

ECE 486: Lab 1 Report

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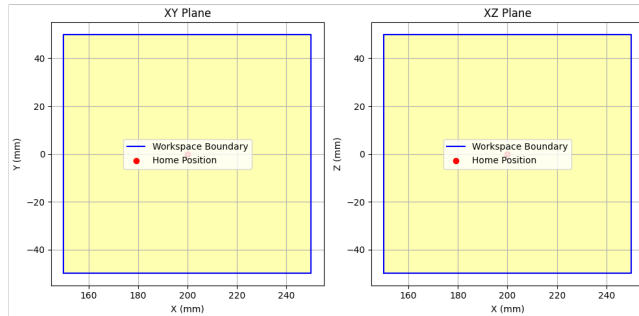
June 3, 2024

1 Part 1

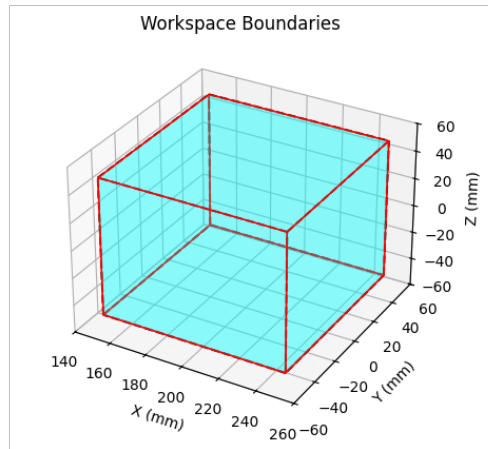
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2 The Workspace

2.1 Workspace Design



(a) Boundaries in XY-XZ Planes



(b) Workspace in XYZ Space

Figure 1: Workspace Boundary Representation

The workspace restrictions are defined to ensure the robot operates within a safe and efficient range. The boundaries chosen are in the x axis is [160,240] mm, y in range of [-40, 40] mm, and z having a range of [-40, 40] mm. The home position as seen in Figure 1, is set at (200, 0, 0).

For a planar robotic arm with three joints, the position (x,y) of the end-effector can be calculated as follows:

L_1, L_2, L_3 : length of the robot's links

$\theta_1, \theta_2, \theta_3$: angle for the joints

L_4, θ_4 : vertical movement length and angle

$$x = L_1 \cos(\theta_1) + L_2 \cos(\theta_1 + \theta_2) + L_3 \cos(\theta_1 + \theta_2 + \theta_3) \quad (1)$$

$$y = L_1 \sin(\theta_1) + L_2 \sin(\theta_1 + \theta_2) + L_3 \sin(\theta_1 + \theta_2 + \theta_3) \quad (2)$$

$$z = L_4 \sin(\theta_4) \quad (3)$$

To verify that any point within the workspace boundaries is reachable, we solve the inverse kinematics equations. The base rotation angle θ_1 can be found using equation:

$$\theta_1 = \arctan(y, x) = \arctan(0, 200) = 0$$

Given $x = 200$ mm and $y = 0$ mm, using $r = \sqrt{x^2 + y^2}, r = \sqrt{200^2 + 0^2} = 200$ mm

Using the Cosine law, we find θ_3 : $\cos(\theta_3) = \frac{r^2 - L_1^2 - L_2^2}{2L_1L_2}$

$$\theta_3 = \cos^{-1} \left(\frac{r^2 - L_1^2 - L_2^2}{2L_1L_2} \right) = \frac{200^2 - 150^2 - 150^2}{2 * 150 * 150} = \cos^{-1} \left(\frac{40000 - 22500 - 22500}{45000} \right) = 180^\circ$$

and

$$\theta_2 = \cos^{-1} \left(\frac{r^2 - L_1^2 - L_2^2}{2L_1L_2} \right) + \arctan(y, x) = \cos^{-1} \left(\frac{200^2 + 150^2 - 150^2}{2 * 200 * 150} \right) + 0 = \cos^{-1} \left(\frac{2}{3} \right) = 48.19^\circ$$

θ_4 can be adjusted based on the specific vertical position.

The above calculations show that for the point (200, 0, 0), the joint angles $\theta_1 = 0^\circ, \theta_2 = 48.19^\circ$ and $\theta_3 = 180^\circ$ allow the end-effector to reach this position (and these angles are well within the allowable angles of the Dobot Magician for the joints [1]), confirming that it falls within the reachable workspace.

2.2 Workspace Validation

Centre of Workspace: (200, 0, 0)

We tested the following points by queuing the movements and observing whether the robot moves or not. For edge cases, we wanted to test the corners of the workspace as well as points where some of the coordinates are in the boundary but the others are not.

Corners of the workspace that were tested: (150, -50, 50), (150, 50, 50), (250, 50, 50), (250, -50, 50), (250, -50, -50), (150, -50, -50), (150, 50, -50), (250, 50, -50)

Additional points that were tested (260, 50, -50), (250, 40, -50), (250, 40, -30), (350, 40, -30)

The Dobot moved the corners of the workspace as expected and for points that were in the workspace, it performed the queued movement. When the coordinates got out of the range of the workspace, the robot refused to move. This shows that the robot only moved within the defined workspace.

3 Trajectory

We chose a circular trajectory of radius 30mm in the XY plane at the maximum z-coordinate which in our case is 50.0. Our trajectory has 50 points and was run 10 times to calculate 10 sets of data on the x,y,z position at each of the 50 points along the trajectory.

The following code was used to generate the trajectory:

```

1 # Function to calculate circle trajectory points
2 def generate_circle_trajectory(radius, center_x, center_y, center_z, num_points):
3     trajectory = []
4     for i in range(num_points):
5         angle = 2 * math.pi * i / num_points
6         x = center_x + radius * math.cos(angle)
7         y = center_y + radius * math.sin(angle)
8         z = center_z
9         trajectory.append((x, y, z))
10    return trajectory
11
12 # Generate circle trajectory
13 radius = 30
14 num_points = 50
15 trajectory = generate_circle_trajectory(radius, xHomePos, yHomePos, zMax, num_points)

```

Listing 1: Generating the circle trajectory

3.1 Proof of twice differentiability

To prove that a circle is twice differentiable, let's consider the standard equation of a circle centered at the origin with radius r (assuming the origin in this case is at the home position):

$$x^2 + y^2 = r^2$$

$$y = \pm\sqrt{r^2 - x^2}$$

Here, $y = f(x) = \sqrt{r^2 - x^2}$ for the upper semicircle and $y = f(x) = -\sqrt{r^2 - x^2}$ for the lower semicircle. We will consider the upper semicircle; the argument for the lower semicircle is similar.

First Derivative

Differentiating $y = \sqrt{r^2 - x^2}$ with respect to x using chain rule:

$$y = \sqrt{r^2 - x^2}$$

$$\frac{dy}{dx} = \frac{d}{dx}(r^2 - x^2)^{1/2} = \frac{1}{2}(r^2 - x^2)^{-1/2} \cdot (-2x) = -\frac{x}{\sqrt{r^2 - x^2}}$$

$$\frac{dy}{dx} = -\frac{x}{\sqrt{r^2 - x^2}}$$

This shows that the first derivative $\frac{dy}{dx}$ exists and is differentiable wherever $r^2 - x^2 \neq 0$.

Second Derivative

Differentiating $\frac{dy}{dx}$ to find the second derivative:

$$\frac{dy}{dx} = -\frac{x}{\sqrt{r^2 - x^2}}$$

Let $u = r^2 - x^2$, so $\sqrt{r^2 - x^2} = \sqrt{u}$. Then,

$$\frac{dy}{dx} = -\frac{x}{\sqrt{u}}$$

Differentiate again with respect to x using the quotient rule:

$$\frac{d}{dx} \left(-\frac{x}{\sqrt{u}} \right) = - \left(\frac{1 \cdot \sqrt{u} - x \cdot \frac{d}{dx}(\sqrt{u})}{u} \right)$$

Calculating $\frac{d}{dx}(\sqrt{u})$:

$$\frac{d}{dx}(\sqrt{u}) = \frac{1}{2\sqrt{u}} \cdot \frac{du}{dx} = \frac{1}{2\sqrt{r^2 - x^2}} \cdot (-2x) = \frac{-x}{\sqrt{r^2 - x^2}}$$

Substituting $\frac{d}{dx}(\sqrt{u})$ back into the expression:

$$\frac{d}{dx} \left(-\frac{x}{\sqrt{u}} \right) = - \left(\frac{\sqrt{u} + x \cdot \frac{x}{\sqrt{u}}}{u} \right) = - \left(\frac{\sqrt{r^2 - x^2} + x \cdot \frac{x}{\sqrt{r^2 - x^2}}}{r^2 - x^2} \right)$$

Substituting $\sqrt{u} = \sqrt{r^2 - x^2}$:

$$\begin{aligned} \frac{d}{dx} \left(-\frac{x}{\sqrt{r^2 - x^2}} \right) &= - \left(\frac{\sqrt{r^2 - x^2} + \frac{x^2}{\sqrt{r^2 - x^2}}}{r^2 - x^2} \right) = - \left(\frac{r^2 - x^2 + x^2}{(r^2 - x^2)\sqrt{r^2 - x^2}} \right) \\ &= - \left(\frac{r^2}{(r^2 - x^2)\sqrt{r^2 - x^2}} \right) = - \frac{r^2}{(r^2 - x^2)^{3/2}} \end{aligned}$$

Therefore, the second derivative is:

$$\frac{d^2y}{dx^2} = -\frac{r^2}{(r^2 - x^2)^{3/2}}$$

Thus, Both the first and second derivatives of y with respect to x exist and are continuous wherever $r^2 - x^2 \neq 0$ (i.e., $x \neq \pm r$). Therefore, the function $y = \sqrt{r^2 - x^2}$ (and similarly $y = -\sqrt{r^2 - x^2}$) is twice differentiable. Since these represent the upper and lower halves of the circle, we can conclude that the circle trajectory is twice differentiable wherever $r^2 - x^2 \neq 0$. When $r^2 - x^2 = 0$, it is a boundary condition that we ignore.

3.2 Error analysis

We used a python script to do error analysis on the coordinate points that were reported by the robot for our circle trajectory. The script does the following:

For error analysis with the theoretical trajectory:

For each run of the trajectory, compute the Euclidean distance between the point in the theoretical trajectory and the corresponding point reported by the robot for that run.

For error analysis between runs:

For each pair of runs (i.e for 10 runs there are 45 total pairs), compute the Euclidean distance between the corresponding points in each run.

The repeatability specification for the Dobot Magician = 0.2 mm [1]. The script checks if the maximum theoretical error and maximum error between runs is below the repeatability specification of 0.2 mm.

Here is the python script. The .csv files can be found in the appendix at the end of the report.

```

1 import numpy
2 import pandas
3 import matplotlib.pyplot as plot
4
5 # Load theoretical points
6 theoretical_trajectory = pandas.read_csv('theoretical_trajectory.csv').values
7 # Load points for each run 1-10
8 robot_trajectories = [pandas.read_csv(f'robot_trajectory_run_{i}.csv').values for i in
    range(1, 11)]
9
10 # Calculate errors between the real robot and the theoretical trajectory
11 errors_to_theoretical = []
12 for run in robot_trajectories:
13     error = numpy.linalg.norm(run - theoretical_trajectory, axis=1)
14     errors_to_theoretical.append(error)
15
16 # Calculate errors between separate runs of the trajectory
17 errors_between_runs = []
18 for i in range(len(robot_trajectories)):
19     for j in range(i+1, len(robot_trajectories)):
20         error = numpy.linalg.norm(robot_trajectories[i] - robot_trajectories[j], axis
    =1)
21         errors_between_runs.append(error)
22
23 # Plot errors between runs and theoretical
24 plot.figure(figsize=(14, 7))
25 for i, error in enumerate(errors_to_theoretical):
26     plot.plot(error, label=f'Run {i+1}')
27 plot.title('Error to Theoretical Trajectory')
28 plot.xlabel('Point')
29 plot.ylabel('Error (mm)')
30 plot.legend()
31 plot.tight_layout()
32 plot.show()
33

```

```

34 # Plot errors between runs
35 plot.figure(figsize=(14, 7))
36 for i, error in enumerate(errors_between_runs):
37     plot.plot(error, label=f'Run Pair {i+1}')
38 plot.title('Error Between Runs')
39 plot.xlabel('Point')
40 plot.ylabel('Error (mm)')
41 plot.legend()
42 plot.tight_layout()
43 plot.show()
44
45 # Compare with Dobot Magician's repeatability
46 repeatability_spec = 0.2 # Repeatability spec in mm
47 max_error_to_theoretical = max([max(error) for error in errors_to_theoretical])
48 max_error_between_runs = max([max(error) for error in errors_between_runs])
49
50 print(f"Max Error to Theoretical Trajectory: {max_error_to_theoretical:.10f} mm")
51 print(f"Max Error Between Runs: {max_error_between_runs:.10f} mm")
52 print(f"Repeatability Spec: {repeatability_spec} mm")
53
54 if max_error_to_theoretical <= repeatability_spec and max_error_between_runs <=
    repeatability_spec:
55     print("The robot meets the repeatability specification.")
56 else:
57     print("The robot does not meet the repeatability specification.")

```

Listing 2: Error analysis code

```

1 Max Error to Theoretical Trajectory: 0.0000362226 mm
2 Max Error Between Runs: 0.0000000000 mm
3 Repeatability Spec: 0.2 mm
4 The robot meets the repeatability specification.

```

Listing 3: Output of error analysis script

Thus, the values reported by the robot meets the repeatability specification. Figures 2 and 3, along with the script output in Listing 3 show how small the errors calculated were.

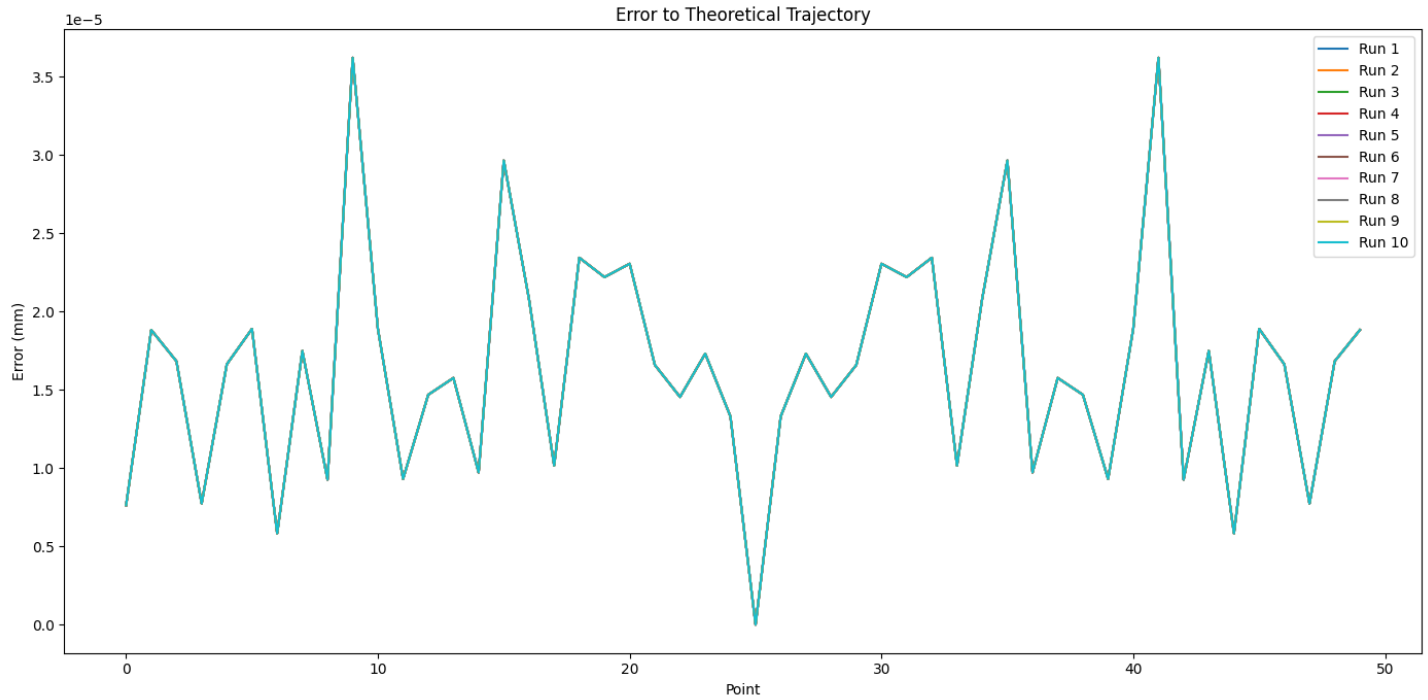


Figure 2: Errors with theoretical points for each run

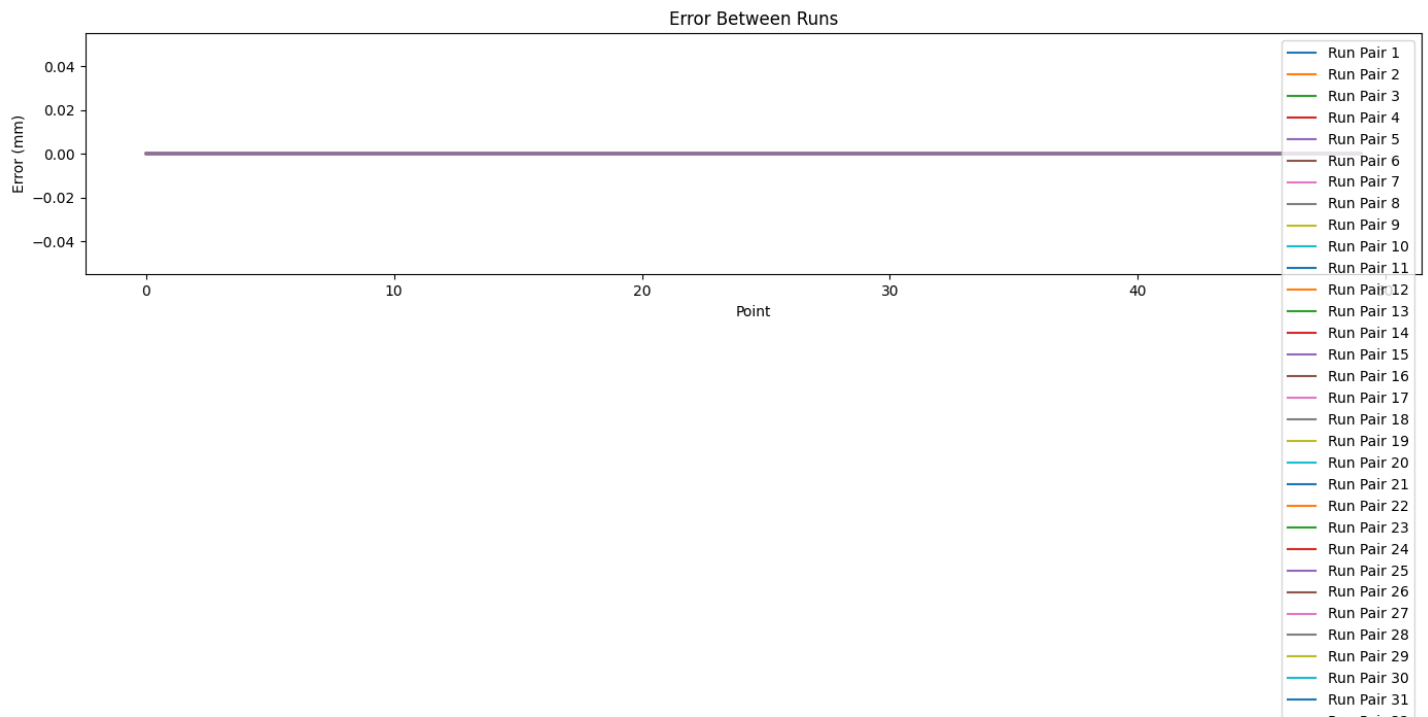


Figure 3: Errors between runs (10 runs mean 45 pairs)

4

References

- [1] RobotLAB, "Dobot Magician Specifications," [Online]. Available: <https://www.robotlab.com/hubfs/Dobot/Dobot%20Magician%20Specifications.pdf>. Accessed on: June 2, 2024.

5 Appendix

```

1 x,y,z
2 230.0,0.0,49.99999237060547
3 229.76345825195312,3.7599971294403076,49.99999237060547
4 229.05747985839844,7.460695743560791,50.00000762939453
5 227.89329528808594,11.04373550415039,50.00000762939453
6 226.28921508789062,14.452611923217773,49.99999237060547
7 224.27049255371094,17.63355827331543,49.99999237060547
8 221.8690643310547,20.536415100097656,50.0
9 219.12271118164062,23.11539649963379,49.99998474121094
10 216.07479858398438,25.329837799072266,50.00000762939453
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13 205.62144470214844,29.468618392944336,49.99999237060547
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21 178.13092041015625,20.53641128540039,50.00000762939453
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24 172.10671997070312,11.043736457824707,50.0
25 170.94248962402344,7.460696697235107,49.99999237060547
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27 170.0,0.0,50.0
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29 170.94248962402344,-7.460696697235107,49.99999237060547
30 172.10671997070312,-11.043736457824707,50.0
31 173.71078491210938,-14.452609062194824,49.99999237060547
32 175.72950744628906,-17.63355827331543,50.00001525878906
33 178.13092041015625,-20.53641128540039,50.00000762939453
34 180.87725830078125,-23.115394592285156,50.00000762939453
35 183.92518615722656,-25.329835891723633,50.0
36 187.2266082763672,-27.144805908203125,49.99998474121094
37 190.72946166992188,-28.531692504882812,49.99999237060547
38 194.37855529785156,-29.46861457824707,49.99999237060547
39 198.11627197265625,-29.9407958984375,50.00000762939453
40 201.88372802734375,-29.94080352783203,49.99999237060547
41 205.62144470214844,-29.468618392944336,49.99999237060547
42 209.27049255371094,-28.531696319580078,49.99999237060547
43 212.77334594726562,-27.14480972290039,50.00001525878906
44 216.07479858398438,-25.329837799072266,50.00000762939453
45 219.12271118164062,-23.11539649963379,49.99998474121094
46 221.8690643310547,-20.536415100097656,50.0
47 224.27049255371094,-17.63355827331543,49.99999237060547
48 226.28921508789062,-14.452611923217773,49.99999237060547
49 227.89329528808594,-11.04373550415039,50.00000762939453
50 229.05747985839844,-7.460695743560791,50.00000762939453
51 229.76345825195312,-3.7599971294403076,49.99999237060547

```

Listing 4: robot_trajectory_run.1.csv

```

1 x,y,z
2 230.0,0.0,49.99999237060547
3 229.76345825195312,3.7599971294403076,49.99999237060547
4 229.05747985839844,7.460695743560791,50.00000762939453
5 227.89329528808594,11.04373550415039,50.00000762939453
6 226.28921508789062,14.452611923217773,49.99999237060547
7 224.27049255371094,17.63355827331543,49.99999237060547
8 221.8690643310547,20.536415100097656,50.0
9 219.12271118164062,23.11539649963379,49.99998474121094
10 216.07479858398438,25.329837799072266,50.00000762939453
11 212.77334594726562,27.14480972290039,50.00001525878906
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13 205.62144470214844,29.468618392944336,49.99999237060547
14 201.88372802734375,29.94080352783203,49.99999237060547
15 198.11627197265625,29.9407958984375,50.00000762939453
16 194.37855529785156,29.46861457824707,49.99999237060547
17 190.72946166992188,28.531692504882812,49.99999237060547
18 187.2266082763672,27.144805908203125,49.99998474121094
19 183.92518615722656,25.329835891723633,50.0
20 180.87725830078125,23.115394592285156,50.00000762939453
21 178.13092041015625,20.53641128540039,50.00000762939453
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30 172.10671997070312,-11.043736457824707,50.0
31 173.71078491210938,-14.452609062194824,49.99999237060547
32 175.72950744628906,-17.63355827331543,50.00001525878906
33 178.13092041015625,-20.53641128540039,50.00000762939453
34 180.87725830078125,-23.115394592285156,50.00000762939453
35 183.92518615722656,-25.329835891723633,50.0
36 187.2266082763672,-27.144805908203125,49.99998474121094
37 190.72946166992188,-28.531692504882812,49.99999237060547
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39 198.11627197265625,-29.9407958984375,50.00000762939453
40 201.88372802734375,-29.94080352783203,49.99999237060547
41 205.62144470214844,-29.468618392944336,49.99999237060547
42 209.27049255371094,-28.531696319580078,49.99999237060547
43 212.77334594726562,-27.14480972290039,50.00001525878906
44 216.07479858398438,-25.329837799072266,50.00000762939453
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46 221.8690643310547,-20.536415100097656,50.0
47 224.27049255371094,-17.63355827331543,49.99999237060547
48 226.28921508789062,-14.452611923217773,49.99999237060547
49 227.89329528808594,-11.04373550415039,50.00000762939453
50 229.05747985839844,-7.460695743560791,50.00000762939453
51 229.76345825195312,-3.7599971294403076,49.99999237060547

```

Listing 5: robot_trajectory_run_2.csv

```

1 x,y,z
2 230.0,0.0,49.99999237060547

```

```

3 229.76345825195312,3.7599971294403076,49.99999237060547
4 229.05747985839844,7.460695743560791,50.00000762939453
5 227.89329528808594,11.04373550415039,50.00000762939453
6 226.28921508789062,14.452611923217773,49.99999237060547
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16 194.37855529785156,29.46861457824707,49.99999237060547
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21 178.13092041015625,20.53641128540039,50.00000762939453
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47 224.27049255371094,-17.63355827331543,49.99999237060547
48 226.28921508789062,-14.452611923217773,49.99999237060547
49 227.89329528808594,-11.04373550415039,50.00000762939453
50 229.05747985839844,-7.460695743560791,50.00000762939453
51 229.76345825195312,-3.7599971294403076,49.99999237060547

```

Listing 6: robot_trajectory_run_3.csv

```

1 x,y,z
2 230.0,0.0,49.99999237060547
3 229.76345825195312,3.7599971294403076,49.99999237060547
4 229.05747985839844,7.460695743560791,50.00000762939453

```

```

5 227.89329528808594,11.04373550415039,50.00000762939453
6 226.28921508789062,14.452611923217773,49.99999237060547
7 224.27049255371094,17.63355827331543,49.99999237060547
8 221.8690643310547,20.536415100097656,50.0
9 219.12271118164062,23.11539649963379,49.99998474121094
10 216.07479858398438,25.329837799072266,50.00000762939453
11 212.77334594726562,27.14480972290039,50.00001525878906
12 209.27049255371094,28.531696319580078,49.99999237060547
13 205.62144470214844,29.468618392944336,49.99999237060547
14 201.88372802734375,29.94080352783203,49.99999237060547
15 198.11627197265625,29.9407958984375,50.00000762939453
16 194.37855529785156,29.46861457824707,49.99999237060547
17 190.72946166992188,28.531692504882812,49.99999237060547
18 187.2266082763672,27.144805908203125,49.99998474121094
19 183.92518615722656,25.329835891723633,50.0
20 180.87725830078125,23.115394592285156,50.00000762939453
21 178.13092041015625,20.53641128540039,50.00000762939453
22 175.72950744628906,17.63355827331543,50.00001525878906
23 173.71078491210938,14.452609062194824,49.99999237060547
24 172.10671997070312,11.043736457824707,50.0
25 170.94248962402344,7.460696697235107,49.99999237060547
26 170.236572265625,3.7599966526031494,50.0
27 170.0,0.0,50.0
28 170.236572265625,-3.7599966526031494,50.0
29 170.94248962402344,-7.460696697235107,49.99999237060547
30 172.10671997070312,-11.043736457824707,50.0
31 173.71078491210938,-14.452609062194824,49.99999237060547
32 175.72950744628906,-17.63355827331543,50.00001525878906
33 178.13092041015625,-20.53641128540039,50.00000762939453
34 180.87725830078125,-23.115394592285156,50.00000762939453
35 183.92518615722656,-25.329835891723633,50.0
36 187.2266082763672,-27.144805908203125,49.99998474121094
37 190.72946166992188,-28.531692504882812,49.99999237060547
38 194.37855529785156,-29.46861457824707,49.99999237060547
39 198.11627197265625,-29.9407958984375,50.00000762939453
40 201.88372802734375,-29.94080352783203,49.99999237060547
41 205.62144470214844,-29.468618392944336,49.99999237060547
42 209.27049255371094,-28.531696319580078,49.99999237060547
43 212.77334594726562,-27.14480972290039,50.00001525878906
44 216.07479858398438,-25.329837799072266,50.00000762939453
45 219.12271118164062,-23.11539649963379,49.99998474121094
46 221.8690643310547,-20.536415100097656,50.0
47 224.27049255371094,-17.63355827331543,49.99999237060547
48 226.28921508789062,-14.452611923217773,49.99999237060547
49 227.89329528808594,-11.04373550415039,50.00000762939453
50 229.05747985839844,-7.460695743560791,50.00000762939453
51 229.76345825195312,-3.7599971294403076,49.99999237060547

```

Listing 7: robot_trajectory_run_4.csv

```

1 x,y,z
2 230.0,0.0,49.99999237060547
3 229.76345825195312,3.7599971294403076,49.99999237060547
4 229.05747985839844,7.460695743560791,50.00000762939453
5 227.89329528808594,11.04373550415039,50.00000762939453
6 226.28921508789062,14.452611923217773,49.99999237060547

```

```

7 224.27049255371094,17.63355827331543,49.99999237060547
8 221.8690643310547,20.536415100097656,50.0
9 219.12271118164062,23.11539649963379,49.99998474121094
10 216.07479858398438,25.329837799072266,50.00000762939453
11 212.77334594726562,27.14480972290039,50.00001525878906
12 209.27049255371094,28.531696319580078,49.99999237060547
13 205.62144470214844,29.468618392944336,49.99999237060547
14 201.88372802734375,29.94080352783203,49.99999237060547
15 198.11627197265625,29.9407958984375,50.00000762939453
16 194.37855529785156,29.46861457824707,49.99999237060547
17 190.72946166992188,28.531692504882812,49.99999237060547
18 187.2266082763672,27.144805908203125,49.99998474121094
19 183.92518615722656,25.329835891723633,50.0
20 180.87725830078125,23.115394592285156,50.00000762939453
21 178.13092041015625,20.53641128540039,50.00000762939453
22 175.72950744628906,17.63355827331543,50.00001525878906
23 173.71078491210938,14.452609062194824,49.99999237060547
24 172.10671997070312,11.043736457824707,50.0
25 170.94248962402344,7.460696697235107,49.99999237060547
26 170.236572265625,3.7599966526031494,50.0
27 170.0,0.0,50.0
28 170.236572265625,-3.7599966526031494,50.0
29 170.94248962402344,-7.460696697235107,49.99999237060547
30 172.10671997070312,-11.043736457824707,50.0
31 173.71078491210938,-14.452609062194824,49.99999237060547
32 175.72950744628906,-17.63355827331543,50.00001525878906
33 178.13092041015625,-20.53641128540039,50.00000762939453
34 180.87725830078125,-23.115394592285156,50.00000762939453
35 183.92518615722656,-25.329835891723633,50.0
36 187.2266082763672,-27.144805908203125,49.99998474121094
37 190.72946166992188,-28.531692504882812,49.99999237060547
38 194.37855529785156,-29.46861457824707,49.99999237060547
39 198.11627197265625,-29.9407958984375,50.00000762939453
40 201.88372802734375,-29.94080352783203,49.99999237060547
41 205.62144470214844,-29.468618392944336,49.99999237060547
42 209.27049255371094,-28.531696319580078,49.99999237060547
43 212.77334594726562,-27.14480972290039,50.00001525878906
44 216.07479858398438,-25.329837799072266,50.00000762939453
45 219.12271118164062,-23.11539649963379,49.99998474121094
46 221.8690643310547,-20.536415100097656,50.0
47 224.27049255371094,-17.63355827331543,49.99999237060547
48 226.28921508789062,-14.452611923217773,49.99999237060547
49 227.89329528808594,-11.04373550415039,50.00000762939453
50 229.05747985839844,-7.460695743560791,50.00000762939453
51 229.76345825195312,-3.7599971294403076,49.99999237060547

```

Listing 8: robot_trajectory_run_5.csv

```

1 x,y,z
2 230.0,0.0,49.99999237060547
3 229.76345825195312,3.7599971294403076,49.99999237060547
4 229.05747985839844,7.460695743560791,50.00000762939453
5 227.89329528808594,11.04373550415039,50.00000762939453
6 226.28921508789062,14.452611923217773,49.99999237060547
7 224.27049255371094,17.63355827331543,49.99999237060547
8 221.8690643310547,20.536415100097656,50.0

```

```

9 219.12271118164062,23.11539649963379,49.99998474121094
10 216.07479858398438,25.329837799072266,50.00000762939453
11 212.77334594726562,27.14480972290039,50.00001525878906
12 209.27049255371094,28.531696319580078,49.99999237060547
13 205.62144470214844,29.468618392944336,49.99999237060547
14 201.88372802734375,29.94080352783203,49.99999237060547
15 198.11627197265625,29.9407958984375,50.00000762939453
16 194.37855529785156,29.46861457824707,49.99999237060547
17 190.72946166992188,28.531692504882812,49.99999237060547
18 187.2266082763672,27.144805908203125,49.99998474121094
19 183.92518615722656,25.329835891723633,50.0
20 180.87725830078125,23.115394592285156,50.00000762939453
21 178.13092041015625,20.53641128540039,50.00000762939453
22 175.72950744628906,17.63355827331543,50.00001525878906
23 173.71078491210938,14.452609062194824,49.99999237060547
24 172.10671997070312,11.043736457824707,50.0
25 170.94248962402344,7.460696697235107,49.99999237060547
26 170.236572265625,3.7599966526031494,50.0
27 170.0,0.0,50.0
28 170.236572265625,-3.7599966526031494,50.0
29 170.94248962402344,-7.460696697235107,49.99999237060547
30 172.10671997070312,-11.043736457824707,50.0
31 173.71078491210938,-14.452609062194824,49.99999237060547
32 175.72950744628906,-17.63355827331543,50.00001525878906
33 178.13092041015625,-20.53641128540039,50.00000762939453
34 180.87725830078125,-23.115394592285156,50.00000762939453
35 183.92518615722656,-25.329835891723633,50.0
36 187.2266082763672,-27.144805908203125,49.99998474121094
37 190.72946166992188,-28.531692504882812,49.99999237060547
38 194.37855529785156,-29.46861457824707,49.99999237060547
39 198.11627197265625,-29.9407958984375,50.00000762939453
40 201.88372802734375,-29.94080352783203,49.99999237060547
41 205.62144470214844,-29.468618392944336,49.99999237060547
42 209.27049255371094,-28.531696319580078,49.99999237060547
43 212.77334594726562,-27.14480972290039,50.00001525878906
44 216.07479858398438,-25.329837799072266,50.00000762939453
45 219.12271118164062,-23.11539649963379,49.99998474121094
46 221.8690643310547,-20.536415100097656,50.0
47 224.27049255371094,-17.63355827331543,49.99999237060547
48 226.28921508789062,-14.452611923217773,49.99999237060547
49 227.89329528808594,-11.04373550415039,50.00000762939453
50 229.05747985839844,-7.460695743560791,50.00000762939453
51 229.76345825195312,-3.7599971294403076,49.99999237060547

```

Listing 9: robot_trajectory_run_6.csv

```

1 x,y,z
2 230.0,0.0,49.99999237060547
3 229.76345825195312,3.7599971294403076,49.99999237060547
4 229.05747985839844,7.460695743560791,50.00000762939453
5 227.89329528808594,11.04373550415039,50.00000762939453
6 226.28921508789062,14.452611923217773,49.99999237060547
7 224.27049255371094,17.63355827331543,49.99999237060547
8 221.8690643310547,20.536415100097656,50.0
9 219.12271118164062,23.11539649963379,49.99998474121094
10 216.07479858398438,25.329837799072266,50.00000762939453

```



```

11 212.77334594726562,27.14480972290039,50.00001525878906
12 209.27049255371094,28.531696319580078,49.99999237060547
13 205.62144470214844,29.468618392944336,49.99999237060547
14 201.88372802734375,29.94080352783203,49.99999237060547
15 198.11627197265625,29.9407958984375,50.00000762939453
16 194.37855529785156,29.46861457824707,49.99999237060547
17 190.72946166992188,28.531692504882812,49.99999237060547
18 187.2266082763672,27.144805908203125,49.99998474121094
19 183.92518615722656,25.329835891723633,50.0
20 180.87725830078125,23.115394592285156,50.00000762939453
21 178.13092041015625,20.53641128540039,50.00000762939453
22 175.72950744628906,17.63355827331543,50.00001525878906
23 173.71078491210938,14.452609062194824,49.99999237060547
24 172.10671997070312,11.043736457824707,50.0
25 170.94248962402344,7.460696697235107,49.99999237060547
26 170.236572265625,3.7599966526031494,50.0
27 170.0,0.0,50.0
28 170.236572265625,-3.7599966526031494,50.0
29 170.94248962402344,-7.460696697235107,49.99999237060547
30 172.10671997070312,-11.043736457824707,50.0
31 173.71078491210938,-14.452609062194824,49.99999237060547
32 175.72950744628906,-17.63355827331543,50.00001525878906
33 178.13092041015625,-20.53641128540039,50.00000762939453
34 180.87725830078125,-23.115394592285156,50.00000762939453
35 183.92518615722656,-25.329835891723633,50.0
36 187.2266082763672,-27.144805908203125,49.99998474121094
37 190.72946166992188,-28.531692504882812,49.99999237060547
38 194.37855529785156,-29.46861457824707,49.99999237060547
39 198.11627197265625,-29.9407958984375,50.00000762939453
40 201.88372802734375,-29.94080352783203,49.99999237060547
41 205.62144470214844,-29.468618392944336,49.99999237060547
42 209.27049255371094,-28.531696319580078,49.99999237060547
43 212.77334594726562,-27.14480972290039,50.00001525878906
44 216.07479858398438,-25.329837799072266,50.00000762939453
45 219.12271118164062,-23.11539649963379,49.99998474121094
46 221.8690643310547,-20.536415100097656,50.0
47 224.27049255371094,-17.63355827331543,49.99999237060547
48 226.28921508789062,-14.452611923217773,49.99999237060547
49 227.89329528808594,-11.04373550415039,50.00000762939453
50 229.05747985839844,-7.460695743560791,50.00000762939453
51 229.76345825195312,-3.7599971294403076,49.99999237060547

```

Listing 10: robot_trajectory_run_7.csv

```

1 x,y,z
2 230.0,0.0,49.99999237060547
3 229.76345825195312,3.7599971294403076,49.99999237060547
4 229.05747985839844,7.460695743560791,50.00000762939453
5 227.89329528808594,11.04373550415039,50.00000762939453
6 226.28921508789062,14.452611923217773,49.99999237060547
7 224.27049255371094,17.63355827331543,49.99999237060547
8 221.8690643310547,20.536415100097656,50.0
9 219.12271118164062,23.11539649963379,49.99998474121094
10 216.07479858398438,25.329837799072266,50.00000762939453
11 212.77334594726562,27.14480972290039,50.00001525878906
12 209.27049255371094,28.531696319580078,49.99999237060547

```

```

13 205.62144470214844,29.468618392944336,49.99999237060547
14 201.88372802734375,29.94080352783203,49.99999237060547
15 198.11627197265625,29.9407958984375,50.00000762939453
16 194.37855529785156,29.46861457824707,49.99999237060547
17 190.72946166992188,28.531692504882812,49.99999237060547
18 187.2266082763672,27.144805908203125,49.99998474121094
19 183.92518615722656,25.329835891723633,50.0
20 180.87725830078125,23.115394592285156,50.00000762939453
21 178.13092041015625,20.53641128540039,50.00000762939453
22 175.72950744628906,17.63355827331543,50.00001525878906
23 173.71078491210938,14.452609062194824,49.99999237060547
24 172.10671997070312,11.043736457824707,50.0
25 170.94248962402344,7.460696697235107,49.99999237060547
26 170.236572265625,3.7599966526031494,50.0
27 170.0,0.0,50.0
28 170.236572265625,-3.7599966526031494,50.0
29 170.94248962402344,-7.460696697235107,49.99999237060547
30 172.10671997070312,-11.043736457824707,50.0
31 173.71078491210938,-14.452609062194824,49.99999237060547
32 175.72950744628906,-17.63355827331543,50.00001525878906
33 178.13092041015625,-20.53641128540039,50.00000762939453
34 180.87725830078125,-23.115394592285156,50.00000762939453
35 183.92518615722656,-25.329835891723633,50.0
36 187.2266082763672,-27.144805908203125,49.99998474121094
37 190.72946166992188,-28.531692504882812,49.99999237060547
38 194.37855529785156,-29.46861457824707,49.99999237060547
39 198.11627197265625,-29.9407958984375,50.00000762939453
40 201.88372802734375,-29.94080352783203,49.99999237060547
41 205.62144470214844,-29.468618392944336,49.99999237060547
42 209.27049255371094,-28.531696319580078,49.99999237060547
43 212.77334594726562,-27.14480972290039,50.00001525878906
44 216.07479858398438,-25.329837799072266,50.00000762939453
45 219.12271118164062,-23.11539649963379,49.99998474121094
46 221.8690643310547,-20.536415100097656,50.0
47 224.27049255371094,-17.63355827331543,49.99999237060547
48 226.28921508789062,-14.452611923217773,49.99999237060547
49 227.89329528808594,-11.04373550415039,50.00000762939453
50 229.05747985839844,-7.460695743560791,50.00000762939453
51 229.76345825195312,-3.7599971294403076,49.99999237060547

```

Listing 11: robot_trajectory_run_8.csv

```

1 x,y,z
2 230.0,0.0,49.99999237060547
3 229.76345825195312,3.7599971294403076,49.99999237060547
4 229.05747985839844,7.460695743560791,50.00000762939453
5 227.89329528808594,11.04373550415039,50.00000762939453
6 226.28921508789062,14.452611923217773,49.99999237060547
7 224.27049255371094,17.63355827331543,49.99999237060547
8 221.8690643310547,20.536415100097656,50.0
9 219.12271118164062,23.11539649963379,49.99998474121094
10 216.07479858398438,25.329837799072266,50.00000762939453
11 212.77334594726562,27.14480972290039,50.00001525878906
12 209.27049255371094,28.531696319580078,49.99999237060547
13 205.62144470214844,29.468618392944336,49.99999237060547
14 201.88372802734375,29.94080352783203,49.99999237060547

```



```

15 198.11627197265625,29.9407958984375,50.00000762939453
16 194.37855529785156,29.46861457824707,49.99999237060547
17 190.72946166992188,28.531692504882812,49.99999237060547
18 187.2266082763672,27.144805908203125,49.99998474121094
19 183.92518615722656,25.329835891723633,50.0
20 180.87725830078125,23.115394592285156,50.00000762939453
21 178.13092041015625,20.53641128540039,50.00000762939453
22 175.72950744628906,17.63355827331543,50.00001525878906
23 173.71078491210938,14.452609062194824,49.99999237060547
24 172.10671997070312,11.043736457824707,50.0
25 170.94248962402344,7.460696697235107,49.99999237060547
26 170.236572265625,3.7599966526031494,50.0
27 170.0,0.0,50.0
28 170.236572265625,-3.7599966526031494,50.0
29 170.94248962402344,-7.460696697235107,49.99999237060547
30 172.10671997070312,-11.043736457824707,50.0
31 173.71078491210938,-14.452609062194824,49.99999237060547
32 175.72950744628906,-17.63355827331543,50.00001525878906
33 178.13092041015625,-20.53641128540039,50.00000762939453
34 180.87725830078125,-23.115394592285156,50.00000762939453
35 183.92518615722656,-25.329835891723633,50.0
36 187.2266082763672,-27.144805908203125,49.99998474121094
37 190.72946166992188,-28.531692504882812,49.99999237060547
38 194.37855529785156,-29.46861457824707,49.99999237060547
39 198.11627197265625,-29.9407958984375,50.00000762939453
40 201.88372802734375,-29.94080352783203,49.99999237060547
41 205.62144470214844,-29.468618392944336,49.99999237060547
42 209.27049255371094,-28.531696319580078,49.99999237060547
43 212.77334594726562,-27.14480972290039,50.00001525878906
44 216.07479858398438,-25.329837799072266,50.00000762939453
45 219.12271118164062,-23.11539649963379,49.99998474121094
46 221.8690643310547,-20.536415100097656,50.0
47 224.27049255371094,-17.63355827331543,49.99999237060547
48 226.28921508789062,-14.452611923217773,49.99999237060547
49 227.89329528808594,-11.04373550415039,50.00000762939453
50 229.05747985839844,-7.460695743560791,50.00000762939453
51 229.76345825195312,-3.7599971294403076,49.99999237060547

```

Listing 12: robot_trajectory_run_9.csv

```

1 x,y,z
2 230.0,0.0,49.99999237060547
3 229.76345825195312,3.7599971294403076,49.99999237060547
4 229.05747985839844,7.460695743560791,50.00000762939453
5 227.89329528808594,11.04373550415039,50.00000762939453
6 226.28921508789062,14.452611923217773,49.99999237060547
7 224.27049255371094,17.63355827331543,49.99999237060547
8 221.8690643310547,20.536415100097656,50.0
9 219.12271118164062,23.11539649963379,49.99998474121094
10 216.07479858398438,25.329837799072266,50.00000762939453
11 212.77334594726562,27.14480972290039,50.00001525878906
12 209.27049255371094,28.531696319580078,49.99999237060547
13 205.62144470214844,29.468618392944336,49.99999237060547
14 201.88372802734375,29.94080352783203,49.99999237060547
15 198.11627197265625,29.9407958984375,50.00000762939453
16 194.37855529785156,29.46861457824707,49.99999237060547

```

```

17 190.72946166992188,28.531692504882812,49.99999237060547
18 187.2266082763672,27.144805908203125,49.99998474121094
19 183.92518615722656,25.329835891723633,50.0
20 180.87725830078125,23.115394592285156,50.00000762939453
21 178.13092041015625,20.53641128540039,50.00000762939453
22 175.72950744628906,17.63355827331543,50.00001525878906
23 173.71078491210938,14.452609062194824,49.99999237060547
24 172.10671997070312,11.043736457824707,50.0
25 170.94248962402344,7.460696697235107,49.99999237060547
26 170.236572265625,3.7599966526031494,50.0
27 170.0,0.0,50.0
28 170.236572265625,-3.7599966526031494,50.0
29 170.94248962402344,-7.460696697235107,49.99999237060547
30 172.10671997070312,-11.043736457824707,50.0
31 173.71078491210938,-14.452609062194824,49.99999237060547
32 175.72950744628906,-17.63355827331543,50.00001525878906
33 178.13092041015625,-20.53641128540039,50.00000762939453
34 180.87725830078125,-23.115394592285156,50.00000762939453
35 183.92518615722656,-25.329835891723633,50.0
36 187.2266082763672,-27.144805908203125,49.99998474121094
37 190.72946166992188,-28.531692504882812,49.99999237060547
38 194.37855529785156,-29.46861457824707,49.99999237060547
39 198.11627197265625,-29.9407958984375,50.00000762939453
40 201.88372802734375,-29.94080352783203,49.99999237060547
41 205.62144470214844,-29.468618392944336,49.99999237060547
42 209.27049255371094,-28.531696319580078,49.99999237060547
43 212.77334594726562,-27.14480972290039,50.00001525878906
44 216.07479858398438,-25.329837799072266,50.00000762939453
45 219.12271118164062,-23.11539649963379,49.99998474121094
46 221.8690643310547,-20.536415100097656,50.0
47 224.27049255371094,-17.63355827331543,49.99999237060547
48 226.28921508789062,-14.452611923217773,49.99999237060547
49 227.89329528808594,-11.04373550415039,50.00000762939453
50 229.05747985839844,-7.460695743560791,50.00000762939453
51 229.76345825195312,-3.7599971294403076,49.99999237060547

```

Listing 13: robot_trajectory_run_10.csv