Probability and statistics.

Master in Cognitive Science. Academic year 2025-2026.
Example sheet 2.

Exercise 1:
A pharmaceutical company decided to make savings on mailing advertisement for clients. Therefore, they decided to randomly stamp "urgent" 3 letters over 5 and the others as "normal".

1. Four letters are sent to a medical center where four doctors are working. What is the probability of:
   - A : "At least one of them get the letter with urgent stamp"
   - B : "exactly 2 doctors get the letter with urgent stamp
2. Let X be the random variable "number of letters with urgent stamp among 10 letters.
   - What is the probability distribution of X?
   - What are its expectation and variance?

Exercise 2:
We denote as X the random variable modeling the number of purchases on the Amazon book website. We assume it is Poisson distributed with parameter λ. We know that the average number of purchases per second is 10.

1. Compute the probability of there being 9, 10 or 11 purchases per second.
2. Compute the probability of there being fewer than 2 purchases per second.
3. Suppose that instead of knowing that the average number of purchases per second is 10, you are told that $P(X = 0) = 0.082$. Find λ and $Var(X)$.

Exercise 3:
Suppose that X has the probability density function $f(x) = c(1 - x^2)$ for $0 \leq x \leq 1$ and $f(x) = 0$ otherwise.

1. Find c.
2. Compute the cumulative density function (cdf).
3. Compute $P(0.1 \leq X \leq 0.9)$.
4. Find $x_{0.95}$ such that $P(X \leq x_{0.95}) = 0.95$.

Exercise 4:
Suppose that the lifetime of light bulb follows an exponential distribution with parameter $\lambda = 0.1$.

1. What is the probability that the lifetime is less than 10.
2. What is the probability that the lifetime is between 5 and 15.
3. Find t such that the probability that the lifetime is greater than t is 0.01.
4. (*) In an experiment you observe the following lifetimes (1.16, 3.51, 7.90, 4.16, 2.07, 1.96) in thousands of hours. Using `Rjags`, obtain the distribution (posterior) of the parameter λ. *Use a Gamma prior distribution*.