



how U doin'?

$$\text{🐼} = \int \text{👶} d\text{🍀}$$

Modeling hw2

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Problems

备用件数	0	0.5	0.6	0.7
1		0.6	0.75	0.9
2		0.7	0.95	1.0
3		0.8	1.0	1.0
4		0.9	1.0	1.0
5		1.0	1.0	1.0
单件价格/元		20	30	40
单件质量/kg		2	4	6

★ 某公司利用钢材和铝材作为原材料,生产两种产品(A和B).单件产品A需消耗钢材6 kg,铝材8 kg,劳动力11 h,利润5 000元(不含工人加班费);单件产品B需消耗钢材12 kg,铝材20 kg,劳动力24 h,利润11 000元(不含工人加班费).该企业目前可提供钢材200 kg,铝材300 kg,劳动力300 h.如果要求工人加班,每小时加班费100元.请制订生产计划,最大化公司的利润和最小化工人加班时间.

12. 某银行营业部设立3个服务窗口,分别为个人业务、公司业务和特殊业务(如外汇和理财等).现有3名服务人员,每人处理不同业务的效率(每天服务的最大顾客数),以及每人处理不同业务的质量(如顾客的满意度)见下表.如何为服务人员安排相应的工作(服务窗口)?

	个人业务	公司业务	特殊业务
最大顾客数			

Model Establishing

Let x = the No. of product A, and y = the No. of product B.

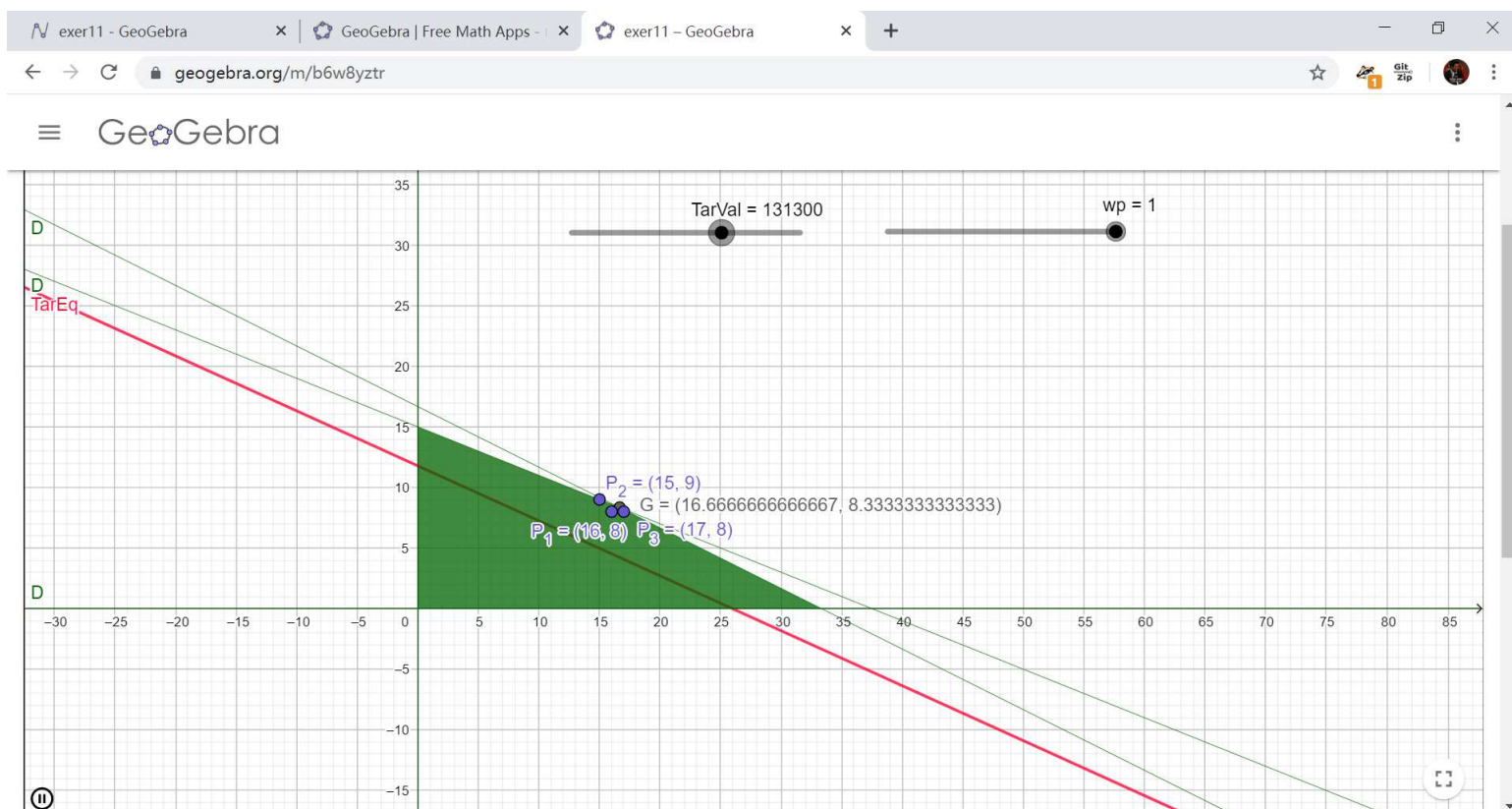
It's not difficult to obtain following model by observing the problem.

```

Max(profit) & Min(overtime);
profit = 5000*x + 11000*y - 100*overtime,
6*x + 12*y <= 200,
8*x + 20*y <= 300,
11*x + 24*y - 300 = overtime.
(Of course, x, y, & overtime >= 0.
Moreover, x & y are integers, which results
in overtime also being an integer.)
  
```

Drawing the Graph

See & play around it online at <https://www.geogebra.org/m/b6w8yztr>.



Solve by Lingo

- See Lingo code as follows

```

model:
! A greedy boss wants maximum profit & minimum overtime for employees;
max = z;

!@param x: the No. of product A
!@param y: the No. of product B;

wp = 0.9;      ! weight of profit. This is decided by the boss;
wt = 1 - wp;  ! weight of overtime;
z = profit * wp + (-overtime) * wt; ! Target optimal (maximum) value;

profit = 5000*x + 11000*y - 100*overtime;
[Steel] 6*x + 12*y <= 200;
[Aluminum] 8*x + 20*y <= 300;
[TotalHours] 11*x + 24*y <= 300 + overtime;

! declare variable x & y as integer only;
@gin(x); @gin(y);

end

```

- Output

Lingo 18.0 - [Solution Report - exer11]

File Edit Solver Window Help

Global optimal solution found.

Objective value:	149301.9
Objective bound:	149301.9
Infeasibilities:	0.000000
Extended solver steps:	0
Total solver iterations:	3
Elapsed runtime seconds:	0.06

Model Class: MILP

Total variables:	5
Nonlinear variables:	0
Integer variables:	2
Total constraints:	6
Nonlinear constraints:	0
Total nonzeros:	15
Nonlinear nonzeros:	0

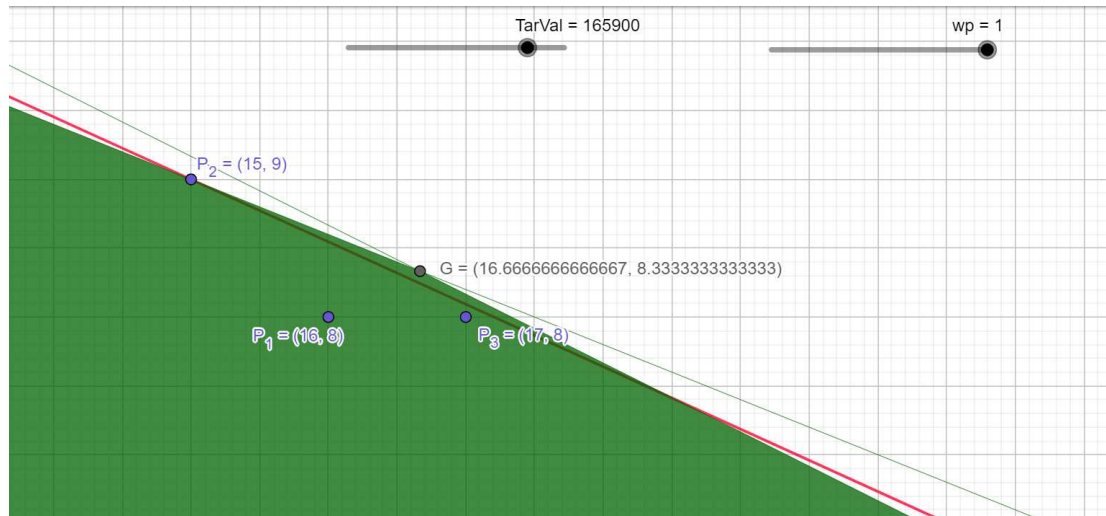
Variable	Value	Reduced Cost
Z	149301.9	0.000000
WP	0.900000	0.000000
WT	0.100000	0.000000
PROFIT	165900.0	0.000000
OVERTIME	81.00000	0.000000
X	15.00000	-3508.900
Y	9.000000	-7737.600

Row	Slack or Surplus	Dual Price
1	149301.9	1.000000
2	0.000000	165981.0
3	0.000000	-81.00000
4	0.000000	1.000000
5	0.000000	0.900000
STEEL	2.000000	0.000000
ALUMINUM	0.000000	0.000000
TOTALHOURS	0.000000	90.10000

For Help, press F1

Ln 15, Col 42 3:24 am

Result & Analysis



Zooming in we shall find when $wp = 1$, Target Equation (red line) will cross point $P_2(15,9)$ as the last integer point in domain D (green area). Moreover, if we try to change wp (simple to do, just to drag wp in the slider), and the result remains unchanged. Let's delve a little bit into the details about why this happens.

By the model,

$$\begin{aligned} \text{profit} &= 5000x + 11000y - 100 * \text{overtime} \\ &= 5000x + 11000y - 100(11x + 24y - 300) \\ &= 3900x + 8600y + 30000, \end{aligned}$$

$$\begin{aligned} z &= \text{profit} * wp + (-\text{overtime}) * (1 - wp) \\ &= \underbrace{(3900x + 8600y + 30000) * wp}_{\text{weight of profit}} - \underbrace{(11x + 24y - 300) * (1 - wp)}_{\text{weight of overtime}}. \quad (\text{TarEq}) \end{aligned}$$

Obviously, even when $wp = 0.1$, $1 - wp = 0.9$, the slope of equation (TarEq) changes little, since the profit weight part has a magnitude of $10^2 \sim 10^3$ and the overtime weight part has a magnitude of at most 10^1 , concerning the coefficients of x & y .

Furthermore, if we try to change the wp in the Lingo program, we can get the same result as that of $wp = 1$.

So, the final solution is to **produce 15 pieces of product A, and 9 pieces of product B, with 81 hours of overtime, generating maximum profit ¥165900, and leaving 2kg of rolled steel, regardless of the wight of profit given by the boss.**

(see also the [Lingo solution report](#) as follows)

Variable	Value	Reduced Cost
Z	149301.9	0.000000
WP	0.9000000	0.000000
WT	0.1000000	0.000000
PROFIT	165900.0	0.000000
OVERTIME	81.00000	0.000000
X	15.00000	-3508.900
Y	9.000000	-7737.600

Row	Slack or Surplus	Dual Price
1	149301.9	1.000000
2	0.000000	165981.0
3	0.000000	-81.00000
4	0.000000	1.000000
5	0.000000	0.9000000
STEEL	2.000000	0.000000
ALUMINUM	0.000000	0.000000
TOTALHOURS	0.000000	90.10000

Lingo solution report