Assignment 4

Question 1.
Given two sequences of integer numbers $r_1, r_2, \ldots, r_n$ and $s_1, s_2, \ldots, s_m$. Then $s_1, s_2, \ldots, s_m$ is called an *increasing subsequence* of $r_1, r_2, \ldots, r_n$ if there exists $1 \leq i_1 < i_2 < \ldots < i_m \leq n$ such that $s_j = r_{i_j}$ for all $1 \leq j \leq n,$ and $s_i < s_{j+1}$ for all $1 \leq j \leq n.$

Design an algorithm using dynamic programming that solves the following problem.

Input: Sequences of integers $s_1, s_2, \ldots, s_n$ and $t_1, t_2, \ldots, t_m$.
Output: A longest increasing common subsequence $r_1, r_2, \ldots, r_k$ of $s_1, s_2, \ldots, s_n$ and $t_1, t_2, \ldots, t_m,$ that is $r_1, r_2, \ldots, r_k$ is an increasing subsequence of $s_1, s_2, \ldots, s_n$ and $t_1, t_2, \ldots, t_m$, and there is no longer sequence that is increasing subsequence of $s_1, s_2, \ldots, s_n$ and $t_1, t_2, \ldots, t_m$.

Describe your algorithm in pseudocode.

Question 2.
The dynamic programming algorithm studied in class that finds a longest common subsequence for 2 strings (of length n and m, respectively) takes time $O(nm)$ and also space $O(nm)$. Adjust the algorithm such that its space complexity is $O(n+m)$ for the case that only the length of a longest common subsequence has to be computed (but not the sequence). Describe your idea clearly in words or pseudocode.

Question 3.
A variant of the Maximum Flow problem is as follows:

Max Flow with minimum capacities
Input: A directed graph $G = (V,E)$ with source s and sink t where each edge is given a minimum capacity c_{min} and a maximum capacity c_{max} with $0 \leq c_{min} < c_{max}$
Question: A maximum st-flow where for each edge e its flow capacity $f(e)$ is either zero or $c_{min}(e) < f(e) < c_{max}(e)$

Describe an algorithm that solves Max Flow with minimum capacities.