Assignment 1 - CSC 467

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1 Application Needs

1.1 Voice

Compressed: Commonly 8kbps
Uncompressed: Commonly 64kbps

1.2 Video

Streaming 480p: 0.5 Mbps

1.3 HD TV

720p: 2.5 Mbps (Amazon)

1.4 Netflix

SD: 1.5 Mpbs
HD: 5.0 Mpbs

2 Erlang-B Derivation

$$p_N = \frac{A^N/N!}{\sum_{n=0}^{N} A^n/n!} \tag{1}$$

$$\frac{p_N}{p_{N-1}} = \frac{\frac{A^N/N!}{\sum_{n=0}^{N} A^n/n!}}{\frac{A^{N-1}/(N-1)!}{\sum_{n=0}^{N-1} A^n/n!}} = \frac{A^N/N!}{\sum_{n=0}^{N} A^n/n!} * \frac{\sum_{n=0}^{N-1} A^n/n!}{A^{N-1}/(N-1)!}$$

$$= \frac{\frac{A^N}{N!}}{\frac{A^{N-1}}{(N-1)}} * \frac{\sum_{n=0}^{N-1} A^n/n!}{\sum_{n=0}^{N} A^n/n!} = \frac{A}{N} * \frac{\sum_{n=0}^{N-1} \frac{A^n}{n!} + \frac{A^N}{N!} - \frac{A^N}{N!}}{\sum_{n=0}^{N} \frac{A^n}{n!}}$$

$$= \frac{A}{N} * (1 - \frac{\frac{A^N}{N!}}{\sum_{n=0}^{N} \frac{A^n}{n!}})$$

Since $p_N = \frac{A^N/N!}{\sum_{n=0}^{N} A^n/n!}$:

$$\frac{p_n}{p_{n-1}} = \frac{A}{N} * (1 - p_n) \tag{2}$$

$$p_n = \frac{A}{N} p_{n-1}(1 - p_n)$$

$$p_n N = A p_{n-1} - A p_n p_{n-1}$$

$$p_n(N + A p_{n-1}) = A p_{n-1}$$

$$p_n = \frac{A p_{n-1}}{N + A p_{n-1}}$$

3 Probablility of Call Blocking

Computing, where $A = 5$, $n = 5$:

$$p_i = \frac{A p_{i-1}}{i + A p_{i-1}}; i = 1...N; p_0 = 1$$

$$p_5 = \frac{5p_4}{5 + 5p_4} = \frac{5\frac{625}{1569}}{5 + 5\frac{625}{1569}} = \frac{625}{2194}$$

$$p_4 = \frac{5p_3}{4 + 5p_3} = \frac{5\frac{125}{236}}{4 + 5\frac{125}{236}} = \frac{625}{1569}$$

$$p_3 = \frac{5p_2}{3 + 5p_2} = \frac{5\frac{25}{37}}{3 + 5\frac{25}{37}} = \frac{125}{236}$$

$$p_2 = \frac{5p_1}{2+5p_1} = \frac{5\frac{5}{6}}{2+5\frac{5}{6}} = \frac{25}{37}$$

$$p_1 = \frac{5*1}{1+5*1} = \frac{5}{6}$$

The probability of blocking is $\frac{11}{38}$.
Asumming the average call holding time is $\frac{1}{\mu} = 3$ minutes the call arrival rate, λ, is:

$$A = \frac{\lambda}{\mu}$$

$$5 = \frac{\lambda}{\frac{1}{3}}$$

$$\lambda = \frac{5}{3}$$

4 Propagation

4.1 $R * d_{prop}$

$$d_{prop} = \frac{RoundTrip}{PropagationSpeed} = \frac{2.0*10^7}{2.0*10^8} = \frac{1}{10}$$

$$R * d_{prop} = (1*10^6) * \frac{1}{10} = 10^5$$

4.2 One Large File

Time taken to send 400,000 bits along a 1 Mbps link:

$$\frac{4*10^5}{10^6} = \frac{4}{10} = 0.4s$$

Time taken for a bit to transfer from A to B:

$$\frac{10^7}{2*10^8} = \frac{1}{20} = 0.05s$$

Since $\frac{0.05}{0.4} = 0.125$.

$$4*10^5*0.125 = 50,000s$$

4.3 Interpretation

The Bandwidth-Delay product is the maximum amount of data on the network circuit at any given time.

5 Queuing

5.1 Circuit Switching

If circuit switching is used on a 2 Mbps line, and each user requires 500 Kbps, only 4 users can be supported at any given time. More users could be supported if it was gauranteed that no more than 4 users would be utilizing the service at a given time.

5.2 Packet Switching

The queuing delay experienced by two users utilizing the link at 500 Kbps will be trivial because there will be no need to buffer load. $user_1 + user_2 = 1$ Mbps while the outgoing link is 2 Mbps.

There will be a delay if five users are simutaneously transmitting, as $\sum_{i=1}^{5} user_i =$ 2.5 Mbps because the link is only able to output 2 Mbps. This means the outgoing link will be saturated, leaving the packets it's not able to service immediately in the buffer, causing delay.

6 Circuit Switched Capacity

6.1 Maximum Usage

The maximum number of connections on the provided topology is $4n$ since each link can support n circuits and there are 4 links which could be utilizing independently.

6.2 Maximum Usage between A and B

Since there are a total of two paths from A to B, both of which are independent from each other, there may be as many as $2n$ simultaneous connections between A and B.

7 Delays In An Internet Router

7.1 Maximum Packet Rate

$$1Gbps = 10^9 \frac{bits}{second}$$

$$10,000 = 10^4 \frac{bits}{packet}$$

$$\frac{10^9}{10^4} = 10^5 \frac{packets}{second} = \mu$$

7.2 Total Packet Arrival Rate to the Router

$$\lambda = 10r$$

7.3 Maximum Value of r

The maximum value for r to have a stable router:

$$\frac{10^5}{10r} = 1$$

$$r = 10^4$$

7.4 Routing Queuing Delay

$$d_{queue} = \frac{1}{\mu - \lambda}$$

Where $\rho = \frac{\lambda}{\mu}$. Since $\mu = 10^5$ from 7.1:

$$\rho = 0.1 = \frac{\lambda}{10^5}, \lambda = 10^4, d_{queue} = \frac{1}{10^5 - 10^4} = \frac{1}{9 * 10^4}$$

$$\rho = 0.2 = \frac{\lambda}{10^5}, \lambda = 2 * 10^4 =, d_{queue} = \frac{1}{10^5 - 2 * 10^4} = \frac{1}{8 * 10^4}$$

$$\rho = 0.4 = \frac{\lambda}{10^5}, \lambda = 4 * 10^4, d_{queue} = \frac{1}{10^5 - 4 * 10^4} = \frac{1}{6 * 10^4}$$

$$\rho = 0.6 = \frac{\lambda}{10^5}, \lambda = 6 * 10^4, d_{queue} = \frac{1}{10^5 - 6 * 10^4} = \frac{1}{4 * 10^4}$$

$$\rho = 0.8 = \frac{\lambda}{10^5}, \lambda = 8 * 10^4, d_{queue} = \frac{1}{10^5 - 8 * 10^4} = \frac{1}{2 * 10^4}$$

7.5 Other Delays

$$d_{nodal} = d_{proc} + d_{queue} + d_{trans} + d_{prop}$$

Where:

$$d_{proc} = 10^{-4} s$$

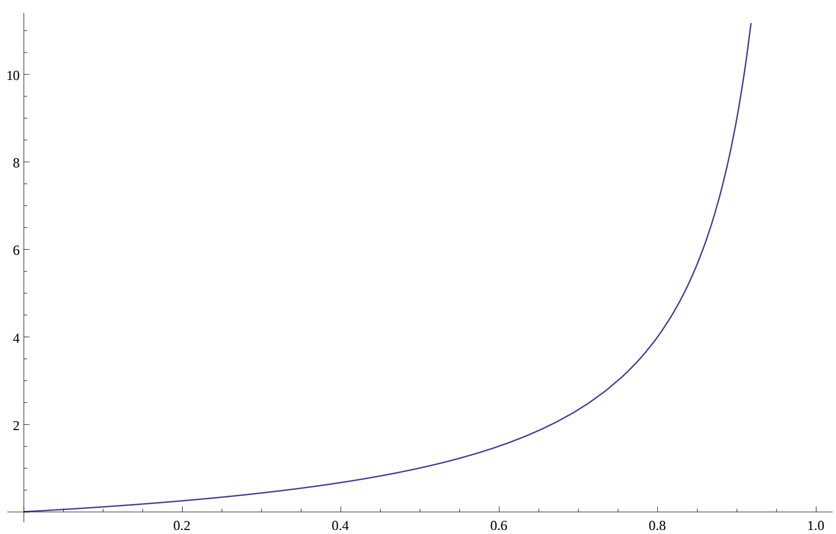
$$d_{queue} = L = \frac{\rho}{1 - \rho}$$

$$d_{trans} = \frac{L}{R} = \frac{10^5}{10^9} = 10^{-4} s$$

$$d_{prop} = \frac{10^6}{2 * 10^8} = 5 * 10^{-3} s$$

Thus:

$$d_{nodal}(\rho) = 10^{-4} + \frac{\rho}{1 - \rho} + 10^{-4} + 5 * 10^{-3} s$$



8 Average Buffer Occupancy

8.1 8 sec

$$L = \frac{\lambda}{u - \lambda}$$

Where $\lambda = \frac{10}{8} = 1.25$, $\mu = \frac{2400}{960} = 2.5$.

$$L = \frac{1.25}{2.5 - 1.25} = 1$$

There is on average 1 960 bit packet in the buffer.

8.2 5 sec.

Where $\lambda = \frac{10}{5} = 2$, $\mu = \frac{2400}{960} = 2.5$.

$$L = \frac{2}{2.5 - 2} = 4$$

There is on average 4 960 bit packets in the buffer.

8.3 16 terminals

Where $\lambda = \frac{16}{8} = 2$, $\mu = \frac{2400}{960} = 2.5$.

$$L = \frac{2}{2.5 - 2} = 4$$

There is on average 4 960 bit packets in the buffer.

8.4 40 terminals, bigger pipes, bigger packets

Where $\lambda = \frac{40}{8} = 5$, $\mu = \frac{9600}{1600} = 6$

$$L\frac{5}{6 - 5} = 5$$

There is on average 5 1600 bit packets in the buffer.

9 File Transfer

9.1 Expected Throughput

The expected throughput is the minimum of the three links, $min(\{R1, R2, R3\}) = R1 = 500Kb/s$.

9.2 Transfer Time

$$4,000,000byte = 3.2 * 10^7 bit$$

$$\frac{3.2 * 10^7}{4 * 10^6} = 8s$$

Therefore the file transfer will take approximately 8 seconds.

9.3 Other Flows

This question lacks details required to appropriately reason about the answer. It's possible that the flow from C to D is prioritized by a QoS implementation and thus recieves a considerable portion of the bandwidth, or that traffic from A to B is prioritized. Furthermore, this implementation could differ at different routers.

Assuming all of the routers use simple bandwidth division (Allowing both transfers equal bandwidth) the throughput would be roughly $\frac{500}{2} = 250Kb/s$.

9.4 R2 Reduced

The expected throughput is now$min(\{R1, R2, R3\}) = R2 = 100Kb/s$, while the transfer time is $\frac{3.2*10^7}{8*10^5} = 40s$.

10 Routers in Series

10.1 ρ_1

$$1Mbyte = 1,000,000bytes$$

Noting all packets are $1000byes$, each router can handle $\frac{1,000,000}{1000} = 1000$packets per second.

$$\rho_1 = \frac{\lambda_1}{\mu_1} = \frac{350}{1000} = \frac{35}{100} = 35\%$$

10.2 ρ_2

$$\rho_2 = \frac{\lambda_2}{\mu_2} = \frac{450 + 350}{1000} = \frac{800}{1000} = 80\%$$

10.3 Delay of Traffic 1

$$w = \frac{1}{\mu - \lambda} = \frac{1}{\mu(1-\rho)}$$

At router 1:

$$w_1 = \frac{1}{\mu_1 - \lambda_1} = \frac{1}{1000 - 350} = \frac{1}{650}$$

Therefore the delay at R1 will be $\frac{1}{650}$ of a second.
At router 2:

$$w_2 = \frac{1}{\mu_2 - \lambda_2} = \frac{1}{1000 - 800} = \frac{1}{200}$$

Therefore the delay at R2 will be $\frac{1}{200}$of a second.
For end-to-end:

$$w_1 + w_2 = \frac{1}{650} + \frac{1}{200} = \frac{17}{2600} s \approx 6.5ms$$

10.4 Delay of Traffic 2

$$w_2 = \frac{1}{\mu_2 - \lambda_2} = \frac{1}{1000 - 800} = \frac{1}{200}$$

It should be $\frac{1}{200}$ assuming the router does not discriminate between traffics 1 and 2.

10.5 How many Packets

$$L_1 = \frac{\lambda}{\mu - \lambda} = \frac{350}{1000 - 350} = \frac{7}{13} \approx 0.5$$

There is generally 1 or less packet in Router 1.

$$L_2 = \frac{\lambda_2}{\mu_2 - \lambda_2} = \frac{800}{1000 - 800} = \frac{800}{200} = 4$$

There is generally 4 packets in router 2.