how U doin'?

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Modeling hw4*

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1 Problems

够生存的现象[24].

7. 据报道,某种山猫在较好、中等及较差的自然环境下,年平均增长率分别为 1.68%, 0.55%和 −4.50%,假定开始时有 100 只山猫,按以下情况讨论山猫数量逐年变化过程及趋势:

(1) 3 种自然环境下 25 年的变化过程(作图).

(2) 如果每年捕获 3 只,会发生什么情况? 山猫会灭绝吗? 如果每年只捕获 1 只呢?

(3) 在较差的自然环境下,如果想使山猫数量稳定在 60 只左右,每年要人工繁殖多少只[73]?

8. 在某种环境下猫头鹰的主要食物来源是田鼠,设田鼠的年平均增长率为 r_1,猫头鹰的存在引起的田鼠增长率的减少与猫头鹰的数量成正比,比例系数为 a_1;猫头鹰的年平均减少率为 r_2;田鼠的存在引起的猫头鹰减少率的增加与田鼠的数量成正比,比例系数为 a_2.建立模型描述田鼠和猫头鹰共处时的数量变化规律,对以下情况作图给出 50 年的变化过程.

(1) 设 $r_1=0.2, r_2=0.3, a_1=0.001, a_2=0.002$,开始时有 100 只田鼠和 50 只猫头鹰.

(2) r_1, r_2, a_1, a_2同上,开始时有 100 只田鼠和 200 只猫头鹰.

(3) 适当改变参数 a_1, a_2(初始值同上).

(4) 求差分方程的平衡点,它们稳定吗[24]?

9. 研究将鹿群放入草场后草和鹿两种群的相互作用.草的生长遵从 logistic 规律,年固有增长率

*This article was typeset by Mark Taylor using the LaTeX document processing system.

2 Model Establishing

Let x_k, y_k be number of rats & owls in the kth year respectively. By observing the problem, we can obtain following difference equations.

$$x_{k+1} = (1 + r_1 - a_1 y_k)x_k,$$

$$y_{k+1} = (1 - r_2 + a_2 x_k)y_k.$$

3 Questions

❓ ❓ ❓

i Let $r_1 = 0.2, r_2 = 0.3, a_1 = 0.001, a_2 = 0.002$, with 100 rats and 50 owls in the beginning.

ii With 100 rats and 200 owls in the beginning, and coefs are the same as (i).

iii Change the arguments aptly (with the same initial values in (ii)).

iv Solve the equilibrium point. Does it stable?

4 Solutions

🔑 🔑 🔑

Let's first determine the equilibrium point (question 4), let it be $E = (\alpha, \beta)$. By the concept, when it goes stable, we have $x_k \to x_{k+1}, y_k \to y_{k+1} ask \to$, i.e. $\frac{x_k}{x_{k+1}} \to 1, \frac{y_k}{y_{k+1}} \to 1$. Thus we have,

$$1 = 1 + r_1 - a_1 y_k,$$

$$1 = 1 - r_2 + a_2 x_k,$$

which indicate $\alpha = \frac{r_1}{a_1} = 200$ and $\beta = \frac{r_2}{a_2} = 150$. Therefore, $E = (200, 150)$.
Let's do a little programming to draw these graphs. In this section, we will see those graphs rendered.

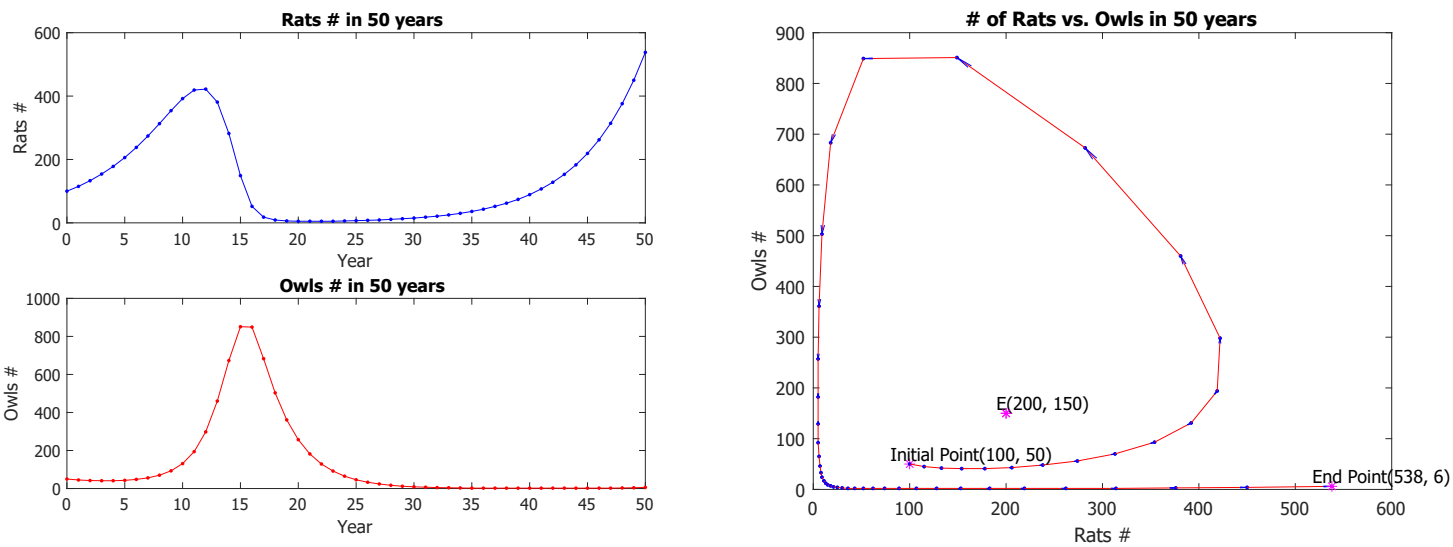


Figure 1: Question 1

See code of question 1, 2, & 4 here💻

And the following graphs are by question 2.

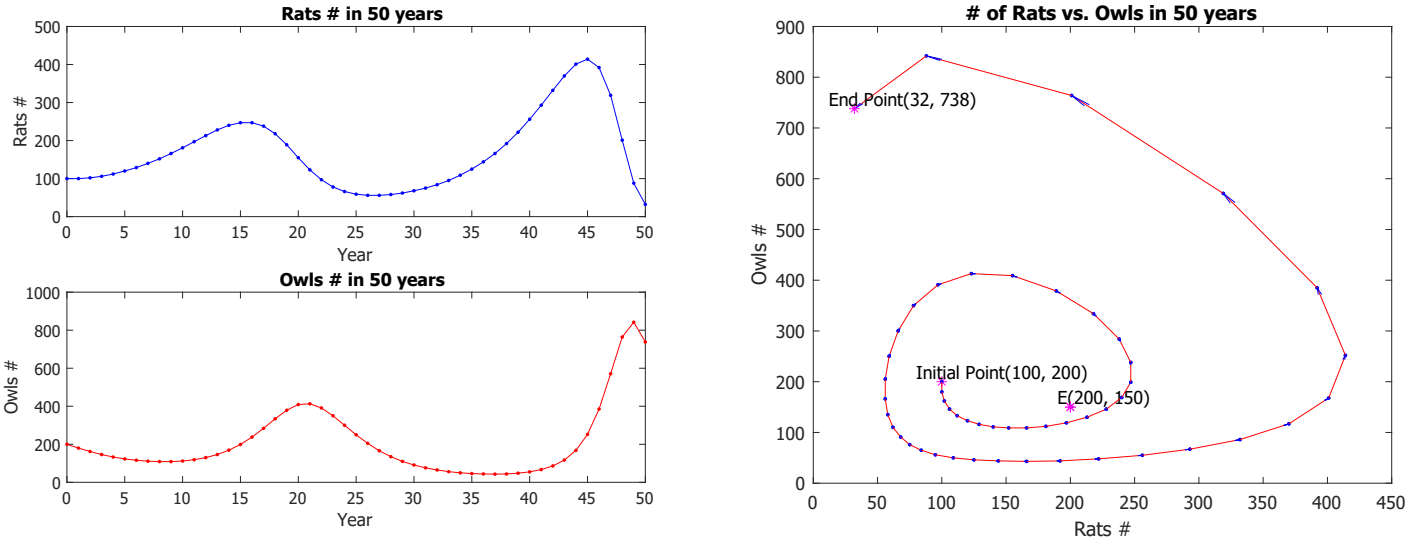


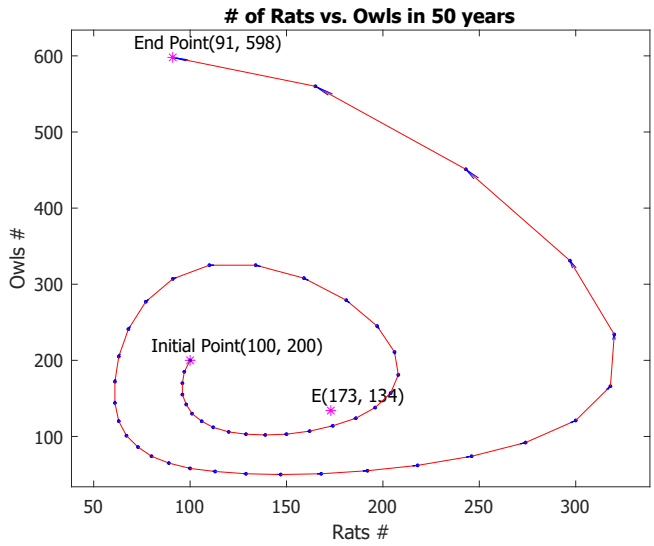
Figure 2: Question 2

In question 3, we plot a dynamic graph, you can run these *MATLAB* programs in the *src* folder and see how it changes. View code here.

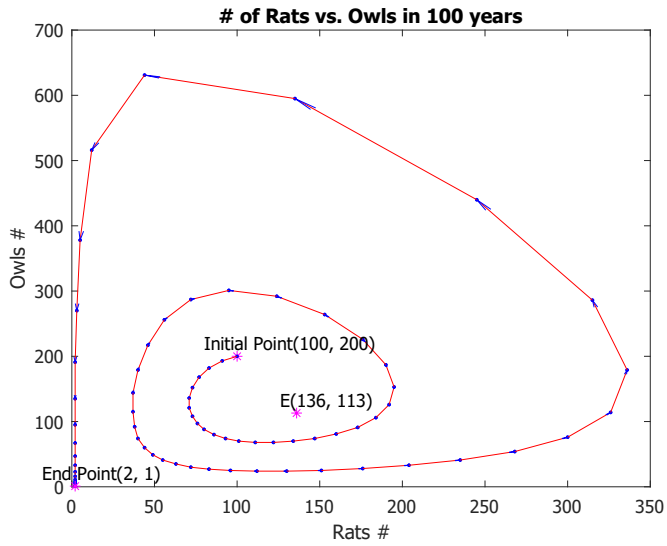
Note that we let a_1 & a_2 satisfy some (reasonable) relationship, say $a_2 = \frac{a_2+0.003}{a_1+0.001} * a_1$. (???) Then change a_1 by a vector.

Finally, we get into question 4, the last one. We shall enlarge the time span so that we can see whether the graph is converging to a specific point (the equilibrium point E) or diverging away form the equilibrium point.

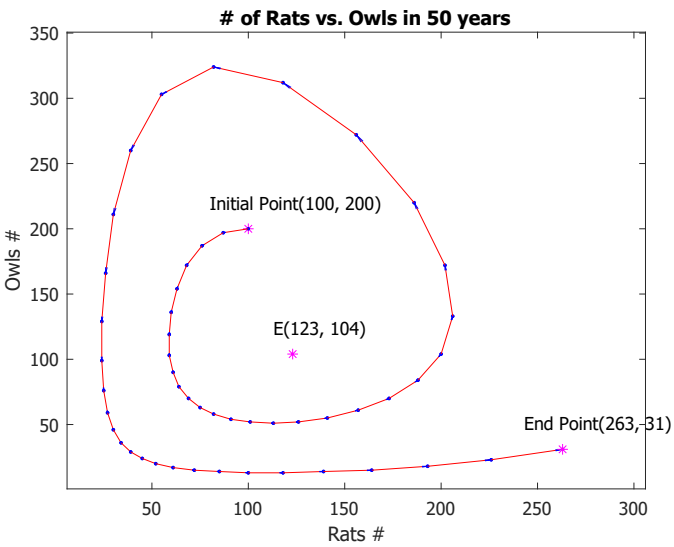
We can see that each of these figures suggests that it's **NOT stable**.



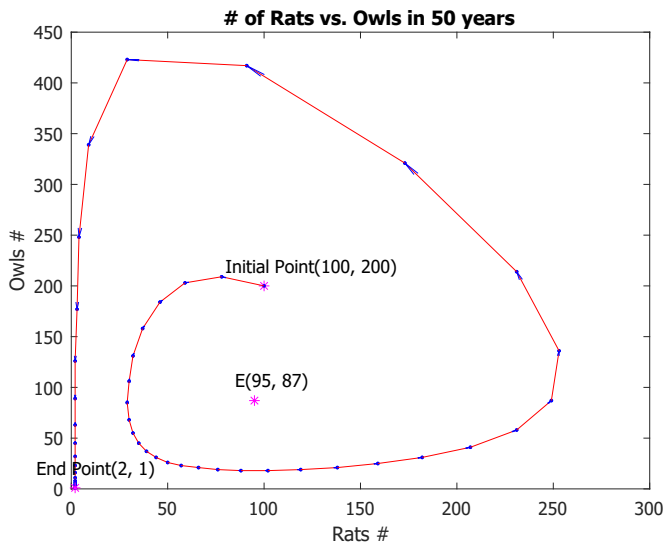
(a) $a_1 = 0.0012,\ a_2 = 0022$



(b) $a_1 = 0.0015,\ a_2 = 0027$

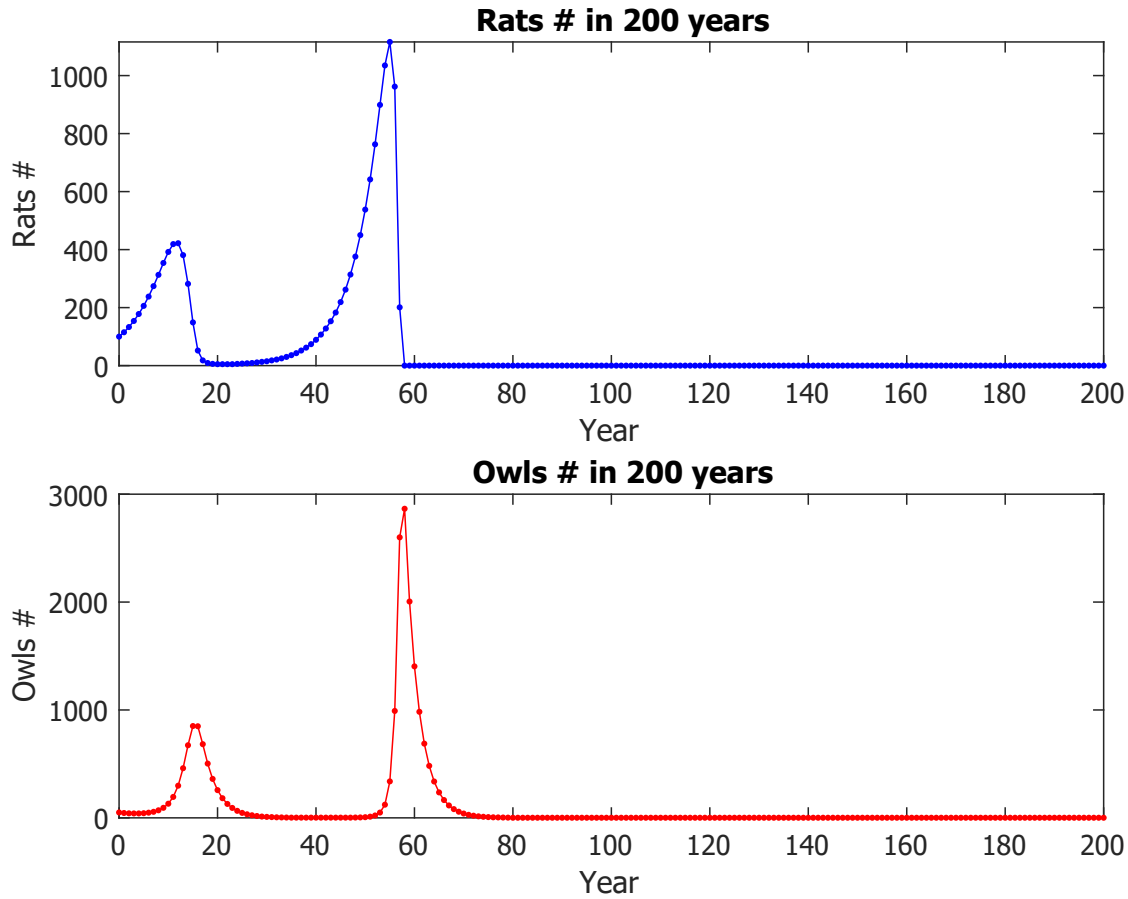


(c) $a_1 = 0.0016,\ a_2 = 0029$

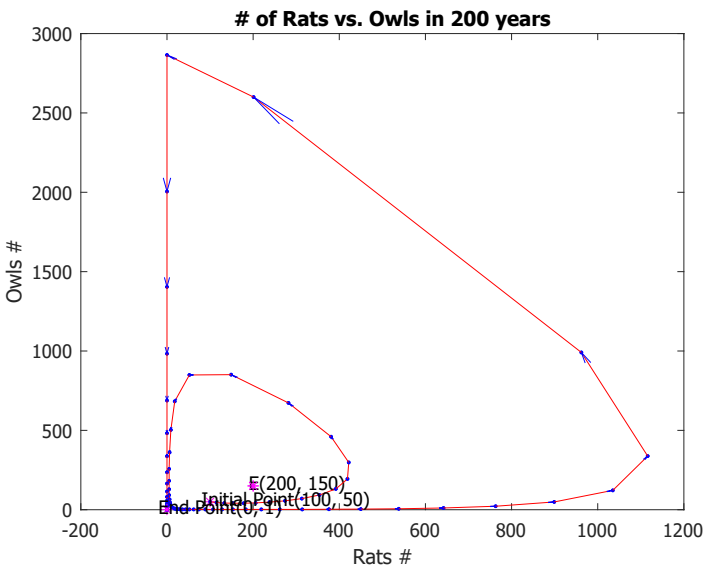


(d) $a_1 = 0.0021,\ a_2 = 0035$

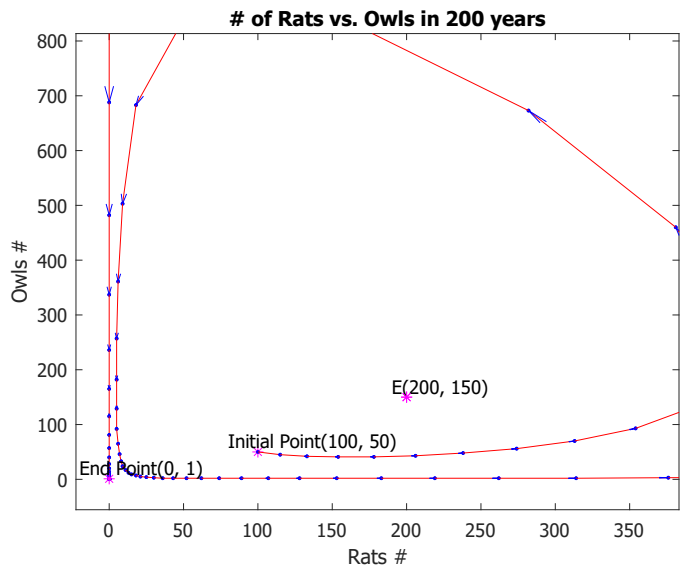
Figure 3: Question 3



(a) Plot of rats # & owls # in 200 years respectively



(b) # of rats vs. owls in 200 years



(c) Roomed in graph of (b)

Figure 4: Question 4

5 Code

5.1 main function

This is the main function that calls other auxiliary functions that serve as modules.

```
close all
% question 1
r1=0.2; r2=0.3;
a1=0.001; a2=0.002;
x0=100; y0=50;
disp('Question 1 --> see figure 1 & 2')
exer8_q124_helper(a1,a2,r1,r2,x0,y0)

% question 2
x0=100; y0=200;
disp('Question 2 --> see figure 3 & 4')
exer8_q124_helper(a1,a2,r1,r2,x0,y0)

% question 3
disp('Question 3 --> see the dynamic graph, figure 5')
exer8_q3_helper() % dynamic graph, script file (click to run)

% question 4
r1=0.2; r2=0.3;
a1=0.001; a2=0.002;
x0=100; y0=50;
years=200;
disp('Question 4 --> see figure 6 & 7')
exer8_q124_helper(a1,a2,r1,r2,x0,y0,years)
```

Listing 1: Main function for solving exercise 8.

5.2 exer8_q124_helper

This is an auxiliary function that solves question 1, 2, & 4. See also graphs of question 1, 2, 4.

```
% programs to solve question 1, 2, & 4 of exercise 8 in page 231
function exer8_q124_helper(a1,a2,r1,r2,x0,y0,years)
% set default args
if nargin<7
    years=50; % for question 1,2
    if nargin<6
        error('Too few arguments')
    end
end

time=0:years;
x=zeros(1,years+1); % # of rats in 50 years
y=zeros(1,years+1); % # of owls in 50 years
x(1)=x0; y(1)=y0; % initial conditions

figure
% let E=(alpha, beta) be the equilibrium point
% let x_{k+1}=x_k, y{k+1}=y_k, then solve them
alpha=round(r1/a1);
beta=round(r2/a2);
plot(alpha,beta,'m*',x(1),y(1),'m*')
msg = sprintf('E(%d, %d)',alpha,beta);
text(alpha-10, beta+20, msg);
for k=1:years
   x(k+1)=(1+r1-a1*y(k))*x(k);
    % in prictice terms, # Of rats & owls are non-negative
    if x(k+1)>0
        x(k+1)=round(x(k+1)); % # can only be integer, e.g. 4.3->4, 4.6->5
    else
```

```
        x(k+1)=0;
    end

    y(k+1)=(1-r2+a2*x(k))*y(k);
    if y(k+1)>0
        y(k+1)=round(y(k+1));
    else
        y(k+1)=0;
    end

    % draw vecters (with arrow) every two points

    hold on
    plot(x(k),y(k),'b.', x(k+1),y(k+1),'b.')
    vectarrow([x(k),y(k)],[x(k+1),y(k+1)])
end
% let E=(alpha, beta) be the equilibrium point
% let x_{k+1}=x_k, y{k+1}=y_k, then solve them
alpha=r1/a1;
beta=r2/a2;
hold on
plot(alpha,beta,'m*')
msg = sprintf('Initial Point(%d, %d)', x(1),y(1));
text(x(1)-20, y(1)+20, msg);

hold on
plot(x(years+1),y(years+1),'m*')
msg = sprintf('End Point(%d, %d)', x(years+1),y(years+1));
text(x(years+1)-20, y(years+1)+20, msg);
xlabel('Rats #'), ylabel('Owls #')
title_str=sprintf('# of Rats vs. Owls in %d years',years);
title(title_str)

figure
subplot(2,1,1)
plot(time,x,'-b.')
xlabel('Year'), ylabel('Rats #')
title_str=sprintf('Rats # in %d years',years);
title(title_str)

subplot(2,1,2)
plot(time,y,'-r.')
xlabel('Year'), ylabel('Owls #')
title_str=sprintf('Owls # in %d years',years);
title(title_str)
end
```

Listing 2: Helper function for solving question 1, 2, & 4.

5.3 exer8_q3_helper

This is an auxiliary function that solves question 3 by drawing a dynamic graph. See also graphs of question 3.

```
% script file (void function) to solve question 3
% of exercise 8 by drawing a dynamic graph
function exer8_q3_helper()
n=20; % draw n graphs
r1=0.2; r2=0.3;
x0=100; y0=200;
a1=linspace(0.001,0.004,n);
a2=(a1+0.003)./(a1+0.001).*a1; % give it some reasonable relationship as you like

years=50;
% years=100;
x=zeros(1,years+1); % # of rats in 50 years
```

```
y=zeros(1,years+1); % # of owls in 50 years
x(1)=x0; y(1)=y0; % initial conditions

figure
for i=1:n
    % let E=(alpha, beta) be the equilibrium point
    % let x_{k+1}=x_k, y{k+1}=y_k, then solve them
    alpha=round(r1/a1(i));
    beta=round(r2/a2(i));
    plot(alpha,beta,'m*',x(1),y(1),'m*')
    msg = sprintf('E(%d, %d)',alpha,beta);
    text(alpha-10, beta+20, msg);
    for k=1:years
        x(k+1)=(1+r1-a1(i)*y(k))*x(k);
        % in prictice terms, # Of rats & owls are non-negative
        if x(k+1)>0
            x(k+1)=round(x(k+1)); % # can only be integer, e.g. 4.3->4, 4.6->5
        else
            x(k+1)=0;
        end

        y(k+1)=(1-r2+a2(i)*x(k))*y(k);
        if y(k+1)>0
            y(k+1)=round(y(k+1));
        else
            y(k+1)=0;
        end

        % draw vecters (with arrow) every two points
        hold on
        plot(x(k),y(k),'b.', x(k+1),y(k+1),'b.')
        vectarrow([x(k),y(k)],[x(k+1),y(k+1)])
    end
    msg = sprintf('Initial Point(%d, %d)', x(1),y(1));
    text(x(1)-20, y(1)+20, msg);
    hold on
    plot(x(years+1),y(years+1),'m*')
    msg = sprintf('End Point(%d, %d)', x(years+1),y(years+1));
    text(x(years+1)-20, y(years+1)+20, msg);
    xlabel('Rats #'), ylabel('Owls #')
    title_str=sprintf('# of Rats vs. Owls in %d years',years);
    title(title_str)

    hold off
    pause(1)
end
```

Listing 3: Helper function for solving question 3.

6 Appendix

Hello from the Beatles.😃