CSC 226 - Assignment 2

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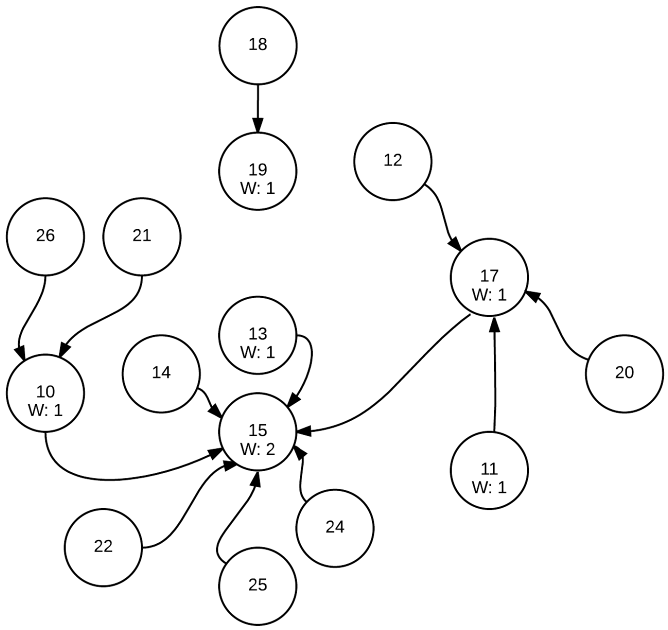
1 Euclidean Minimum Spanning Trees

Any of the minimum spanning tree algorithms can be used for this problem. Since performance is not a limiting concern, the simplest case is to build a set of edges from each point to each other, then perform Boruvka's algorithm.

```
Input: A set of points S with (X,Y) values.
Build a set of edges from each point to each other.
Give each edge weight based on distance between points.
Initialize a forest T to be a set of one-vertex trees
While T has more than one component:
  For each component C of T:
    Begin with an empty set of edges S
    For each vertex v in C:
      Find the cheapest edge(s) from v to a vertex
          outside of C
      If there is more than one cheapest edge:
        Choose the first in lexicographic order
      Add the edge to S
    Add the cheapest edge in S to T
Output: T, the euclidean minimum spanning tree of S.
```

2 Union-Find Data Structure



3 AVL Trees

3.1 Internal Nodes

We want the minimum, so let us examine the least balanced tree possible. Let $N(h)$ be the minimum number of nodes.

$$N(0) = 1$$

$$N(1) = 2$$

$$N(2) = 4$$

If can be observed that:

$$N(h) = 1 + N(h-1) + N(h-2)$$

This follows from the fact that a tree of height h must be composed of at least a root node, a tree of height $h-1$, and a tree of height $h-2$

Therefore, a tree of height 9 must have:

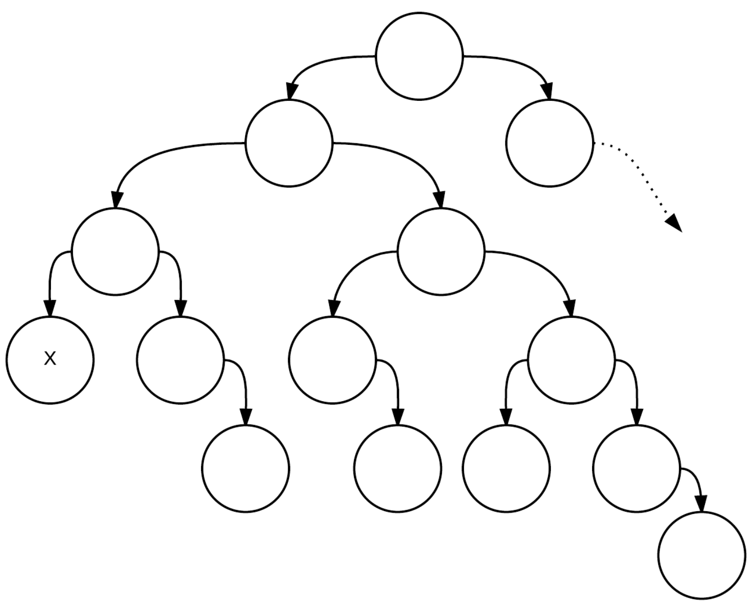
$$N(9) = 143$$

```
fn n(h: int) -> int {
    match h {
        h if h == 0 => 1,
        h if h == 1 => 2,
        h => 1 + n(h-1) + n(h-2)
    }
}

fn main() {
    println!("{}", n(9))
}
```

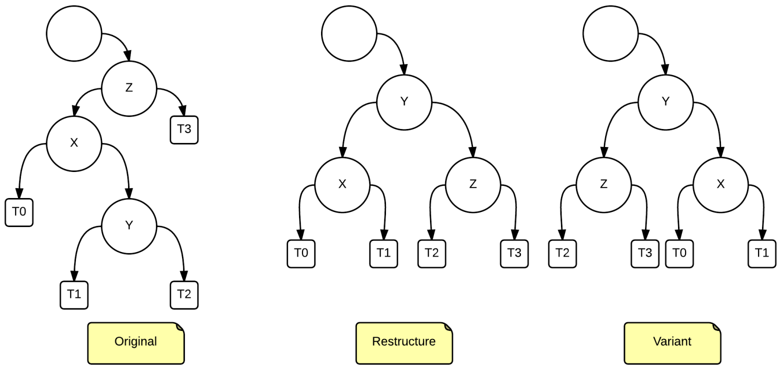
3.2 Number of Restructures

In a tree of height 9 would have a maximum of 9 restructures in the case where the tree had a structure like so:



In the case of removing 'x', each resulting restructure will require further rebalance all the way up the tree.

4 AVL Trees



The variant proposed fails to maintain the ordering of $z > y > x$, while the original restructure does. Because of this, the **in-ordering** of the AVL tree is violated. Since an AVL tree must be a valid Binary Search Tree, this is not a valid restructuring method.