

Introduction to Probability and statistics

Master in Cognitive Science 2025-2026

Example sheet 1

Exercise 1:

Suppose there are n people in a room. Assume that each day of the year is equally likely, and that a year has 365 days.

1. What is the probability that at least two people have a common birthday ?
2. Write an R function to visualize this probability as a function of n .
3. What is the smallest number of people in the room such that this probability is greater than or equal to 0.5 ?

Exercise 2:

A company is forming a 5-member executive board. There are 16 applicants, 6 of whom are men. If the board is formed by selecting randomly from the applicants, what is the probability of selecting 3 men and 2 women.

Exercise 3:

A multiple choice exam has 10 questions. For each question, there are 4 possible answers, only one is correct.

1. How many possible answer sheets are there?
2. Answering randomly, what is the probability of getting at least 6 correct answers ?

Exercise 4:

A six-sided regular dice is rolled twice. The sample space is the following:

$$\Omega = \{(x, y) : x = 1, 2, \dots, 6 \text{ and } y = 1, 2, \dots, 6\},$$

where x and y are respectively the result of the first and second roll. Thus, the sample space has 36 elements, each of which is a pair of integers. Determine the probability of the following events:

- $A = \{(x, y) \in \Omega : x = 1\}$,
- $B = \{(x, y) \in \Omega : y \geq 4\}$,
- $C = \{(x, y) \in \Omega : x + y = 7\}$,
- $A \cup B$,
- $B \cap C$,
- B^c .

Exercise 5:

50% of the adult population in the US has hypertension (high blood pressure). Suppose that a new non-invasive test for diagnosing hypertension has been designed based on using heart rate variability along with blood pressure measurements. The new test will classify 25% of people with hypertension as not having hypertension and 15% of people without hypertension as having hypertension.

1. What is the prevalence of hypertension in the US adult population?
2. What is the sensitivity of the test?
3. What is the specificity of the test?

Exercise 6:

We analyze a soil sample from a waste dump. With probability 0.4 we find arsenic. With probability 0.3, we find the lead. With probability 0.1 we find both. Note that “finding arsenic” does not mean “finding arsenic only”. If we find arsenic, we may or may not find also lead.

1. What is the probability that we find lead but not arsenic?
2. What is the probability that we find arsenic but not lead?
3. What is the probability that we find at least one of them?
4. What is the probability that we find neither?

Exercise 7:

In a certain town there are 100 taxicabs: one is blue, and 99 are green. One night, a cab collides with a bicycle, and flees the scene. An eyewitness says that the colliding cab was blue. However, from previous research we know that in similar situations, a blue car is seen as blue with probability 0.9; and a green car is seen as blue with probability 0.08.

1. What is the probability that the colliding cab was indeed blue?
2. Do you think that the driver of the only blue cab in town should be convicted, based on this evidence alone?

Exercise 8:

In a certain place it rains on one third of the days. The local evening newspaper attempts to predict whether or not it will rain the following day. Three quarters of rainy days and three fifths of dry days are correctly predicted by the previous evening’s paper. Given that this evening’s paper predicts rain, what is the probability that it will actually rain tomorrow?

Exercise 9:

A game consists in picking a dice in a box among 10 similar-looking dices. All dices look the same but there are actually three types of dice.

- There are 6 dices of type A which are fair dices with $P(6|A) = 1/6$ (where $Pr(6|A)$ is the probability of getting a 6 in a throw of a type A die).
- There are 2 dices of type B which are biased with $P(6|B) = 0.8$.
- There are 2 dices of type C which are biased with $P(6|C) = 0.04$.

Find the conditional probability that the picked dice is of type B given that it gives a 6.