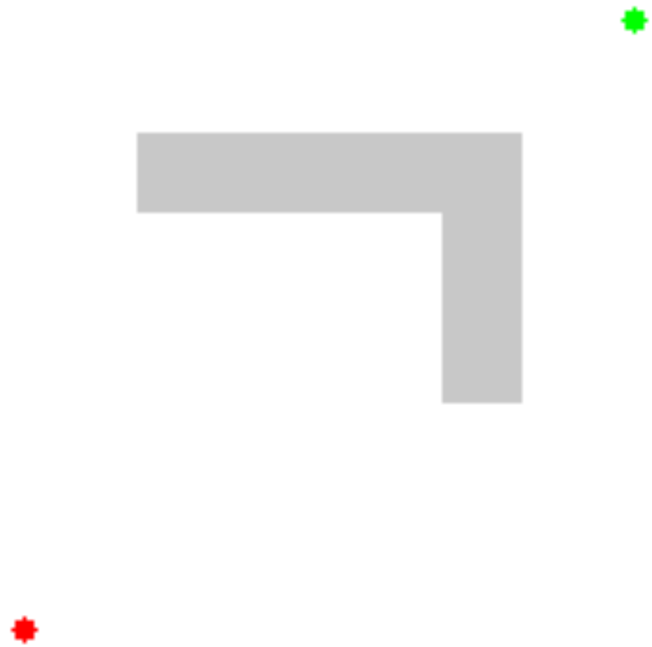
- Suppose we have a collection of admissible heuristics $h_1(n), h_2(n), \ldots, h_m(n)$, but none of them dominates the others
- How can we combine them?

$$h(n) = \max\{h_1(n), h_2(n), \ldots, h_m(n)\}$$

Weighted A* search

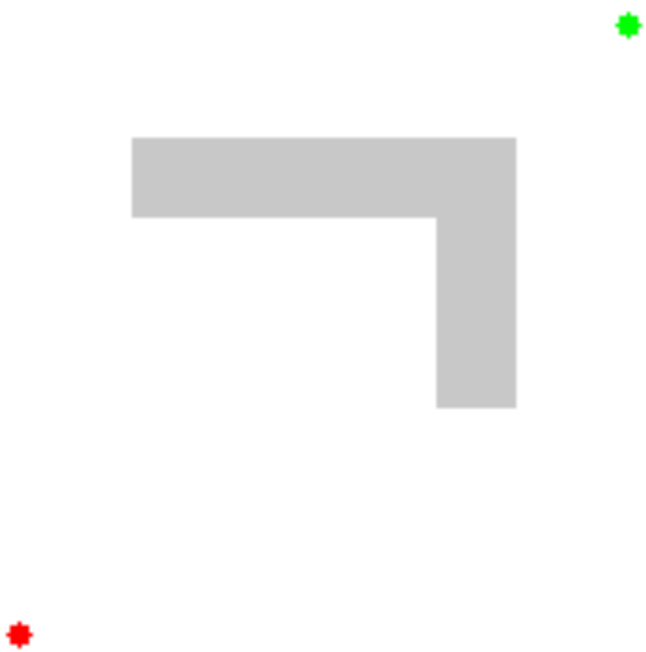
- Idea: speed up search at the expense of optimality
- Take an admissible heuristic, "inflate" it by a multiple $\alpha > 1$, and then perform A* search as usual
- Fewer nodes tend to get expanded, but the resulting solution may be suboptimal (its cost will be at most α times the cost of the optimal solution)

Example of weighted A* search



Heuristic: 5 * Euclidean distance from goal
Source: Wikipedia

Example of weighted A* search



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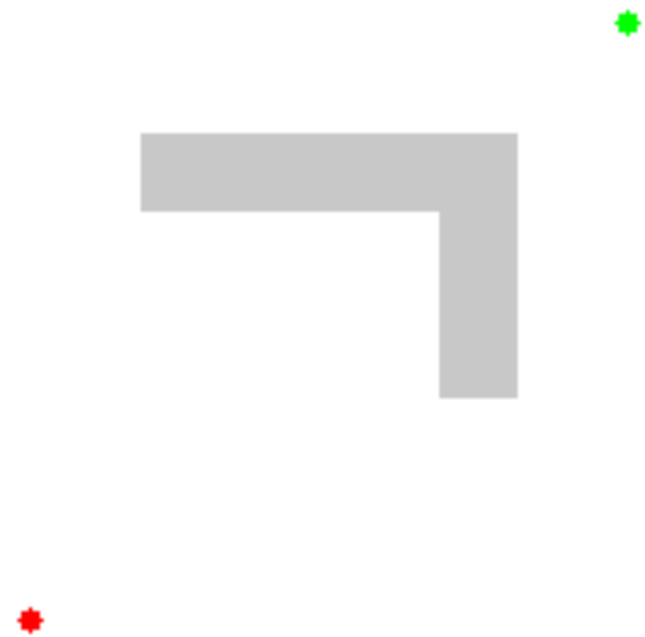


Compare: Exact A*

Memory-bounded search

- The memory usage of A* can still be exorbitant
- How to make A* more memory-efficient while maintaining completeness and optimality?

- IDA* (Iterative Deepening A*)
- SMA* (Simplified Memory Bounded A*)
  - Forget some subtrees but remember the best f-value in these subtrees and regenerate them later if necessary

- Problems: memory-bounded strategies can be complicated to implement, suffer from "thrashing"
  - repeated pruning and regeneration of the same few nodes

SMA*

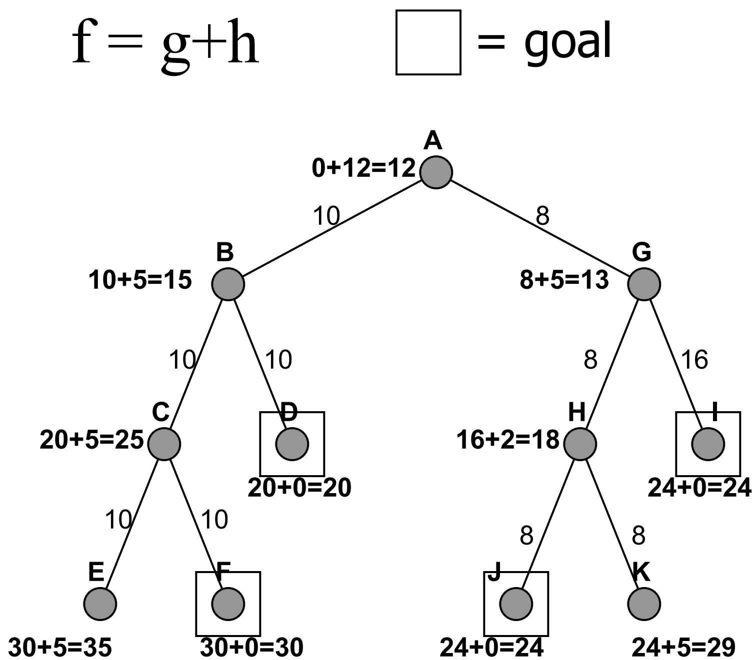
- Optimizes A* to work within reduced memory
- Key Idea:
  - IF memory full for extra node
  - Remove highest f-value leaf
  - Remember best-forgotten child in each parent node
- Generate Children 1 by 1
  - Expanding: add <u>1 child at a time</u> to QUEUE – Avoids memory overflow
  - Allows monitoring if nodes need deletion
- Too long paths: Give up
  - **Extending** path **cannot fit** in **memory**: give up
- Set **f-value** node to infinity
  - **Remembers:** path cannot be found here

SMA*

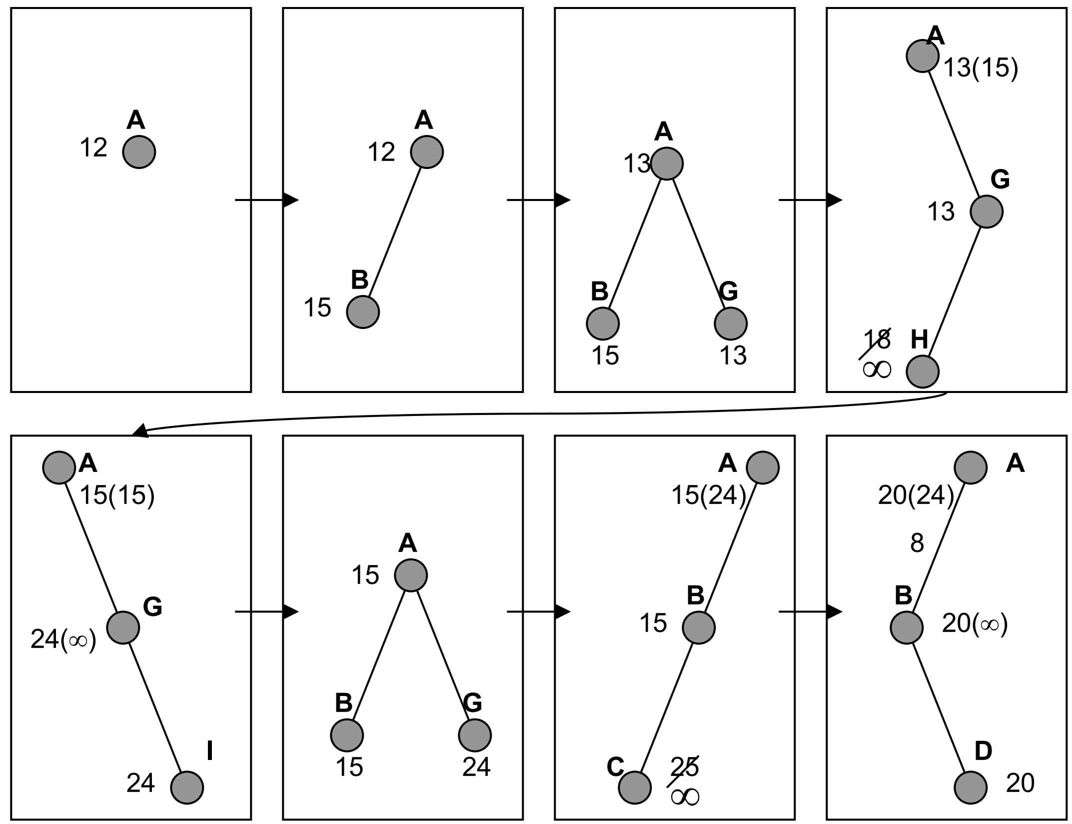
- Adjust f-values
  - IF all children M_i of node N have been explored
  - AND for all i: $f(M_i) > f(N)$
  - THEN reset (through N means through children)
    - $f(N) = \min\{f(M_i) \mid M_i \text{ child of } N\}$

Simple Memory-bounded A* (SMA*)

Search *space*



Progress of SMA* (with enough memory to store just 3 nodes).
Each node is labeled with its *current f*-cost.
Values in parentheses show the value of the best forgotten descendant (child node).



Optimal & complete if enough memory
Can be made to signal when the best solution found might not be optimal (e.g., if J=19)

```
function SMA*(problem) returns a solution sequence
  inputs: problem, a problem
  local variables: Queue, a queue of nodes ordered by f-cost

  Queue ← MAKE-QUEUE({MAKE-NODE(INITIAL-STATE[problem])})
  loop do
     if Queue is empty then return failure
     n ← deepest least-f-cost node in Queue
     if GOAL-TEST(n) then return success
     s ← NEXT-SUCCESSOR(n)
     if s is not a goal and is at maximum depth then
         f(s) ← ∞
     else
         f(s) ← MAX(f(n), g(s)+h(s))
     if all of n's successors have been generated then
         update n's f-cost and those of its ancestors if necessary
     if SUCCESSORS(n) all in memory then remove n from Queue
     if memory is full then
         delete shallowest, highest-f-cost node in Queue
         remove it from its parent's successor list
         insert its parent on Queue if necessary
     insert s on Queue
  end
```

Figure 4.12 Sketch of the SMA* algorithm. Note that numerous details have been omitted in the interests of clarity.

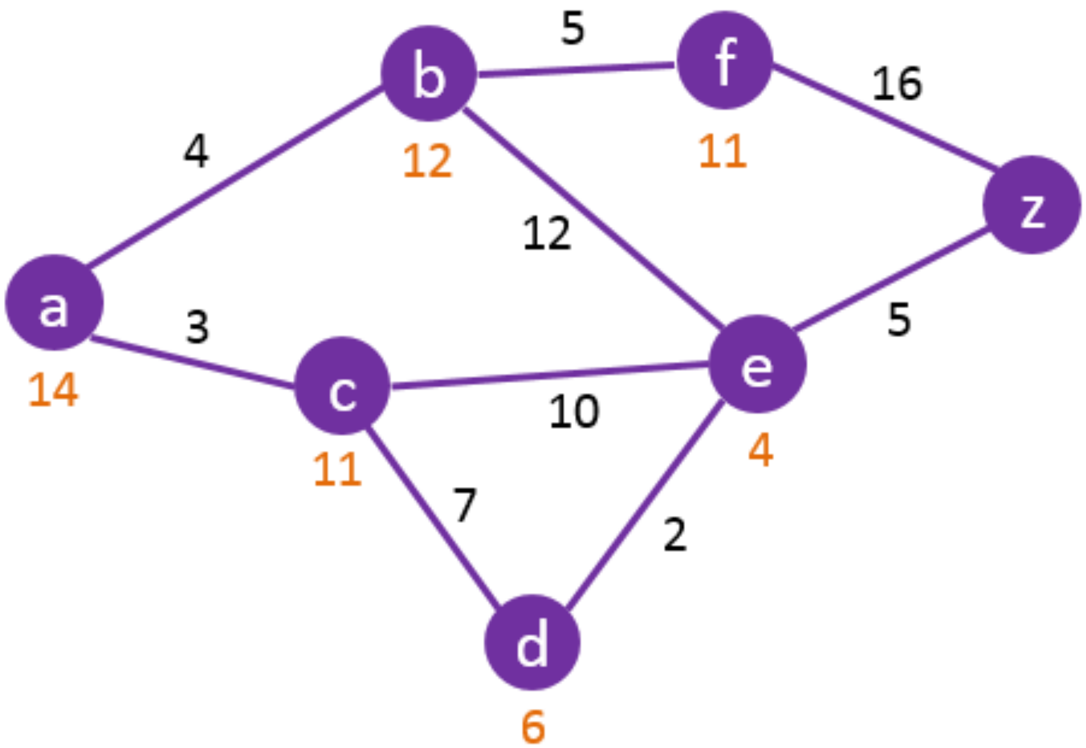
Uninformed search strategies

Algorithm	Complete?	Optimal?	Time complexity	Space complexity
BFS	Yes	If all step costs are equal	$O(b^d)$	$O(b^d)$
UCS	Yes	Yes	Number of nodes with $g(n) \leq C^*$	
DFS	No	No	$O(b^m)$	$O(bm)$
IDS	Yes	If all step costs are equal	$O(b^d)$	$O(bd)$

b: maximum branching factor of the search tree
d: depth of the optimal solution
m: maximum length of any path in the state space
C^*: cost of optimal solution

All search strategies

Algorithm	Complete?	Optimal?	Time complexity	Space complexity
BFS	Yes	If all step costs are equal	$O(b^d)$	$O(b^d)$
UCS	Yes	Yes	Number of nodes with $g(n) \leq C^*$	
DFS	No	No	$O(b^m)$	$O(bm)$
IDS	Yes	If all step costs are equal	$O(b^d)$	$O(bd)$
Greedy	No	No	Worst case: $O(b^m)$ Best case: $O(bd)$	
A*	Yes	Yes	Number of nodes with $g(n)+h(n) \leq C^*$	



A* Search Algorithm

What is the shortest path to travel from A to Z?

Numbers in orange are the heuristic values, distances in a straight line (as the crow flies) from a node to node Z.