pyxel

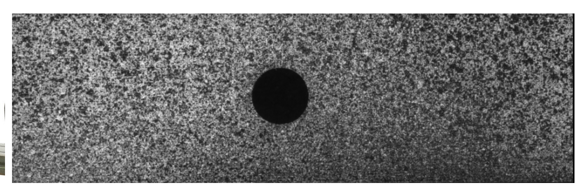
quick tutorial and exercises

May 10, 2019

Goal of the session

1. Understand an implementation of the DIC (or Image Registration) problem
   https://github.com/jcpassieux/pyxel
2. Implement a Tikhonov regularization based on the laplace operator
3. Implement an elastic regularization based on mechanical equilibrium
4. Perform an experimental displacement driven simulation
5. Validation of the above mechanical model wrt to experimental field
6. Identify a constitutive parameter from fullfield measurement

Let us consider an openhole glass/epoxy specimen subjected to a tensile test. The experiment is instrumented with a CCD camera.

A set of images is taken before and during the mechanical test. The corresponding images are given in folder `data/dic_composite/`.

1 Step by step tutorial

1.1 Start by importing some useful libraries

```
In [3]: import numpy as np
        import matplotlib.pyplot as plt
```

```python
import scipy.sparse.linalg as splalg
import scipy as sp
import pyxel as px
import os
```

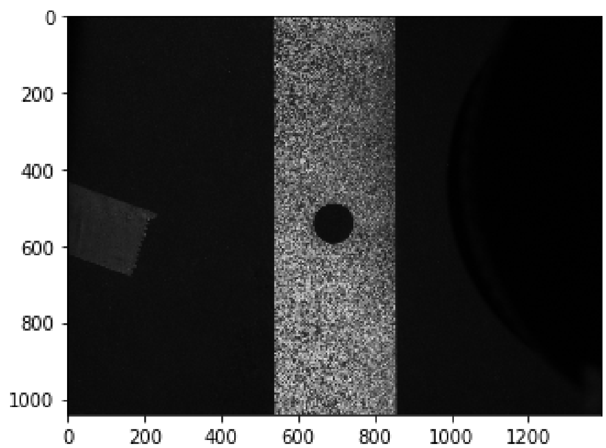
1.2 Naming and loading images

Here a list of 11 images named like *zoom-0053.tif*. The first one being the reference state image.

```python
In [6]: imnums=np.array([53,54,57,58,61,62,65,66,69,70,75])
        imagefile=os.path.join('data','dic_composite','zoom-0%03d_1.tif')
```

Load and **plot** reference image

```python
In [7]: imref = imagefile % imnums[0]
        f=px.Image(imref).Load()
        f.Show()
```



Load the penultimate image

```python
In [8]: imdef = imagefile % imnums[-2]
        g = px.Image(imdef).Load()
```

1.3 Mesh and associated camera model

An example with a quadrilateral ABAQUS mesh using meter (m) unit.

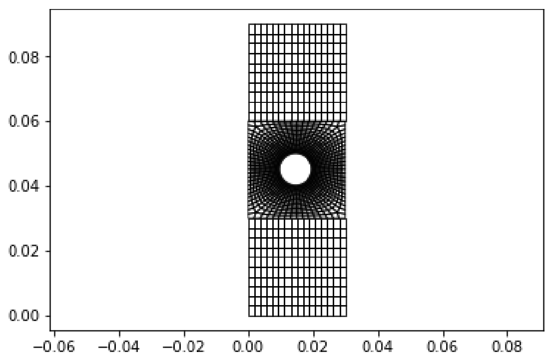
```python
In [9]: meshfile=os.path.join('data','dic_composite','abaqus_q4_m.inp')
        m=px.ReadMeshINP(meshfile)
        l0=0.005      # regularization length
```

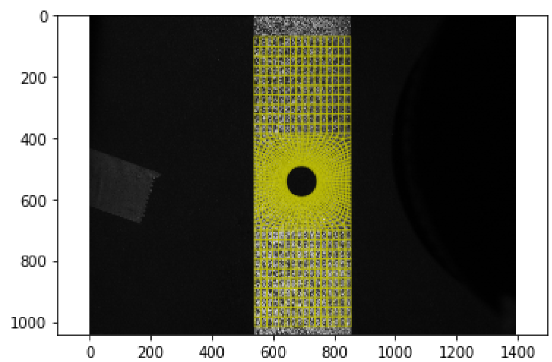
Calibration or **loading** the camera model parameters

```
In [10]: #cam=px.MeshCalibration(f,m,[1,2])
         #print(cam.get_p())
         # or
         cam=px.Camera(np.array([ 1.05168768e+04,  5.13737634e-02,
                                 -9.65935782e-02, -2.65443047e-03]))
```

Different ways to **plot the mesh**: (1) the mesh or (2) its projection on the image.

```
In [11]: # Visualization of the mesh alone
         m.Plot()
         # Plot Mesh on the reference image
         px.PlotMeshImage(f,m,cam)
```





1.4 Preprocessing

Build the **connectivity** table Build the **quadrature** rule + Compute **FE basis functions** and derivatives

```
In [12]: m.Connectivity()
         m.DICIntegration(cam)
```

Optionnal **multiscale initialization** of the displacement field

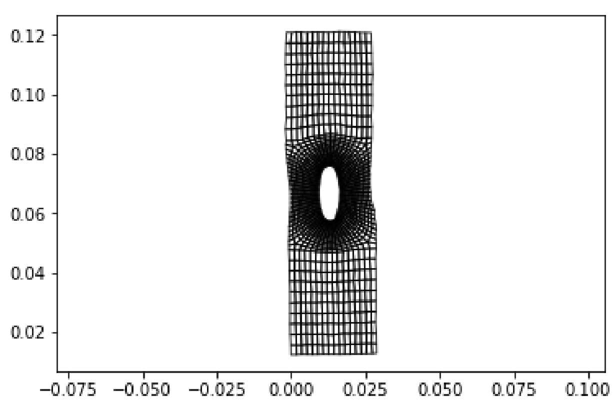
```
In [13]: U0=px.MultiscaleInit(m,f,g,cam,3)
```

```
SCALE  3
Iter #  1 | disc/dyn=10.44 % | dU/U=1.00e+00
Iter #  2 | disc/dyn=7.54 % | dU/U=4.86e-01
Iter #  3 | disc/dyn=5.02 % | dU/U=2.41e-01
Iter #  4 | disc/dyn=3.80 % | dU/U=9.59e-02
Iter #  5 | disc/dyn=3.51 % | dU/U=3.52e-02
Iter #  6 | disc/dyn=3.45 % | dU/U=1.29e-02
Iter #  7 | disc/dyn=3.44 % | dU/U=4.88e-03
Iter #  8 | disc/dyn=3.44 % | dU/U=1.96e-03
Iter #  9 | disc/dyn=3.43 % | dU/U=8.36e-04
```

```
Iter # 10 | disc/dyn=3.43 % | dU/U=3.80e-04
Iter # 11 | disc/dyn=3.43 % | dU/U=1.84e-04
Iter # 12 | disc/dyn=3.43 % | dU/U=9.48e-05
Iter # 13 | disc/dyn=3.43 % | dU/U=5.10e-05
Iter # 14 | disc/dyn=3.43 % | dU/U=2.83e-05
Iter # 15 | disc/dyn=3.43 % | dU/U=1.60e-05
Iter # 16 | disc/dyn=3.43 % | dU/U=9.12e-06
SCALE  2
Iter #  1 | disc/dyn=4.89 % | dU/U=6.09e-02
Iter #  2 | disc/dyn=4.11 % | dU/U=1.53e-02
Iter #  3 | disc/dyn=4.07 % | dU/U=5.11e-03
Iter #  4 | disc/dyn=4.06 % | dU/U=1.90e-03
Iter #  5 | disc/dyn=4.06 % | dU/U=7.52e-04
Iter #  6 | disc/dyn=4.06 % | dU/U=3.10e-04
Iter #  7 | disc/dyn=4.06 % | dU/U=1.32e-04
Iter #  8 | disc/dyn=4.06 % | dU/U=5.71e-05
Iter #  9 | disc/dyn=4.07 % | dU/U=2.53e-05
Iter # 10 | disc/dyn=4.07 % | dU/U=1.14e-05
Iter # 11 | disc/dyn=4.07 % | dU/U=5.18e-06
SCALE  1
Iter #  1 | disc/dyn=3.68 % | dU/U=3.89e-02
Iter #  2 | disc/dyn=3.00 % | dU/U=5.69e-03
Iter #  3 | disc/dyn=2.99 % | dU/U=1.15e-03
Iter #  4 | disc/dyn=2.99 % | dU/U=2.86e-04
Iter #  5 | disc/dyn=2.99 % | dU/U=8.02e-05
Iter #  6 | disc/dyn=2.99 % | dU/U=2.44e-05
Iter #  7 | disc/dyn=2.99 % | dU/U=7.87e-06
```

Plot the coarse initialization

```
In [14]: m.Plot(U0,30)
```



1.5 DIC Solver without regularization

Initialization of U and DIC engine. **Assembly** and **factorization** of DIC operator **H**.

```
In [15]: U=U0.copy()
         dic=px.DICEngine()
         H=dic.ComputeLHS(f,m,cam)                  # DIC operator assembly
         H_LU=splalg.splu(H)                        # LU Decomposition
```

Gauss-Newton iterations (without regularization)

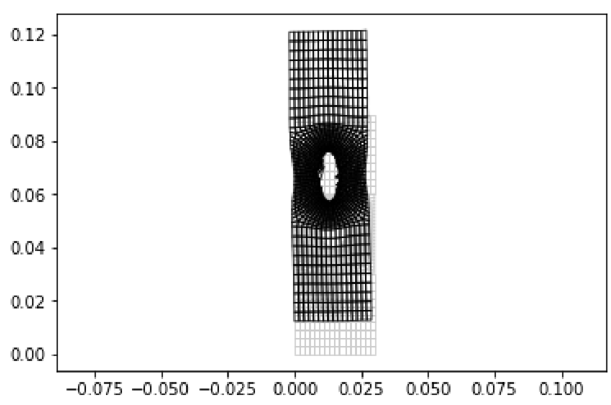
```
In [16]: for ik in range(0,30):
             [b,res]=dic.ComputeRHS(g,m,cam,U)      # DIC rhs assembly
             d=H_LU.solve(b)                        # Forward and backward substitution
             U+=d                                   # Displacement update
             err=np.linalg.norm(d)/np.linalg.norm(U)
             print("Iter # %2d | disc/dyn=%2.2f %% | dU/U=%1.2e" % (ik+1
                                                   ,np.std(res)/dic.dyn*100,err))
             if err<1e-3:
                 break
```

```
Iter #  1 | disc/dyn=1.22 % | dU/U=1.49e-02
Iter #  2 | disc/dyn=0.96 % | dU/U=1.45e-03
Iter #  3 | disc/dyn=0.96 % | dU/U=2.07e-04
```

1.6 Postprocessing

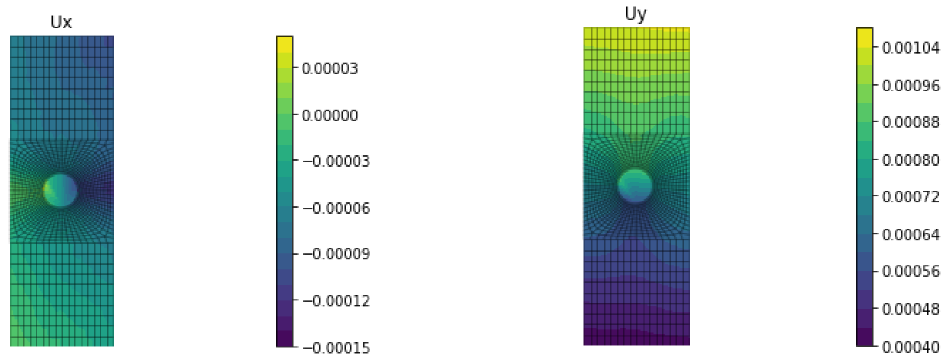
Visualization: **Scaled deformation of the mesh**

```
In [17]: m.Plot(edgecolor='#CCCCCC')
         m.Plot(U,30)
```



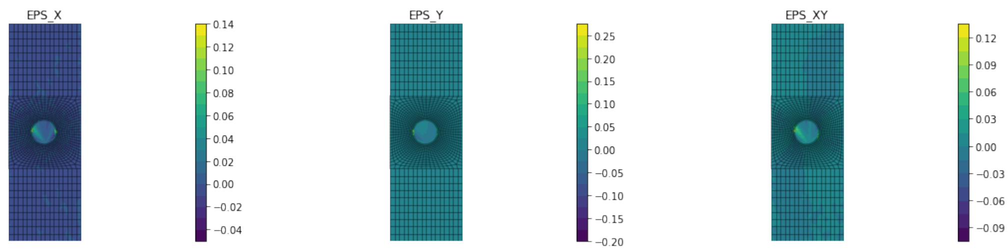
Visualization: **displacement fields**

```
In [18]: m.PlotContourDispl(U)
```



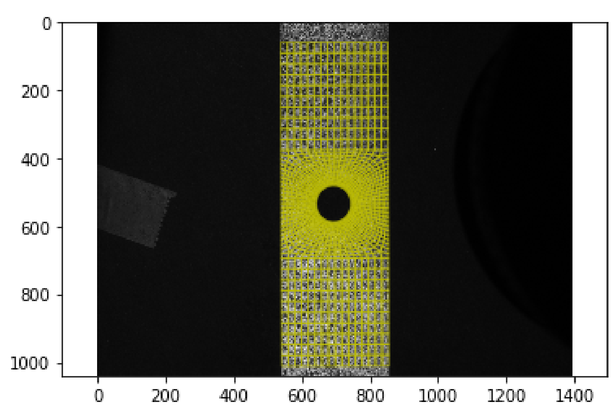
Visualization: **strain fields**

```
In [19]: m.PlotContourStrain(U)
```



Plot deformed **Mesh** on deformed state **image**

```
In [20]: px.PlotMeshImage(g,m,cam,U)
```



2 Exercises

2.1 Time resolved DIC analysis

Perform the measurement for each time steps and plot the corresponding displacement fields.

```
In [22]: U=np.zeros(m.ndof)
         #TODO
```

2.2 DIC with Tikhonov regularization

Reminder: a regularization term is added to the DIC functionnal $j(\mathbf{d})$. The regularized problem becomes:

$$\widetilde{j}(\mathbf{d}) = j(\mathbf{d}) + \frac{\alpha}{2}\, \mathbf{u}^T \mathbf{A}\ \mathbf{u}$$

with $\mathbf{u} = \mathbf{u}_0 + \mathbf{q}$, the running approximation $\mathbf{u}_0$ being fixed. The parameter α can be interpreted as a filter cut-off. Many techniques exists to define a "good" value. For the sake of simplicity, try to find a value by trial and error. The stationnarity of the regularized functionnal with respect to displacement correction $\mathbf{q}$ reads:

$$\mathbf{H}\ \mathbf{d} = \mathbf{b} \qquad \Rightarrow \qquad (\mathbf{H} + \alpha\ \mathbf{A})\ \mathbf{d} = \mathbf{b} - \alpha\ \mathbf{A}\ \mathbf{u}_0$$

```
In [23]: imdef = imagefile % imnums[-2]
         g = px.Image(imdef).Load()
         U=U0.copy()
         A=m.Tikhonov()
         #TODO
```

2.3 DIC with Elastic Regularization

Same technique as before, but with a different regularization operator $\mathbf{A}$:

$$\mathbf{A} = \mathbf{K}^T \mathbf{D}\ \mathbf{K} \qquad \textit{with} \qquad \mathbf{D} = \begin{bmatrix} \mathbf{I}_{ni} & 0 \\ 0 & \mathbf{0}_{nb} \end{bmatrix}$$

$\mathbf{D}$ is a diagonal matrix with null diagonal term for non-free boundary nodes and a one instead. Define **Stiffness/Hooke tensor** (ortotropic in this case)

```
In [27]: El=1.0e+10
         Et=1.9e+10
         vtl=0.18
         Glt=1.0e+09
         vlt=vtl*El/Et
         alp=1/(1-vlt*vtl)
         hooke=np.array([[alp*El     , alp*vtl*El,0    ],
                         [alp*vlt*Et , alp*Et    ,0    ],
                         [0          , 0         ,2*Glt]])
```

Assemble **stiffness matrix**

```
In [28]: K=m.Stiffness(hooke)
```

Select or load the non-free boundary nodes (here two lines)

```
In [29]: # repb1=px.SelectMeshLine(m)
         # repb2=px.SelectMeshLine(m)
         # or
         repb1=np.array([ 12,  13, 221, 222, 223, 224, 225, 226, 227, 228, 229,
                                                230, 231, 232, 233, 234, 235])
         repb2=np.array([ 14,  15, 254, 255, 256, 257, 258, 259, 260, 261, 262,
                                                263, 264, 265, 266, 267, 268])
         repb=np.append(repb1,repb2)
         dofb=m.conn[repb,:].ravel()
         D=np.ones_like(U)
         D[dofb]=0
         D=sp.sparse.diags(D)
         A=K.T.dot(D.dot(K))
```

FE-DIC with elastic regularization

```
In [ ]: #TODO
```

2.4 Experimental displacement driven simulation

Here, the goal is to perform a FE simulation on the same domain (same mesh) with measured boundary conditions. Find displacement $\mathbf{v}$ such that:

$$\mathbf{K}\ \mathbf{v} = \mathbf{0} \qquad with \qquad \mathbf{v}|_{\partial_u \Omega} = \mathbf{u}|_{\partial_u \Omega}$$

```
In [ ]: #TODO
```

2.5 Model Validation

Plot the distance map between simulated and measured displacements fields. When using the same FE basis for both simulation and measurement, it simply consists in comparing displacement dof-by-dof:

$$\mathbf{dis} = abs(\mathbf{u} - \mathbf{v})$$

```
In [31]: #TODO
```

2.6 Constitutive parameter identification

For the sake of simplicity, let's try to identify only the Poisson's ratio ν_{tl} using a very basic FEMU approach. The functionnal reads:

$$\mathcal{J}^{femu} = \frac{1}{2} \|\mathbf{u} - \mathbf{v}(\mathbf{p})\|^2$$

$\mathbf{u}$ being the experimental displacement and $\mathbf{v}$ the simulated displacement fields that depends on the vector of constitutive parameters $\mathbf{p}$

To linearize the problem, the parameter $\mathbf{p}$ is sought in the form of $\mathbf{p} = \mathbf{p}_0 + \mathbf{q}$, the running approximation $\mathbf{p}_0$ begin fixed. The stationnarity of the above functionnal with respect to constitutive parameter update $\mathbf{q}$ reads:

$$\frac{\partial \mathbf{v}}{\partial \mathbf{p}}^T \frac{\partial \mathbf{v}}{\partial \mathbf{p}} \mathbf{q} = -\frac{\partial \mathbf{v}}{\partial \mathbf{p}}^T (\mathbf{u} - \mathbf{v}(\mathbf{p}_0))$$

The sensitivities $\frac{\partial \mathbf{v}}{\partial \mathbf{p}}$ (derivative of the displacement with respect to the parameter) is computed numerically by finite differences.

```
In [30]: #TODO (may require more time than is available...)
```