Modeling hw2

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Problems

备用件数	0	0.5	0.6	0.7
	1	0.6	0.75	0.9
	2	0.7	0.95	1.0
	3	0.8	1.0	1.0
	4	0.9	1.0	1.0
	5	1.0	1.0	1.0
单件价格/元		20	30	40
单件质量/kg		2	4	6

某公司利用钢材和铝材作为原材料,生产两种产品(A 和 B).单件产品 A 需消耗钢材 6 kg,铝材 8 kg,劳动力 11 h,利润 5 000 元(不含工人加班费);单件产品 B 需消耗钢材12 kg,铝材 20 kg,劳动力 24 h,利润 11 000 元(不含工人加班费).该企业目前可提供钢材200 kg,铝材 300 kg,劳动力 300 h. 如果要求工人加班,每小时加班费 100 元.请制订生产计划,最大化公司的利润和最小化工人加班时间.

12. 某银行营业部设立 3 个服务窗口,分别为个人业务、公司业务和特殊业务(如外汇和理财等).现有 3 名服务人员,每人处理不同业务的效率(每天服务的最大顾客数),以及每人处理不同业务的质量(如顾客的满意度)见下表.如何为服务人员安排相应的工作(服务窗口)?

最大顾客数

|	个人业务	公司业务	特殊业务

Model Establishing

Let x: = the No. of product A, and y: = the No. of product B.
It's not difficult to obtain following model by observing the problem.

```
Max(profit) & Min(overtime);
    profit = 5000*x + 11000*y - 100*overtime,
    6*x + 12*y <= 200,
    8*x + 20*y <= 300,
    11*x + 24*y - 300 = overtime.
    (Of course, x, y, & overtime >= 0.
     Moreover, x & y are integers, which results
     in overtime also being an integer.)
```

Drawing the Graph

See & play around it online at https://www.geogebra.org/m/b6w8yztr.
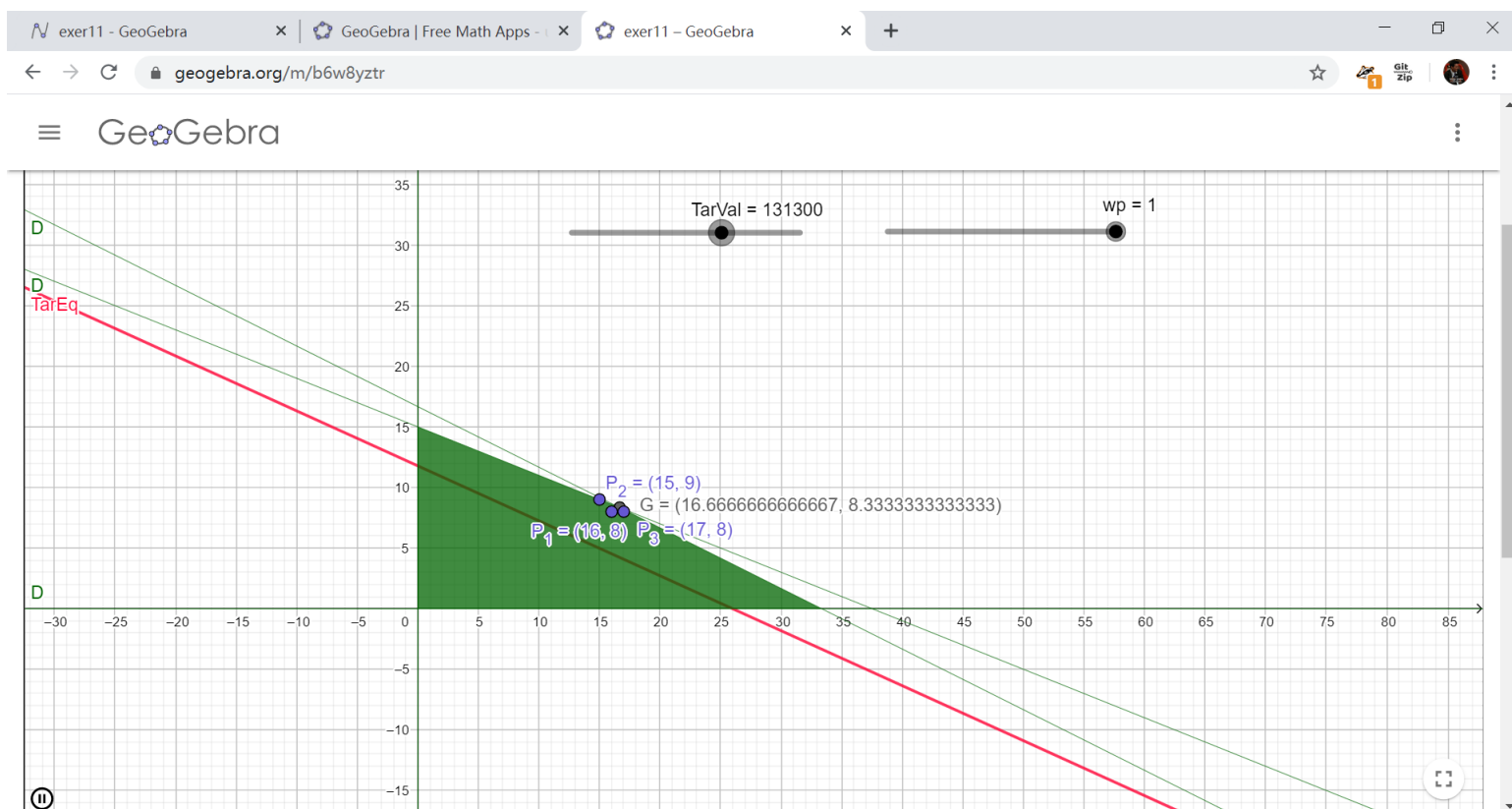
Solve by Lingo

o   See Lingo code as follows

```
model:
! A greedy boss wants maximum profit & minimum overtime for employees;
max = z;
  !@param x: the No. of product A
   @param y: the No. of product B;

  wp = 0.9;    ! weight of profit. This is decided by the boss;
  wt = 1 - wp; ! weight of overtime;
  z = profit * wp + (-overtime) * wt; ! Target optimal (maximum) value;

  profit = 5000*x + 11000*y - 100*overtime;
  [Steel] 6*x + 12*y <= 200;
  [Aluminum] 8*x + 20*y <= 300;
  [TotalHours] 11*x + 24*y <= 300 + overtime;

  ! declare variable x & y as integer only;
  @gin(x); @gin(y);

end
```

o   Output


```
Global optimal solution found.
Objective value:                    149301.9
Objective bound:                    149301.9
Infeasibilities:                    0.000000
Extended solver steps:                     0
Total solver iterations:                   3
Elapsed runtime seconds:                0.06

Model Class:                            MILP

Total variables:              5
Nonlinear variables:          0
Integer variables:            2

Total constraints:            6
Nonlinear constraints:        0

Total nonzeros:              15
Nonlinear nonzeros:           0


          Variable           Value        Reduced Cost
                 Z        149301.9            0.000000
                WP       0.9000000            0.000000
                WT       0.1000000            0.000000
            PROFIT        165900.0            0.000000
          OVERTIME        81.00000            0.000000
                 X        15.00000           -3508.900
                 Y        9.000000           -7737.600

               Row    Slack or Surplus        Dual Price
                 1        149301.9            1.000000
                 2        0.000000            165981.0
                 3        0.000000           -81.00000
                 4        0.000000            1.000000
                 5        0.000000           0.9000000
             STEEL        2.000000            0.000000
          ALUMINUM        0.000000            0.000000
        TOTALHOURS        0.000000            90.10000
```
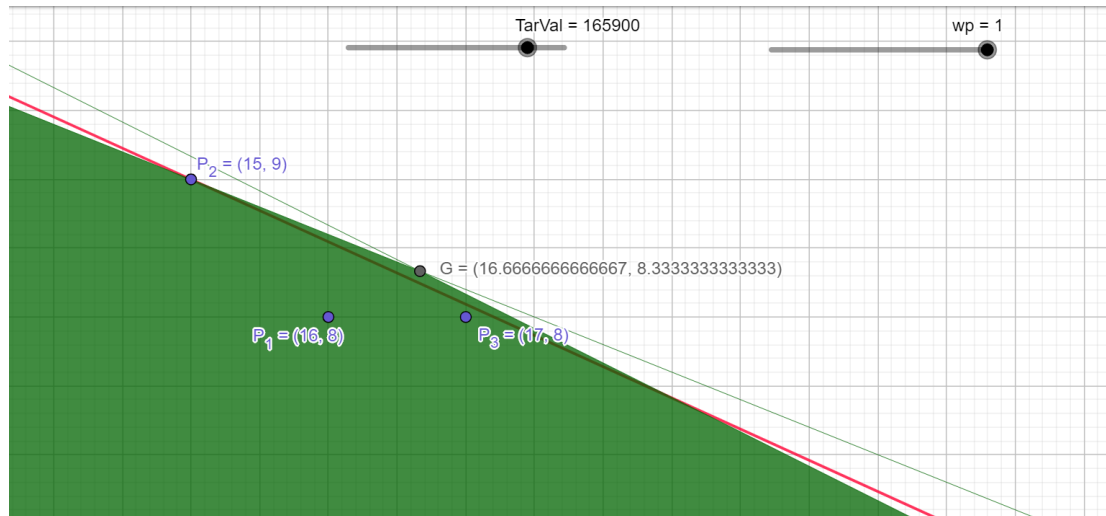
Result & Analysis

Zooming in we shall find when $wp = 1$, Target Equation (red line) will cross point $P_2(15,9)$ as the last integer point in domain D (green area). Moreover, if we try to change wp (simple to do, just to drag wp in the slider), and the result remains unchanged. Let's delve a little bit into the details about why this happens.
By the model,

$$profit = 5000x + 11000y - 100 * overtime$$
$$= 5000x + 11000y - 100(11x + 24y - 300)$$
$$= 3900x + 8600y + 30000,$$

$$z = profit * wp + (-overtime) * (1 - wp)$$
$$= \underbrace{(3900x + 8600y + 30000) * wp}_{\text{weight of profit}} - \underbrace{(11x + 24y - 300) * (1 - wp)}_{\text{weight of overtime}}. \quad \text{(TarEq)}$$

Obviously, even when $wp = 0.1, 1 - wp = 0.9$, the slope of equation (TarEq) changes little, since the profit weight part has a magnitude of $10^2 \sim 10^3$ and the overtime weight part has a magnitude of at most 10^1, concerning the coefficients of x & y.

Furthermore, if we try to change the wp in the Lingo program, we can get the same result as that of $wp = 1$.

So, the final solution is to **produce 15 pieces of product A, and 9 pieces of product B, with 81 hours of overtime, generating maximum profit ¥165900, and leaving 2kg of rolled steel, regardless of the wight of profit given by the boss.**
(see also the Lingo solution report as follows)

```
    Variable            Value        Reduced Cost
           Z          149301.9          0.000000
          WP         0.9000000          0.000000
          WT         0.1000000          0.000000
      PROFIT          165900.0          0.000000
    OVERTIME          81.00000          0.000000
           X          15.00000         -3508.900
           Y          9.000000         -7737.600

         Row   Slack or Surplus        Dual Price
           1          149301.9          1.000000
           2         0.000000          165981.0
           3         0.000000         -81.00000
           4         0.000000          1.000000
           5         0.000000         0.9000000
       STEEL          2.000000          0.000000
    ALUMINUM         0.000000          0.000000
  TOTALHOURS         0.000000          90.10000
```

Lingo solution report