Assignment 2

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1 On-Off Model

What is the average data rate?

Assuming a period of 20 seconds, the source will be on for 5 seconds, then off for 15 seconds. The total amount of data sent will be $5s * \frac{1}{2}(1.5 * 10^6 \frac{b}{s})$.

$$Average = \frac{5s * \frac{1}{2}(1.5 * 10^6 \frac{b}{s})}{20s} = 187,500\frac{b}{s}$$

Therefore the average data rate of the source is $187,500\frac{bits}{second}$

How many packets are sent by this source when it is on?

Each burst is 5 seconds, each packet is 2000 bits, and the data is sent at a rate of $\frac{1}{2}(1.5 * 10^6 \frac{b}{s})$

$$Burst = \frac{5 * \frac{1}{2}(1.5 * 10^6 \frac{b}{s})}{2000\frac{b}{packet}} = 1875 packets$$

Therefore a burst contains 1875 packets of 2000 bits each.

2 Scheduling

2.1 WRR Cycle Size

An adequate frame size is 10, since all packets are of unit weight (1). $1 + 2 + 3 + 4 = 10$.

$$\{A, B, B, C, C, C, D, D, D, D\}$$

This is chosen because it is the simplest implementation. A less bursty rotation which acts more similar to WFQ is:

$$\{C, A, B, D, D, C, B, D, C, D\}$$

2.2 WRR Bandwidth

All packets are of unit weight (1). Flow A recieves $\frac{1}{10}$ of the link rate. Flow B recieves $\frac{2}{10}$ of the link rate. Flow C recieves $\frac{3}{10}$ of the link rate, while Flow D recieves $\frac{4}{10}$ of the link rate.

2.3 WFQ Transmission Sequence

Using $\frac{1}{r_i}$ to determine the weights, we assign $w_A = 10, w_B = 5, w_C = 3.3, w_D = 2.5$. All queues start at finishing time 0 since all packets have already arrived, no queues start empty.

The tables below represent the sent packet of a given round, as well as the finishing times of all queues $A...D$.

R1	0	1	2	3	4	5	6	7	8	9
A	10	10	10	10	10	10	10	10	10	10
B	0	5	5	5	5	5	10	10	10	10
C	0	0	3.3	3.3	3.3	6.6	6.6	6.6	9.9	9.9
D	0	0	0	2.5	5	5	5	7.5	7.5	10
Sent:	A	B	C	D	D	C	B	D	C	D

R2	10	11	12	13	14	15	16	17	18	19
A	10	20	20	20	20	20	20	20	20	20
B	10	10	15	15	15	15	20	20	20	20
C	13.2	13.2	13.2	13.2	13.2	16.5	16.5	16.5	19.8	19.8
D	10	10	10	12.5	15	15	15	17.5	17.5	20
Sent:	C	A	B	D	D	C	B	D	C	D

R3	20	21	22	23	24	25	26	27	28	29
A	20	30	30	30	30	30	30	30	30	30
B	20	20	25	25	25	25	30	30	30	30
C	23.1	23.1	23.1	23.1	23.1	26.4	26.4	26.4	29.7	29.7
D	20	20	20	22.5	25	25	25	27.5	27.5	30
Sent:	C	A	B	D	D	C	B	D	C	D

R4	30	31	32	33	34	35	36	37	38	39
A	30	40	40	40	40	40	40	40	40	40
B	30	30	35	35	35	35	40	40	40	40
C	33	33	33	33	33	36.3	36.3	36.3	39.6	39.6
D	30	30	30	32.5	35	35	35	37.5	37.5	40
Sent:	C	A	B	D	D	C	B	D	C	D

The packets are correctly transmitted in accordance with their weights. See below.

2.4 WFQ Bandwidth

$$sent_A = \frac{4}{40} = \frac{1}{10}$$
$$sent_B = \frac{8}{40} = \frac{2}{10}$$
$$sent_C = \frac{12}{40} = \frac{3}{10}$$
$$sent_D = \frac{16}{40} = \frac{4}{10}$$

2.5 Comparison

Packets get bursty in the WRR scheduler when they are of high weight, starving the other connections. The WFQ schedule attempts control bursty-ness by transmitting packets more evenly.

Delay in the simple WRR schedule can become high with many connections because even a high weight connection may need to wait an entire poll cycle before getting to re-transmit. The WFQ scheduler limits delay by allowing other queues to transmit at regular intervals.

2.6 Mimicing the Order

The WFQ schedule can be mimiced by rearranging the packet transmission order into:

$$\{C, A, B, D, D, C, B, D, C, D\}$$

This could be achieved by determining the finishing times of one "round" of WFQ and distributing the transmission times in WRR appropriately.

2.7 Variable Packet Sizes

Using $\frac{1}{r_i}$ to determine the weights, we assign $w_A = 10, w_B = 5, w_C = 3.3, w_D = 2.5$. All queues start at finishing time 0 since all packets have already arrived, no queues start empty. Finishing times are computed via $t_i = t_{i-1} + w_f * s_f$ where $\{s_1 = 50, s_2 = 100, s_3 = 500, s_4 = 500\}$

The tables below represent the sent packet of a given round, as well as the finishing times of all queues $A...D$.

R1	0	1	2	3	4	5	6	7	8	9
A	500	500	500	500	1000	1000	1500	1500	1500	2000
B	0	500	500	500	500	1000	1000	1500	1500	1500
C	0	0	1650	1650	1650	1650	1650	1650	1650	1650
D	0	0	0	1250	1250	1250	1250	1250	2500	2500
Sent:	A	B	C	D	A	B	A	B	D	A

R2	10	11	12	13	14	15	16	17	18	19
A	2000	2000	2500	2500	2500	3000	3000	3500	3500	3500
B	2000	2000	2000	2500	2500	2500	3000	3000	3500	3500
C	1650	3300	3300	3300	3300	3300	3300	3300	3300	4950
D	2500	2500	2500	2500	3750	3750	3750	3750	3750	3750
Sent:	B	C	A	B	D	A	B	A	B	C

R3	20	21	22	23	24	25	26	27	28	29
A	4000	4000	4000	4500	4500	5000	5000	5000	5500	5500
B	3500	4000	4000	4000	4500	4500	5000	5000	5000	5500
C	4950	4950	4950	4950	4950	4950	4950	6600	6600	6600
D	3750	3750	5000	5000	5000	5000	5000	5000	5000	5000
Sent:	A	B	D	A	B	A	B	C	A	B

R4	30	31	32	33	34	35	36	37	38	39
A	5500	6000	6000	6500	6500	6500	7000	7000	7000	7500
B	5500	5500	6000	6000	6500	6500	6500	7000	7000	7000
C	6600	6600	6600	6600	6600	6600	6600	6600	8250	8250
D	6250	6250	6250	6250	6250	7500	7500	7500	7500	7500
Sent:	D	A	B	A	B	D	A	B	C	A

The packets are correctly transmitted in accordance with their weights. See below.

2.8 WFQ Variable Packet Bandwidth

$$bandwidth_A = \frac{15}{40} * 50 = 18.75 \rightarrow \frac{18.75}{(18.75 + 35 + 62.5 + 75)} \approx 9.8\%$$

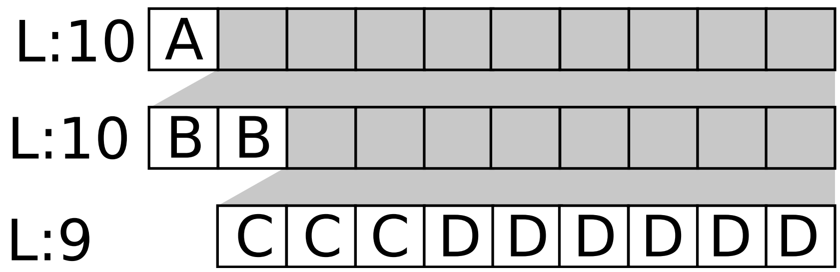
$$sent_B = \frac{14}{40} * 100 = 35 \rightarrow \frac{35}{191.25} \approx 18.3\%$$

$$sent_C = \frac{5}{40} * 500 = 62.5 \rightarrow \frac{62.5}{191.25} \approx 32.7\%$$

$$sent_D = \frac{6}{40} * 500 = 75 \rightarrow \frac{75}{191.25} \approx 39.2\%$$

3 Heirarchical Scheduling

The following heirarchy results:



The schedule produced has the following first two frames of length 10.

$$F_{1,2} = \{A, B, B, C, C, C, D, D, D, D\}, \{A, D, B, B, D, C, C, C, D, D\}$$

4 Self Clock Fair Queuing

The flow table is as follows, starting at t_o until t_{17} and the values of the columns are the finishing times of the queues.

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
A	10	10								20								30
B	0	15								15								15
	A	B								A								A

5 Kleinrock's Conservation Theorem

$$\rho_{1a}w_{1a} + \rho_{1b}w_{1b} = \rho_{2a}w_{2a} + \rho_{2b}w_{2b}$$

$$(0.2)(0.3) + (0.4)(0.4) = (0.2)(0.1) + (0.4)(x)$$

$$\frac{(0.2)(0.3) + (0.4)(0.4) - (0.2)(0.1)}{0.4} = x = 0.5s$$

The mean delay of the other connection will change to 0.5s.

6 Max-min Allocation

Setting $C = 100Mb/s$, $n = 6$:

$$R(f_1) = 1Mb \qquad (N = 5, C = 99)$$

$$R(f_2) = 2Mb \qquad (N = 4, C = 97)$$

$$R(f_3) = 10Mb \qquad (N = 3, C = 87)$$

$$R(f_4) = 20Mb \qquad (N = 2, C = 67)$$

$$R(f_5) = 33.5Mb \qquad (N = 1, C = 33.5)$$

$$R(f_6) = 33.5Mb \qquad (N = 0, C = 0)$$

7 Weighted Max-min allocation

A flow with a weight $w > 1$ can be considered w different flows. First, normalize all the weights such that the lowest weight is 1. Allow n to equal the sum of all weights, since they can be seen as many different flows. Then, use an iterative allocation process:

1. Determine $u = \frac{C}{n}$. u is the Fair Share Unit.
2. Allow each flow to consume up to w_i fair share units. For example, if $u = 2, w_1 = 1, w_2 = 2, C = 6$ then $R(f_1) = 2, R(f_2) = 4$.
3. Decrement C by the amount allocated and decrement N by w_i.
4. Iterate using any residual C along with any unsatisfied flows. n will be the sum of the weights of the unsatisfied flows.

Example: Q6

Iteration 1

Setting $C = 100Mbit/s$, $w_{\{1,\ldots,6\}} = \{1, 1, 2, 2, 4, 10\}$, $n = \sum w = 20$.
The Fair Share Unit is $u = \frac{C}{n} = 5\frac{Mb}{s}$.

$$R(f_1) = 1\frac{Mb}{s}$$

$$R(f_2) = 2\frac{Mb}{s}$$

$$R(f_3) = 2 * 5 = 10\frac{Mb}{s}$$

$$R(f_4) = 2 * 5Mb = 10\frac{Mb}{s} \qquad (Unsatisfied)$$

$$R(f_5) = 4 * 5Mb = 20\frac{Mb}{s} \qquad (Unsatisfied)$$

$$R(f_6) = 10 * 5Mb = 50\frac{Mb}{s} \qquad (Unsatisfied)$$

There is $C = 7\frac{Mb}{s}$ residual.

Iteration 2

Setting $C = 7\frac{Mb}{s}$, $w_{\{4,5,6\}} = 1, 2, 5$, $n = 7$.
The Fair Share Unit is $u = \frac{C}{n} = 1\frac{Mb}{s}$.

$$R_2(F_4) = 1\frac{Mb}{s}$$

$$R_2(F_5) = 2 * 1\frac{Mb}{s} = 2\frac{Mb}{s}$$

$$R_3(F_6) = 5 * 1\frac{Mb}{s} = 5\frac{Mb}{s}$$

Resulting in the final allocations of:

$$R(f_1) = 1\frac{Mb}{s}$$

$$R(f_2) = 2\frac{Mb}{s}$$

$$R(f_3) = 10\frac{Mb}{s}$$

$$R(f_4) = 11\frac{Mb}{s}$$

$$R(f_5) = 22\frac{Mb}{s}$$

$$R(f_6) = 55\frac{Mb}{s}$$

8 Delays Experienced by Flows

8.1 Packet Service Time for Class 1

$\mu_1 = \frac{1Mb}{500b} = 2000$

$$\frac{1}{\mu_1} = \frac{1}{2000}s$$

8.2 Packet Service Time for Class 2

$\mu_1 = \frac{1Mb}{1000b} = 1000$

$$\frac{1}{\mu_2} = \frac{1}{1000}s$$

8.3 Class 1 at 25, Class 2 at 50

Setting $\rho_1 = \frac{\lambda_1}{\mu_1} = \frac{25}{2000}, \rho_2 = \frac{50}{1000}$. $E\{R\} = \frac{25(\frac{2}{2000^2})+50(\frac{2}{1000^2})}{2} = 0.00005625$.
Expected wait times:

$$E(\{w_1\} = \frac{E(R)}{1 - \frac{25}{2000}} = 0.00005696s$$

$$E\{w_2\} = \frac{E(R)}{1 - \frac{50}{1000}} = 0.00005921s$$

Expected number in queue:

$$E\{n_1\} = \lambda_1 E\{w_1\} = 25 * 0.00005696 = 0.001424$$

$$E\{n_2\} = \lambda_2 E\{w_2\} = 50 * 0.00005921 = 0.002961$$

Utilization:

$$\rho_1 + \rho_2 = \frac{125}{2000} = \frac{1}{16}$$

8.4 Class 1 at 50, Class 2 at 50

Setting $\rho_1 = \frac{\lambda_1}{\mu_1} = \frac{50}{2000}, \rho_2 = \frac{50}{1000}$. $E\{R\} = \frac{50(\frac{2}{2000^2})+50(\frac{2}{1000^2})}{2} = 0.0000625$. Expected wait times:

$$E(\{w_1\} = \frac{E(R)}{1-\frac{50}{2000}} = 0.00006410s$$

$$E\{w_2\} = \frac{E(R)}{1-\frac{50}{1000}} = 0.00006579s$$

Expected number in queue:

$$E\{n_1\} = \lambda_1 E\{w_1\} = 50 * 0.00006410 = 0.003205$$

$$E\{n_2\} = \lambda_2 E\{w_2\} = 50 * 0.00006579 = 0.003290$$

Utilization:

$$\rho_1 + \rho_2 = \frac{150}{2000} = \frac{3}{40}$$

8.5 Class 1 at 100, Class 2 at 50

Setting $\rho_1 = \frac{\lambda_1}{\mu_1} = \frac{100}{2000}, \rho_2 = \frac{50}{1000}$. $E\{R\} = \frac{100(\frac{2}{2000^2})+50(\frac{2}{1000^2})}{2} = 0.000075$. Expected wait times:

$$E(\{w_1\} = \frac{E(R)}{1-\frac{100}{2000}} = 0.00007895s$$

$$E\{w_2\} = \frac{E(R)}{1-\frac{50}{1000}} = 0.00007895s$$

Expected number in queue:

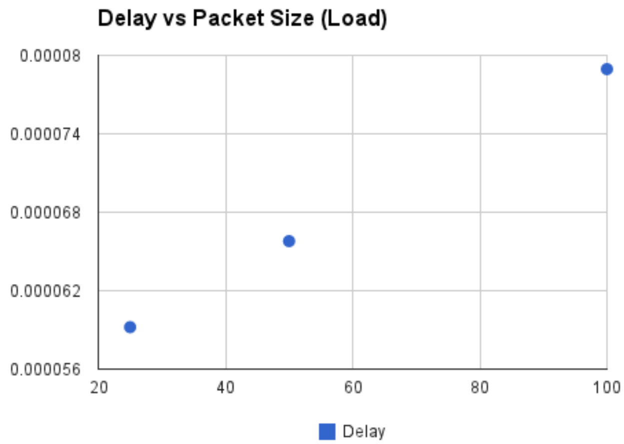
$$E\{n_1\} = \lambda_1 E\{w_1\} = 100 * 0.00007895 = 0.007895$$

$$E\{n_2\} = \lambda_2 E\{w_2\} = 50 * 0.00007895 = 0.0039475$$

Utilization:

$$\rho_1 + \rho_2 = \frac{200}{2000} = \frac{1}{10}$$

8.6 Change in Class 1 leads to...



$$\frac{\delta}{\delta c_2}\left(\frac{\frac{c_1(\frac{2}{2000^2})+c_2(\frac{2}{1000^2})}{2}}{1-\frac{c_2}{1000}}\right)=\frac{500c_1(\frac{1}{2000000})+1}{(c_2-1000)^2}$$

For example, if $c_2 = 50$, $c_1 = \frac{500(1)(\frac{1}{2000000})+1}{(50-1000)^2} = \frac{4001}{3610000000}s$.